AMPLITUDES OF SEISMIC WAVES REFLECTED AND REFRACTED AT THE EARTH'S CORE

Thesis by

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SUMMARY

The Zoeppritz equations are used to determine the energy partition in seismic waves reflected and refracted at the earth's core. First, the assumption is made that the core is fluid and hence does not transmit any shear waves. Then the four possible cases are treated; an incident longitudinal wave in the mantle against the core, an incident longitudinal wave in the core against the mantle, and an incident shear wave both of the SV type (particles vibrating in the plane of incidence) and of the SH type (particles vibrating perpendicular to the plane of incidence) in the mantle against the core. The square roots of relative energy partitioned among the waves of the above cases are plotted against angle of incidence.

The horizontal and vertical components of the displacement produced at the surface of the earth by seismic waves reflected and refracted at the earth's core are calculated by means of the formula $A = CTf\sqrt{\frac{\tan i_o}{\sin \Delta}\,\frac{di_o}{d\Delta}}$, which gives the amplitude of the incident wave. C is a constant whose value depends on the energy at the source, and is different for longitudinal and shear waves, T is the period, io the angle of incidence at a distance Δ from the source, and f is the square root of the relative energy calculated earlier. The values of A for a certain type of wave are multiplied by the ratios $\frac{u}{A}$ and $\frac{w}{A}$ in order to obtain the horizontal and vertical components of surface displacement. u is the horizontal component and w the vertical. u and w are plotted against Δ.

Lastly the variation of u and w with Δ are determined for P, PP, S, and SS. Then, depending on whether the seismic wave reflected or refracted at the earth's core started out from the source as a longitudinal or a shear wave, the ratios of u and w of that wave to the u and w of the corresponding direct wave are calculated. This does away with the necessity of determining the value of C in the amplitude equation. However, for application of these results to seismograms the ratio of the periods must be introduced, as in the calculations the period of all waves was assumed to be 1.00 second, and the magnification of the ground by the instrument must be considered.

CHAPTER I

INTRODUCTION

The reflection and refraction of seismic waves has been treated extensively in the past by various authors. Among them are Knott (12,13), Wiechert (16), Zoeppritz (17), Bollasse (3), Jeffreys (11), Gutenberg (5), Berlage (1), Blut (2), and Macelwane (14,15). It is not the intent of the following thesis to repeat the calculations contained in their work but to use some of their results and then to study the reflection and refraction of seismic waves at the earth's core exclusively.

The results of this research consist of the calculated vertical and horizontal displacements produced at the surface of the earth by seismic waves reflected from and transmitted by the earth's core. Certain simplifying assumptions were necessary. No attempt has been made as yet to compare these calculated displacements with actual measurements made from seismograms of recorded earthquakes.

The first purpose of these calculations was to obtain results which would aid in interpreting points on seismograms where confusion is caused by almost simultaneous arrival of various waves. The second purpose was to facilitate the discrimination between branches of phases which follow each other at short intervals but with different amplitudes. In addition, comparison of the calculated displacements with measured amplitudes in seismograms may provide information concerning the elastic properties at the boundary between core and mantle.

Finally, one of the important assumptions made in this work was that the core of the earth did not transmit shear waves. This is in line with the measurements from seismograms, which show that shear waves if transmitted at all are very feeble. Thus another purpose of these calculations was to aid the investigation of the problem of whether the earth's core, though not necessarily a fluid, resembles a fluid in not transmitting shear waves.

CHAPTER II

THEORY

In the reflection and refraction of plane waves at a discontinuity between two media of different elastic properties, continuity requires that the normal and tangential stresses and strains should be respectively equal on both sides of that discontinuity. For the general case this gives six boundary conditions. The equations describing these boundary conditions for an incident longitudinal and an incident shear wave are known as the Zoeppritz equations. They are given below without derivation, since if desired the details of this derivation may be studied in the original paper by Zoeppritz (17), in the Handbuch der Geophysik of Gutenberg (5), or in the treatment by Macelwane (14) in Theoretical Seismology, Part I - Geodynamics. It should therefore be emphasized that all assumptions and conditions used in the derivation of the Zoeppritz equations are also valid in the following work.

The notation used is taken from the article by Macelwane (15) in Physics of the Earth - VI except for changing the symbol for density from the Greek "rho" to the English "p". P refers to a longitudinal wave and S to a shear wave. There are two types of shear waves considered in this research. One is the SV type (the direction of vibration is in the plane of incidence) and the other is the SH type (the direction of vibration is perpendicular to the plane of incidence). The rest of the notation is given in the following table:

Wave	Velocity	Amplitude	Angle with Normal to Discontinuity
Incident P	V_{p1}	A	a
Incident S	V_{s1}	B	b
Reflected P	V_{p1}	C	c
Reflected S	V_{s1}	D	d
Refracted P	V_{p2}	E	e
Refracted S	V_{s2}	F	f

The four derived waves from an incident longitudinal wave in medium 1 are shown in Fig. 1. The Zoeppritz equations for this case are:

Normal strains, $A \cos a - C \cos a + D \sin b - E \cos e - F \sin f = 0.$
Tangential strains, $A \sin a + C \sin a + D \cos b - E \sin e + F \cos f = 0.$
Normal stresses, $-A\cos 2b - C\cos 2b + \left(D \frac{V_{s1}}{V_{p1}}\right) \sin 2b + E\frac{P_2}{P_1}\left(\frac{V_{p2}}{V_{p1}}\right) \cos 2f$

$$+ F \frac{P_2}{P_1}\left(\frac{V_{s2}}{V_{p1}}\right) \sin 2f = 0.$$

Tangential stresses, $-A \sin 2a + C \sin 2a + D\left(\frac{V_{p1}}{V_{s1}}\right) \cos 2b$

$$+ E \frac{P_2}{P_1}\left(\frac{V_{s2}}{V_{s1}}\right)^2 \frac{V_{p1}}{V_{p2}} \sin 2e - F \frac{p_2}{P_1}\left(\frac{V_{s2}}{V_{s1}}\right)^2 \frac{V_{p1}}{V_{s2}} \cos 2f = 0.$$

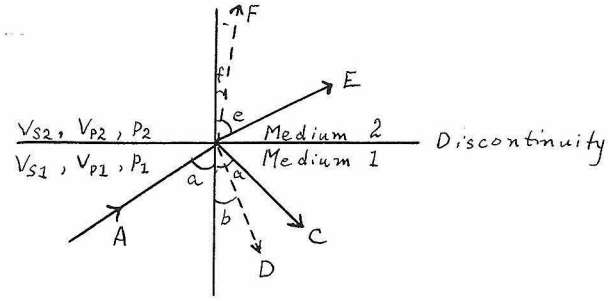


Fig. 1. The Four Derived Waves from Incident Longitudinal Wave.

The next case, that of an incident shear wave (SV) in medium 1, is illustrated in Fig.2. The Zoeppritz equations for an incident shear wave of the SV type are as follows:

Normal strains, $-B \sin b - C \cos a + D \sin b - E \cos e - F \sin f = 0.$
Tangential strains, $B \cos b + C \sin a + D \cos b - E \sin e + F \cos f = 0.$
Normal stresses, $B \sin 2b - C \frac{V_{p1}}{V_{s1}} \cos 2b + D \sin 2b + E \frac{P_2 V_{p2}}{P_1 V_{s1}} \cos 2f$

$$+ F \frac{P_2 V_{s2}}{P_1 V_{s1}} \sin 2f = 0.$$

Tangential stresses, $-B \cos 2b + C\frac{V_{s1}}{V_{p1}} \sin 2a + D \cos 2b + E\frac{P_2 V_{s2}^2}{P_1 V_{s1} V_{p2}} \sin 2e$

$$- F \frac{P_2 V_{s2}}{P_1 V_{s1}} \cos 2f = 0.$$

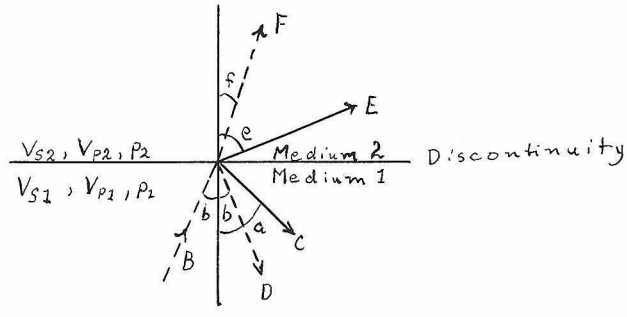


Fig. 2. The Four Derived Waves from an Incident Shear Wave (SV type).

An incident shear wave of the SH type in medium 1 can generate only a reflected SH wave and a refracted SH wave. This does away with the normal strains equation and the normal stresses equation, and the Zoeppritz equations become:

Tangential strains, $B + D - F = 0$.

Tangential stresses, $B - D - \frac{P_2 V_{s2} (\cos f)}{P_1 V_{s1} (\cos b)} F = 0$.

The solution of the Zoeppritz equations for the reflection and refraction of seismic waves at the earth's core is simplified by assuming that the energy of shear waves in the core is negligible. This is in agreement with the best evidence from seismograms. The value of the amplitude of the incident P or S wave, either A or B, is taken as 1.00. For a particular angle of incidence a or b the angles of reflection and refraction may be found by Snell's Law:

$$\frac{\sin a}{V_{p1}} = \frac{\sin b}{V_{s1}} = \frac{\sin c}{V_{p1}} = \frac{\sin d}{V_{s1}} = \frac{\sin e}{V_{p2}} = \frac{\sin f}{V_{s2}} .$$

The velocity of P waves just outside the core is 13.7 km. per sec., and inside it is 8.0 km. per sec. The velocity of S waves just outside the core is 7.25 km. per sec. The values are from p. 278 of Gutenberg (10) and p. 134 of Gutenberg and Richter (9). From p. 341 of Gutenberg (10) the density p just outside the core is 5.4 gm. per cc., and just inside it is 10.1 gm. per cc.

The various angles of reflection and refraction are worked out for an incident P or S for a and b varying from 0° to 90°. The four cases considered are for an incident longitudinal wave in the mantle against the core, an incident longitudinal wave in the core against the mantle, an incident shear wave of the SV type in the mantle against the core, and an incident shear wave of the SH type in the mantle against the core. Substitution of the values of these angles, the values of the velocities and density, and the above assumptions reduces the Zoeppritz equations for each of the four cases considered to four sets of three simultaneous linear equations of the following form:

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3.$$

The constants a, b, c, and d contain the trigonometric functions of the angles, and V_{s1}, V_{p1}, etc.; whereas the unknowns x, y, and z correspond to the amplitudes of the reflected and refracted waves. As will be shown in Chapter III on Calculations and Results these sets of simultaneous equations for each of the four cases may be solved for the unknown amplitudes for values of the incident angle a or b between 0° and 90° by means of the ordinary determinant method for the solution of third order determinants. Five place logarithms are used in these and following calculations. It should be understood that x, y, and z are the ratios of the amplitudes of the reflected and refracted waves to the amplitude of the incident P or S wave.

The next step is the investigation of the partition of energy at the earth's core for each of the four cases studied. Blut (2) carried Zoeppritz' studies on the amplitudes further and developed equations for the distribution of the incident energy among the derived waves for the general case of an incident P or S wave. His equations may also be studied in Macelwane (14). They are therefore given here without derivation:

For an incident P wave in medium 1 against medium 2,

$$1 = \frac{C^2}{A^2} + \frac{D^2 \sin 2b}{A^2 \sin 2a} + \frac{E^2 P_2 \sin 2e}{A^2 P_1 \sin 2a} + \frac{F^2 P_2 \sin 2f}{A^2 P_1 \sin 2a}.$$

A, C, D, E, F are the true amplitudes. By using the values of the amplitude ratios calculated as described, the energy distribution for an incident P wave becomes:

$\frac{C^2}{A^2}$ = relative energy going into the reflected P wave,

$\frac{D^2 \sin 2b}{A^2 \sin 2a}$ = relative energy going into the reflected S wave,

$\frac{E^2 P_2 \sin 2e}{A^2 P_1 \sin 2a}$ = relative energy going into the refracted P wave,

$\frac{F^2 P_2 \sin 2f}{A^2 P_1 \sin 2a}$ = relative energy going into the refracted S wave.

For an incident S wave in medium 1 against medium 2 Blut's equation is

$$1 = \frac{C^2 \sin 2a}{B^2 \sin 2b} + \frac{D^2}{B^2} + \frac{E^2 P_2 \sin 2e}{B^2 P_1 \sin 2b} + \frac{F^2 P_2 \sin 2f}{B^2 P_1 \sin 2b}.$$

Again B, C, D, E, F are the true amplitudes. Thus by using the amplitude ratios determined earlier, the energy distribution for an incident S wave becomes:

$\frac{C^2 \sin 2a}{B^2 \sin 2b}$ = relative energy going into the reflected P wave,

$\frac{D^2}{B^2}$ = relative energy going into the reflected S wave,

$\frac{E^2 P_2 \sin 2e}{B^2 P_1 \sin 2b}$ = relative energy going into the refracted P wave,

$\frac{F^2 P_2 \sin 2f}{B^2 P_1 \sin 2b}$ = relative energy going into the refracted S wave.

Thus Blut's equations not only give the means of calculating the relative energy distribution in each of the four cases considered at the earth's core, as the angle of incidence of the incident wave varies from 0° to 90°, but also provide a check for

all of the calculations up to this point, since the sums of the relative energies for a certain angle of incidence must be very close to 1.00.

The values of the relative energy going into the various reflected and refracted waves in each of the four cases are plotted against angle of incidence as it goes from 0° to 90°. Also the square roots of these relative energies are plotted against angle of incidence, since they had to be used in further calculations.

The second part of the research for this thesis consisted of the determination of the horizontal and vertical components of the displacement produced at the surface of the earth by certain waves reflected and refracted at the earth's core. The first assumption made was that the source of these body waves was a shock at the surface of the earth. Hence for the following work a zero focal depth is assumed. The next assumption is that any discontinuities between the core and the surface of the earth have a negligible effect upon the passage of the body waves considered. Also in the case of waves transmitted by the core, it was assumed that no appreciable discontinuities exist within the core.

The amplitude of an incoming seismic wave at the surface of the earth is given by

$$A = C\ T\ f \sqrt{e^{-\int k dD \frac{\tan i_o}{\sin \Delta} \frac{d i_o}{d \Delta}}} \quad .$$

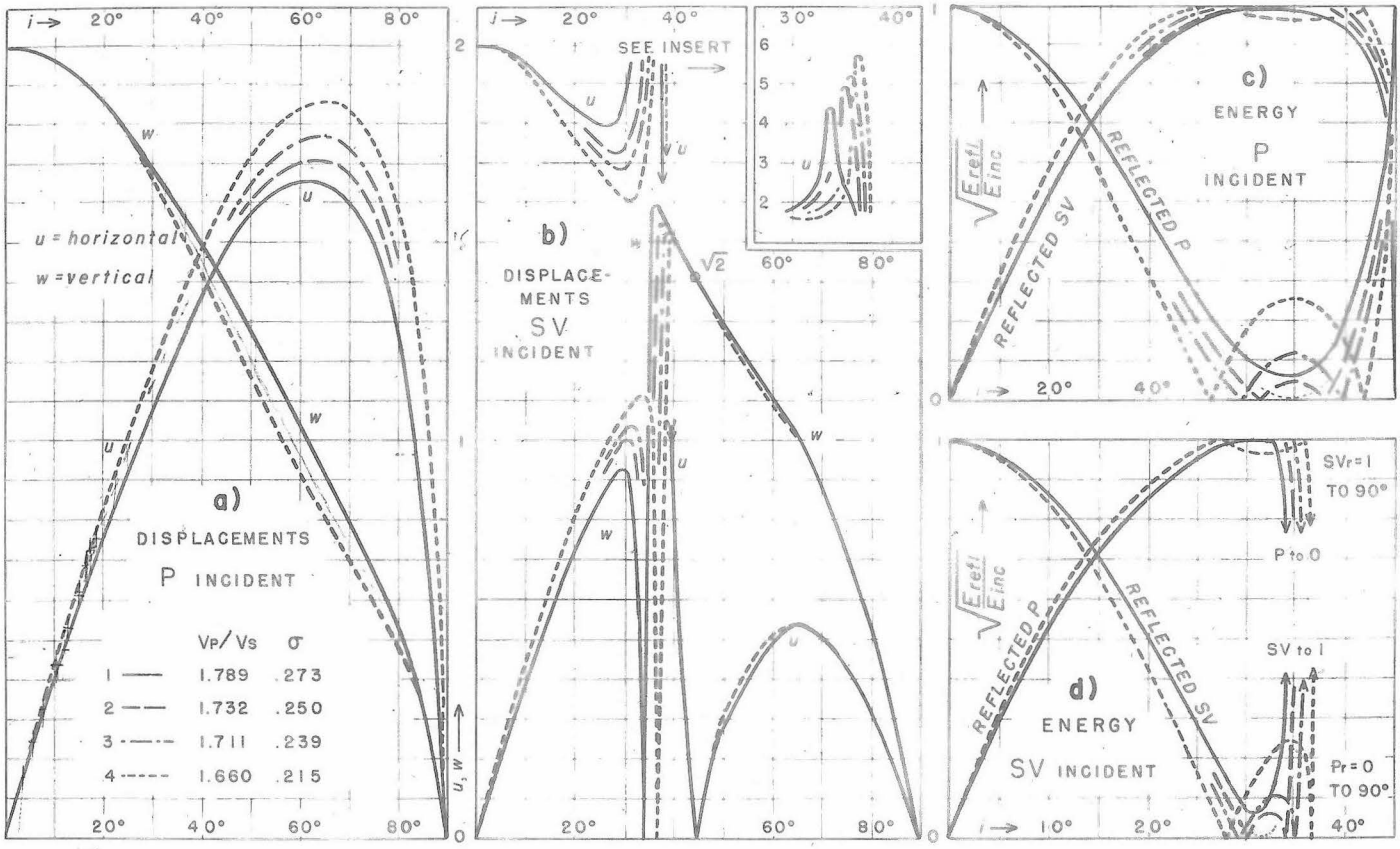
Its derivation may be found on p. 59 of Gutenberg (5). All assumptions involved in its derivation should therefore be understood to apply here. The exponential term represents the absorption which has taken place in the incident wave. Since the value of the absorption constant k is not known accurately and since the absorption is relatively small anyway, this term is taken as 1.00 in the following work. dD is the element of path.

As used in these calculations then, the amplitude equation may be restated as

$$A = C\ T\ f \sqrt{\frac{\tan i_o}{\sin \Delta} \frac{d i_o}{d \Delta}} ,$$

where A is the amplitude of the incident wave, i_o is its angle of incidence with a normal to the earth's surface, and Δ is the angular distance from the source of the wave to its point of emergence. C is a constant whose value depends upon the energy at the focus of the shock. In addition, it very likely differs for P and S waves and in the following calculations was assumed equal to π. T is the period and for all waves considered was taken as 1.00 second. f is the product of a number of the square root of relative energy values, calculated in the first part of this research, each of which represents the fraction of energy remaining after a reflection or refraction at the core or at the surface of the earth.

The required component of the displacement at the earth's surface is computed by multiplying A by a ratio that depends, for longitudinal waves, upon the angle of incidence, Poisson's ratio in the surface layer, and the direction of the required component relative to the plane of propagation; and for shear waves this ratio also depends upon the polarization. If u denotes the horizontal component of displacement in the plane of propagation, v the horizontal component perpendicular to the plane of propagation, and w the vertical component, then $\frac{u}{A}$ and $\frac{w}{A}$ for shear waves of the SH type are zero whereas $\frac{v}{A}$ is always 2.00. The values of $\frac{u}{A}$ and $\frac{w}{A}$ for an incident P wave and an incident SV wave have been calculated for $i_o = 0^o$ to 90° by Dr. Gutenberg in a paper entitled "The Energy Ratio of Reflected and Refracted Seismic Waves" now in press for the Seismological Society of America Bulletin. A copy of Fig. 3 of that article is included here. It is from this that the values of $\frac{u}{A}$ and $\frac{w}{A}$ used in this research were taken for $\sigma = .239$. Also it will be noticed that this figure contains a plot of the square root of the partition of relative energy for waves reflected from the surface of the earth both for an incident P and an SV wave. These values were used in calculating the f of body waves which had been reflected at least once from the earth's surface in the course of their path.



GUTENBERG REFLECTIONS & REFRACTIONS 1943 FIG. 3

It is now evident that the necessary factors in the computation of vertical and horizontal components of the displacement produced at the earth's surface by a certain body wave are accounted for except i_o and Δ . For nearly all of the calculations the value of i_o at a certain Δ was determined from travel time curves plotted from data contained in "On Seismic Waves I" and in "On Seismic Waves IV" by Gutenberg and Richter (6 and 9). The relation used is $\sin i_o = \frac{V_o}{\bar{V}_\Delta}$, where V_o is the true velocity of a P or S wave at the surface and $\bar{V}_\Delta$ is the apparent velocity of that wave at a particular Δ as measured from the tangent to the travel time curve at that Δ . This equation is derived from Fig. 3:

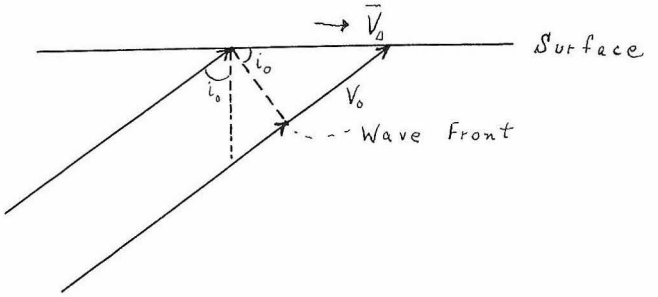


Fig. 3. Wave Front and Angle of Incidence for a P or S Wave.

It is clear that $\sin i_o = \frac{V_o\, dt}{V_\Delta\, dt} = \frac{V_o}{V_\Delta}$.

The value of V_o was taken from page 134 of Gutenberg and Richter (9) to be 5.5 km. per sec. for P and 3.2 km. per sec. for S. After i_o has been calculated for the Δ range of a particular wave, an i_o vs. Δ curve is plotted. The slope to this curve gives the value of the differential $\frac{di_o}{d\Delta}$, where both di_o and $d\Delta$ are in degrees. It should be noted therefore that the radical part of the amplitude of an incident wave, $f\sqrt{\frac{\tan i_o}{\sin \Delta}\frac{di_o}{d\Delta}}$, is a pure number.

The value of the angle of incidence of a certain wave at the core of the earth, i_c, is determined from the well known equation

$$\frac{r_s \sin i_o}{V_s} = \frac{r_c \sin i_c}{V_c},$$

where V_s is the velocity of the wave at the surface, (5.5 or 3.2 km. per sec.), V_c is the velocity at the core (13.7 km. or 7.25km. per sec.), r_c is the radius of the core of the earth and r_s is the radius of the surface of the earth. The derivation of this equation may be found in Gutenberg (5) or in Macelwane (13). r_s is taken as 6,366 km. and r_c as 3,446 km. from p. 135 of Gutenberg and Richter (9).

This i_c is the angle of incidence which determines the value of f for a given wave from the plots of square root of relative energy against angle of incidence in the first part of this research.

The vertical and horizontal components, u and w, of the surface displacement produced by waves reflected and refracted at the earth's core were calculated and plotted against Δ in Chapter III for the following groups of body waves; Group I, PcP, PcS, ScS, ScP, Group II, P'(PKP), PKS, SKS, SKP, Group III, PKKP, PKKS, SKKS, SKKP, and Group IV, P'P' AND P'P'P' (i.e., PKPPKP and PKPPKPPKP). Since the path of each of these waves is illustrated with a sketch in Chapter III, no attempt will be made to define them here.

In connection with these calculations for body waves, u and w vs. Δ for the direct P and SV waves were determined. This permits avoiding the determination of the constant C in the amplitude equation. Ratios of u and w for the waves of Group I to the u and w of P or SV were plotted against Δ. The components of P were used if the wave started from its source as longitudinal, and the u and w of SV were used if the wave started from its source as a shear wave. This is due to the fact, mentioned earlier, that the value of C depends on the energy at the source and is different for a P and S wave. Thus the u and w ratios against Δ were plotted for $\frac{PcP}{P}$, $\frac{PcS}{P}$, $\frac{ScS}{SV}$, and $\frac{ScP}{SV}$.

The range of Group II and Group III waves was beyond that of the direct P or SV. Consequently the u and w values for PP and SS (SV type) were worked out. These are the direct P and SV waves which have been reflected once from the earth's surface at the midpoint between their source and emergence. Then the ratios of the u and w of Group II and Group III to the u and w of PP or SS were calculated. Again the precaution was observed to use PP in the denominator if the wave was longitudinal near its source and SS if the wave was transverse near its source.

This removes the necessity of calculating the values of C for Groups II and III since, for example, the wave PKKP would obviously have the same C as the wave PP. Hence in the ratio $\frac{u_{PKKP}}{u_{PP}}$ or $\frac{w_{PKKP}}{w_{PP}}$ it is cancelled out. In this particular section of the calculations the supposition was made that C in SH and in SV is the same.

The ratios of u and w of P'P' and P'P'P' to the u and w of both P and PP were also calculated. The values of some miscellaneous ratios such as that of the u and w of SS to SV, PP to P, and SV to SH are included in their appropriate place in Chapter III. These are of interest in the study of the boundary conditions at the earth's core. It should be noted in the examination of the u and w ratios using P, SV, PP, and SS that the values of u and w for those four waves were determined only at Δ intervals of 10 degrees on the average. Thus the plots of u and w for those four against Δ are smoothed curves and take no account of minor irregularities in the travel time curves of P, SV, PP, and SS. This may be noted particularly by comparing the plot of $\frac{u_{PP}}{u_P}$ and $\frac{w_{PP}}{w_P}$ on p. 133 with the plots of in Figs. 6 and 7 of Gutenberg and Richter (7). The graphs of the $\frac{PP}{P}$ ratios of this research are clearly smoothed curves of Figs. 6 and 7 of "On Seismic Waves II" (7) and do not indicate the irregularities shown in that work.

It should be noted that for application of these results to seismograms the ratio of the periods must be introduced, since in the calculations the period of all waves was assumed to be 1.00 second, and the magnification of the ground movement by the instrument must be considered.

CHAPTER III

CALCULATIONS AND RESULTS

No attempt will be made to describe or discuss the results of this research since it is believed that an understanding and knowledge of the material in this chapter may be obtained only from an intensive examination of the various graphs and calculations. A complete summary of the maxima and minima and other characteristics of the u and w curves for the various waves would be redundant and too extensive.

The calculations and plots of u and w against Δ are arranged in the following order:

Part I Calculation of the Partition of the Relative Energy at the Earth's Core.

- Case I. Incident Longitudinal Wave in the Mantle Against the Core.
- Case II. Incident Longitudinal Wave in the Core Against the Mantle.
- Case III. Incident Shear Wave (SV type) in the Mantle Against the Core.
- Case IV. Incident Shear Wave (SH type) in the Mantle Against the Core.

Part II Calculation of the Vertical and Horizontal Components of the Ground Displacement at the Earth's Surface Produced by Seismic Waves Reflected and Refracted at the Earth's Core.

Group I. Waves Reflected Once from the Surface of the Core.

1. PcP
2. PcS
3. ScS a. SV type b. SH type
4. ScP

Group II. Waves Transmitted once through the Core.

1. P'(PKP)
2. PKS
3. SKS
4. SKP

Group III Waves Reflected once from the Interior Surface of the Core.

1. PKKP
2. PKKS
3. SKKS
4. SKKP

Group IV. Waves Transmitted Two or Three Times by the Core of the Earth.

1. P'P'
2. P'P'P'

Part III Calculation of the Vertical and Horizontal Components of the Ground Displacement Produced by Direct Seismic Waves and of the Ratios of the Vertical and Horizontal Components of the Body Waves to those of the Corresponding Direct Waves.

1. P and its u and w Ratios
2. SV and its u and w Ratios
3. PP and its u and w Ratios
4. SS (SV type) and its u and w Ratios

Some comment is necessary upon the fact that the values of u and w for waves transmitted by the earth's core are designated as approaching infinity at certain distances. This is to be regarded purely as a mathematical statement, since at certain points ($\Delta = 180^{o}$) sin Δ in the amplitude equation becomes zero and, besides, at focal points of certain waves $\frac{di_o}{d\Delta}$ becomes infinity. Either of these two factors gives a value of infinity to the amplitude of that wave transmitted by the core. In the physical sense this merely means that the value of the amplitude becomes much greater than the amplitudes at neighboring distances on that particular plot. At focal points, for example, actual measurements show that the amplitudes become for certain waves as much as twenty times the value of amplitudes at distances on both sides of the focal points.

Part I. Calculation of the Partition of the Relative Energy at the Earth's Core.

Case I. Incident Longitudinal Wave in the Mantle against the Core.

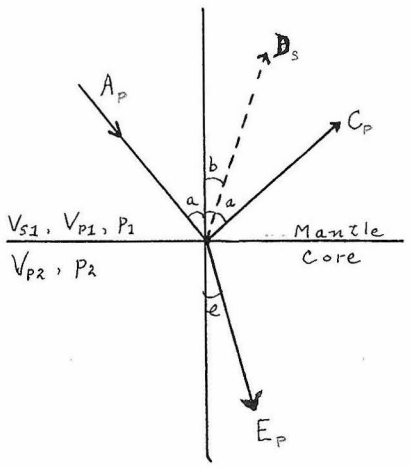


Fig. 4. The Three Derived Waves from the Incident Longitudinal Wave.

In this case:

$V_{S2} = 0$ and $\sin f = 0$.

The tangential strains equation disappears,

V_{p1} = 13.7 km. per sec.
V_{p2} = 8.0 km. per sec.
V_{s1} = 7.25km. per sec.
P_1 = 5.4 gm. per cm.^{3}
P_2 = 10.1 gm. per cm.^{3}

The Zoeppritz Equations become:

Normal strains, $A \cos a - C \cos a + D \sin b - E \cos e = 0.$

Normal stresses, $-A \cos 2b - C \cos 2b + D\left(\frac{V_{s1}}{V_{p1}}\right) \sin 2b + E \frac{P_2}{P_1} \frac{V_{p2}}{V_{p1}} = 0.$

Tangential stresses, $-A \sin 2a + C \sin 2a + D\left(\frac{V_{p1}}{V_{s1}}\right) \cos 2b = 0.$

Taking A as 1.00:

$(\cos a) - (\cos a)\, C + (\sin b) D + (-\cos e)\, E = 0$

$(-\cos 2b) - (\cos 2b) C + \left(\frac{V_{s1}}{V_{p1}} \sin 2b\right) D + \left(\frac{P_2 V_{p2}}{P_1 V_{p1}}\right) E = 0$

$(-\sin 2a) + (\sin 2a) C + \left(\frac{V_{p1}}{V_{s1}} \cos 2b\right) D = 0.$

Substituting the initial values, the equations become

$(-\cos a) C + (\sin b) D + (-\cos e) E = -\cos a$

$(-\cos 2b) C + (.5292 \sin 2b) D + 1.092\, E = \cos 2b$

$(\sin 2a) C + (1.8896 \cos 2b) D = \sin 2a.$

These are of the order $a_i x + b_i y + c_i z = d_i$.

Hence

$$C = \frac{\begin{vmatrix} (-\cos a) & (\sin b) & (-\cos e) \\ (\cos 2b) & (.5292 \sin 2b) & (1.092) \\ (\sin 2a) & (1.8896 \cos 2b) & (0) \end{vmatrix}}{\begin{vmatrix} (-\cos a) & (\sin b) & (-\cos e) \\ (-\cos 2b) & (.5292 \sin 2b) & (1.092) \\ (\sin 2a) & (1.8896 \cos 2b) & (0) \end{vmatrix}},$$

$$D = \frac{\begin{vmatrix} (-\cos a) & (-\cos a) & (-\cos e) \\ (-\cos 2b) & (\cos 2b) & (1.092) \\ (\sin 2a) & (\sin 2a) & (0) \end{vmatrix}}{\begin{vmatrix} \text{same as for C} \end{vmatrix}},$$

and $$E = \frac{\begin{vmatrix} (-\cos a) & (\sin b) & (-\cos a) \\ (-\cos 2b) & (.5292 \sin 2b) & (\cos 2b) \\ (\sin 2a) & (1.8896 \cos 2b) & (\sin 2a) \end{vmatrix}}{\begin{vmatrix} \text{same as for C} \end{vmatrix}}.$$

Or $C =$

$$\frac{2.064(\cos 2b)\cos a - 1.8896(\cos^2 2b)\cos e + 1.092(\sin 2a)\sin b + .5292 \sin 2b (\cos e) \sin 2a}{2.064(\cos a)\cos 2b + 1.8896(\cos^2 2b)\cos e + 1.092(\sin 2a)\sin b + .5292 \sin 2b (\cos e) \sin 2a},$$

$D =$

$$\frac{2 \cos 2b \sin 2a \cos e}{\text{same as for C}},$$

and $E =$

$$\frac{3.7792 \cos^2 2b \cos a + 2 \sin 2a \cos 2b \sin b}{\text{same as for C}}.$$

a	C/A	e	E/A	b	D/A
0°	.04412	0°	.95596	0°	.00000
10°	.06402	5°49'	.94238	5°16'	.17231
20°	.12013	11°31'	.90298	10°26'	.32031
30°	.20170	16°59'	.84070	15°21'	.42554
40°	.29225	22°03'	.76083	19°53'	.47976
50°	.37436	26°34'	.67027	23°55'	.48572
60°	.42384	30°23'	.57598	27°17'	.45549
70°	.40410	33°17'	.48231	29°49'	.40099
80°	.20654	35°06'	.36881	31°25'	.31455
83°	.05399	35°25'	.31559	31°41'	.27018
84°	-.01724	35°30'	.29273	31°45'	.25084
85°	-.10242	35°34'	.26599	31°49'	.22812
89°	-.71487	35°43'	.08371	35°56'	.07191
90°	-1.00000	35°44'	.00000	31°57'	.00000

The check equation of energy for an incident compressional wave is given on p. 178 of Macelwane (14) as

$$1 = \frac{C^2}{A^2} + \frac{D^2 \sin 2b}{A^2 \sin 2a} + \frac{E^2 P_2 \sin 2e}{A^2 P_1 \sin 2a} + \frac{F^2 P_2 \sin 2f}{A^2 P_1 \sin 2a},$$

Since for this case F = 0, the energy equation becomes

$$1 = \frac{C^2}{A^2} + \frac{D^2 \sin 2b}{A^2 \sin 2a} + \frac{E^2 P_2 \sin 2e}{A^2 P_1 \sin 2a}$$

or $$1 = \frac{E_{rp}}{E_{ip}} + \frac{E_{rs}}{E_{ip}} + \frac{E_{fp}}{E_{ip}} .$$

Where E_{ip} = energy of incident compressional wave,
E_{rp} = " " reflected " "
E_{fp} = " " refracted " "
and E_{rs} = " " reflected shear " .

Therefore $$\frac{E_{rp}}{E_{ip}} = \frac{C^2}{A^2},$$

$$\frac{E_{rs}}{E_{ip}} = \frac{D^2}{A^2} \frac{\sin 2b}{\sin 2a},$$

and $$\frac{E_{fp}}{E_{ip}} = \frac{E^2 P_2 \sin 2e}{A^2 P_1 \sin 2a},$$

where each one of these equations denotes relative energy.

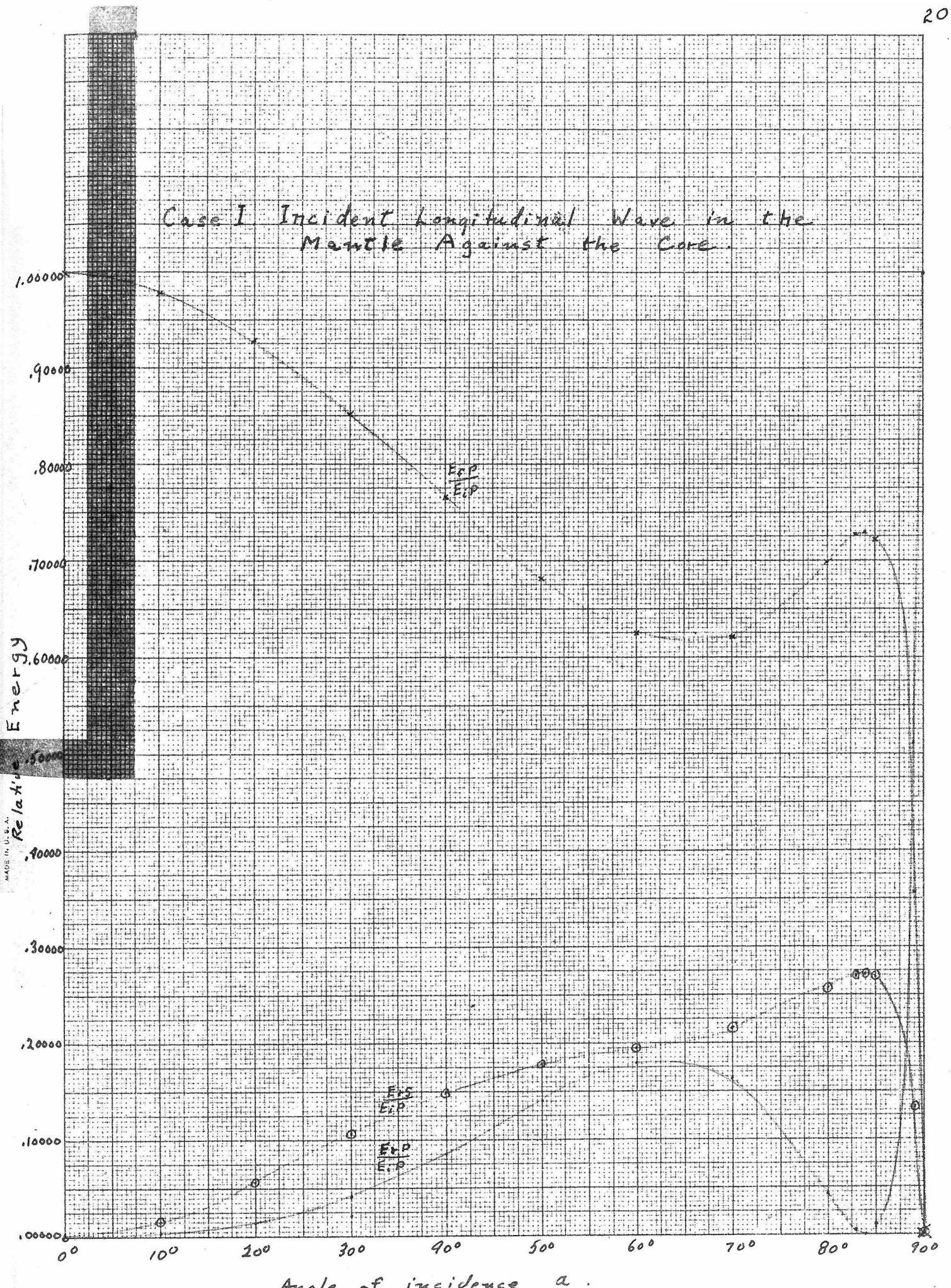
a	$\frac{E_rP}{E_iP}$	$\frac{E_rS}{E_iP}$	$\frac{E_fP}{E_iP}$	Total
0°	.00195	.00000	.99805	=1.00000
10°	.00410	.01587	.97933	=0.99930
20°	.01443	.05685	.92830	=0.99958
30°	.04069	.10675	.85284	=1.00028
40°	.08541	.14950	.76508	=0.99999
50°	.14015	.17757	.68263	=1.00035
60°	.17964	.19520	.62524	=1.00008
70°	.16330	.21582	.62107	=1.00019
80°	.04266	.25737	.69986	=0.99989
83°	.00291	.26973	.72731	=0.99995
84°	.00030	.27084	.72888	=1.00002
85°	.01049	.26851	.72110	=1.00010
89°	.51104	.13301	.35604	=1.00009
90°	1.00000	.00000	.00000	=1.00000

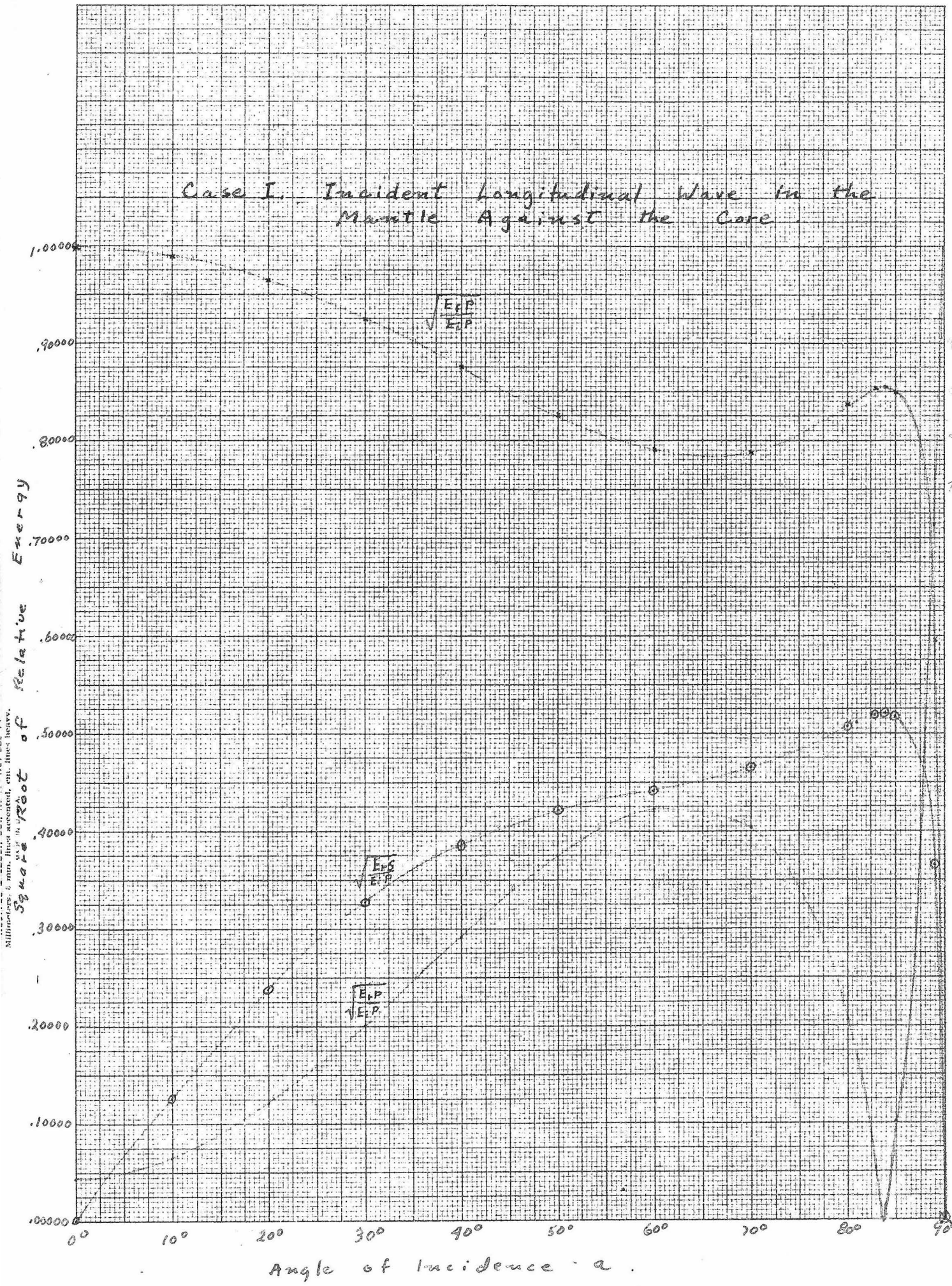
A plot of these relative energies against a is on p. 20.

a	$\sqrt{\frac{E_rP}{E_iP}}$	$\sqrt{\frac{E_rS}{E_iP}}$	$\sqrt{\frac{E_fP}{E_iP}}$
0°	.04416	.00000	.99903
10°	.06403	.12598	.98963
20°	.12013	.23843	.96350
30°	.20172	.32673	.92350
40°	.29225	.38665	.87470
50°	.37436	.42139	.82622
60°	.42385	.44181	.79073
70°	.40411	.46457	.78808
80°	.20654	.50732	.83658
83°	.05399	.51936	.85282
84°	.01732	.52043	.85374
85°	.10242	.51818	.84918
89°	.71487	.36471	.59669
90°	1.00000	.00000	.00000

A plot of square roots of relative energies against a is on p. 21.

Knott (13) in 1919 used calculations similar to the above but with different equations to determine the energies of the reflected and refracted waves in terms of the energy of the incident wave. The plot on p. 189 of his article for an incident longitudinal wave in the mantle against the core is like the plot on p. 20. The slight differences are due to the fact that Knott used V_{s1} = 6.88 km. per sec., V_{p1} = 12.89 km. per sec., V_{p2} = 10.15 km. per sec., and $P_1 = P_2$, which differ from the values used in this work.


Case I Incident Longitudinal Wave in the Mantle Against the Core.
1.00000
.90000
.80000
.70000
.60000
.50000
.40000
.30000
.20000
.10000
.00000
Relative Energy
$\frac{E_{rP}}{E_{iP}}$
$\frac{E_{rS}}{E_{iP}}$
$\frac{E_{rP}}{E_{iP}}$
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
Angle of incidence a.


Case I. Incident Longitudinal Wave in the Mantle Against the Core
1.00000
.90000
.80000
.70000
.60000
.50000
.40000
.30000
.20000
.10000
.00000
Square Root of Relative Energy
$\sqrt{\frac{E_1P'}{E_1P}}$
$\sqrt{\frac{E_1S}{E_1P}}$
$\sqrt{\frac{E_1P}{E_1P}}$
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
Angle of Incidence - a.

Case II. Incident Longitudinal Wave in the Core against the Mantle.

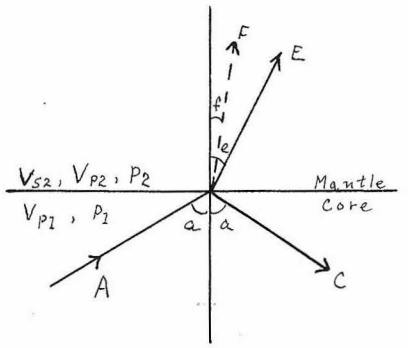


Fig. 5. The Three Derived Waves from the Incident Longitudinal Wave

In this case:

P_1 = 10.1 gm. / cm.^{3}
P_2 = 5.4 gm. / cm.^{3}
V_{p1} = 8.0 km. / sec.
V_{p2} = 13.7 km. / sec.
V_{s2} = 7.25 km. / sec.

The D terms disappear.

V_{s1} = 0 and sin b = 0.

The tangential strains equation disappears.

The Zoeppritz Equations become:

Normal strains, $A \cos a - C \cos a - E \cos e - F \sin f = 0$,

Normal stresses, $-A - C + E \frac{P_2}{P_1} \frac{V_{p2}}{V_{p1}} \cos 2f + F \frac{P_2}{P_1} \frac{V_{s2}}{V_{p1}} \sin 2f = 0$,

Tangential stresses, $E V_{s2} \sin 2e - F V_{p2} \cos 2f = 0$.

Taking A as 1.00:

$\cos a - (\cos a)C - (\cos e)E - (\sin f)F = 0$

$-1 - C + \left(\frac{P_2 V_{p2}}{P_1 V_{p1}} \cos 2f\right)E + \left(\frac{P_2}{P_1} \frac{V_{s2}}{V_{p1}} \sin 2f\right)F = 0$

$(V_{s2} \sin 2e)E - (V_{p2} \cos 2f)F = 0$.

Substituting the initial values, the equations become

$(-\cos a)C + (-\cos e)E + (-\sin f)F = -\cos a$

$(-1)C + (.91558 \cos 2f)E + (.48453 \sin 2f)F = 1$

$(7.25 \sin 2e)E + (-13.7 \cos 2f)F = 0$.

These are of the order $a_i x + b_i y + c_i z = d_i$.

Hence

$$C = \frac{\begin{vmatrix} (-\cos a) & (-\cos e) & (-\sin f) \\ (1) & (.91558 \cos 2f) & (.48453 \sin 2f) \\ (0) & (7.25 \sin 2e) & (-13.7 \cos 2f) \end{vmatrix}}{\begin{vmatrix} (-\cos a) & (-\cos e) & (-\sin f) \\ (-1) & (.91558 \cos 2f) & (.48453 \sin 2f) \\ (0) & (7.25 \sin 2e) & (-13.7 \cos 2f) \end{vmatrix}},$$

$$E = \frac{\begin{vmatrix} (-\cos a) & (-\cos a) & (-\sin f) \\ (-1) & (1) & (.48453 \sin 2f) \\ (0) & (0) & (-13.7 \cos 2f) \end{vmatrix}}{\begin{vmatrix} \text{same as for C} \end{vmatrix}},$$

$$\text{and } F = \frac{\begin{vmatrix} (-\cos a) & (-\cos e) & (-\cos a) \\ (-1) & (.91558 \cos 2f) & (1) \\ (0) & (7.25 \sin 2e) & (0) \end{vmatrix}}{\begin{vmatrix} \text{same as for C} \end{vmatrix}}.$$

Or C =

$$\frac{12.544(\cos a)\cos^2 2f + 3.5128 \sin 2f (\sin 2e)\cos a - 13.7(\cos 2f)\cos e - 7.25(\sin 2e)\sin f}{12.544(\cos a)\cos^2 2f + 3.5128 \sin 2f (\sin 2e)\cos a + 13.7(\cos 2f)\cos e + 7.25(\sin 2e)\sin f},$$

E =

$$\frac{27.4 \cos 2f \cos a}{\text{same as for C}},$$

and F =

$$\frac{14.5 \sin 2e \cos a}{\text{same as for C}}.$$

a	C/A	e	E/A	f	F/A
0°	-.04407	0°	1.04450	0°	.00000
10°	-.05268	17°18'	1.03200	9°3'	.32628
20°	-.07572	35°51'	1.00760	18°3'	.62656
30°	-.08681	58°54'	1.07370	26°57'	.85302
33°	-.05039	68°52'	1.25270	29°35'	.86974
35°	.09536	79°11'	1.81500	31°19'	.77032
35°44'	1.00000	90°00'	4.96520	31°57'	.00000
		sine e			
37°	-.28761-i.44748	1.0306	---	33°3'	1.1217+i.38981
40°	-.40145-i.15631	1.1008	---	35°38'	1.2509+i.13951
50°	-.39659-i.00088	1.3118	---	43°58'	1.2458+i.00079
60°	-.45959-i.02392	1.4832	---	51°42'	1.1455+i.01877
70°	-.59473-i.06417	1.6092	---	58°23'	.92894+i.03738
80°	-.78206-i.06653	1.6865	---	63°11'	.55153+i.02059
90°	-1.00000	1.7125	---	64°59'	.00000

The check equation of energy in this case becomes, using the same notation as in Case 1,

$$1 = \frac{C^2}{A^2} + \frac{E^2P_2 \sin 2e}{A^2P_1 \sin 2a} + \frac{F^2P_2 \sin 2f}{A^2P_1 \sin 2a},$$

where

$$\frac{E_rP}{E_iP} = \frac{C^2}{A^2},$$

$$\frac{E_fS}{E_iP} = \frac{F^2P_2 \sin 2f}{A^2P_1 \sin 2a},$$

and

$$\frac{E_fP}{E_iP} = \frac{E^2P_2 \sin 2e}{A^2P_1 \sin 2a}.$$

It should be noted at this point that for certain angles of incidence a, the values of the amplitude ratios $\frac{C}{A}$ and $\frac{F}{A}$ become complex. This indicates merely a change of phase among the incident and reflected and refracted waves. The absolute values of these complex amplitude ratios are used in the relative emergy calculations.

a	$\frac{E_rP}{E_iP}$	$\frac{E_fP}{E_iP}$	$\frac{E_fS}{E_iP}$	Total
0°	.00194	.99806	.00000	= 1.00000
10°	.00278	.94548	.05170	= .99996
20°	.00573	.80175	.19239	= .99987
30°	.00754	.62950	.36297	= 1.00001
33°	.00254	.61743	.38014	= 1.00011
35°	.00910	.69098	.29983	= .99991
$35^\circ 44'$**	1.00000	.00000	.00000	= 1.00000
37°	.28297*	---	.71710*	= 1.00007
40°	.18560*	---	.81450*	= 1.00010
50°	.15728*	---	.84210*	= .99938
60°	.21179*	---	.78822*	= 1.00001
70°	.35782*	---	.64189*	= .99971
80°	.61604*	---	.38351*	= .99955
90°	1.00000	---	.00000	= 1.00000

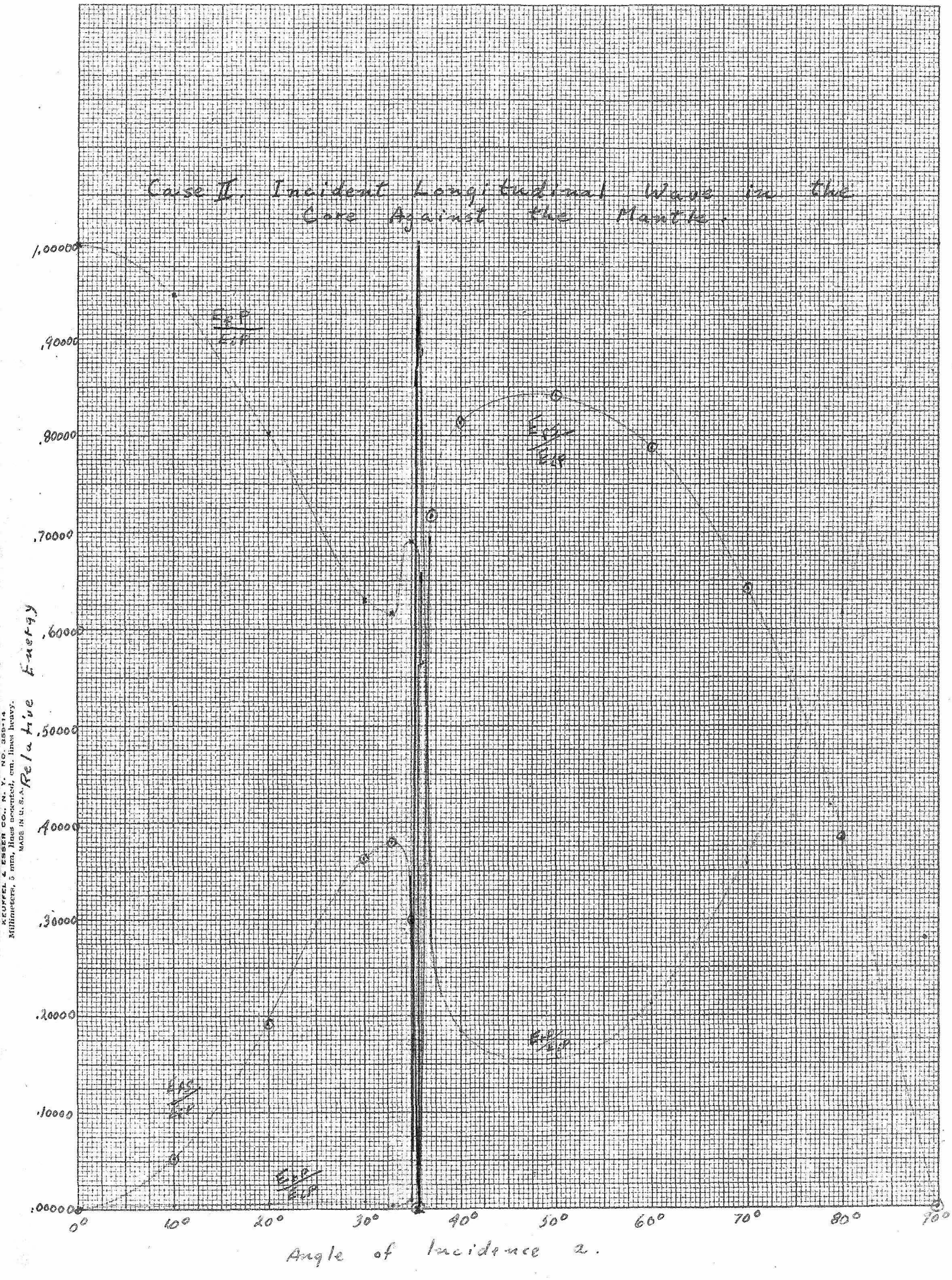
A plot of these relative energies against a is on p. 26.

a	$\sqrt{\frac{E_rP}{E_iP}}$	$\sqrt{\frac{E_fP}{E_iP}}$	$\sqrt{\frac{E_fS}{E_iP}}$
0°	.04405	.99903	.00000
10°	.05273	.97238	.22738
20°	.07570	.89542	.43863
30°	.08683	.79342	.60248
33°	.05040	.78578	.61656
35°	.09537	.83126	.54758
$35^\circ 44'$	1.00000	.00000	.00000
37°	.53195	---	.84682
40°	.43081	---	.90250
50°	.39659	---	.91766
60°	.46021	---	.88783
70°	.59818	---	.80118
80°	.78490	---	.61929
90°	1.00000	---	.00000

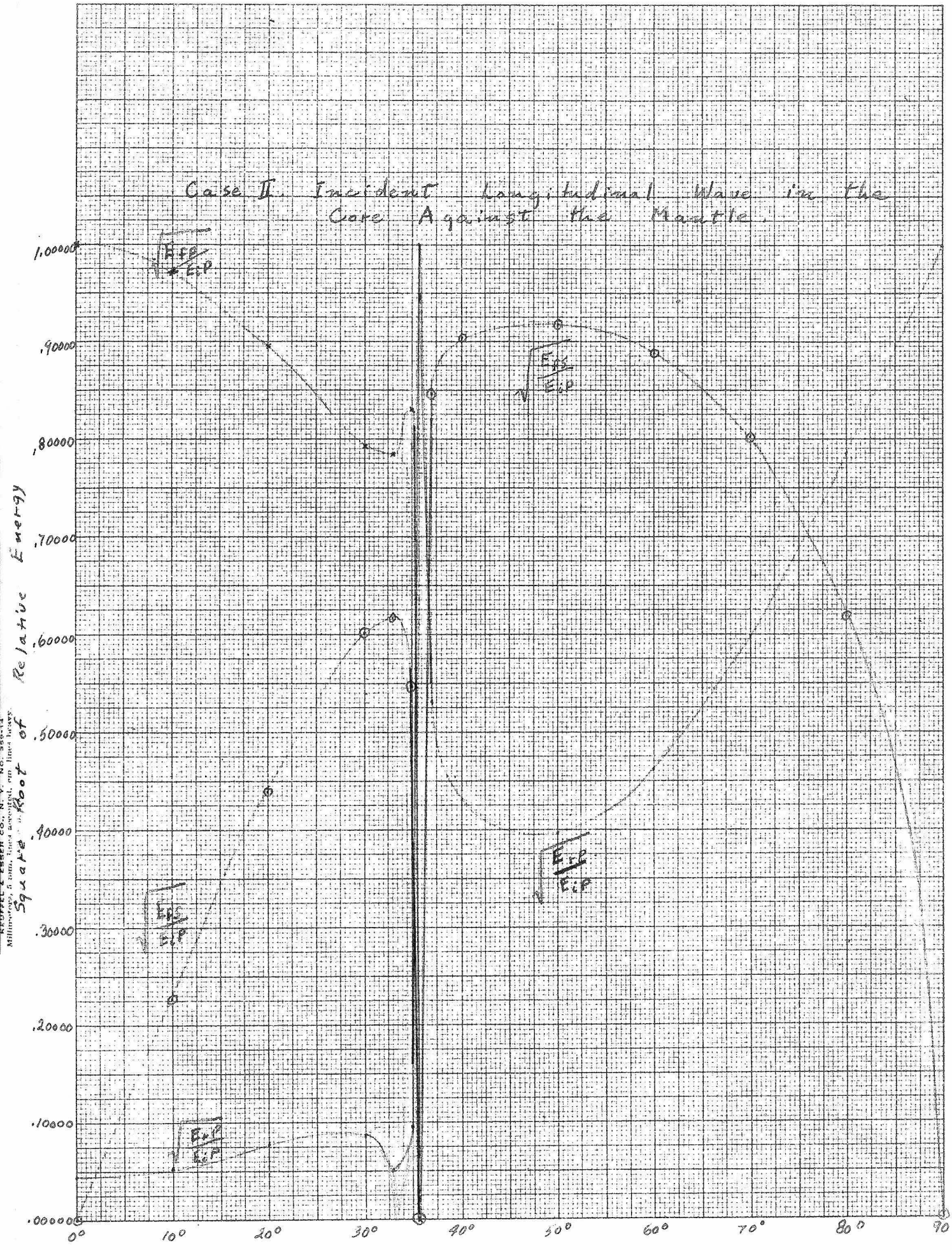
A plot of the square roots of relative energies against a is on p. 27.

The plot on p. 26 is similar to the graph on p. 189 of Knott (13) for an incident longitudinal wave in the core against the mantle. As in Case I, differences between the two are slight and due to the different values used by Knott for the densities and the velocities of the waves.

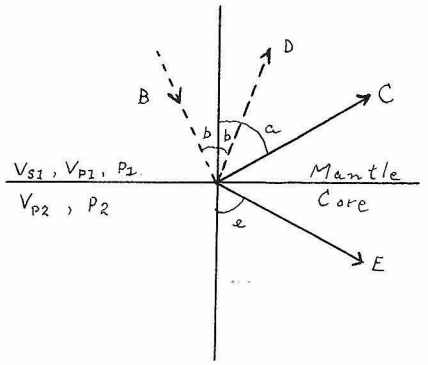
** Actually $35^\circ 43.7'$.
* "Absolute value" .


Case II. Incident Longitudinal Wave in the Core Against the Mantle.
1.00000
.90000
.80000
.70000
.60000
.50000
.40000
.30000
.20000
.10000
.00000
Relative Energy
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
Angle of Incidence a.
KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm, lines accented, cm. lines heavy.
MADE IN U.S.A.

Case II. Incident Longitudinal Wave in the Core Against the Mantle.



Case III. Incident Shear Wave (SV type) in the Mantle against the Core.



In this case:

P_2 = 10.1 gm./cm.^{3}
P_1 = 5.4 gm./cm.^{3}
V_{p1} = 13.7 km./sec.
V_{p2} = 8.0 km./sec.
V_{s1} = 7.25 km./sec.
V_{s2} = 0 and sin f = 0.

The tangential strains equation disappears.

Fig. 6. The Three Derived Waves from the Incident Shear Wave.

The Zoeppritz Equations become:

Normal strains, $-B \sin b - C \cos a + D \sin b - E \cos e = 0$.

Normal stresses, $B \sin 2b - C \frac{V_{p1}}{V_{s1}} \cos 2b + D \sin 2b + E \frac{P_2}{P_1} \frac{V_{p2}}{V_{s1}} = 0$.

Tangential stresses, $-B \cos 2b + C \frac{V_{s1}}{V_{p1}} \sin 2a + D \cos 2b = 0$,

Letting B = 1:

$$- \sin b - (\cos a)C + (\sin b)D - (\cos e)E = 0$$

$$\sin 2b - \left(\frac{V_{p1}}{V_{s1}} \cos 2b\right) C + (\sin 2b)D + \left(\frac{P_2}{P_1} \frac{V_{p2}}{V_{s1}}\right) E = 0$$

$$-\cos 2b + \left(\frac{V_{s1}}{V_{p1}} \sin 2a\right) C + (\cos 2b)D = 0,$$

or

$$(-\cos a)C + (\sin b)D + (-\cos e)E = \sin b$$

$$\left(-\frac{V_{p1}}{V_{s1}} \cos 2b\right) C + (\sin 2b)D + \left(\frac{P_2}{P_1} \frac{V_{p2}}{V_{s1}}\right) E = -\sin 2b$$

$$\left(\frac{V_{s1}}{V_{p1}} \sin 2a\right) C + (\cos 2b)D = \cos 2b.$$

These are of the order $a_i x + b_i y + c_i z = d_i$.

Hence, using the initial conditions,

$$C = \frac{\begin{vmatrix} (\sin b) & (\sin b) & (-\cos e) \\ (-\sin 2b) & (\sin 2b) & (2.0639) \\ (\cos 2b) & (\cos 2b) & (0) \end{vmatrix}}{\begin{vmatrix} (-\cos a) & (\sin b) & (-\cos e) \\ (-1.8897 \cos 2b) & (\sin 2b) & (2.0639) \\ (.5292 \sin 2a) & (\cos 2b) & (0) \end{vmatrix}},$$

$$D = \frac{\begin{vmatrix} (-\cos a) & (\sin b) & (-\cos e) \\ (-1.8897 \cos 2b) & (-\sin 2b) & (2.0639) \\ (.5292 \sin 2a) & (\cos 2b) & (0) \end{vmatrix}}{\begin{vmatrix} (-\cos a) & (\sin b) & (-\cos e) \\ (-1.8897 \cos 2b) & (\sin 2b) & (2.0639) \\ (.5292 \sin 2a) & (\cos 2b) & (0) \end{vmatrix}},$$

$$\text{and } E = \frac{\begin{vmatrix} (-\cos a) & (\sin b) & (\sin b) \\ (-1.8897 \cos 2b) & (\sin 2b) & (-\sin 2b) \\ (.5292 \sin 2a) & (\cos 2b) & (\cos 2b) \end{vmatrix}}{\begin{vmatrix} (-\cos a) & (\sin b) & (-\cos e) \\ (-1.8897 \cos 2b) & (\sin 2b) & (2.0639) \\ (.5292 \sin 2a) & (\cos 2b) & (0) \end{vmatrix}},$$

Or

$$C = \frac{2 \cos e \sin 2b \cos 2b}{2.0639(\cos a)\cos 2b + 1.8897(\cos e)\cos^2 2b + 1.0922(\sin 2a)\sin b + .5292 \sin 2a (\sin 2b)\cos e},$$

$$D = \frac{2.0639(\cos a)\cos 2b + 1.8897(\cos e)\cos^2 2b + 1.0922(\sin 2a)\sin b - .5292 \sin 2a (\sin 2b)\cos e}{\text{same as for C}},$$

and

$$E = \frac{-2 \cos a \sin 2b \cos 2b - 1.0584 \sin 2a \sin 2b \sin b}{\text{same as for C}}.$$

b	D/B	e	E/B	a	C/B
0°	1.00000	0°	.00000	0°	.00000
10°23'	.93593	11°28'	-.18130	19°55'	.17670
21°47'	.75438	24°10'	-.36270	44°32'	.33635
30°00'	.63789	33°30'	-.43418	70°53'	.55279
31°00'	.65224	34°38'	-.41033	76°43'	.68983
31°57'06"	1.00000	35°43'50"	.00000	90°00'	2.16050
33°00'	.50720-i.17700	36°56'	-.56598-i.20324	sin a 1.02920	---
40°00'	.39654-i.01189	45°11'	-.66603-i.01312	1.21460	---
50°00'	.43823-i.00570	57°42'	-.68620-i.00696	1.44750	---
60°00'	.64794-i.01509	72°52'	-.68998-i.02958	1.63650	---
64°00'	.83840-i.00482	82°38'	-.70127-i.02089	1.69840	---
64°59'30"	1.00000	90°00'	-.74268	1.71250	---
66°00'	.98712-i.15996	sin e 1.00800	---	1.72630	---
70°00'	.95838-i.28565	1.03690	---	1.77570	---
80°00'	.97440-i.22479	1.08670	---	1.86090	---
90°00'	1.00000	1.10370	---	1.88970	---

The check equation of energy for an incident shear wave is given on p. 178 of Macelwane (14) as

$$1 = \frac{C^2 \sin 2a}{B^2 \sin 2b} + \frac{D^2}{B^2} + \frac{E^2 P_2 \sin 2e}{B^2 P_1 \sin 2b} + \frac{F^2 P_2 \sin 2f}{B^2 P_1 \sin 2b} .$$

Since F = 0 for Case III, the energy equation becomes

$$1 = \frac{C^2 \sin 2a}{B^2 \sin 2b} + \frac{D^2}{B^2} + \frac{E^2 P_2 \sin 2e}{B^2 P_1 \sin 2b} ,$$

where

$$\frac{E_r P}{E_i S} = \frac{C^2 \sin 2a}{B^2 \sin 2b},$$

$$\frac{E_r S}{E_i S} = \frac{D^2}{B^2} ,$$

and

$$\frac{E_f P}{E_i S} = \frac{E^2 P_2 \sin 2e}{B^2 P_1 \sin 2b} ,$$

using the same notation as for Case I.

b	$\frac{E_{rS}}{E_{iS}}$	$\frac{E_{rP}}{E_{iS}}$	$\frac{E_{fP}}{E_{iS}}$	Total
0°	1.00000	.00000	.00000	= 1.00000
10°23'	.87554	.05654	.06757	= .99965
21°47'	.56909	.16413	.26671	= .99993
30°00'	.40689	.21837	.37481	= 1.00007
31°00'	.42540	.24104	.33356	= 1.00000
31°57'06"	1.00000	.00000	.00000	= 1.00000
33°00'	.28858*	---	.71128*	= .99986
40°00'	.15738*	---	.84280*	= 1.00018
50°00'	.19207*	---	.80816*	= 1.00023
60°00'	.42007*	---	.57998*	= 1.00005
64°00'	.70293*	---	.29712*	= 1.00005
64°59'30"	1.00000	---	.00000	= 1.00000
66°00'	.99999*	---	---	= .99999
70°00'	1.00009*	---	---	= 1.00009
80°00'	1.00001*	---	---	= 1.00001
90°00'	1.00000	---	---	= 1.00000

As in Case II the absolute values of the amplitude ratios have to be used to calculate the relative energy values for this range of b. A plot of these relative energies against b is on p. 32.

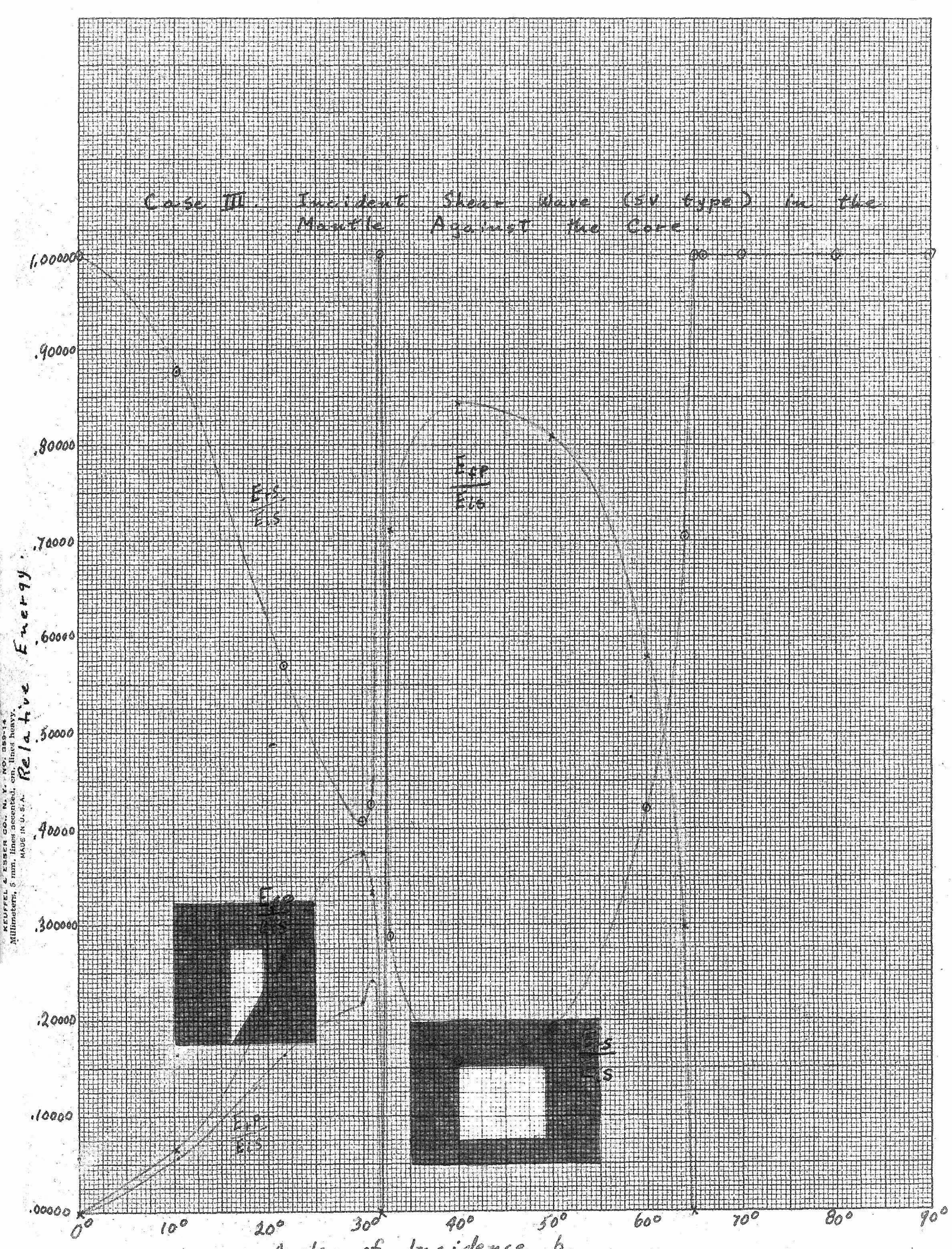
b	$\sqrt{\frac{E_{rS}}{E_{iS}}}$	$\sqrt{\frac{E_{rP}}{E_{iS}}}$	$\sqrt{\frac{E_{fP}}{E_{iS}}}$
0°	1.00000	.00000	.00000
10°23'	.93570	.23778	.25994
21°47'	.75438	.40514	.51644
30°00'	.63789	.46730	.61223
31°00'	.65223	.49097	.57755
31°57'06"	1.00000	.00000	.00000
33°00'	.53720	---	.84338
40°00'	.39672	---	.91804
50°00'	.43826	---	.89898
60°00'	.63337	---	.76157
64°00'	.83842	---	.54509
64°59'30"	1.00000	---	.00000
66°00'	1.00000	---	---
70°00'	1.00005	---	---
80°00'	1.00000	---	---
90°00'	1.00000	---	---

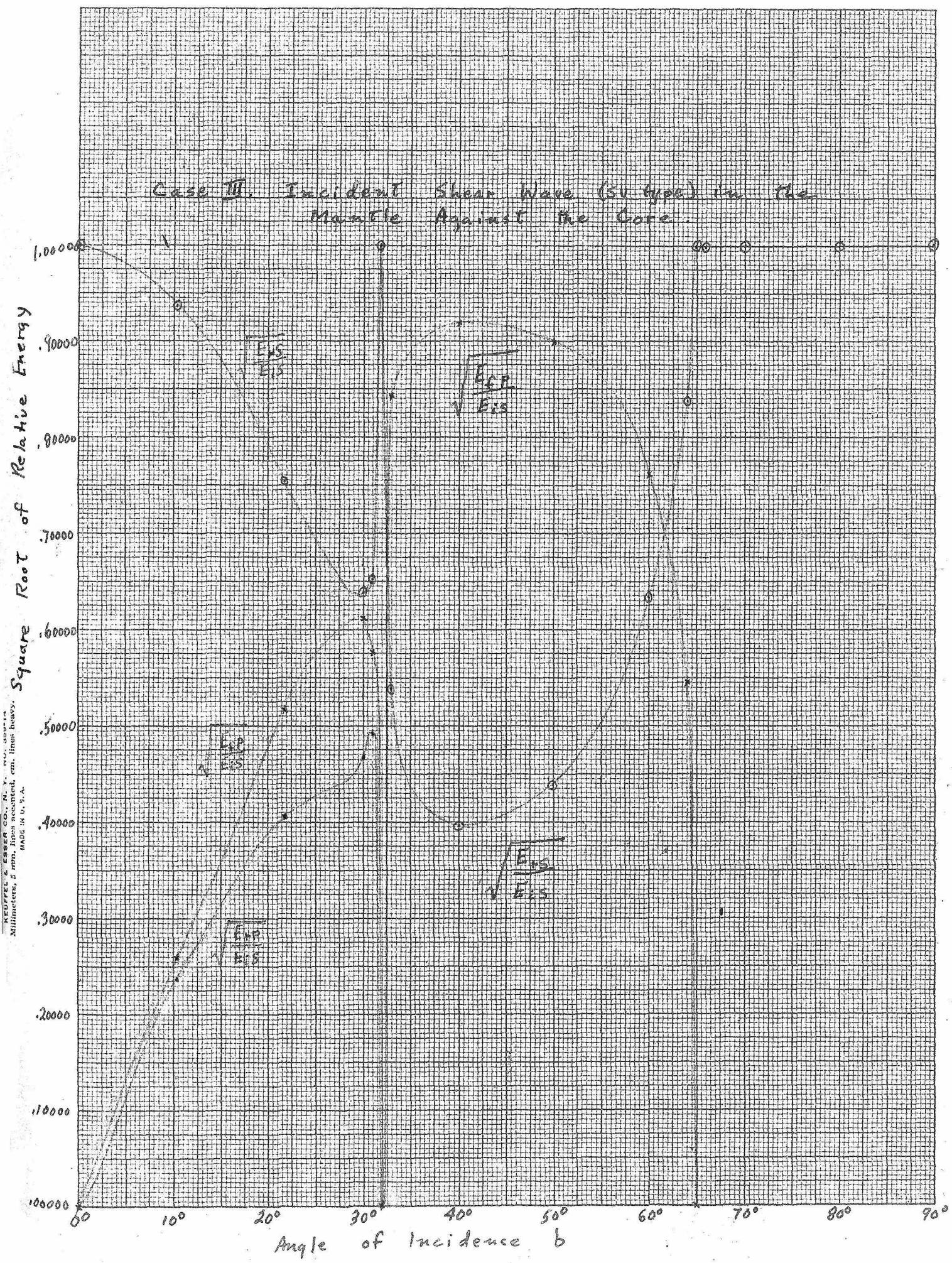
A plot of the square roots of relative energy values against b is on p. 33.

---

* "absolute value".

Case III. Incident Shear Wave (SV type) in the Mantle Against the Core.




Case III. Incident Shear Wave (SV type) in the Mantle Against the Core.
Square Root of Relative Energy
1.00000
.90000
.80000
.70000
.60000
.50000
.40000
.30000
.20000
.10000
.00000
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
Angle of Incidence b
$\sqrt{\frac{E_{rS}}{E_{iS}}}$
$\sqrt{\frac{E_{fP}}{E_{iS}}}$
$\sqrt{\frac{E_{rP}}{E_{iS}}}$
$\sqrt{\frac{E_{rS}}{E_{iS}}}$
$\sqrt{\frac{E_{rP}}{E_{iS}}}$
KEUFFEL & ESSER CO., N. Y. NO. 359-14
Millimeters, 5 mm. lines accented, cm. lines heavy.
MADE IN U. S. A.

There is a definite difference between the plot on p. 32 of this work and the graph on p. 189 of Knott (13) for an incident shear (SV) wave in the mantle against the core. On the plot on p. 32 of these calculations there is one critical angle ($b=31^{\circ}57'06''$) at which the relative energy going into the reflected P and the refracted P becomes zero and relative energy going into the reflected shear wave becomes 1.00. In Knott's graph for the same case there is no such "zero point", but instead at approximately $b=31^{\circ}$ the relative energy going into the reflected P becomes zero, that into the refracted P about .82, and that into the reflected SV about .17. Knott assumed the limiting case of an earth consisting of an elastic shell which passes abruptly into a non-rigid elastic core (that is, a core consisting of elastic, greatly compressed liquid) of the same compressibility and density. This is essentially the same assumption in this work except that the density of the core is taken as almost twice that of the surrounding mantle. The differences in the graphs for the incident shear (SV) wave in the mantle against the core must therefore result from the different density and velocity values and from the different methods of calculation.

Case IV. Incident Shear Wave (SH type) in the Mantle against the Core.

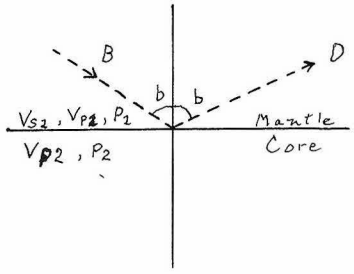


Fig. 7. An Incident Shear Wave of the SH Type.

In this case:

Since there is no refracted shear wave, the tangential strains equation disappears and only the tangential stresses equation is left from the two equations on p. 5 for an incident shear (SH) wave. Also F=0.

The tangential stresses equation,

$$B - D - \frac{P_2 V_{s2} \cos f}{P_1 V_{s1} \cos b} F = 0,$$

becomes

$$B = D .$$

thus

$$\frac{E_{rS}}{E_{iS}} = \frac{D^2}{B^2} = 1.$$

Hence the relative energy $\frac{E_{rS}}{E_{iS}}$ going into the reflected shear wave and the square root of the relative energy are always 1.00 for Case IV, and a plot of them against angle of incidence b would be a straight line.

Part II. Calculation of the Vertical and Horizontal Components of the Ground Displacement at the Earth's Surface Produced by Seismic Waves Reflected and Refracted at the Earth's Core.

Group I. Waves Reflected Once From the Surface of the Core.

1. PcP

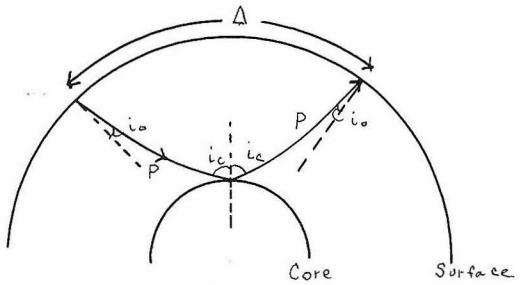


Fig. 8. Sketch to Illustrate the Path of PcP.

i_c	sin i_c	[1] Δ	[2] sin i_o	i_o	tan i_o	[3] $\frac{d\, i_o}{d\, \Delta}$
0°	.00	.00°	.00000	0°	.0000	.268
6°	.10	4.60°	.02173	1°15'	.0218	.260
9°	.15	7.00°	.03260	1°52'	.0326	.260
10°	.17	8.00°	.03694	2° 7'	.0370	.260
20°	.34	16.60°	.07389	4°14'	.0740	.241
30°	.50	25.40°	.10866	6°14'	.1092	.209
40°	.64	34.60°	.13937	8°01'	.1408	.168
50°	.77	45.40°	.16647	9°35'	.1688	.128
60°	.87	57.20°	.18820	10°51'	.1917	.0786
70°	.94	71.40°	.20421	11°47'	.2086	.0290
80°	.98	88.00°	.21401	12°21'	.21895	.0235
84°	.99	95.00°	.21612	12°29'	.2214	.0145
90°	1.00	103.00°	.21732	12°33'	.2226	.0053

---

[1] Taken from Table 7, pp. 103-104 of Gutenberg and Richter (9).

[2] V_p at surface of earth = 5.5 km. per sec. V_p at surface of core = 13.7 km. per sec. r_s = radius of earth = 6,366 km. r_c = radius of core = 3,446 km.

$$\frac{r_c \sin i_o}{(V_p)_s} = \frac{r_c \sin i_c}{(V_p)_c}$$

$$\sin i_o = \frac{(V_p)_s\, r_c}{(V_p)_c\, r_s} \quad \sin i_c = \frac{3446}{6366} \times \frac{5.5}{13.7} \quad \sin i_c = .21732 \sin i_c.$$

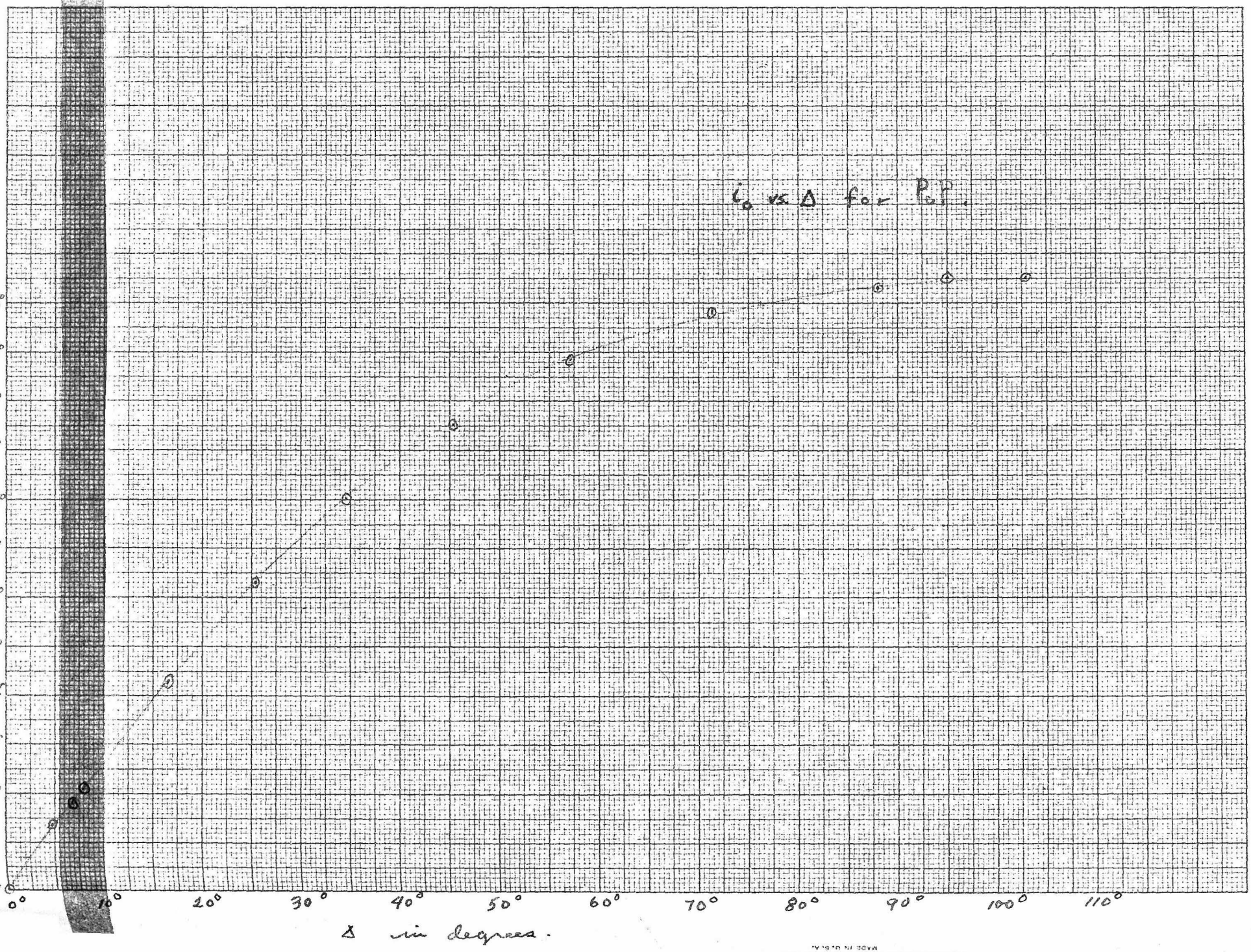
[3] Taken from the plot of i_o against Δ on p. 37.

Δ	$\sqrt{\frac{\tan i_o}{\sin \Delta}\frac{di_o}{d\Delta}}$	[2] f	[3] $A=CT\sqrt{\frac{\tan i_o}{\sin \Delta}\frac{di_o}{d\Delta}}$
.00°	[1].268	.04416	.0372
4.60°	.2675	.05300	.0446
7.00°	.2640	.06150	.0510
8.00°	.2630	.06403	.0530
16.60°	.2500	.12013	.0945
25.40°	.231	.20172	.1461
34.60°	.204	.29225	.1875
45.40°	.1738	.37436	.2050
57.20°	.1338	.42385	.1780
71.40°	.0801	.40411	.1018
88.00°	.0583	.20654	.0379
95.00°	.0568	.01732	.0031
103.00°	.0348	1.00000	.1092

Δ	[4] $\frac{u}{A}$	[4] $\frac{w}{A}$	[5] u	[5] w
.00°	.00	2.00	.0000	.0744
4.60°	.04	2.00	.00179	.0892
7.00°	.09	2.00	.00459	.1020
8.00°	.11	2.00	.00583	.1060
16.60°	.15	2.00	.01417	.1990
25.40°	.24	1.980	.0351	.2897
34.60°	.32	1.975	.0601	.3710
45.40°	.38	1.970	.0780	.4040
57.20°	.44	1.960	.0783	.3490
71.40°	.48	1.950	.0489	.1985
88.00°	.50	1.945	.01895	.0739
95.00°	.52	1.940	.00161	.00601
103.00°	.53	1.940	.0579	.2122

---

1 Evaluated on p. 38.
2 Taken from p. 19, column 1.
3 C = π and T = 1 sec.
4 Taken from Reflections and Refractions 1943, graphs of Dr. Gutenberg (copy on p. 10).
5 A plot of u and w against Δ is on p. 39.


i_o vs Δ for P.P.
12.00°
11.00°
10.00°
9.00°
8.00°
7.00°
6.00°
5.00°
4.00°
3.00°
2.00°
1.00°
0.00°
i_o in degrees
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
Δ in degrees.

Evaluation of $\sqrt{\frac{\tan i_o}{\sin \Delta} \frac{di_o}{d\Delta}}$ at $\Delta = 0^o$.

For small angles $\tan i_o = i_o^*$, where i_o^* is in radians.
and also $\sin \Delta = \Delta^*$, " Δ^* " " " ,

From the plot on p. 37 $\frac{d\, i_o}{d\, \Delta} = \frac{2.68}{10}$, where di_o and $d\Delta$ are in degrees .

It is evident from the plot on p. 37 that for $\Delta = 0^o$ to 15° the plot of i_o against Δ is a straight line.

Hence at $\Delta \cong 0^o$ $\frac{di_\Delta}{d\Delta} = C = .268$.

Integrating , $i_o^* = C\Delta^* + C'$.

when $i_o^* = 0$, $\Delta^* = 0$ and hence $C' = 0$.

Therefore $i_o^* = C\Delta^*$

or $\frac{i_\Delta^*}{\Delta^*} = C = .268$,

Therefore at $\Delta \cong 0^o$:

$$\sqrt{\frac{\tan i_o}{\sin \Delta} \frac{d\, i_o}{d\, \Delta}} = \sqrt{\frac{i_o^*}{\Delta^*} \frac{d\, i_o}{d\, \Delta}} = \sqrt{C \frac{d\, i_o}{d\, \Delta}} = \frac{d\, i_o}{d\, \Delta} = .268 .$$

It will be noted that u = w = o at $\Delta = 94.5^o$ in the graph on p. 39. This results from the fact that there is a critical angle of incidence at the core for PcP at that Δ . That is, f or $\sqrt{\frac{E_r P}{E_i P}}$ is zero at a = 83.7^o, which occurs for PcP at $\Delta = 94.5^o$. This may be seen in the plot on p. 21.

u and w vs. Δ for
P.C.P.
u = horizontal displacement.
w = vertical displacement.

u and w. Displacements Ground

.4500
.4000
.3500
.3000
.2500
.2000
.1500
.1000
.0500
.0000

0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110°

Δ in degrees

2. PcS

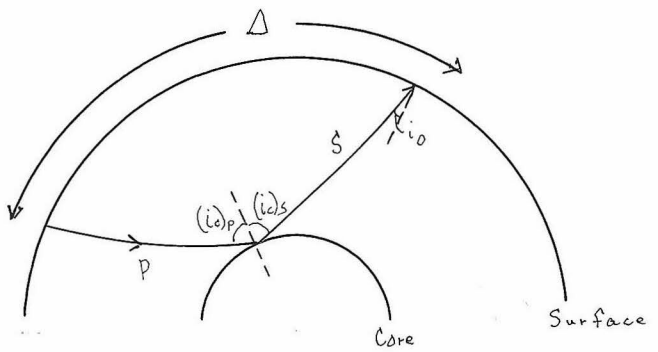


Fig. 9. Sketch to Illustrate the Path of PcS.

$(i_c)_p$	$(\sin i_c)_p$	[1] Δ_p	$(i_c)_s$	[2] $(\sin i_c)_s$	[1] Δ_s	Δ (Total Δ)
0°	.00	.00°	0°	.00	.00°	.00°
10°	.17	4.00°	5°16'	0.10	2.40°	6.40°
20°	.34	8.30°	10°26'	0.18	4.40°	12.70°
30°	.50	12.70°	15°21'	0.26	6.40°	19.10°
40°	.64	17.30°	19°53'	0.34	8.50°	25.80°
50°	.77	22.70°	23°55'	0.41	10.40°	33.10°
60°	.87	28.60°	27°17'	0.46	11.80°	40.40°
70°	.94	35.70°	29°49'	0.50	13.10°	48.80°
80°	.98	44.00°	31°25'	0.520	13.70°	57.70°
84°	.99	47.50°	31°45'	0.526	13.90°	61.40°
90°	1.00	51.20°	31°57'	0.530	14.00°	65.20°

---

[1] Taken from Table 7, pp. 103-104 of Gutenberg and Richter (9).
[2] V_s at surface of core = 7.25 km. 1 sec. V_s at surface of earth = 3.2 km. per sec. r_o = radius of surface of earth=6,366 km. r_c = radius of surface of core = 3,446 km.

Hence: $$\frac{r_o \sin i_o}{(V_s)_o} = \frac{r_c (\sin i_c)_s}{(V_s)_c},$$

and $$\sin i_o = \frac{3.2}{7.25} \times \frac{3,446}{6,366} \times (\sin i_c)_s = .23892 \ (\sin i_c)_s.$$

For the determination of i_o then,

$$\sin i_o = .23892 \ (\sin i_c)_s .$$

$(\sin i_c)_s$ is of course determined by Snell's Law:

$$\frac{(\sin i_c)_s}{(V_s)_c} = \frac{\sin (i_c)_p}{(V_p)_c},$$

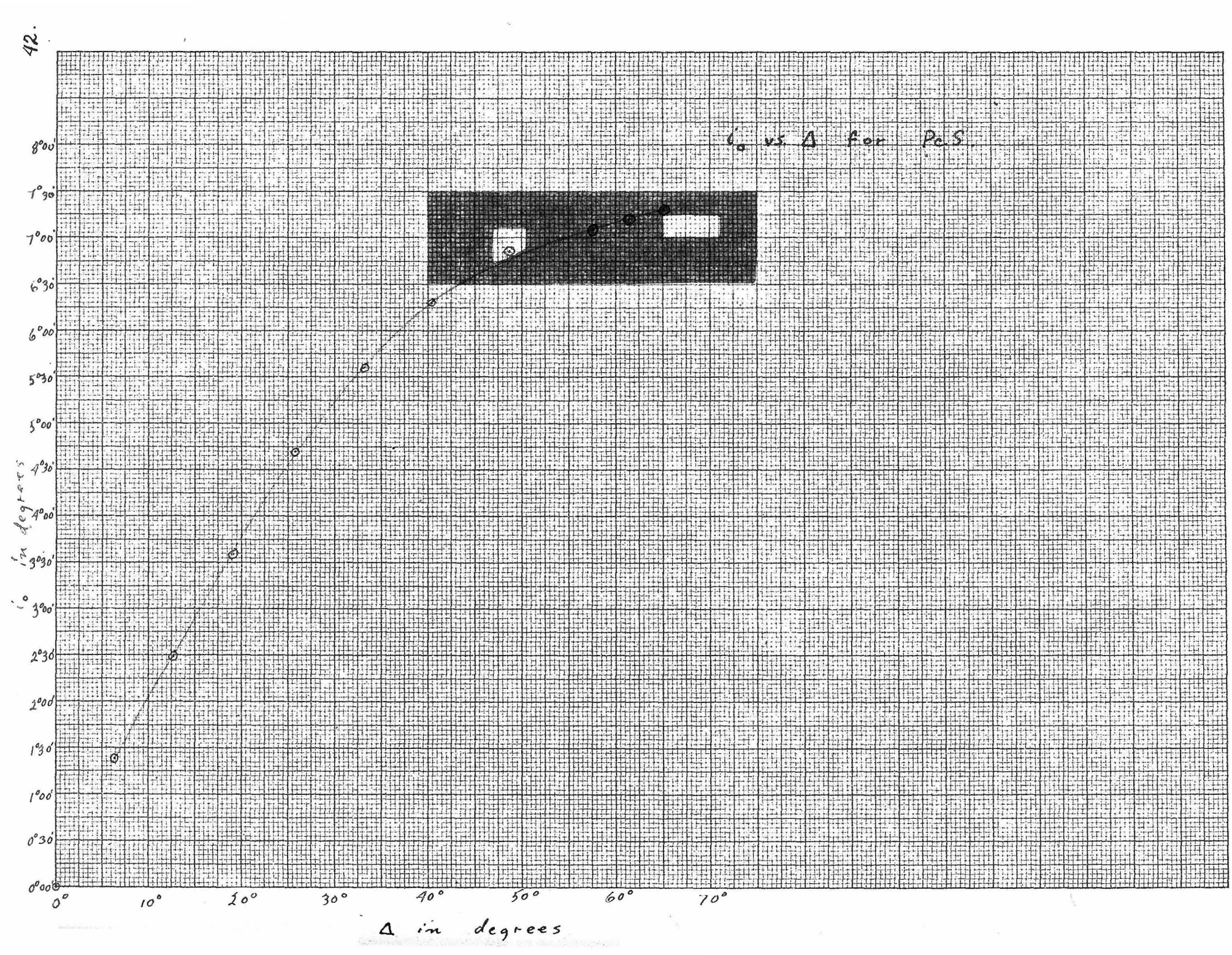
$$(\sin i_c)_s = \frac{7.25}{13.7} (\sin i_c)_p = .529 (\sin i_c)_p .$$

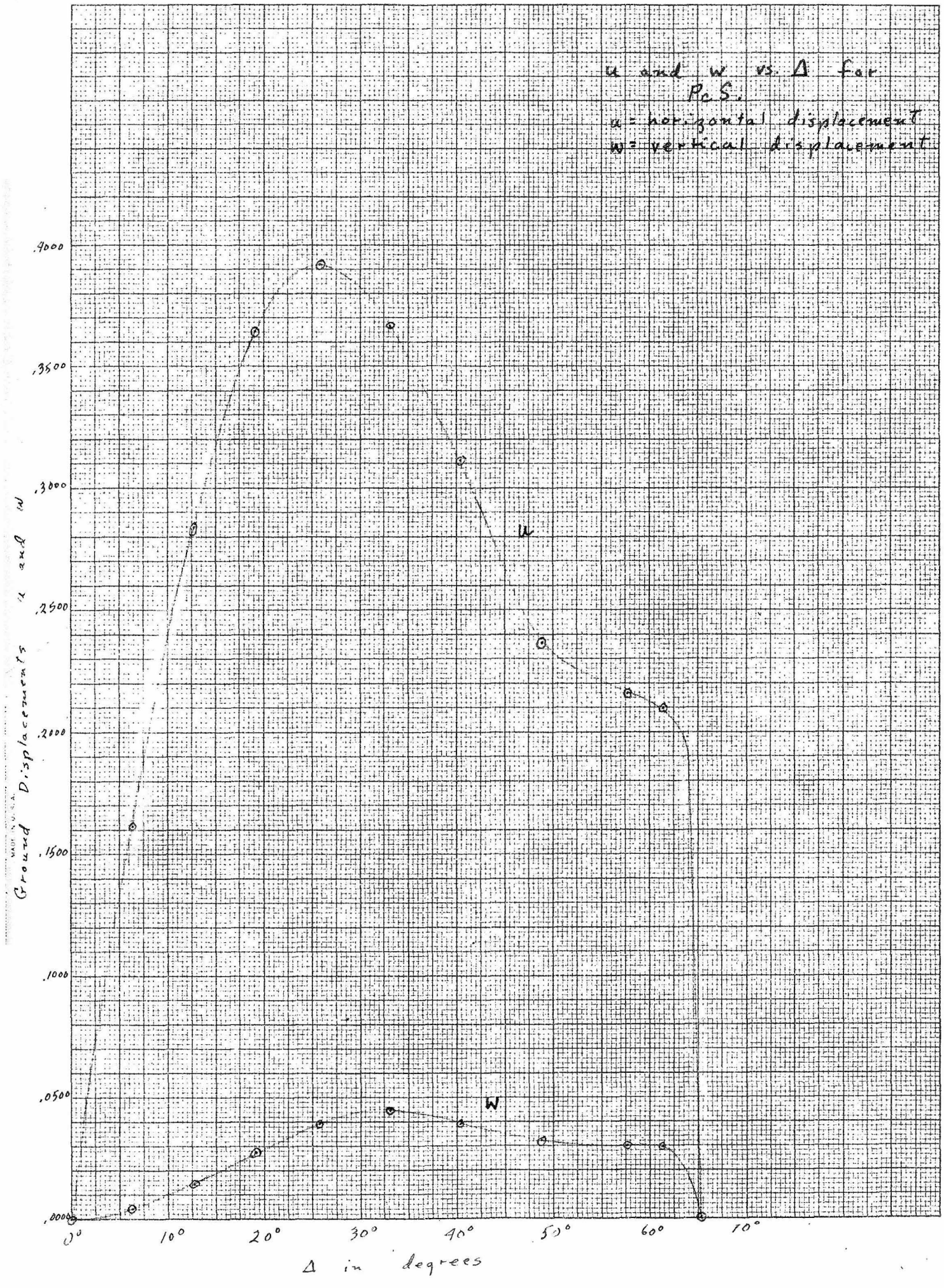
Δ	sin i_o	i_o	tan i_o	[1] $\frac{d\, i_o}{d\Delta}$	[3] $\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{d i_o}{d\Delta}}$	[2] $\mathcal{F}_f$
.00°	.00000	.00°	.0000	.217	.217	.00000
6.40°	.02389	1°22'	.0239	.1945	.2045	.12598
12.70°	.04301	2°28'	.0431	.175	.1892	.23843
19.10°	.06212	3°34'	.0623	.166	.1775	.32673
25.80°	.08213	4°43'	.0825	.139	.1622	.38665
33.10°	.09796	5°37'	.0934	.1145	.1401	.42139
40.40°	.10991	6°19'	.1107	.0760	.1139	.44181
48.80°	.11946	6°52'	.1204	.0419	.0820	.46457
57.70°	.12424	7° 8'	.1252	.0320	.0689	.50732
61.40°	.12567	7°13'	.1266	.0295	.0653	.52043
65.20°	.12663	7°17'	.1278	.0265	.0611	.00000

Δ	[4] $A = CTf\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{d i_o}{d\Delta}}$	[5] $\mathcal{F}_u/A$	[5] $\mathcal{F}_w/A$	[6] $\mathcal{F}_u$	[6] $\mathcal{F}_w$
.00°	.0000	2.00	0.00	.0000	.0000
6.40°	.0809	2.00	.05	.1618	.00404
12.70°	.1418	2.00	.10	.2836	.01418
19.10°	.1822	2.00	.15	.3644	.02735
25.80°	.1970	1.99	.20	.392	.0394
33.10°	.1852	1.98	.23	.367	.0445
40.40°	.1573	1.975	.25	.311	.03935
48.80°	.1197	1.973	.27	.2365	.03235
57.70°	.1095	1.970	.275	.2157	.03015
61.40°	.1068	1.965	.280	.2100	.0299
65.20°	.0000	1.960	.280	.0000	.0000

---

1 Taken from the plot of i_o against Δ on p. 42.
2 Taken from p. 19, column 2.
3 Calculated as on p. 38.
4 $C = \pi$ and $T = 1$ sec.
5 Taken from Reflections and Refractions 1943, graphs of Dr. Gutenberg (copy on p. 10).
6 A plot of u and w against Δ is on p. 43.


ϵ_0 vs. Δ for Pc.S.
8°00'
7°30'
7°00'
6°30'
6°00'
5°30'
5°00'
4°30'
4°00'
3°30'
3°00'
2°30'
2°00'
1°30'
1°00'
0°30'
0°00
ϵ_0 in degrees
0°
10°
20°
30°
40°
50°
60°
70°
Δ in degrees


u and w vs. Δ for P.C.S.
u = horizontal displacement
w = vertical displacement
.4000
.3500
.3000
.2500
.2000
.1500
.1000
.0500
.0000
Ground Displacements u and w
u
w
0°
10°
20°
30°
40°
50°
60°
70°
Δ in degrees

3. ScS

a. SV type

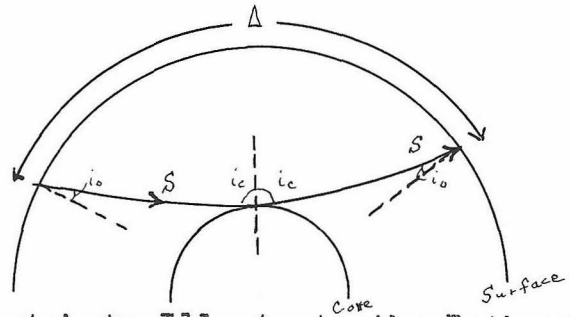


Fig. 10. Sketch to Illustrate the Path of ScS.

i_c	sin i_c	[1] Δ	[2] sin i_o	i_o	tan i_o	[3] $\frac{d\, i_o}{d\, \Delta}$
0°	.00	0.0°	.00000	0°	.0000	.286
10°23'	.18	8.8°	.04301	2°28'	.0431	.279
21°47'	.37	18.6°	.08840	5°04'	.0887	.246
30°00'	.50	26.2°	.11946	6°52'	.1204	.213
31°57'06"	.53	28.0°	.12663	7°17'	.1278	.202
40°00'	.64	35.4°	.15291	8°48'	.1548	.179
50°00'	.77	46.6°	.18397	10°36'	.1871	.145
60°00'	.87	58.8°	.20787	12°00'	.2126	.100
64°59'30"	.91	65.4°	.21742	12°33'	.2226	.0725
70°00'	.94	72.6°	.22459	12°59'	.2306	.0490
80°00'	.98	88.0°	.23415	13°33'	.2410	.0265
90°00'	1.00	101.0°	.23892	13°49'	.2459	.0155

---

[1] Taken from Table 7, p. 103 of Gutenberg and Richter (9).

[2] V_s at surface of core = 7.25 km. per sec. V_s at surface of earth = 3.20 km. per sec. r_o = radius surface of earth = 6,366 km. r_c = radius surface of core = 3,446 km.

$$\frac{r_o \sin i_o}{(V_s)_o} = \frac{r_c \sin i_c}{(V_s)_c},$$

$$\sin i_o = \frac{3.2}{7.25} \times \frac{3,446}{6,366} \times \sin i_c = .23892 \sin i_c.$$

[3] From the plot of i_o vs. Δ on p. 46.

Δ	$\sqrt{\frac{\tan i_o}{\sin \Delta}\frac{d\, i_o}{d\, \Delta}}$	[1] $\frac{1}{4}f$	$^2A = CTf\sqrt{\frac{\tan i_o}{\sin \Delta}\frac{d\, i_o}{d\, \Delta}}$
0.00°	[5].286	1.00000	.899
8.80°	.280	.93570	.825
18.60°	.262	.75438	.621
26.20°	.242	.63789	.485
28.00°	.2345	1.00000	.738
35.40°	.2190	.39672	.274
46.60°	.1930	.43826	.266
58.80°	.158	.63337	.314
65.40°	.134	1.00000	.422
72.60°	.1085	1.00005	.342
88.00°	.0800	1.00000	.251
101.00°	.0629	1.00000	.196

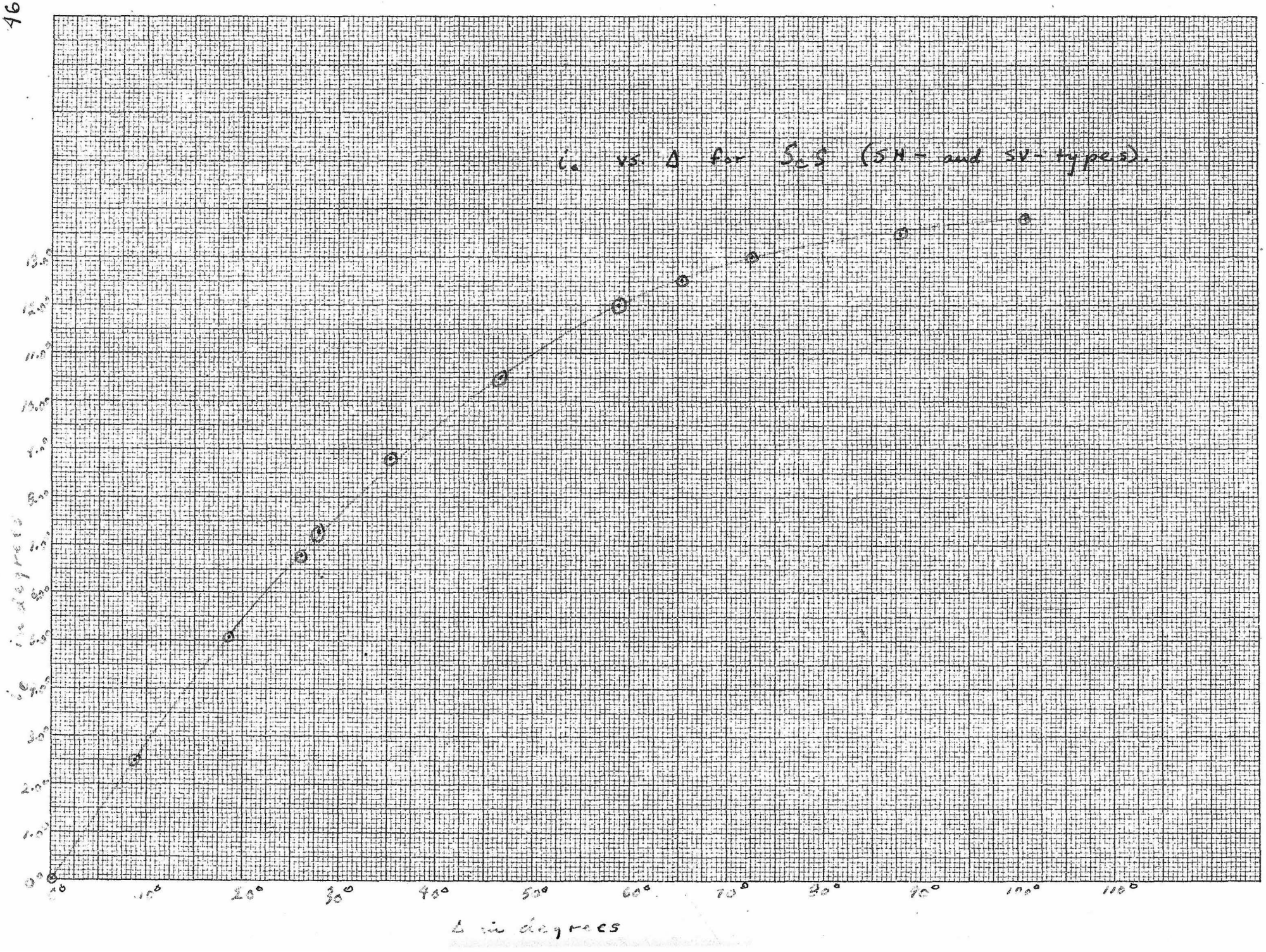
Δ	[3] $\frac{u}{A}$	[3] $\frac{w}{A}$	[4] $\frac{4}{A}u$	[4] $\frac{4}{A}w$
0.00°	2.00	.00	1.798	.0000
8.80°	2.00	.10	1.650	.0825
18.60°	1.985	.20	1.232	.1241
26.20°	1.980	.25	.961	.1213
28.00°	1.975	.275	1.459	.2045
35.40°	1.970	.350	.540	.0960
46.60°	1.940	.440	.516	.117
58.80°	1.910	.500	.600	.157
65.40°	1.900	.520	.803	.219
72.60°	1.890	.540	.646	.1845
88.00°	1.880	.560	.472	.1405
101.00°	1.875	.570	.368	.1118

---

[1] Taken from p. 31, column 1.
[2] $C = \pi$ and $T = 1$ sec.
[3] Taken from Reflections and Refractions 1943, graphs of Dr. Gutenberg (copy on p. 10).
[4] A plot of u and w against Δ is on p. 47.
[5] For small angles $\frac{\tan i_o}{\sin \Delta} = \frac{i_o^*}{\Delta^*}$, where i_o^* and Δ^* are in radians. for $\Delta \cong 0°$, $\frac{d\, i_o}{d\, \Delta}$ = constant = .286 as may be seen by referring to p. 46.

Hence $\frac{i_o^*}{\Delta^*} = C = .286$,

and $\sqrt{\frac{\tan i_o}{\sin \Delta}\frac{d\, i_o}{d\, \Delta}} = \sqrt{\frac{i_o^*}{\Delta^*}\frac{d\, i_o}{d\, \Delta}} = \sqrt{.286 \times .286} = .286$ for $\Delta \cong 0°$


i_e vs. Δ for S_{KS} (SH- and SV-types).
i_e in degrees
Δ in degrees
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°

u and w vs. Δ for
ScS (SV-type).
u = horizontal displacement.
w = vertical displacement.

Ground Displacements u and w

2.0000
1.9000
1.8000
1.7000
1.6000
1.5000
1.4000
1.3000
1.2000
1.1000
1.0000
.9000
.8000
.7000
.6000
.5000
.4000
.3000
.2000
.1000
.0000

u
w

0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110°

Δ in degrees.

The maximum in the plot of u and w against Δ for ScS (SV type) at $\Delta = 28^\circ$ is due to the total reflection of energy from the core of the earth at this particular Δ. That is, f = 1.00 for ScS (SV) (see graph on p. 33) at the angle of incidence $b = 31^\circ 57' 06''$ at the surface of the core. This value of b occurs for ScS (SV) when $\Delta = 28^\circ$.

3. ScS

b. SH type

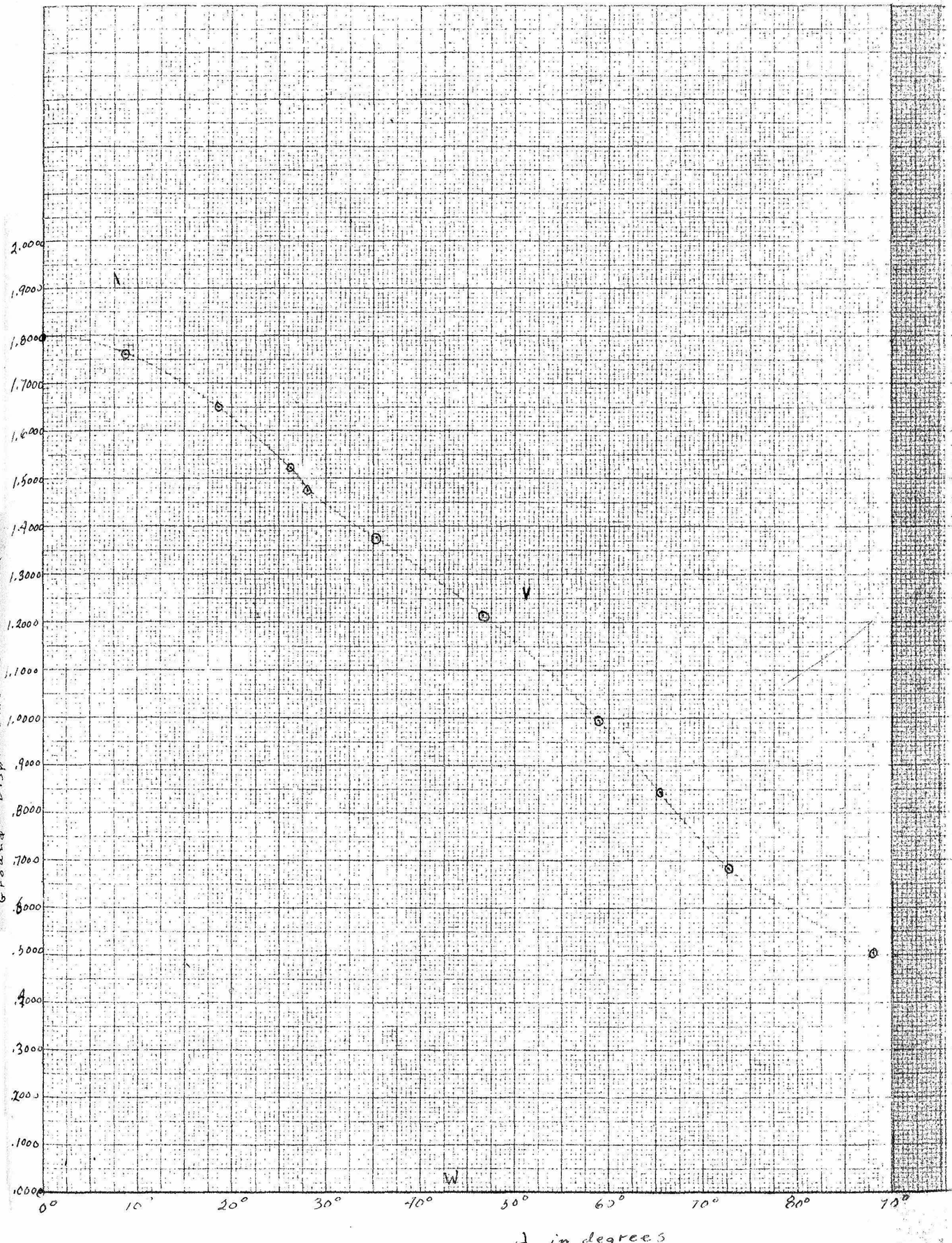
For this case the same i_c, sin i_c, Δ, sin i_o, i_o, cos i_o, and $\frac{di_o}{d\Delta}$ can be used as for ScS of the SV type on p. 44. Also for the SH type, $\frac{W}{A}$ is always 0. Hence W_* is always 0.

Δ	$\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{di_o}{d\Delta}}$	[1] f	[2] $A=CTf\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{di_o}{d\Delta}}$	[3] $\frac{v}{A}$	[4] v
0.00°	.286	1.00	.899	2.00	1.798
8.80°	.280	"	.880	"	1.760
18.60°	.262	"	.825	"	1.650
26.20°	.242	"	.761	"	1.522
28.00°	.2345	"	.738	"	1.476
35.40°	.2190	"	.689	"	1.378
46.60°	.1930	"	.607	"	1.214
58.80°	.1580	"	.497	"	.994
65.40°	.134	"	.422	"	.844
72.60°	.1085	"	.342	"	.684
88.00°	.0800	"	.251	"	.502
101.00°	.0624	"	.196	"	.392

---

1 Refer to p. 34.
2 $C = \pi$, $T = 1$ sec.
3 This value is mentioned on p.9.
4 A plot of v against Δ is on p. 49.

u and w vs. Δ for
SaS (SH-type).
V = horizontal displacement
W = vertical displacement



4. ScP

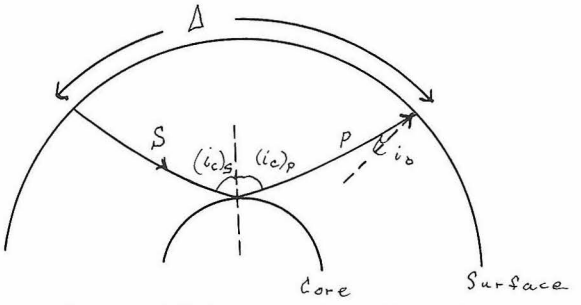


Fig. 11. Sketch to Illustrate the Path of ScP.

[1]$(i_c)_s$	$(\sin i_c)_s$	[2] Δ	[1]$(i_c)_p$	$(\sin i_c)_p$	[2] Δ_p	Δ(Total)
0°	.00	0°	0°	.00	0°	0°
10°23'	.18	4.4°	19°55'	.34	8.3°	12.7°
21°47'	.37	9.3°	44°32'	.70	19.6°	28.9°
30°00'	.50	13.1°	70°53'	.94	35.7°	48.8°
31°00'	.515	13.5°	76°43'	.97	41.0°	54.5°
31°57'06"	.53	14.0°	90°00'	1.00	51.5°	65.5°

Δ	[3]$\sin i_o$	i_o	$\tan i_o$	[4] $\frac{d\, i_o}{d\, \Delta}$
0°	.00000	0°	.0000	.329
12.7°	.07389	4°14'	.0740	.311
28.9°	.16213	8°45'	.1539	.249
48.8°	.20428	11°47'	.2086	.0751
54.5°	.21081	12°10'	.2156	.0464
65.5°	.21732	12°33'	.2226	.0275

---

[1] Taken from p. 30, columns 1 and 6 respectively.
[2] Taken from Table 7, pp. 103-104 of Gutenberg and Richter (9).
[3] V_p at surface = 5.5 km. per sec.
V_p at surface of core = 13.7 km. per sec.
r_s = radius surface of earth = 6,366 km.
r_c = radius surface of core = 3,446 km.

$$\frac{r_s \sin i_o}{(V_p)_s} = \frac{r_c (\sin i_c)_p}{(V_p)_c},$$

$$\sin i_o = \frac{(V_p)_s\, r_c}{(V_p)_c\, r_s} \qquad (\sin i_c)_p = \frac{3446}{6366} \times \frac{5.5}{13.7} (\sin i_c)_p,$$

or $\sin i_o = .21732\ (\sin i_c)_p$.

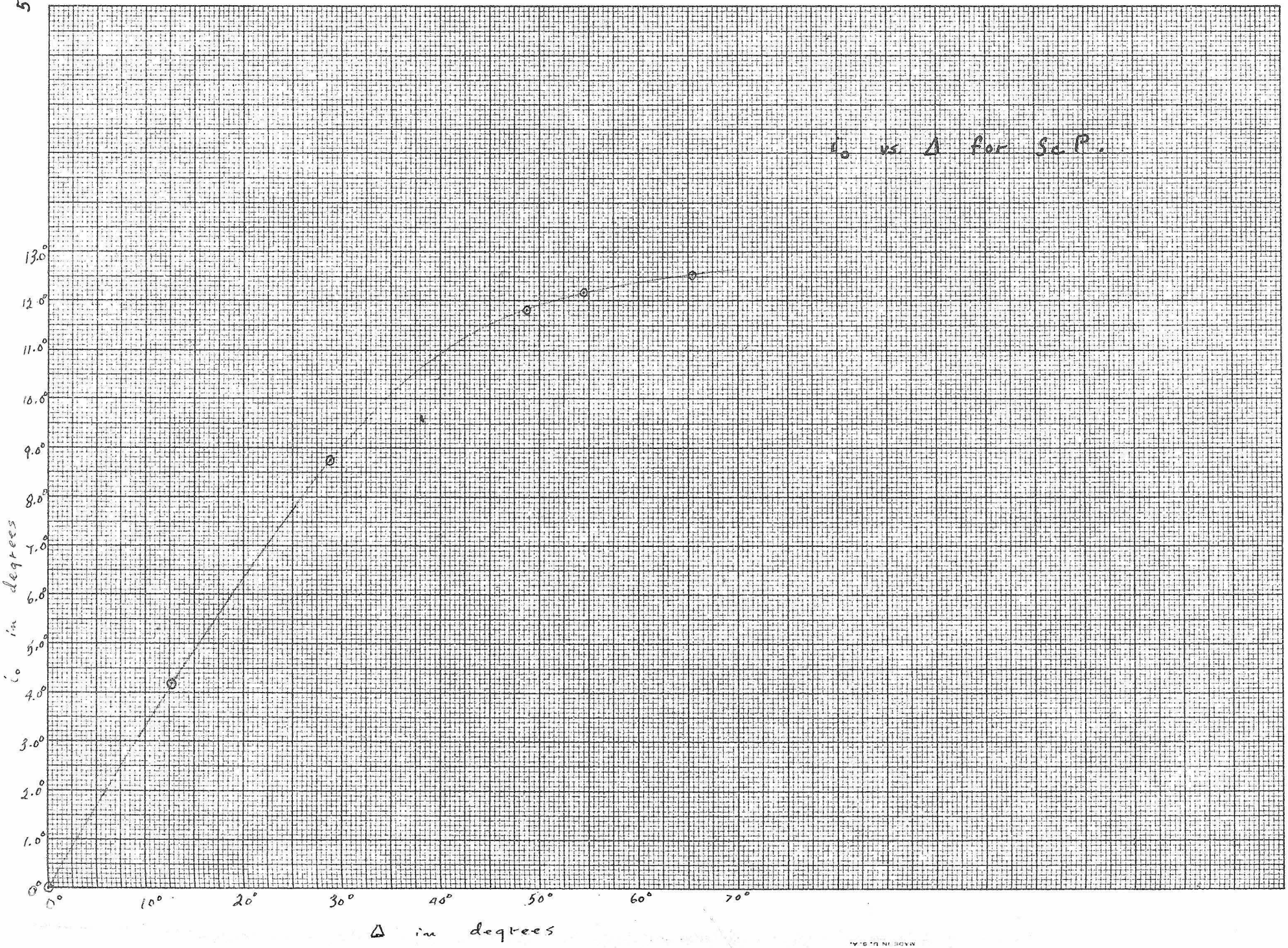
[4] Taken from the plot of i_o against Δ on p. 52.

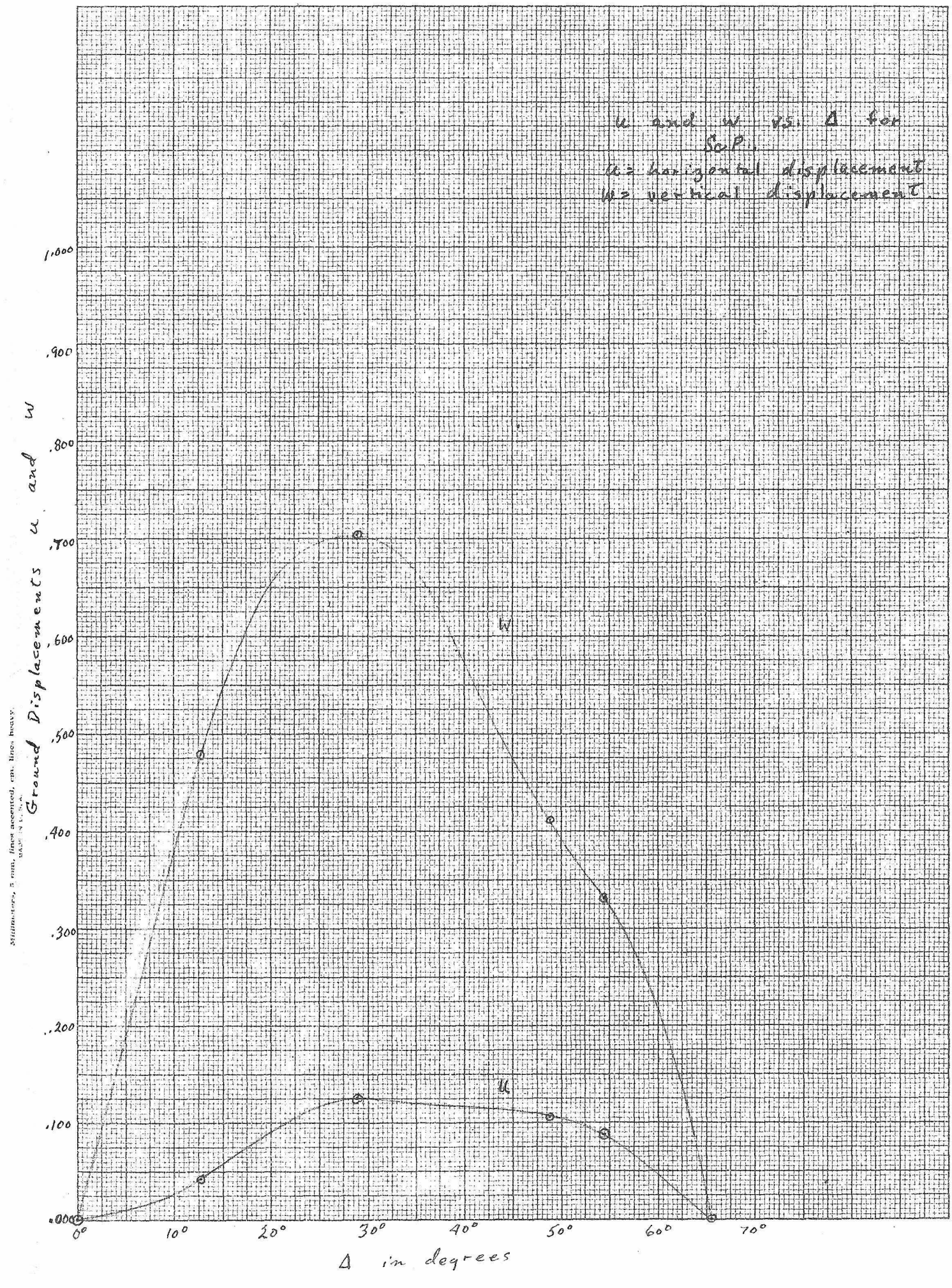
Δ	$\sqrt{\frac{\tan i_o}{\sin \Delta}\frac{d\, i_o}{d\, \Delta}}$ [1]	f [2]	$A = CTf\sqrt{\frac{\tan i_o}{\sin \Delta}\frac{d\, i_o}{d\, \Delta}}$ [3]
0 °	.329	.00000	.0000
12.7°	.323	.23778	.2413
28.9°	.282	.40514	.359
48.8°	.144	.46730	.211
54.5°	.111	.49097	.1712
65.5°	.0824	.00000	.0000

Δ	$\frac{u}{A}$ [4]	$\frac{w}{A}$ [4]	$\mathfrak{S}_u$ [5]	$\mathfrak{S}_w$ [5]
0 °	.00	2.00	.0000	.0000
12.7°	.18	1.98	.0435	.478
28.9°	.35	1.96	.1257	.7045
48.8°	.50	1.95	.1055	.412
54.5°	.51	1.94	.0874	.3325
65.5°	.52	1.93	.0000	.0000

---

1 Calculated as on p. 38.
2 Taken from p. 31, column 2.
3 $C = \pi$ and $T = 1$ sec.
4 Taken from Reflections and Refractions 1943, graphs of Dr. Gutenberg (copy on p. 10).
5 A plot of u and w against Δ is on p. 53.


i_o vs. Δ for Sc P.
13.0°
12.0°
11.0°
10.0°
9.0°
8.0°
7.0°
6.0°
5.0°
4.0°
3.00°
2.0°
1.0°
0°
i_o in degrees
0°
10°
20°
30°
40°
50°
60°
70°
Δ in degrees


u and w vs. Δ for S of P.
u = horizontal displacement
w = vertical displacement
Ground Displacements u and w
1.000
.900
.800
.700
.600
.500
.400
.300
.200
.100
.000
w
u
0°
10°
20°
30°
40°
50°
60°
70°
Δ in degrees

Group II. Waves Transmitted once through the Core.

1. P'(PKP)

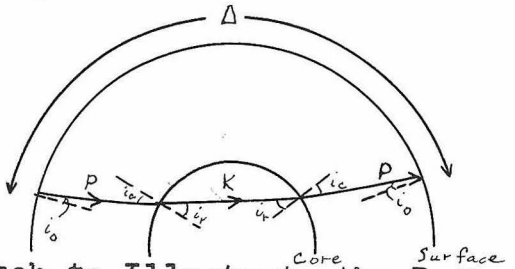


Fig. 12. Sketch to Illustrate the Path of P'(PKP)

A travel-time curve for P'(PKP) is on p. 56. It is plotted from the data of Table 23 on p. 124 of Gutenberg and Richter (9). From this, sin i_o is determined by sin $i_o = \frac{V_o}{\overline{V}_\Delta}$, where V_o is the true velocity of P waves at the surface of the earth (5.5 km. per sec.) and $\overline{V}_\Delta$ is the apparent velocity of P' waves taken from the curve at a particular Δ. Sin i_c is determined from

$$\frac{r_s \sin i_o}{(V_p)_s} = \frac{r_c \sin i_c}{(V_p)_c},$$

$$\sin i_o = \frac{(V_p)_s\, r_c}{(V_p)_c\, r_s} \sin i_c = \frac{3446}{6366} \times \frac{5.5}{13.7} \sin i_c = .21732 \sin i_c.$$

$$\sin i_c = \frac{\sin i_o}{.21732}.$$

In the following tabulation P_2' extends from 180°-142½°, an unnamed segment from 142½°-169°, the reversed segment from 169°-110°, P" from 110°-142½°, and P' or P_1' from 142½°-180°. These branches of PKP are tabulated in that order respectively. Between Δ = 110° and 130°, P" and the reversed segment are too close together to be plotted separately.

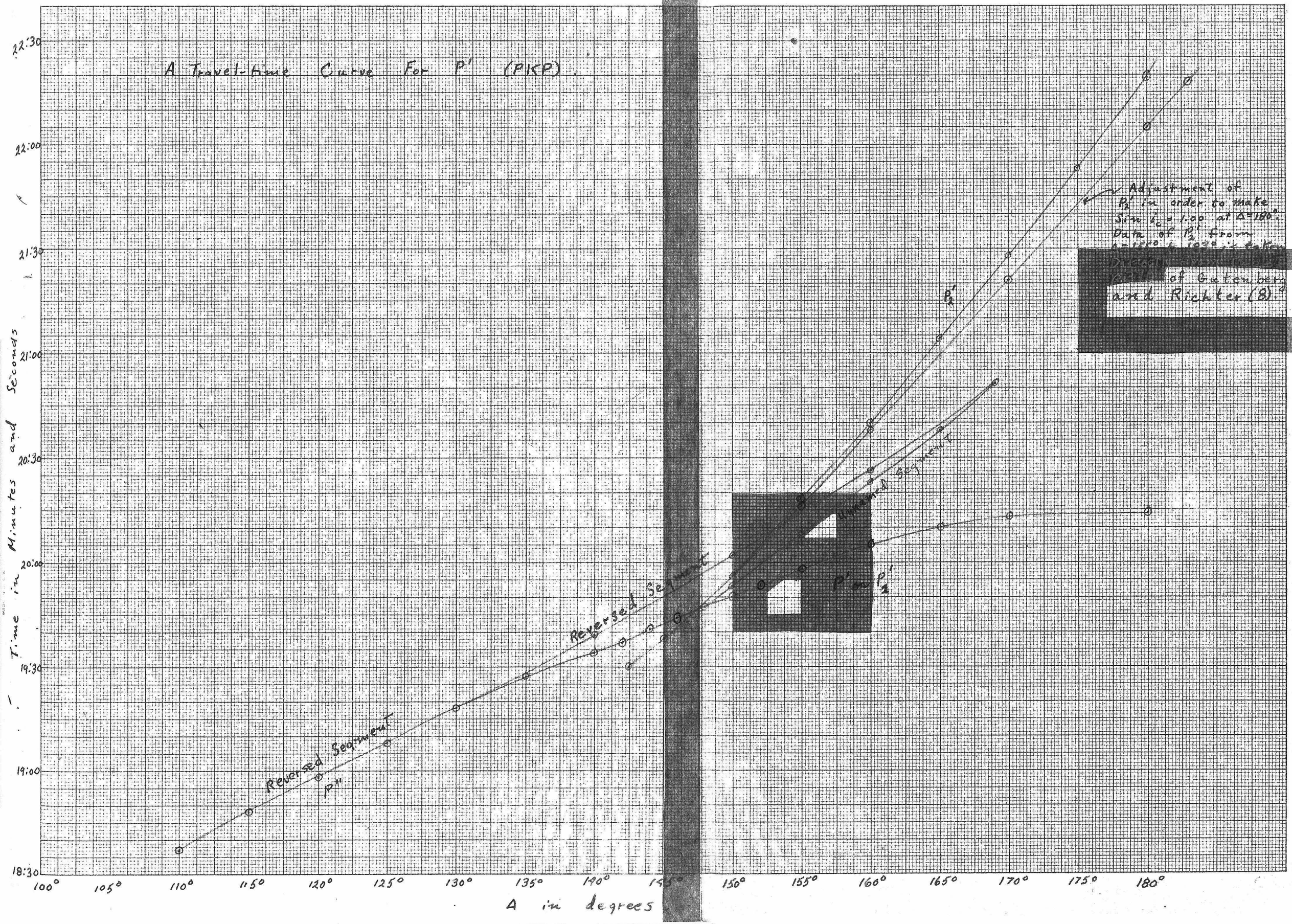
Δ	$\overline{V}_\Delta$	sin i_o	i_o	$\frac{1}{2}I_o$	$\frac{d I_o}{d \Delta}$	tan I_o	sin i_c	i_c
180°	25.38km/sec.	.2163	12°30'	12°30'	.0300	.2217	.998	86°23'
175°	25.63 "	.2146	12°24'	12°24'	.0300	.2199	.989	81°30'
170°	26.05 "	.2110	12°11'	12°11'	.0300	.2159	.974	76°54'
165°	26.30 "	.2091	12°04'	12°04'	.0300	.2138	.964	74°35'
160°	26.62 "	.2064	11°55'	11°55'	.0300	.2110	.952	72°11'
155°	27.03 "	.2037	11°45'	11°45'	.0300	.2080	.939	69°54'
150°	27.40 "	.2010	11°36'	11°22'	.1168	.2010	.907	65°06'
145°	36.65 "	.1633	9°24'	10°14'	.3490	.1805	.819	54°59'
142½°	37.00 "	.1485	8°32'	9°15'	∞	.1629	.740	47°44'
142½°	37.00 "	.1485	8°32'	9°15'	∞	.1629	.740	47°44'
145°	35.25 "	.1558	8°58'	8°44'	.0268	.1536	.700	44°26'
150°	36.55 "	.1502	8°38'	8°38'	.02215	.1518	.692	43°47'
155°	37.35 "	.1472	8°28'	8°28'	.01768	.1489	.689	43°33'
160°	37.80 "	.1455	8°22'	8°22'	.01323	.1471	.670	42°04'

[1] I_o is taken from the smoothed curve of i_o vs. Δ on p. 57.

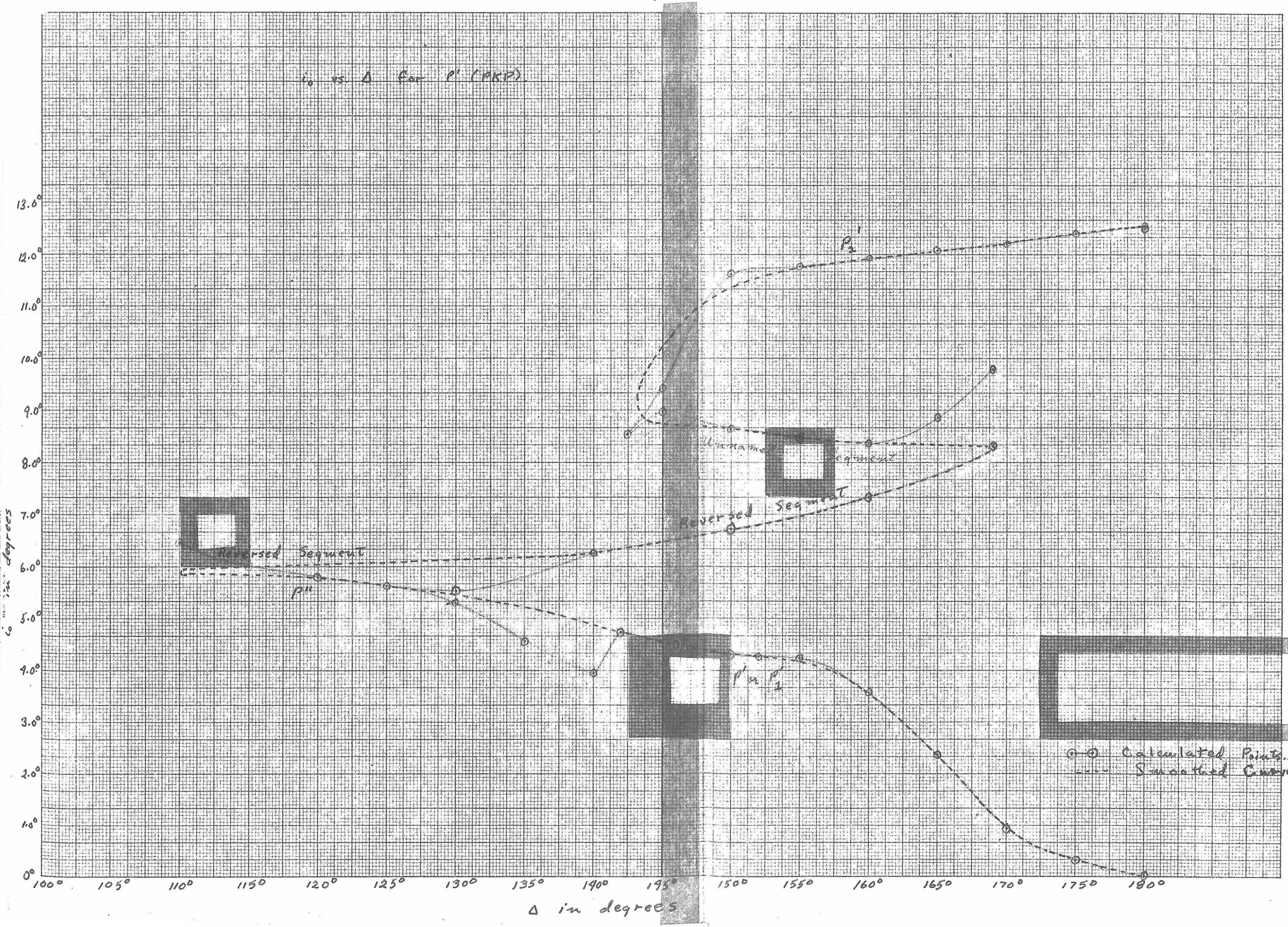
Δ	$\bar{V}_\Delta$	sin i_o	i_o	I_o	$^1\frac{d I_o}{d \Delta}$	tan I_o	^{2}sini_c	i_c
165°	35.70km/sec.	.1540	8°52'	8°20'	.00664	.1465	.668	41°55'
169°	32.33 "	.1699	9°47'	8°18'	∞	.1459	.664	41°36'
169°	38.10 "	.1442	8°18'	8°18'	∞	.1459	.664	41°36'
160°	43.10 "	.1276	7°20'	7°20'	.0848	.1287	.588	36°01'
150°	46.95 "	.1171	6°43'	6°43'	.0498	.1178	.540	32°41'
140°	50.45 "	.1090	6°15'	6°15'	.0333	.1095	.502	30°08'
130°	57.25 "	.0962	5°31'	6°08'	.01002	.1075	.492	29°28'
120°	54.40 "	.1010	5°48'	6°02'	.01002	.1057	.484	28°57'
110°	49.00 "	.1122	6°27'	5°53'	∞	.1031	.472	28°10'
110°	49.00 "	.1122	6°27'	5°53'	∞	.1031	.472	28°10'
115°	52.80 "	.1042	5°59'	5°50'	.01002	.1022	.468	27°54'
120°	54.40 "	.1010	5°48'	5°48'	.02498	.1016	.465	27°43'
125°	56.25 "	.0979	5°37'	5°37'	.0350	.0983	.451	26°48'
130°	59.55 "	.0924	5°18'	5°28'	.0416	.0957	.438	25°59'
135°	69.40 "	.0794	4°33'	5°13'	.0550	.0913	.419	24°46'
140°	79.40 "	.0693	3°58'	4°52'	.0718	.0851	.391	23°01'
142°	66.90 "	.0824	4°44'	4°44'	.0664	.0828	.379	22°16'
146°	69.80 "	.0788	4°31'	4°31'	.0550	.0790	.3625	21°15'
150°	73.50 "	.0749	4°18'	4°18'	.0333	.0752	.3450	20°11'
152°	74.05 "	.0743	4°16'	4°16'	.0316	.0746	.3415	19°58'
155°	74.50 "	.0738	4°14'	4°10'	.0567	.0729	.335	19°34'
160°	88.90 "	.0619	3°33'	3°33'	.955	.0620	.285	16°34'
165°	132.80 "	.0414	2°22'	2°22'	.2639	.0413	.191	10°59'
170°	336.50 "	.0163	0°56'	0°56'	.2165	.0163	.0751	4°18'
175°	1000.00 "	.0055	0°19'	0°19'	.0854	.0055	.0253	1°27'
180°	∞ "	.0000	0°00'	0°00'	.0665	.0000	.0000	0°00'

---

1 Taken from the plot of i_o vs. Δ on p. 57.
2 Sin i_c is calculated from sin I_o.


A Travel-time Curve For P' (PKP).
Time in Minutes and Seconds
22:30
22:00
21:30
21:00
20:30
20:00
19:30
19:00
18:30
100°
105°
110°
115°
120°
125°
130°
135°
140°
145°
150°
155°
160°
165°
170°
175°
180°
Δ in degrees
Reversed Segment
P''
Reversed Segment
P'
Unnamed Segment
P'₂
Adjustment of P₂' in order to make Sin i_c = 1.00 at Δ=180°. Data of P₂' from
of Gutenberg
and Richter (8).

i_0 vs. Δ for ρ' (PKP)



Δ	[1] f_1	[2] i_r	[3] f_2	[4] f	$A = CTf\sqrt{\frac{\tan i_c di_c}{\sin \Delta d\Delta}}$ [5]	$\frac{u}{A}$ [6]	$\frac{w}{A}$ [6]	u [7]	w [7]
180°	.832	35°35'	.360	.299	∞	.530	1.930	∞	∞
175°	.844	35°14'	.760	.691	.555	.525	1.935	.291	1.072
170°	.817	34°37'	.828	.676	.412	.520	1.940	.2145	.801
165°	.800	34°12'	.820	.656	.326	.520	1.940	.1695	.632
160°	.793	33°47'	.800	.634	.271	.515	1.945	.1397	.527
155°	.788	33°14'	.786	.620	.238	.510	1.950	.1218	.465
150°	.784	31°56'	.786	.615	.419	.500	1.955	.2095	.820
145°	.805	28°33'	.805	.648	.669	.470	1.960	.3145	1.312
142½°	.835	25°36'	.839	.701	∞	.410	1.965	∞	∞
142½°	.835	25°36'	.839	.701	∞	.410	1.965	∞	∞
145°	.850	24°09'	.855	.726	.1908	.400	1.970	.0764	.376
150°	.854	23°50'	.858	.732	.1878	.390	1.975	.0734	.371
155°	.855	23°42'	.859	.735	.1832	.390	1.975	.0715	.362
160°	.863	23°01'	.866	.747	.1780	.385	1.980	.0686	.353
165°	.864	22°57'	.867	.749	.1445	.385	1.980	.0556	.286
169°	.865	22°46'	.868	.751	∞	.385	1.980	∞	∞
169°	.865	22°46'	.868	.751	∞	.385	1.980	∞	∞
160°	.897	20°04'	.894	.803	.451	.320	1.985	.1445	.896
150°	.914	18°22'	.907	.829	.283	.290	1.985	.0822	.562
140°	.925	17°02'	.918	.849	.2015	.280	1.985	.0564	.400
130°	.928	16°41'	.920	.854	.1008	.280	1.985	.0282	.200
120°	.930	16°25'	.922	.856	.0941	.275	1.985	.0258	.1865
110°	.933	16°00'	.926	.865	∞	.275	1.985	∞	∞
110°	.933	16°00'	.926	.865	∞	.275	1.985	∞	∞
115°	.934	15°52'	.927	.866	.0919	.275	1.985	.0253	.1820
120°	.935	15°43'	.928	.869	.1488	.275	1.985	.0409	.2955
125°	.938	15°15'	.931	.874	.1790	.260	1.990	.0466	.3570
130°	.942	14°50'	.935	.880	.2005	.250	1.990	.0502	.399
135°	.947	14°11'	.942	.892	.236	.240	1.990	.0566	.470

[1] f_1 is the square root of energy refracted into the core at the angle i_c and is taken from the plot of $\sqrt{\frac{E_f P}{E_i P}}$ against angle of incidence on p. 21.

[2] $\frac{\sin i_r}{8.0} = \frac{\sin i_c}{13.7}$ or $\sin i_r = .584 \sin i_c$.

[3] f_2 is the square root of energy refracted into the mantle at the angle i_r and is taken from the plot of $\sqrt{\frac{E_f P}{E_i P}}$ against angle of incidence on p. 27.

[4] $f = f_1 f_2$.

[5] $C = \pi$ and $T = 1$ sec.

[6] Taken from Reflections and Refractions 1943, graphs of Dr. Gutenberg (p.10).

[7] A plot of u and w against Δ is on p. 61 and p. 62.

Δ	f_1	i_r	f_2	f	$A=CTf\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{di_o}{d\Delta}}$	$\frac{u}{A}$	$\frac{w}{A}$	u	w
140°	.954	13°11'	.948	.904	.2775	.210	1.990	.0584	.554
142°	.957	12°46'	.952	.911	.272	.200	1.990	.0544	.542
146°	.960	12°14'	.956	.919	.256	.200	1.990	.0513	.510
150°	.963	11°37'	.960	.925	.2065	.195	1.990	.0404	.412
152°	.964	11°29'	.961	.926	.2075	.195	1.990	.0409	.414
155°	.966	11°16'	.963	.931	.2910	.195	1.990	.0568	.580
160°	.975	9°35'	.973	.949	.562	.170	1.995	.0956	1.122
165°	.988	6°22'	.989	.976	.633	.120	1.995	.0760	1.262
170°	.996	2°31'	.998	.995	.446	.060	2.000	.0268	.892
175°	.998	0°51'	.999	.996	.231	.040	2.000	.00924	.461
180°	.999	0°00'	.999	.998	[1].2085	.000	2.000	.0000	.417

[1] At $\Delta = 180°$, $\tan i_o = 0$. Hence the value of A for P'_1 at this point must be approximated in the following manner:

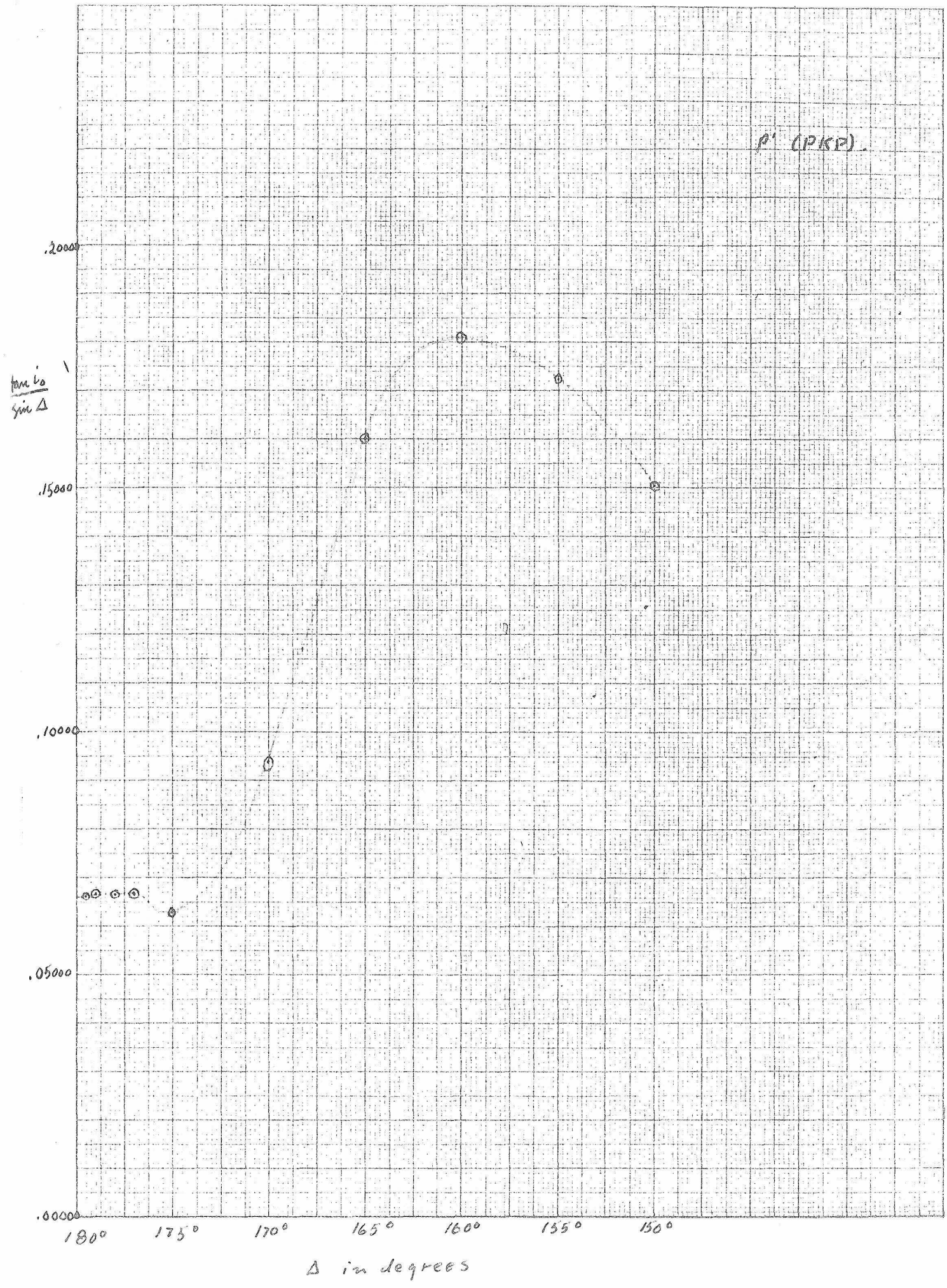
Δ	$\frac{\tan I_o}{\sin}$
150°	.1501
155°	.1725
160°	.1810
165°	.1600
170°	.0939
175°	.0631
177°	.0668
179°	.0667
178°	.0668
179°30'	.0664

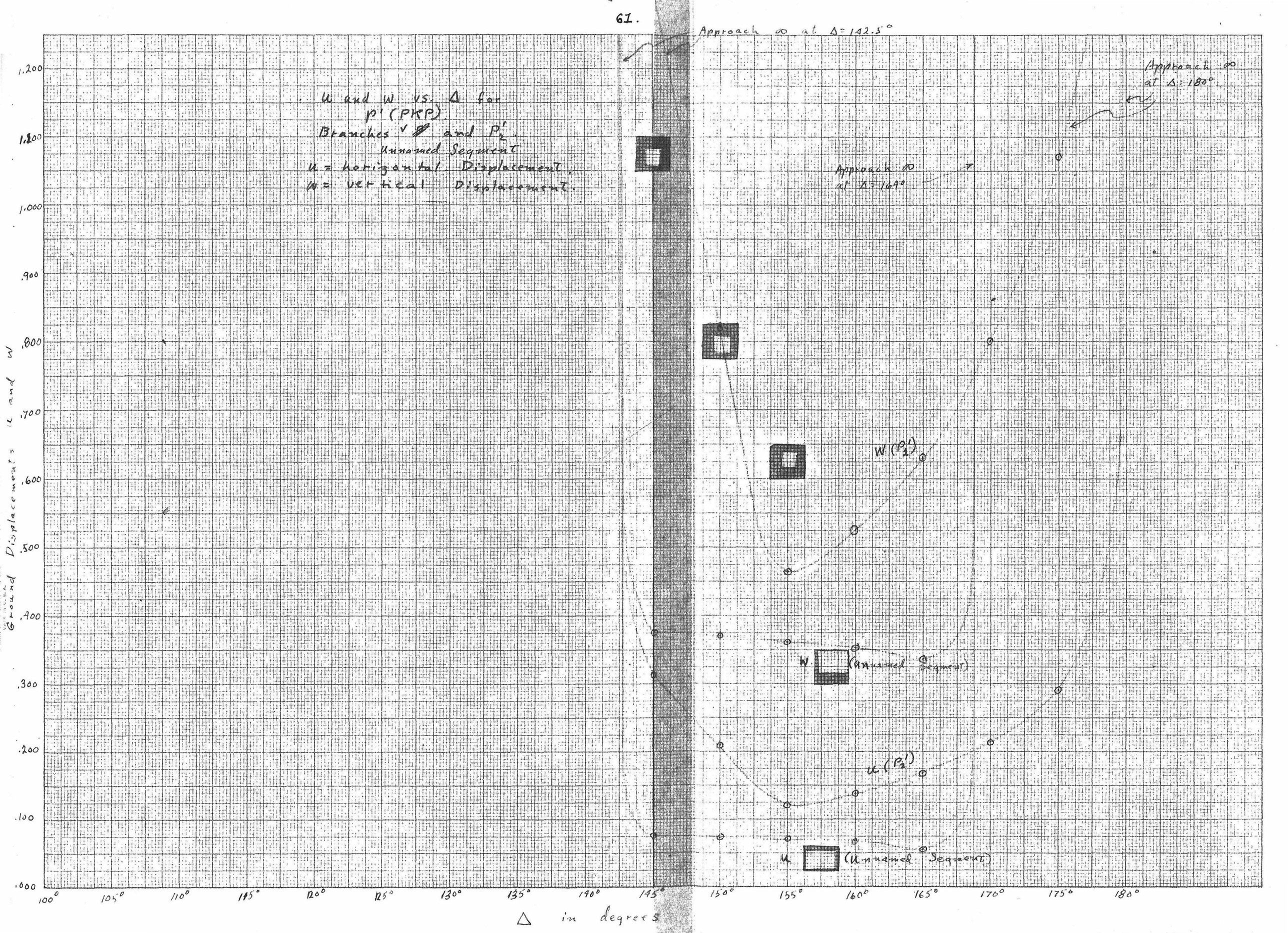
A plot of these values is on p. 60.

It is evident that in the vicinity of $\Delta = 180°$, the ratio $\frac{\tan I_o}{\sin \Delta}$ approaches a constant value .0665. Hence that value will be used in the calculation of A at $\Delta = 180°$. This gives

$$A = CTf\sqrt{.0665 \times .0665} = CTf\ .0665 = .2085\ ,$$

It should be pointed out that amplitudes of P' or P'_1, especially P", depend to a great extent upon the change of velocity with depth $\left(\frac{dV}{dr}\right)$ inside the core, which is only approximately known. There is still no final solution to the problem of whether V changes abruptly or gradually, though very fast, at a depth of 5,000 km.


P' (PKP)
.20000
.15000
.10000
.05000
.00000
dt/d Δ sin Δ
180°
175°
170°
165°
160°
155°
150°
Δ in degrees



Approach ∞ at Δ = 142.5°
Approach ∞ at Δ = 180°
Approach ∞ at Δ = 169°
u and w vs. Δ for p' (PKP)
Branches ✓ and P'₂
Unnamed Segment
u = horizontal Displacement.
w = vertical Displacement.
W (P'₂)
W (Unnamed Segment)
u (P'₂)
u (Unnamed Segment)
Ground Displacements u and w
1.200
1.100
1.000
.900
.800
.700
.600
.500
.400
.300
.200
.100
.000
100°
105°
110°
115°
120°
125°
130°
135°
140°
145°
150°
155°
160°
165°
170°
175°
180°
Δ in degrees

u and w vs. Δ for P' (PKP).
Brackets P'' and P_1' or P' and the Reversed Segment.

Approach ∞ at Δ = 110°

Approach ∞ at Δ = 169°

1,262

W (reversed Segment)

U (reversed Segment)

W (P'' and P_1' or P')

U (P'' and P_1' or P')

Ground Displacements in U and W

Δ in degrees

2. PKS

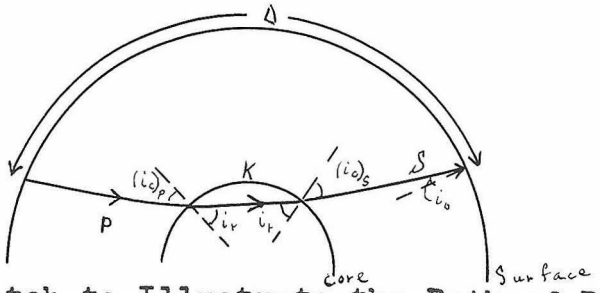


Fig. 13. Sketch to Illustrate the Path of PKS

A travel-time curve for PKS and SKP is on p. 64. It is plotted from the data of Table 25 on p. 126 of Gutenberg and Richter (9). From this, sin i_o is determined by $\sin i_o = \frac{V_o}{\bar{V}_\Delta}$, where V_o is the true velocity of S waves at the surface of the earth (3.2 km. per sec.) and $\bar{V}_\Delta$ is the apparent velocity of PKS waves at the surface taken from the curve at a particular Δ. $\sin(i_c)_s$ is determined from

$$\frac{r_s \sin i_o}{(V_s)_s} = \frac{r_c \sin(i_c)_s}{(V_s)_c},$$

$$\text{or } \sin i_o = \frac{(V_s)_s\, r_c}{(V_s)_c\, r_s} \sin(i_c)_s = \frac{3.2}{7.25} \times \frac{3,446}{6,366} \sin(i_c)_s,$$

$$\text{or } \sin i_o = .23892 \sin(i_c)_s .$$

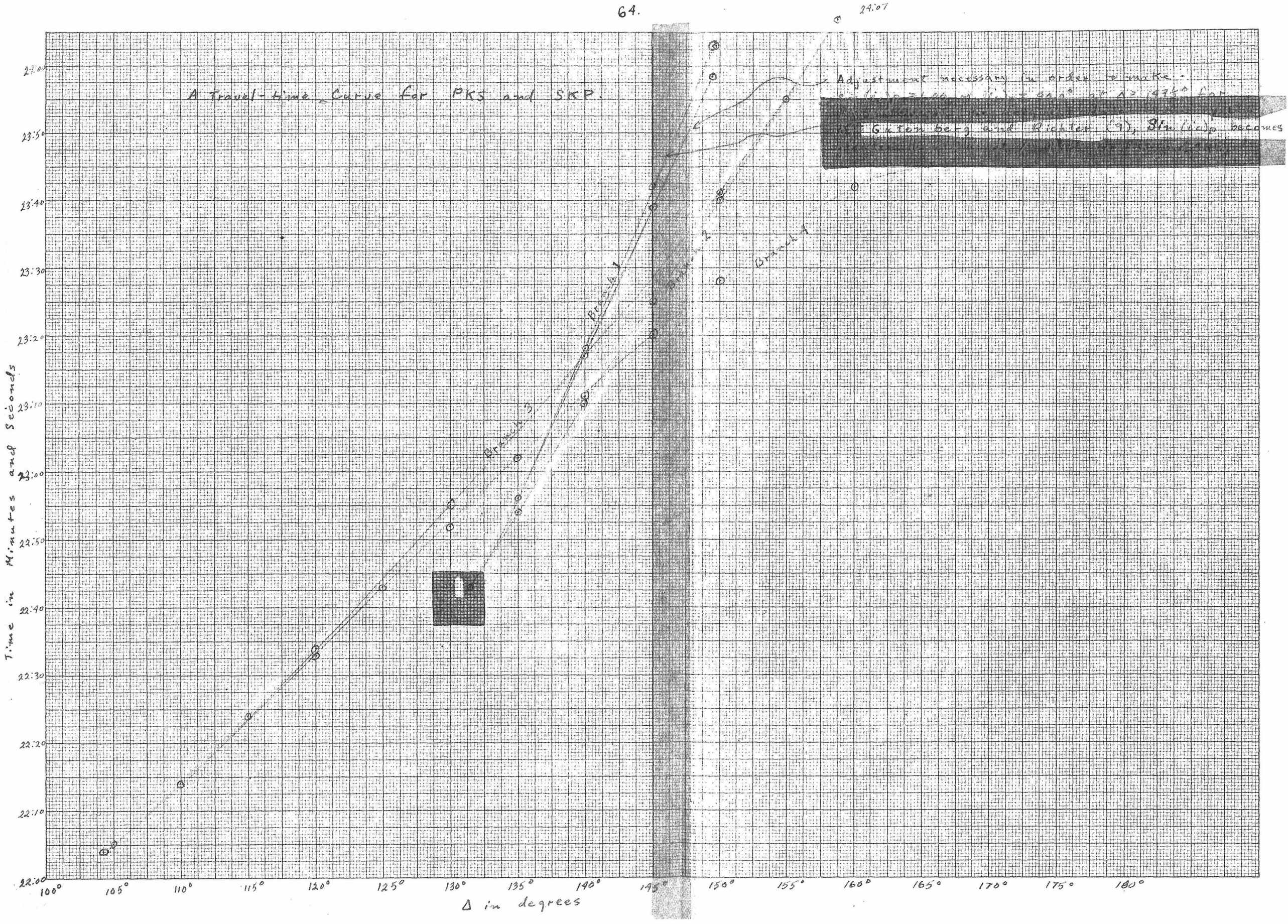
$$\text{Hence } \sin(i_c)_s = \frac{\sin i_o}{.23892} .$$

Δ	$\bar{V}_\Delta$	sin i_o	i_o	tan i_o	$\frac{d\, i_o}{d\, \Delta}$ [1]	sin $(i_c)_s$	$(i_c)_s$	[2] sin i_r	i_r
144½°	25.3km/sec.	.1262	7°15'	.1272	.00834	.529	31°56'	.584	35°44'
145°	25.5 "	.1253	7°12'	.1263	.01248	.524	31°36'	.578	35°19'
140°	25.8 "	.1239	7°07'	.1249	.01415	.518	31°12'	.572	34°53'
135°	26.1 "	.1225	7°02'	.1234	.0717	.513	30°52'	.566	34°28'
131½°	34.7 "	.0924	5°18'	.0928	∞	.386	22°42'	.426	25°13'
131½°	34.7 "	.0924	5°18'	.0928	∞	.386	22°42'	.426	25°13'
135°	35.01 "	.0915	5°15'	.0919	.01835	.382	22°27'	.421	24°54'
140°	35.70 "	.0898	5°09'	.0901	.01668	.375	22°03'	.414	24°27'
145°	36.80 "	.0871	5°00'	.0875	.01499	.364	21°21'	.401	23°38'
150°	37.00 "	.0866	4°58'	.0869	.0133	.362	21°13'	.399	23°31'
155°	37.25 "	.0860	4°56'	.0863	.00834	.359	21°02'	.396	23°20'

1 Taken from a plot of i_o against Δ on p. 66.

2 $\frac{\sin i_r}{8.00} = \frac{\sin(i_c)_s}{7.25}$,

$$\sin i_r = \frac{8.00}{7.25} \sin(i_c)_s = 1.102 \sin(i_c)_s .$$


A Travel-time Curve for PKS and SKP.
Adjustment necessary in order to make
Gutenberg and Richter (9), Sin (ic)p becomes
Branch 1
Branch 2
Branch 3
Branch 4
24:07
24:00
23:50
23:40
23:30
23:20
23:10
23:00
22:50
22:40
22:30
22:20
22:10
22:00
Time in Minutes and Seconds
100°
105°
110°
115°
120°
125°
130°
135°
140°
145°
150°
155°
160°
165°
170°
175°
180°
Δ in degrees

Δ		$\bar{V}_\Delta$	sin i_o	i_o	tan i_o	$\frac{d\, i_o}{d\, \Delta}$	sin$(i_c)_s$	$(i_c)_s$	sini_r	i_r
158 3/4°	37.63	km/sec.	.0851	4°53'	.0854	∞	.3555	20°49'	.392	23°05'
158 3/4°	37.63	"	.0851	4°53'	.0854	∞	.3555	20°49'	.392	23°05'
150°	41.75	"	.0767	4°24'	.0770	.0626	.3205	18°42'	.3535	20°42'
140°	49.10	"	.0652	3°44'	.0653	.0325	.272	15°47'	.3000	17°27'
130°	51.80	"	.0618	3°33'	.0620	.01668	.2585	14°59'	.285	16°34'
120°	53.80	"	.0595	3°25'	.0597	.00751	.2485	14°23'	.274	15°54'
110°	55.00	"	.0582	3°20'	.0582	.00083	.2430	14°04'	.268	15°33'
104¼°	55.20	"	.0580	3°19.5'	.0581	∞	.2423	14°01'	.2675	15°31'
104¼°	55.20	"	.0580	3°19.5'	.0581	∞	.2423	14°01'	.2675	15°31'
110°	55.50	"	.0577	3°18'	.0577	.00167	.241	13°57'	.266	15°26'
115°	55.80	"	.0574	3°17'	.0574	.00333	.240	13°53'	.2645	15°20'
120°	56.10	"	.0571	3°16'	.0571	.00467	.2385	13°48'	.263	15°15'
125°	56.60	"	.0565	3°14'	.0565	.00834	.236	13°39'	.260	15°04'
130°	58.10	"	.0551	3°10'	.0553	.01582	.230	13°18'	.254	14°43'
135°	59.40	"	.0539	3°05'	.0539	.0242	.2255	13°02'	.298	14°22'
140°	61.70	"	.0519	2°59'	.0521	.0317	.217	12°32'	.239	13°50'
145°	67.70	"	.0473	2°43'	.0475	.0350	.1978	11°25'	.218	12°36'
150°	72.10	"	.0444	2°33'	.0445	.0467	.1858	10°42'	.205	11°50'
155°	76.60	"	.0418	2°24'	.0419	.0784	.1748	10°04'	.1928	11°07'
160°	111.00	"	.0288	1°39'	.0288	.1883	.1203	6°55'	.1327	7°38'
170°	926.00	"	.00346	0°12'	.00349	.0492	.0145	0°50'	.01599	0°55'
180°		"	.00000	0°00'	.00000	.0100	.0000	0°00'	.00000	0°00'

Note: In the above data, from $\Delta = 149\frac{1}{2}^\circ$ to $131\frac{1}{2}^\circ$ constitutes branch 1 of the PKS - SKP travel-time curve, from $\Delta = 131\frac{1}{2}^\circ$ to 158 3/4° constitutes branch 2, from Δ = 158 3/4° to $\Delta = 104\frac{1}{4}^\circ$, constitutes branch 3, and from $\Delta = 104\frac{1}{4}^\circ$ to 180° constitutes branch 4. These numbers 1,2,3,4 were assigned arbitrarily to the branches of the travel-time curve on p. 64.

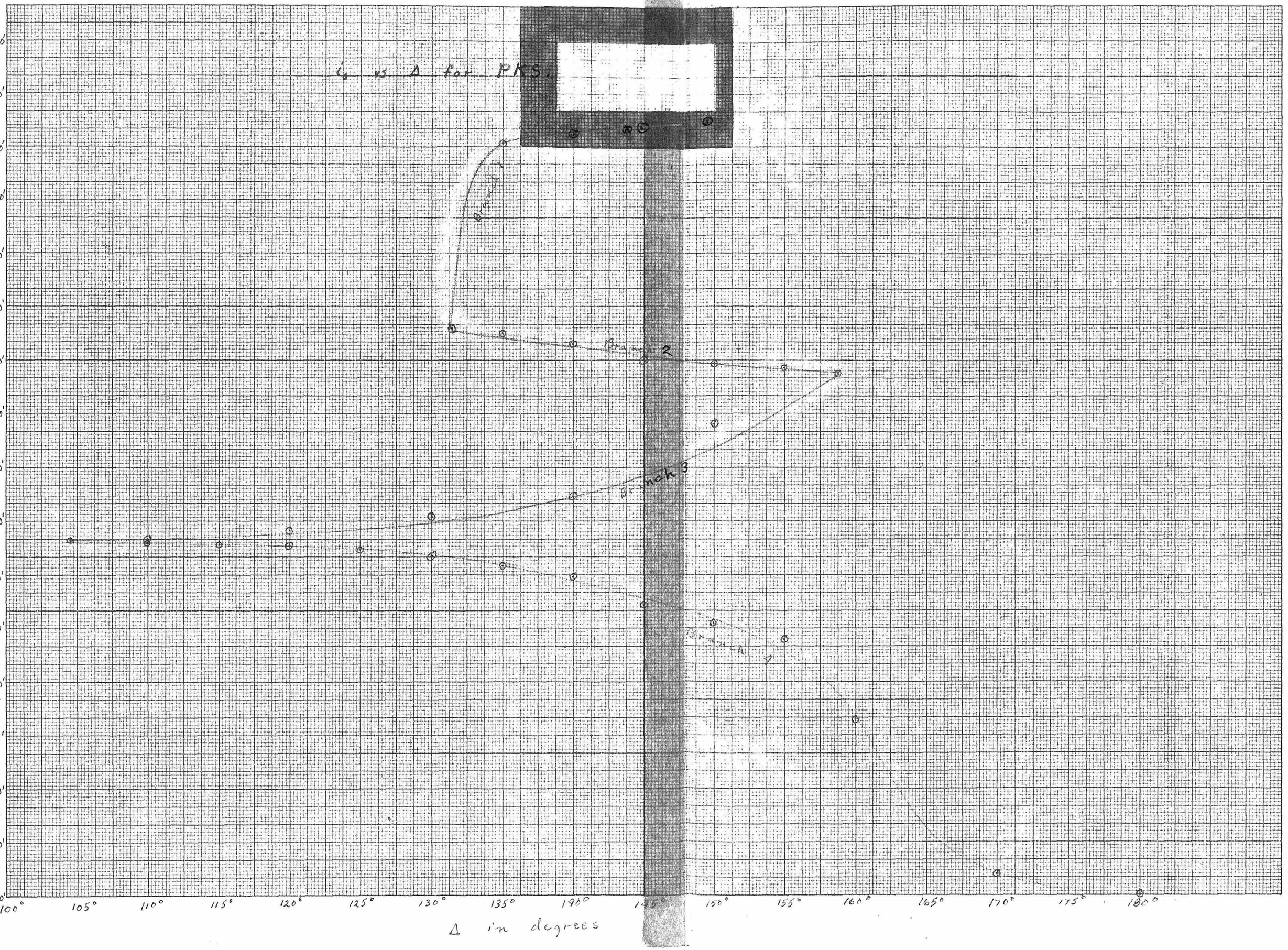
In the following data on p. 67 sin(i_c) is calculated from

$$\frac{\sin(i_c)_p}{13.7} = \frac{\sin i_r}{8.0},$$

$$\text{or} \quad \sin(i_c)_p = 1.712 \sin i_r.$$

Also, f_1 is the square root of the energy refracted into the core at the angle (i_c) and f_2 is the square root of the energy refracted into the mantle at the angle i_r. f is the product of f_1 and f_2.

i_0 vs Δ for PKS.



Δ	$\sin(i_c)_p$	$(i_c)_p$	[1] f_1	[2] f_2	f	[3] A	[4] $\frac{u}{A}$	[4] $\frac{w}{A}$	[5] u	[5] w
149½°	1.000	90°	.000	.000	.000	.000	1.97	.315	.0000	.000
145°	.991	82°19'	.850	.2365	.201	.03325	1.97	.315	.0656	.0105
140°	.981	78°49'	.828	.549	.454	.0750	1.975	.310	.1480	.02325
135°	.971	76°10'	.813	.595	.484	.1693	1.975	.310	.3345	.0525
131½°	.731	46°58'	.838	.534	.447	∞	1.980	.240	∞	∞
131½°	.731	46°58'	.838	.534	.447	∞	1.980	.240	∞	∞
135°	.721	46°08'	.842	.530	.446	.0683	1.980	.240	.1352	.0164
140°	.710	45°14'	.846	.521	.441	.0670	1.985	.235	.1330	.01572
145°	.688	43°28'	.855	.506	.433	.0651	1.985	.230	.1291	.01498
150°	.682	43°00'	.857	.503	.431	.0650	1.985	.230	.1289	.01493
155°	.679	42°46'	.858	.500	.429	.0556	1.985	.220	.1102	.01222
158 3/4°	.672	42°13'	.862	.496	.428	∞	1.985	.215	∞	∞
158 3/4°	.672	42°13'	.862	.496	.428	∞	1.985	.215	∞	∞
150°	.605	37°14'	.891	.453	.404	.1248	1.990	.200	.2482	.02495
140°	.514	30°56'	.921	.397	.365	.0657	1.990	.190	.1308	.01248
130°	.488	29°13'	.928	.377	.350	.0404	1.990	.180	.0804	.00727
120°	.470	28°02'	.934	.364	.340	.0243	1.990	.175	.0484	.00426
110°	.459	27°19'	.936	.356	.333	.0075	1.990	.170	.0149	.00128
104¼°	.458	27°16'	.937	.355	.332	∞	1.990	.170	∞	∞
104¼°	.458	27°16'	.937	.355	.332	∞	1.990	.170	∞	∞
110°	.456	27°08'	.9375	.354	.3315	.0106	1.995	.165	.02115	.00175
115°	.454	27°00'	.938	.353	.331	.0151	1.995	.165	.03015	.00249
120°	.451	26°48'	.939	.350	.3285	.01812	1.995	.165	.0362	.00299
125°	.446	26°29'	.940	.346	.325	.0245	1.995	.160	.0489	.00392
130°	.435	25°47'	.943	.338	.3185	.0338	1.995	.160	.0675	.00541
135°	.425	25°09'	.945	.331	.313	.0424	1.999	.150	.0847	.00636
140°	.410	24°12'	.949	.308	.292	.0465	1.999	.145	.0930	.00675
145°	.3735	21°56'	.958	.290	.278	.0471	1.999	.145	.0943	.00684
150°	.351	20°33'	.962	.271	.260	.0526	1.999	.135	.1051	.00712
155°	.330	19°16'	.967	.256	.247	.0684	1.999	.120	.1367	.00821
160°	.227	13°07'	.984	.175	.172	.0681	2.000	.075	.1362	.00511
170°	.0274	1°34'	.998	.024	.024	.00237	2.000	.040	.00474	.000095
180°	.0000	0°00'	.999	.000	.000	.0000	2.000	.000	.0000	.0000

---

1 Taken from the plot of $\sqrt{\frac{E_r P}{E_i P}}$ vs. angle of incidence on p. 21.

2 " " " " " $\sqrt{\frac{E_f S}{E_i P}}$ " " " " " p. 27.

3 $A = CTf\sqrt{\frac{\tan i_o}{\sin \Delta}\frac{d\, i_o}{d\, \Delta}}$ where $C = \frac{1}{\pi}$ and T = 1 sec.

4 Taken from Reflections and Refractions 1943, graphs of Dr. Gutenberg (copy on p. 10).

5 A plot of u and w against Δ is on p. 68 and p. 69.

u and w vs Δ for PKS.

Branch 1 and Branch 2.

u = horizontal Displacement
w = vertical displacement

Approach ∞ at Δ = 131½°

Approach ∞ at Δ = 158 ¾°

Ground Displacements u and w

.4000
.3500
.3000
.2500
.2000
.1500
.1000
.0500
.0000

u (Branch 1)

u (Branch 2)

w (Branch 1)

w (Branch 2)

100° 105° 110° 115° 120° 125° 130° 135° 140° 145° 150° 155° 160° 165° 170° 175° 180°

Δ in degrees

u and w vs. Δ for PKS.
Branch 3 and Branch 4.
u = horizontal displacement.
w = vertical displacement.

Approach ∞ at Δ = 114½°

Approach ∞ at Δ = 158½°

Ground Displacements u and w

.4000
.3500
.3000
.2500
.2000
.1500
.1000
.0500
.0000

u (Branch 3)
W (Branch 4)
W (Branch 3)
W (Branch 4)

100° 105° 110° 115° 120° 125° 130° 135° 140° 145° 150° 155° 160° 165° 170° 175° 180°

Δ in degrees

3. SKS

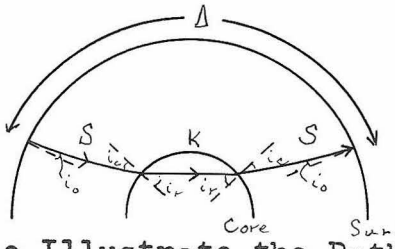


Fig. 14. Sketch to Illustrate the Path of SKS.

A travel-time curve for SKS is on p. 71. It is plotted from the data of Table 24 on p. 125 of Gutenberg and Richter (9). From this, sin i_o is determined by sin $i_o = \frac{V_o}{V_\Delta}$, where V_o is the true velocity of S waves at the surface of the earth (3.2 km. per sec.) and V_Δ is the apparent velocity of SKS waves at the surface taken from the curve at a particular Δ. Only the data from $\Delta = 80^\circ - 130^\circ$ in the first column of Table 24 and from $\Delta = 135^\circ - 150^\circ$ in the second column of Table 24 are used; that is, only two branches to the travel-time curve. Sin i_c is determined from $\frac{r_s \sin i_o}{(V_s)_s} = \frac{r_c \sin i_c}{(V_s)_c}$.

$$\sin i_o = \frac{(V_s)_s\, r_c}{(V_s)_c\, r_s} \sin i_c = \frac{3.2}{7.25} \times \frac{3,446}{6,366} \sin i_c .$$

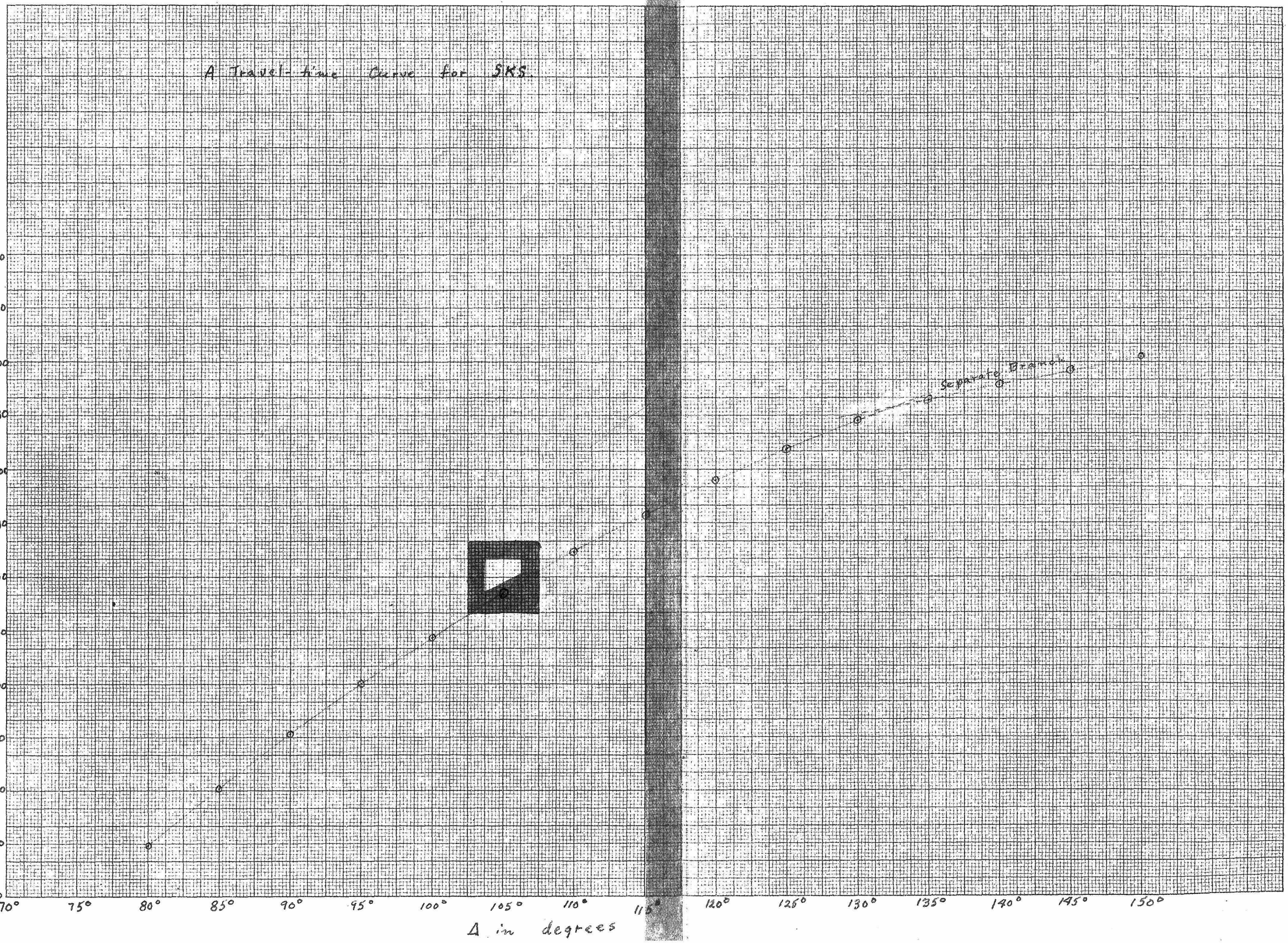
or $\sin i_o = .23892 \sin i_c$.

Hence $\sin i_c = \frac{\sin i_o}{.23892}$.

Δ	$\bar{V}_\Delta$	sin i_o	i_o	tan i_o	$\frac{d\, i_o}{d\, \Delta}$ [1]	sin i_c	i_c
80°	16.82km/sec.	.1901	10°58'	.1938	.110	.796	52°45'
85°	17.75 "	.1802	10°23'	.1832	.1168	.754	48°56'
90°	18.80 "	.1702	9°48'	.1727	.1282	.713	45°34'
95°	20.55 "	.1558	8°58'	.1578	.1168	.652	40°42'
100°	21.75 "	.1472	8°28'	.1489	.0953	.616	38°01'
105°	23.10 "	.1385	7°58'	.1399	.1068	.580	35°27'
107°	23.60 "	.1355	7°47'	.1367	.1232	.567	34°32'
108°	24.10 "	.1328	7°38'	.1340	.1755	.555	33°43'
109° (109.7°)	24.95 "	.1282	7°22'	.1293	.1985	.537	32°29'
109°42'	25.30 "	.1264	7°16'	.1275	.1835	.529	31°57'
110°	25.50 "	.1254	7°12'	.1263	.1802	.525	31°40'
115°	28.50 "	.1122	6°27'	.1131	.0818	.469	27°58'
120°	30.00 "	.1066	6°07'	.1072	.0968	.448	26°37'
125°	33.15 "	.0967	5°33'	.0972	.1285	.404	23°50'
130°	38.3 "	.0836	4°48'	.0839	.1732	.3495	20°27'
135°	60.0 "	.0533	3°03'	.0533	.0884	.2230	12°53'
140°	67.3 "	.0476	2°44'	.0477	.0305	.1992	11°29'
145°	69.4 "	.0462	2°39'	.0463	.0217	.1931	11°08'
150°	74.1 "	.0432	2°29'	.0434	.0384	.1806	10°29'

[1] Taken from a plot of i_o against Δ on p. 73.

A Travel-time Curve for SKS.



Δ	[1] f_1	[2] i_r	[3] f_2	[4] f	[5] $A = CTf\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{di_o}{d\Delta}}$	[6] $\frac{u}{A}$	[6] $\frac{w}{A}$	[7] u	[7] w
80°	.885	61°31'	.879	.778	.3605	1.920	.490	.693	.1765
85°	.903	56°12'	.907	.819	.3789	1.930	.450	.731	.1701
90°	.914	54°49'	.916	.837	.392	1.940	.420	.761	.1645
95°	.920	45°58'	.916	.843	.358	1.960	.390	.703	.1397
100°	.914	42°51'	.912	.834	.314	1.965	.350	.618	.1099
105°	.895	39°48'	.900	.805	.315	1.970	.340	.621	.1071
107°	.883	38°41'	.892	.786	.329	1.970	.340	.649	.1119
108°	.868	37°44'	.882	.765	.379	1.975	.320	.749	.1212
109°	.427	36°18'	.379	.166	.0859	1.975	.310	.1695	.0266
109°42'	.000	35°40'	.051	.000	.0000	1.975	.310	.00000	.00000
110°	.500	35°23'	.286	.143	.0701	1.975	.310	.1385	.0217
115°	.605	31°08'	.609	.368	.1170	1.980	.270	.2315	.0316
120°	.593	29°40'	.598	.355	.1218	1.985	.250	.2420	.0304
125°	.560	26°25'	.555	.311	.1205	1.985	.240	.239	.0289
130°	.490	22°40'	.490	.240	.104	1.990	.210	.207	.0218
135°	.316	14°14'	.325	.103	.0265	1.995	.160	.0529	.00424
140°	.284	12°41'	.292	.083	.0124	1.995	.150	.0247	.00186
145°	.275	12°18'	.283	.078	.0103	2.000	.140	.0206	.00144
150°	.261	11°29'	.265	.0692	.0125	2.000	.130	.0250	.00163

---

[1] f_1 is the square root of the energy refracted into the core at the angle i_c. It is taken from a plot of $\sqrt{\frac{E_f P}{E_i S}}$ against angle of incidence on p. 33.

[2] $\frac{\sin i_r}{8.0} = \frac{\sin i_c}{7.25}$ or $\sin i_r = 1.102 \sin i_c$.

[3] f_2 is the square root of the energy refracted into the mantle at the angle i_r. It is taken from a plot of $\sqrt{\frac{E_f S}{E_i P}}$ against angle of incidence on p. 27.

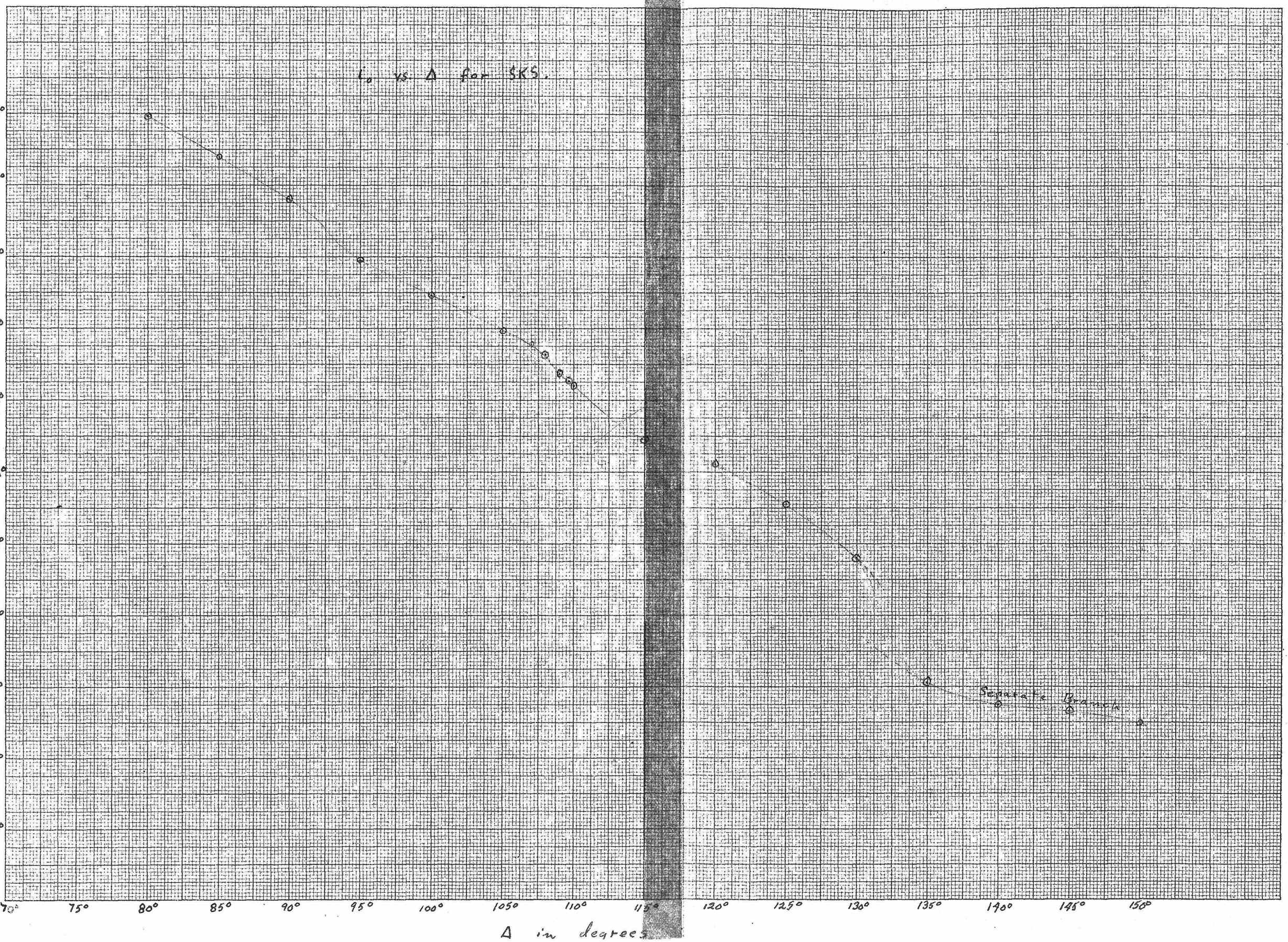
[4] $f = f_1 f_2$.

[5] $C = \pi$ and $T = 1$ sec.

[6] Taken from Reflections and Refractions 1943, graphs of Dr. Gutenberg (copy on p.10).

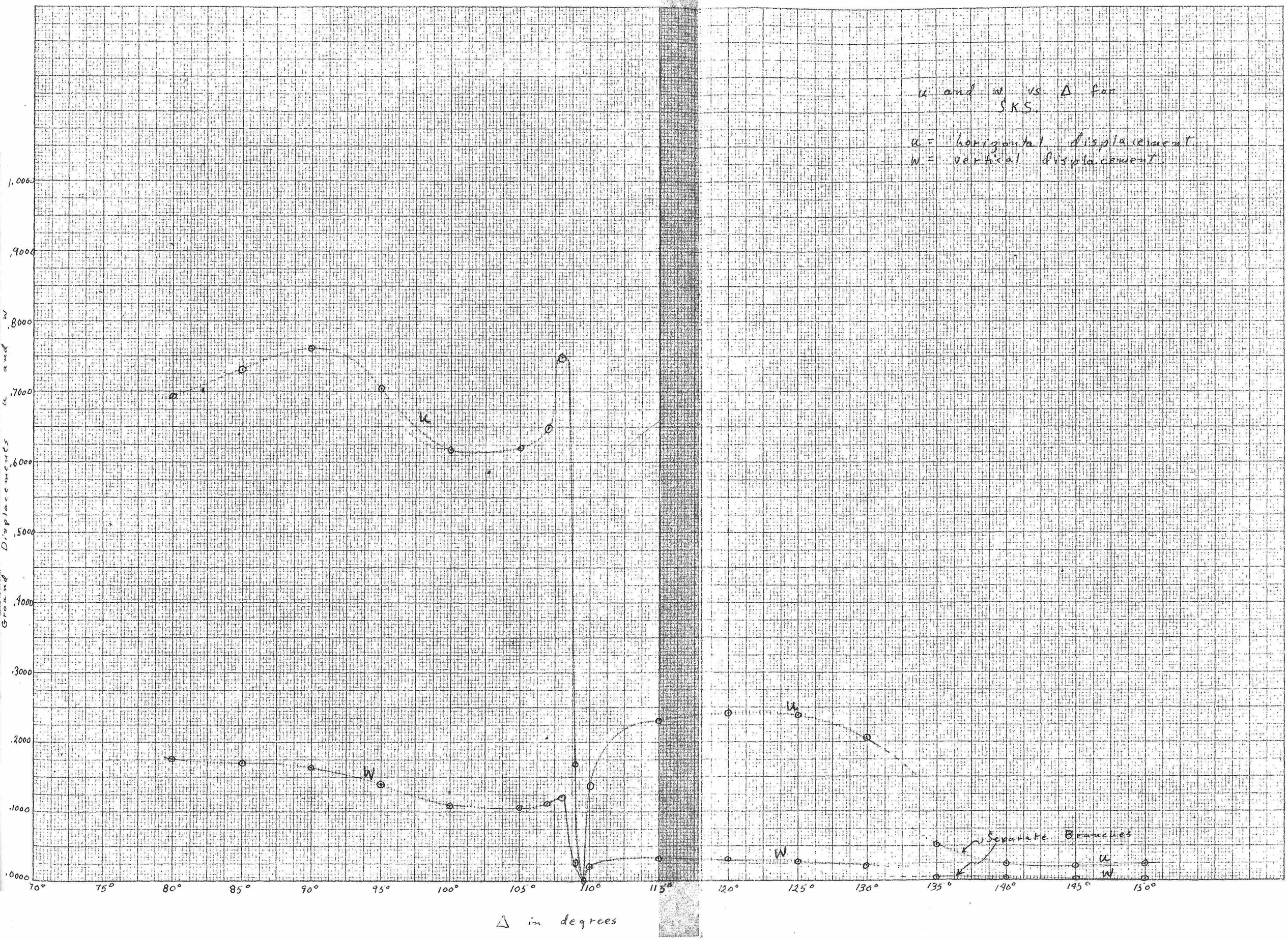
[7] A plot of u and w against Δ is on p. 74. The values $u = w = 0$ at $\Delta = 109°42'$ correspond to an angle of incidence at the core, i_c, for which $f_2 = .00$.

t_0 vs. Δ for SKS.



u and w vs. Δ for SKS.

u = horizontal displacement
w = vertical displacement



4. SKP

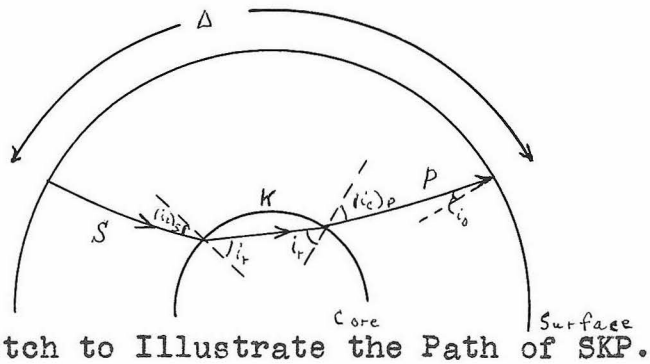


Fig. 15. Sketch to Illustrate the Path of SKP.

The apparent velocity $\overline{V}_\Delta$ of SKP waves at the surface is taken from the travel-time curve for PKS and SKP on p. 64 and is the same as the apparent velocity of the PKS waves. From this, $\sin i_o = \frac{V_o}{\overline{V}_\Delta}$, where V_o is the true velocity of P waves at the earth's surface (5.5 km. per sec.). $\sin (i_c)$ is determined from

$$\frac{r_s \sin i_o}{(V_p)_s} = \frac{r_c \sin (i_c)_p}{(V_p)_c} ,$$

$$\sin i_o = \frac{(V_p)_s \, r_c}{(V_p)_c \, r_s} \sin (i_c)_p = \frac{3446}{6366} \times \frac{5.5}{13.7} \sin (i_c)_p ,$$

$$\sin i_o = .21732 \sin (i_c)_p ,$$

$$\sin (i_c)_p = \frac{\sin i_o}{.21732} .$$

The following tabulation (similar to that for PKS) corresponds to four branches. From $\Delta = 149\frac{1}{2}^\circ$ to $131\frac{1}{2}^\circ$ constitutes branch 1 of the PKS-SKP travel-time curve, from $\Delta = 131\frac{1}{2}^\circ$ to $158\ 3/4^\circ$ constitutes branch 2, from $\Delta = 158\ 3/4^\circ$ to $\Delta = 104\frac{1}{4}^\circ$ constitutes branch 3, and from $\Delta = 104\frac{1}{4}^\circ$ to 180° constitutes branch 4. These are arbitrarily assigned numbers.

Δ	$\overline{V}_\Delta$	$\sin i_o$	i_o	$\tan i_o$	$\frac{d\, i_o}{d\, \Delta}$	$\sin (i_c)_p$	$(i_c)_p$	$\sin i_r$	i_r
$149\frac{1}{2}^\circ$	25.3km/sec.	.2173	$12^\circ 33'$	.2226	.01833	1.000	$90^\circ 00'$	.584	$35^\circ 44'$
145°	25.5 "	.2155	$12^\circ 27'$	.2208	.02165	.991	$82^\circ 19'$	.578	$35^\circ 19'$
140°	25.8 "	.2130	$12^\circ 18'$	.2180	.0317	.981	$78^\circ 49'$	.573	$34^\circ 58'$
135°	26.1 "	.2105	$12^\circ 09'$	.2153	.1700	.971	$76^\circ 10'$	.567	$34^\circ 32'$
$131\frac{1}{2}^\circ$	34.7 "	.1582	$9^\circ 06'$	.1596	∞	.728	$46^\circ 43'$	.425	$25^\circ 09'$
$131\frac{1}{2}^\circ$	34.7 "	.1582	$9^\circ 06'$	.1596	∞	.728	$46^\circ 43'$	.425	$25^\circ 09'$
135°	35.01 "	.1570	$9^\circ 02'$	.1590	.03335	.723	$46^\circ 18'$	.421	$24^\circ 54'$
140°	35.70 "	.1541	$8^\circ 52'$	.1560	.02835	.710	$45^\circ 14'$	.414	$24^\circ 27'$
145°	36.80 "	.1494	$8^\circ 36'$	.1512	.0250	.688	$43^\circ 28'$	.401	$23^\circ 38'$
150°	37.00 "	.1485	$8^\circ 32'$	.1501	.02165	.684	$43^\circ 09'$	.399	$23^\circ 31'$
155°	37.25 "	.1476	$8^\circ 29'$	.1492	.01498	.679	$42^\circ 46'$	.396	$23^\circ 20'$

Δ	$\overline{V_\Delta}$	$\sin i_o$	i_o	$\tan i_o$	$\frac{di_o}{d\Delta}$ [1]	$\sin(i_c)_p$	$(i_c)_p$	$\sin i_r$ [2]	i_r
158 3/4°	37.63km/sec.	.1461	8°24'	.1477	∞	.673	42°18'	.393	23°08'
158 3/4°	37.63 "	.1461	8°24'	.1477	∞	.673	42°18'	.393	23°08'
150°	41.75 "	.1318	7°34'	.1328	.1050	.606	37°18'	.3535	20°42'
140°	49.10 "	.1119	6°25'	.1125	.0434	.515	31°00'	.3005	17°28'
130°	51.80 "	.1061	6°05'	.1066	.0250	.489	29°16'	.285	16°34'
120°	53.80 "	.1022	5°52'	.1028	.01498	.471	28°06'	.275	15°58'
110°	55.00 "	.1000	5°44.4'	.1055	.00834	.460	27°23'	.2685	15°35'
104 1/4°	55.20 "	.0998	5°43.7'	.1003	∞	.459	27°19'	.268	15°33'
104 1/4°	55.20 "	.0998	5°43.7'	.1003	∞	.459	27°19'	.268	15°33'
110°	55.50 "	.0992	5°42'	.0998	.00334	.456	27°08'	.266	15°26'
115°	55.80 "	.0986	5°40'	.0992	.00500	.454	27°00'	.2645	15°20'
120°	56.10 "	.0982	5°38'	.0986	.00667	.452	26°52'	.264	15°18'
125°	56.60 "	.0973	5°35'	.0978	.01498	.448	26°37'	.262	15°11'
130°	58.10 "	.0948	5°26'	.0951	.03000	.436	25°51'	.255	14°46'
135°	59.40 "	.0928	5°19'	.0931	.0367	.427	25°17'	.249	14°25'
140°	61.70 "	.0893	5°07'	.0895	.0484	.411	24°16'	.240	13°53'
145°	67.70 "	.0814	4°40'	.0816	.0650	.3745	22°00'	.2185	12°37'
150°	72.10 "	.0764	4°23'	.0767	.0934	.3515	20°35'	.205	11°50'
155°	76.60 "	.0719	4°07'	.0721	.1818	.331	19°20'	.193	11°08'
160°	111.00 "	.0496	2°51'	.0498	.2485	.228	13°11'	.133	7°39'
170°	926.00 "	.00594	0°20'	.00582	.1182	.0273	1°34'	.01592	0°55'
180°	∞	.00000	0°00'	.00000	.0000	.0000	0°00'	.00000	0°00'

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[1] Taken from the plot of i_o against Δ on p. 77.

[2]
$$\frac{\sin i_r}{8.0} = \frac{\sin(i_c)_p}{13.7},$$

$$\sin i_r = \frac{8.0}{13.7} \sin (i_c)_p = .584 \sin(i_c)_p .$$

On p. 78 $\sin (i_c)_s$ is determined from

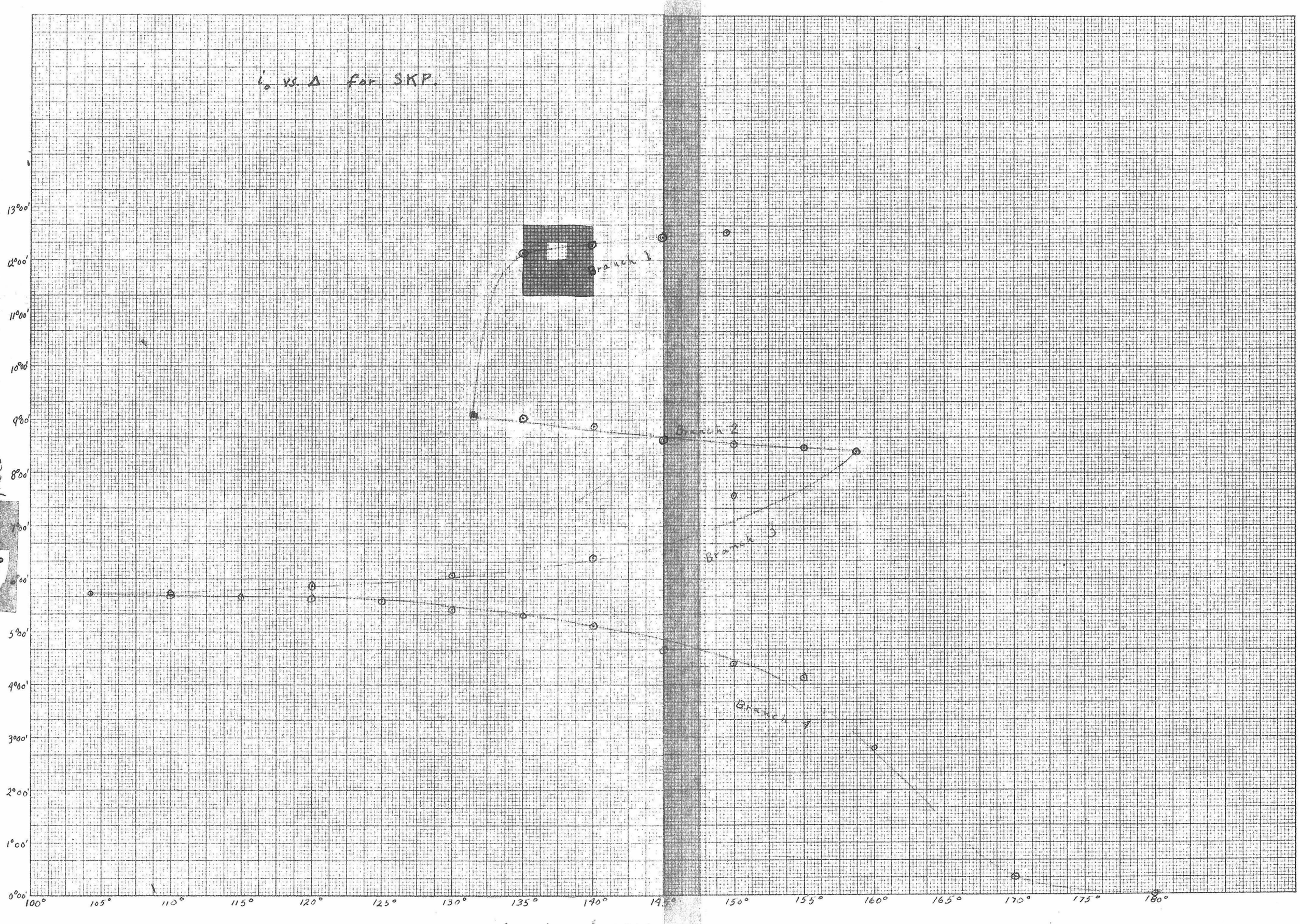
$$\frac{\sin(i_c)_s}{7.25} = \frac{\sin i_r}{8.00},$$

$$\sin(i_c)_s = \frac{7.25}{8.00} \sin i_r = .906 \sin i_r .$$

f_1 is the square root of the energy refracted into the core at the angle $(i_c)_s$ and f_2 is the square root of the energy refracted into the mantle at the angle i_r.

f is the product of f_1 and f_2.

i_0 vs Δ for SKP.



Δ	$\sin(i_c)_s$	$(i_c)_s$	$^1f_1$	$^2f_2$	f	^3A	$^4\frac{u}{A}$	$^4\frac{w}{A}$	^5u	^5w
149½°	.529	31°56'	.000	.000	.000	.0000	.54	1.94	.000	.000
145°	.524	31°36'	.213	.472	.1005	.0289	.535	1.94	.01545	.0561
140°	.518	31°12'	.456	.829	.378	.1233	.520	1.945	.0642	.2400
135°	.513	30°52'	.582	.828	.481	.3440	.500	1.945	.1720	.6690
131½°	.385	22°39'	.537	.844	.454	∞	.400	1.960	∞	∞
131½°	.385	22°39'	.537	.844	.454	∞	.400	1.960	∞	∞
135°	.382	22°27'	.536	.846	.4535	.1232	.390	1.965	.0481	.2423
140°	.375	22°03'	.525	.852	.446	.1161	.385	1.965	.0448	.2285
145°	.364	21°21'	.513	.861	.442	.1131	.380	1.970	.0431	.2230
150°	.3615	21°12'	.505	.862	.435	.1109	.375	1.970	.0416	.2185
155°	.359	21°02'	.503	.864	.434	.0995	.370	1.970	.0368	.1959
158 3/4°	.356	20°49'	.497	.866	.430	∞	.370	1.970	∞	∞
158 3/4°	.356	20°49'	.497	.866	.430	∞	.370	1.970	∞	∞
150°	.320	18°40'	.447	.888	.397	.2080	.350	1.975	.0729	.411
140°	.272	15°47'	.376	.914	.3435	.0943	.290	1.980	.02735	.1868
130°	.258	14°57'	.358	.921	.330	.0613	.280	1.980	.01654	.1212
120°	.249	14°25'	.345	.926	.3195	.0423	.270	1.985	.01142	.0841
110°	.2435	14°06'	.339	.929	.3145	.02965	.260	1.985	.00772	.0589
104¼°	.2425	14°02'	.337	.930	.3135	∞	.255	1.985	∞	∞
104¼°	.2425	14°02'	.337	.930	.3135	∞	.255	1.985	∞	∞
110°	.241	13°57'	.336	.931	.313	.01851	.250	1.985	.00464	.03680
115°	.240	13°53'	.335	.932	.312	.02295	.245	1.985	.00562	.0445
120°	.239	13°50'	.334	.9325	.311	.0269	.240	1.985	.00646	.0534
125°	.2375	13°44'	.330	.933	.3075	.0410	.235	1.990	.00965	.0816
130°	.231	13°21'	.325	.935	.3035	.0581	.220	1.990	.01279	.1158
135°	.2255	13°02'	.317	.939	.2975	.0652	.200	1.990	.01302	.1297
140°	.2175	12°34'	.308	.944	.2905	.0752	.190	1.990	.01428	.1495
145°	.198	11°25'	.282	.953	.2685	.0815	.180	1.990	.01465	.1621
150°	.1858	10°42'	.267	.958	.2555	.0960	.175	1.990	.01679	.1909
155°	.1748	10°04'	.253	.964	.244	.1349	.170	1.990	.02293	.2682
160°	.1203	6°55'	.176	.983	.173	.1032	.130	1.995	.01343	.2062
170°	.01442	0°50'	.025	.998	.0249	.00493	.040	2.000	.000197	.00986
180°	.00000	0°00'	.000	.999	.0000	.00000	.000	2.000	.0000	.0000

1 Taken from the plot of $\sqrt{\frac{E_f P}{E_i S}}$ vs. angle of incidence on p. 33.

2 Taken from the plot of $\sqrt{\frac{E_f P}{E_i P}}$ vs. angle of incidence on p. 27.

3 $A = CTf \sqrt{\frac{\tan i_o}{\sin \Delta} \frac{d\, i_o}{d\, \Delta}}$ where $C = \pi$ and $T = 1$ sec.

4 Taken from Reflections and Refractions, 1943, graphs of Dr. Gutenberg (p. 10).

5 A plot of u and w against Δ is on p. 79 and p. 80.

u and w vs. Δ for
SKP.
Branch 1 and Branch 2.

u = horizontal displacement.
w = vertical displacement.

w = .869 at Δ = 135°

Approach ∞ at Δ = 131.5°

Approach ∞ at Δ = 158.99°

w (Branch 2)

w (Branch 1)

u (Branch 1)

u (Branch 2)

Displacements u and w
Ground Distance

.3000
.2500
.2000
.1500
.1000
.0500
.0000

100° 105° 110° 115° 120° 125° 130° 135° 140° 145° 150° 155° 160° 165° 170° 175° 180°

Δ in degrees

u and w vs. Δ for SHP.

Branch 3 and Branch 4.

u = horizontal displacement

w = vertical displacement

Approach ∞ at Δ = 104 1/2°

Approach ∞ at Δ = 158 2/3°

w (Branch 3)

w (Branch 4)

u (Branch 3)

u (Branch 4)

Ground Displacements

.5000 .4500 .4000 .3500 .3000 .2500 .2000 .1500 .1000 .0500 .0000

100° 105° 110° 115° 120° 125° 130° 135° 140° 145° 150° 155° 160° 165° 170° 175° 180°

Δ in degrees

Group III. Waves Reflected Once from the Interior Surface of the Core

1. PKKP

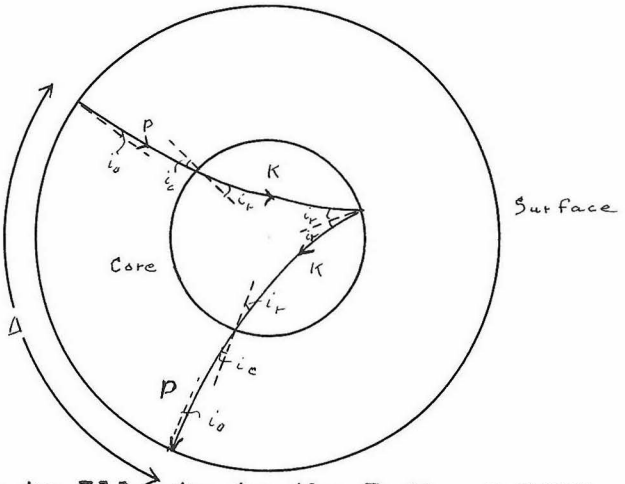


Fig. 16. Sketch to Illustrate the Path of PKKP.

The apparent velocity $\overline{V_\Delta}$ of the PKKP waves at the surface is taken from the travel-time curve of PKKP on p. 82. The data used for the curve is from Table 27 on p. 127 of Gutenberg and Richter (9). Only the data in the first column of that table is used, from $\Delta = 120^\circ$ to $\Delta = 95^\circ$ to calculate $\overline{V_\Delta}$. $\sin i_o = \frac{V_o}{\overline{V_\Delta}}$, where V_o is the true velocity of P waves at the earth's surface (5.5 km. per sec.) $\sin i_c$ is determined from

$$\frac{r_s \sin i_o}{(V_p)_s} = \frac{r_c \sin i_c}{(V_p)_c},$$

$$\sin i_o = \frac{(V_p)_s\, r_c}{(V_p)_c\, r_s} \sin i_c = \frac{3446}{6336} \times \frac{5.5}{13.7} \sin i_c,$$

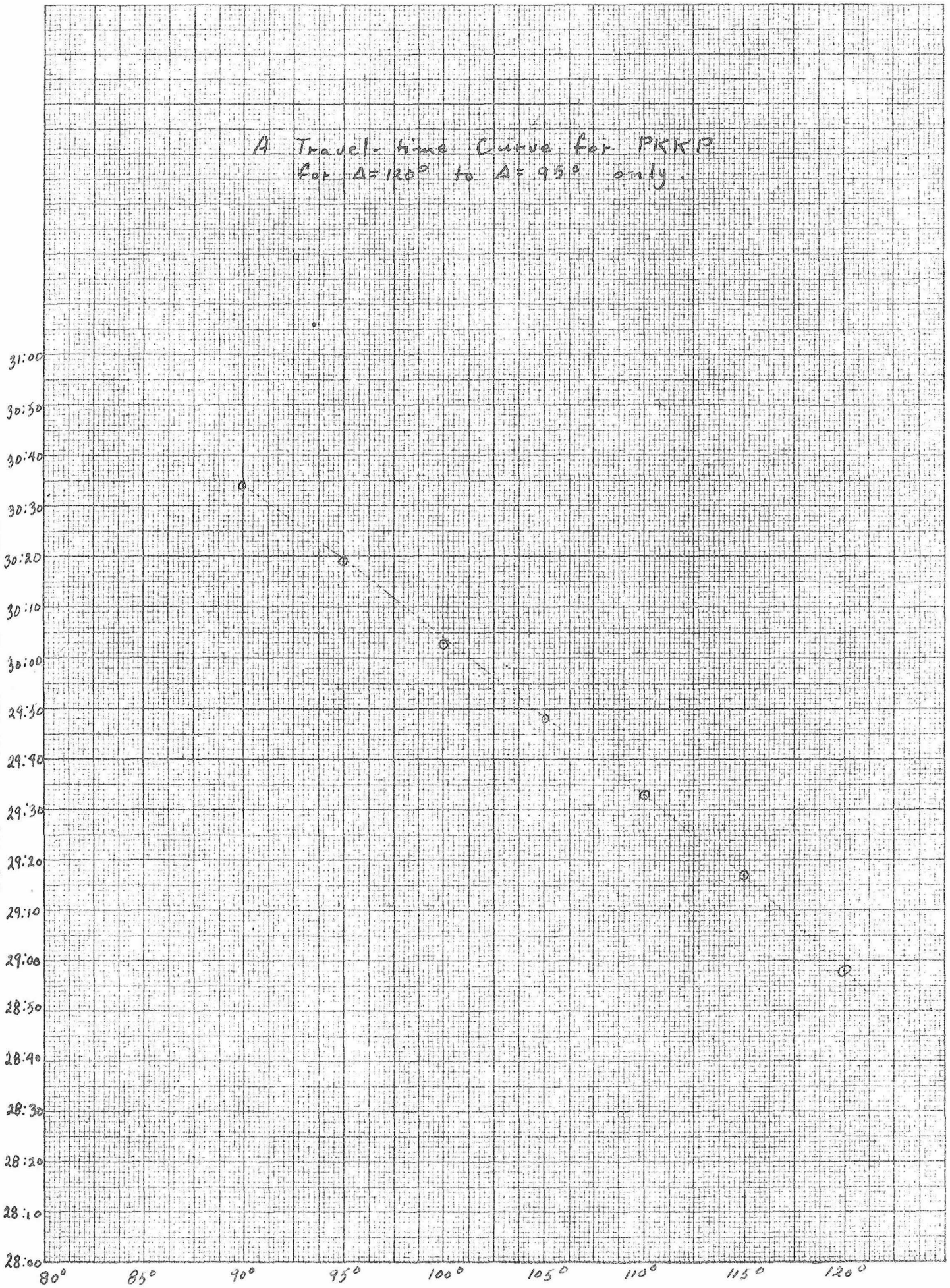
$$\sin i_o = .21732 \sin i_c,$$

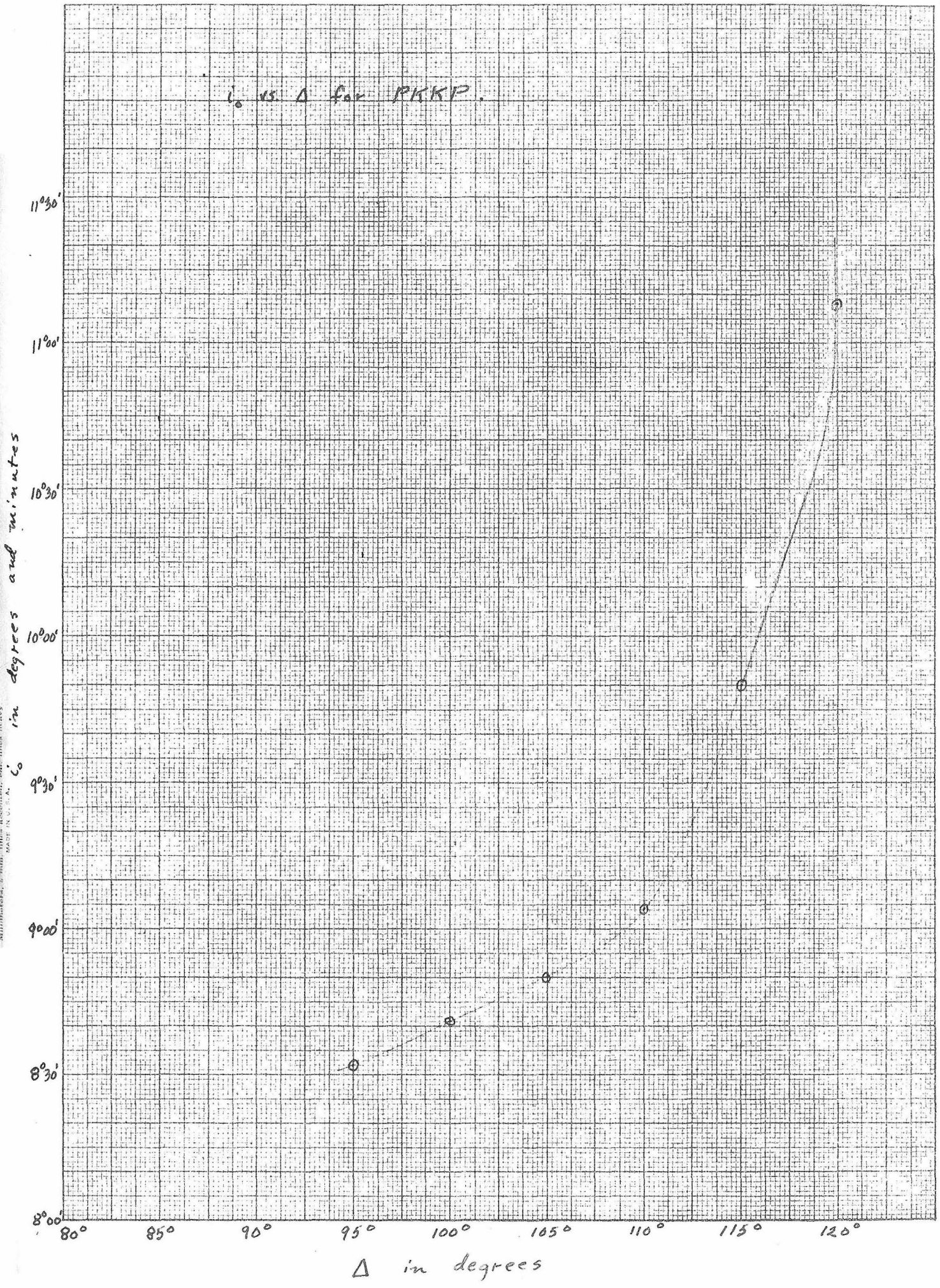
or $$\sin i_c = \frac{\sin i_o}{.21732}.$$

Δ	$\overline{V_\Delta}$	$\sin i_o$	i_o	$\tan i_o$	$\frac{di_o}{d\Delta}$ [1]	$\sin i_c$	i_c	$\sin i_r$	i_r
120°	28.45km/sec.	.1931	11°08'	.1968	∞ [2]	.889	62°45'	.518	31°12'
115°	32.18 "	.1709	9°50'	.1733	.1983	.786	51°49'	.459	27°19'
110°	34.90 "	.1576	9°04'	.1596	.0859	.725	46°28'	.423	25°01'
105°	35.80 "	.1535	8°50'	.1554	.0333	.706	44°55'	.412	24°12'
100°	36.40 "	.1510	8°41'	.1527	.0310	.695	44°02'	.405	23°53'
95°	37.00 "	.1484	8°32'	.1501	.0292	.6835	43°07'	.3981	23°28'

[1] Taken from the plot of i_o vs. Δ on p. 83.

[2] $\frac{di_o}{d\Delta} = \infty$ at $\Delta = 120^\circ$. This is due to the fact that 120° is a focal point for PKKP, as may be seen by referring to the data in Table 27 of Gutenberg and Richter (9).


A Travel-time Curve for PKKP
for Δ=120° to Δ=95° only.
31:00
30:50
30:40
30:30
30:20
30:10
30:00
29:50
29:40
29:30
29:20
29:10
29:00
28:50
28:40
28:30
28:20
28:10
28:00
Time in Minutes and Seconds
80°
85°
90°
95°
100°
105°
110°
115°
120°
Δ in degrees


i_o vs. Δ for PKKP.
11°30'
11°00'
10°30'
10°00'
9°30'
9°00'
8°30'
8°00'
i_o in degrees and minutes
80°
85°
90°
95°
100°
105°
110°
115°
120°
Δ in degrees

Δ	[1] f_1	[2] f_2	[3] f_3	[4] f	[5] A	[6] $\frac{u}{A}$	[6] $\frac{w}{A}$	[7] u	[7] w
120°	.785	.081	.788	.0510	∞	.480	1.95	∞	∞
115°	.816	.089	.818	.0594	.0363	.420	1.96	.01522	.0712
110°	.840	.087	.844	.0616	.0234	.390	1.97	.00914	.0461
105°	.848	.086	.859	.0624	.01435	.380	1.975	.00544	.02835
100°	.852	.085	.856	.0620	.01351	.375	1.975	.00507	.0267
95°	.857	.0840	.862	.0620	.01297	.370	1.975	.00479	.0256

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[1] f_1 is the square root of the energy refracted into the core at the angle i_c. It is taken from the plot of $\sqrt{\frac{E_f P}{E_i P}}$ vs. angle of incidence on p. 21.

[2] f_2 is the square root of the energy reflected back into the core at the angle i_r. It is taken from the plot of $\sqrt{\frac{E_r P}{E_i P}}$ vs. angle of incidence on p. 27.

$\sin i_r$ is determined from

$$\frac{\sin i_r}{8.0} = \frac{\sin i_c}{13.7},$$

$$\sin i_r = \frac{8.0}{13.7} \sin i_c,$$

$$\text{or } \sin i_r = .584 \sin i_c,$$

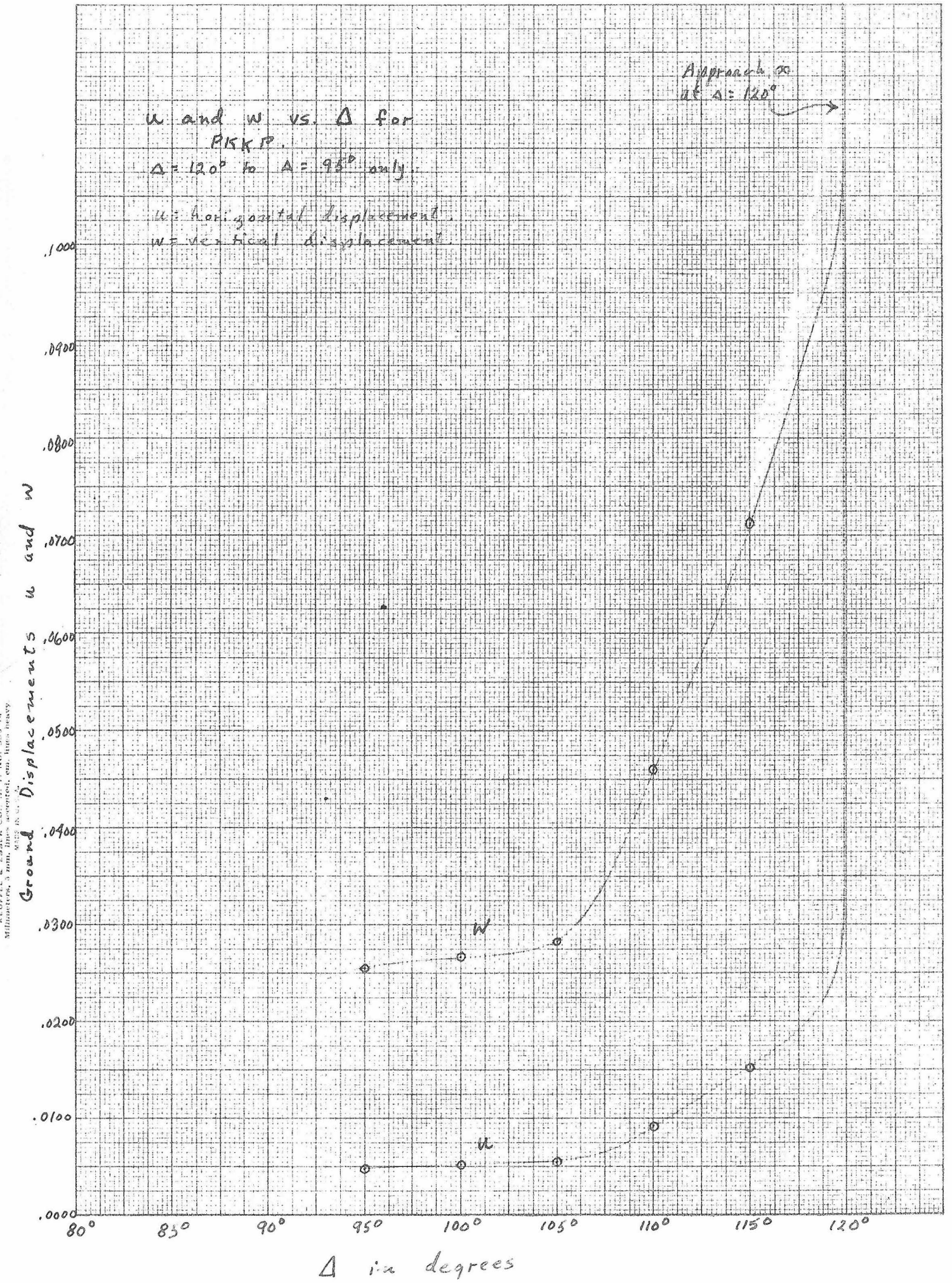
[3] f_3 is the square root of the energy refracted into the mantle at the angle i_r. It is taken from the plot of $\sqrt{\frac{E_f P}{E_i P}}$ vs. angle of incidence on p. 27.

[4] $f = f_1 f_2 f_3$.

[5] $A = CTf \sqrt{\frac{\tan i_o}{\sin \Delta} \frac{d\, i_o}{d \Delta}}$, with $C = \frac{1}{\pi}$ and $T = 1$ sec.

[6] Taken from Reflections and Refractions 1943, graphs of Dr. Gutenberg (copy on p. 10).

[7] A plot of u and w against Δ is on p. 85.


u and w vs. Δ for
PIKP.
Δ = 120° to Δ = 95° only.
u: horizontal displacement.
w: vertical displacement.
Approach ∞
at Δ = 120°
.1000
.0900
.0800
.0700
.0600
.0500
.0400
.0300
.0200
.0100
.0000
Ground Displacements u and w
w
u
80°
85°
90°
95°
100°
105°
110°
115°
120°
Δ in degrees

2. PKKS.

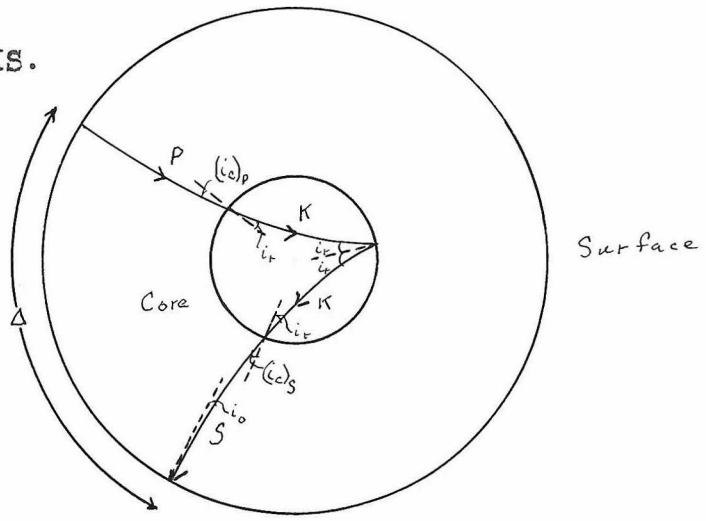


Fig. 17. Sketch to Illustrate the Path of PKKS.

The apparent velocity $\bar{V}_\Delta$ of the PKKS waves at the surface is taken from the travel-time curve of PKKS and SKKP on p. 87. The data used for the curve is from Table 28 on p. 127 of Gutenberg and Richter (9). Only the first column of that table is used, from $\Delta = 132^\circ$ through $\Delta = 100^\circ$. From this, $\sin i_o = \frac{V_o}{\bar{V}_\Delta}$, where V_o is the true velocity of S waves at the earth's surface (3.2 km. per sec.).

$\sin (i_c)_s$ is determined from

$$\frac{r_s \sin i_o}{(V_s)_s} = \frac{r_c \sin (i_c)_s}{(V_s)_c},$$

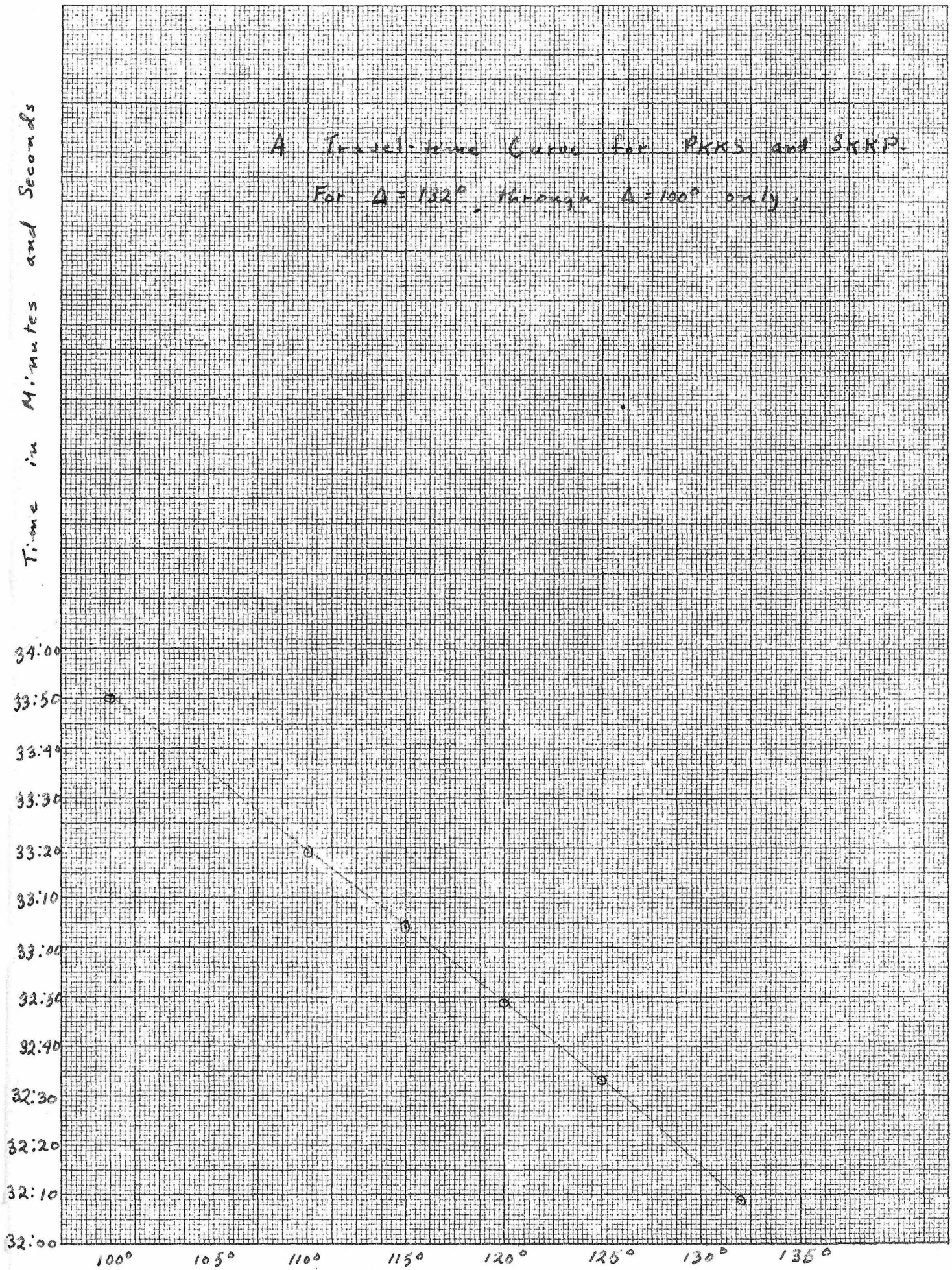
$$\sin i_o = \frac{(V_s)_s\, r_c}{(V_s)_c\, r_s} \sin(i_c)_s = \frac{3.2}{7.25} \times \frac{3,446}{6,366} \sin(i_c)_s,$$

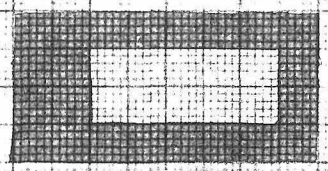
$$\sin i_o = .23892 \sin(i_c)_s,$$

$$\sin(i_c)_s = \frac{\sin i_o}{.23892}.$$

Δ	$\bar{V}_\Delta$	$\sin i_o$	i_o	$\tan i_o$	$^1\frac{di_o}{d\Delta}$	$\sin(i_c)_s$	$(i_c)_s$
132°	30.18km/sec.	.1060	$6^\circ05'$	.1066	.109	.4435	$26^\circ20'$
130°	30.80 "	.1038	$5^\circ57'$	.1042	.1067	.434	$25^\circ43'$
125°	34.18 "	.0937	$5^\circ23'$	.0942	.0834	.392	$23^\circ05'$
120°	34.92 "	.0917	$5^\circ16'$	.0922	.02785	.384	$22^\circ35'$
115°	36.30 "	.0882	$5^\circ04'$	.0887	.01017	.369	$21^\circ39'$
110°	36.52 "	.0876	$5^\circ02'$	.0881	.00684	.3665	$21^\circ30'$
105°	36.78 "	.0871	$5^\circ00'$	.0875	.00667	.3645	$21^\circ23'$
100°	37.00 "	.08655	$4^\circ58'$	.0869	.00650	.362	$21^\circ13'$

1 Taken from the plot of i_o vs. Δ on p. 88.


A Travel-time Curve for PKKS and SKKP.
For Δ = 132°, through Δ = 100° only.
Time in Minutes and Seconds
34:00
33:50
33:40
33:30
33:20
33:10
33:00
32:50
32:40
32:30
32:20
32:10
32:00
100°
105°
110°
115°
120°
125°
130°
135°



i_o vs. Δ for PKKS

i_o in degrees and minutes

$6°30'$
$6°00'$
$5°30'$
$5°00'$
$4°30'$
$4°00'$

$100°$ $105°$ $110°$ $115°$ $120°$ $125°$ $130°$ $135°$

Δ in degrees

Δ	[1] $\sin i_r$	i_r	[2] $\sin(i_c)_p$	$(i_c)_p$	[3] f_2	[4] f_2	[5] f_3	[6] f
132°	.489	29°16'	.839	57°02'	.798	.088	.592	.0416
130°	.479	28°37'	.821	55°11'	.804	.089	.586	.0419
125°	.433	25°40'	.743	47°59'	.832	.088	.542	.0397
120°	.424	25°05'	.726	46°33'	.840	.087	.532	.0388
115°	.407	24°01'	.698	44°16'	.851	.086	.514	.0375
110°	.4045	23°52'	.694	43°57'	.854	.0855	.512	.0373
105°	.402	23°42'	.690	43°38'	.855	.0850	.508	.0368
100°	.399	23°31'	.684	43°09'	.857	.0845	.504	.0364

Δ	[7] $A=CTf\sqrt{\frac{\tan i_o d i_o}{\sin\Delta\, d\Delta}}$	[8] $\frac{u}{A}$	[8] $\frac{w}{A}$	[9] u	[9] w
132°	.01639	1.980	.250	.03245	.00410
130°	.01579	1.980	.245	.03125	.00387
125°	.01220	1.985	.240	.02420	.00293
120°	.00665	1.985	.235	.01320	.00156
115°	.00372	1.985	.215	.00739	.00080
110°	.00297	1.985	.210	.00590	.000624
105°	.00284	1.990	.205	.00566	.000583
100°	.00274	1.990	.200	.00546	.000548

---

[1]

$$\frac{\sin i_r}{8.00} = \frac{\sin (i_c)_s}{7.25},$$

$$\sin i_r = \frac{8.00}{7.25} \sin (i_c)_s = 1.102 \sin (i_c)_s .$$

[2]

$$\frac{\sin (i_c)_p}{13.7} = \frac{\sin i_r}{8.00},$$

$$\sin (i_c)_p = \frac{13.7}{8.00} \sin i_r = 1.712 \sin i_r .$$

[3] f_1 is the square root of the energy refracted into the core at the angle $(i_c)_p$. It is taken from the plot of $\sqrt{\frac{E_f P}{E_i P}}$ vs. angle of incidence on p. 21.

[4] f_2 is the square root of the energy reflected back into the core at the angle i_r. It is taken from the plot of $\sqrt{\frac{E_r P}{E_i P}}$ vs. angle of incidence on p. 27.

[5] f_3 is the square root of the energy refracted into the mantle at the angle i_r. It is taken from the plot of $\sqrt{\frac{E_f S}{E_i P}}$ vs. angle of incidence on p. 27.

[6] $f = f_1 f_2 f_3$.

[7] $C = \pi$ and $T = 1$ sec.

[8] Taken from Reflections and Refractions 1943, graphs of Dr. Gutenberg. (Copy on p. 10).

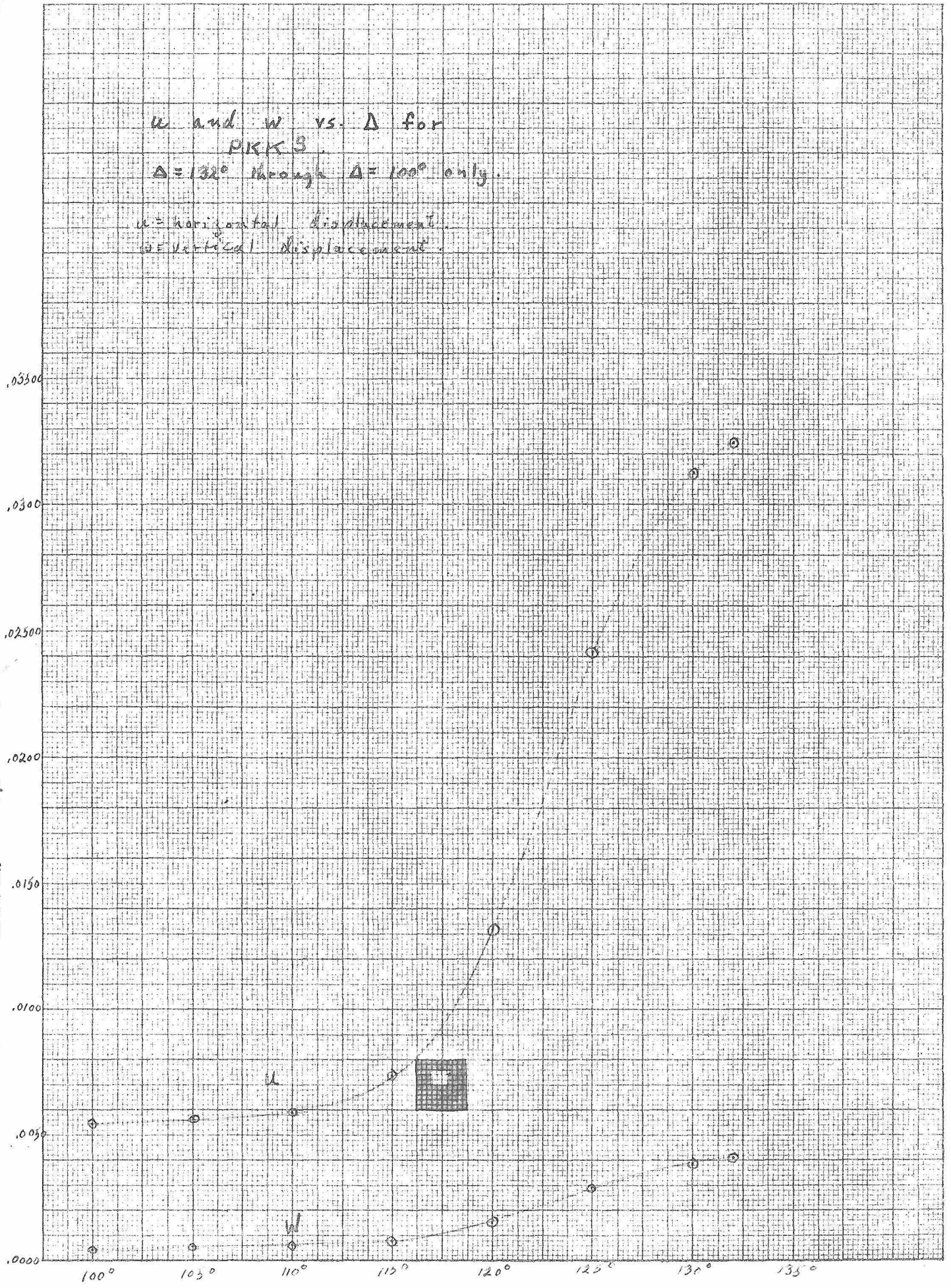
[9] A plot of u and w against Δ is on p. 90.

u and w vs. Δ for PKKS.

Δ = 132° through Δ = 100° only.

u = horizontal displacement.

w = vertical displacement.



3. SKKS

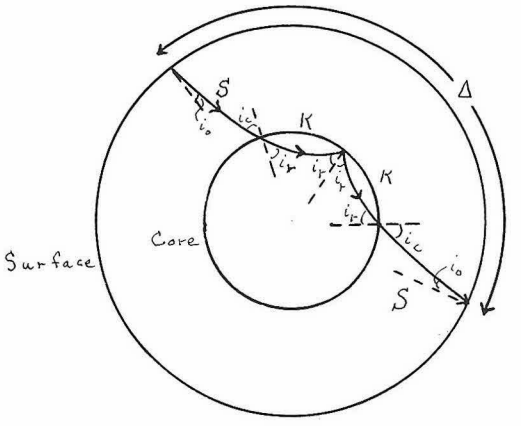


Fig. 18. Sketch to Illustrate the Path of SKKS.

The apparent velocity $\overline{V_\Delta}$ of the SKKS waves at the surface is taken from the travel-time curve of SKKS on p. 92. The data used for the curve is from Table 30 on p. 129 of Gutenberg and Richter (9). Only the data from $\Delta = 100^\circ$ through $\Delta = 165^\circ$ is used (calculated values). From this, $\sin i_o = \frac{V_o}{\overline{V_\Delta}}$ where V_o is the true velocity of S waves at the earth's surface (3.2 km. per sec.).
Sin i_c is determined from

$$\frac{r_s \sin i_o}{(V_s)_s} = \frac{r_c \sin i_c}{(V_s)_c},$$

$$\sin i_o = \frac{(V_s)_s\, r_c}{(V_s)_c\, r_s} \sin i_c = \frac{3.2}{7.25} \times \frac{3,446}{6,366} \sin i_c,$$

$$\sin i_o = .23892 \sin i_c,$$

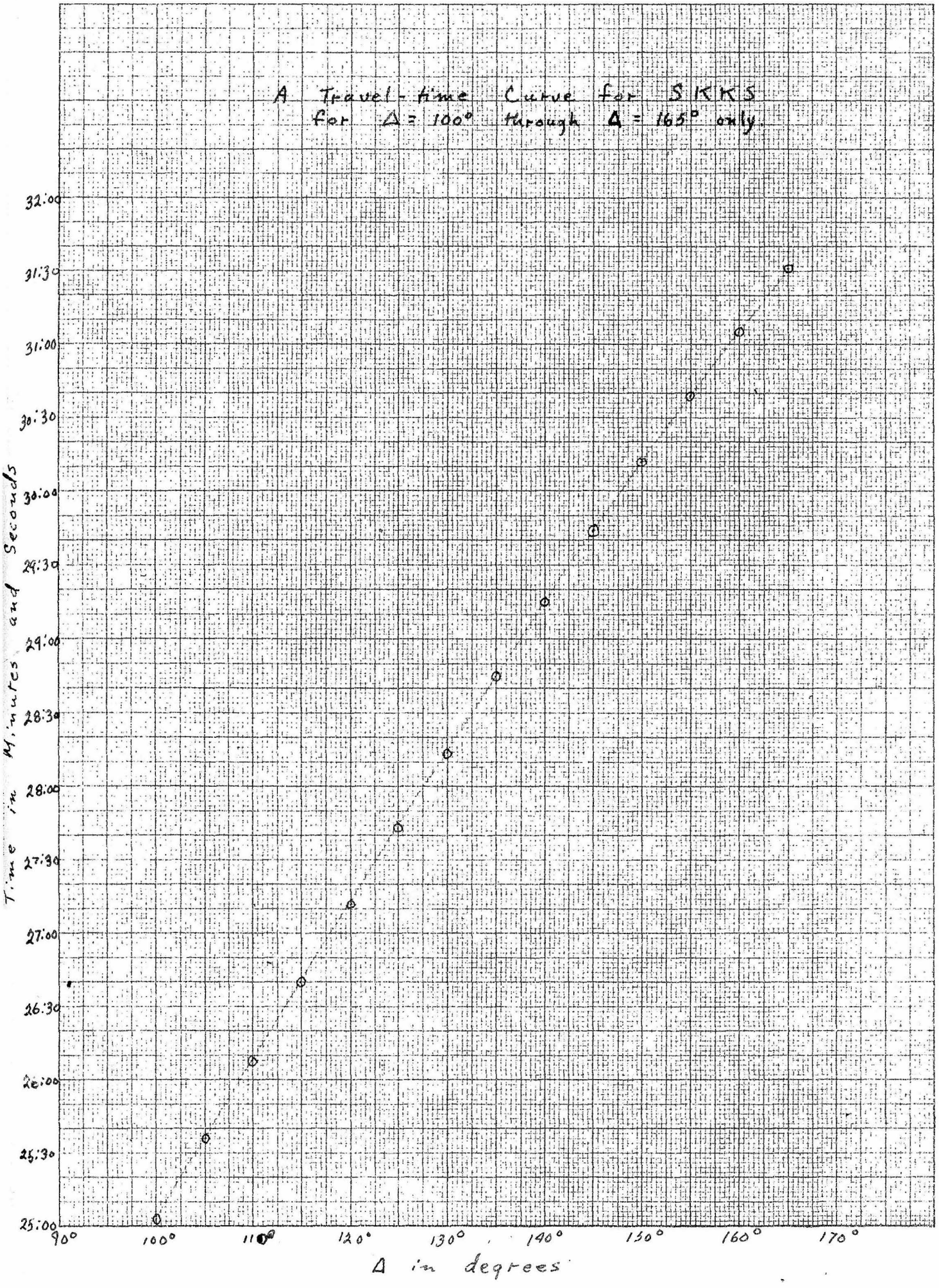
$$\sin i_c = \frac{\sin i_o}{.23892}.$$

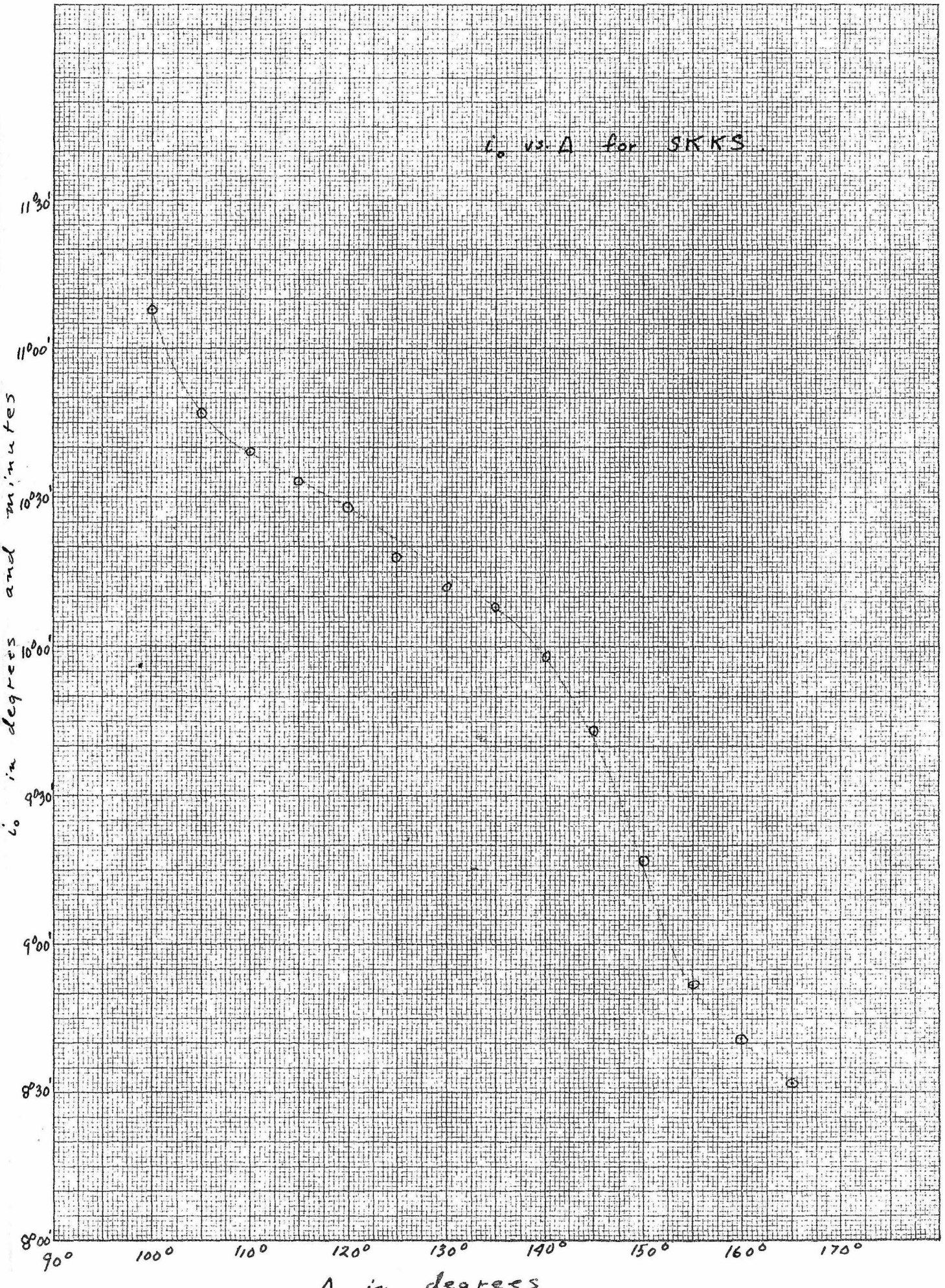
Δ	$\overline{V_\Delta}$	$\sin i_o$	i_o	$\tan i_o$	[1] $\frac{di_o}{d\Delta}$	$\sin i_c$	i_c	[2] $\sin i_r$	i_r
100°	16.55km/sec.	.1931	11°08'	.1968	.0968	.808	53°54'	.891	63°00'
105°	17.08 "	.1872	10°47'	.1905	.0425	.784	51°38'	.864	59°46'
110°	17.32 "	.1847	10°39'	.1881	.01882	.772	50°32'	.851	58°19'
115°	17.48 "	.1830	10°33'	.1862	.01832	.765	49°54'	.844	57°34'
120°	17.61 "	.1817	10°28'	.1847	.02035	.759	49°23'	.836	56°43'
125°	17.89 "	.1789	10°18'	.1817	.02165	.747	48°20'	.824	55°29'
130°	18.03 "	.1772	10°12'	.1799	.02300	.741	47°49'	.817	54°47'

[1] Taken from the plot of i_o vs. Δ on p. 93.

[2] $$\frac{\sin i_r}{8.00} = \frac{\sin i_c}{7.25},$$

$$\sin i_r = \frac{8.00}{7.25} \sin i_c = 1.102 \sin i_c.$$


A Travel-time Curve for SKKS
for Δ = 100° through Δ = 165° only.
32:00
31:30
31:00
30:30
30:00
29:30
29:00
28:30
28:00
27:30
27:00
26:30
26:00
25:30
25:00
Time in Minutes and Seconds
90°
100°
110°
120°
130°
140°
150°
160°
170°
Δ in degrees


i_o vs. Δ for SKKS
11°30'
11°00'
10°30'
10°00'
9°30'
9°00'
8°30'
8°00'
i_o in degrees and minutes
90°
100°
110°
120°
130°
140°
150°
160°
170°
Δ in degrees

Δ	$\bar{V}_\Delta$	$\sin i_o$	i_o	$\tan i_o$	$\frac{di_o}{d\Delta}$	$\sin i_c$	i_c	$\sin i_r$	i_r
135°	18.19km/sec	.1759	10°08'	.1787	.02535	.735	47°18'	.811	54°12'
140°	18.49 "	.1730	9°58'	.1757	.04120	.724	46°23'	.799	53°02'
145°	18.97 "	.1687	9°43'	.1712	.06840	.705	44°50'	.777	50°59'
150°	19.82 "	.1612	9°17'	.1635	.09810	.675	42°27'	.745	48°10'
155°	20.75 "	.1541	8°52'	.1560	.05370	.645	40°10'	.711	45°19'
160°	21.15 "	.1511	8°41'	.1527	.03135	.632	39°12'	.696	49°06'
165°	21.55 "	.1485	8°32'	.1501	.02920	.621	38°23'	.685	43°14'

Δ	[1] f_1	[2] f_2	[3] f_3	[4] f	[5] A	[6] $\frac{u}{A}$	[6] $\frac{w}{A}$	[7] u	[7] w
100°	.875	.499	.867	.378	.1650	1.92	.47	.317	.0776
105°	.892	.462	.888	.365	.1051	1.025	.46	.2025	.0484
110°	.898	.445	.898	.358	.0691	1.930	.45	.1332	.0311
115°	.901	.436	.901	.355	.0685	1.930	.445	.1322	.0305
120°	.902	.429	.904	.349	.0723	1.930	.440	.1394	.0318
125°	.905	.422	.909	.347	.0757	1.930	.435	.1460	.0329
130°	.907	.418	.910	.345	.0799	1.935	.430	.1544	.03435
135°	.908	.413	.912	.342	.0861	1.935	.425	.1665	.0366
140°	.911	.407	.915	.339	.1132	1.940	.420	.2198	.0476
145°	.916	.400	.916	.3355	.1502	1.945	.410	.2925	.0616
150°	.920	.396	.917	.334	.1882	1.950	.390	.3670	.0735
155°	.919	.398	.915	.3345	.1478	1.955	.385	.289	.0569
160°	.917	.402	.914	.337	.1250	1.960	.380	.245	.0475
165°	.915	.405	.912	.3375	.1380	1.960	.375	.2708	.0518

---

[1] f_1 is the square root of the energy refracted into the core at the angle i_c. It is taken from the plot of $\sqrt{\frac{E_f P}{E_i S}}$ vs. angle of incidence on p. 33.

[2] f_2 is the square root of the energy reflected back into the core at the angle i_r. It is taken from the plot of $\sqrt{\frac{E_r P}{E_i P}}$ vs. angle of incidence on p. 27.

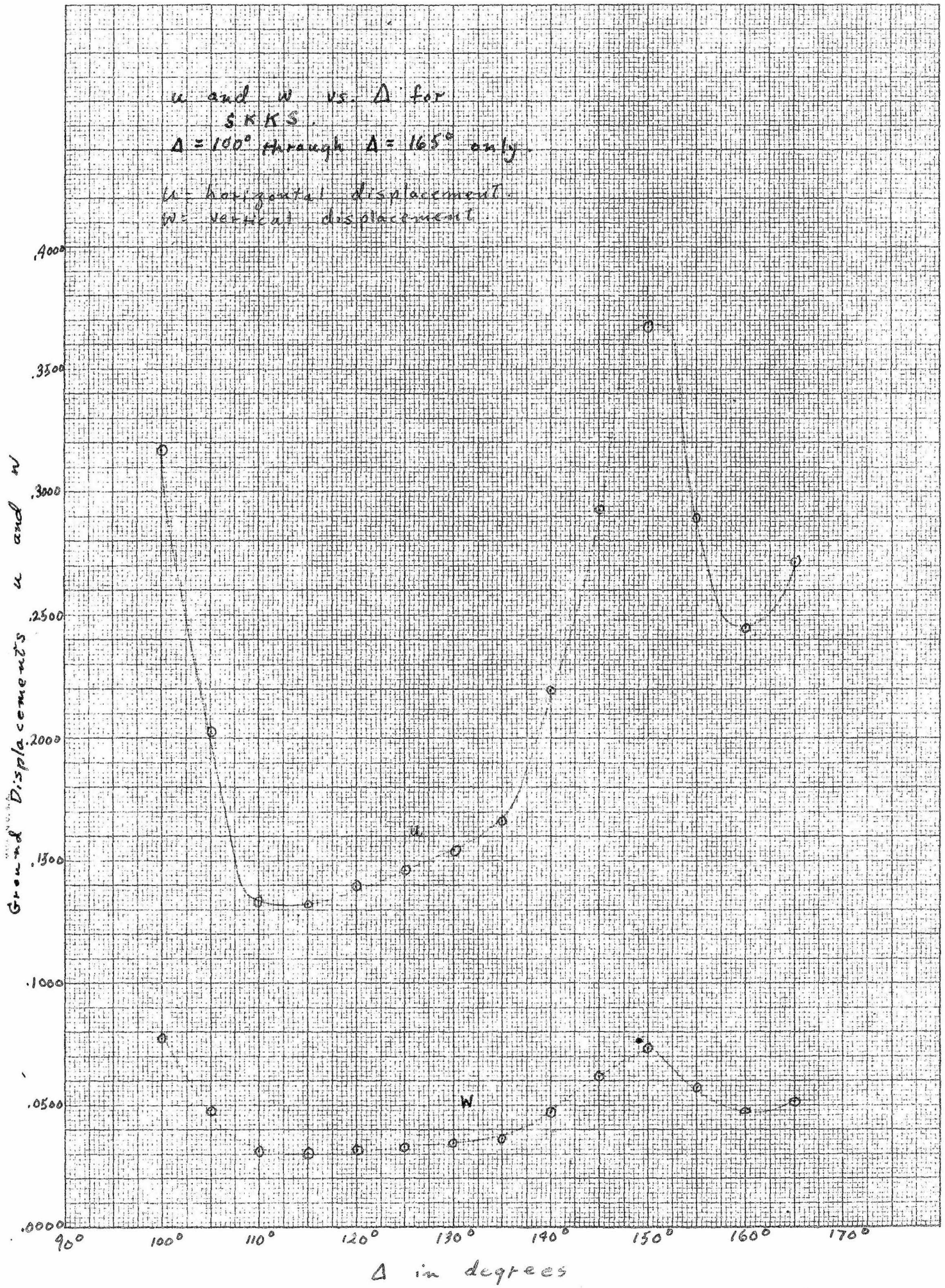
[3] f_3 is the square root of the energy refracted into the mantle at the angle i_r. It is taken from the plot of $\sqrt{\frac{E_f S}{E_i P}}$ vs. angle of incidence on p. 27.

[4] $f = f_1 f_2 f_3$.

[5] $A = CTf \sqrt{\frac{\tan i_o}{\sin \Delta} \frac{di_o}{d\Delta}}$, where $C = \pi$ and $T = 1$ sec.

[6] Taken from Reflections and Refractions, 1943, graphs of Dr. Gutenberg (copy on p. 10).

[7] A plot of u and w against Δ is on p. 95.


u and w vs. Δ for SKKS.
Δ = 100° through Δ = 165° only.
u = horizontal displacement.
w = vertical displacement.
Ground Displacements u and w
.4000
.3500
.3000
.2500
.2000
.1500
.1000
.0500
.0000
u
w
90°
100°
110°
120°
130°
140°
150°
160°
170°
Δ in degrees

4. SKKP

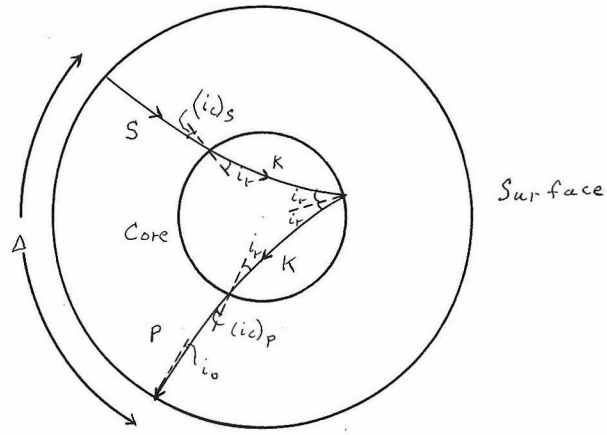


Fig. 19. Sketch to Illustrate the Path of SKKP.

The apparent velocity of the SKKP waves at the surface is taken from the travel-time curve of PKKS and SKKP on p. 87. It is the same as the apparent velocity of PKKS waves. As in the case of PKKS, only the data from $\Delta = 132^\circ$ through $\Delta = 100^\circ$ is used. From this, $\sin i_o = \frac{V_o}{\bar{V}_\Delta}$, where V_o is the true velocity of P waves at the earth's surface (5.5 km. per sec.).

Sin $(i_c)_p$ is determined from

$$\frac{r_s \sin i_o}{(V_p)_s} = \frac{r_c \sin(i_c)_p}{(V_p)_c} ,$$

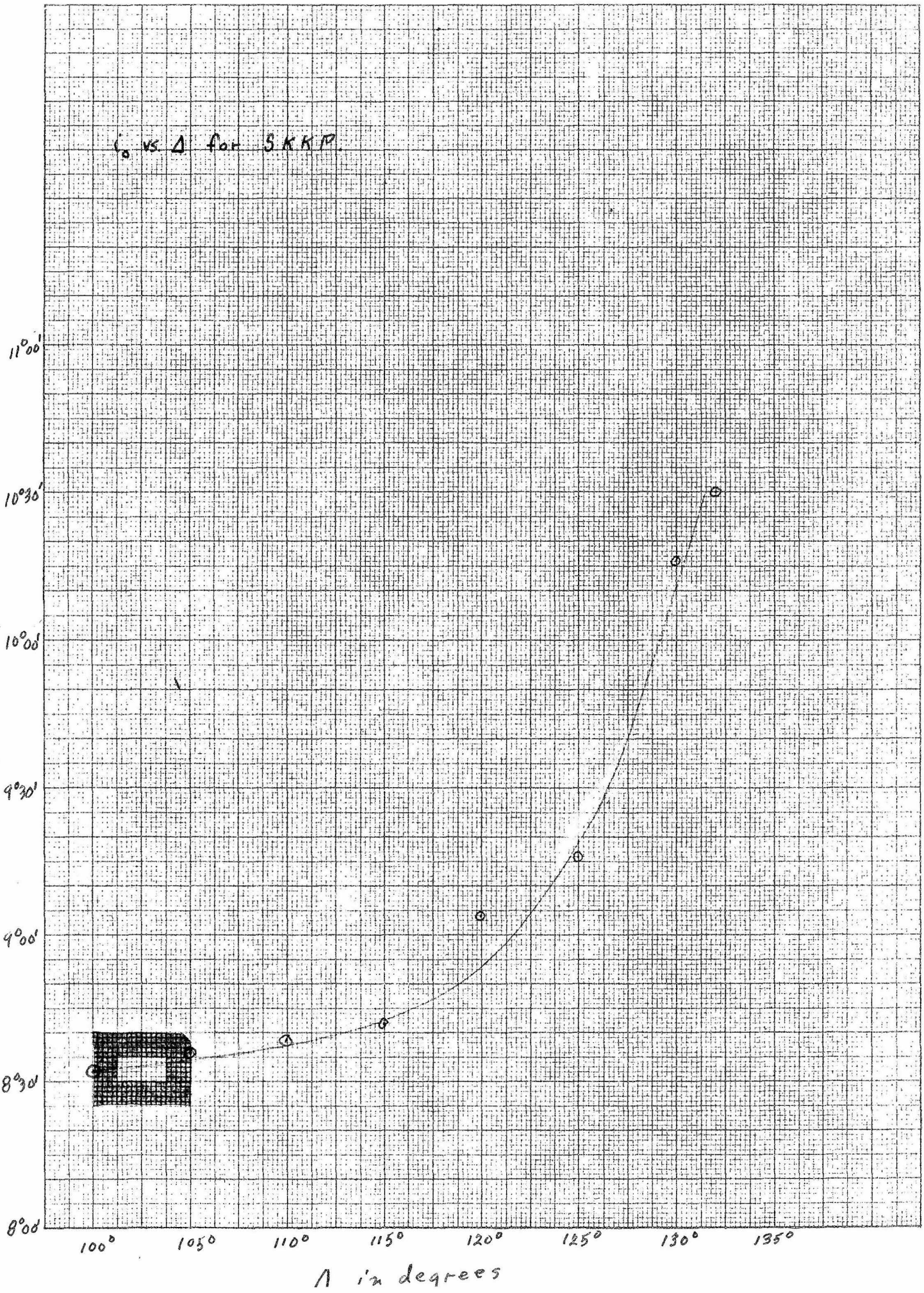
$$\sin i_o = \frac{(V_p)_s r_c}{(V_p)_c r_s} \quad \sin(i_c)_p = \frac{3446}{6366} \times \frac{5.5}{13.7} \quad \sin(i_c)_p ,$$

$$\sin i_o = .21732 \quad \sin(i_c)_p ,$$

$$\text{or } \sin(ic)_p = \frac{\sin i_o}{.21732} .$$

Δ	$\bar{V}_\Delta$	$\sin i_o$	i_o	$\tan i_o$	$\frac{1}{d}\frac{di_o}{\Delta}$	$\sin(i_c)_p$	$(i_c)_p$
132°	30.18km/sec.	.1822	10°30'	.1853	.2368	.840	57°09'
130°	30.80 "	.1782	10°16'	.1811	.2168	.821	55°11'
125°	34.18 "	.1610	9°16'	.1632	.1129	.741	47°49'
120°	34.92 "	.1575	9°04'	.1596	.0550	.725	46°28'
115°	36.30 "	.1514	8°42'	.1530	.02335	.697	44°11'
110°	36.52 "	.1504	8°39'	.1521	.01314	.693	43°52'
105°	36.78 "	.1496	8°36'	.1512	.0085	.689	43°33'
100°	37.00 "	.1485	8°32'	.1501	.00634	.684	43°09'

[1] Taken from the plot of i_o vs. Δ on p. 97.


i_0 vs. Δ for SKKP.
i_0 in degrees and minutes
11°00'
10°30'
10°00'
9°30'
9°00'
8°30'
8°00'
100°
105°
110°
115°
120°
125°
130°
135°
Δ in degrees

Δ	[1] $\sin i_r$	i_r	[2] $\sin(i_c)_s$	$(i_c)_s$	[3] f_1	[4] f_2	[5] f_3	[6] f
132°	.490	29°20'	.449	26°22'	.592	.088	.800	.0416
130°	.480	28°41'	.435	25°47'	.585	.089	.804	.0418
125°	.433	25°40'	.392	23°05'	.546	.088	.838	.0403
120°	.424	25°05'	.384	22°35'	.536	.087	.845	.03945
115°	.407	24°01'	.3685	21°37'	.515	.086	.856	.0379
110°	.405	23°53'	.367	21°32'	.514	.085	.857	.03735
105°	.402	23°42'	.364	21°21'	.511	.0845	.860	.0371
100°	.399	23°31'	.361	21°10'	.506	.0840	.861	.0366

Δ	[7] $A=CTf\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{di_o}{d\Delta}}$	[8] $\frac{u}{A}$	[8] $\frac{w}{A}$	[9] u	[9] w
132°	.0317	.47	1.95	.0149	.0619
130°	.0297	.45	1.96	.01335	.0582
125°	.0190	.40	1.97	.00761	.0375
120°	.01247	.385	1.975	.00480	.0246
115°	.00751	.380	1.975	.00285	.01481
110°	.00541	.375	1.975	.00203	.0107
105°	.00423	.370	1.975	.001565	.00835
100°	.00358	.365	1.975	.001309	.00708

---

[1]
$$\frac{\sin i_r}{8.00} = \frac{\sin(i_c)_p}{13.7},$$
$$\sin i_r = .584 \sin(ic)_p,$$

[2]
$$\frac{\sin(i_c)_s}{7.25} = \frac{\sin i_r}{8.00},$$
$$\sin(i_c)_s = .906 \sin i_r.$$

[3] f_1 is the square root of the energy refracted into the core at the angle $(i_c)_s$. It is taken from the plot of $\sqrt{\frac{E_f P}{E_i S}}$ vs. angle of incidence on p. 33.

[4] f_2 is the square root of the energy reflected back into the core at the angle i_r. It is taken from the plot of $\sqrt{\frac{E_r P}{E_i P}}$ vs. angle of incidence on p. 27.

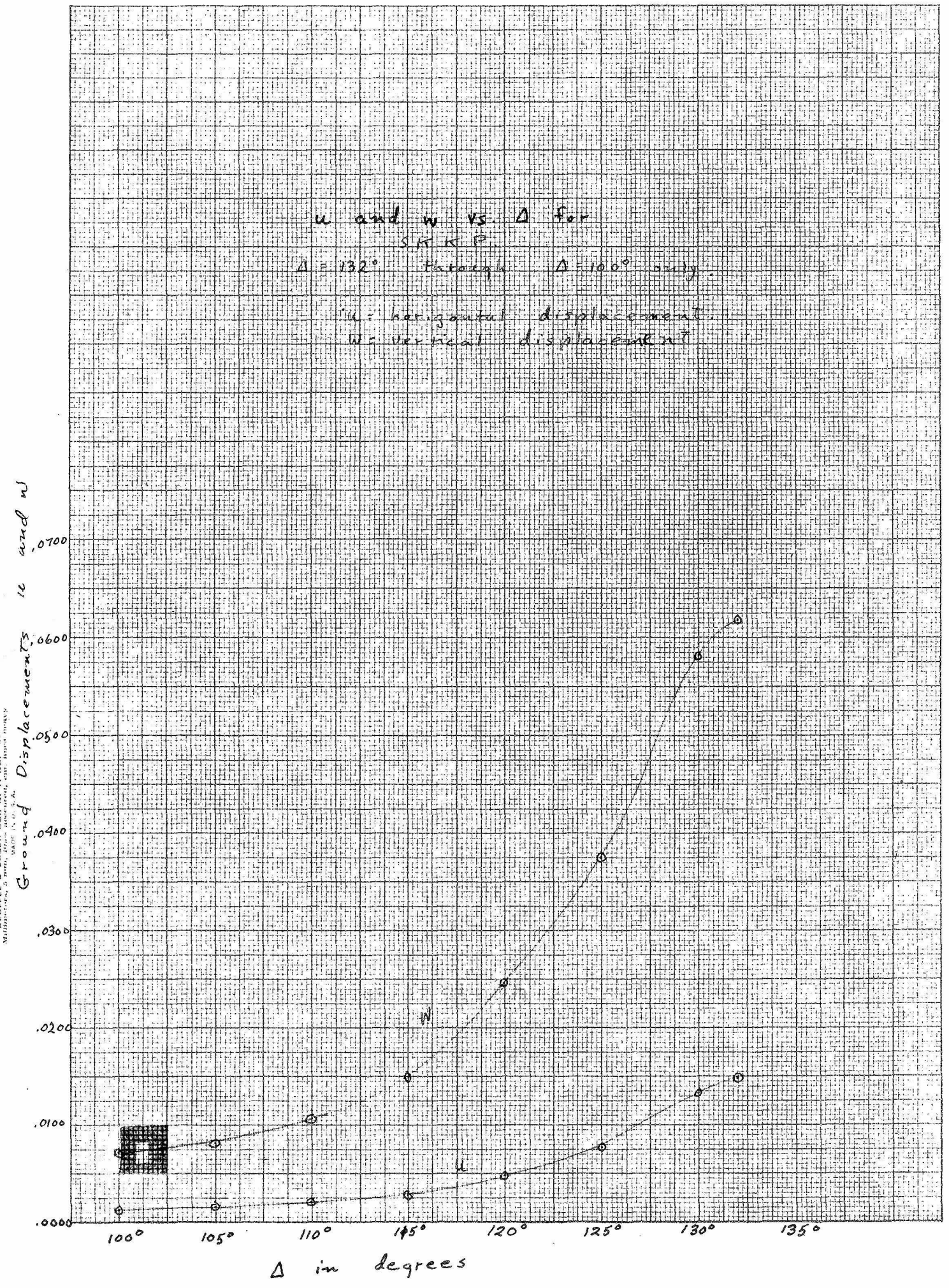
[5] f_3 is the square root of the energy refracted into the mantle at the angle i_r. It is taken from the plot of $\sqrt{\frac{E_f P}{E_i P}}$ vs. angle of incidence on p. 27.

[6] $f = f_1 f_2 f_3$.

[7] $C = \pi$ and $T = 1$ sec.

[8] Taken from Reflections and Refractions, 1943, graphs of Dr. Gutenberg (copy on p. 10).

[9] A plot of u and w against Δ is on p. 99.


u and w vs. Δ for
SKKP.
Δ = 132° through Δ = 100° only.
u = horizontal displacement.
w = vertical displacement
Ground Displacements in u and w
.0700
.0600
.0500
.0400
.0300
.0200
.0100
.0000
w
u
100°
105°
110°
115°
120°
125°
130°
135°
Δ in degrees

Group IV. Waves Transmitted Two or Three Times by the Core of the Earth.

1. P'P'

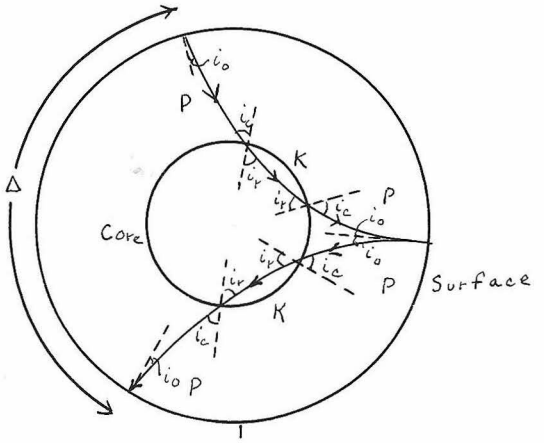


Fig. 20. Sketch to Illustrate the Path of P'P'.

For the case of P'P' the angle of incidence at a particular Δ is the same as the angle of incidence of P' at $\frac{1}{2}$ that particular Δ. Also $\frac{d\, i_o}{d\, \Delta}$ for P'P' at a particular Δ is $\frac{1}{2}$ the $\frac{d\, i_o}{d\, \Delta}$ for P' at $\frac{1}{2}$ that particular Δ. This is because as P' goes from 179° to 180°, the change in the angle of incidence is the same as for P'P' when it goes from 2° to 0° (or 358° to 360°). In the following data Δ is given as the shorter distance from point of origin to point of emergence. Thus a Δ of 150° for P' becomes 300° for P'P' or 60° in the following material. It is obvious that f, the square root of the energy in the P'P' wave arriving at the surface at a particular delta is the square of the f for the P' wave at $\frac{1}{2}$ that particular Δ times the square root of the energy reflected from the surface at that i_o at $\frac{1}{2}$ the Δ of the P'P'.

Δ	[1] i_o	[1] $\tan i_o$	[1] $\frac{di_o}{d\Delta}$	$\sin\Delta$	[2] $\sqrt{\frac{E\,refl}{E\,inc}}$	[*] $(f_{P'})^2$	[3] f	[4] A	[5] $\frac{u}{A}$	[5] $\frac{w}{A}$	[6] u	[6] w
0°	12°30'	.2217	.015	.000	.920	.0899	.0827	∞	.530	1.930	∞	∞
20°	12°11'	.2159	.015	.342	.925	.457	.423	.1295	.520	1.940	.0674	.2515
40°	11°55'	.2110	.015	.643	.930	.401	.373	.0821	.515	1.945	.0423	.1595
60°	11°22'	.2010	.0584	.866	.935	.378	.354	.1294	.500	1.955	.0648	.2535
75°	9°15'	.1629	∞	.966	.950	.491	.466	∞	.410	1.965	∞	∞
75°	9°15'	.1629	∞	.966	.950	.491	.466	∞	.410	1.965	∞	∞

---

1 Taken from the data on p. 54 and p. 55.

2 $\sqrt{\frac{E\ refl.}{E\ inc.}}$ is the square root of the energy reflected from the land surface at i_o at $\frac{1}{2}$ the Δ of the arriving P'P' wave. It is taken from Reflections and Refractions 1943, graphs of Dr. Gutenberg (copy on p. 10).

* Taken from p. 58 and p. 59.

3 $f = (f_{P'})^2 \sqrt{\frac{E\ reflec.}{E\ inc.}}$

4 $A = CTf \sqrt{\frac{\tan i_o}{\sin\Delta}\,\frac{di_o}{d\Delta}}$ where $C = \pi$, $T = 1.0$ sec.

5 Taken from the data on p. 58 and p. 59. $\frac{u}{A}$, for example, at 40° (320°) for P'P' is the same as $\frac{u}{A}$ for P' at 160°.

6 Plotted against Δ on p. 102 and p. 103.

Δ	i_o	$\tan i_o$	$\frac{di_o}{d\Delta}$	$\sin\Delta\sqrt{\frac{E_{refl.}}{E_{inc.}}}$	$(f_{p'})^2$	f	A	$\frac{u}{A}$	$\frac{w}{A}$	u	w
60°	8°38'	.1518	.01108	.866	.955	.535	.511	.071	.390	1.975	.0277	.140
40°	8°22'	.1471	.00662	.643	.960	.558	.536	.0655	.385	1.980	.0252	.1298
22°	8°18'	.1459	∞	.375	.965	.565	.545	∞	.385	1.980	∞	∞
22°	8°18'	.1459	∞	.375	.965	.565	.545	∞	.385	1.980	∞	∞
40°	7°20'	.1287	.0424	.643	.970	.645	.626	.1815	.320	1.985	.0581	.362
60°	6°43'	.1178	.0249	.866	.975	.686	.670	.1221	.290	1.985	.0354	.2425
80°	6°15'	.1095	.0167	.985	.980	.720	.706	.0957	.280	1.985	.0268	.1898
100°	6°08'	.1075	.00501	.985	.983	.729	.716	.0526	.280	1.985	.01473	.1045
120°	6°02'	.1057	.00501	.866	.985	.734	.722	.0561	.275	1.985	.01541	.1114
140°	5°53'	.1031	∞	.643	.987	.748	.739	∞	.275	1.985	∞	∞
140°	5°53'	.1031	∞	.643	.987	.748	.739	∞	.275	1.985	∞	∞
120°	5°48'	.1016	.01249	.866	.988	.755	.746	.0896	.275	1.985	.0246	.1778
100°	5°28'	.0957	.0208	.985	.989	.775	.766	.1082	.250	1.990	.0271	.2155
80°	4°52'	.0851	.0359	.985	.990	.816	.808	.1412	.210	1.990	.0297	.2815
60°	4°18'	.0752	.0167	.866	.992	.855	.849	.1018	.195	1.990	.01985	.2028
40°	3°33'	.0620	.0978	.643	.995	.900	.895	.2739	.170	1.995	.0466	.546
20°	0°56'	.0163	.1083	.342	.999	.990	.989	.223	.060	2.000	.0134	.446
0°	0°00'	.0000	.0333	.000	1.000	.996	.996	.1041[1]	.000	2.000	.0000	.208

---

[1] At $\Delta = 0°$, $\tan i_o = 0$ and $\sin\Delta = 0$. Hence the value of A here is determined by taking the value of $\frac{\tan i_o}{\sin\Delta}$ when i_o and Δ are both quite small. Using the data on p. 59 for determining A of P' at $\Delta = 180°$;

Δ	$(\tan i_o)_{P'P'}$	$\sin\Delta$	$\frac{\tan i_o}{\sin\Delta}$
20°	.0163	.342	.0977
10°	.0055	.174	.0316
6°	.00349	.1045	.0334
4°	.00233	.0698	.0334
2°	.00116	.0349	.0333
1°	.00058	.0174	.03335

It is evident that in the vicinity of $\Delta = 0°$, $\frac{\tan i_o}{\sin\Delta}$ becomes .0333 for P'P'.

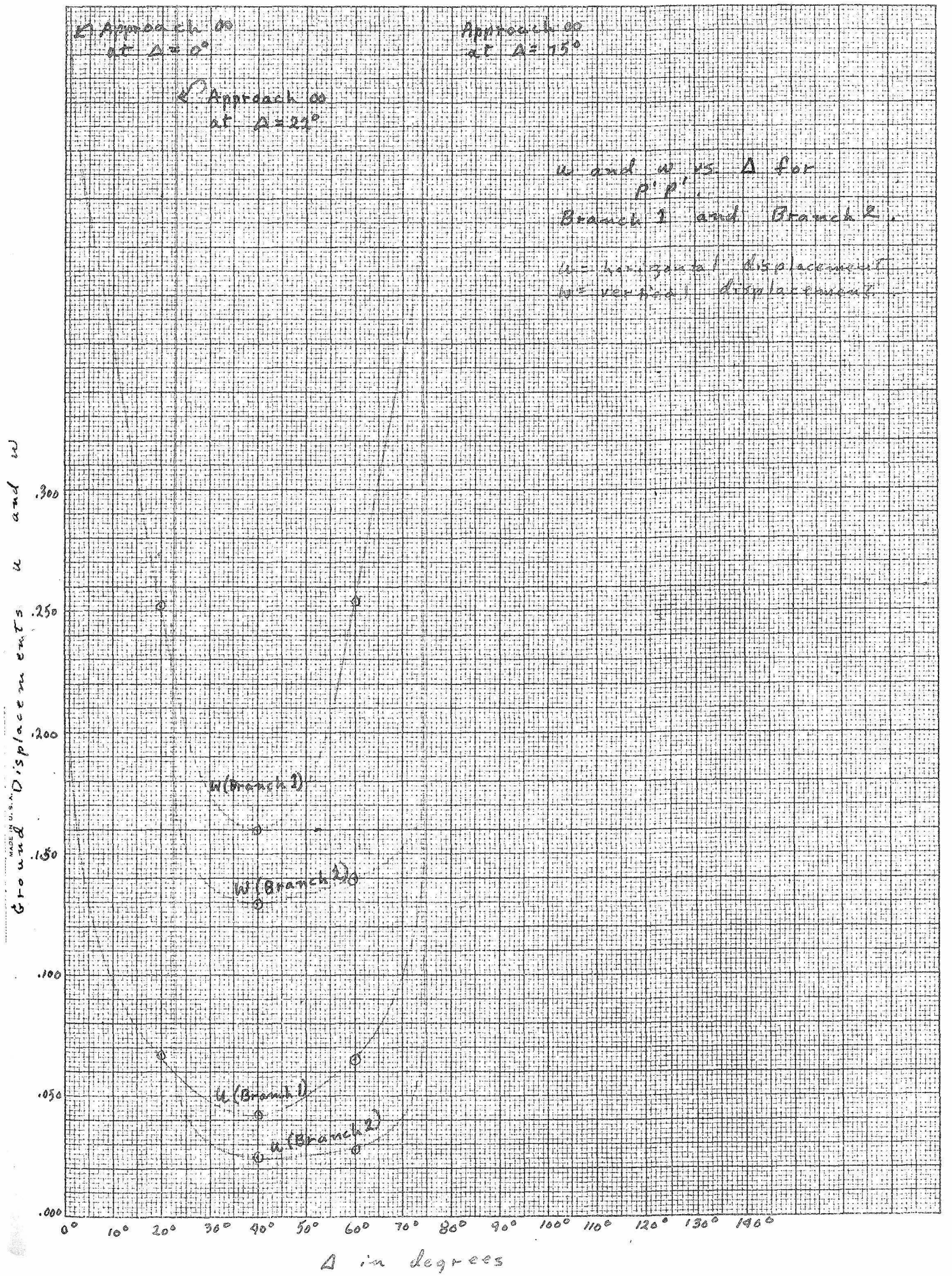
Thus

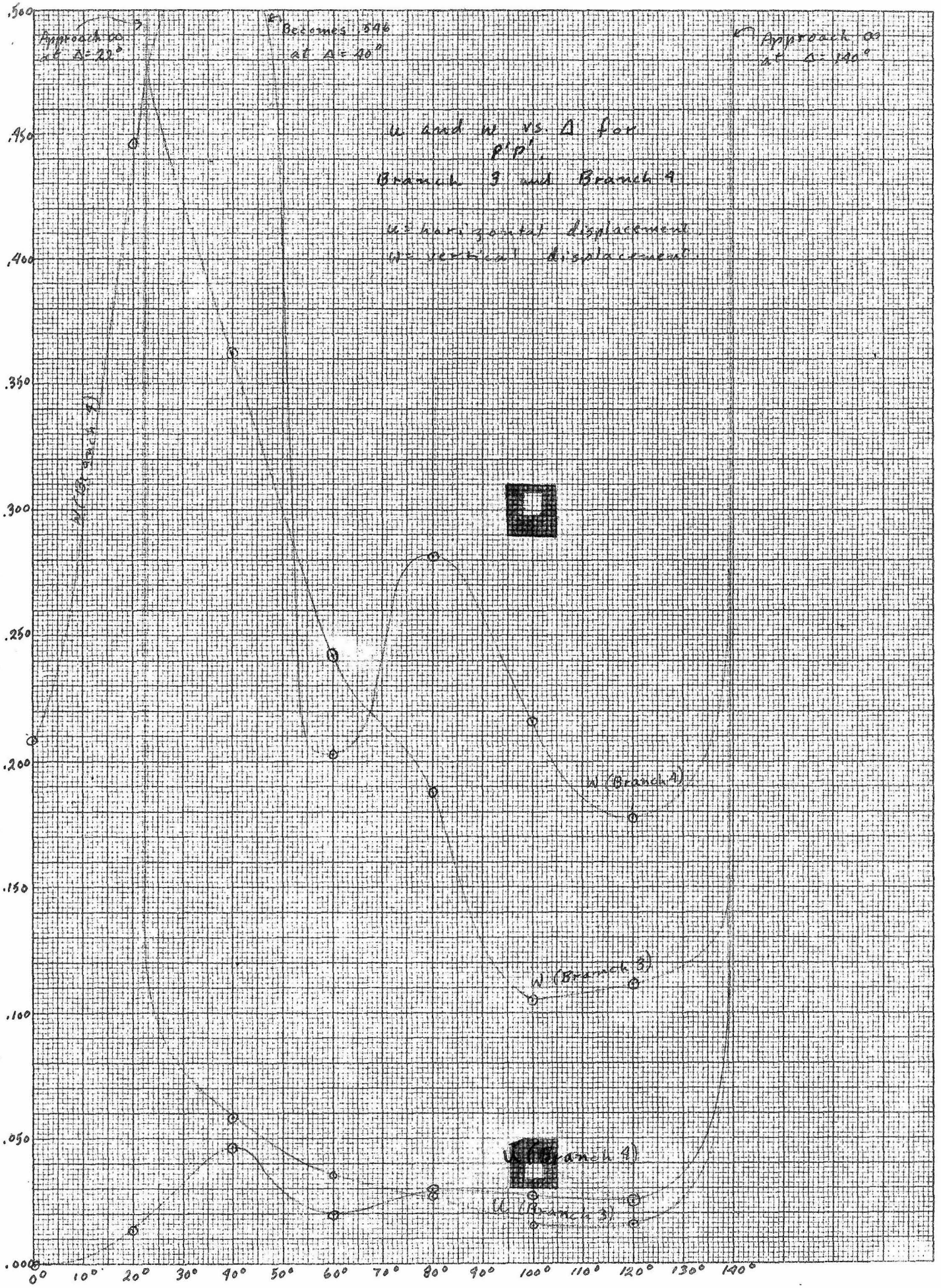
$$A = CTf\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{di_o}{d\Delta}} = \pi\ .996\sqrt{(.0333)(.0333)} = \pi\,(.996)(.0333),$$

or A = .1041 .

This is, of course, an approximation.

Note: In the above data for P'P', $\Delta = 0° - 75°$ is labeled Branch 1. It corresponds to the P'_2 branch of P'. $\Delta = 75° - 22°$ is labeled Branch 2 (the unnamed segment of P'_1 corresponds to it). $\Delta = 22° - 140°$ is labeled Branch 3 (reversed segment of P'). $\Delta = 140° - 0°$ is labeled Branch 4 (corresponds to the P" and P'_2 branch of P').


Approach ∞ at Δ = 0°
Approach ∞ at Δ = 15°
Approach ∞ at Δ = 24°
u and w vs. Δ for P' P' Branch 1 and Branch 2.
u = horizontal displacement
w = vertical displacement
Ground Displacements u and w
.300
.250
.200
.150
.100
.050
.000
W (Branch 1)
W (Branch 2)
u (Branch 1)
u (Branch 2)
0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140°
Δ in degrees


.500
.450
.400
.350
.300
.250
.200
.150
.100
.050
.000
0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140°
Δ in degrees
Ground Displacements u and w
Approach as Δ → 22°
Becomes .596 at Δ = 40°
Approach as Δ → 140°
u and w vs Δ for p'p', Branch 3 and Branch 4
u = horizontal displacement
w = vertical displacement
w (Branch 3)
w (Branch 4)
u (Branch 4)
u (Branch 3)

2. P'P'P'

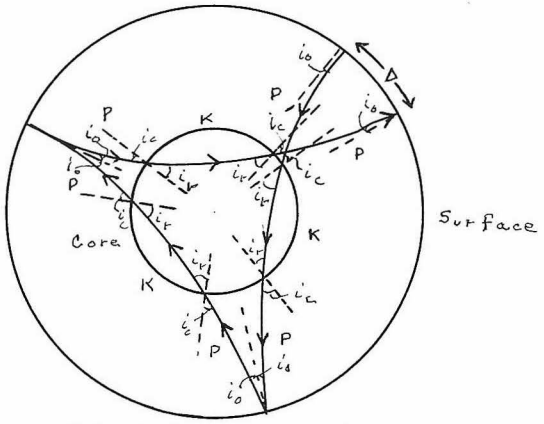


Fig. 21.

Sketch to Illustrate the Path of P'P'P'.

For the case of P'P'P' the angle of incidence at a particular Δ is the same as the angle of incidence of P' at 1/3 that particular Δ. In this data as in the case of P'P', Δ is given as the shorter distance from origin to point of emergence. It therefore ranges from -30° (330°) to + 180°(540°).

For P'P'P' $\frac{di_o}{d\Delta}$ at a particular Δ is 1/3 the $\frac{di_o}{d\Delta}$ for P' at 1/3 the particular Δ. This is because as P' goes from 179° to 180° the change in the angle of incidence is the same as for P'P'P' when it goes from 177° to 180° (537° to 540°).

f_p the square root of the energy in the P'P'P' wave arriving at the surface at a particular Δ, is the product of $f_{p'}{}^3$ and $\left(\sqrt{\frac{E_{refl.}}{E_{inc.}}}\right)^2$. $f_{p'}$ is the square root of the energy arriving in the P' wave at 1/3 the Δ of the P'P'P' wave. $\sqrt{\frac{E_{refl.}}{E_{inc.}}}$ is the square root of the energy reflected from the ~~land~~ surface at 1/3 and 2/3rd's of the Δ of the P'P'P' wave.

Δ	[1] i_o	[1] $\tan i_o$	[1] $\frac{di_o}{d\Delta}$	$\sin\Delta\sqrt{\frac{E_{refl.}}{E_{inc.}}}$ [*]	$\left(\sqrt{\frac{E_{refl.}}{E_{inc.}}}\right)^2$ [*]	$(f_{p'})^3$ [**]	f	[2] A	[3] $\frac{u}{A}$	[3] $\frac{w}{A}$	[4] u	[4] w
180°	12°30'	.2217	.010	.000	.845	.0268	.02265	∞	.530	1.930	∞	∞
150°	12°11'	.2159	.010	.500	.855	.3085	.264	.0545	.520	1.940	.0283	.1058
120°	11°55'	.2110	.010	.866	.865	.254	.2197	.0340	.515	1.945	.0175	.0661
90°	11°22'	.2010	.0389	1.000	.874	.2323	.203	.0565	.500	1.955	.0282	.1105
67.5°	9°15'	.1629	∞	.924	.902	.344	.3102	∞	.410	1.965	∞	∞
67.5°	9°15'	.1629	∞	.924	.902	.344	.3102	∞	.410	1.965	∞	∞

[1] Taken from the data on p. 54 and p. 55.

[*] Taken from Reflections and Refractions, 1943, graphs of Dr. Gutenberg (copy on p.10)

[**] $f_{p'}{}^3$ is taken from the data on p. 58 and p. 59.

[2] $A = CTf\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{di_o}{d\Delta}}$ where $C = \pi$ and $T = 1.0$ sec.

[3] Taken from the data on p. 58 and p. 59. $\frac{u}{A}$, for example, at 90° (450°) for P'P'P' is the same as $\frac{u}{A}$ for P' at 150°.

[4] Plotted against Δ on p. 106 and p. 107.

Δ	i_o	$\tan i_o$	$\frac{di_o}{d\Delta}$	$\sin\Delta\left(\sqrt{\frac{E_{refl.}}{E_{inc.}}}\right)^2 (f_{p'})^3$		f	A	$\frac{u}{A}$	$\frac{w}{A}$	u	w
90°	8°38'	.1518	.00738	1.000	.911	.391	.356	.0374	.390	1.975	.0146	.0740
120°	8°22'	.1471	.00441	.866	.022	.417	.385	.0332	.385	1.980	.0128	.0658
147°	8°18'	.1459	∞	.545	.930	.424	.394	∞	.385	1.980	∞	∞
147°	8°18'	.1459	∞	.545	.930	.424	.394	∞	.385	1.980	∞	∞
120°	7°20'	.1287	.0283	.866	.940	.517	.486	.0992	.320	1.985	.0317	.1965
90°	6°43'	.1178	.0166	1.000	.951	.568	.531	.0741	.290	1.985	.0215	.147
60°	6°15'	.1095	.0111	.866	.960	.611	.586	.0690	.280	1.985	.0193	.137
30°	6°08'	.1075	.00334	.500	.965	.621	.600	.0504	.280	1.985	.0141	.100
0°	6°02'	.1057	.00334	.000	.970	.628	.609	∞	.275	1.985	∞	∞
-30°	5°53'	.1031	∞	.500	.974	.647	.630	∞	.275	1.985	∞	∞
-30°	5°53'	.1031	∞	.500	.974	.647	.630	∞	.275	1.985	∞	∞
0°	5°48'	.1016	.00833	.000	.976	.656	.641	∞	.275	1.985	∞	∞
30°	5°28'	.0957	.0138	.500	.978	.682	.666	.1073	.250	1.990	.0269	.214
60°	4°52'	.0851	.0239	.866	.980	.737	.721	.1100	.210	1.990	.0231	.219
90°	4°18'	.0752	.0111	1.000	.984	.790	.775	.0704	.195	1.990	.01372	.140
120°	3°33'	.0620	.0650	.866	.990	.854	.845	.1815	.170	1.995	.0308	.362
150°	0°56'	.0163	.0722	.500	.998	.985	.983	.1500	.060	2.000	.00901	.300
180°	0°00'	.0000	.0222	.000	1.000	.994	.994	.0694[1]	.000	2.000	.000	.1388

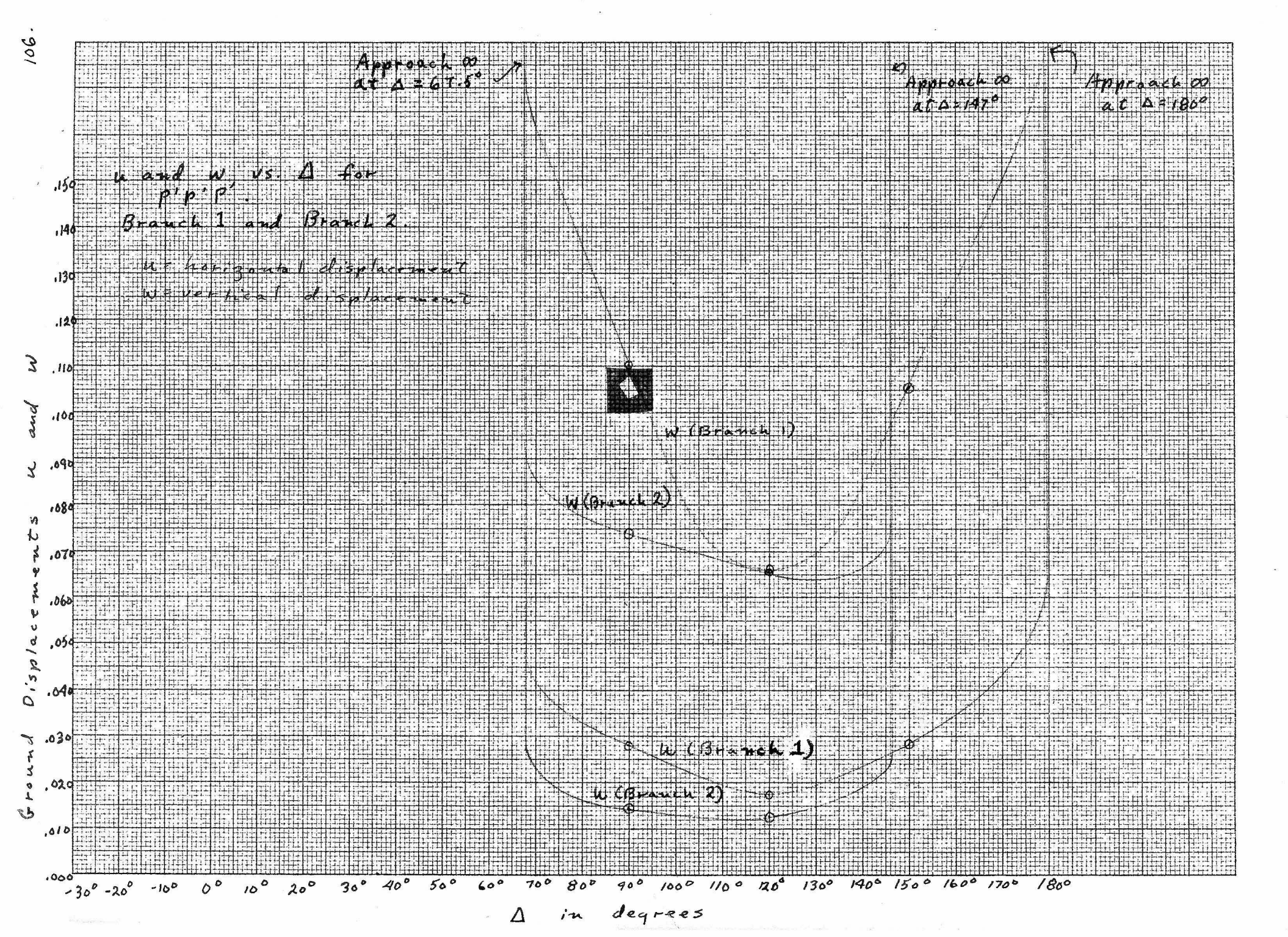
---

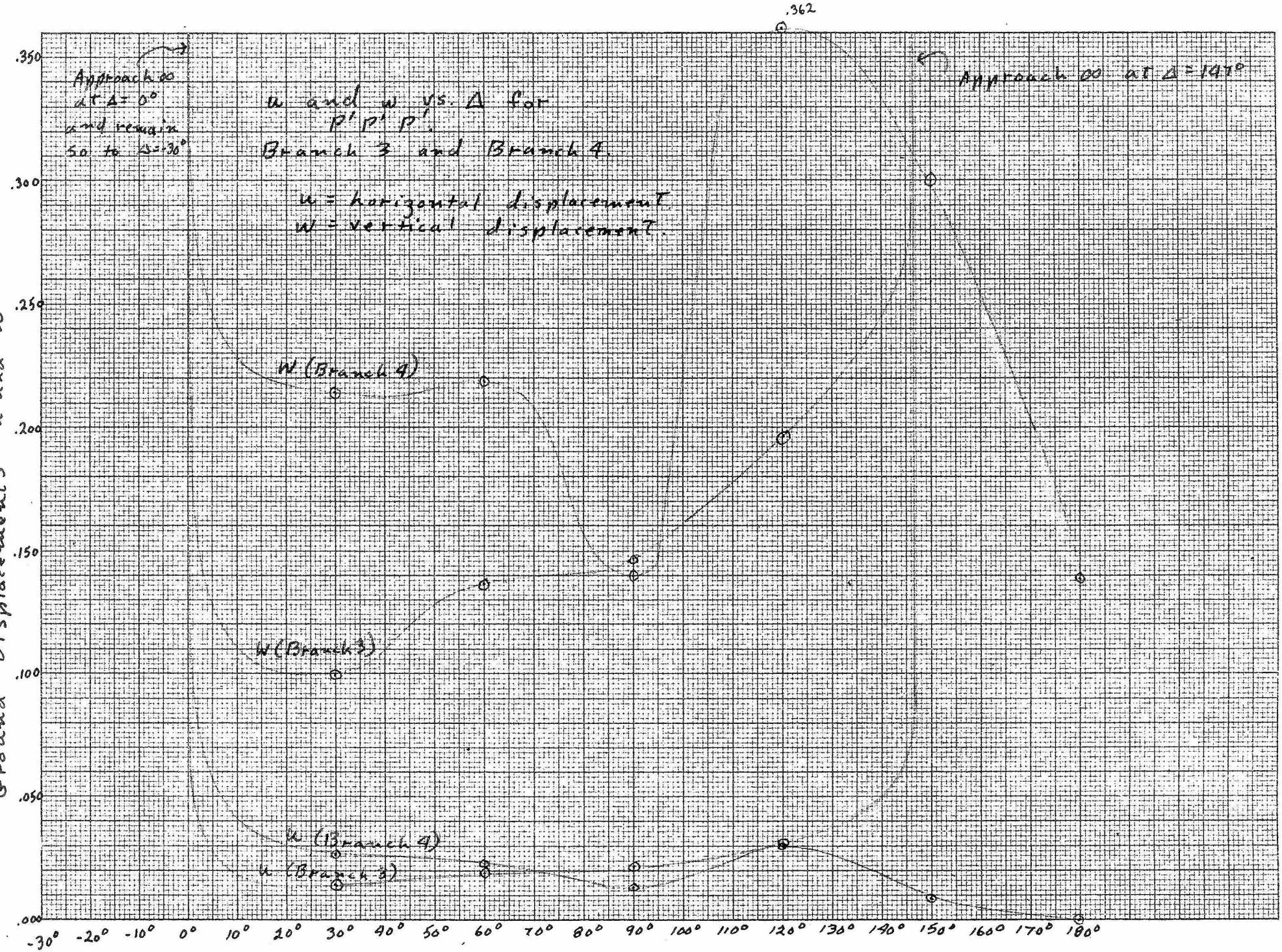
[1] At $\Delta = 180°$, $\tan i_o = 0$ and $\sin = 0$. Hence the value of A here is determined by taking the value of $\frac{\tan i_o}{\sin \Delta}$ when i_o and Δ are both very small. Using the data on p. 59 for determining A of P' at $\Delta = 180°$:

Δ	$(\tan i_o)_{P'P'P'}$	$\sin\Delta$	$\frac{\tan i_o}{\sin\Delta}$
150°	.0163	.500	.0326
165°	.0055	.259	.0212
171°	.00349	.156	.0224
174°	.00233	.105	.02225
177°	.00116	.0523	.0222
178.5°	.00058	.0262	.0222

It is evident that in the vicinity of $\Delta = 180°$, $\frac{\tan i_o}{\sin \Delta}$ becomes .0222 for P'P'P'. Hence $A = \pi \times .994 \sqrt{(.0222)(.0222)} = .0694$, an approximation.

Note: In the above data for P'P'P', $\Delta = 180° - 67.5°$ is labeled Branch 1, $\Delta = 67.5° - 147°$ is labeled Branch 2, $\Delta = 147°$ to $-30°$ Branch 3, and $\Delta = -30°$ to $180°$ Branch 4.. Branch 1 corresponds to the P_2' branch of P', Branch 2 corresponds to the unnamed segment branch of P', Branch 3 to the reversed segment branch of P', and Branch 4 to the P'' and P_1' branch of P'.


Approach ∞ at Δ = 67.5°
Approach ∞ at Δ = 147°
Approach ∞ at Δ = 180°
u and w vs. Δ for p' p · P'.
Branch 1 and Branch 2.
u = horizontal displacement
w = vertical displacement
w (Branch 1)
w (Branch 2)
w (Branch 1)
w (Branch 2)
Ground Displacements u and w
.150
.140
.130
.120
.110
.100
.090
.080
.070
.060
.050
.040
.030
.020
.010
.000
-30° -20° -10° 0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140° 150° 160° 170° 180°
Δ in degrees



Millimeter, 3 mm. lines accented, cm. lines heavy.
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Part III. Calculation of the Vertical and Horizontal Components of the Ground Displacement Produced by Direct Seismic Waves and of the Ratios of the Vertical and Horizontal Components of the Body Waves to Those of the Corresponding Direct Waves.

1. P and its u and w Ratios.

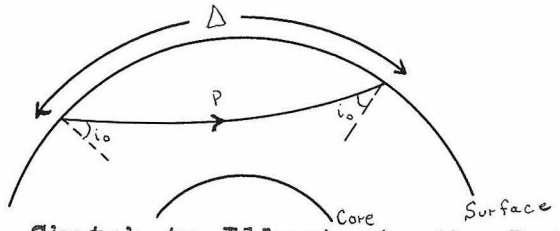


Fig. 22. Sketch to Illustrate the Path of P.

For the core of P, $\sin i_o = \frac{V_o}{\overline{V_\Delta}}$, where $\overline{V_\Delta}$ is the apparent velocity of P, as measured by the tangent to the travel-time curve at an arcual epicentral distance Δ, and V_o is the true velocity of P at the surface of the earth (5.5 km. per sec.).

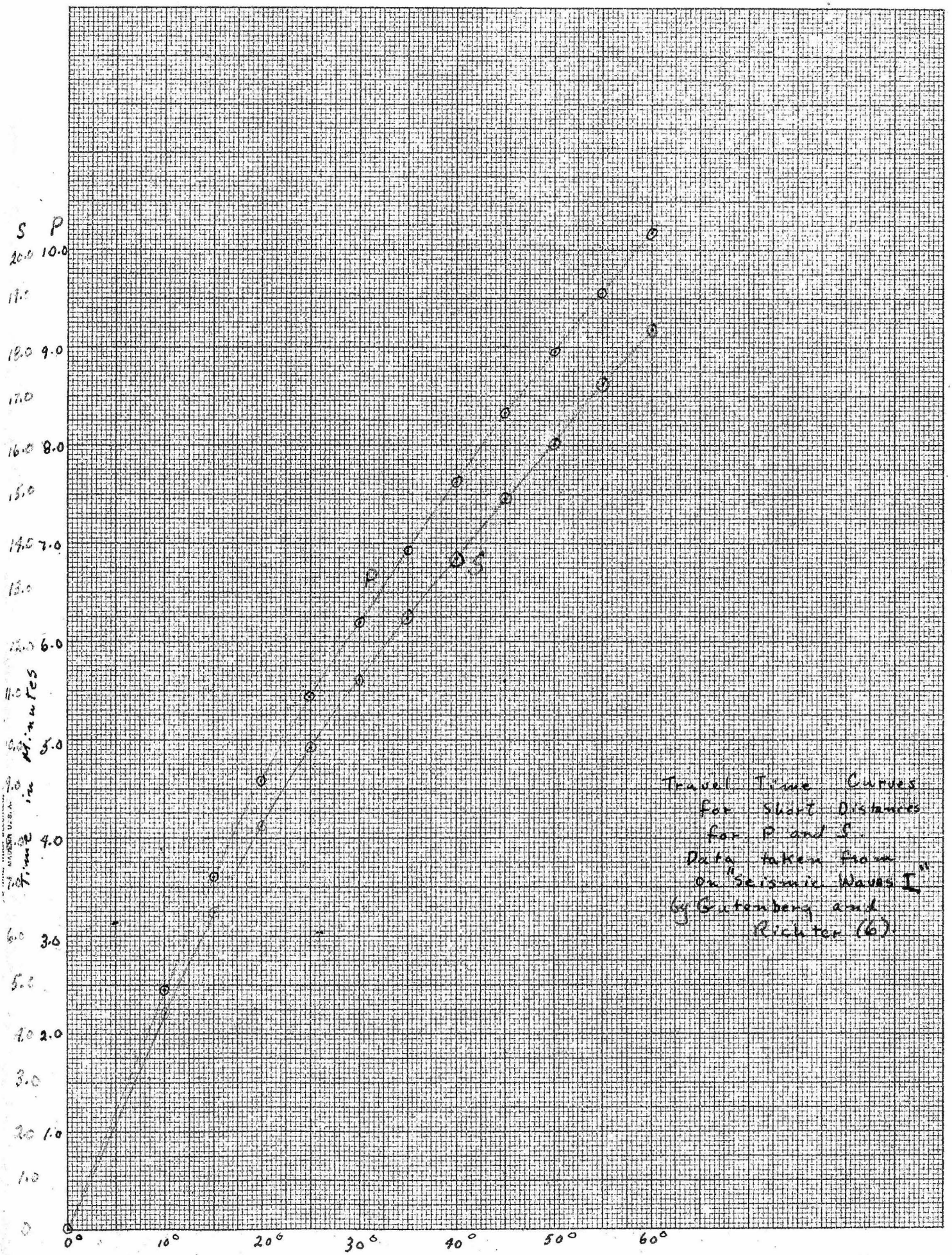
Δ	[1] $\overline{V_\Delta}$	$\sin i_o$	i_o	$\tan i_o$	[2] $\frac{di_o}{d\Delta}$	[3] A	[4] $\frac{u}{A}$	[4] $\frac{w}{A}$
15°	* 8.81km/sec	.6243	38°38'	.7992	1.345	6.39	1.34	1.52
20°	*10.28 "	.5350	32°21'	.6334	.910	4.07	1.20	1.63
30°	*12.43 "	.4435	26°16'	.4935	.325	1.78	.96	1.77
40°	*13.39 "	.4108	24°15'	.4505	.220	1.232	.90	1.80
50°	*14.80 "	.3716	21°49'	.4003	.210	1.04	.85	1.82
60°	16.02 "	.3433	20°05'	.3656	.206	.929	.78	1.85
70°	18.81 "	.2924	17°00'	.3057	.202	.806	.65	1.90
80°	20.35 "	.2703	15°41'	.2808	.1975	.752	.61	1.91
90°	22.82 "	.2410	13°57'	.2484	.1825	.671	.58	1.93
100°	25.90 "	.2124	12°16'	.2174	.0875	.437	.50	1.94
103°	25.95 "	.2120	12°14'	.2168	.0200	.214	.50	1.94

[1] Taken from the travel-time curve in "On Seismic Waves I" by Gutenberg and Richter (6). *Taken from a plot on p. 109 of Δ vs. t for P and S from data in "On Seismic Waves I" for $\Delta = 0° - 60°$. This travel-time curve is smooth and does not consider irregularities, as may be noted by referring to Fig. 92 p. 202 of Gutenberg (5) and Figs. 6 and 7 and pp. 332-344 of Gutenberg and Richter (7).

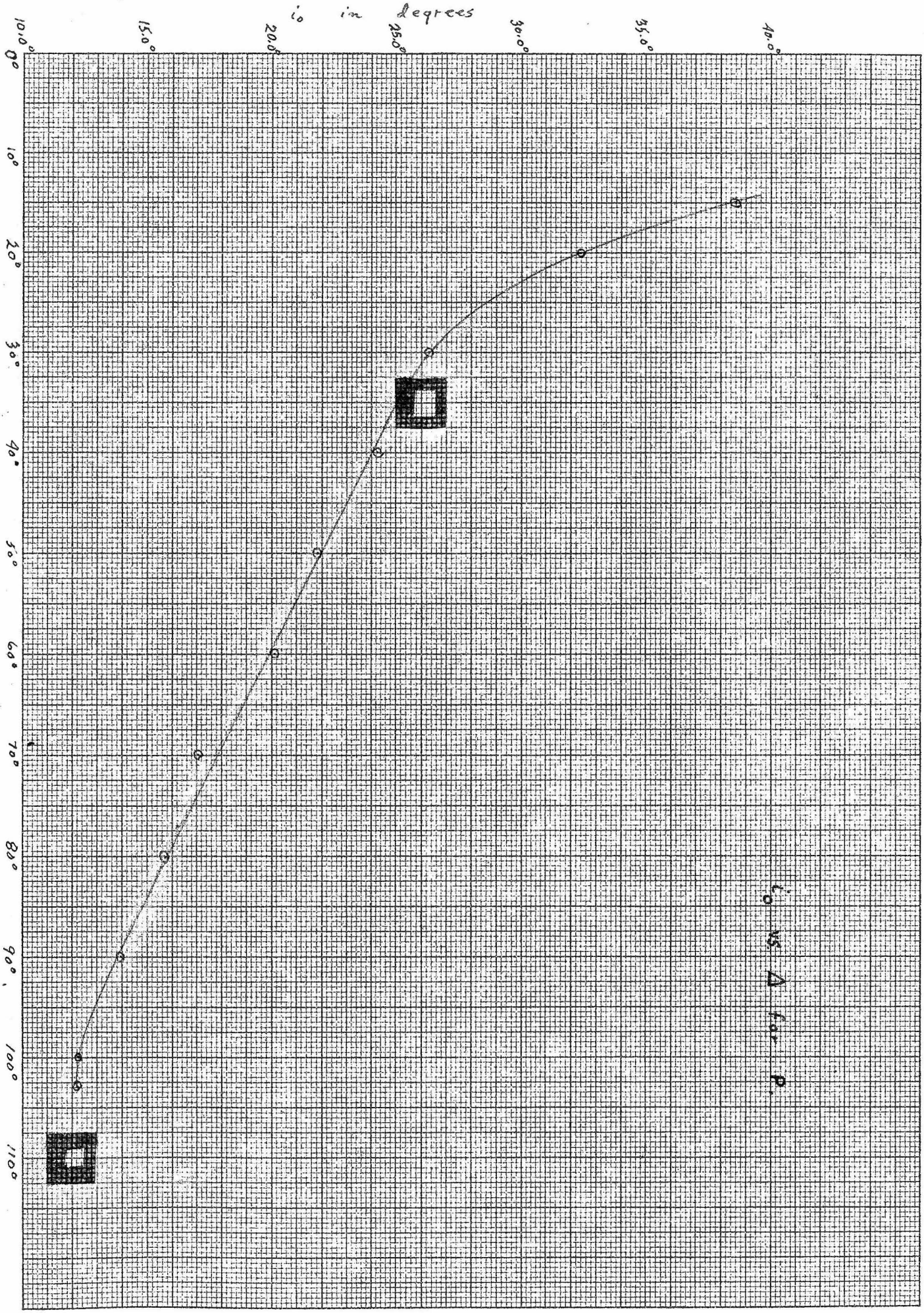
[2] Taken from a plot of i_o vs. Δ on p. 110.

[3] $A = CTf \sqrt{\frac{\tan i_o}{\sin \Delta} \frac{di_o}{d\Delta}}$, where $C = \pi$, $T = 1$ sec., and $f = 1.00$.

[4] Taken from Reflections and Refractions, 1943, graphs of Dr. Gutenberg. (Copy on p. 10.)



Travel Time Curves for Short Distances for P and S. Data taken from "Seismic Waves I" by Gutenberg and Richter (6).


i in degrees
10.0°
15.0°
20.0°
25.0°
30.0°
35.0°
40.0°
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
Δ in degrees
i vs Δ for D.

Δ	^{1}u	^{1}w
15°	8.57	9.73
20°	4.89	6.64
30°	1.71	3.15
40°	1.108	2.22
50°	.885	1.894
60°	.724	1.717
70°	.524	1.531
80°	.458	1.435
90°	.389	1.295
100°	.2185	.849
103°	.1071	.416

Ratios of the u and w of the appropriate waves to the u and w of P.
1. $\frac{PcP}{P}$. 2. $\frac{PcS}{P}$.

Δ	$^{2}\frac{u_{PcP}}{u_P}$	$^{2}\frac{w_{PcP}}{w_P}$
15°	.00153	.01751
20°	.00382	.03021
30°	.02698	.10224
40°	.06745	.18548
50°	.09166	.20821
60°	.11300	.20240
70°	.12492	.17410
80°	.09848	.12629
90°	.03877	.04369
100°	.1416	.13786
103°	.33186	.31171
94.5°	.00000	.00000

Δ	$^{3}\frac{u_{PcS}}{u_P}$	$^{3}\frac{w_{PcS}}{w_P}$
15°	.03694	.00184
20°	.06733	.00406
25.8°	.14463	.00807
30°	.21516	.01322
40°	.31388	.01997
50°	.26526	.01751
60°	.23954	.01433
65.2°	.00000	.0000

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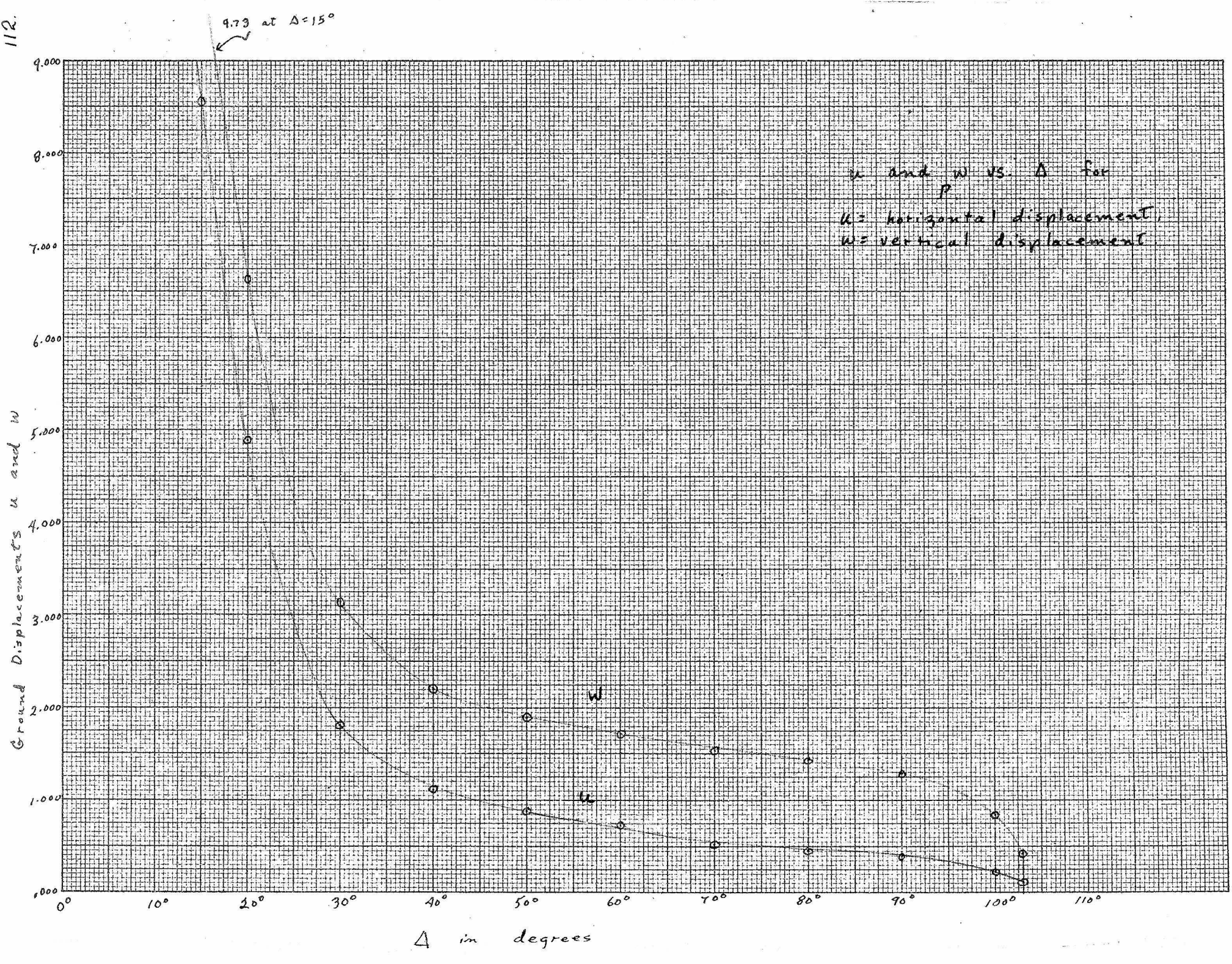
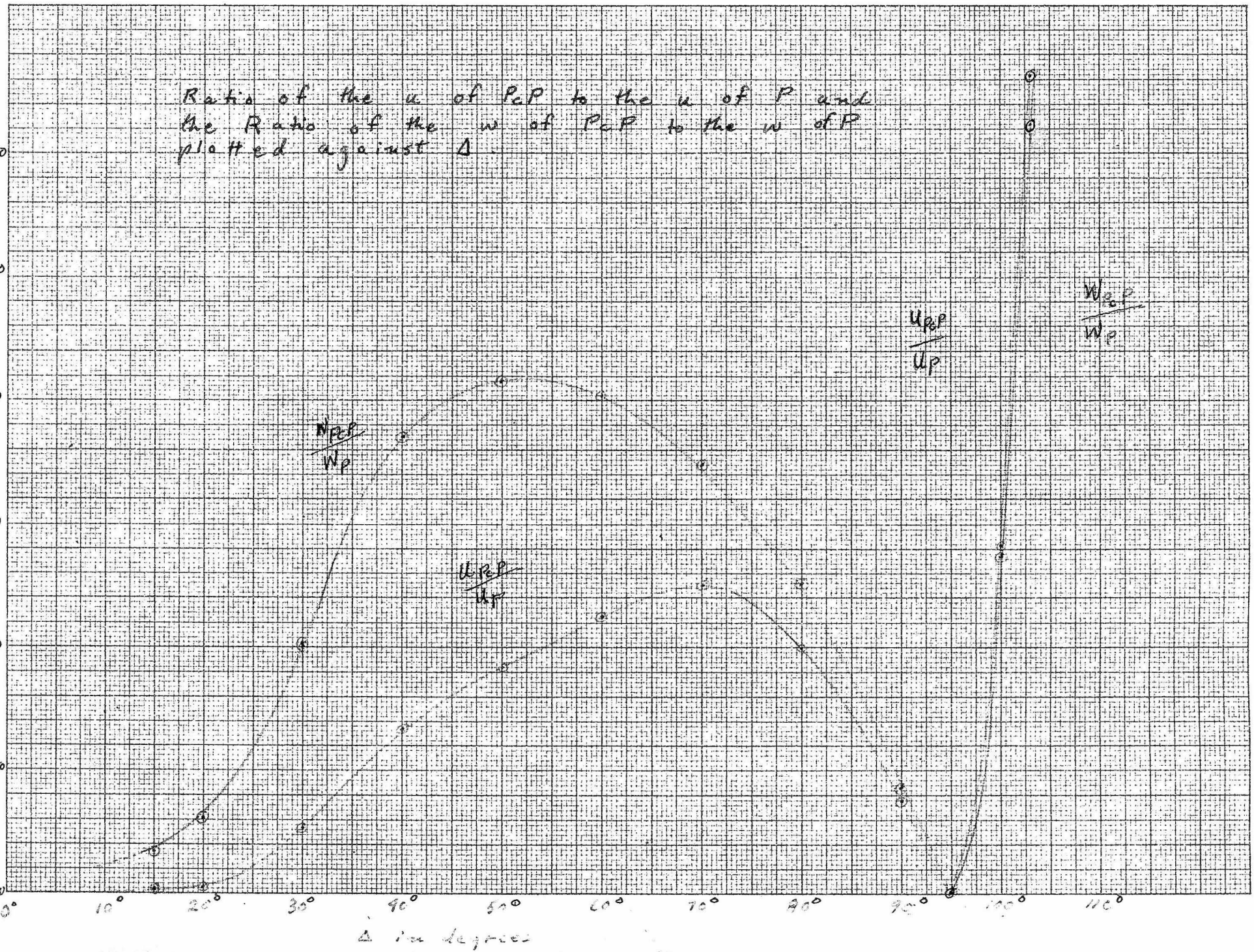
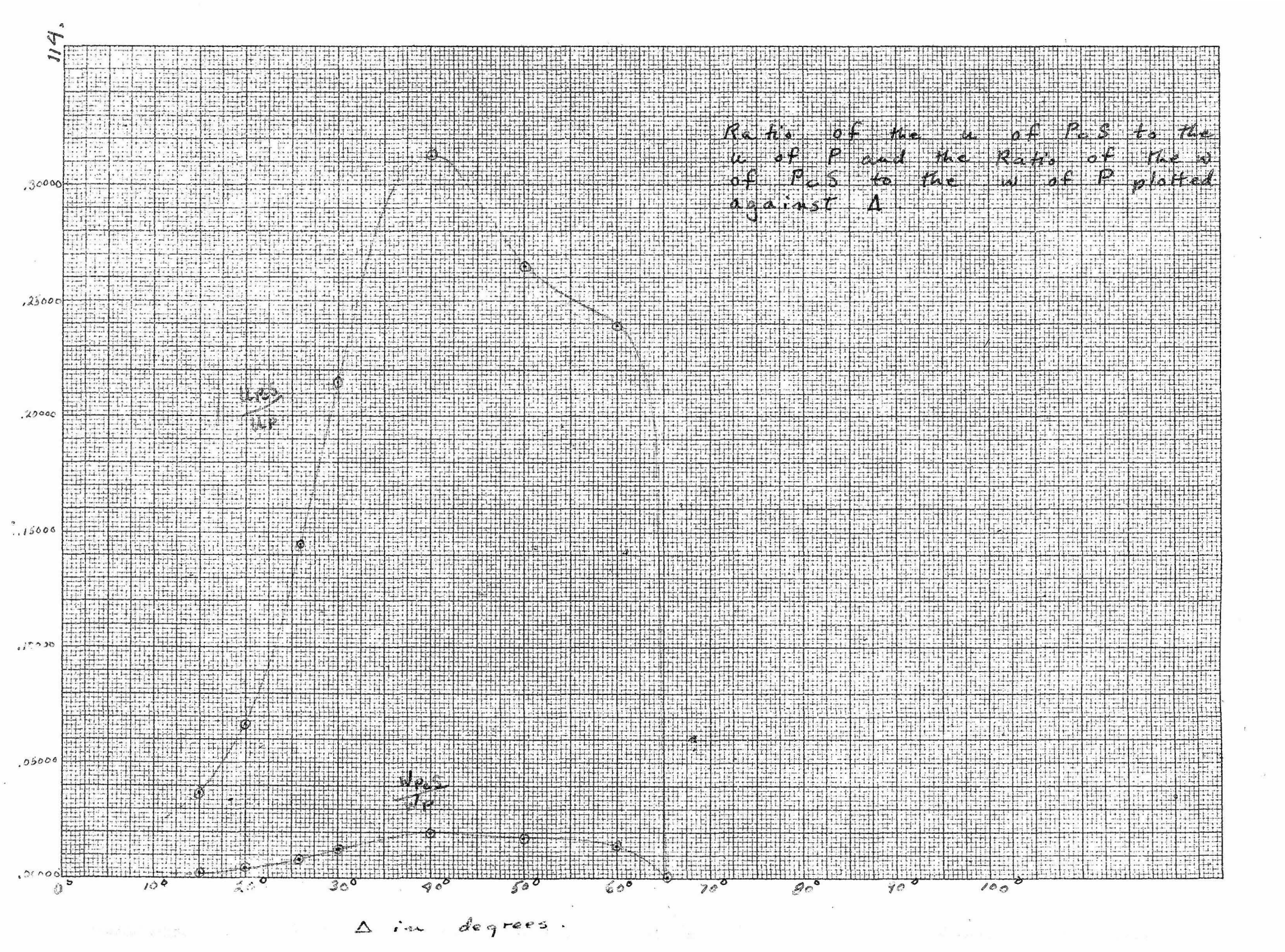
1 A plot of u and w against Δ is on p. 112.
2 A plot of these against Δ is on p. 113.
3 A plot of these against Δ is on p. 114.


9.73 at Δ=15°
u and w vs. Δ for p
u: horizontal displacement,
w: vertical displacement.
w
u
9.000
8.000
7.000
6.000
5.000
4.000
3.000
2.000
1.000
.000
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
Ground Displacements u and w
Δ in degrees

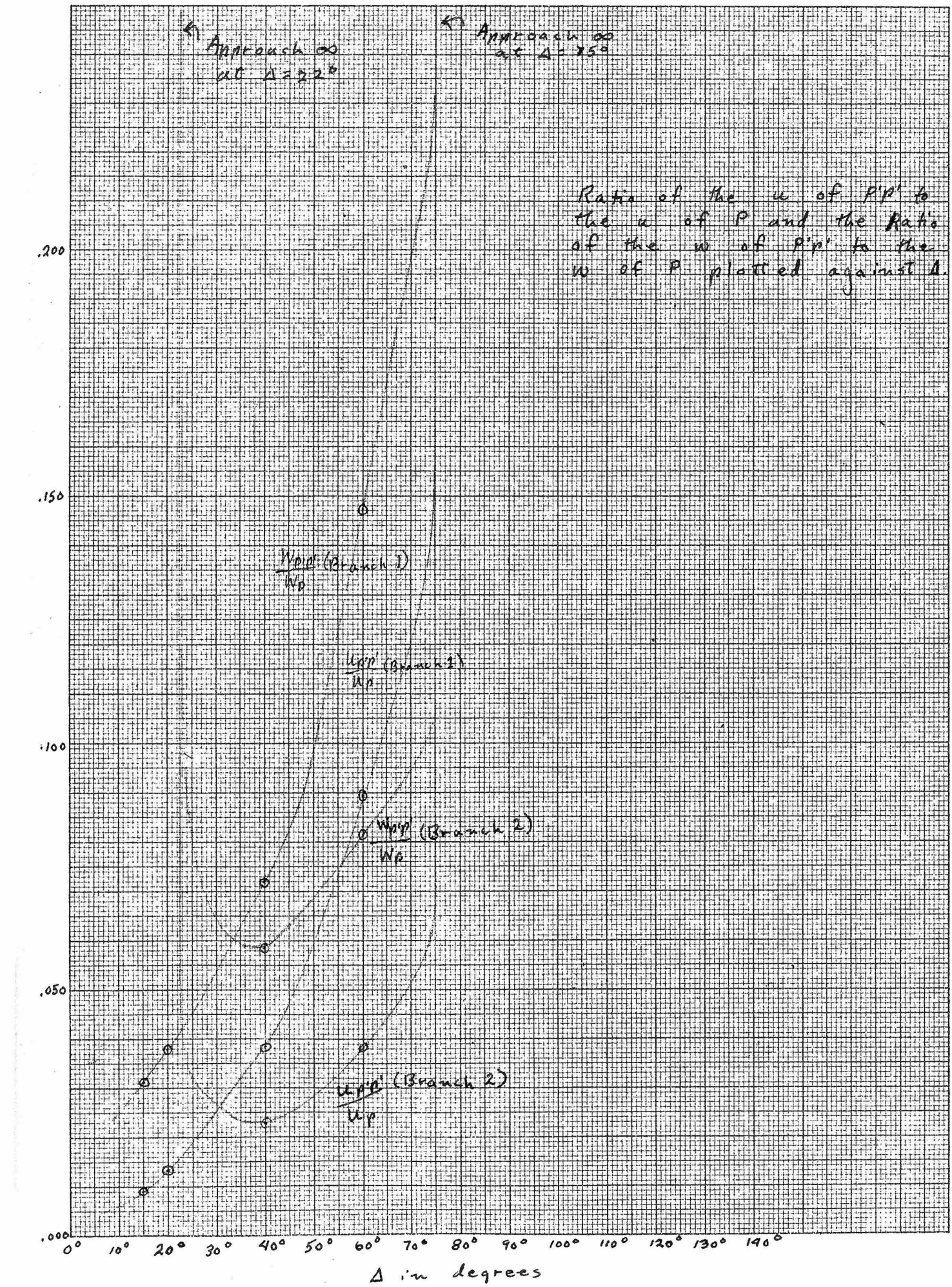

Ratio of the u of PcP to the u of P and the Ratio of the w of PcP to the w of P plotted against Δ.
.30000
.25000
.20000
.15000
.10000
.05000
.00000
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
Δ in degrees
Wpcp / Wp
Upcp / Up
Wpcp / Wp
Upcp / Up

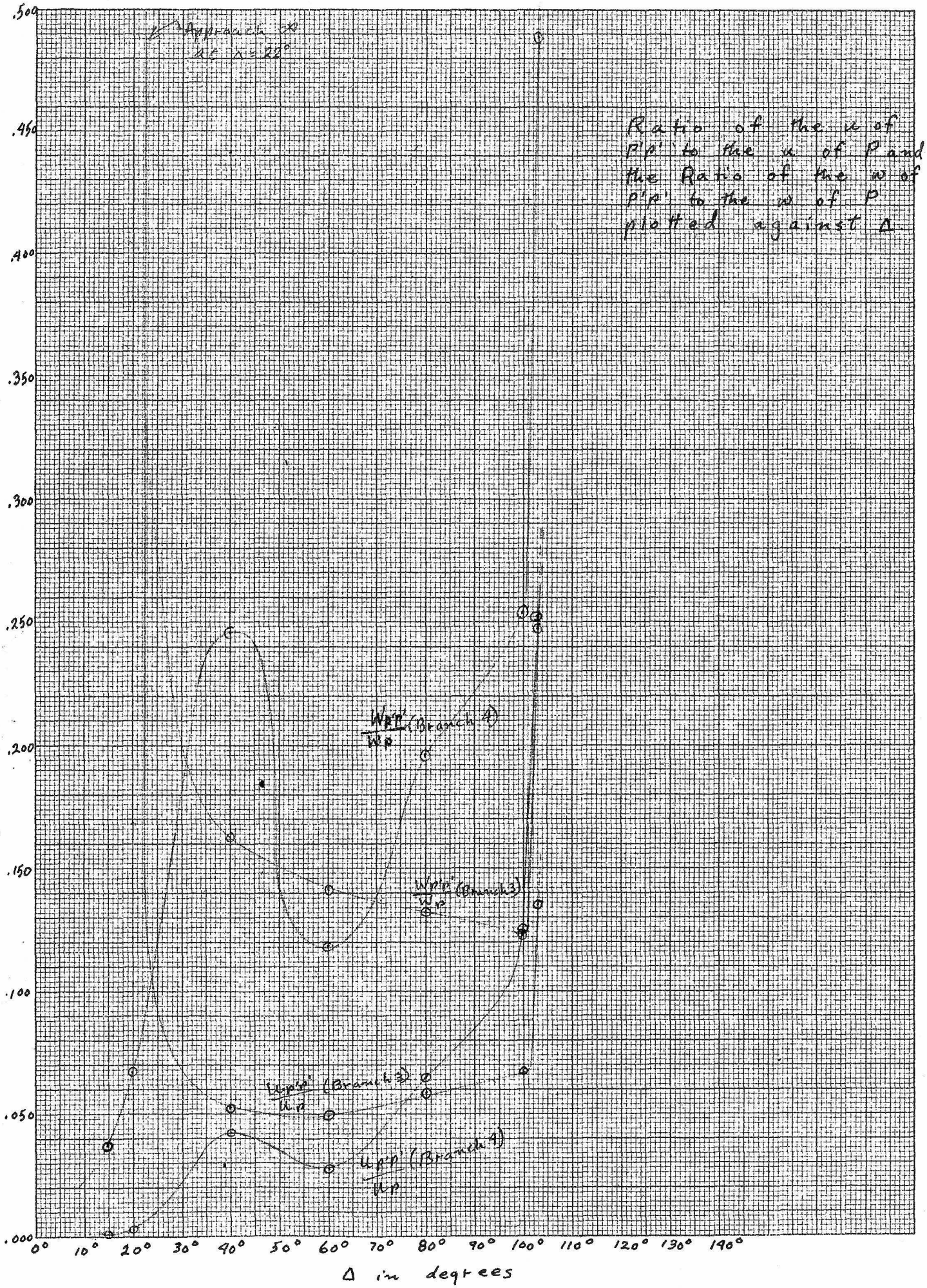

Ratio of the w of Pcs to the w of P and the Ratio of the w of Pcs to the w of P plotted against Δ
.30000
.25000
.20000
.15000
.05000
0°
100
200
300
400
500
600
700
800
900
1000
Δ in degrees.

3. $\frac{P'P'}{P}$ for $\Delta = 15^\circ - 103^\circ$.

Δ	$^1u_{P'P'}$	$^1w_{P'P'}$	^2u_P	^2w_P	$^3\frac{u_{P'P'}}{u_P}$	$^3\frac{w_{P'P'}}{w_P}$
15°	.078	.3035	8.57	9.73	.0091	.0312
20°	.0674	.2515	4.89	6.64	.0138	.0380
40°	.0423	.1595	1.108	2.22	.0382	.0720
60°	.0648	.2535	.724	1.717	.0896	.1476
75°	∞	∞	.510	1.500	∞	∞
75°	∞	∞	.510	1.500	∞	∞
60°	.0277	.40	.724	1.717	.0383	.0816
40°	.0252	.1298	1.108	2.22	.0228	.0585
22°	∞	∞	3.920	5.55	∞	∞
22°	∞	∞	3.920	5.55	∞	∞
40°	.0581	.362	1.108	2.22	.0524	.1629
60°	.0354	.2425	.724	1.717	.0490	.1412
80°	.0268	.1898	.458	1.435	.0585	.1322
100°	.01473	.1045	.2185	.849	.0674	.1232
103°	.01450	.1050	.1071	.416	.1351	.252
103°	.0265	.2035	.1071	.416	.247	.489
100°	.0271	.2155	.2185	.849	.1239	.254
80°	.0297	.2815	.458	1.435	.0648	.196
60°	.01985	.2028	.724	1.717	.0275	.1182
40°	.0466	.546	1.108	2.22	.0421	.246
20°	.0134	.446	4.89	6.64	.00274	.0673
15°	.0070	.365	8.57	9.73	.00082	.0375

[1] Taken from the data on pp. 100-101 and the graphs on pp. 102-103.
[2] Taken from the data on p. 111 and the plot on p. 112.
[3] Plotted on p. 116 and p. 117.

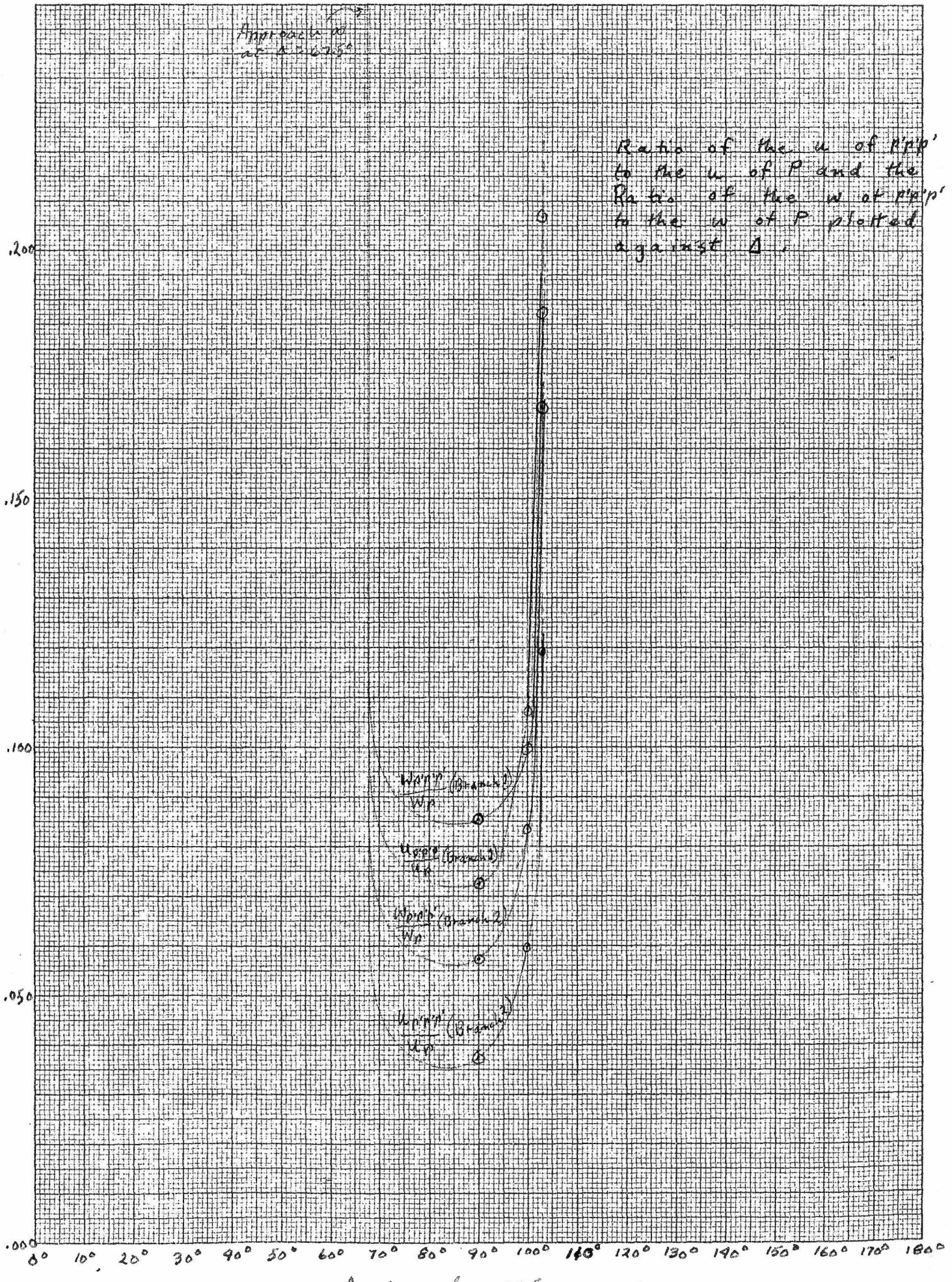

Approach ∞ at Δ = 22°
Approach ∞ at Δ = 15°
Ratio of the w of P'P' to the w of P and the Ratio of the w of P'P' to the w of P plotted against Δ.
.200
.150
.100
.050
.000
$\frac{W_{PP'}}{W_P}$ (Branch 1)
$\frac{W_{P'P'}}{W_P}$ (Branch 1)
$\frac{W_{P'P'}}{W_P}$ (Branch 2)
$\frac{W_{P'P'}}{W_P}$ (Branch 2)
0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140°
Δ in degrees

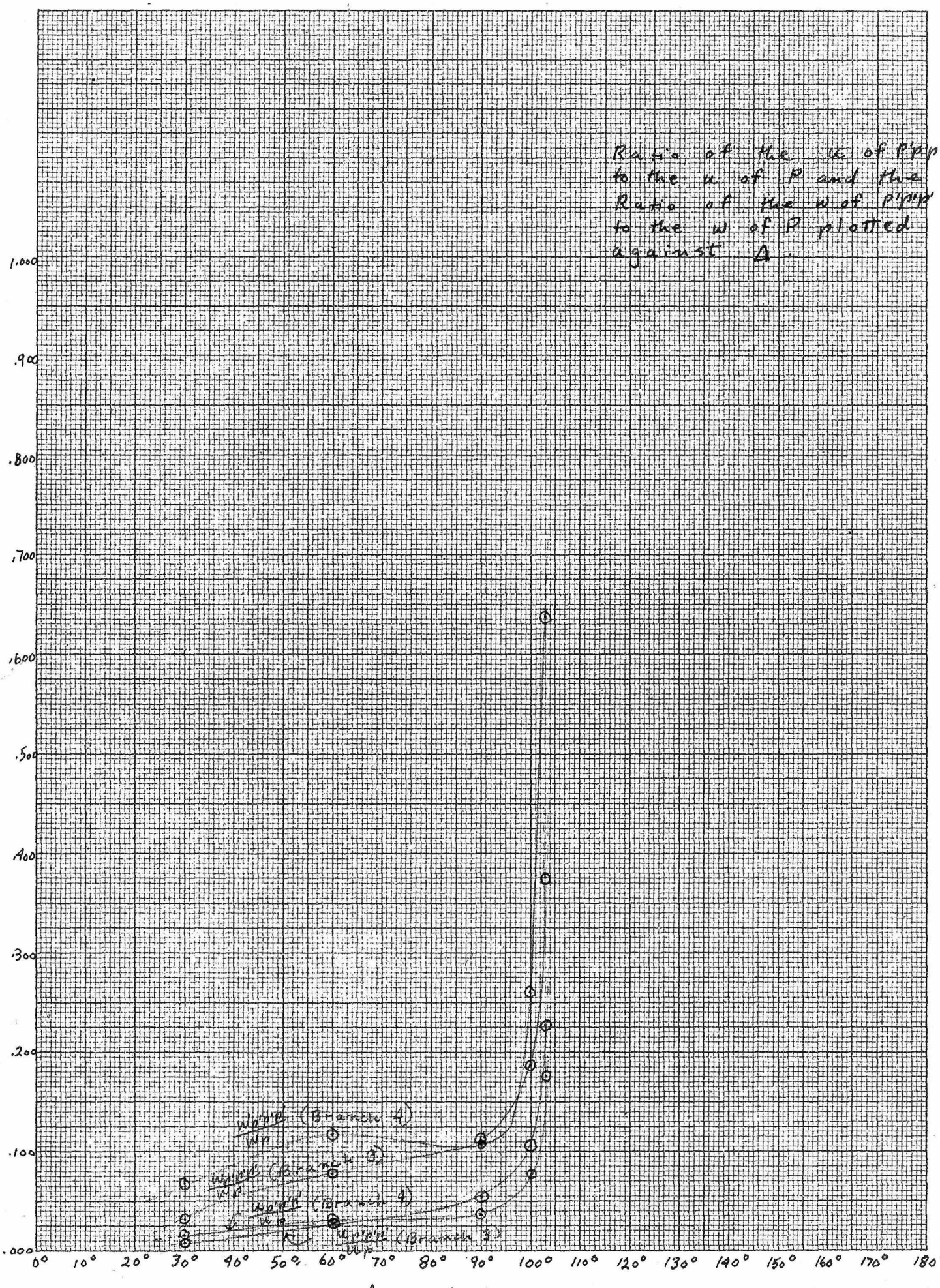

Approach to Δ = 22°
Ratio of the w of P'P' to the w of P and the Ratio of the w of P'P' to the w of P plotted against Δ
.500
.450
.400
.350
.300
.250
.200
.150
.100
.050
.000
0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140°
Δ in degrees
W_P'P' / W_P (Branch 4)
W_P'P' / W_P (Branch 3)
u_P'P' / u_P (Branch 3)
u_P'P' / u_P (Branch 4)

4. $\frac{P'P'P'}{P}$ for $\Delta = 30^\circ - 103^\circ$.

Δ	$^1u_{P'P'P'}$	$^1w_{P'P'P'}$	^2u_P	^2w_P	$^3\frac{u_{P'P'P'}}{u_P}$	$^3\frac{w_{P'P'P'}}{w_P}$
103°	.0222	.0781	.1071	.416	.207	.1875
100°	.0235	.0845	.2185	.849	.1073	.0996
90°	.0282	.1105	.389	1.295	.0726	.0854
67.5°	∞	∞	.575	1.610	∞	∞
67.5°	∞	∞	.575	1.610	∞	∞
90°	.0146	.0740	.389	1.295	.0375	.0571
100°	.0131	.0710	.2185	.849	.0599	.0836
103°	.0128	.0701	.1071	.416	.1192	.1682
103°	.0243	.156	.1071	.416	.2265	.375
100°	.0235	.161	.2185	.849	.1075	.1898
90°	.0215	.147	.389	1.295	.0554	.1135
60°	.0193	.137	.724	1.717	.0267	.0799
30°	.0141	.100	1.71	3.15	.00825	.0317
30°	.0269	.214	1.71	3.15	.01571	.068
60°	.0231	.219	.724	1.717	.03190	.1273
90°	.01372	.140	.389	1.295	.0353	.1081
100°	.0170	.222	.2185	.849	.0778	.2618
103°	.0190	.265	.1071	.416	.1771	.6375

[1] Taken from the data on pp. 104-105 and the graphs on pp. 106-107.
[2] Taken from the data on p. 111 and the graph on p. 112.
[3] Plotted on p. 119 and p. 120.


Approach to ∞ at Δ = 167.5°
Ratio of the w of P'P'P' to the w of P and the Ratio of the w of P'P'P' to the w of P plotted against Δ.
.200
.150
.100
.050
.000
0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140° 150° 160° 170° 180°
Δ in degrees


Ratio of the w of P'pp' to the w of P and the Ratio of the w of P'pp' to the w of P plotted against Δ.
1.000
.900
.800
.700
.600
.500
.400
.300
.200
.100
.000
0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140° 150° 160° 170° 180°
Δ in degrees

2. SV and its u and w Ratios.

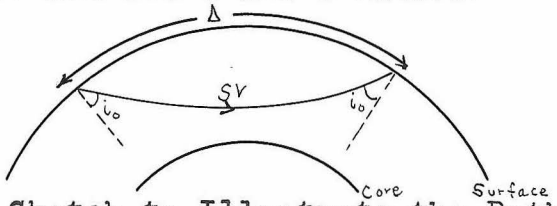


Fig. 23. Sketch to Illustrate the Path of SV.

For the case of SV, $\sin i_o = \frac{V_o}{V_\Delta}$, where V_o is the true velocity of SV waves (3.2 km. per sec.), and $\overline{V_\Delta}$ is the apparent velocity of SV, as measured by the tangent to the time-distance curve at an arcual epicentral distance Δ .

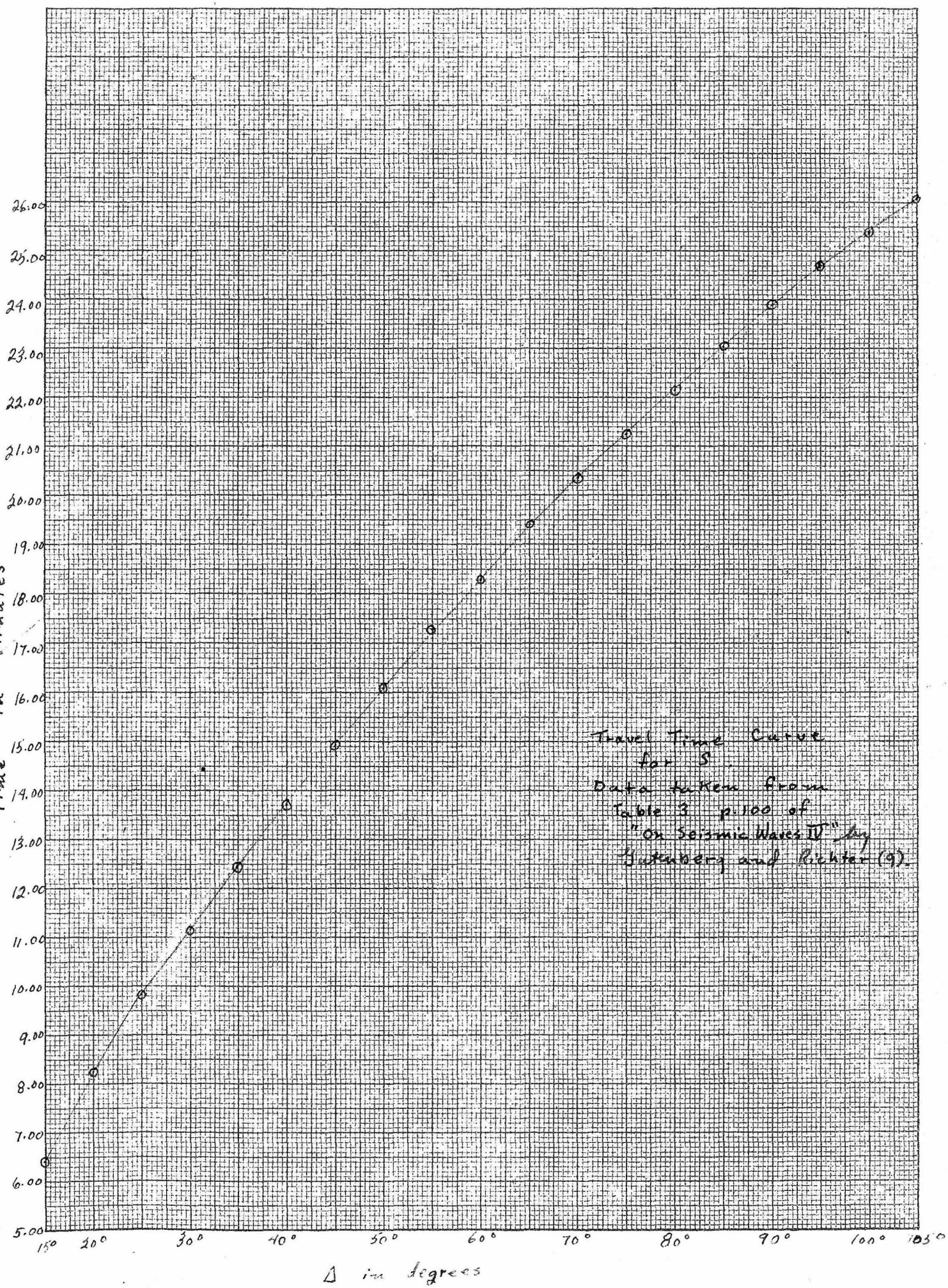
Δ	[1] $\overline{V_\Delta}$	$\sin i_o$	i_o	$\tan i_o$	[2] $\frac{di_o}{d\Delta}$	$A=CTf\sqrt{\frac{\tan i_o}{\sin \Delta}\frac{di_o}{d\Delta}}$
15°	4.88km/sec.	.655	40°55'	.8667	1.290	6.53
19°	5.44 "	.588	36°00'	.7265	1.165	5.06
20°	5.58 "	.574	35°00'	.7002	1.080	4.67
30°	7.05 "	.454	27°00'	.5095	.415	2.02
40°	7.64 "	.419	24°46'	.4614	.195	1.178
50°	8.15 "	.39205	23°05'	.4262	.190	1.019
60°	8.93 "	.3583	21°00'	.3839	.180	.886
70°	10.00 "	.320	18°40'	.3378	.165	.766
75°	10.12 "	.316	18°25'	.3330	.145	.704
80°	10.52 "	.304	17°42'	.3191	.125	.633
85°	10.71 "	.2985	17°22'	.3127	.115	.598
90°	10.98 "	.2918	16°58'	.3051	.110	.576
100°	12.32 "	.2600	15°04'	.2692	.105	.532
101°	12.65 "	.2530	14°39'	.2614	.103	.518

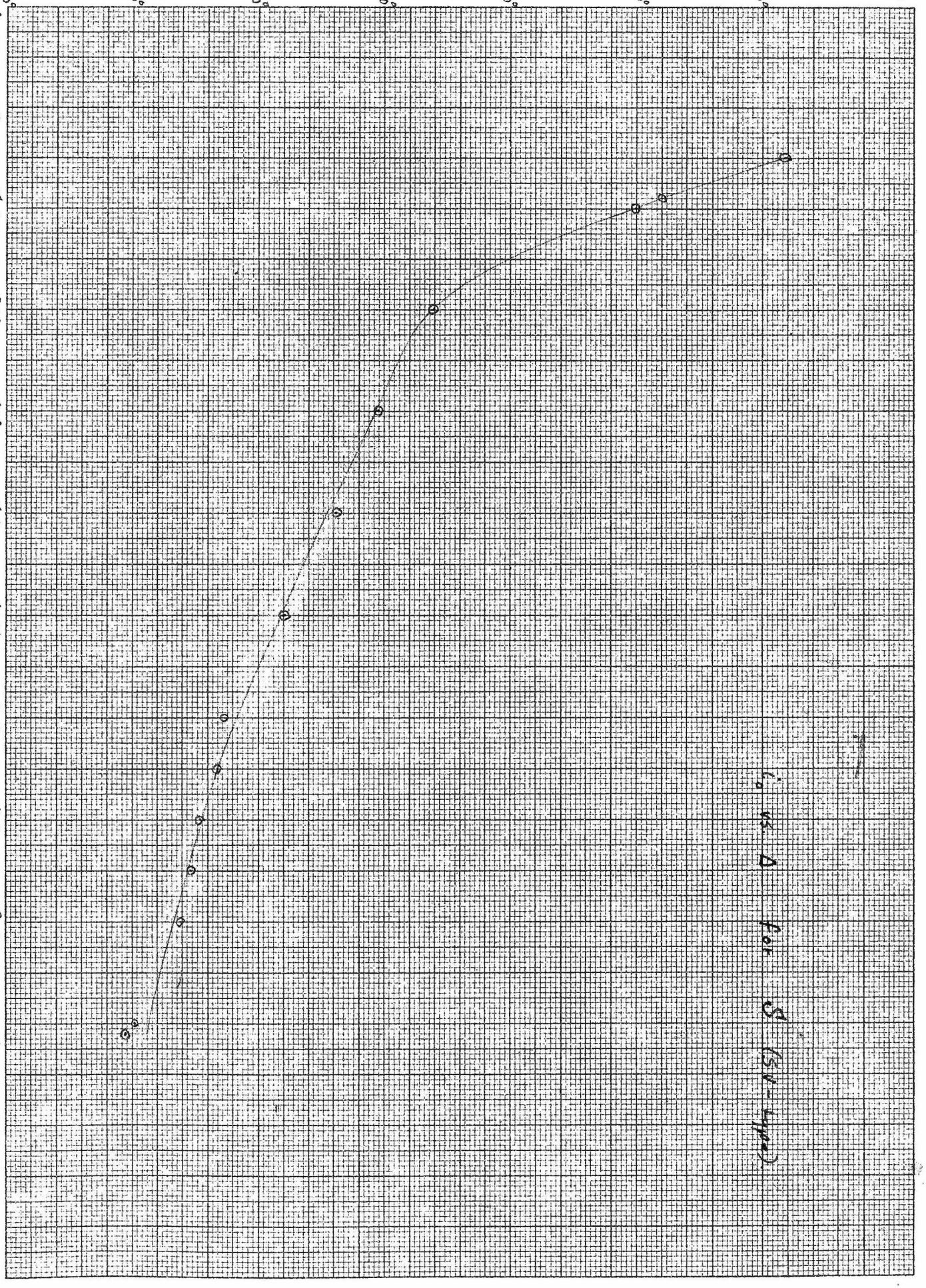
---

[1] Taken from the travel-time curve on p. 122. The data for this curve is taken from Table 3, p. 100 of Gutenberg and Richter (9). Column C is used. This travel-time curve is smooth and does not consider irregularities, which may be noted by referring to Fig. 92, p. 202 of Gutenberg (5) and pp. 332-344 of Gutenberg and Richter (7).

[2] Taken from a plot of i_o vs. Δ on p. 123.

[3] $C = \pi$, $T = 1$ sec., and $f = 1.00$.


Travel Time Curve
for S
Data taken from
Table 3 p.100 of
"On Seismic Waves IV" by
Gutenberg and Richter (9).
Time in Minutes
26.00
25.00
24.00
23.00
22.00
21.00
20.00
19.00
18.00
17.00
16.00
15.00
14.00
13.00
12.00
11.00
10.00
9.00
8.00
7.00
6.00
5.00
15°
20°
30°
40°
50°
60°
70°
80°
90°
100°
105°
Δ in degrees


i₀ in degrees
10.0°
15.0°
20.0°
25.0°
30.0°
35.0°
40.0°
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
Δ in degrees
i₀ vs. Δ for SI (SV-type).

Δ	$\frac{1_u}{A}$	$\frac{1_w}{A}$	2_u	2_w
15°	.55	1.48	3.59	9.67
19°	5.20	.50	26.35	2.535
20°	3.10	.00	14.49	.000
30°	1.70	.97	3.435	1.959
40°	1.72	.92	2.025	1.084
50°	1.73	.90	1.761	.917
60°	1.78	.81	1.575	.717
70°	1.81	.74	1.388	.568
75°	1.82	.72	1.280	.506
80°	1.825	.71	1.155	.449
85°	1.830	.69	1.094	.413
90°	1.840	.68	1.061	.392
100°	1.86	.60	.991	.319
101°	1.87	.59	.970	.3055

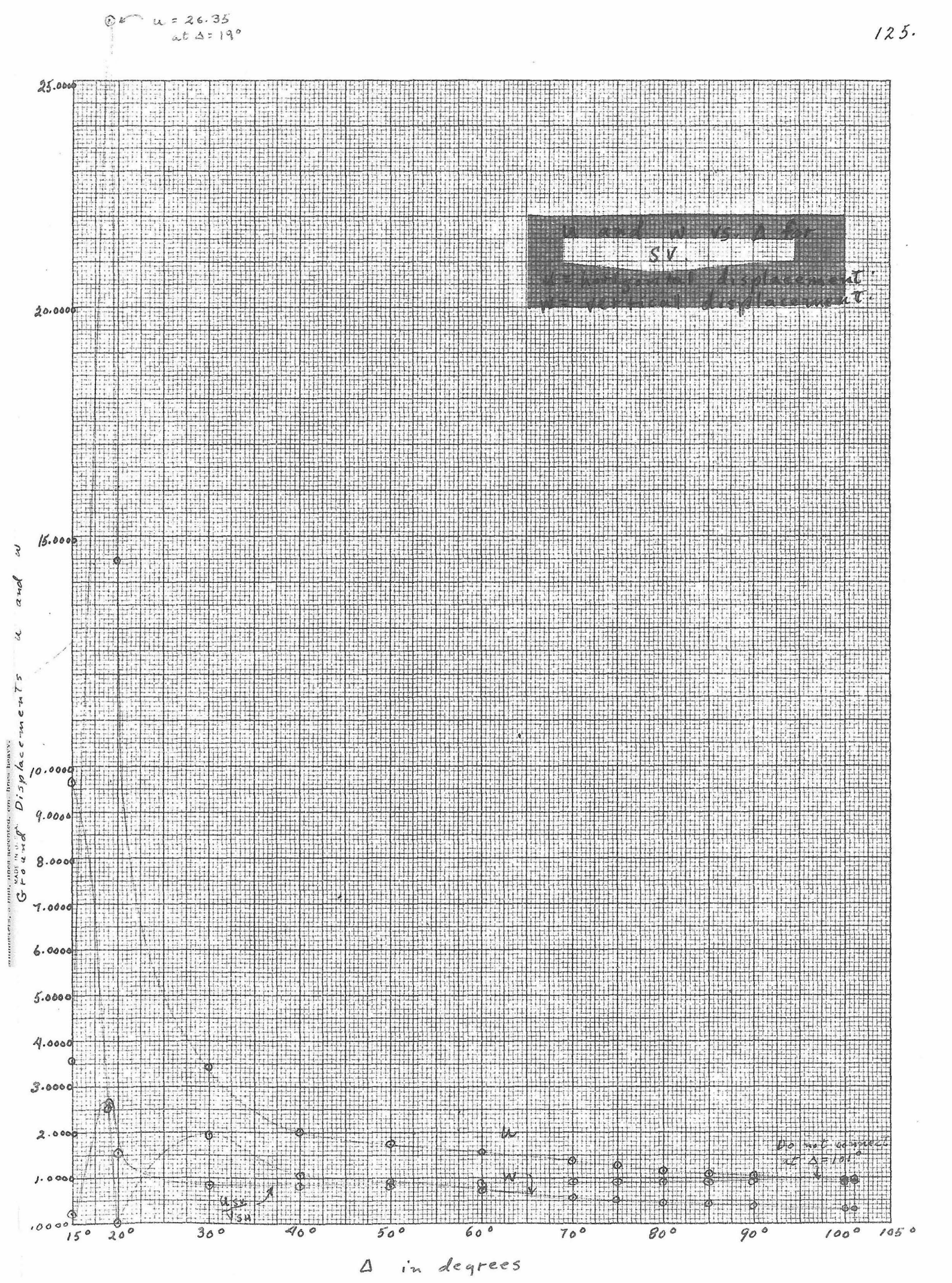
Ratios of the u and w of the appropriate waves to the u and w of S.

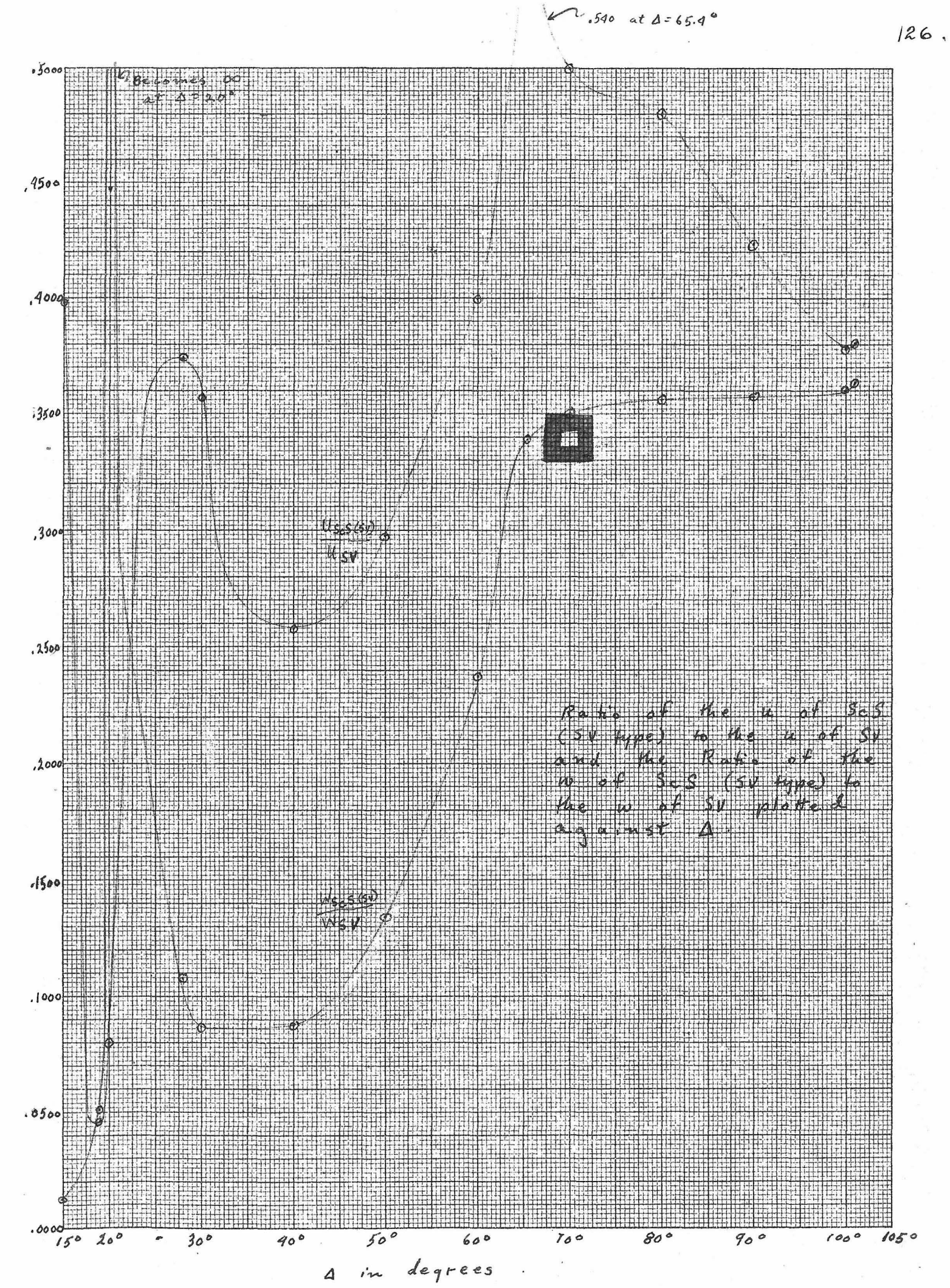
1. $\frac{\text{ScS (SV type)}}{\text{SV}}$,

Δ	u_{SV}	[3] $u_{ScS(SV)}$	[4] $\frac{u_{ScS(SV)}}{u_{SV}}$	w_{SV}	[3] $w_{ScS(SV)}$	[1] $\frac{w_{ScS(SV)}}{w_{SV}}$
15°	3.59	1.429	.398	9.67	.122	.01261
19°	26.35	1.220	.0463	2.535	.128	.0505
20°	14.49	1.160	.0801	.000	.125	∞
28°	3.90	1.460	.3745	1.900	.205	.1079
30°	3.435	1.225	.357	1.959	.170	.0869
40°	2.025	.522	.2578	1.084	.095	.0875
50°	1.761	.525	.2978	.917	.123	.1340
60°	1.575	.630	.3998	.717	.170	.2375
65.40°	1.480	.800	.540	.650	.220	.339
70°	1.388	.695	.500	.568	.200	.350
80°	1.155	.555	.480	.449	.160	.356
90°	1.061	.450	.424	.392	.140	.357
100°	.991	.375	.3785	.319	.115	.3605
101°	.970	.370	.381	.3055	.111	.3635

---

[1] Taken from Reflections and Refractions, 1943, graphs of Dr. Gutenberg (copy on p. 10).
[2] A plot of u and w vs. Δ is on p. 125.
[3] From the plot on p. 47.
[4] Plotted on p. 126.


u = 26.35
at Δ = 19°
u and w vs Δ for
SV.
Ground Displacements u and w
Δ in degrees
25.0000
20.0000
15.0000
10.0000
9.0000
8.0000
7.0000
6.0000
5.0000
4.0000
3.0000
2.0000
1.0000
.0000
15°
20°
30°
40°
50°
60°
70°
80°
90°
100°
105°
u
w
v
Do not connect
at Δ = 101°


.540 at Δ = 65.4°
.5000
.4500
.4000
.3500
.3000
.2500
.2000
.1500
.1000
.0500
.0000
Becomes ∞ at Δ = 20°
$\frac{U_{ScS(SV)}}{U_{SV}}$
$\frac{W_{ScS(SV)}}{W_{SV}}$
Ratio of the u of ScS (SV type) to the u of SV and the Ratio of the w of ScS (SV type) to the w of SV plotted against Δ.
15° 20° 30° 40° 50° 60° 70° 80° 90° 100° 105°
Δ in degrees

2. $\frac{ScS\ (SH\ type)}{SV}$.

Δ	u_{SV}	$^1v_{ScS(SH)}$	$\frac{^2v_{ScS(SH)}}{u_{SV}}$	$^1w_{ScS(SH)}$	$\frac{^2w_{ScS(SH)}}{w_{SV}}$
15°	3.59	1.699	.473	.00	.00
19°	26.35	1.642	.0625	.00	.00
20°	14.49	1.629	.1124	.00	.00
30°	3.435	1.445	.421	.00	.00
40°	2.025	1.311	.648	.00	.00
50°	1.761	1.160	.659	.00	.00
60°	1.575	.970	.615	.00	.00
70°	1.388	.739	.533	.00	.00
80°	1.155	.581	.504	.00	.00
90°	1.061	.486	.458	.00	.00
100°	.991	.402	.406	.00	.00
101°	.970	.395	.407	.00	.00

3. $\frac{ScP}{SV}$.

Δ	u_{SV}	$^3u_{ScP}$	$\frac{^4u_{ScP}}{u_{SV}}$	w_{SV}	$^3w_{ScP}$	$\frac{^4w_{ScP}}{w_{SV}}$
15°	3.59	.059	.01642	9.67	.546	.0565
19°	26.35	.085	.003265	2.535	.635	.2505
20°	14.49	.090	.00621	.000	.654	∞
28.9°	3.61	.125	.03465	1.940	.704	.362
30°	3.435	.124	.03615	1.959	.701	.3578
40°	2.025	.116	.0573	1.084	.571	.526
50°	1.761	.104	.0590	.917	.390	.425
60°	1.575	.045	.02858	.717	.215	.2996
65.5°	1.48	.000	.0000	.640	.000	.000

4. $\frac{SV}{SH}$.

that is, assuming C_{SV} and C_{SH} are the same. Actually, to obtain the correct ratios from those here for $\frac{SV}{SH}$, they would have to be multiplied by $\frac{C_{SV}}{C_{SH}}$.

$A = CTf\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{di_o}{d\Delta}}$ is the same for both SV and SH; $\frac{u}{A}$ and $\frac{w}{A}$ are both 0 for SH but $\frac{v}{A}$ is always 2.00. Hence $\frac{w_{SV}}{w_{SH}} = 0$, but $\frac{u_{SV}}{v_{SH}} = \frac{A \times \frac{u}{A}\ (SV)}{A \times \frac{v}{A}\ (SH)} = \frac{\frac{u}{A}}{\frac{v}{A}} = \frac{\frac{u}{A}}{2}$. Hence $\frac{u_{SV}}{v_{SH}}$ is $\frac{1}{2}$ the values of $\frac{u}{A}$ given for SV on p. 124. These are plotted against Δ on p. 125.

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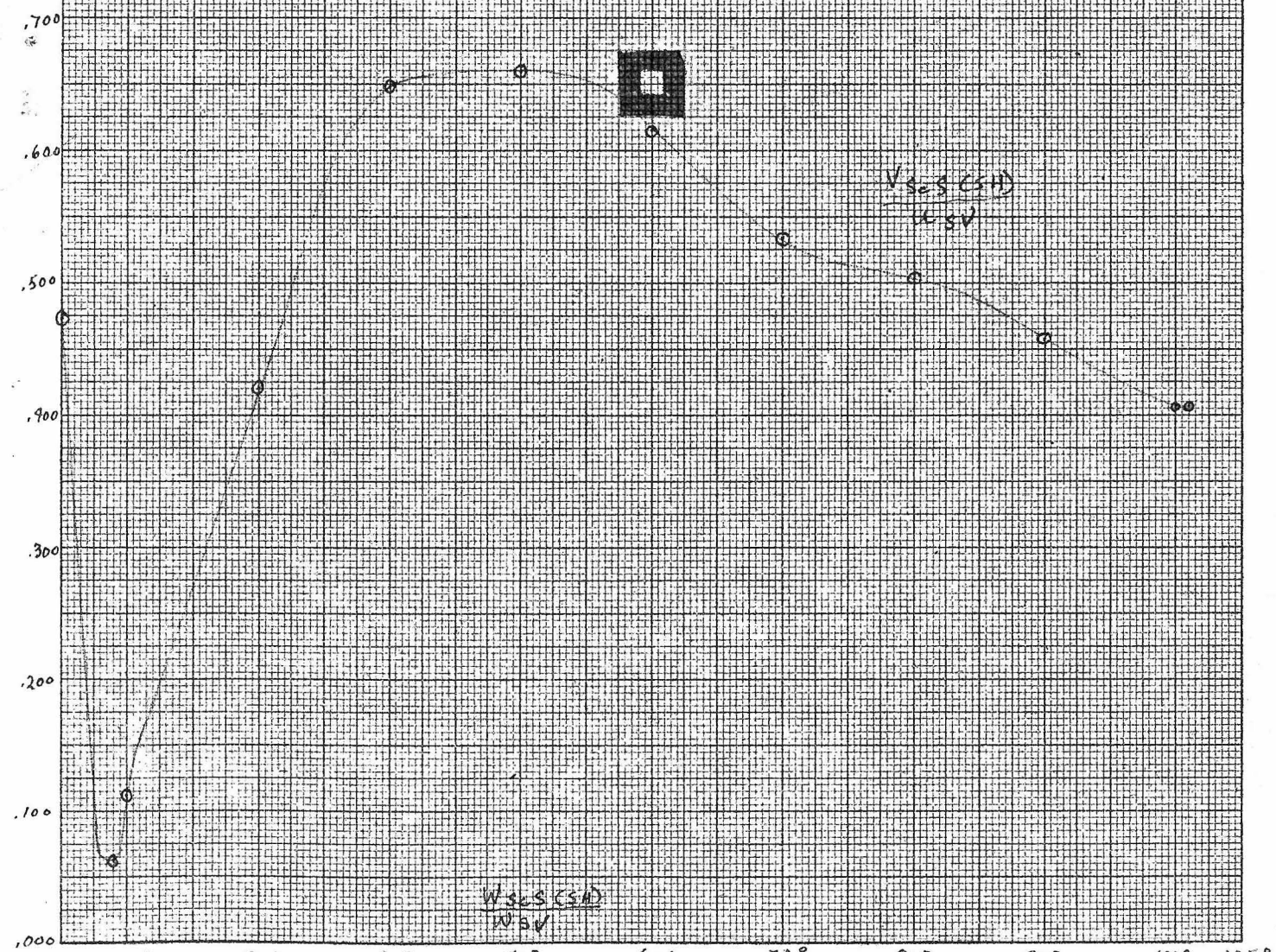
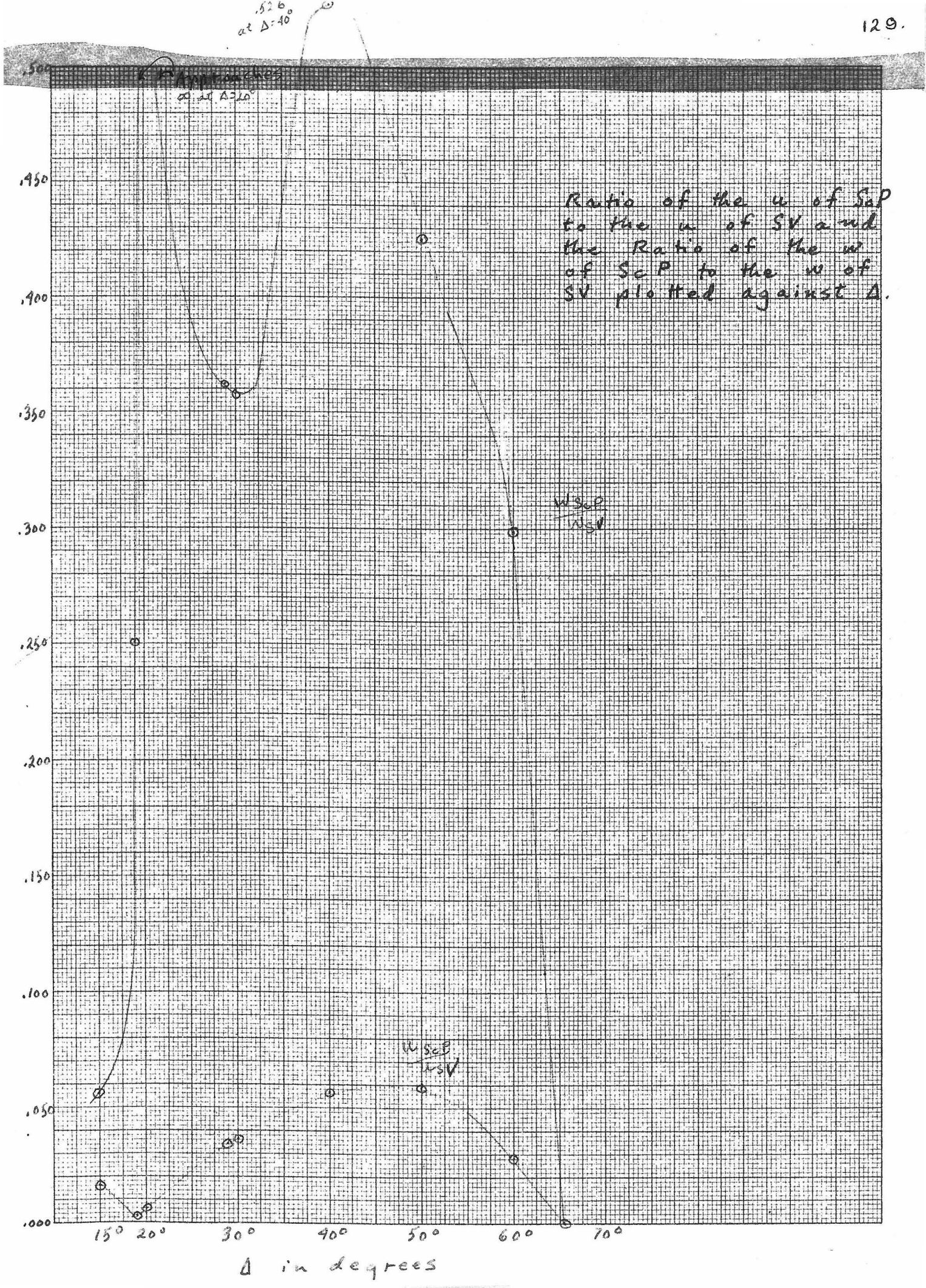
1 Taken from the plot on p. 49.
2 Plotted on p. 128.
3 Taken from the plot on p. 53.
4 Plotted on p. 129.

Ratio of the w of ScS (SH type) to the w of SV and the Ratio of the w of ScS (SH type) to the w of SV plotted against Δ.





Ratio of the w of ScP to the w of SV and the Ratio of the w of ScP to the w of SV plotted against Δ.

3. PP and its u and w Ratios.

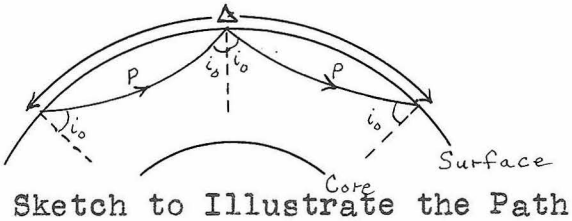


Fig. 24. Sketch to Illustrate the Path of PP.

For the case of PP the angle of incidence at a particular Δ is the same as the angle of incidence of P at $\frac{1}{2}$ that particular Δ. Also $\frac{di_o}{d\Delta}$ for PP at a particular Δ is $\frac{1}{2}$ $\frac{di_o}{d\Delta}$ for P at $\frac{1}{2}$ that particular Δ. This is because as P goes from 89° to 90° the change in the angle of incidence is the same as for PP when it goes from 178° to 180°. Thus for PP di_o is the same as for P but $d\Delta$ is twice the $d\Delta$ for P.

Δ	[1] i_o	$\tan i_o$	[2] $\frac{di_o}{d\Delta}$	$\sin\Delta$	[3] f	[4] A	[5] $\frac{u}{A}$	[5] $\frac{w}{A}$	[6] u	[6] w
30°	38°38'	.7992	.6725	.500	.390	1.268	1.34	1.52	1.70	1.928
40°	32°21'	.6334	.455	.643	.540	1.136	1.20	1.63	1.362	1.851
50°	28°36'	.5452	.285	.766	.635	.900	1.08	1.70	.974	1.530
60°	26°16'	.4935	.1625	.866	.700	.671	.96	1.77	.644	1.188
70°	25°06'	.4684	.1125	.940	.710	.526	.93	1.785	.489	.941
80°	24°15'	.4505	.110	.985	.740	.522	.90	1.80	.470	.940
90°	22°57'	.4235	.1075	1.000	.770	.516	.875	1.81	.452	.936
100°	21°49'	.4003	.105	.985	.790	.513	.850	1.82	.436	.934
110°	20°58'	.3832	.104	.940	.805	.520	.815	1.835	.424	.955
120°	20°05'	.3656	.103	.866	.810	.531	.780	1.85	.414	.984
130°	18°48'	.3404	.102	.766	.825	.551	.715	1.875	.394	1.032
140°	17°00'	.3057	.101	.643	.875	.604	.650	1.90	.392	1.147
150°	16°44'	.3007	.09988	.500	.880	.678	.630	1.905	.426	1.291
160°	15°41'	.2808	.09875	.342	.890	.799	.610	1.91	.488	1.528
170°	14°42'	.2624	.0950	.174	.895	1.061	.595	1.92	.633	2.09
180°	13°57'	.2484	.09125	.000	.900	∞	.580	1.93	∞	∞

---

[1] Taken from the data on p. 108 and from the graph on p. 110. Thus any irregularities in a travel-time curve for PP are smoothed over, as was the case for P.

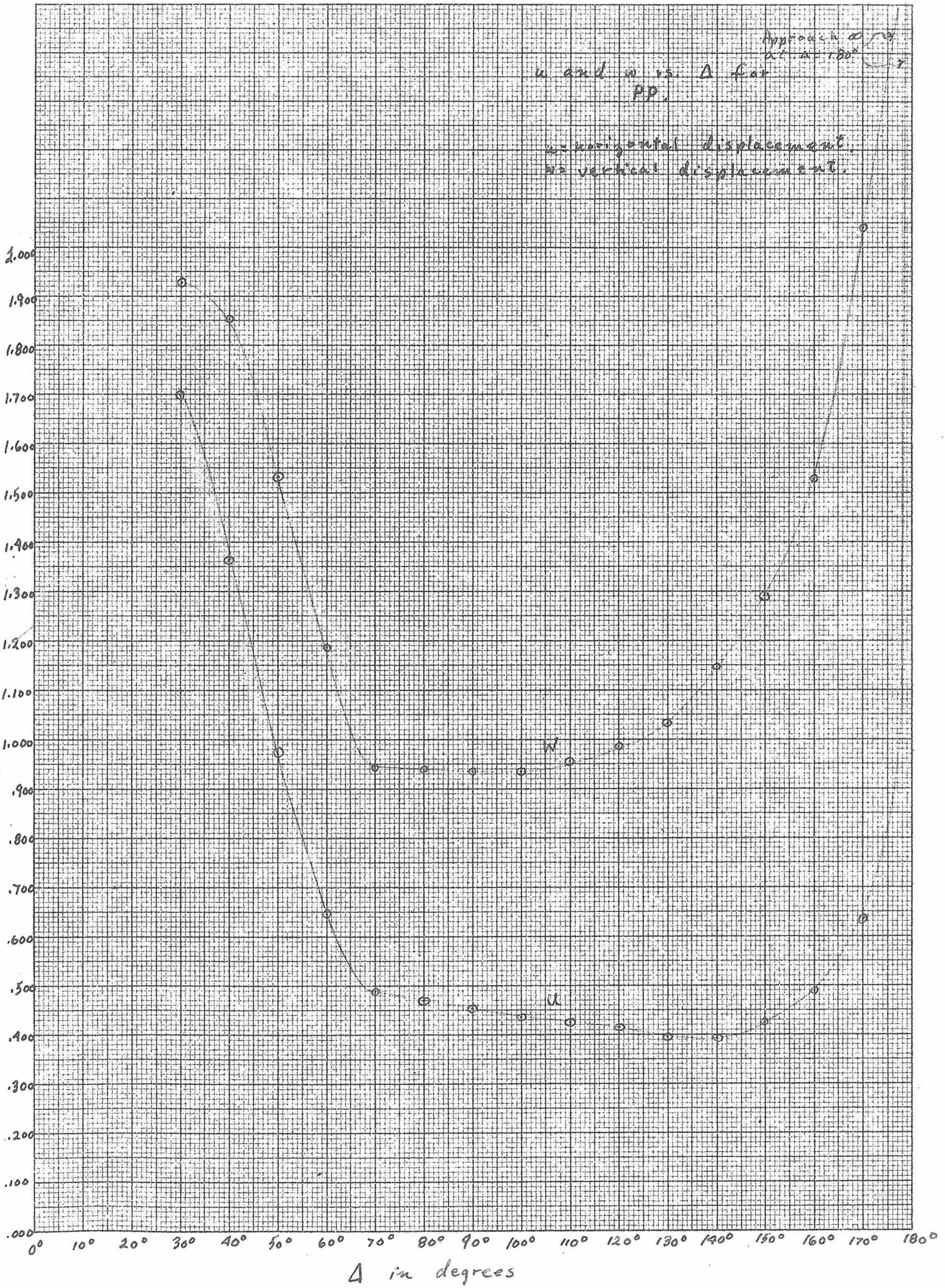
[2] Taken from the data on p. 108.

[3] $f = \sqrt{\frac{E_{reflected}}{E_{inc.}}}$ at the earth's surface at $\frac{1}{2}$ the Δ for PP. It is taken from Reflections and Refractions, 1943, graphs of Dr. Gutenberg (copy on p. 10).

[4] $A = CTf\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{di_o}{d\Delta}}$ where $C = \pi$, $T = 1.0$ sec.

[5] Taken from the data on p. 108. $\frac{u}{A}$, for example, at $\Delta = 180°$ for PP is the same as $\frac{u}{A}$ for P at 90°.

[6] A plot of u and w against Δ is on p. 131.


Approach ∞ at Δ = 180°
u and w vs. Δ for PP.
u = horizontal displacement.
w = vertical displacement.
Ground Displacements u and w
2.000
1.900
1.800
1.700
1.600
1.500
1.400
1.300
1.200
1.100
1.000
.900
.800
.700
.600
.500
.400
.300
.200
.100
.000
0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140° 150° 160° 170° 180°
Δ in degrees
W
U

Ratios of the u and w of the appropriate waves to the u and w of PP.

1. $\frac{PP}{P}$.

Δ	^1u_P	^1w_P	$^2u_{PP}$	$^2w_{PP}$	$\frac{^3u_{PP}}{u_P}$	$\frac{^3w_{PP}}{w_P}$
30°	1.71	3.15	1.70	1.928	.995	.612
40°	1.108	2.22	1.362	1.851	1.231	.835
50°	.885	1.894	.974	1.530	1.100	.808
60°	.724	1.717	.644	1.188	.889	.691
70°	.524	1.531	.489	.941	.934	.614
80°	.458	1.435	.470	.940	1.024	.654
90°	.389	1.295	.452	.936	1.161	.724
100°	.2185	.849	.436	.934	1.995	1.100
103°	.1071	.416	.432	.938	4.020	2.252

2. $\frac{P'(PKP)}{PP}$.

Δ	$^4u_{P'}$	$^4w_{P'}$	$^5u_{PP}$	$^5w_{PP}$	$\frac{^6u_{P'}}{u_{PP}}$	$\frac{^6w_{P'}}{w_{PP}}$
180°	∞	∞	∞	∞	.1638*	.1747*
170°	.2145	.801	.633	2.04	.339	.3925
160°	.1397	.527	.488	1.528	.286	.345
150°	.2095	.820	.426	1.291	.492	.634
142½°	∞	∞	.397	1.176	∞	∞
142½°	∞	∞	.397	1.176	∞	∞
150°	.0734	.371	.426	1.291	.1721	.2875
160°	.0686	.353	.488	1.528	.1405	.2310
169°	∞	∞	.630	1.977	∞	∞
169°	∞	∞	.630	1.977	∞	∞
160°	.1445	.896	.488	1.528	.296	.586
150°	.0822	.562	.426	1.291	.1927	.435
140°	.0564	.400	.392	1.147	.1438	.349
130°	.0282	.200	.394	1.032	.0716	.1935
120°	.0258	.1865	.414	.984	.0624	.1899
110°	∞	∞	.424	..955	∞	∞
110°	∞	∞	.424	.955	∞	∞
120°	.0409	.2955	.414	.984	.0989	.3005
130°	.0502	.399	.394	1.032	.1274	.387
140°	.0584	.554	.392	1.147	.1489	.483
150°	.0404	.412	.426	1.291	.0947	.319

---

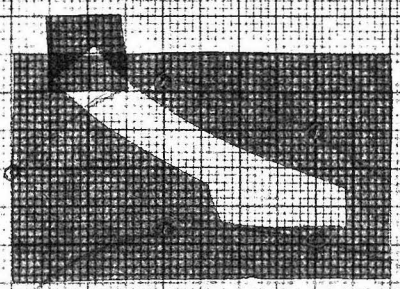
1 From the data on p. 111.
2 From the data on p. 130 and the graph on p. 131.
3 Plotted on p. 133. It will be noted that these graphs, as would be expected, are smoothed curves of Figs. 6 and 7 of Gutenberg and Richter (7).
4 Taken from the data on p. 58 and p. 59.
5 Taken from the data on p. 130 and the graph on p. 131.
6 Plotted on p. 135 and p. 136.
* See note on p. 134.

Ratio of the w of PP to the w of P and the Ratio of the w of PP to the w of P plotted against Δ.

4.500
4.000
3.500
3.000
2.500
2.000
1.500
1.000
.500
.000

30° 40° 50° 60° 70° 80° 90° 100° 110°

Δ in degrees



$\frac{w_{PP}}{w_P}$

$\frac{w_{PP}}{w_P}$

Δ	$u_{P'}$	$w_{P'}$	u_{PP}	w_{PP}	$\frac{u_{P'}}{u_{PP}}$	$\frac{w_{P'}}{w_{PP}}$
160°	.0956	1.122	.488	1.528	.1958	.736
170°	.0268	.892	.633	2.04	.0424	.436
180°	.0000	.417	∞	∞	.000	.000

3. $\frac{PKS}{PP}$,

Δ	$^1u_{PKS}$	$^1w_{PKS}$	$^2u_{PP}$	$^2w_{PP}$	$\frac{^3u_{PKS}}{u_{PP}}$	$\frac{^3w_{PKS}}{w_{PP}}$
$149\frac{1}{2}^\circ$	.000	.000	.420	1.285	.000	.000
140°	.1480	.02325	.392	1.147	.378	.02025
135°	.3345	.0525	.393	1.082	.853	.0485
$131\frac{1}{2}^\circ$	∞	∞	.3935	1.048	∞	∞
$131\frac{1}{2}^\circ$	∞	∞	.3935	1.048	∞	∞
140°	.1330	.01572	.392	1.147	.339	.01371
150°	.1289	.01493	.426	1.291	.3025	.01155
$158\ 3/4^\circ$	∞	∞	.480	1.493	∞	∞
$158\ 3/4^\circ$	∞	∞	.480	1.493	∞	∞
150°	.2482	.02495	.426	1.291	.584	.0193
140°	.1308	.01248	.392	1.147	.3335	.01088
130°	.0804	.00727	.394	1.032	.2037	.00705

---

1 Taken from the data on p. 67.
2 Taken from the data on p. 130 and the graph on p. 131.
3 Plotted on p. 138 and p. 139.
* Actually $\frac{u_{P'}}{u_{PP}}$ and $\frac{w_{P'}}{w_{PP}}$ are the indeterminate form $\frac{\infty}{\infty}$ here. They are approximated in the following manner, however, from the data in connection with P' on p. 58.

at 180° (for P_2') $u_{P'} = \frac{.299 \times \pi \sqrt{.2217 \times .03}}{\sqrt{\sin \Delta}}$ $\frac{u}{A} = \frac{.0406}{\sqrt{\sin \Delta}}$,

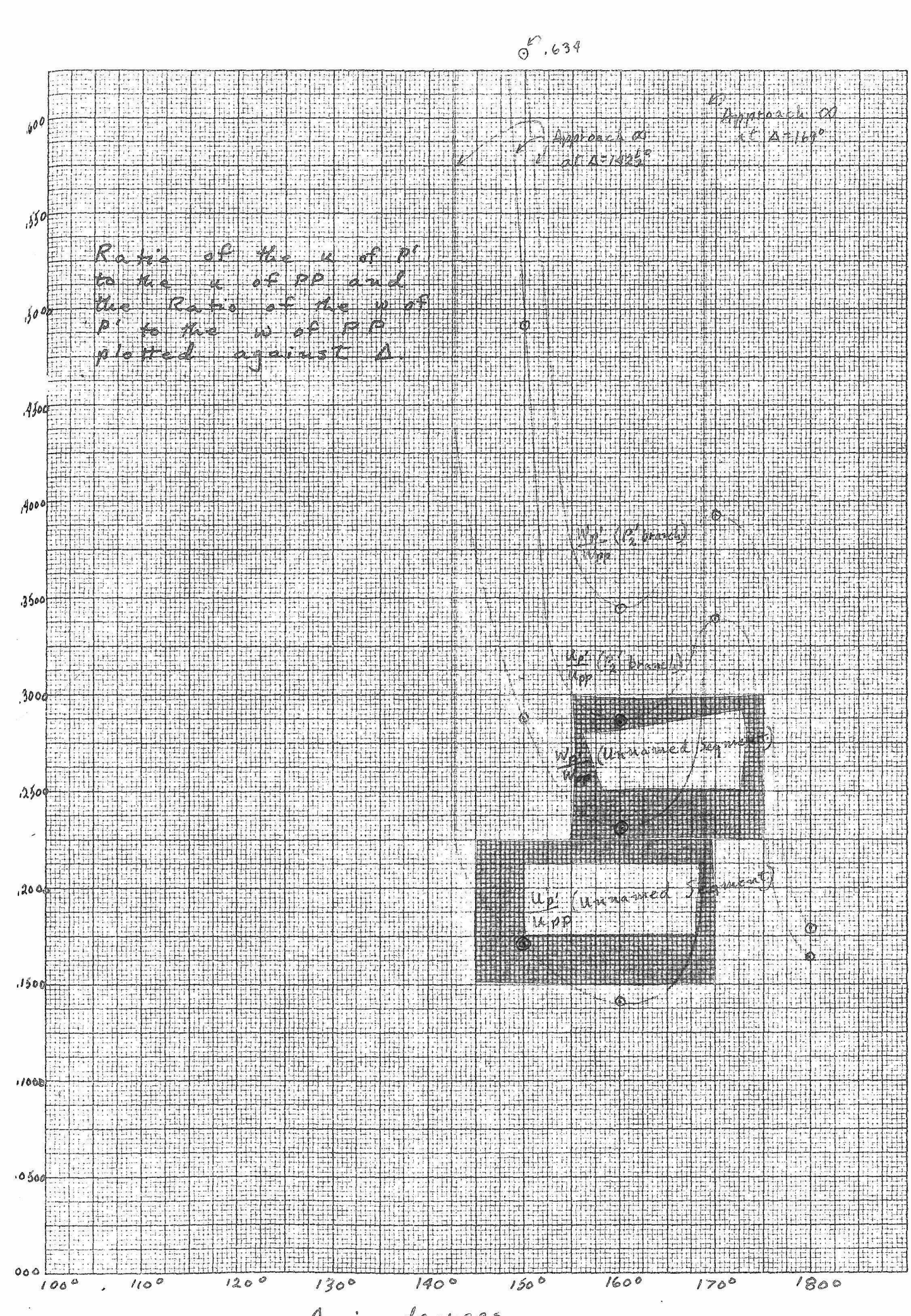
and $w_{P'} = \frac{.299 \times \pi \sqrt{.2217 \times .03}}{\sqrt{\sin \Delta}}$ $\frac{w}{A} = \frac{.1479}{\sqrt{\sin \Delta}}$,

at 180° (for PP)

$u_{PP} = \frac{.900 \times \pi \sqrt{.2484 \times .09125}}{\sqrt{\sin \Delta}}$ $\frac{u}{A} = \frac{.248}{\sqrt{\sin \Delta}}$,

and $w_{PP} = \frac{.900 \times \pi \sqrt{.2484 \times .09125}}{\sqrt{\sin \Delta}}$ $\frac{w}{A} = \frac{.824}{\sqrt{\sin \Delta}}$,

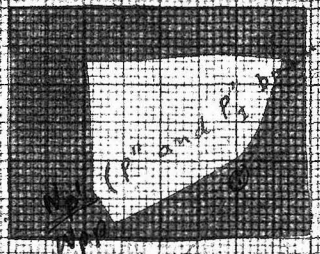
Thus, even though $\sin \Delta = 0$ for both PP and P' (actually the end of the P_2' branch) at 180°, it is possible to approximate $\frac{u_{P'}}{u_{PP}}$ and $\frac{w_{P'}}{w_{PP}}$ since $\sqrt{\sin \Delta}$ cancels out.


.634
Approach α at Δ=169°
Approach α at Δ=182½°
Ratio of the w of p' to the w of pp and the Ratio of the w of p' to the w of PP plotted against Δ.
Wp'/Wpp (P'2 branch)
Up'/Upp (P'2 branch)
Wp'/Wpp (Unnamed segment)
Up'/Upp (Unnamed segment)
.4500
.4000
.3500
.3000
.2500
.2000
.1500
.1000
.0500
.000
100°
110°
120°
130°
140°
150°
160°
170°
180°
Δ in degrees

Approach ∞ at Δ = 110°

Approach ∞ at Δ = 169°

Ratio of the w of P' to the w of PP' and the Ratio of the w of P' to the w of PP' plotted against Δ.





.800
.700
.600
.500
.400
.300
.200
.100
.000

100° 110° 120° 130° 140° 150° 160° 170° 180°

Δ in degrees

Δ	u_{PKS}	w_{PKS}	u_{PP}	w_{PP}	$\frac{u_{PKS}}{u_{PP}}$	$\frac{w_{PKS}}{w_{PP}}$
120°	.0484	.00426	.414	.984	.1171	.00434
110°	.0149	.00128	.424	.955	.0352	.00134
104¼°	∞	∞	.431	.939	∞	∞
104¼°	∞	∞	.431	.939	∞	∞
110°	.02115	.00175	.424	.955	.0499	.001831
120°	.0362	.00299	.414	.984	.0876	.00304
130°	.0675	.00541	.394	1.032	.1710	.00524
140°	.0930	.00675	.392	1.147	.2365	.00589
150°	.1051	.00712	.426	1.291	.2470	.00550
155°	.1367	.00821	.452	1.387	.3023	.00591
160°	.1362	.00511	.488	1.528	.2790	.00335
170°	.00474	.000095	.633	2.04	.0075	.000046
180°	.00000	.0000	∞	∞	.0000	.0000

4. $\frac{PKKP}{PP}$.

Δ	$^{1}u_{PKKP}$	$^{1}w_{PKKP}$	$^{2}u_{PP}$	$^{2}w_{PP}$	$\frac{^{3}u_{PKKP}}{u_{PP}}$	$\frac{^{3}w_{PKKP}}{w_{PP}}$
120°	∞	∞	.414	.984	∞	∞
115°	.01522	.0712	.421	.962	.0362	.0740
110°	.00914	.0461	.424	.955	.02155	.0484
105°	.00544	.02835	.431	.940	.01258	.03015
100°	.00507	.0267	.436	.934	.01161	.02865
95°	.00479	.0256	.445	.935	.01075	.02735

5. $\frac{PKKS}{PP}$,

Δ	$^{4}u_{PKKS}$	$^{4}w_{PKKS}$	$^{5}u_{PP}$	$^{5}w_{PP}$	$\frac{^{6}u_{PKKS}}{u_{PP}}$	$\frac{^{6}w_{PKKS}}{w_{PP}}$
132°	.03245	.00410	.3938	1.051	.0826	.00389
130°	.03125	.00387	.394	1.032	.0795	.00375
120°	.01320	.00156	.414	.984	.0319	.001587
110°	.00590	.000624	.424	.955	.01391	.000653
100°	.00546	.000548	.436	.934	.01251	.000587

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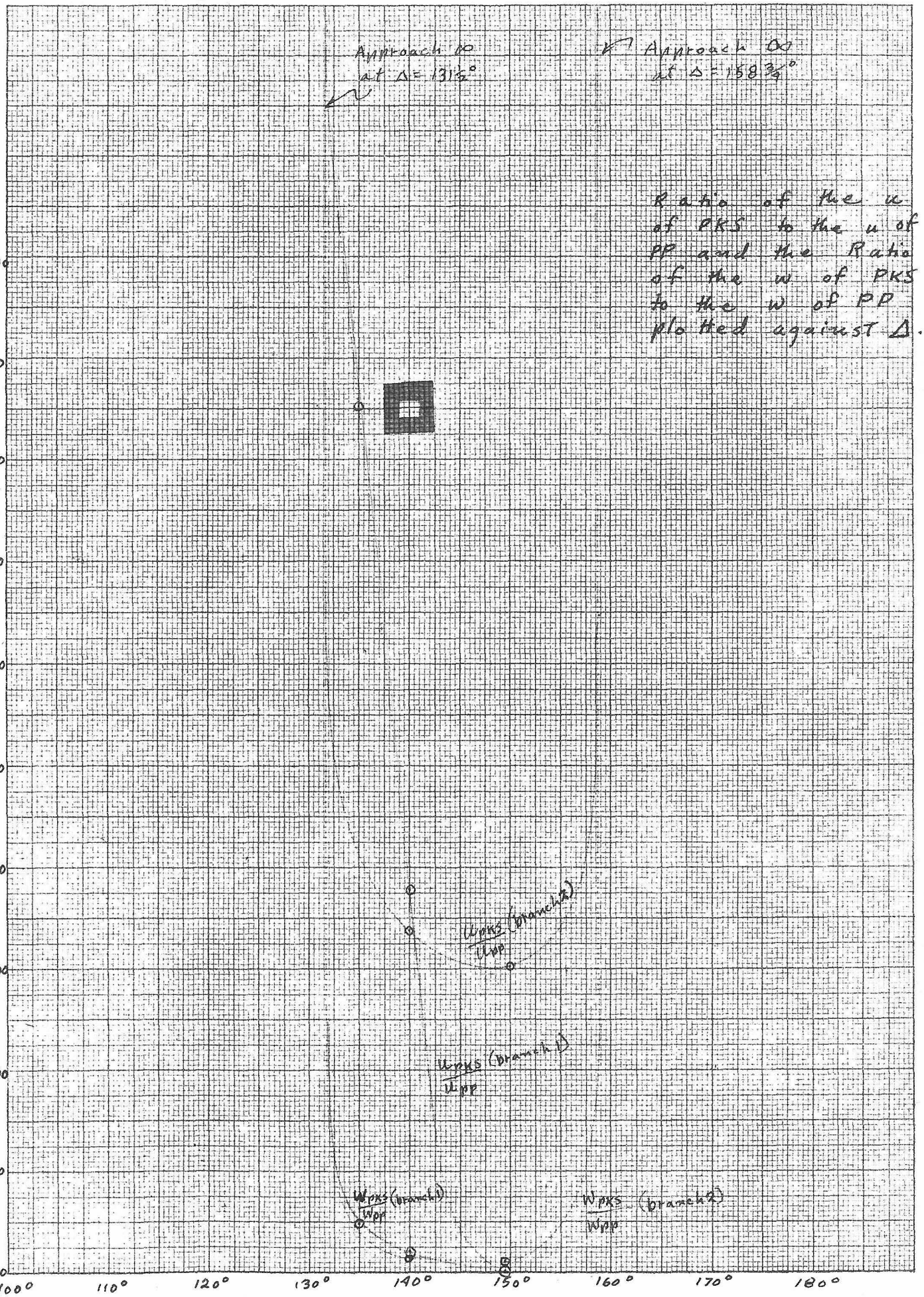
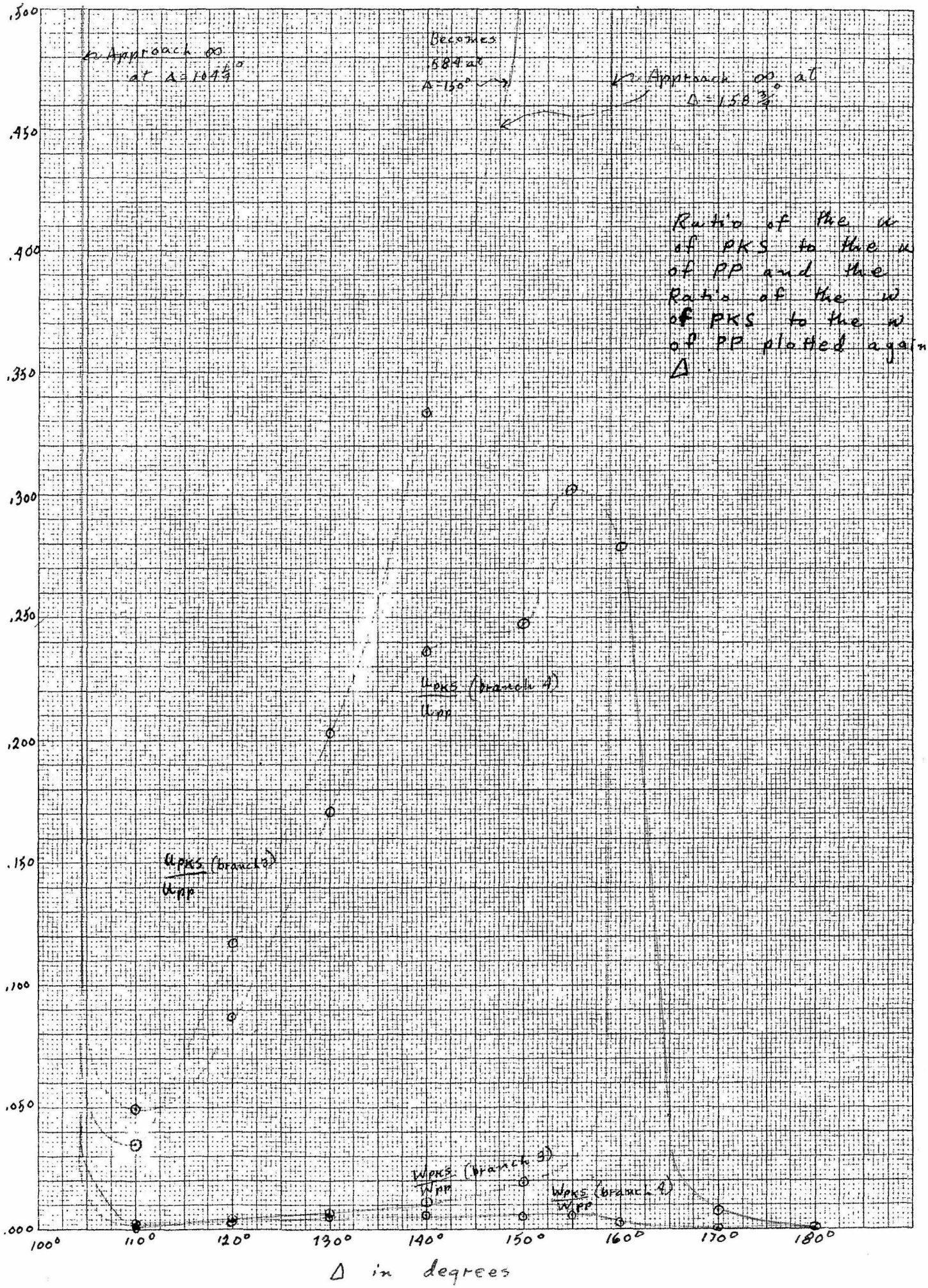
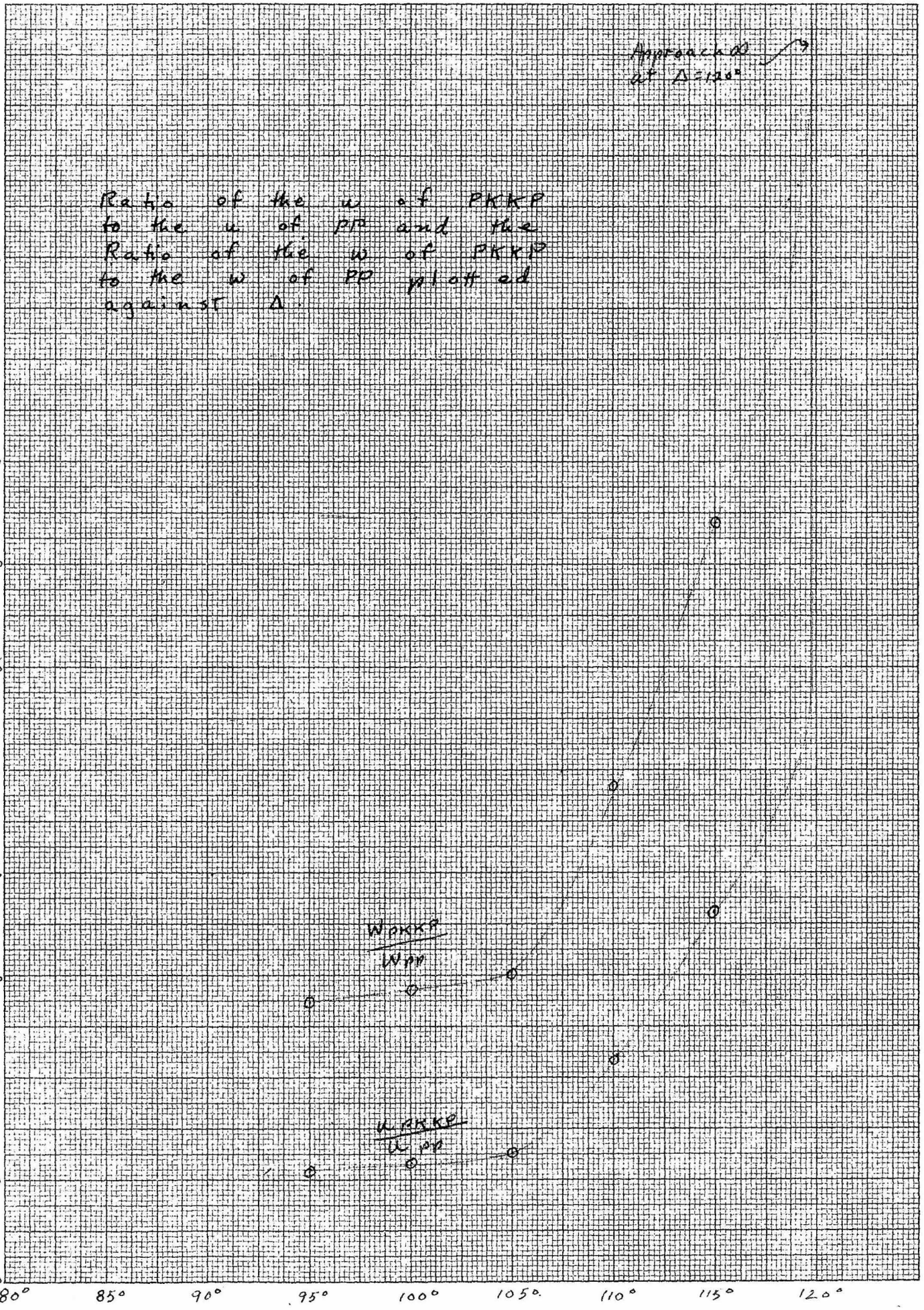
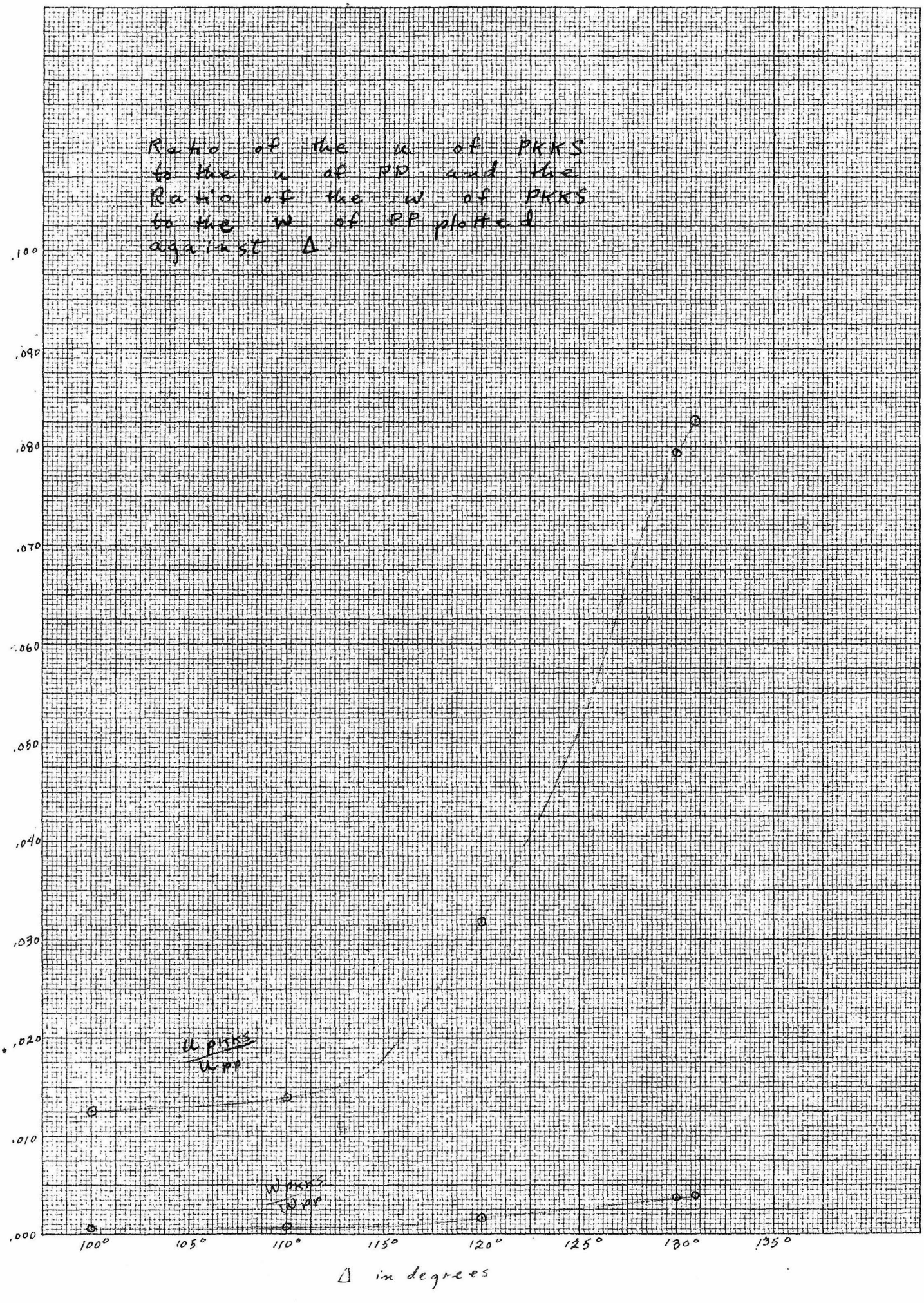
1 Taken from the data on p. 84.
2 Taken from the data on p. 130 and the graph on p. 131.
3 Plotted on p. 140.
4 Taken from the data on p. 89.
5 Taken from the data on p. 130 and the graph on p. 131.
6 Plotted on p. 141.


Approach ∞ at Δ = 131½°
Approach ∞ at Δ = 158 3/4°
Ratio of the u of PKS to the u of PP and the Ratio of the w of PKS to the w of PP plotted against Δ.
1.000
.900
.800
.700
.600
.500
.400
.300
.200
.100
.000
100°
110°
120°
130°
140°
150°
160°
170°
180°
Δ in degrees
u PKS (branch 1) / u PP
u PKS (branch 2) / u PP
W PKS (branch 1) / W PP
W PKS (branch 2) / W PP


.500
.450
.400
.350
.300
.250
.200
.150
.100
.050
.000
100°
110°
120°
130°
140°
150°
160°
170°
180°
Approach ∞ at Δ=104½°
Becomes .584 at Δ=150°
Approach ∞ at Δ=158¾°
Ratio of the w of PKS to the w of PP and the Ratio of the w of PKS to the w of PP plotted against Δ.
$\frac{u_{PKS}\ (branch\ 4)}{u_{PP}}$
$\frac{u_{PKS}\ (branch\ 3)}{u_{PP}}$
$\frac{w_{PKS}\ (branch\ 3)}{w_{PP}}$
$\frac{w_{PKS}\ (branch\ 4)}{w_{PP}}$
Δ in degrees


Approaching ∞ at Δ=120°
Ratio of the ω of PKKP to the ω of PP and the Ratio of the ω of PKKP to the ω of PP plotted against Δ.
.100
.090
.080
.070
.060
.050
.040
.030
.020
.010
.000
W PKKP / W PP
W PKKP / W PP
80°
85°
90°
95°
100°
105°
110°
115°
120°
Δ in degrees


Ratio of the w of PKKS to the w of PP and the Ratio of the w of PKKS to the w of PP plotted against Δ.
.100
.090
.080
.070
.060
.050
.040
.030
.020
.010
.000
100°
105°
110°
115°
120°
125°
130°
135°
Δ in degrees

6. $\frac{P'P'}{PP}$ for $\Delta = 30^\circ - 140^\circ$.

Δ	$^1u_{P'P'}$	$^1w_{P'P'}$	$^2u_{PP}$	$^2w_{PP}$	$\frac{^3u_{P'P'}}{u_{PP}}$	$\frac{^3w_{P'P'}}{w_{PP}}$
30°	.0477	.175	1.70	1.928	.0281	.0909
40°	.0423	.1595	1.362	1.851	.0311	.0861
60°	.0648	.2535	.644	1.188	.1007	.2135
75°	∞	∞	.480	.9405	∞	∞
75°	∞	∞	.480	.9405	∞	∞
60°	.0277	.140	.644	1.188	.0430	.1178
40°	.0252	.1298	1.362	1.851	.0185	.0702
30°	.0300	.1365	1.70	1.928	.0176	.0709
30°	.0765	.421	1.70	1.928	.0450	.2185
40°	.0581	.362	1.362	1.851	.0426	.1955
60°	.0354	.2425	.644	1.188	.055	.2042
80°	.0268	.1898	.470	.940	.057	.2020
100°	.01473	.1045	.436	.934	.0338	.1120
120°	.01541	.1114	.414	.984	.0373	.1135
140°	∞	∞	.392	1.147	∞	∞
140°	∞	∞	.392	1.147	∞	∞
120°	.0246	.1778	.414	.984	.0595	.1805
100°	.0271	.2155	.436	.934	.0621	.2305
80°	.0297	.2815	.470	.940	.0633	.2996
60°	.01985	.2028	.644	1.188	.0309	.1708
40°	.0466	.546	1.362	1.851	.0342	.2945
30°	.0325	.509	1.70	1.928	.0191	.2638

7. $\frac{P'P'P'}{PP}$ for $\Delta = 30^\circ - 180^\circ$.

Δ	$^4u_{P'P'P'}$	$^4w_{P'P'P}$	$^2u_{PP}$	$^2w_{PP}$	$\frac{^5u_{P'P'P'}}{u_{PP}}$	$\frac{^5w_{P'P'P}}{w_{PP}}$
180°	∞	∞	∞	∞	.00720	.00789
150°	.0283	.1058	.426	1.291	.0664	.0819
120°	.0175	.0661	.414	.984	.0423	.0672
90°	.0282	.1105	.452	.936	.0624	.1179
67.5°	∞	∞	.502	.965	∞	∞
67.5°	∞	∞	.502	.965	∞	∞

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[1] Taken from the data on p. 100-101 and the graphs on p. 102-103.
[2] Taken from the data on p. 130 and the graph on p. 131.
[3] Plotted on p. 144 and p. 145.
[4] Taken from the data on p. 104-105 and the graphs on p. 106-107.
[5] Plotted on p. 146 and p. 147.

Δ	$u_{P'P'P'}$	$w_{P'P'P'}$	u_{PP}	w_{PP}	$\frac{u_{P'P'P'}}{u_{PP}}$	$\frac{w_{P'P'P'}}{w_{PP}}$
90°	.0146	.0740	.452	.936	.0323	.0790
120°	.0128	.0658	.414	.984	.0310	.0669
147°	∞	∞	.410	1.243	∞	∞
147°	∞	∞	.410	1.243	∞	∞
120°	.0317	.1965	.414	.984	.0767	.1998
90°	.0215	.147	.452	.936	.0475	.1570
60°	.0193	.137	.644	1.188	.03003	.1151
30°	.0141	.100	1.70	1.928	.00830	.0519
30°	.0269	.214	1.70	1.928	.01581	.1110
60°	.0231	.219	.644	1.188	.0359	.1842
90°	.01372	.140	.452	.936	.0304	.1496
120°	.0308	.362	.414	.984	.0746	.368
150°	.00901	.300	.426	1.291	.0211	.232
180°	.000	.1388	∞	∞	.0000	.000

Evaluation of $\frac{u_{P'P'P'}}{u_{PP}}$ and $\frac{w_{P'P'P'}}{w_{PP}}$ at $\Delta = 180^\circ$ for branch 1:

At $\Delta = 180^\circ$ $\quad u_{P'P'P'} = \pi \times .02265 \sqrt{\frac{.2217}{\sin 180^\circ}} \times .01 \times .53 = \frac{.001778}{\sqrt{\sin 180^\circ}}$,

" $\quad u_{PP} = \pi \times .900 \sqrt{\frac{.2484}{\sin 180^\circ} \times .09125} \times .58 = \frac{.247}{\sqrt{\sin 180^\circ}}$.

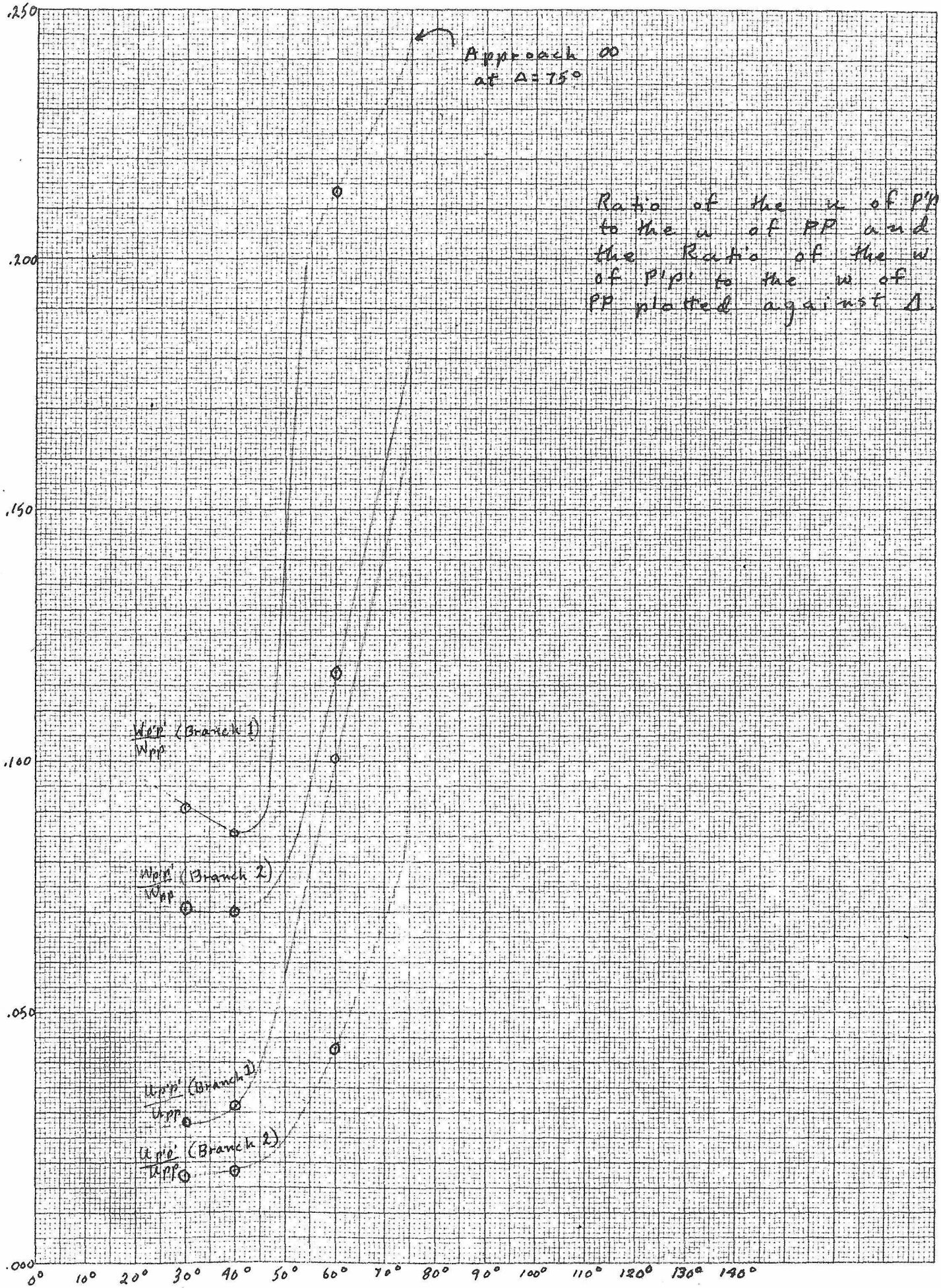
Hence $\quad \frac{u_{P'P'P'}}{u_{PP}} = .00720$.

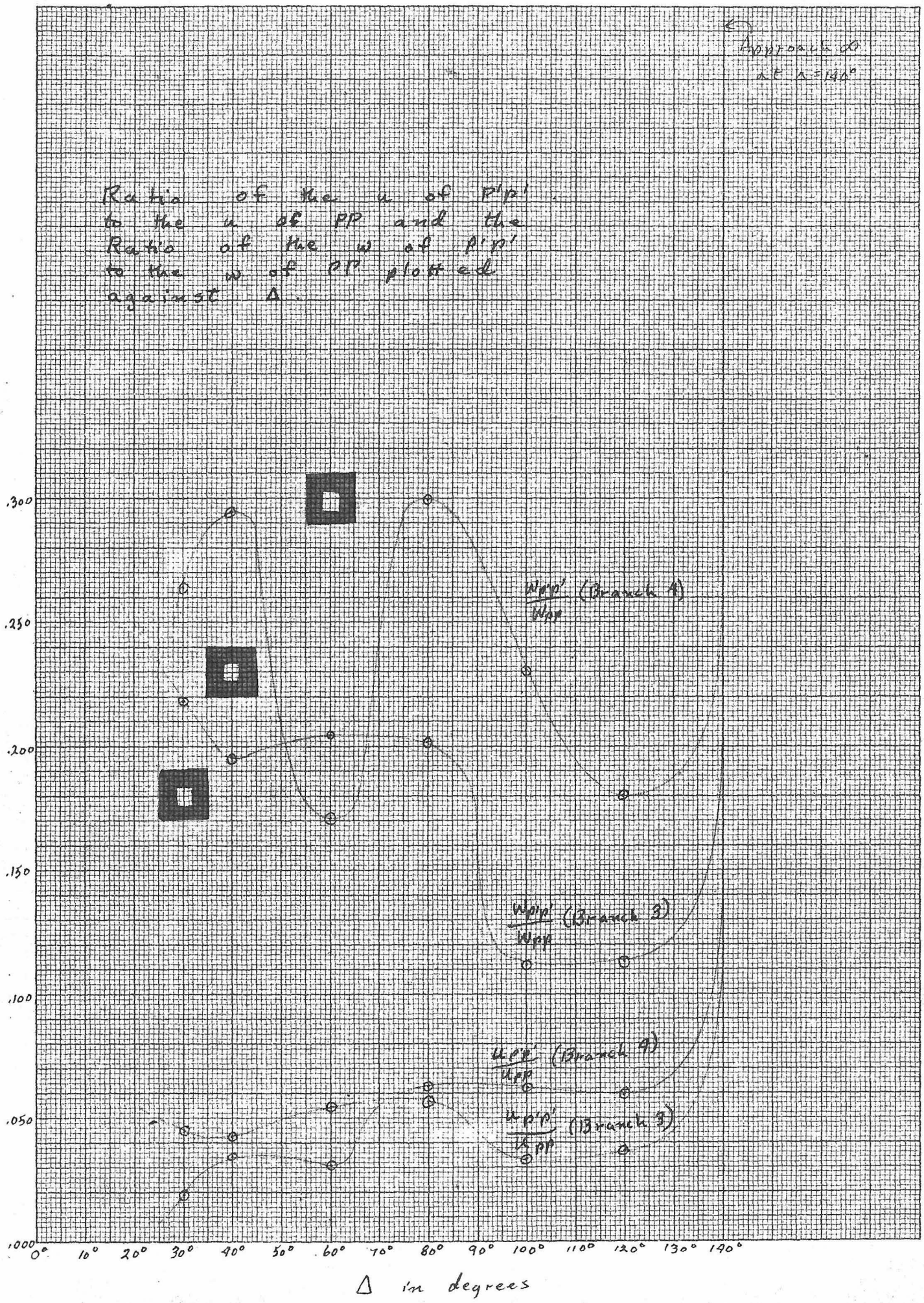
At $\Delta = 180^\circ$ $\quad w_{P'P'P'} = \pi \times .02265 \sqrt{\frac{.2217}{\sin 180^\circ} \times .01} \times 1.930 = \frac{.00649}{\sqrt{\sin 180^\circ}}$,

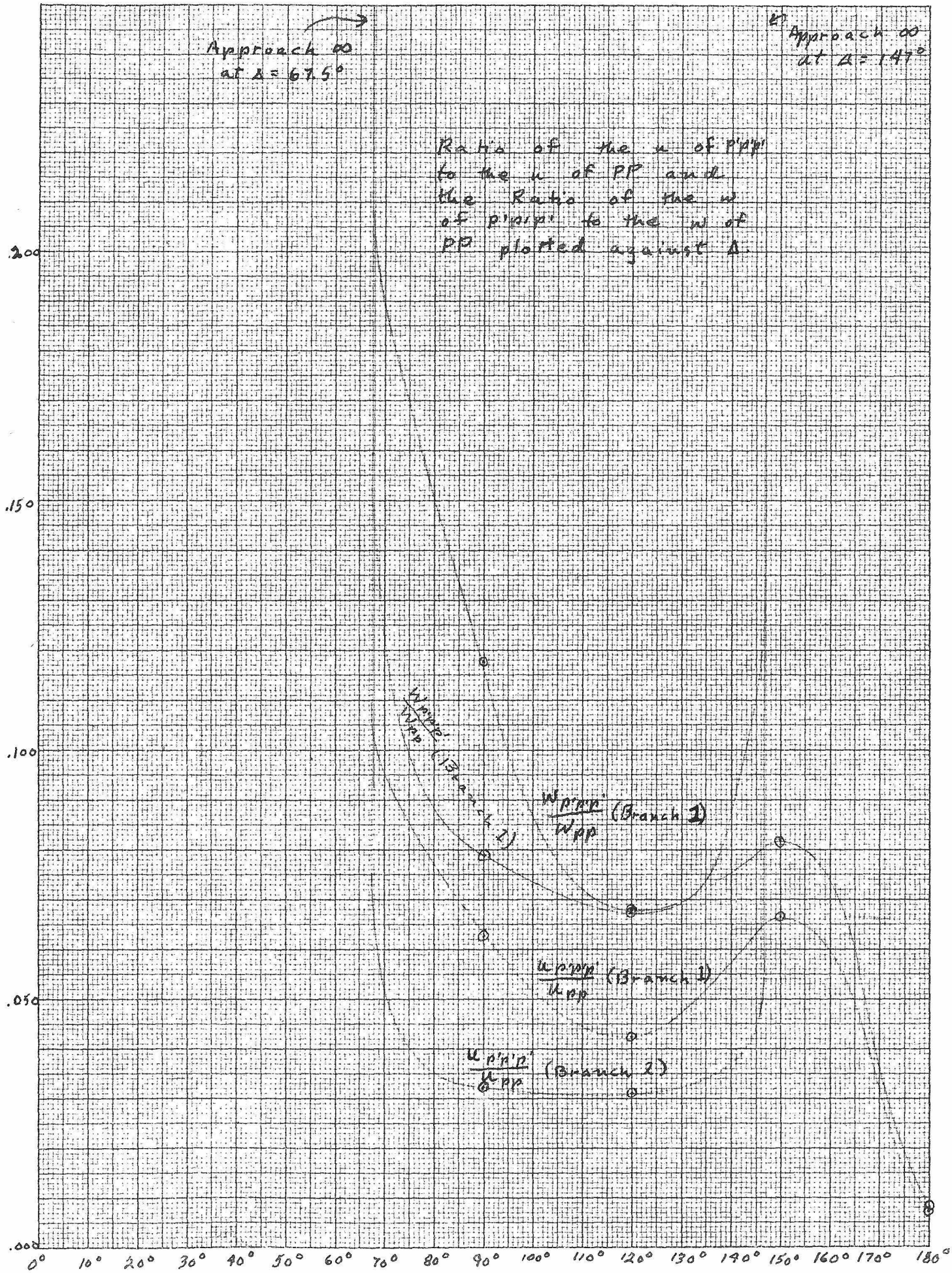
" $\quad w_{PP} = \pi \times .900 \sqrt{\frac{.2484}{\sin 180^\circ} \times .09125} \times 1.93 = \frac{.824}{\sqrt{\sin 180^\circ}}$.

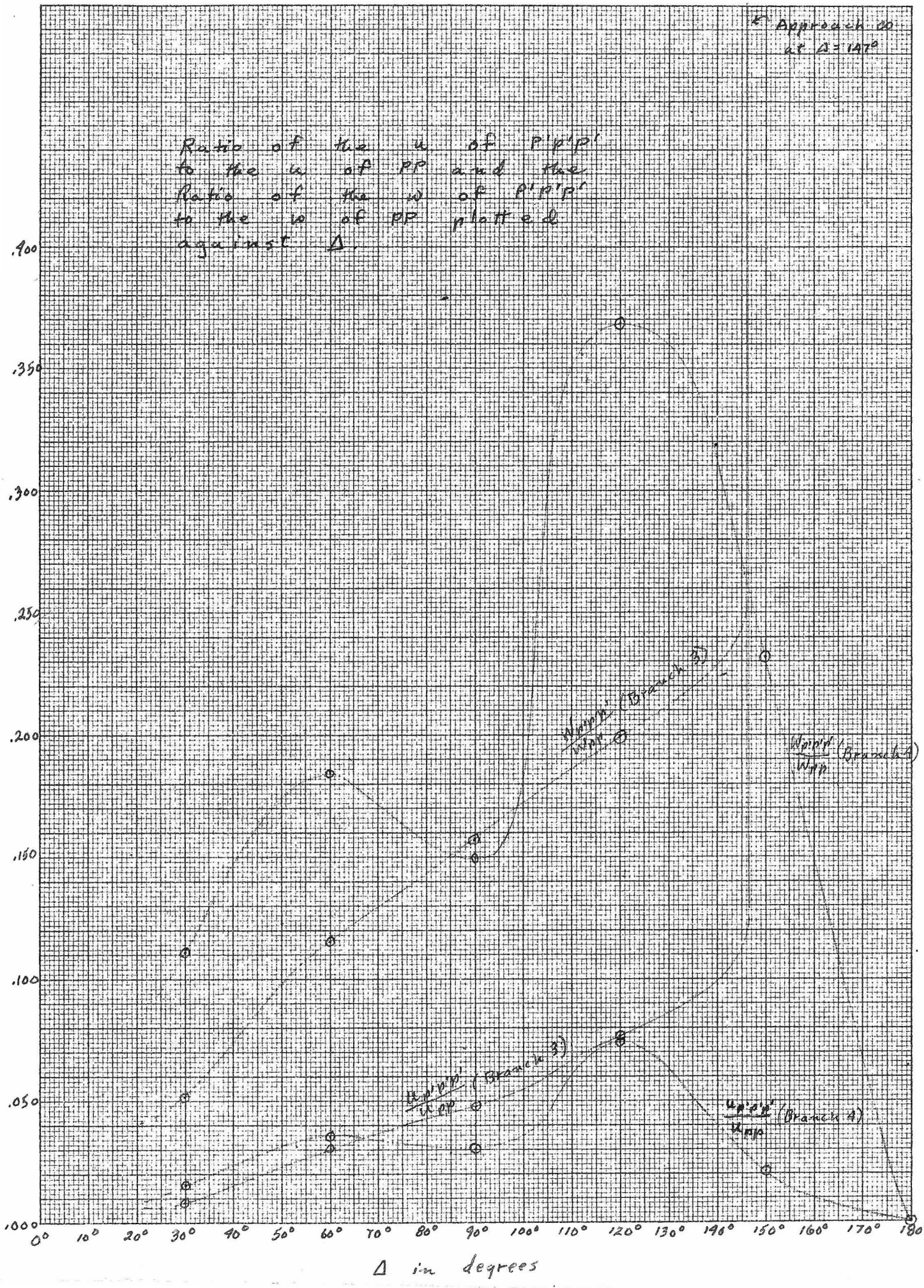
Hence $\quad \frac{w_{P'P'P'}}{w_{PP}} = .00789$.

It should be remembered that these are evaluations of the indeterminate form $\frac{\infty}{\infty}$, and as such are only approximations.


.250
.200
.150
.100
.050
.000
Approach ∞ at Δ = 75°
Ratio of the w of P'p' to the w of PP and the Ratio of the w of p'p' to the w of PP plotted against Δ.
W'p' / Wpp (Branch 1)
W'p' / Wpp (Branch 2)
W'p'' / Wpp (Branch 1)
W'p'' / Wpp (Branch 2)
0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140°
Δ in degrees


Approach ∞ at Δ = 140°
Ratio of the u of P'p' to the u of PP and the Ratio of the w of P'p' to the w of PP plotted against Δ.
.300
.250
.200
.150
.100
.050
.000
0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140°
Wp'p' / Wpp (Branch 4)
Wp'p' / Wpp (Branch 3)
up'p' / upp (Branch 4)
up'p' / upp (Branch 3)
Δ in degrees


Approach ∞ at Δ = 67.5°
Approach ∞ at Δ = 147°
Ratio of the w of p'p'p' to the w of PP and the Ratio of the w of p'p'p' to the w of PP plotted against Δ.
.200
.150
.100
.050
.000
$\frac{w_{p'p'p'}}{w_{pp}}$ (Branch 1)
$\frac{w_{p'p'p'}}{w_{pp}}$ (Branch 2)
0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140° 150° 160° 170° 180°
Δ in degrees


Approach to
at Δ = 147°
Ratio of the w of P'p'p'
to the w of pp and the
Ratio of the w of p'p'P'
to the w of pp plotted
against Δ.
.900
.350
.300
.250
.200
.150
.100
.050
.000
0° 10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120° 130° 140° 150° 160° 170° 180°
Δ in degrees
(Branch 3)
(Branch 4)
(Branch 3)
(Branch 4)

4. SS (SV type) and its u and w Ratios

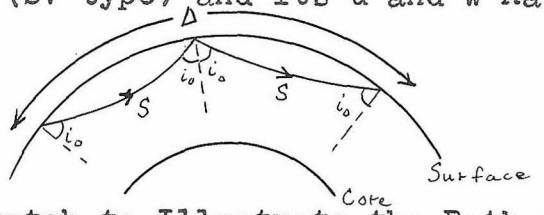


Fig. 25. Sketch to Illustrate the Path of SS.

As in the case of PP, the angle of incidence of SS at a particular Δ is the same as the angle of incidence of SV at $\frac{1}{2}$ that particular Δ. $\frac{di_o}{d\Delta}$ for SS at a particular Δ is $\frac{1}{2}$ the $\frac{di_o}{d\Delta}$ for SV at $\frac{1}{2}$ that particular Δ. The reason for this is the same as in the case of PP. The SS treated in this case is the SV type.

Δ	[1] i_o	$\tan i_o$	[2] $\frac{di_o}{d\Delta}$	$\sin\Delta$	[3] f	[4] A	[5] $\frac{u}{A}$	[5] $\frac{w}{A}$	[6] u	[6] w
30°	40°55'	.8667	.645	.500	1.00	3.319	.55	1.48	1.822	4.91
38°	36°00'	.7265	.5825	.6157	.31	.806	5.20	.50	4.19	.403
40°	35°00'	.7002	.540	.643	.00	.000	3.10	.00	.000	.000
44.4°	32°30'	.6371	.500	.700	.10	.2123	1.75	1.20	.372	.255
50°	29°50'	.5750	.395	.766	.025	.0428	1.69	1.30	.0724	.0556
52.6°	29°00'	.5543	.350	.792	.000	.0000	1.68	1.00	.0000	.0000
60°	27°00'	.5095	.2075	.866	.110	.1208	1.70	.97	.2055	.1172
70°	25°33'	.4781	.1050	.940	.180	.1310	1.71	.94	.2240	.1231
80°	24°46'	.4614	.0975	.985	.250	.1678	1.72	.92	.2897	.1544
90°	23°42'	.4390	.0962	1.000	.280	.1807	1.725	.91	.3120	.1644
100°	23°05'	.4262	.0950	.985	.300	.1909	1.73	.90	.3303	.1719
110°	21°48'	.4000	.0925	.940	.370	.2302	1.755	.855	.405	.1970
120°	21°00'	.3839	.0900	.866	.400	.2510	1.78	.81	.447	.2030
130°	20°00'	.3420	.0862	.766	.460	.2842	1.795	.775	.510	.2202
140°	18°40'	.3378	.0825	.643	.510	.333	1.81	.74	.604	.2464
150°	18°25'	.3330	.0725	.500	.530	.3665	1.82	.72	.668	.264
160°	17°42'	.3191	.0625	.342	.570	.4340	1.825	.71	.791	.3078
170°	17°22'	.3127	.0575	.174	.590	.5960	1.830	.69	1.09	.411
180°	16°58'	.3051	.0550	.000	.600		1.840	.68	∞	∞

---

[1] Taken from the data on p. 121 and the graph on p. 123. The values of the SS (SV-type) amplitudes are thus smoothed and rid of irregularities in the same way as those for SV.

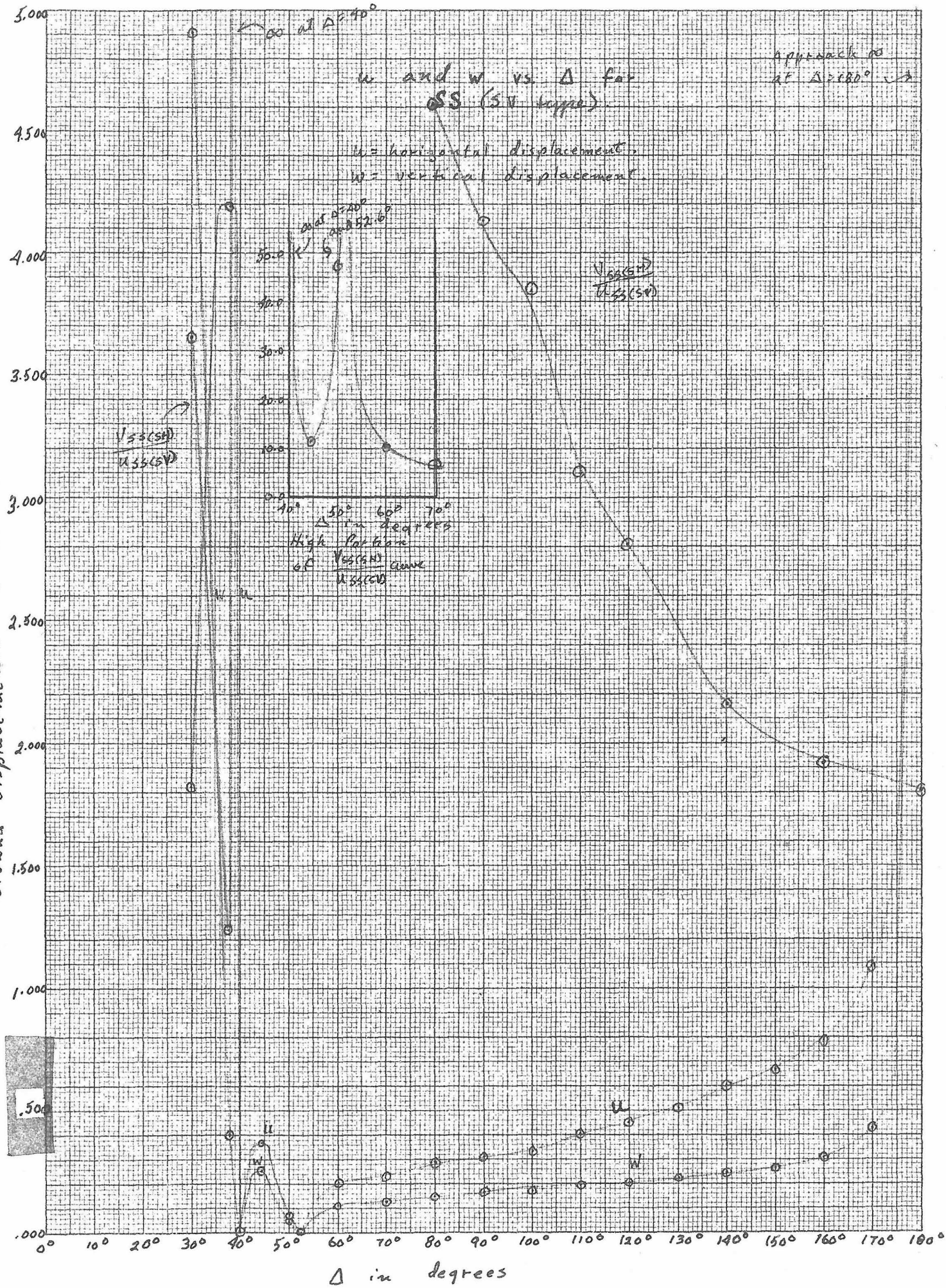
[2] Taken from the data on p. 121.

[3] $f = \sqrt{\frac{E_{reflected}}{E_{inc.}}}$ at the earth's surface at $\frac{1}{2}$ the Δ for SS. It is taken from Reflections and Refractions, 1943, graphs of Dr. Gutenberg (copy on p. 10).

[4] $A = CTf\sqrt{\frac{\tan i_o}{\sin\Delta}\frac{di_o}{d\Delta}}$ where $C = \pi$, $T = 1.0$ sec.

[5] Taken from the data on p. 124. $\frac{u}{A}$, for example, at 180° for SS is the same as $\frac{u}{A}$ for SV at 90°.

[6] Plotted on p. 149.


u and w vs. Δ for SS (SV type).
u = horizontal displacement.
w = vertical displacement.
∞ at Δ = 40°
approach ∞ at Δ = 180°
∞ at Δ = 40° and 52.6°
Vss(SH) / Uss(SV)
High Portion of Vss(SH) / Uss(SV) curve
Δ in degrees
Ground Displacements u and w.
Δ in degrees

Ratios of the u and w of the appropriate waves to the u and w of SS (SV type).

1. $\frac{SS\ (SV)}{SV}$.

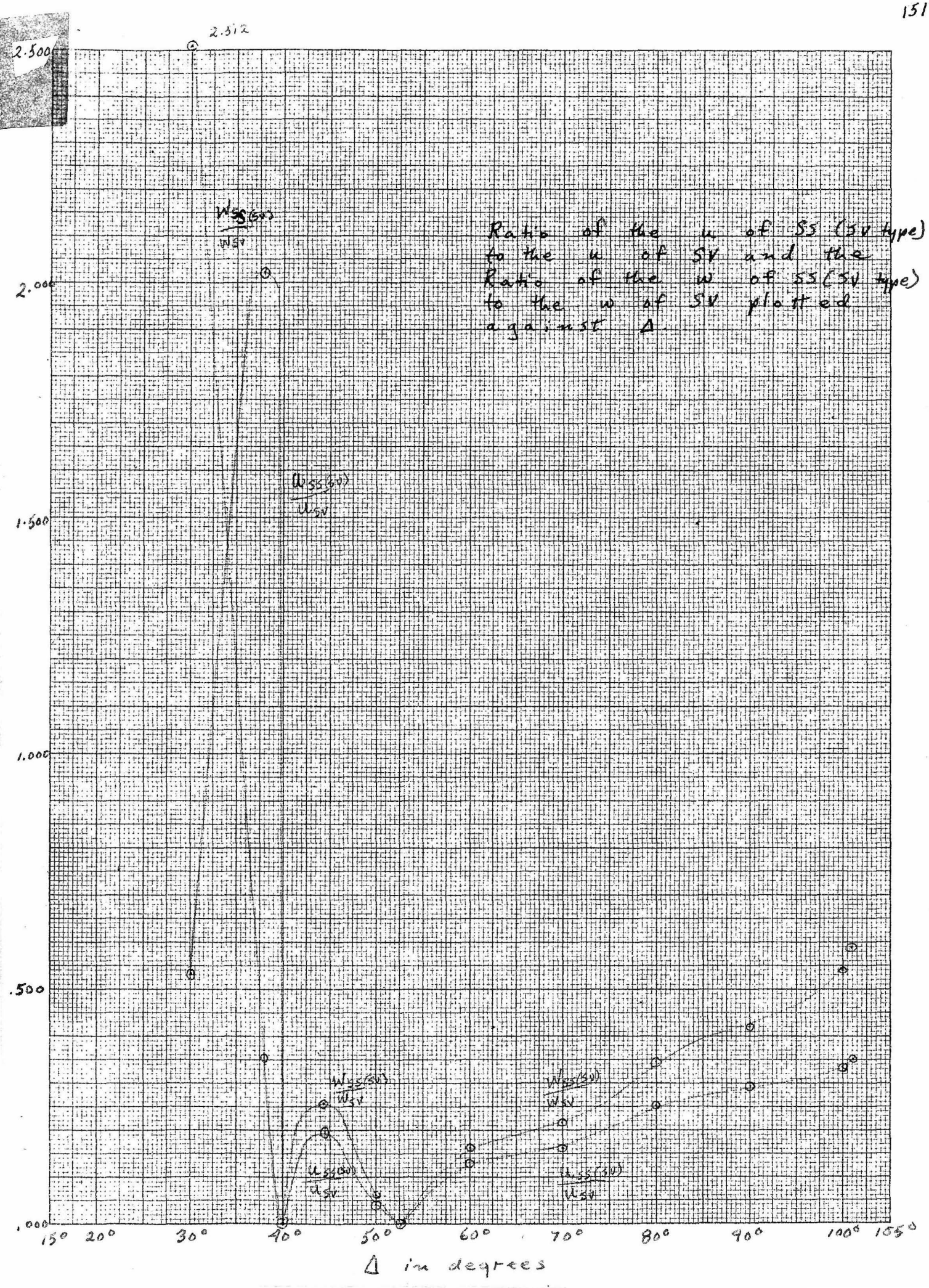
Δ	$^1u_{SS(SV)}$	$^1w_{SS(SV)}$	$^2u_{SV}$	$^2w_{SV}$	$\frac{^3u_{SS(SV)}}{u_{SV}}$	$\frac{^3w_{SS(SV)}}{w_{SV}}$
30°	1.822	4.91	3.435	1.959	.531	2.512
38°	4.19	.403	2.07	1.130	2.021	.3562
40°	.00	.00	2.025	1.084	.000	.0000
44.4°	.372	.255	1.870	.990	.1987	.2575
50°	.0724	.0556	1.761	.917	.0410	.0606
52.6°	.000	.000	1.69	.900	.0000	.0000
60°	.2055	.1172	1.575	.717	.1302	.1637
70°	.224	.1231	1.388	.568	.1614	.217
80°	.2897	.1544	1.155	.449	.2508	.345
90°	.3120	.1644	1.061	.392	.294	.420
100°	.3303	.1719	.991	.319	.3327	.539
101°	.340	.1800	.970	.3055	.3502	.590

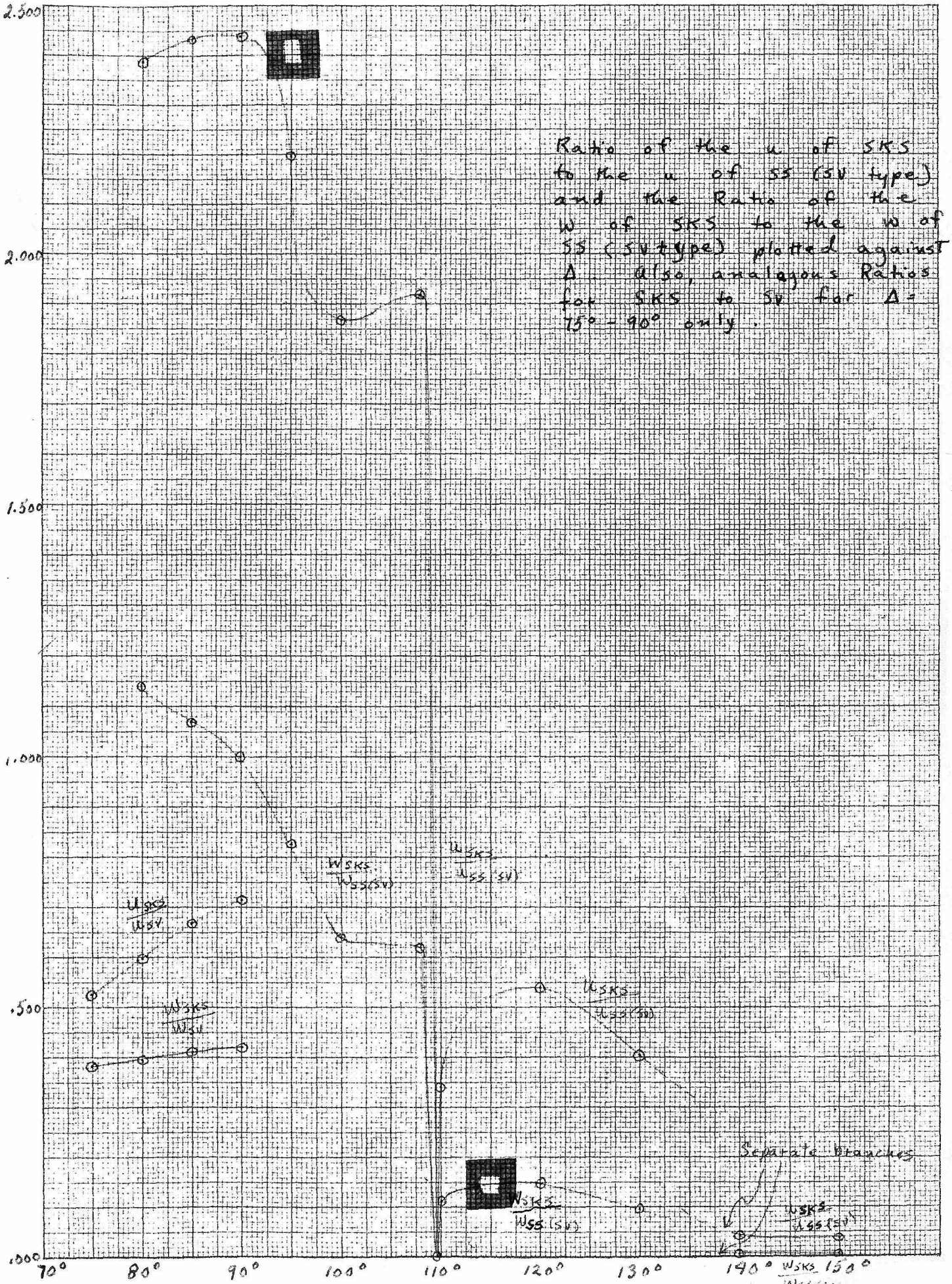
2. $\frac{SKS}{SS(SV)}$.

Δ	$^4u_{SKS}$	$^4w_{SKS}$	$^2u_{SS(SV)}$	$^2w_{SS(SV)}$	$\frac{^5u_{SKS}}{u_{SS(SV)}}$	$\frac{^5w_{SKS}}{w_{SS(SV)}}$
80°	.693	.1765	.2897	.1544	2.39	1.141
85°	.731	.1701	.3000	.1590	2.435	1.070
90°	.761	.1645	.3120	.1644	2.438	1.001
95°	.703	.1397	.3200	.1695	2.197	.825
100°	.618	.1099	.3303	.1719	1.869	.640
108°	.749	.1212	.3900	.1950	1.919	.622
109.7°	.000	.0000	.400	.1965	.000	.000
110°	.1385	.0217	.405	.1970	.342	.1101
120°	.2420	.0304	.447	.203	.541	.1497
130°	.207	.0218	.510	.2202	.406	.0990
140°	.0247	.00186	.604	.2464	.041	.0075
150°	.0250	.00163	.668	.264	.0374	.0062

---

1 Taken from the data on p. 148 and the graph on p. 149.
2 Taken from the data on p. 124 and the graph on p. 125.
3 Plotted on p. 151. The effect of T should be considered here since SS(SV) has a much larger period than SV.
4 Taken from the data on p. 72.
5 Plotted on p. 152. The effect of T should be considered here since SS(SV) has a much larger period than SKS.


2.500
2.000
1.500
1.000
.500
1.000
2.512
Ratio of the w of SS (SV type) to the w of SV and the Ratio of the w of SS (SV type) to the w of SV plotted against Δ
150 200 300 400 500 600 700 800 900 1000 1550
Δ in degrees


2.500
2.000
1.500
1.000
.500
.000
70°
80°
90°
100°
110°
120°
130°
140°
150°
Δ in degrees
Ratio of the w of SKS to the w of SS (SV type) and the Ratio of the w of SKS to the w of SS (SV type) plotted against Δ. Also, analagous Ratios for SKS to SV for Δ = 75° - 90° only.
W_SKS / W_SS(SV)
W_SKS / W_SV
W_SKS / W_SH
Separate Branches

3. $\frac{SKS}{SV}$ for $\Delta = 75^\circ$ through $\Delta = 90^\circ$ only.

Δ	$^1u_{SKS}$	$^1w_{SKS}$	$^2u_{SV}$	$^2w_{SV}$	$^3\frac{u_{SKS}}{u_{SV}}$	$^3\frac{w_{SKS}}{w_{SV}}$
75°	.676*	.1950*	1.28	.506	.528	.385
80°	.693	.1765	1.55	.449	.599	.394
85°	.731	.1701	1.094	.413	.669	.412
90°	.761	.1645	1.061	.392	.716	.420

4. $\frac{SKP}{SS(SV)}$.

Δ	$^4u_{SKP}$	$^4w_{SKP}$	$^5u_{SS(SV)}$	$^5w_{SS(SV)}$	$^6\frac{u_{SKP}}{u_{SS(SV)}}$	$^6\frac{w_{SKP}}{w_{SS(SV)}}$
$149\frac{1}{2}^\circ$	.000	.000	.672	.263	.000	.000
140°	.0642	.240	.604	.2464	.1061	.974
135°	.172	.669	.552	.2310	.3112	2.892
$131\frac{1}{2}^\circ$	∞	∞	.527	.225	∞	∞
$131\frac{1}{2}^\circ$	∞	∞	.527	.225	∞	∞
140°	.0448	.2285	.604	.2464	.0744	.927
150°	.0416	.2185	.668	.264	.0625	.829
$158\ 3/4^\circ$	∞	∞	.780	.300	∞	∞
$158\ 3/4^\circ$	∞	∞	.780	.300	∞	∞
150°	.0729	.411	.668	.264	.1091	1.557
140°	.02735	.1868	.604	.2464	.0453	.757
130°	.01654	.1212	.510	.2202	.03244	.551
120°	.01142	.0841	.447	.2030	.02558	.414
110°	.00772	.0589	.405	.1970	.01903	.2982
$104\frac{1}{4}^\circ$	∞	∞	.360	.193	∞	∞
$104\frac{1}{4}^\circ$	∞	∞	.360	.193	∞	∞
110°	.00464	.0368	.405	.197	.01144	.1868
120°	.00646	.0534	.447	.203	.01443	.2625
130°	.01279	.1158	.510	.2202	.02507	.5300
140°	.01428	.1495	.604	.2464	.02365	.606
150°	.01679	.1909	.668	.264	.02515	.724
160°	.01343	.2062	.791	.3078	.01699	.672
170°	.00197	.00986	1.09	.411	.00018	.02397
180°	.00000	.00000	∞	∞	.00000	.00000

---

1 Taken from the data on p. 72. *Obtained from the plot of u_{SKS} and w_{SKS} vs. Δ on p. 74 by interpolating back to $\Delta = 75^\circ$.

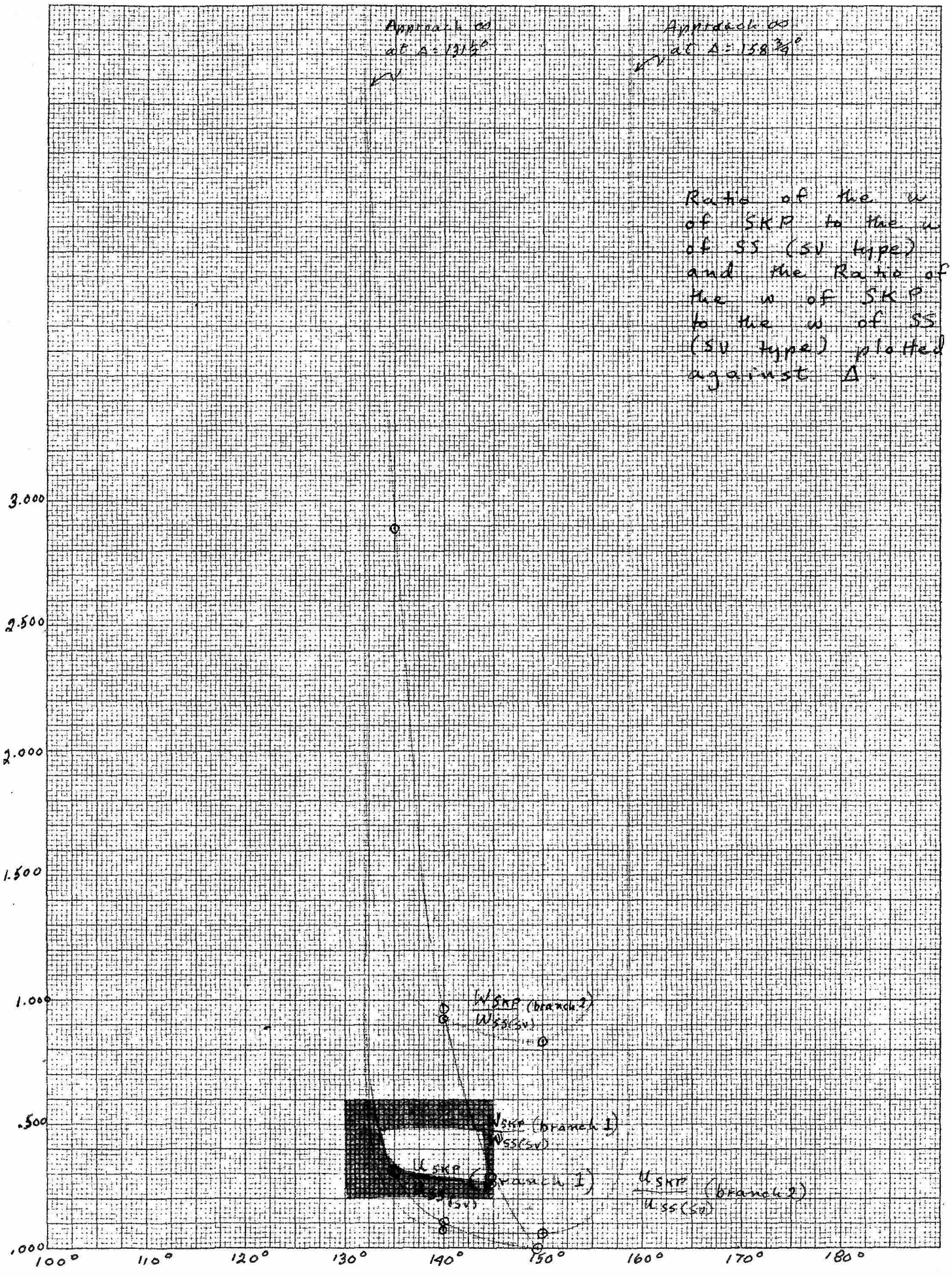
2 Taken from the data on p. 124.

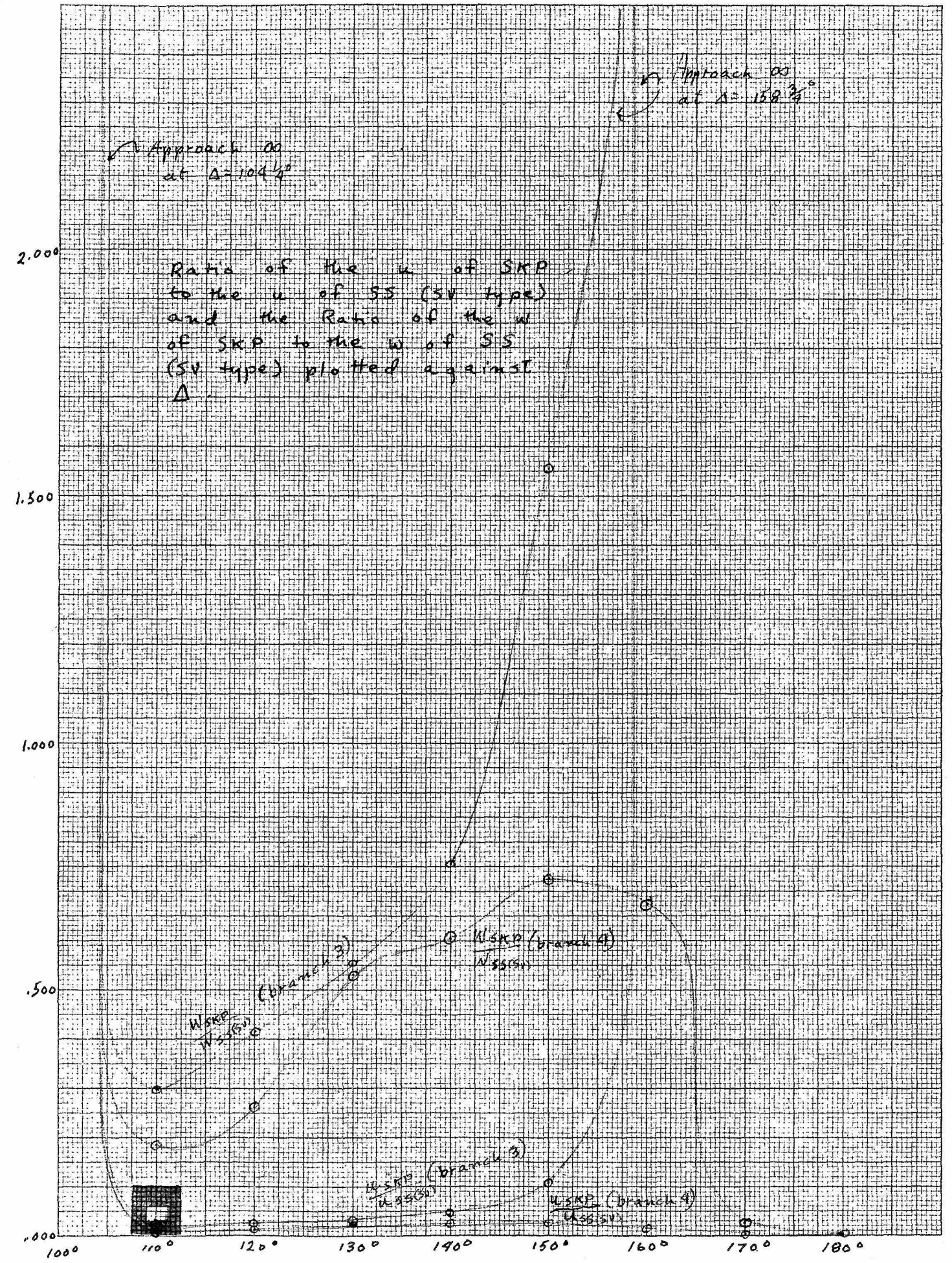
3 Plotted on p. 152.

4 Taken from the data on p. 78.

5 Taken from the data on p. 148 and the graph on p. 149.

6 Plotted on p. 154 and p. 155.


Approach ∞ at Δ = 131½°
Approach ∞ at Δ = 158 3/4°
Ratio of the w of SKP to the w of SS (SV type) and the Ratio of the w of SKP to the w of SS (SV type) plotted against Δ.
3.000
2.500
2.000
1.500
1.000
.500
.000
100°
110°
120°
130°
140°
150°
160°
170°
180°
Δ in degrees
W SKP (branch 2) / W SS(SV)
W SKP (branch 1) / W SS(SV)
W SKP (branch 1) / W SS(SV)
W SKP (branch 2) / W SS(SV)


Approach ∞ at Δ = 158 3/4°
Approach ∞ at Δ = 104 1/4°
Ratio of the u of SKP to the u of SS (SV type) and the Ratio of the w of SKP to the w of SS (SV type) plotted against Δ.
2.000
1.500
1.000
.500
.000
1000
110°
120°
130°
140°
150°
160°
170°
180°
Δ in degrees
W SKP / W SS(SV) (branch 4)
(branch 3)
W SKP / W SS(SV)
u SKP / u SS(SV) (branch 3)
u SKP / u SS(SV) (branch 4)

5. $\frac{SKKS}{SS(SV)}$.

Δ	$^1u_{SKKS}$	$^1w_{SKKS}$	$^2u_{SS(SV)}$	$^2w_{SS(SV)}$	$^3\frac{u_{SKKS}}{u_{SS(SV)}}$	$^3\frac{w_{SKKS}}{w_{SS(SV)}}$
100°	.317	.0776	.3303	.1719	.961	.451
110°	.1332	.0311	.405	.1970	.329	.1578
120°	.1394	.0318	.447	.203	.312	.1566
130°	.1544	.03435	.510	.2202	.303	.1557
140°	.2198	.0476	.604	.2464	.364	.1929
150°	.367	.0735	.668	.264	.550	.2781
160°	.245	.0475	.791	.3078	.3097	.1542
165°	.2708	.0518	.903	.357	.3000	.1451

6. $\frac{SKKP}{SS(SV)}$.

Δ	$^4u_{SKKP}$	$^4w_{SKKP}$	$^2u_{SS(SV)}$	$^2w_{SS(SV)}$	$^5\frac{u_{SKKP}}{u_{SS(SV)}}$	$^5\frac{w_{SKKP}}{w_{SS(SV)}}$
100°	.001309	.00708	.3303	.1719	.00396	.0412
110°	.00203	.0107	.405	.1970	.00501	.0543
120°	.00480	.0246	.447	.203	.01072	.1211
130°	.01335	.0582	.510	.2202	.02619	.2642
132°	.0149	.0619	.535	.225	.02782	.2745

7. $\frac{SS(SH)}{SS(SV)}$.

Δ	$^6v_{SS(SH)}$	$^7u_{SS(SV)}$	$^8\frac{v_{SS(SH)}}{u_{SS(SV)}}$		$^6v_{SS(SH)}$	$^7u_{SS(SV)}$	$^8\frac{v_{SS(SH)}}{u_{SS(SV)}}$
30°	6.65	1.822	3.65	80°	1.33	.2897	4.61
38°	5.21	4.19	1.24	90°	1.29	.3120	4.13
40°	4.68	.00	∞	100°	1.27	.3303	3.85
44.4°	4.25	.372	11.4	110°	1.25	.405	3.09
50.0°	3.42	.0724	47.3	120°	1.255	.447	2.81
52.6°	3.11	.000	∞	140°	1.305	.604	2.16
60.0°	2.19	.2055	10.67	160°	1.52	.791	1.92
70.0°	1.45	.2240	6.48	180°	∞	∞	^{9}1.803

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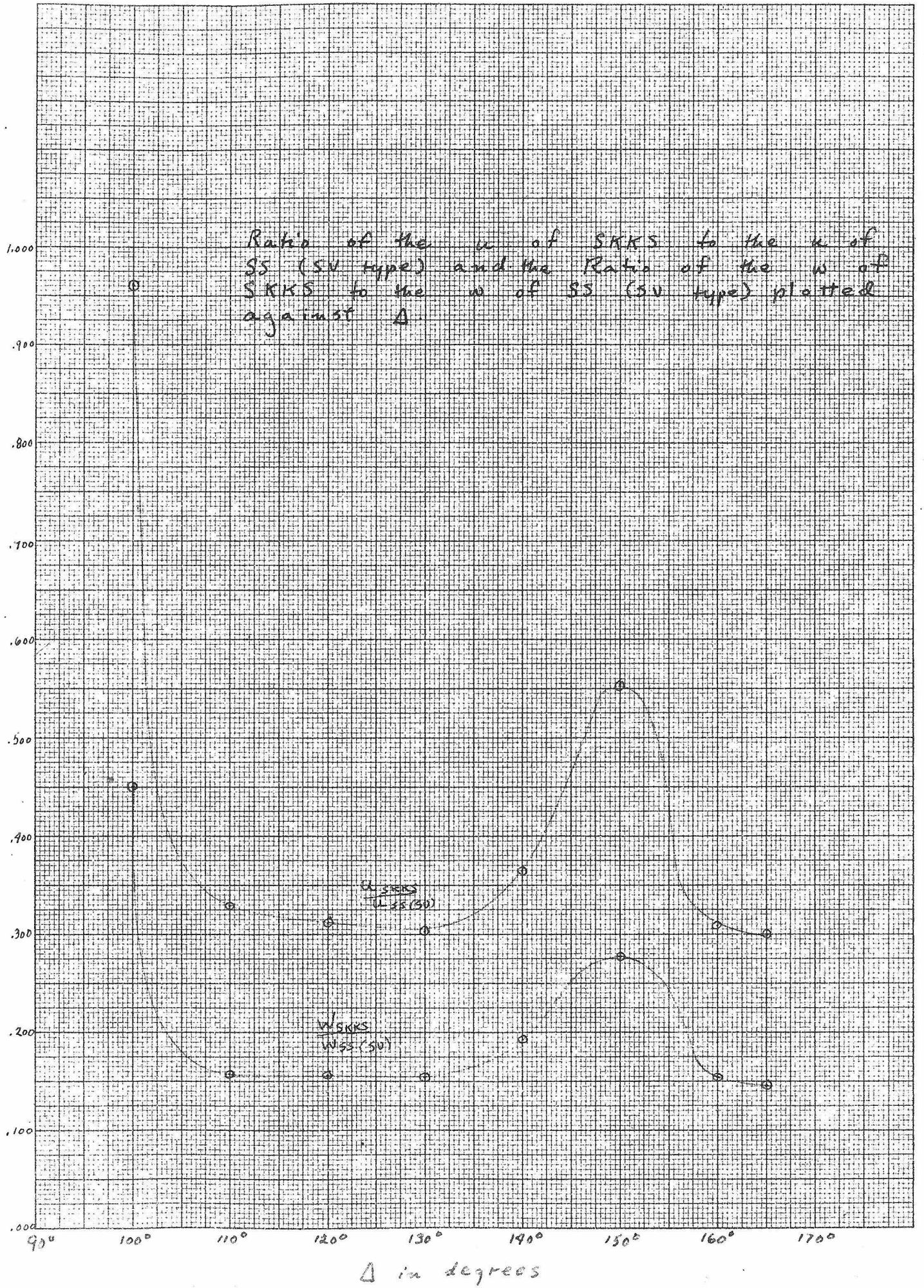
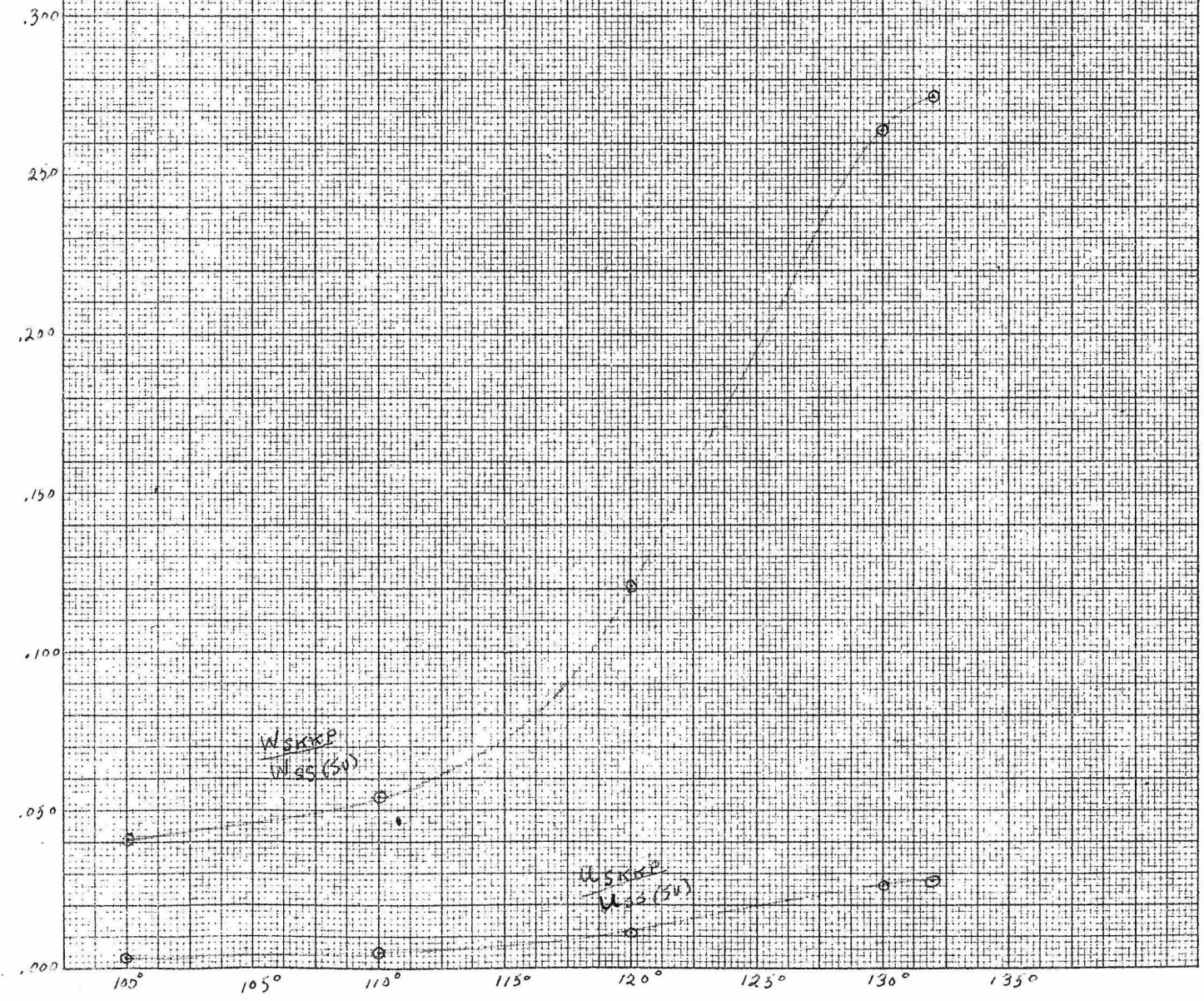
[1] Taken from the data on p. 94.
[2] Taken from the data on p. 148 and the graph on p. 149.
[3] Plotted on p. 157.
[4] Taken from the data on p. 98.
[5] Plotted on p. 158.
[6] i_o, Δ, and $\frac{di_o}{d\Delta}$ are the same for SS(SH) as SS(SV). f, however, is always 1.00 and $\frac{v}{A}$ always 2.00 for SS(SH). $\frac{u}{A}$ and $\frac{w}{A}$ for SS(SH) are always 0.00. Thus the data on p. 148 is used in calculating $^vSS(SH)$.
[7] From the data on p. 148.
[8] Plotted on p. 149.
[9] Calculated as on p. 134.

It should be noted that the values of C for ~~SS(SH)~~ SH and SV ~~SS(SV)~~ are assumed to be equal in these calculations. Actually, to obtain the correct ratios for $\frac{SS(SH)}{SS(SV)}$, those given here would have to be multiplied by $\frac{C_{SH}}{C_{SV}}$.


Ratio of the ω of SKKS to the ω of SS (SV type) and the Ratio of the ω of SKKS to the ω of SS (SV type) plotted against Δ
1.000
.900
.800
.700
.600
.500
.400
.300
.200
.100
.000
90°
100°
110°
120°
130°
140°
150°
160°
170°
Δ in degrees
ω SKKS / ω SS (SV)
W SKKS / W SS (SV)

Ratio of the ω of SKKP to the ω of SS (SV type) and the Ratio of the ω of SKKP to the ω of SS (SV type) plotted against Δ.



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