

*Chapter 5***CONCLUDING REMARKS AND FUTURE WORK**

The inverted-flag configuration was first proposed as a performance-improving alternative to the conventional flag used in piezoelectric energy harvesters. Other applications have emerged, however, as their study has provided further insights into the configuration. An example is the use of inverted flags as vortex generators to enhance the heat transfer in heat exchangers (Chen et al., 2018; Li et al., 2019; Park et al., 2016; Yu et al., 2018). The study of inverted flags has, additionally, been found to be relevant to the understanding of natural phenomena such as the flutter of leaves in the wind (Fan et al., In press; Zhou et al., 2019), which possess a clamped-free configuration and present varying angles to the flow. The emergence of these applications has generated an increased interest in the inverted-flag configuration, resulting in the development of an extensive literature. The behavior and mechanics of the inverted flag are, nevertheless, not yet fully understood.

The first part of this thesis has researched aspects of the inverted flag's mechanics that are essential to its characterization and had been previously unexplored. Chapter 2 was devoted to inverted flags of very low aspect ratio, which were shown to undergo a saddle-node bifurcation instead of a divergence instability followed by a vortex induced vibration. Chapter 3 focused on the effect of a moderate angle of attack on the dynamics of the flag. Regimes analogous to those existent at zero angle of attack were shown to be present, with the flapping regime being divided into two distinct branches. Chapter 4 delved into the interaction between two inverted flags that are placed in a side-by-side arrangement and highlighted the presence of an energetically favorable symmetric flapping mode among other coupled dynamics.

Several outstanding topics have, however, not been addressed in the current work, which has additionally raised numerous new questions, many of which remain unanswered. Some of these topics are highlighted here. A detailed description of the added mass and flow damping experienced by the flag will undoubtedly aid in the prediction of the lock-off of the flapping regime, as well as the development of a more rigorous theoretical framework for the flag's dynamics. This is, however, an arduous task; many related studies have been performed on vortex induced vibrations of different geometries without a complete answer being available to

date. An additional phenomenon that has been only lightly investigated is the vortex formation on the flag's leading edge and the process by which the initial transients give rise to the resulting limit cycle oscillations. The identification of parameters that result in optimal vortex formation may, moreover, be useful in the design and dimensioning of the piezoelectric energy harvesters. In relation to natural phenomena, the use of non-uniform flexibility and porosity in the flag will deliver a more faithful description of leaf-like structures. Its use may also be conducive to increased performance in engineering applications.

The most prominent deficit in the existing literature is the lack of experimental flow visualizations of the fluid surrounding the inverted flag. Up to date, only two such analyses, both of which were performed in water, have been reported. The corresponding flags had an unspecified aspect ratio and $\mu = 4 - 6 \cdot 10^{-3}$ (Kim et al., 2013), and $AR=3$ and $\mu = 7 \cdot 10^{-3}$ (Yu et al., 2017) and were placed at zero angle of attack. The observation of the vortex dynamics and quantitative analysis of the flow for flags of different aspect ratios, angles of attack and arrangements would provide significant insights into the topics presented in this thesis. In particular, the vortex formation and scale behind flags of low aspect ratios would provide a rationale for the lack of flapping in very low aspect ratio flags. The wake patterns and shedding timing would aid in elucidating the mechanics behind the lower branch of the flapping regime, as well as clarifying the distinction between branches in the $AR=2$ case. They may be additionally valuable to interpreting the transitions occurring at the marked $\kappa^{1/2} = 1.5$ and $\kappa^{1/2} = 2$ velocities as well as the emergence of the chaotic and deflected regimes. The observation of vortex shedding modes would be particularly relevant in the case of coupled flags, were each coupled dynamical mode is expected to be associated to a different wake pattern.

Overall, the inverted-flag configuration examined throughout this text has been shown to possess striking dynamical characteristics and constitutes an outstanding representation of the complexity of coupled solid-fluid interactions. Although many advances have been made in recent years, its behavior is yet to be fully explained, with the continued investigation of the inverted flag configuration remaining a promising line for future work.