RELATIVE f-VALUES OF SELECTED LINES IN THE ABSORPTION SPECTRUM OF CHROMIUM I

Thesis by

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ABSTRACT

Relative f- and gf-values for 410 lines in the spectrum of neutral chromium have been measured in electric furnace absorption spectra by the method developed by R. B. King for obtaining relative gf-values from the equivalent widths of weak absorption lines. Most of the transitions having lower excitation potentials less than 3.00 volts, in the wave length region λλ 3800 - 5500 have been measured. Some lines from terms as high as 3.45 volts are included, though most of these were too weak to permit accurate determination of their gf-values.

Some information was obtained on three lines which have not as yet been classified, and is included with the hope that it may prove of assistance in the classification of these lines.

Wave lengths were found for two lines which did not agree with those which had previously been assigned. These were measured with reference to known lines and corrected values are suggested.

The methods used in these measurements are described in some detail and suggestions for improvement made. Sources of error are discussed and the measures taken to reduce error from these sources are described.

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I. INTRODUCTION

The project discussed in this paper was undertaken as a continuation of the work of R.B. King on the determination of relative f-values of absorption lines. In particular, the methods which he developed and successfully applied to the spectra of iron[1], titanium[2,3], vanadium[4], and nickel[5], were used to study the absorption spectrum of neutral chromium (Cr I).

Fairly complete analyses of the chromium spectrum are now available, and most of the important lines have been identified. However, except for the excellent visual estimates of A.S. King[6,7], and the measurements of a few isolated multiplets by Frerichs[8,9], but little quantitative study of line intensities appears to have been made.

Interestingly enough, many of the important multiplets in this spectrum are found in the wave length region between λ4000 and λ5500. There are also enough of the weaker lines within these wave length limits that a full range of intensities is available for curve of growth studies. From an astrophysical point of view, this region has an advantage over the shorter wave lengths in being comparatively free of crowding. It is still quite difficult to study, but improvements in gratings and in photographic emulsions are overcoming this, and good quantitative studies of absorption line strengths of these wave lengths in stellar spectra are becoming easier. Thus it is possible that as more work is done in this longer wave length range, chromium will acquire an astrophysical importance out of proportion to its relative abundance in stellar objects.

II. RESUME OF THEORY

The f-value of a given spectral line is a constant which is characteristic of the transition producing the line. Therefore it is fundamentally important in any quantitative studies of line intensities. These values first appeared with the application of quantum theory to the study of line formation processes. In the older, classical, theory, it had been assumed that spectral lines were produced by linear oscillators, and the intensity of a line was therefore expressed as a function of the number of these oscillators involved in its production.

When the subject was considered from the point of view of quantum theory, it became apparent that the lines were produced, not by an oscillation, but by a transition of an electron from one energy state to another. However, it was found that much that was useful in the older theory could be retained simply by expressing the number of "equivalent oscillators" in terms of the number of electrons, or atoms, available in the initial energy state. Thus if N_c represent the number of classical oscillators, and N the number of atoms in the initial energy state (in some convenient unit of volume), then

$$N_c = f\ N \qquad (1)$$

It is quite obvious that the ratio f will be proportional to the transition probability relating the number of atoms in a given initial state to the number of transitions from that state producing the line. Therefore this "f-value" will be constant for a given transition, and because of its relationship to both theories, it is a convenient form in which to express the probability of the transition which produces the line.

Since these f-values are proportional to transition probabilities, and these in turn may be calculated from the diagonal components of the matrix relating the two energy levels between which the transition occurs, it is theoretically possible to calculate them for any desired transition. In practice, however, it has been found impractical to do so except for some of the simpler atoms.

A more successful approach for the complex atoms has been to determine these f-values experimentally. Since they are closely related to line intensities in either emission or absorption, also to the mean lifetime of the atom initially in an excited state and producing emission lines, and also to the absorption coefficient which in turn is related to the index of refraction, a variety of experimental methods are available for their determination. Korff and Breit[10] review several of these which have been used, and an excellent treatment is also given by Mitchell and Zemansky[11], as well as by Unsöld[12].

The determination of absolute f-values requires a knowledge of the number of atoms N, available in the initial state, and as this determination is usually quite difficult, such values are customarily obtained for relatively few lines of a spectrum. Determinations for the other lines are then made by finding relative f-values since these do not require an absolute determination of N. These relative determinations can be made even though the lines do not originate in the same energy levels, for if a condition of thermodynamic equilibrium exists, the relative numbers of atoms in different energy levels are given by the Boltzmann relation:

$$\frac{N_i}{N_j} = \frac{g_i}{g_j} e^{-\frac{h\nu_{ij}}{kT}} \qquad (2)$$

where N_i and N_j represent the number of atoms in each of the states i and j respectively, the g's are "statistical weights" given by $(2J + 1)$ for each state, h and k are the Planck and Boltzman constants, respectively, ν_{ij} is the frequency of the line produced by a transition between the states, and T is the absolute temperature.

A more convenient form for relative studies is obtained by multiplying both sides by f_{ik}, letting j represent a suitable reference state indicated by the subscript "o", and then rearranging to give:

$$g_i f_{ik} = \frac{g_o}{N_o} e^{\frac{hc\Delta\bar{\nu}}{kT}} N_i f_{ik} \qquad (3)$$

where $\Delta\bar{\nu}$ is the difference in wave number between the state i and the reference state. Therefore if Nf-values can be measured for several lines which are produced simultaneously under conditions of thermal equilibrium, by correcting these for the different populations of the different levels according to the Boltzman relation, it is possible to find relative gf-values for these lines.

Since gf-values rather than f-values are obtained by this procedure, and these in turn can often be applied directly in analytical work, a tabulation of relative gf-values is often more convenient than one of f-values. Both forms are given in the tabulation in this report.

A. The Curve of Growth

To obtain Nf for a given line it is necessary to know the functional relationship between this product (which is equal to N_c, or the number of equivalent oscillators), and the resultant strength of the absorption line. This strength is often expressed in terms of "equivalent width", measured in wavelength units, and which is defined as the width of a line of equivalent area having a rectangular profile and which completely absorbs the continuum.

As the value of Nf is increased, it is found that the equivalent width does not increase in direct proportion, but rather follows a "curve of growth" such as shown in figure 1.

Allen[13] and Menzel[14] found that by plotting the ratio of equivalent width to wave length (W_λ/λ), or rather log (W_λ/λ), against a function of Nf, the resulting curve was independent of wave length. Van der Held[15] divided the equivalent width by the "Doppler" width and obtained a curve which was independent of both the wave length and temperature. Ladenberg[16] introduced as a suitable function of Nf, a value given the symbol C, defined as:

$$C = k_o Nfl \qquad (4)$$

where l is the length of path through the "active region", Nf

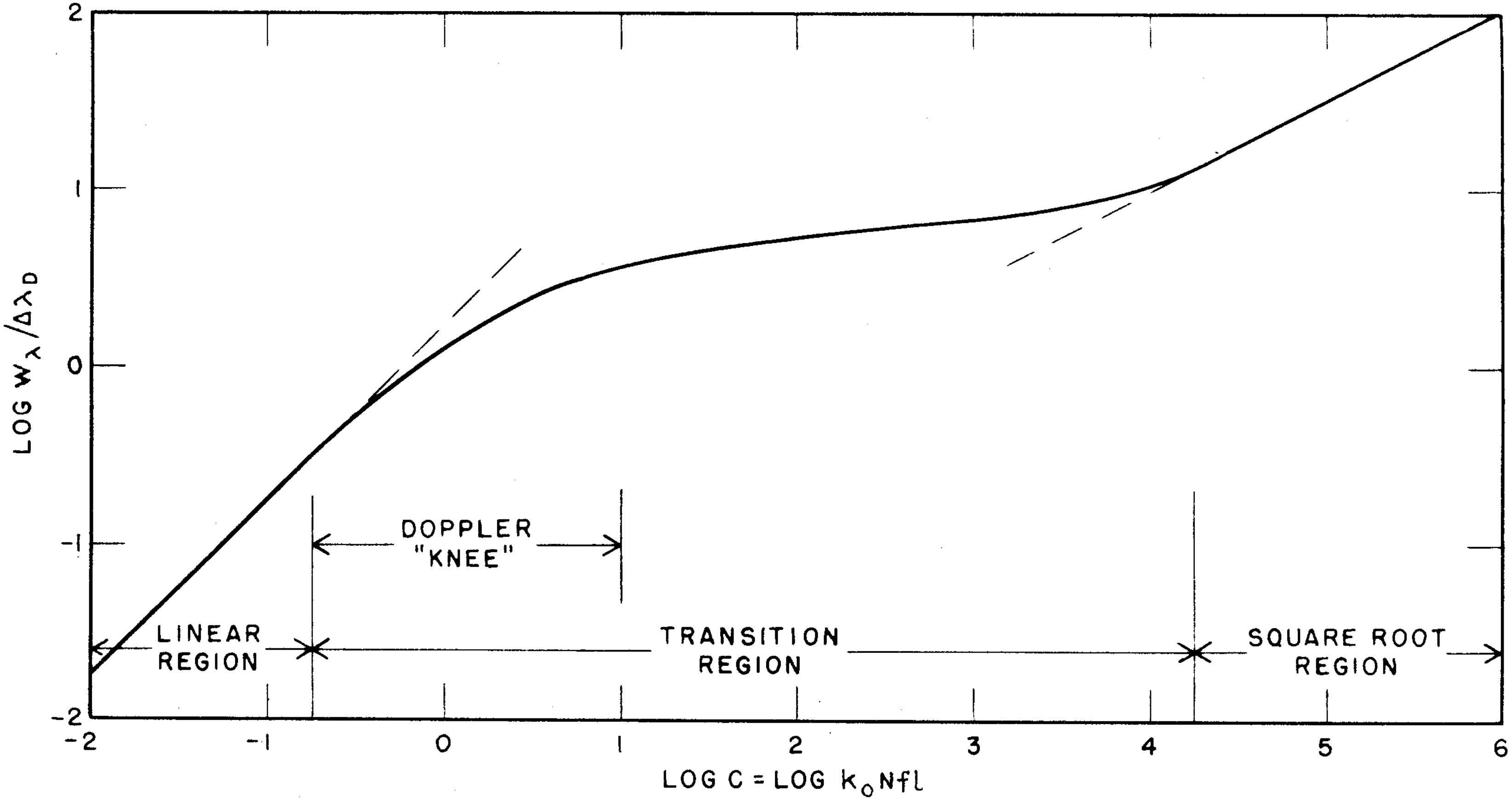

FIG. I CURVE OF GROWTH

is the number of oscillators per unit of volume, and k_o is the "classical" absorption coefficient at the center of the line, given by

$$k_o = \frac{\sqrt{\pi}\, e^2}{mc^2} \frac{\lambda_o^2}{\Delta\lambda_D} \times 10^{-8} \tag{5}$$

e and m being the charge and mass, respectively, of the electron, and λ_o being the wave length at the center of the line.

Van der Held gave the Doppler width as well as the equivalent width in frequency units, but for this work it is more convenient to use units of wave length, for which the Doppler width becomes

$$\Delta\lambda_D = \frac{\lambda}{c}\sqrt{\frac{2RT}{M}} \tag{6}$$

where c is the velocity of light, R the gas constant, and M the atomic weight. Thus the curve of growth shown in figure 1 is a plot of log $W_\lambda/\Delta\lambda_D$ against log C.

Three distinct regions are immediately apparent. At very low intensities only the Doppler effect resulting from thermal motion of the atoms beed be considered. For values of $C \ll 1$, the relation is approximately linear and is given by the equation

$$W_\lambda = \frac{\pi e^2}{mc^2} \lambda_o^2 \times 10^{-8}\ Nfl \tag{7}$$

At higher values of C, natural damping and pressure broadening become important. It can be shown that if broadening can be considered as primarily due to natural damping, the equivalent width increases proportionally to the square root of Nfl, and this is found experimentally to be approximately valid in the region represented by the right side of figure 1.

The region between the "linear" and "square root" portions of the curve, where all three effects, Doppler effect, damping, and pressure broadening must be considered, is usually relatively flat, so is unsuited for graphical reduction of data. Therefore it need not concern us here, except that it often is desirable to work well up on the "knee" between the linear portion and this transition region. For this, it has been found suitable to use a relationship derived by Ladenberg[16] for Doppler broadening and given by Unsöld in the form

$$\frac{W_\lambda}{2\Delta\lambda_D} = \frac{\sqrt{\pi}}{2} C \left(1 - \frac{C}{2!\sqrt{2}} + \frac{C}{3!\sqrt{3}} - \frac{C}{4!\sqrt{4}} + - \cdot \cdot \frac{C}{(n-1)!\sqrt{(n-1)}}\right) \tag{8}$$

Although this is supposed to be valid only for values of C less than 3, it was found that under favorable conditions as described later, it could be used with values as high as C = 10 and still give results which were accurate within limits of error established by other features of the experiment.

III. METHOD

In brief, the method employed to determine relative f-values of lines of Cr I was that of obtaining equivalent widths of absorption lines which were weak enough to be on or near the linear portion of the curve of growth, and from the known relationship between equivalent widths and values of Nfl in this region, finding the relative f-values or gf-values for the lines.

The absorption lines were produced by passing a beam of light from a suitable lamp through a resistance type electric furnace in which pure chromium metal had been vaporized. The particular furnace used in this project employed a graphite tube heated to the desired temperature by a heavy electric current, the chromium vapor in this tube providing an optically thin absorbing layer which produced the desired lines with little difficulty.

This method has the advantage of simplicity, both as to theory and to apparatus. When used with a furnace of the type which was employed in this project, there is practically no uncertainty about the temperature at which the lines are formed, and this is very important because of the critical manner in which the Boltzmann relation (equation 2) affects the relative number of active atoms in different energy levels.

A disadvantage in this method, particularly in absorption studies, is the limited range of intensities which can be compared directly because of the characteristics of the curve of growth. The equivalent widths of very weak lines are hard to measure because of irregularity in the continuum, and stronger lines begin to enter the transition region of the curve of growth where determination of the corresponding Nfl values is uncertain. Between these limits, it is usually possible to compare lines having relative intensities not greater than 10:1. Thus for analyses of the type described here, where the range of intensities is of the order of 100,000:1, it is necessary to "step" down, in several steps, each of which adds to the uncertainty of the comparison.

Chromium proved to be quite well adapted to this step-by-step process, with multiplets of suitable intensities located closely enough together that all comparisons could be made on plates exposed with the same spectrograph. Some steps required very long "reaches", and so could not be determined with all the accuracy desired, but if the spectrum had not provided the intermediate multiplets about as they were arranged, comparison of strong and weak lines would have been much more difficult if it had been possible at all.

A photographic method of recording the data was used in preference to the photo-tube method developed by Carter[17]. The difficulties of applying Carter's method to absorption studies have not yet been solved, and it was desired to study the absorption lines rather than the emission lines of the Chromium spectrum for King[1] had found that lines from the ground state showed severe self-reversal in emission due to the cooler gases at the end of the tube.

Also, chromium has a relatively low boiling point, so was expected to give more trouble by boiling away rapidly than had been experienced with the metals studied previously. This difficulty definitely appeared at the higher temperatures and it was advantageous to have the integrated effect of the photographic plate as a record of determinations which certainly would have been difficult to obtain photoelectrically.

The photographic procedure of course had the disadvantage common to all quantitative photographic studies in that it was necessary to calibrate each plate so that exposure densities could be reduced to relative light intensities. Further, the "grain" of the plate imposed a definite lower limit on the strength of a line which could be measured reliably, and added to the uncertainty of the measurements of the stronger lines. These matters are discussed in greater detail later.

IV. BASIS OF SELECTING LINES FOR MEASUREMENT

More than 600 lines of the chromium I spectrum are listed in the "Revised Multiplet Table of Astrophysical Interest"[18] (referred to henceforth as the RMT) as being in the region of astrophysical interest and as being strong enough and with low enough excitation potential to be suitable for study by this method. It was decided to concentrate attention on (1) those lines which would be of most interest astrophysically, giving a maximum range of intensities, and (2) those lines which could be used as a reference framework with which future comparisons could be made. Enough measurements were made of lines in the red and near infra-red regions to determine that these regions could be reached without particular difficulty. Also some exploratory work was done in the blue and near ultra-violet regions, but because of the wealth of lines located in the range between λ4000 and λ5500 most of the studies were made on these. Even in this range, no attempt was made to measure those lines which required special techniques. For example a few were found in unfavorable relationship to the C_2 and CN bands excited by the graphite tube. The CN bands, in particular, could be reduced by use of a helium or other non-nitrogen atmosphere, but so few lines were involved that this was not done.

V. APPARATUS

The apparatus used in obtaining the photographic exposures consisted principally of (A) an electric furnace of the carbon-resistance type, (B) a suitable light source, (C) a spectrograph, and (D) an optical pyrometer for reading the furnace temperature. The arrangement is shown in figure 2.

A. The Furnace

The furnace used was the large hood furnace designed by A.S. King[19], located at the Mount Wilson Observatory Laboratory. This consists essentially of a graphite tube 10 inches long, $\frac{1}{2}$ in. I.D., and 3/4 in. O.D., clamped between suitable connecting electrodes which in turn are supplied from a heavy duty transformer capable of furnishing 2000 amperes or more of 60 cycle alternating current.

Electrodes and conductors are water cooled and a cooling jacket surrounds the tube. Cooling jackets, electrodes and tube are enclosed in a large steel hood which is provided with quartz windows opposite the ends of the tube. Hood and furnace are supported on a 3/4 in. thick cast iron base, and an air tight seal is provided by covering this base with a 0.125 in. sheet of copper, grooved to receive a double-edged tongue around the base of the hood. It was found possible to evacuate the region within the hood to a pressure of 5 mm. Hg. or less, and most of the runs were taken at such pressures.

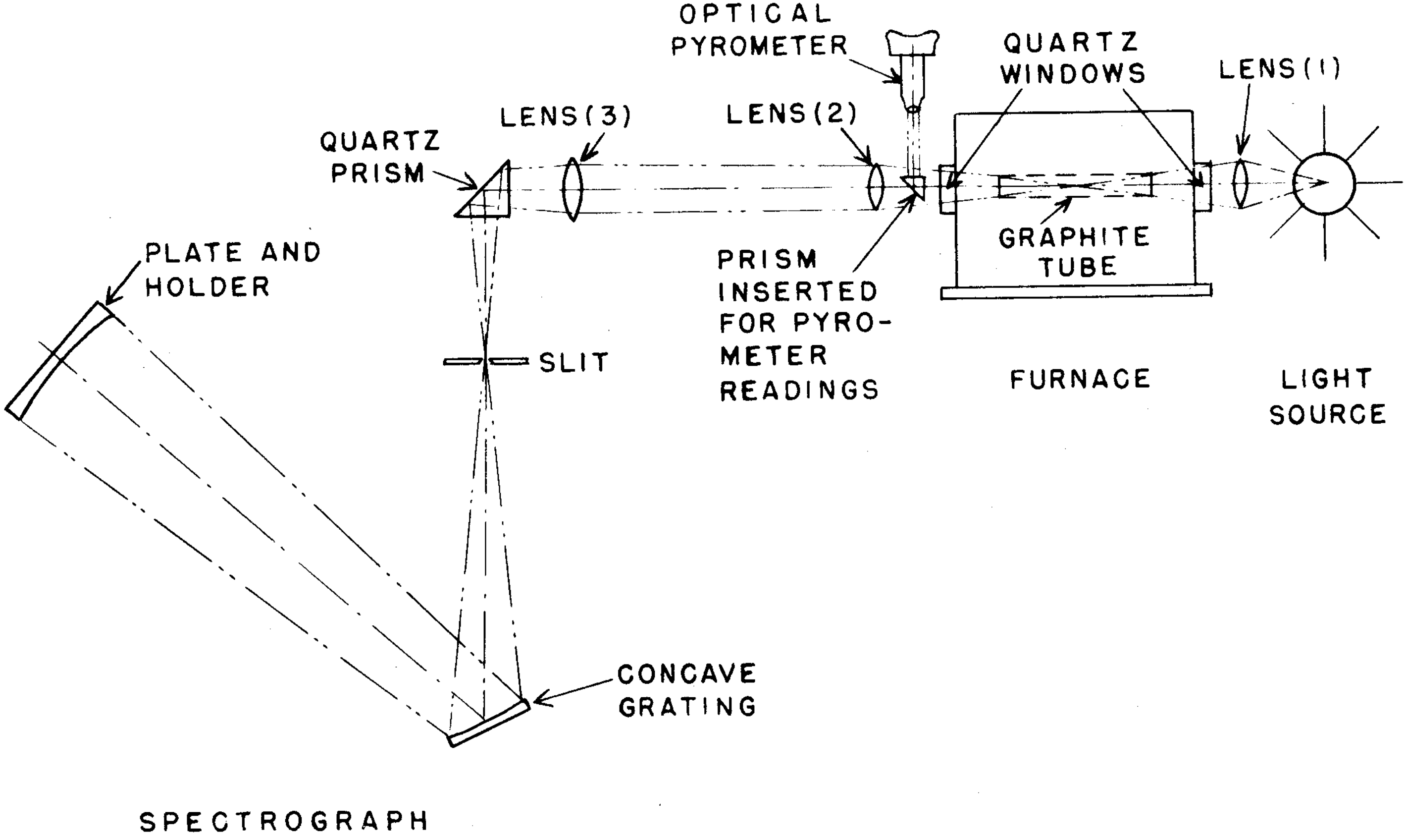

FIG. 2 ARRANGEMENT OF OPTICAL SYSTEM FOR ABSORPTION STUDIES

1. Tubes

The graphite tubes were accurately turned from rods of pure graphite in the shops of the Mount Wilson Observatory. This turning had to be done with a high order of uniformity to avoid formation of hot spots or rings as the tube was heated.

In previous work, it had been found possible to use the same tube for several successive runs. As it was operated, the tube would lose some of its material by sublimation so that on later runs it required less current to reach high temperatures. This was a desirable feature when high temperatures were to be reached quickly. However, chromium presented several difficulties at these higher temperatures, and although tubes could be used several times if not heated above about 2300°C., any heated much above this were unsuited for a second run.

Apparently chromium combines readily with the carbon of the tube, and whereas pure chromium melts at 1615°C., and boils at 2200°C., the resulting carbide has melting and boiling points of 1890°C. and 2800°C., respectively.

The result is an eroding process which appears to take place as follows: as the chromium melts it is absorbed by the carbon, wetting the walls well up the sides of the tube. This forms a layer of chromium carbide, some of which is under the remaining liquid chromium. As the melting point of the carbide is reached, it rises through the chromium and floats on it so that when the chromium begins to boil, its vapor has difficulty in escaping through this liquid carbide layer. At temperatures under 2500°C. little difficulty was experienced, but at higher temperatures the bubbling became quite violent and often would expel material from the tube. The net effect was the removal of carbon from the tube walls and this erosion was so irregular that it was impractical to use a tube the second time when it had once been heated above 2300°C. with a full charge.

A second difficulty developed in that the liquid chromium spread readily along the tube and formed a conducting path of much lower resistance than the carbon, making it very difficult to obtain high temperatures when a heavy charge of chromium was used. Since the chromium also boiled away rapidly at high temperatures, it was desirable to raise the temperature as quickly as possible to these higher values, and to do so with a fairly large charge of chromium in the tube. Otherwise the vapor would disappear before an exposure could be made.

After several ideas had been tried, a technique was developed which made it possible to obtain several exposures at temperatures up to and even above 3000°K. It was found possible to reduce the thickness of the tube walls slightly by having the shop turn down the outside diameter over the central portion of the tube, leaving the ends which would be clamped by the electrodes unchanged. Two different reductions were used, one in which the diameter was decreased 0.060 in., and one with it decreased 0.030 in. The thinner walled one worked well, giving high temperatures even more quickly than a well "burned" tube, but it was very difficult, even with the fine facilities and skill available in the Observatory shop, to obtain the requisite uniformity with this thin a tube wall. The smaller reduction was therefore used as a compromise, and worked quite satisfactorily. The standard tubes with uniform wall thickness were, of course, used for low temperature runs or when small charges of chromium were used.

Different ways of distributing the charge along the tube were tried to reduce difficulties from conduction, but all of these tended to cause non-uniform heating so were abandoned.

A "boat" of tantalum was tried, as this metal has a melting point of 2850°C., and thus should be suitable for all except the highest temperatures. However, it was found that liquid chromium dissolved the boat quickly at a temperature of about 1800°C. Tungsten was not tried as it appeared probable that it would react similarly.

A crystal of magnesium oxide was hollowed out and tried as a boat also, but at about 2300°C. quite a violent reaction appeared to take place, and it too disappeared.

The most successful solution to the bubbling problem was found to be that of partially lining the tube with a thin layer of chromium carbide by using uniformly distributed small charges, heating to about 1800°C. so that the chromium is well absorbed by the walls, then recharging and re-heating two or three times, each time rotating the tube slightly so that about one-third of the inner circumference was coated. If the charges were small, it seemed that the resulting layer of carbide would not "float", or at least not enough so that it caused serious bubbling, yet it was enough to protect the tube against further erosion.

This method of tube lining was very crude, yet it seemed successful enough to warrant the suggestion that in future work, tubes be lined carefully, say by an evaporation process. Then, if reduced wall thicknesses are used for higher temperature runs, the special difficulties experienced with erosion and conductivity in this project should be largely eliminated.

2. Leaks

At the beginning of experimental work it was found difficult to pump the furnace down to low enough pressure. Serious leaks were found to have developed around the quartz window on the side toward the lamp, and also around the air pump connection. The seals around the conductors and cooling pipes had been coated with glyptal and seemed to hold satisfactorily.

The leak at the window was apparently due to heat from the lamps, so a sheet of asbestos was cut to fit closely around the window and a good wax seal then held without further trouble. The air pump connection was sealed with glyptol but the vibration from the pump seemed to keep working it loose. At any rate it continued to give trouble and should be taken down and tightened well before further work is done. Asbestos sheeting was laid over the glyptal seals inside the hood, though the design of the furnace is such that little heat reaches these from the furnace itself. Asbestos shields were also inserted between the electrodes and water jacket to help maintain uniform temperature along the outside of the tube walls.

3. Heating of Base Plate

This furnace was designed to remain cool during long runs with high current, but it was found that the length of continuous run was limited by a rising temperature in the base plate, which seemed to concentrate near the two conductor entrances. It was noticed that this heating was not reduced by the introduction of asbestos sheets between the base plate and the furnace tube. Thus it was felt that the heating might be due to eddy currents induced in the base plate by the high currents in the conductors.

Rough calculation of these eddy currents showed that those induced by the vertical conductors would cause negligible heating in the copper sheet, and insufficient heating in the cast iron base plate to account for the observed effect. However, these conductors bend immediately below the plate in an arc of 5 in. radius, becoming horizontal about 5 inches below the plate. Magnetic fields from this arc, and from the horizontal portion will also induce eddy currents in the plate, and furthermore these fields will have a vertical component, so that the comparatively large magnetic induction in the base plate will induce currents in the copper sheet. When these effects were added to those of the vertical conductors, the result appeared sufficient to account for the observed heating. If such is the case, this heating can be reduced materially be rearranging the conductors so that they extend vertically below the furnace a distance sufficient to make the effects from the horizontal portions negligible.

4. Charging and Recharging the Furnace

Only very small charges of chromium were required to study the stronger lines. In fact for comparisons of the two strongest multiplets, the charge consisted of a single grain of chromium powder, probably weighing not more than 5 milligrams. It was necessary to boil this away for several minutes before these lines became weak enough to fall on the linear part of the curve of growth.

Normally, however, a charge of between one and two grams was used. The crystals of pure metal were reduced to powder in a mortar, then distributed along the tube by means of a special charging spatula.

It was found desirable in runs where the temperature did not exceed 2300°C. to recharge the tube without opening the furnace. This could be done by removing one of the quartz windows with its mounting, and charging with the long spatula. If 1 to 1½ grams of chromium powder were added after every 15 minutes of operation at temperatures between 2000° and 2300°, or after 25-30 minutes of operation at temperatures below 2000°, as many as eight exposures, with each exposure requiring as long as 10 minutes, could be taken on a single plate. This could be done without disturbing the optical system so the overall saving of time was significant.

B. Light Sources

The absorption method requires a light source having a brightness temperature considerably above that of the furnace tube. Carter[20] considers that a temperature difference of the order of 100°-150° is sufficient. Careful inspection of several of the earlier exposures showed that these values were certainly a minimum and that good practice which will allow for ordinary fluctuations of the light source would require temperature differences between the lamp and tube of not less than 200° at a furnace temperature of 2500°C., and 300° at 2750°C. Throughout this work, margins of this magnitude were maintained over the whole region being photographed and no appreciable difficulty was experienced with re-emission in the lines.

In addition to the temperature requirement, suitable light sources also must (1) provide a reasonably uniform continuum free of absorption lines and comparatively free of emission lines in the regions being observed, (2) have an area such that with a magnification of not more than three or four, its image covers approximately a centimeter length of the spectrograph slit (this follows from considerations of the effect of astigmatism of the grating on exposure time), (3) be steady enough so that test plates can be used reliably to determine exposure time, (4) not require close attention, and (5) have a life expectancy sufficiently long that failure is not probable during the course of a run requiring an hour or more.

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Needless to say, a suitable light source, especially for high temperature work, is very difficult to find, and an ideal one has not yet appeared. Those which received special attention in this project are described briefly here.

1. Tungsten Lamps

Tungsten lamps met all of the above requirements for furnace temperatures not exceeding 2400°C. Where exposures would extend into the ultraviolet, a 50-amp. Philips model 6001 Cinema projection lamp with quartz bulb was used. Where glass was satisfactory, the 30 amp. mogul base 4-filament projection lamp manufactured by the General Electric Co. was satisfactory. In each case, the lamp was operated at about 10% above rated current, thereby decreasing exposure time by about 30%. Although lamp life was also decreased, it was not shortened enough to cause inconvenience.

These lamps were mounted so that an image of the <u>inside surface</u> of a filament coil fell along the slit. A slight shift in image position would bring the image of the outside surface onto the slit however, and since its brightness was very much less than that of the inner surface, care had to be exercised that the image did not shift during exposure. As heating of the filament supports could cause them to expand sufficiently to cause serious shift in image position, it was customary to heat the lamp at full intensity for several minutes before taking the exposure.

2. The Carbon Arc

The carbon arc provided high enough temperatures but had two serious drawbacks. Its comparatively unstable operation required considerable attention and made exposure times uncertain. Also troublesome band lines, particularly those of C_2 and CN, were superimposed on the continuum. Fortunately these bands were absorbed to some extent by the furnace tube so that at some temperatures the continuum in these band regions was actually better with the carbon arc than with some other light source. Generally, however, the continuum provided by the arc was not as uniform as that from an incandescent source.

The difficulty of unstable operation was largely overcome by obtaining a discarded motion picture projection arc of low intensity type. This had adequate housing to reduce convection currents, and was provided with an automatic feed which reduced the amount of attention required. In fact, when once adjusted, it would often operate for thirty minutes or more with no attention other than an occasional check on the position of the image of the crater on the slit.

This lamp did not give the high temperature obtainable with a high intensity arc, but its temperature was higher than could be read with the pyrometer, which meant that it exceeded 3100°C. Further, the high intensity design would have required special types of carbons, and this lamp used those which were on hand.

An attempt was made to use copper coated carbons of the type employed in motion picture projection work. These gave steadier light, but contained impurities which gave strong emission lines through the whole spectrum. Tests showed that to yield a sufficiently clean continuum, carbons do not have to be spectroscopically pure but must be of a good grade of graphite.

It was found best to operate the lamp in the manner for which it had been designed, with the large positive carbon toward the furnace. This required the use of a mirror to reflect an image of the positive crater through the optical system. The reflector mounted in the lamp was spherical so distorted the image badly, and also was back silvered which made it unsuited for ultraviolet exposures. For these reasons it was removed and a small plane, front-surfaced mirror, about 50 mm. square, was mounted on the reflector frame in such a manner as to reflect light from the positive crater into a lens placed between the lamp and the furnace.

This carbon arc was well adapted for most of the work in the region between λ4000 and λ5400. Its intensity was not as great as that of the Bol lamp over much of this region, but it was roughly three or four times that of the tungsten lamps.

3. The Bol Lamp

A high pressure mercury arc designed by C. Bol[21,22], and manufactured in Holland by the Philips Co, was used on some of the earlier exposures but failed soon after the work started. It did not give a true continuum, but at the high pressure (about 200 atmospheres) at which it operated, the mercury lines were broadened to such a degree that they provided a slowly varying "pseudo-continuum" which was quite satisfactory. The very high intensity (1800 candles/mm^2) made short exposures possible even in regions where the continuum was relatively weak or where the grating of the spectrograph was slow.

The varying continuum gave rise to some difficulties in exposure technique and also in the reduction of data. In order to obtain suitable density for all portions of the spectral region on a given plate, several exposures of different time lengths were made in as rapid succession as possible. Since the Cr vapor would be boiling away rapidly, successive exposure times were usually increased to the maximum required to expose the weaker parts of the continuum, then decreased again through the same steps. Comparison of values obtained with similar exposure time than gave a check on the effect of the boiling away, and disclosed effects of any slight temperature variations.

The cause of failure of the original Bol lamp was never determined, though some difficulty had been experienced with the cooling system used in conjunction with it. Distilled water was used to avoid coating the inner surfaces of the water jacket, and this was circulated by an electrically driven centrifugal pump. It is possible that a slight leak in the pump allowed an air bubble to pass at a critical moment. The lamp failed as it was being started, and a bubble at this crucial instant could certainly have contributed to failure. However, with temperatures and pressures of the magnitude existing in such a lamp, failure can be expected at any time.

Efforts were made immediately to obtain a replacement of the Bol lamp, but the Philips Co. had discontinued its manufacture and no one else made this particular type. A similar lamp produced by the General Electric Co., and known as their type AH-6, operates at much lower pressures and so does not provide the essential pseudo-continuum.

Finally the Huggins Laboratories, at Menlo Park, California, agreed to duplicate the lamp as closely as they could. This duplicate was designed to be used in the standard mounting manufactured by the Huggins Laboratories for their line of mercury arc lamps. The result was fairly satisfactory, though the Huggins lamp apparently did not have quite the pressure or intensity of the Philips lamp. Exposure time was of the order of two or three times as long, and the continuum was not as well spread out. However, it was possible to use it satisfactorily for several critical exposures.

Rather than risk more difficulty with the centrifugal pump, water from the city mains was used for cooling, and the lamp was then carefully rinsed for a period of thirty minutes or so after each run by pumping distilled water through it.

4. Tests on Other Types of Lamps

In the course of looking for a suitable replacement for the Philips-Bol lamp, several of the recent lamp developments were tested and for one reason or another found to be unsuitable.

a. The Point-O-Lite Lamp

This lamp, manufactured by Ediswan Ltd., London, uses an arc discharge in an inert gas to a small tungsten electrode which is kept incandescent by the discharge. Its brightness temperature was very little, if any, above that of an overloaded tungsten lamp, and the 150 c.p. model which was tested had too small an area to give reasonable exposure times.

b. Western Union Lamps

The Water Mill Electronic Research Laboratories of the Western Union Co. have developed a series of very intense lamps using an arc discharge between electrodes of compressed zirconium oxide.[23] Tests with a 100-watt bulb type indicated that the continuum was suitable except in the blue and ultraviolet where it was rich with zirconium lines. The brightness temperature was above that of the tungsten filaments, registering about 2800° with the pyrometer. However, the surface area of this model was again too small to send enough light into the spectrograph to give reasonable exposure times.

Inquiry concerning larger models revealed that a 1000-watt bulb type had been built experimentally, but that effort was now being concentrated on an open air type of arc. Arrangements were made for a test on one of these new lamps and through the kind offices of W.D. Buckingham of the Water Mill Laboratories, one was made available. It gave a very brilliant light, quite steady for an arc, and its brightness temperature was above that which could be read with the pyrometer. However, it was found that in addition to the lines appearing in the spectrum of the bulb type lamps, this open air arc gave troublesome emission lines through the green and red regions. There remains the possibility that the 1000-watt bulb type might avoid these difficulties.

c. The Schulz Lamp

The most interesting of the lamps examined was one of German manufacture, designed by Paul Schulz[24]. This produces a continuum as a result of recombination (free-bound radiation) and "Bremestrahlung" (free-free radiation). Because of the close packing of term levels immediately below the ionization limit in the noble gas atoms, these provide the best source for such radiation. For these, the ionization potential appearing as the excitation potential of the continuum is only slightly higher than the excitation potential of the lines originating in these closely packed term levels. Thus the yield of continuum radiation increases rapidly with the current, whereas the yield of the line radiation remains approximately constant. Xenon is found to give the highest yield of this type of radiation of any of the noble gases, so this lamp is essentially a high pressure discharge in Xenon.

Two different sizes have been developed, but unfortunately both are very hard to obtain in this country. However, it was possible to make a test with the smaller size, which was operated at a maximum current of 8 amperes, and at approximately 30 volts.

This model had matched conical electrodes of tungsten, and could be operated on either alternating or direct current. With direct current, a very bright spot appeared in the discharge column, about 1/3 of the way from the cathode to the anode. The image of this spot was held on the slit of the spectrograph, though it wandered back and forth along the slit with the irregularity characteristic of an unstabilized arc discharge.

Starting was accomplished as with the other bulb type arc lamps by brushing the anode with a high voltage discharge from a "leak detector". A series lamp bank maintained the current at 8.0 amperes during the test.

Exposures were taken of the $\lambda 3000$ to $\lambda 3800$ region in the second order. This small lamp gave an exposure time of approximately one third that required with an overloaded quartz-tungsten lamp, despite the fact that the image on the slit was one-half or less as wide as the image from the tungsten lamp. This checks roughly with the observations of Baum and Dunkleman[25]. The continuum was excellent, very free of lines, and brightness temperature was well above the limit of the pyrometer.

Principally because of its small size, the lamp which was tested did not give as short an exposure time as either the Bol lamp or the carbon arc. Therefore no attempt was made to obtain a longer use of it for project work.

The larger model, designed to operate on 30 amperes, thus using approximately 1000 watts, is rated to give a luminance of 230 candles per mm.^{2} for an arc width of 2 mm. Unfortunately one was not available for test, but these ratings are adequate to make it compare favorably with the Bol lamp as an intense light source. Indications are that its continuum is much more uniform. Also the larger model is provided with a stabilizing coil which should reduce difficulties from shifting arc position. Every indication is that life expectancy considerably exceeds that of the Bol lamp, and since it does not require water cooling, its operation is much simpler. It may not be a complete answer to the problem of a suitable light source for high temperature work, but it appears to be the most promising of all sources which were tested, and if the larger model ever becomes available, it should certainly be tried.

C. The Spectrograph

The spectrograph used was the 15-foot vertical Rowland mounting in the laboratory of the Mount Wilson Observatory[26]. This has a concave grating ruled by J.D. Anderson having a ruled area 5.1 x 10.6 cm., with 590 lines/mm. Dispersion in the second order was approximately 0.5425 mm. per angstrom unit. The grating was very weak when used in the second order above λ4500 which added to the difficulty of obtaining good exposures in the important region near λ5200.

A slit width of approximately 0.0050 mm. was used for most of the exposures, though it was found necessary to open it to 0.0055 mm. to decrease exposure time in the weaker regions. It was found that the sensitive alignment of the slit with the grating rulings could be disturbed easily, especially while adjusting slit height. Special care had to be taken while making these adjustments not to disturb this alignment, and each test plate taken of the slit focus was carefully checked for the "flags" at the ends of the lines which would indicate misalignment.

The plate holder was originally equipped with a shutter of thin brass which wound on rollers at each end. This kept breaking at the ends so was replaced with a shutter of beryllium copper which worked much better. Care had to be taken never to turn this shutter beyond the "open" and "closed" marks if trouble was to be avoided.

D. The Optical System

The optical system consisted of three lenses and a prism as shown in figure 2. The prism was of quartz and was used to deflect the beam from horizontal to vertical. The lens (3) was of quartz with 78 cm. focal length, providing the proper aperture to fill the grating with light at the distance required to focus a parallel beam on the slit. The other two lenses were of quartz or glass as required by the wave length region under study.

To bring as much light as possible from the source to the slit and onto the grating, it was necessary (1) to focus the image of the source as nearly midway along the tube as possible thus equalizing the apertures at the tube ends and allowing maximum light to traverse the tube, (2) have the beam between lens 2 and 3 as nearly parallel as possible, which requires that lens 3 be exactly 78 cm. along the optical path from the slit, and (3) have the optical axis from the source to the prism as nearly horizontal as possible so that a truly vertical beam strikes the grating. This requires accurate adjustment of the height of the prism.

It was found best to align the system by starting with the source approximately in the position calculated to give an image at the slit magnified three or four times. It was then aligned with the optic axis by sighting along the axis of the tube without lenses in position. Lens 1 was then inserted and alighned to center the image in the tube. Then lens 2 was inserted at a distance from the mid-point of the tube equal to its focal length. Prism height and position of lens 3 were then adjusted to meet requirements (2) and (3) above. Readjustments were continued until a sharp image of maximum intensity was obtained on the slit and the illumination of the grating was uniform over the ruled surface.

E. Photographic Plates

To obtain a plate suitable for microphotometric analysis it was usually necessary to expose the following series of plates: (1) a test plate to indicate the best focal position of the slit, (2) a test plate to indicate proper exposure time, (3) the record plate, (4) a test plate to indicate proper light intensity for calibration, and (5) a calibration plate.

Except for the first, which should be low contrast, the others required high contrast, fine grain emulsions. It was found that Eastman spectrographic plates type IV-O were satisfactory for work below λ4200, types IV-C, IV-G, and IV-J for the region $\lambda 4000$ to $\lambda 5500$, and some exposures were taken above $\lambda 5500$ using IV-F plates, with one extending to $\lambda 7700$ exposed on an N-IV plate. Cramer's Contrast plates were also used in the same regions as the IV-O plates, though they seemed to extend a little farther toward the longer wave lengths than the IV-O plates.

Development in most instances was with D-19 solution at 20°C. To keep grain as fine as possible development time was never allowed to exceed 3 minutes, and often was held as low as 2½ minutes.

1. Test Plates

It was found that the slight temperature variations in the spectrograph pit apparently affected the focus so that readjustment was necessary from one day to the next. The shutter on the plate holder could be set narrow enough that ten or more exposures could be taken on a single plate, and with slit height changed 1 mm. between successive exposures, this covered adequate range to indicate the best setting. The source was usually an iron arc, placed between the second and third lens (fig. 2) with its own lens so that parallel light would enter lens 3 and a sharp image of the arc stream would be focused on the slit. Plates were of slow contrast type, such as Eastman 30, and exposure times in the second order under $\lambda 4000$ were of the order of thirty seconds.

The test plate used to indicate proper exposure time was of the same type as would be used in the record exposures, and was taken immediately prior to the heating of the furnace after all final adjustments had been made so that conditions were as nearly those of the record exposure as possible. Each successive exposure on this test plate was then made twice as long as the one preceding it. Usually there was enough variation in the continuum that accurate interpolation for correct time could be made easily. It was found that usually it was best to increase the record exposures by about 10% over the indications of the test plate for heating the furnace apparently caused a slight decrease in intensity of the continuum.

2. Calibration

Because the density of an exposed photographic plate is not proportional to the intensity of light falling on the plate, it is necessary in all photometric work to prepare calibration plates which will indicate the functional relation between density and intensity. In this work, the calibration plate was of the same stock as the record plate and was developed with it, under as nearly identical conditions as possible. Exposure was made by replacing the regular slit with a "step slit" which had nine different openings that could be moved successively into the light beam. The ratios between these openings was accurately known, so assuming the light beam to be of uniform intensity over the small region passing through the slit, the light reaching the plate would be in proportion to the area of the slit opening. A tungsten lamp having a ribbon filament was used as a light source and held at very constant intensity during the course of exposures.

It was found that the d.c. laboratory supply was steadier as a source for this lamp than the regulated a.c. supply which also was available. Control was by means of a series lamp bank which contained enough carbon lamps to keep the current constant within a few hundredths of an ampere.

Lamp intensity was adjusted on the basis of previous experience or from results of a test plate so that the central opening of the step slit gave approximately the density found on the record exposures.

F. The Optical Pyrometer

Furnace temperatures were read with a Leeds and Northrup optical pyrometer, model 8622-C, Serial 526261, having scales which read directly in degrees Centigrade. Readings were taken by sighting the pyrometer into the furnace tube, using a small right-angle prism inserted in the beam from the furnace. While this reading was being taken, a shield was placed over the window toward the light source so that no light from the lamp entered the tube. It was found that readings could be taken with an accuracy of about 3° on the "low" scale (up to about 1250°C.), of about 5° on the "high" scale (up to 1850°C.), and of about 10° on the "xH" scale which had a maximum reading of about 3000°C.

After several plates had been obtained, it was found that readings were not in agreement when taken on different scales in regions where they overlapped. Therefore the instrument was taken to the Leeds and Northrup agency in Los Angeles where each scale was calibrated against their standard instrument. A new standard lamp was then installed after which each scale was found to agree with correct values as given by the U.S. Bureau of Standards certificate with the Leeds and Northrup standard within reading error of the instrument.

The calibrations taken before replacement of the standard lamp were then used to correct the readings which had been taken previously. Fortunately, all important readings had been taken at temperatures where the calibration corrections were small, and a sufficient number of plates were taken with the corrected instrument to provide sufficient information without using any that was at all uncertain.

Since the pyrometer readings were taken through the prism and also through the quartz window at the end of the furnace, it was necessary to obtain the temperature correction for each. In previous work of this kind, it had been assumed that the temperature correction of the prism was constant, but with the wide range of temperatures used in this project, such an assumption was no longer valid. Therefore it became necessary to prepare a correction curve for the prism and this was done by reading the temperature of the filament of a ribbon filament lamp first through the prism and then directly, while lamp temperature was held constant. While the equipment was set up, the older correction curves for the window were also checked and found to be correct within reading error of the instrument.

G. The Microphotometer

The microphotometer used in all of the reduction was the one designed by Dunham at the Mount Wilson Observatory. In it a condensed beam of light passed through a narrow region of the exposed plate as the plate was carried along at a uniform speed on a worm driven carriage. After leaving the plate the light passed through a microscope and slit to a photo-cell. The resulting electrical impulse actuated a galvanometer which used a mirror and light beam to make a trace on sensitized paper carried on a revolving drum driven in synchronism with the carriage. Thus traces were obtained whose abscissae were proportional to wave length differences, and whose ordinates varied with the density of exposed emulsion in the path of the light beam.

Position of the "clear" line, indicating density where the emulsion had not been exposed, and of the "dark" line, formed when no light reached the photo-cell, were recorded at several points along each trace. The continuum formed an irregular line (due to grain, defects, dust, etc.) with absorption lines appearing as dips toward the "clear" line.

Calibration plates were traced by having the light beam move rapidly across the width of the plate at positions corresponding to different wave lengths. Each such trace would then appear as a series of rectangles whose heights were proportional to the densities of the corresponding exposures. From these it was possible as described later to construct "characteristic" curves relating the intensity, I, or rather log I, to the density of exposure as recorded by the microphotometer. Since a change in the electrical adjustments of the microphotometer would affect the shape of the characteristic curves, it was necessary to re-trace the calibration plates whenever a period of time elapsed between photometerings of the same plate.

This microphotometer seemed to require a very long time to reach equilibrium. Thus it was necessary to check constantly the positions of the "dark" and "clear" lines.

The focus was very critical and poor results were obtained any time it was not in exact adjustment. Slight warping in some plates made it necessary to refocus on each successive reading while other plates did not give this trouble. Refocusing required the rechecking of "clear" and "dark" lines, and thus slowed down the operation considerably.

The chief disadvantage of this instrument is the amount of time required to reduce the resulting traces to actual intensity values. Because of the characteristic curves, it is difficult to obtain accurately the areas of lines which cannot be approximated as triangles, and even the fitting of triangular lines is complicated when the base and apex are not both on the linear portion of the characteristic curve. Mr. H.W. Babcock has developed a microphotometer that gives intensity ratios directly, but it requires specially prepared calibration plates which were not convenient to prepare with this work, and furthermore his instrument was so busily engaged during the whole course of this project that no attempt was made to use it. However, its use would have made possible a more accurate measurement of the stronger lines and thus would have improved the accuracy of the "step-down" process.

VI. REDUCTION OF DATA

A. Procedure

Reduction of data from the microphotometer tracings consisted of the following steps: (1) prepraration of characteristic curves, (2) marking traces and lines, (3) scaling, (4) reducing scale readings to intensities, (5) obtaining equivalent widths, (6) finding number of equivalent oscillators, (7) finding relative gf-values for each exposure, and (8) reducing all gf-values to a common reference and finding relative f-values.

1. Preparation of Characteristic Curves

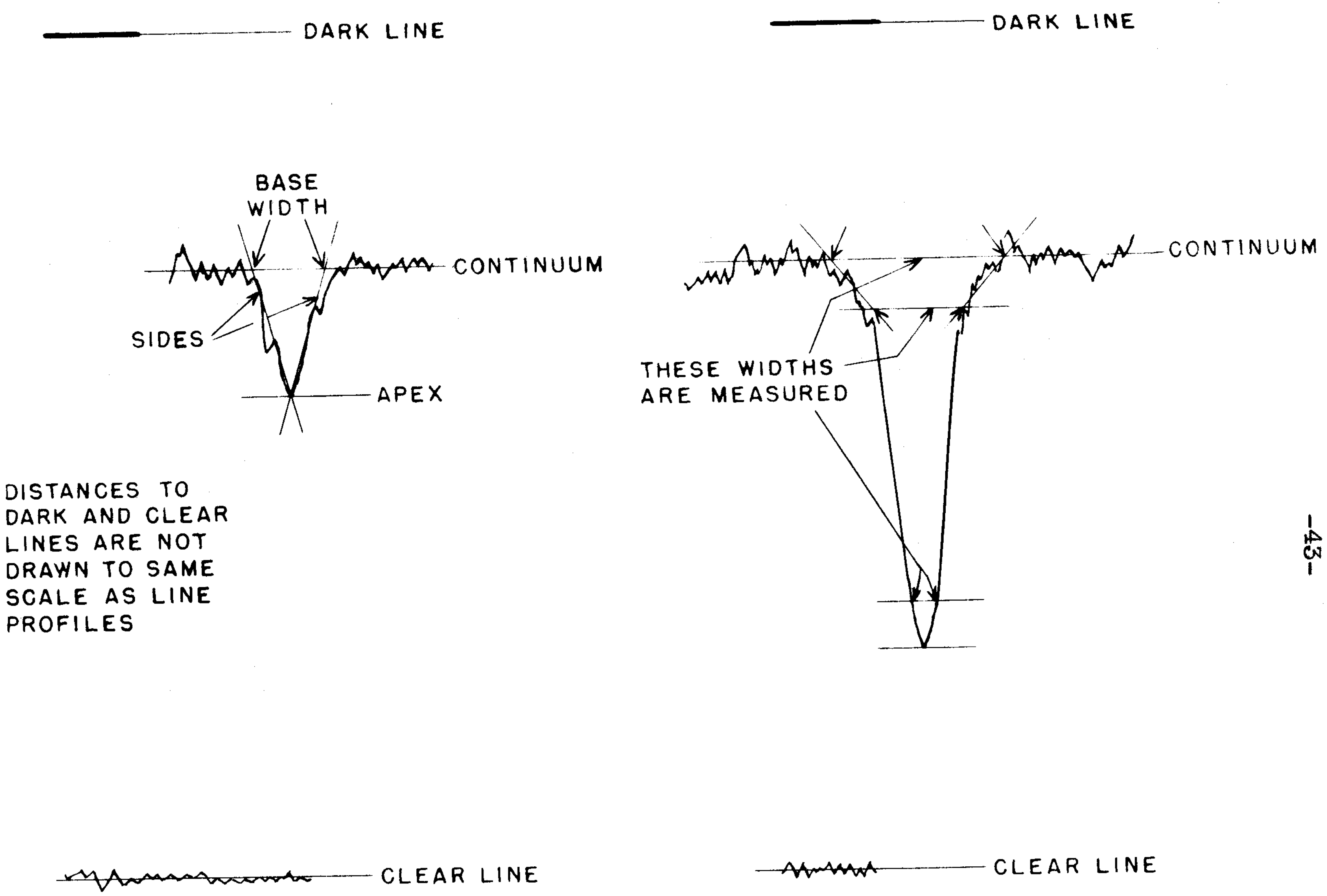
Tracings were taken at several suitable wave lengths depending upon distribution of lines in the exposure. Each tracing was marked by first extending the "clear" and "dark" lines the whole length of the sheet. Then sharp horizontal lines were marked through the midpoint of each calibration step. The proportionate distance of this mark from the "clear" to the "dark" was then read off by placing a 10 cm. scale diagonally on the trace with the zero on the clear line and the 10 on the dark. Positions of each marked line were then read off to the nearest 1/10 mm.

Readings for each step were plotted as ordinates against the log I of the corresponding step slit opening. Smooth curves were then drawn to connect these points. By using the same graph paper ruling and french curve throughout the course of this work it was found possible to keep these curves uniform and to detect discrepancies which might have caused serious error.

2. Marking Traces and Lines

The "clear" and "dark" lines were extended across the traces as described above. Then the continuum was carefully marked in the vicinity of each line. As discussed later, this was the most important of all the markings, for a slight misplacement would cause an error in base width and also in depth of the measured line. With a varying continuum, and with sharp fluctuations due to grain in the emulsion, accurate marking was extremely difficult. It was found that continued experience was the best guide and that it was not well to depend upon a single level of the continuum for different lines unless these were close together, for it was found that the continuum varied with wave length in all exposures to a greater degree than was at first realized.

Finally, the line itself was marked as shown in figure 3. For weak lines, a simple triangle as shown in 3a was adequate, but if any widening at the tip or evidence of "wings" was observed, additional marks were used as shown in 3b. When lines extended into the "knees" of the characteristic curves, it was found advisable to use several horizontal lines, even though they appeared to be triangular for the distortion in such cases often proved to be very serious. Further discussion of this matter is given under "Sources of Error".



DISTANCES TO DARK AND CLEAR LINES ARE NOT DRAWN TO SAME SCALE AS LINE PROFILES

WEAK LINE (E.W. = 0.008A.)

(a)

STRONGER LINE (E.W. = 0.02A)

(b)

FIG. 3 MARKING OF LINE PROFILES

3. Scaling

Proportional distances of the continuum, apex, and any other horizontal lines from the "clear" to "dark" lines were scaled in the same manner as on the calibration traces. The base widths, and also the widths of the line at any other horizontal marking, were also measured by using a divider and transferring the width to an accurate millimeter scale.

The base width for all weak lines on a given exposure should be independent of the line intensities, and in fact should vary approximately directly with wave length. This is evident from consideration of the line formed on the exposure as being a "fold" or product of the instrumental "line", and the spectral line. The instrumental line, for very narrow slit opening, has a base width $d\lambda$ which can be obtained from the relationship for resolving power R of a grating as

$$d\lambda \doteq 2R \tag{9}$$

But R for a grating is constant, and therefore $d\lambda$ varies directly with λ . Even with finite slit widths, the proportion will be approximately valid as shown by Baly[27].

If the spectral line is produced principally by doppler effect, as is the case for the weak lines considered here, its width for any given temperature is seen from equation (6) to be proportional to λ also. The spectral line profile under these conditions is a Gaussian curve, and assuming that the instrumental line can also be approximated by a Gaussian curve, as is usually done, the half width of the product will be the square root of the sum of the squares of the individual half-widths. Or, if they are more correctly treated as Voigt profiles as discussed by van de Hulst and Reesinck[28], the resulting half-width will be a linear combination of the component widths. In either treatment, the resultant half-width, or similarly the base width, will be proportional to the wave length for a given exposure with small slit width and closely approximate to this with finite slit widths.

On the basis of these considerations, base widths for all triangular lines in a single exposure were averaged, or rather the ratio of base width to wave length was averaged, and a factor obtained which could be multiplied by the wave length to give a mean base width at that wave length. Since the temperature factor in the doppler width was much less than the scatter in measured base widths, all the weak lines on a plate were usually averaged together. Use of these mean base widths improved the constancy of the determinations considerably.

4. Reducing Scale Readings to Intensities

Scale readings of the horizontal markings gave relative densities and since these were assumed to be the same on the record plate as on the calibration plate where the corresponding log I is known, log I for each reading could be obtained from the characteristic curve for the plate at the wave length of the line. Only differences of log I were of interest, and therefore the log I scale could have an arbitrary reference point.

5. Obtaining Equivalent Widths

The depth of a line at any point where the intensity is I is conveniently given by (1-r) where

$$r = I/I_0 \qquad (10)$$

I_0 being the intensity of the continuum at the wave length under consideration if the line were not present. Thus (1-r) could be obtained by finding the differences of the two logarithms, usually in the form $\Delta \log I = \log I_0 - \log I$, finding the antilogarithm of the negative of this value, and subtracting it from unity. This calculation was expedited by preparing a list of values of (1-r) corresponding to $\Delta \log I$ in a range from 0.008 to 0.400 (which was found sufficient to handle most of the computations).

It is now apparent that the equivalent width of a line in Angstrom units will be given by its "area" when all vertical distances are in terms of (1-r), and horizontal widths are in Angstrom units. Therefore all widths must be reduced by a factor given by the dispersion of the spectrograph and the magnifying power of the microphotometer. For most of the work on this project this magnifying power was 100:1, which gave a scale factor for second order exposures of 54.25 mm. per Angstrom unit.

Thus for a triangular line of base width b and maximum depth (1-r), obtained in the second order, the equivalent width was

$$W = \frac{1}{2}\,\frac{b(1-r)}{54.25} = \frac{b(1-r)}{108.5} \qquad (11)$$

If the exposure was taken in the first order, the denominator would be half as large.

6. Finding Number of Equivalent Oscillators

Since but few lines are weak enough to be on the truly linear part of the curve of growth, it was necessary to use a theoretical curve to obtain values of C from equivalent widths. As mentioned previously, it was found that equation 8 was satisfactory for plotting this curve.

Increased accuracy in this critical step was obtained by plotting a single curve very carefully and to large scale as described under "Special Methods". Values of log $\Delta\lambda_D$ were then tabulated for each exposure and for various wave lengths. To find the value of C for a given line, a distance on the vertical scale corresponding to the proper value of log $\Delta\lambda_D$ was set on a pair of dividers. These were then moved into position with points in a line parallel with the vertical graph rulings, and with one point on the plotted curve while the other was on the ordinate corresponding to log W_λ. In this manner log $\Delta\lambda_D$ was effectively subtracted from log W_λ, and the resulting log $(W_\lambda/\Delta\lambda_D)$ entered in the curve in a single operation. Manipulating the dividers in this way was much simpler than plotting several different curves and increased accuracy resulted from the use of a larger graph than would have been practical if separate ones had been required.

Once the value of C had been determined in this manner, Nfl was found by multiplying C by $1/k_o$. Thus values of $1/k_o$

were required for each exposure and wave length also, and these were tabulated with the values of log $\Delta\lambda_o$ by using the numerical forms of equations (6) and (5), respectively, as follows:

$$\log \Delta\lambda_o = 0.77563 + \log \sqrt{T} + \log \lambda - 8 \qquad (12)$$

$$1/k_o = 1.194 \sqrt{T}/\lambda \times 10^{13} \qquad (13)$$

A slide rule was used for the calculation of equation (13) with the final readings being taken as log Nfl.

7. Finding Relative gf-values for Each Exposure

If the exposure is homogeneous, that is, if all portions have been exposed for the same length of time, and if the furnace conditions were those of thermal equilibrium, it is apparent that gf-values as given for each line in terms of observed Nfl values according to equation (3), will have the same proportionality constant. Therefore relative gf-values will be given by first applying the appropriate Boltzmann factor, $e^{\frac{hc\bar{\nu}}{kT}}$, to the Nfl values, and then finding the ratios between the results. The simplest way of handling this factor was to tabulate the logarithms of $\frac{hc\bar{\nu}}{k}$ for each term level. Then, since log T had already been obtained for each exposure, it could be subtracted for those levels giving lines in that exposure, and the antilogarithms would be the appropriate Boltzmann factors. This tabulation could be done quickly at the same time as were those for log $\Delta\lambda_D$ and for $1/k_o$.

8. Reducing All gf-values to a Common Reference

Reduction of all values to a common reference was made by first establishing a "base" to which selected important lines could be related, then using these lines as references for a secondary set of weaker lines, until finally a reference set for all intensities had been established.

It was natural to select as a base, the multiplet containing the "raie ultime" as given by Meggers[29], and listed in the RMT as No. 1. This is composed of transitions from the ground state given by $a^7S - z^7P^o$ consisting of three lines, λ4254, λ4275, and λ4290. A total of eight exposures were made with intensities suitable for reduction by this method. After Nfl values had been obtained, ratios between each pair were found for each exposure. Since all transitions were from a singlet ground state, no Boltzmann factor was required. The mean values of each ratio were then found and these were adjusted by the method of least squares to give the best relative gf-values for the three lines.

These checked closely the theoretical values given by White[30] and also those obtained by Estabrook[31] in his studies of absolute f-values for chromium.

Succeeding multiplets were likewise reduced, first by finding the best ratios between the lines within a multiplet, then finding the mean ratios between one of the "adjusted" lines in each of the two multiplets for each exposure having lines of measurable strength in common. Weighted means of these inter-multiplet ratios were used, with the weighting taking into consideration the regions of the curve of growth where the contributing lines appeared, the precision with which they agreed with one another, and the general quality of the exposures, including such factors as uniformity of furnace temperature, density of exposure, and fineness of grain.

When multiplet ratios were obtained in this manner before intermultiplet comparisons were made, all of the readable exposures of a multiplet could be taken into account rather than only those which also contained lines of the reference multiplet. Also, by establishing these ratios on the basis of readings taken on the most suitable part of the curve of growth, it was often possible to adjust and use readings taken on other parts satisfactorily. For example, it was found that adjusted readings well up on the "knee" of the curve usually checked in intermultiplet comparisons with readings taken on the more linear part. Thus by measuring strong lines as accurately as possible and then reducing "looseness" by adjusting according to pre-determined ratios, it was possible to make comparative readings on exposures that otherwise would have had no value. Methods by which this adjusting was done are discussed under "Special Methods".

After several of the more important multiplets had been carefully compared with each other in this way their lines were used as references in exposures containing other lines. When several reference lines appeared in a single exposure, the values for these measured in that exposure were first adjusted to the ratios previously obtained for them, and then any one of these adjusted values could be used as a reference with which to compare readings of all the other lines on that exposure. Since these comparisons gave gf-values relative to a common base, it was then possible to average all such values obtained for a given line on several different exposures, giving weights to the various readings depending upon the quality of the exposure. These weighted average relative gf-values were then tabulated, and relative f-values obtained from them by dividing by the statistical weight of the lower term level of each line.

B. Special Methods

Little change was made in the techniques which had been developed by Dr. King, though one or two modifications might be worth mentioning.

1. Double Photometering

It was found worth while to photometer each line profile on each exposure at least twice. In fact, it was shown that if sufficient photometerings were made, errors due to grain in the emulsion were practically eliminated and that excellent results could be obtained. However, this required considerable time so that a compromise between accuracy and time expenditure had to be made.

After trying several systems, it was found that photometering each line on each exposure twice using different parts of the continuum, probably gave maximum results for time expended. This provided a check on each profile and quickly showed up serious errors due to irregular grain or other plate imperfections. In about eighty percent of the cases, the photometerings checked with a few percent. In the remainder the discrepancies ranged from mild to serious. Often it was evident upon close inspection that irregular grain had distroted one of the profiles, so that it could either be remeasured or discarded and the better profile used. In a very few cases the discrepancy could not be resolved and, if it was not too serious, the mean value was used with a low weight being given the result. If too bad, the reading was discarded.

This extra photometering and measuring increased the time over that required for single photometering by approximately 50%, for not all of the work had to be duplicated. For example with triangular lines, similar readings of $\Delta \log I$ could be averaged and a single computation carried on from this point. With stronger lines, it was necessary to find the "areas" or equivalent widths before averaging.

2. Use of a Single Curve of Growth

The use of the curve of growth in reducing equivalent widths to Nfl values was found to be a critical step in the reduction, principally because of the difficulty of reading values from the smaller curves accurately enough to make best use of available data. Yet if seperate curves were to be constructed for each set of temperatures and wave lengths, it would not be practical to construct each one to large scale.

This difficulty was overcome by using a single large curve and auxiliary tabulations of $\log \Delta\lambda_D$ and $1/k_0$ as described above. The curve was plotted for values of C from

0.1 to 10 and for log $W_\lambda/\Delta\lambda_D$ from 9.300 to 0.600 on semilogarithmic graph paper approximately two feet square, making it possible to read log $W_\lambda/\Delta\lambda_D$ to three decimal places, and C to three significant figures.

Tabulations could be quickly prepared using interpolations from key values for wave lengths in increments of not more than 50 Angstroms and for temperatures corresponding to each exposure. A separate table giving log $\Delta\lambda_D$ and $1/k_o$ for each exposure was thus prepared for each plate. In this way, each line practically was given its individual values, yet total time required, including that of preparing the tabulations, for reducing equivalent widths to Nfl values in this manner averaged less than one minute per line.

3. Applications of the Methods of Least Squares

Without going into details concerning underlying theory and the resulting derivations, for these are adequately treated in any text on the subject, the following equations are given here in the form which was found convenient for averaging and adjusting the readings obtained in this work.

a. Given several ratios between sets of readings such that their product is unit. If each ratio is obtained in each of a series of readings, what are the best values which should be assigned to each one?

Letting r_1, r_2, $r_3 \dots r_n$, represent the ratios with the stipulation that $r_j = 1$, then if u_{11}, u_{12}, $u_{13} \dots u_{1k}$ represent the values of the ratio r_1 as determined by a series of k readings, u_{21}, $u_{22} \dots$ represent the values of r2, etc. it can be shown that the best values of the r's will be obtained by first finding the arithmetic mean of each set of readings, $\bar{u}_1$, $\bar{u}_2$, etc. and then finding r_j from the equation

$$\log r_j = \log \bar{u}_j - \frac{1}{n}\sum_{i=1}^{n} \log \bar{u}_i$$

For example, if a triplet has values of Nfl given for its three lines by a, b, and c respectively, it is possible to take $r_1 = a/b$, $r_2 = b/c$, and $r_3 = c/a$. Where more than three lines are involved, any set of ratios may be used, but care must be exercised that these do not contain a factor which itself is unity, otherwise the best values for the factor may not agree with those for the total product. Often it is cumbersome to use all the ratios, in which case the best ones relating the best determined lines, or several which are of approximately the same strength, can be used. Once the ratios between these selected lines are established, the others can be related to them by first adjusting each set of readings as given under "b", then finding the ratio of each of the other lines to one of these adjusted values, and obtaining the mean of these ratios.

b. Given a set of accepted ratios between a set of readings, how can the readings be best adjusted to match these ratios?

This may be done either with the readings themselves or with their logarithms. If the readings are given by a series of observations whose arithmetic means are u, v, w, ..., and if the corresponding readings, a, b, c, ..., respectively, should have the relationship that $b = a\, r_1$, $c = a\, r_2$, etc., then it can be shown that

$$a\ (1 + r_1^2 + r_2^2 + +) = u + r_1 v + r_2 w + + \qquad (14)$$

If all values are given in terms of their logarithms, then

$$\log a = \frac{1}{n}\ (\log u + \log v + \log w + + - \sum_{j=1}^{n} \log r_j) \qquad (15)$$

c. Given two sets of readings, such as two sets of Nfl values obtained from two different exposures, whose relative ratios are known, and which are known to have a constant ratio between corresponding readings (such as would be true for lines having the same Boltzmann factor in the two exposures). What is the best adjustment to make if (a) the ratio if unknown, (b) if it is known?

If the ratio is unknown, it can be shown that the best results are obtained by adjusting each set of readings to the given ratios as outlined in "b" above, and the ratio will then appear as a common ratio between corresponding readings.

If the ratio is already known, say, for example, it has the value m, the best value for one of the readings, say a_1, will be given by

$$\begin{aligned} a_1(1 + m^2)(1 + r_1^2 + r_2^2 + +) = u_1 + r_1 v_1 + r_2 w_2 + + \\ + m(u_2 + r_1 v_2 + r_2 w_2 + +) \end{aligned} \qquad (16)$$

where the subscripts on the a, u, v, and w designate the two different sets, and the r's are as given in "b".

VII. RESULTS

Relative gf-values were measured for 410 lines in the spectrum of neutral chromium. Furnace temperatures from 1100°C. to 2850°C. were used and some lines having lower excitation potentials as high as 3.45 volts appeared strongly enough to be measured. A total of 35 plates containing 199 individual exposures were made and 168 of the exposures were photometered and reduced.

Measurements were made in the region from λ3190 to λ7470, though only 74 lines were measured below λ3800, and only 9 were measured having wave lengths greater than λ5500.

Two lines were found to have been assigned wave lengths which did not check with observations, so corrections of these values were obtained. Three unclassified lines were observed and measurements made which may assist in their classification.

A. Tabulation of Results

Results of the measurements are tabulated in Table 5 which follows the arrangement used by King[3] and uses the system of designation adopted in the RMT[18]. The first two columns of the table, headed "Mult. No." and "Designation" include, first, the multiplet number in parenthesis assigned in the RMT; next, the over-all multiplet designation, followed by the inner quantum numbers, or j-values, of the individual lines in the multiplet. When the gf-values of all the lines in the multiplet have not been measured, an asterisk (*) follows the multiplet number. Since all multiplets which were measured are listed in the RMT, no modification of the numbering arrangement is necessary.

The third column of the table gives the laboratory wave lengths corresponding to the transitions listed in the second column. Wave lengths are from the RMT except for two unclassified lines and one line whose wave length in the RMT was in error. These three values are taken from the MIT Wave Length Tables[32]. A dagger (†) before the wave length indicates that the line is a blend of two or more transitions in the chormium spectrum.

The fourth column gives the estimated arc intensity of the line as given in the RMT. These give a good indication of the relative intensities at a considerably higher excitation temperature than that used in the furnace and so may be useful in forming judgment as to the behavior of blends of high-and low-level lines. Arc intensities followed by the letter "w" are widened by hyperfine structure in the arc. The use of parentheses in the intensity column indicates that a reference has been used which is different than that used for the main portion of the spectrum, and these intensity values are not to be compared with others which are not in parentheses.

The fifth column in the table lists the relative gf-values derived from the furnace absorption spectra. The scale used for these is, of course, an arbitrary one, though it is arranged so that all values can easily be referred to that for λ4254.

The sixth column lists the relative f-values which are derived from the relative gf-values simply by dividing the gf-value for each line by the statistical weight ($g = 2j + 1$) for the lower term of the transition producing that line. Where lines are blended so that it is improbable that this transition is the only one contributing to the measured intensity of the line, the f-value and gf-value are placed in parentheses. Where, however, the contribution of one of the components of such a blend is probably negligible at furnace temperatures, no parentheses are used.

The seventh column gives the weights assigned to the measurements, and these differ slightly from those used by King. The letter "A" is used only for those lines which have been referred to the intensities of multiplet (1) by measurements on not less than 10 good exposures, and also by using methods of least squares to weight and adjust the values so that overall probable error in the comparison with the values of multiplet (1) is of the order of 5% or less. The letter "B" is used for those results of at least five independent measurements which are of such quality that the probable error in relative values for lines within the same multiplet is of the order of 5%, and for comparisons with lines in other multiplets is of the order of 10% or less. The use of a small "r" following the "A" and "B" weights indicates that these lines were used as references in establishing the values for the other lines. Therefore comparative values between these reference lines had been established with more care than for other lines of comparable weight. The letter "C" indicates that measurements have been obtained from at least three exposures, and that these gf-values should therefore be reliable within a probable error of 15% or so. The letter "D" indicates that the measurements were not reliable or that data from only one or two exposures were available. About four out of five of these values with "D" weight will be dependable within an error of plus of minus 10%, but some may be off by as much as 50%. The letter "M" in the weight column indicates that the line was too weak to make reliable measurement possible although the tabulated value is almost certainly as large or larger than the true gf-value of the line. It was found that where comparisons could be made with measurements of stronger profiles of the same line, that correct values were usually from 80% to 90% as large as given by measurements on these weak lines. Also, if the correct values were not at least one-half as large, the line would be too weak to be observable. Therefore tabulated values which have been assigned an "M " weight can be expected to have actual values of from 50% to 100% of those assigned, with greater probability being given the higher percentages.

The eighth column is used for reference to notes which follow the table. Multiplets are separated by double spacing, and a horizontal line across the table indicates a change in lower level of the transitions.

B. Comparison with Theoretical Values

A comparison with the gf-values obtained for some of the multiplets with theoretical values given by White[30] has been made in Table 1.

Values for multiplets (1) and (7) are obviously ver close to those given by White and this comparison shows the accuracy attainable by the method used in this project when a sufficient number of readings were taken. Eight different exposures were used to obtain the values for multiplet (1) and 30 photometer tracings were analyzed for each line. Values for multiplet (7) were determined from 11 different exposures with an average of 37 photometer tracings measured for each of the three lines.

The transition $a^7S_3 - y^7P^o_2$ in multiplet (4) is of interest because its ratio if apparently higher than predicted by theory. A.S. King[6] noticed this in the first estimates he made of the intensities of these lines, and in the RMT this line is indicated as being a little stronger than its counterpart in multiplet (1). Therefore there is reason to believe that the lack of agreement with theory is real.

Multiplet (18) was used as an intermediate step for comparison between lines in multiplets (1), (4), and (7) and all of the weaker lines. Therefore a total of thirty different exposures and 323 photometer tracing profiles were analyzed and weighted by the methods of least squares to obtain the values given in Table 1. These were found to check with those of multiplet (7) within a probable error of less than 3%, though

TABLE 1

OBSERVED RELATIVE gf-VALUES FOR SELECTED MULTIPLETS COMPARED WITH THEORETICAL VALUES FOR L-S COUPLING

(Theoretical values are given in parentheses)

Multiplet (1) $a^7S - z^7P^o$

	$^7P^o_2$	$^7P^o_3$	$^7P^o_4$
$^7S_3$	56.6 (55.6)	78.8 (77.8)	100.0 (100.0)

Multiplet (4) $a^7S - y^7P^o$

	$^7P^o_2$	$^7P^o_3$	$^7P^o_4$
$^7S_3$	59.7 (55.6)	78.2 (77.8)	100.0 (100.0)

Multiplet (7) $a^5S - z^5P^o$

	$^5P^o_1$	$^5P^o_2$	$^5P^o_3$
$^5S_2$	42.8 (42.8)	71.2 (71.4)	100.0 (100.0)

Table 1 Continued

Multiplet (18) $a^5D - z^5P^o$

	$^5P^o_1$	$^5P^o_2$	$^5P^o_3$
D_0	11.4 (11.1)		
D_1	26.5 (25.0)	8.8 (8.3)	
D_2	19.4 (19.4)	31.4 (42.4)	4.0 (3.7)
D_3		48.0 (51.9)	25.6 (25.9)
D_4			90.8 (100.0)

Multiplet (94) $z^7P^o - e^7D$

	$^7D_1$	$^7D_2$	$^7D_3$	$^7D_4$	$^7D_5$
$^7P^o_2$	27.6 (27.3)	30.6 (30.3)	22.7 (18.2)		
$^7P^o_3$		15.2 (15.2)	37.5 (39.8)	49.0 (51.1)	
$^7P^o_4$			10.4 (5.7)	33.3 (30.7)	71.2 (100.0)

of course there is the possibility that individual values will not check this closely.

Multiplet (94) was selected at random to indicate the results obtained after several "steps" were taken, for though the gf-values of the lines in this multiplet are relatively high, it has a lower excitation potential of 2.90 volts and so the lines were comparatively weak on most of the exposures. Several good tracings were obtained with $z^7P^o_4 - e^7D_3$ (λ5329.12) and $z^7P^o_4 - e^7D_5$ (λ5328.339) together, and each of these showed that the ratio of gf-values could not be much greater than 2:1 between the stronger and the weaker line. Therefore the low value observed for λ5328 is probably not due to observational error.

C. Unclassified Lines

One line which was listed in the RMT as unclassified and two lines which were not included in the RMT but which were listed in the MIT Wave length Tables were measured though all were very weak so that measurements should be considered to have an "M" weight as discussed above.

Fortunately each of the three lines could be compared on each exposure on which it appeared with λ4126.521 in multiplet (35) for which good measurements were available. Therefore ratios of Nhf values could be obtained as given in Table 2.

TABLE 2

COMPARISONS OF Nhf VALUES OF UNCLASSIFIED LINES WITH THOSE OF 4126.521

λ	Listing	Temp. °C.	Ratio Nhf-values
4126.925	RMT	2580°	0.098
4126.925	RMT	2645°	0.107
4126.925	RMT	2750°	0.175
4128.405	MIT	2580°	0.091
4031.130	MIT	2645°	0.089

D. Correction of Wave Lengths Assigned Two Lines

Two lines were found which had been assigned wave lengths that did not agree with those found on the exposures made for this project. Since it was possible to measure small wave length differences on the microphotometer tracings to within 0.01 Angstrom unit, and to estimate with some degree of accuracy to 1/10 of this amount, the distance of each of these lines from two known reference lines was measured on at least six tracings from three or more exposures, and the mean value of these determinations is suggested as an improved value to be assigned the wave length.

Fortunately for each line, two suitable reference lines were found within a distance of less than 2 A.U. and results of the measurements were such that probable errors could be assigned. These results are given in Table 3.

TABLE 3

CORRECTION OF WAVE LENGTHS ASSIGNED TWO LINES

Transition and Mult.	Assigned Wave Length	Lines Used as Reference	No. Meas.	Corrected Wave Length
$a^5G_6 - y^5H^o_6$ (38)	3969.061	Cr 3969.748 Fe 3969.261	6	2969.084 ± 0.001
$z^7P^o_3 - e^7D_3$	5297.976	Cr 5296.686 Cr 5298.269	18	5298.018 ± 0.002

E. The "Raie Ultime"

Meggers[29] indicates that λ4254 should be the strongest line in the chromium spectrum, and this is understandable from the data available at the time he published this information. However, it is definitely evident that the transitions from $a^7S - y^7P^o$ are several times as strong as those from $a^7S - z^7P^o$ as shown by a comparison of the gf-values for comparable lines in multiplets (1) and (4) in Table 5.

Therefore when proper consideration is given to the fact that lines in the shorter wave length regions appear weaker on photographic plates, it evident that the strongest line observable in the chromium spectrum at low excitation temperatures will be λ3578.694 rather and the one suggested by Meggers.

It is of interest to note that A.S. King's first estimates gave comparatively low intensities for the lines in multiplet (4), but that as better techniques have been devised for taking into account the effects of the photographic plate, this multiplet was assigned larger and larger values, until in the RMT it is assigned intensities equal to those for multiplet (1). Now it seems quite conclusive that the lines in multiplet (4) are several times as intense as those in the other multiplet.

F. Sources of Error

Principle sources of error and uncertainty were found to be (1) grain and imperfections in the photographic plate, (2) errors in marking and scaling the lines, (3) uncertainty in temperature readings, (4) variation of experimental conditions during exposure, (5) microphotometer errors, and (6) imperfect line profiles resulting from (a) blends with other lines, (b) emission or absorption lines in the source, (c) re-emission in the line.

Errors of a random type could be treated statistically by taking several exposures of each line and by photometering each line profile several times. This was done for most of the lines and results were consistent enough to indicate that the most serious uncertainties were of a random type. However, some systematic error was definitely present, and this could be cumulative under the step-by-step process used in reducing strong and weak lines to the same scale, with the result that serious error might be found in comparisons between lines where the ratio of intensities was large.

Comparative study of several exposures of the same set of lines taken under different experimental conditions made it possible to detect most of the systematic errors and either to modify the procedure to eliminate them, or to treat them in such a manner that their effect would be minimized.

1. Grain and Other Imperfections

The irregularity of the grain is readily observable in the microphotometer trace and causes serious difficulty in two ways. First, it makes the location of the clear line and of the continuum uncertain, and, secondly, it often deforms the line profile so that marking of the line will be faulty. Fine imperfections in the emulsion give similar effects. Some of the Eastman plates, in particular, had areas which seemed to be peppered with fine bubbles or imbedded dust grains. Where these were large enough to be seen or to cause noticeable irregularity in the tracing, results were discarded, but they may have caused faulty measurements in other portions of the exposure where their effects were not so noticeable.

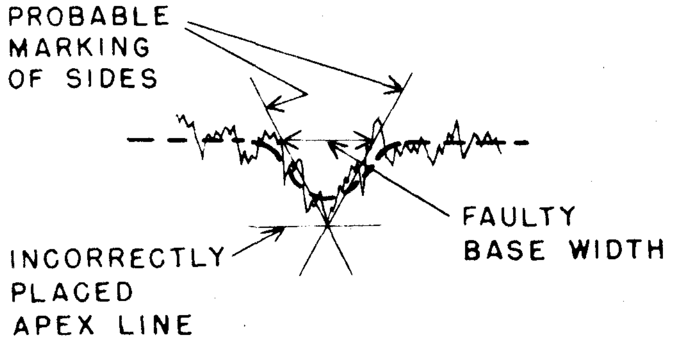
The uncertainty due to grain irregularity which normally exists in the location of the position of the clear line does not usually introduce much error in the line measurements. However, if the plate is underexposed, or if measurements are made on the tip of a deep line, some readings may fall near the clear line, and these will also lie along the toe of the characteristic curve. Under such conditions, slight uncertainty in the position of the clear line can be very serious.

Uncertainty in locating the position of the continuum introduces a double error, for it affects both the depth and the base width of the measured line. This can be especially serious in weak lines where the total depth of the line may not be much greater than the uncertainty in the position of the continuum.

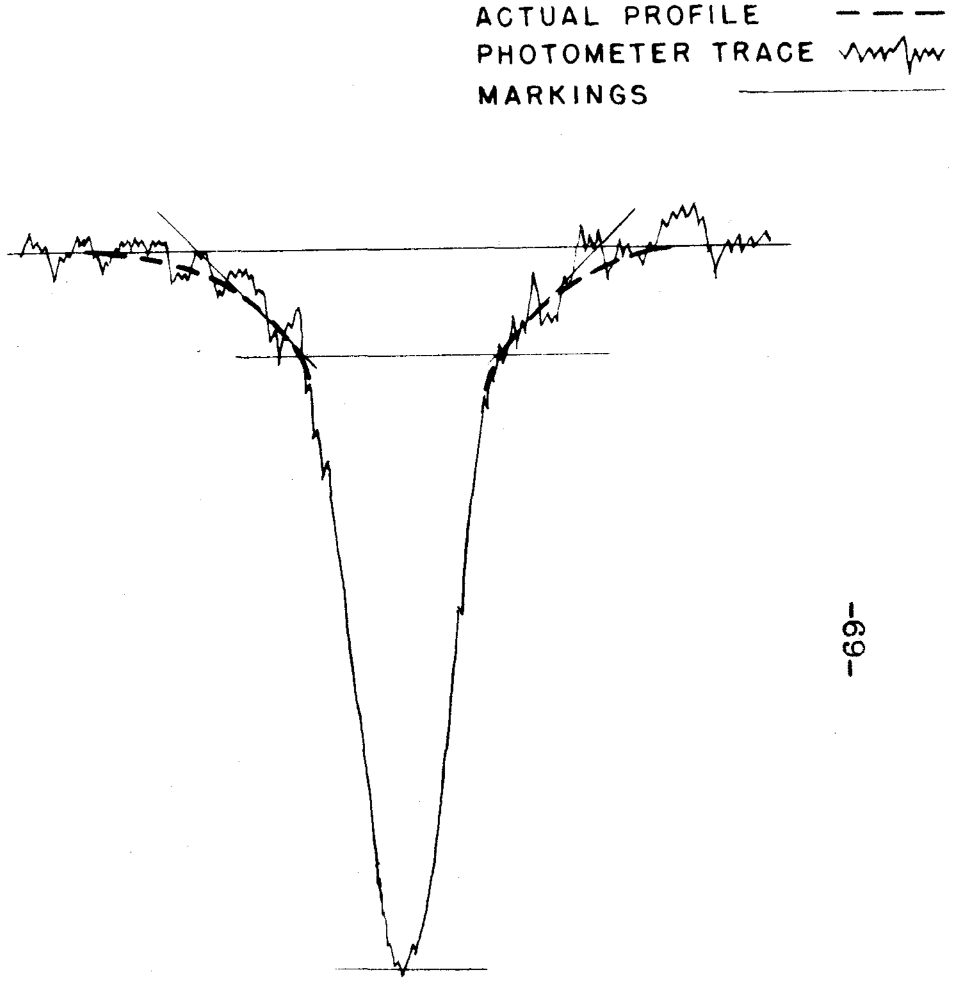
Very weak lines usually appeared to be narrower than slightly stronger ones, although as already discussed, line widths for all of the weaker lines should be practically independent of line strengths. That this effect may be due to the irregularity of the continuum can best be explained by the diagrams shown in figure 4a and 4b. In these diagrams a dashed line is used to show the profile the line would have if no irregularities were present, with the irregular solid line representing the type of deviation usually found in a photometer trace. The solid straight lines show the markings that would usually be made in the measurement of these profiles.

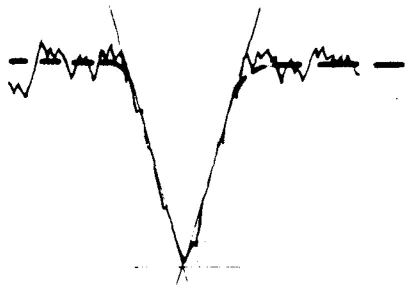
It is immediately evident that for very weak lines of the kind shown in figure 4a, the irregularities will mask an appreciable portion of the line width, but when the sides of the profile are steeper as in figure 4b, most of the actual width will be discernable. Therefore in computing mean base widths it was necessary to use only measurements on lines whose profiles were definitely formed. Then by using these mean base widths in the calculation of equivalent widths for the weaker lines, error due to the "narrowing" effect shown in figure 4a could be avoided.

An error in the measurements of the depths of these very weak lines is caused by the uncertainty of the true location of the center of the profile. In all likelihood the depth will be measured to the lowest point of a nearby grain irregularity, and this will give an equivalent width which is almost certainly



a) EFFECT ON MARKING OF
A VERY WEAK LINE (E.W. = 0.002A.)





b) EFFECT ON MARKING OF
A STRONGER LINE (E.W. = 0.009A)

c) EFFECT IN WINGS OF
A STRONG LINE (E.W. = 0.088)

FIG. 4 EFFECTS OF IRREGULARITIES IN PHOTOMETER TRACE
DUE TO GRAIN IN THE PLATE.

greater than that which should be assigned to the line.

While the error due to "narrowing", and that due to uncertainty of central line depth will therefore be in opposite directions, they are unfortunately not related to each other and cannot be expected to cancel. The only safe procedure is to use mean base widths and then to recognize that resulting measurements will in general be larger than they should be.

Whenever possible, the use of measurements obtained from these very weak profiles (equivalent width less than 0.005 A.) was avoided. However, for some of the lines, such measurements were the only ones available. They are tabulated with the others, but are given a weight of "M" as already discussed, indicating that the assigned value is a maximum one.

The irregularity in the continuum also masks the wings of strong lines as shown in figure 4c. Therefore, unless some care is used, the equivalent widths assigned such lines are apt to be smaller than they should be. It was found, however, that if lines were measured by using trapezoidal approximations, the trapezoid next to the continuum would have some area outside of the line profile, and this apparently was usually enough to compensate for that lost in the extended wings. At any rate, measurements which were carefully made in this manner showed no systematic tendency to give values which were too small.

Triangular approximations were found to be satisfactory for lines having equivalent widths up to about 0.015 A., though slight broadening was apparent in those which were stronger than about 0.01 A., and to allow for this it was best to use actual base widths rather than mean base widths for lines having equivalent widths between 0.01 A. and 0.015 A. When any broadening of the tip or definite formation of wings became evident, as it usually did for lines having equivalent widths greater than 0.015 A., the method of measurement shown in figure 3b was used.

2. Errors in Marking and Scaling the Lines

Since the lines which had the best profiles for measurement had depths which usually did not exceed 20 mm. on the tracing, it was evident that small errors in marking and scaling would give large percentage errors in the results. These should be completely random, however, and were reduced by using the measurements from several different exposures and by taking several photometer tracings of the same profile.

It was necessary to approximate the areas of the stronger lines by some method in which the number of readings which had to be reduced through the characteristic curve was kept a minimum yet which gave results that were accurate within a few percent.

Several methods of approximation were compared by applying them to line profiles which had been measured accurately by point-by-point reduction of ordinates. The results are given in Table 4. The standard methods (Simpson's, Weddle's, and Gauss') are described in any good book on numerical approximation. The use of Voigt profiles is discussed by van de Hulst and Reesinck[28].

TABLE 4

COMPARISON OF AREA OF LINE PROFILES OBTAINED BY USING DIFFERENT METHODS OF APPROXIMATION

Method	No. Base Divisions	Area Profile 1		Area Profile 2	
		Area	%Error	Area	%Error
Point-by-point		1886	0.0	3306	0.0
Simpson's	8	1842	-2.3	3360	+1.6
Simpson's	16	1890	+0.2	3350	+1.3
Weddle's	6	2268	+20.2	3163	-4.3
Weddle's	12	1855	-1.6	3192	-3.3
Gauss'	6	1710	-9.3	3039	-7.9
Gauss'	5	2186	+15.9	3000	-9.1
Gauss'	10	1881	-0.3	3206	-3.0
Voigt Profiles	(2 pts.)	2040	+8.2	3340	+1.0
Trapezoids	(4 pts.)	1890	+0.2	3297	-0.3

Profile 1 was of a line which was only slightly broadened (equivalent width 0.0189 A.), though wings were definitely in evidence. Profile 2 was of a stronger line (equivalent width 0.033 A.) with definite broadening at the tip and well extended wings.

For each of the standard methods, the base width was divided as required by the method, at certain specified points. The depth of the line at each of these points was then measured from the profile and reduced in the usual manner.

In applying Voigt profiles it is necessary to measure the width of the line at half depth and at 0.1 depth. This avoids the uncertainty of measuring the base width at the continuum, though to locate these required points, it is necessary to work back through the characteristic curve after finding the central depth of the line.

In applying the trapezoid method, marking was done as shown in figure 3b, except that a fourth horizontal line was used about midway between the upper two making a closer approximation of the curved wings possible.

It is evident that the trapezoid method gives results which are comparable with the best obtained by any of the other methods, and with much less work. Interestingly enough, when division into trapezoids was made vertically rather than horizontally, results were no better than with the standard methods, for the uncertainty of the intersection of the vertical division lines with the steeply sloping profile is one of the primary sources of error.

Although horizontal division with the standard methods would probably have given better results than the vertical division, the extra work required to obtain division points from the density values on the trace made this impractical.

The Voigt profile method had this difficulty and it also required the use of another curve which introduced additional uncertainty. However, the Voigt method is not much more laborious than the trapezoid method, so if time were taken to plot the required curves to larger scale than given in the publication, this method may prove to be practical.

The use of trapezoidal division had certain definite advantages over the others. First, several measurements were taken of the width of the line at various points. This reduced error from grain fluctuations and from errors of measurement which were always appreciable percentages of these widths. Second, the trapezoidal approximation in the wings seems to just about compensate for the wing region lost in the continuum. At any rate, this method was adopted, and as many trapezoids as required by the profile were used. The results from measurements of different photometer tracings of the same profile usually fluctuated by less than 5%.

3. Uncertainty in Temperature Readings

The temperature of the furnace could be controlled so that it remained constant within a few degrees during exposures of 5 minutes or so. Also, as King[1] and Carter[20] found by extensive observation on conditions of thermal equilibrium within the graphite tube, the temperature along the tube was constant except within a few centimeters of each end. This was verified by observing the tube walls with the pyrometer at various places along the tube. In no instance, except when hot rings were forming prior to tube failure, was a measurable temperature variation observed.

Exposures were compared to see if any effects which might be attributable to lack of thermal equilibrium could be observed. None were found of a magnitude in excess of the uncertainty of measurements. Therefore it can be safely assumed that the experimental conditions approximated those of thermal equilibrium closely enough that equation (2) could be applied, and that deviations from such conditions were generally of a random type.

Portions of the tube wall which had absorbed the chromium had an emissivity which did not approximate that of a black body and so temperature readings with the optical pyrometer would not have been correct. However, these portionswere easily detectable with the pyrometer so that no error need to have occurred from this source.

A slight misalignment of the prism and pyrometer, dust or finger marks on the prism or window are found to reduce the temperature readings critically, so that it was necessary to keep the optical system clean and in good alignment for each reading. One exposure had an error in temperature which might have been caused by misalignment of the prism, but all the others checked where comparisons could be made within the usual 10% or so random fluctuation. Fortunately the faulty exposure did not have to be used for comparisons where the Boltzmann factor was required. A study of the plates taken prior to the calibration of the instrument and which were known to have faulty temperature values showed that comparatively small errors in temperature determination quickly revealed themselves by giving systematic differences in the relative gf-values for low-level as compared with high-level lines, and where these could be checked against values obtained on plates having correctly known temperatures, an estimate of the error in the temperature of the faulty plate could be made. This information was used principally to check against possibility of faulty temperature readings for on none of the plates used for important inter-level comparisons was such temperature error in evidence.

4. Variation of Experimental Conditions During Exposure.

During exposures extending over a period of ten to fifteen minutes, the chromium vapor could be expected to boil away appreciably. If this boiling away takes place at a uniform rate, the effect will be to shift all the lines along the curve of growth toward the left. Assuming that the temperature is constant, all the lines will move at the same rate along the curve, for the ratios of Nhf will remain constant. This will mean that lines on the linear portion will weaken more rapidly than those on the transition portion. However, the effect on the ratios of line strengths will be of the second order, due only to the differences in the curvature of the portions of the curve passed over during the exposure. This can be seen by assuming that the portion of the curve traversed by the line during an exposure can be approximated by a straight line connecting the initial and final points. Although these lines will have different slopes for spectral lines on different portions of the curve of growth, the mean value of each log C will correspond to the mean value of the corresponding log $W_\lambda/\Delta\lambda_o$ and since the differences in the values of log C for two spectral lines will be the same whether boiling away takes place or not, the ratios of the integrated measured line strengths will therefore give correct ratios of C under the assumption of linearity in the portion of the curve each line traverses.

Due to differences in curvature of the different portions of the curve, there is a small error introduced in the measured ratios of Nhf values by the boiling away of the vapor. But comparisons of these ratios from long and short exposures for lines in the same regions of the curve of growth showed that this effect was much smaller than was anticipated. Even for ten minute exposures at high temperatures the effect is not serious enough to be noticeable.

Temperature fluctuations during exposure were found to have a much more serious effect, for then it is difficult to apply a suitable mean temperature. The form of the Boltzman factor is such that lines originating in different low levels will move at different rates along the curve of growth as the temperature fluctuates. What is more serious, the rate at which they move will no longer be uniform, even with a uniform change in temperature.

Fortunately the temperature fluctuations could be kept small enough that the largest differences between Boltzman factors would change only a percent or two at the most during any exposure. Thus any errors which were introduced were small compared with those from other sources.

In order to equalize density of exposure at the two ends of some plates, it was found best to expose one half of the plate longer than the other. Thus some trouble both from boiling away and from temperature variation was introduced. In taking these exposures it was customary to make the longer exposure first one time, and the shorter one first the next time. By comparing results from one exposure to the next, it was then possible to observe these effects for there would be a systematic difference in the gf-values obtained for the two ends of the plate. If the vapor pressure was appreciably greater at the beginning of the exposure than at the end, the gf-values of the lines exposed for the shorter period of time would be relatively greater when this short exposure was made concurrent with the beginning of the longer one than when it was made at the end with the longer one. One series of exposures represented an extreme case in which the short exposure was for 4 minutes and the long one for 15 minutes, at a temperature of 2500°C. It was found that gf-values obtained for lines exposed for the 4 minute period, relative to those for lines exposed for 15 minutes, were on the average about 6% greater when the 4 minute exposure coincided with the first portion of the longer exposure than when it coincided with the last portion.

The effect of temperature fluctuation was evident on exposures taken at temperatures well below the melting point of chromium for under these conditions the vapor pressure would be a function of the temperature rather than of time. The exposures taken to compare multiplets (1) and (4) were made at about 1250°C. with the shorter wave length region exposed for 8 minutes and the longer one for only 3 minutes. On one of these exposures the mean temperature of the longer portion was 1 degree below that of the shorter one, while on a succeeding exposure it was less than one degree above that of the shorter one. Even this small difference in conditions produced systematic differences in the ratios obtained between the two multiplets. While this difference was small in this particular instance, averaging less than 3%, it showed how seriously small temperature fluctuations can affect the value of N and therefore, under some conditions, the gf-values obtained by considering N as constant.

From the comparisons which have just been described and many others of a similar kind, it was concluded that if the temperature did not change seriously during an exposure, the errors introduced by temperature fluctuations and by the boiling away of the vapor would probably be considerably less than the errors due to measurement, provided that comparative values were obtained from lines exposed for the same length of time. More or less serious error might exist in comparisons made between lines on different portions of "split" exposures where one had been given more exposure time than the other.

It was possible to use filters to equalize density, and while these usually increased total exposure time, enough plates were exposed using these filters to provide a check on the others so that no lines having differing exposure times were compared unless the comparison could be checked on exposures where the time was the same for both lines. Once a sufficient number of reference lines had been established, each portion of a "split" exposure could be analyzed just as if it had been a separate exposure, and comparisons made with reference lines which appeared on that portion of the exposure.

5. Microphotometer Errors

If the microphotometer is in good adjustment, and calibration plates are traced with exactly the same instrumental characteristics as used for the plates exposed to the furnace spectrum, instrumental errors should be negligible. It was found, however, that the electrical circuit seemed to take a long time to reach an equilibrium condition where there would no longer be a drift of the zero (dark line) position, or a change in the damping of the galvanometer. Extraneous stray vibrations whose source no one could seem to locate often gave trouble also.

The drift in the zero position could be corrected by taking several dark and clear line marks on each trace. However, at times the zero position shifted suddenly, and such shifts caused seriously faulty results. If the galvanometer was sluggish this affected the form of the line profile, usually giving a skewed form to the line. Such a condition, if large enough to be noticeable, could be corrected by adjusting the damping shunt, but there is evidence that some profiles were spoiled from this cause. An underdamped galvanometer tended to make the profile too deep but also gave a high shoulder at one of the wings so this situation seldom passed unnoticed.

As mentioned before, about one profile out of every five was faulty, either because of the grain irregularities or from the microphotometer difficulties described above.

The most serious microphotometer error, however, was introduced when the instrument was slightly out of focus, for it was found that profiles taken with poor focus had areas which were too small by factors as high as 50%. A slight warp in the plates sometimes made refocusing on each profile necessary, and in some cases sharp focus was lost during the tracing of a single profile. Fortunately it was easy to detect bad focus, and once the importance of correct focus was realized better quality profiles were obtained. It was usually necessary to re-check both the dark and clear line positions after re-focusing.

6. Imperfect Line Profiles

A few profiles were faulty due to causes which were not instrumental. Some lines were blended, either with other chromium lines, or with lines of some impurity. Usually but little could be done to resolve such blends, though more careful handling of the chromium or use of purer metal might have removed the impurities sufficiently to make possible the measurement of the chromium lines which were usually blended with lines of these impurities. Some of the blends were between chromium lines originating in different levels, and of course these would have different relative strengths on exposures taken at different temperatures. Some blends were found to be quite well resolved at some temperatures whereas at others the component lines could not be distinguished. Where two lines originating from the same level were so closely blended that they could not be resolved, the blended line was measured, usually with its own base width rather than with the mean base width for the blended lines were usually found to be wider than the others.

A few lines were found to be blended with much stronger lines, and if they were measurable at all, the treatment suggested by Thackeray[33] was used. The validity of his method was strikingly confirmed in the case of two lines, λ5297.976 and λ5298.44 in multiplet (94), which have lower excitation potentials of 2.89 volts, and λ5298.269 in multiplet (18) having a lower e.p. of 0.98 volts. The two lines with the higher excitation potential were very weak at low temperatures, and λ 5298 was narrow enough at these temperatures that there was no appreciable blending. Therefore good quality determinations of gf-values could be made. At a temperature of 2710°C., however, the higher level lines were found well blended with the wings of λ5298. By following Thackeray's suggestions for the measurement of such blended lines, the results were found to check satisfactorily with those obtained at lower temperatures, whereas measurements referred directly to the continuum gave values which were about 50% too high for these blended lines.

Lines in the vicinity of the C_2 and CN bands were generally more irregular in profile than those outside of these band regions. This is probably because weak band lines are present, and if this be true, these irregularities will have similar positions and shapes in all exposures which are made under conditions conducive to the formation of the band lines. Where these irregularities did not appear to be too serious, mean base widths obtained from outside of the band regions were used with maximum depth to obtain the area of the line. Fortunately many of the lines in the band regions appeared at temperatures low enough that the bands caused no difficulty. Most of the high level lines in these regions, however, could not be measured because of these band irregularities, and probably their measurement will only be possible when special techniques are used to eliminate the bands at the high temperatures required to excite these lines.

But little difficulty was encountered from re-emission in the lines. A few of the stronger lines showed some evidence of re-emission on some exposures, but its effect could be minimized by using the trapezoidal marking which has already been described. The area affected by this re-emission is then only the small triangle at the tip of the line, and is usually a very small portion of the total area.

VIII. SUGGESTIONS FOR FUTURE WORK

A. More Complete Study of the Spectrum

This study is, as can be seen, not at all complete. Those lines having wave lengths less than λ3800 have not been measured on enough exposures to give reliable results. Good reference lines must be established in this region and the stronger and weaker lines need to be measured, for in the survey of this region made here only the lines of intermediate strength were considered.

Many of the weaker lines, particularly those originating in the higher levels, that are listed in Table 5 can be measured more satisfactorily if higher temperatures are used, and there seems to be no reason that such temperatures cannot be obtained. It should be possible, therefore, to complete a study of all lines with lower excitation potentials up to about 3.00 volts and stronger lines with excitation potentials as high as 3.45 volts, by applying the same techniques as used in this project, following suggestions for improving the characteristics of the tubes as given in this report.

Measurements using the same techniques can also be extended without apparent difficulty to the far red and infra-red regions. It has already been shown that the most difficult jumps can be bridged by using the first order of the 15' spectrograph, and by using moderately high temperatures, enough of the lines in the regions beyond λ7700 can be brought out to make measurements to at least λ10000 quite possible. When the astrophysical importance of this region is compared with that of the violet, an extension of measurements into the infra-red should probably be given priority over further study of the violet and ultra-violet regions.

Of course, special techniques may be applied to give measurements for the few lines usually masked in the CN and C_2 bands, but whether the results are worth the effort seems questionable.

B. Application of Results

These measured line strengths and relative f- and gf-values cover a very wide range of intensities and so should be useful for constructing curves of growth for solar and stellar spectra. It may in some cases be possible to extend these curves from the linear portion well into the square root region and thus to obtain definite information of the damping constants which cannot be done by fitting two or more portions of curves together as has heretofore been necessary with data for other elements.

A more thorough analysis of the comparison between observed and theoretical f-values should be made, for several of the multiplets have been measured accurately enough so that such comparisons should reveal any systematic differences that may exist.

C. Use of Other Techniques

It has still not been shown to be impossible to adapt the photocell method to absorption studies. With the use of a lamp such as the large model Schulz lamp, and the application of a suitable optical or electrical feedback to minimize effects of lamp noise, it seems that most of the difficulties encountered by Carter[20] might be overcome. By using two photocells arranged so that two lines could be studied simultaneously, much of the tedious work of this type of study could be eliminated. Whether such a technique could be applied to the study of chromium might be questionable, however, because of the rapidity with which chromium boils away at temperatures required for study of most of the lines. A further study of the boiling away process itself might reveal information of value, particularly with regard to the behavior of the lines as they move down the curve of growth.

Most of the transitions to the lower levels have been analyzed with some thoroughness in this project, and so there is no reason that future work might not be done in emission instead of in absorption. Comparative advantages of the two methods are discussed by Carter[20] and will not be repeated here, except to point out that emission studies will probably give better results and involve less technical difficulties in the measurement of the numerous lines having lower excitation potentials of between 3.0 and 3.5 volts.

It is hoped that the quantitative analysis of the line strengths in the chromium spectrum which was begun in this project, will be continued until a complete analysis is available. In many respects, the data obtained for this spectrum should prove to be very useful and convenient for astrophysical purposes.

TABLE 5

RELATIVE gf- AND f-VALUES FOR LINES OF Cr I

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(1)	$a^7S-z^7P^o$3-4	4254.346	1000	1000	143	Ar	
	3-3	4274.803	800	788	112	Ar	
	3-2	4289.721	700	566	81.0	Ar	
(2)	$a^7S-z^5P^o$3-3	3732.032	50	21.9	3.13	D	
	3-2	3730.807	40	17.0	2.43	D	
(3)	$a^7S-z^7D^o$3-4	3615.645	30	6.40	0.915	D	
	3-3	3635.281	10	3.34	0.477	D	
(4)	$a^7S-y^7P^o$3-4	3578.687	1000	3322	476	B	
	3-3	3593.488	900	2601	373	B	
	3-2	3605.333	750	1986	284	B	
(5)	$a^7S-y^5P^o$3-3	3351.966	12	20.3	2.90	D	
	3-2	3379.171	15	7.00	1.00	D	
(6)	$a^5S-z^7P^o$2-3	6330.101	40	2.29	0.458	B	
	2-2	6362.874	30	1.57	0.314	B	
(7)	$a^5S-z^5P^o$2-3	5208.436	500	3955	790	Ar	
	2-2	5206.039	300	2815	563	Ar	
	2-1	5204.518	200	1692	338	Ar	
(8)	$a^5S-z^7D^o$2-3	5021.903	25	2.88	0.575	Br	
	2-2	5051.900	40	8.34	1.67	Ar	
	2-1	5072.920	60	12.4	2.48	Ar	
(9)	$a^5S-y^7P^o$2-3	4942.495	200	27.2	5.44	Ar	
	2-2	4964.928	100	18.0	3.60	Ar	
(10)	$a^5S-y^5P^o$2-3	4496.862	100	342	68.4	Ar	
	2-2	4545.956	50	224	44.8	Ar	
	2-1	4580.056	40	116	23.2	Ar	
(11)	$a^5S-z^5D^o$2-3	3833.49	4	12.5	2.50	D	
	2-2	3852.58	15	9.11	1.82	D	
(13)	$a^5S-z^3D^o$2-3	3192.12	5	69.0	13.8	C	
	2-2	3210.62	2	16.6	3.32	D	

TABLE 5 - Continued

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(16)*	$a^5D-z^7P^o$3-3	6537.921	20	1.05	0.150	C	
	2-2	6501.212	15	0.800	0.160	C	
	4-3	6630.015	25	2.08	0.231	C	
	3-2	6572.900	15	0.665	0.095	C	
(18)	$a^5D-z^5P^o$4-3	5409.791	500	775	86.1	Br	
	3-2	5345.807	500	410.7	58.8	Ar	
	2-1	5296.686	100	165.8	33.6	Ar	
	3-3	5348.319	350	272.0	38.8	Ar	
	2-2	5298.269	100	268.9	53.8	Ar	
	1-1	5264.152	200	218.5	73.1	Ar	
	2-3	5300.749	75	34.64	6.94	Ar	
	1-2	5265.722	100	75.25	24.4	Ar	
	0-1	5247.564	150	97.61	97.6	Ar	
(19)*	$a^5D-z^7D^o$4-5	5122.121	30	5.75	0.639	Br	
	3-4	5112.490	25	3.37	0.481	C	
	4-4	5168.63	8	1.03	0.115	M	
	3-3	5151.83	12	2.65	0.379	D	
	2-2	5138.71	10	2.10	0.420	D	
	3-2	5183.41	10	4.93	0.706	M	
(20)*	$a^5D-y^7P^o$3-3	5068.290	35	5.04	0.720	B	
	2-2	5048.752	25	4.96	0.992	B	
	4-3	5123.465	35	7.96	0.885	Ar	
	3-2	5091.890	30	4.47	0.640	Br	
	2-3	5025.54	10	2.50	0.500	D	
	1-2	5019.20	20	2.23	0.745	C	
(21)	$a^5D-y^5P^o$4-3	4646.174	100	771	80.7	Ar	
	3-2	4652.158	100	623	89.1	Ar	
	2-1	4651.285	75	214	42.8	Ar	
	3-3	4600.752	75	306	43.7	Ar	
	2-2	4616.137	75	272	54.4	Ar	
	1-1	4626.188	65	278	92.6	Ar	
	2-3	4565.512	50	49.0	9.80	Ar	
	1-2	4591.394	60	118	39.4	Ar	
	0-1	4613.373	60	119	119	Ar	
(22)	$a^5D-z^5F^o$4-5	4351.770	100	1932	215	Ar	
	3-4	4344.507	100	864	123	Ar	
	2-3	4339.450	75	706	141	Ar	
	1-2	4337.566	75	368	123	Ar	
	0-1	4339.718	60	214	214	Ar	
	4-4	4384.977	75	218	24.2	Ar	
	3-3	4371.279	75	653	93.4	Ar	
	2-2	4359.631	75	300	60.0	Ar	
	1-1	4351.051	75	215	71.8	Ar	

TABLE 5 - Continued

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(22)	$a^5D-z^5F^o$4-3	4412.250	40	32.9	3.65	Ar	
	(cont.) 3-2	4391.753	40	45.7	6.54	Br	
	2-1	4373.254	35	52.7	11.6	Br	
(23)*	$a^5D-z^5D^o$4-4	3919.159	100	670	74.5	D	
	3-3	3908.755	100	444	63.5	D	
	2-2	3902.915	50	342	68.4	C	
	1-1	3903.164	25	103	34.4	C	
	4-3	3941.490	60	352	39.2	B	
	3-2	3928.636	60	468	68.0	D	
	2-1	3921.022	50	318	63.6	D	
	1-0	3916.243	25	183	61.0	C	
	3-4	3886.789	50	390	55.7	C	
	1-2	3885.218	40	274	91.4	D	
	0-1	3894.035	40	124	124	C	
(24)	$a^5D-z^3P^o$3-2	3831.032	12	56.5	8.07	B	
	2-1	3849.534	40	126	25.2	C	
	1-0	3852.218	30	118	39.3	C	
	2-2	3806.55	5	26.9	5.38	D	
	1-1	3832.32	5	16.9	5.64	C	
	1-2	3789.723	15	68.5	22.8	C	
	0-1	3823.522	12	66.5	66.5	B	
(25)*	$a^5D-z^3D^o$4-3	3266.634	7	68.5	7.61	B	
	2-1	3259.60	1	18.8	3.76	C	
	3-3	3244.115	7	90.5	12.92	B	
	2-2	3245.485	10	95.0	19.0	B	
	1-1	3247.274	4	70.0	23.4	B	
	2-3	3226.55	4	36.5	7.10	C	
	1-2	3233.234	7	44.0	14.7	B	
	0-1	3240.951	6	51.5	51.5	B	
(30)	$a^5G-x^5P^o$4-3	4872.02	18	425	47.2	D	
	3-2	4885.776	75	2480	354	D	
(31)*	$a^5G-y^5F^o$6-5	4789.354	75	4460	343	Br	
	5-4	†4829.376	100	3680	351	Br	1
	4-3	4861.842	75	1510	168	B	
	3-2	4888.530	40	1220	175	B	
	2-1	4903.239	70	1400	280	B	
	5-5	†4790.337	20	745	67.6	C	2
	4-4	†4829.376	100	3680	409	Br	1
	3-3	4861.205	35	725	104	D	
	2-2	4887.73	25	345	69.0	M	
	4-5	†4790.337	20	745	82.7	C	2

TABLE 5 - Continued

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(32)*	$a^5G-z^5H^o$6-7	4571.676	40	6450	495	C	
	5-6	4601.021	30	4870	443	D	
	4-5	4621.893	45	1630	181	D	
	3-4	4637.182	40	1930	276	C	
	2-3	4648.126	25	3020	607	D	
	6-6	4600.104	40	1970	152	D	
	5-5	†4621.963	45	3500	318	D	3
	4-4	†4637.772	40	2190	244	D	4
	3-3	4648.868	35	1840	263	D	
	5-4	†4637.772	40	2190	198	D	4
	4-3	4649.461	45	690	76.7	M	
(33)	$a^5G-z^5G^o$6-6	4526.466	75	20600	1580	C	
	5-5	†4530.755	100	17900	1630	D	5
	4-4	†4535.721	60	14600	1620	D	6
	3-3	4540.502	50	9450	1350	D	
	2-2	4544.619	50	10400	2080	D	
	6-5	4529.851	25	1820	140	D	
	5-4	4535.721	60	14600	1330	D	
	4-3	4541.071	30	3080	342	C	
	3-2	4545.335	25	2520	360	D	
	5-6	†4527.339	40	3860	351	D	7
	4-5	†4530.739	100	17900	1990	D	5,8
	3-4	4535.146	35	3640	520	C	
	2-3	4539.788	30	3510	703	D	
(34)*	$a^5G-x^5D^o$3-2	4561.20	2	1600	228	M	
(35)*	$a^5G-y^5G^o$6-6	4126.521	20	7550	580	Br	
	5-5	†4153.816	25	6890	625	B	9
	4-4	†4163.625	20	1220	136	C	10
	3-3	4191.271	25	4090	585	B	
	2-2	4203.590	18	2760	551	C	
	6-5	4153.067	9	1740	134	C	
	5-4	†4163.625	20	1220	111	C	10
	4-3	4191.750	10	1450	161	C	
	3-2	4204.19	8	963	137	C	
	5-6	4127.302	7	380	34.6	M	
	4-5	†4135.816	25	6890	766	B	9
	2-3	4190.66	4	642	128	C	
(36)*	$a^5G-x^5F^o$6-5	4033.263	6	2400	185	D	
	5-4	†4037.294	10	2260	206	D	11
	3-2	4046.760	6	1350	193	M	
	5-5	†4033.95	3	655	59.5	M	12
	4-4	†4037.294	10	2260	252	D	11
	4-5	†4033.95	3	655	72.8	M	12

TABLE 5 - Continued

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(37)	a^{5}G-z^{3}H$^\circ$5-6	4027.103	20	3770	343	B	
	4-5	†4026.166	18	5260	585	B	13
	3-4	4025.012	15	3760	536	B	
	5-5	†4026.166	18	5260	479	B	13
	4-4	†4025.44	5	500	55.6	M	14
	5-4	†4025.44	5	500	46.5	M	14
(38)	a^{5}G-y^{5}H$^\circ$6-7	3963.690	100	91500	7040	Br	
	5-6	3969.748	70	67000	6090	Br	
	4-5	†3976.665	100	62100	6900	B	15
	3-4	3983.907	100	47700	6810	Br	
	2-3	3991.123	80	40600	8120	Br	
	6-6	3969.061	18	9610	740	Br	
	5-5	†3976.665	100	62100	5650	B	15
	4-4	†3984.338	25	12600	1400	B	16
	3-3	3991.673	25	10900	1560	Br	
	6-5	3976.01	5	595	45.7	M	
	5-4	†3984.338	25	12600	1140	B	16
	4-3	3992.11	4	467	51.9	M	
(40)*	a^{5}G-w^{5}F$^\circ$5-4	†3816.173	20	4680	425	D	17
	5-5	†3818.481	25	5900	536	C	18
	4-4	†3816.173	20	4680	520	D	17
	4-5	†3818.481	25	5900	656	C	18
(43)	a^{5}G-x^{5}G$^\circ$6-6	3743.884	50	84300	6480	C	
	5-5	†3743.578	45	71700	6510	C	19
	4-4	†3748.998	50	55700	6200	C	20
	3-3	3757.662	50	23800	3400	C	
	2-2	3768.240	35	25500	5100	C	
	6-5	3742.968	18	10600	815	C	
	5-4	†3748.998	50	55700	5240	C	20
	4-3	3758.044	15	11000	1220	D	
	3-2	3768.734	15	11100	1580	D	
	5-6	3744.490	18	7500	681	C	
	4-5	†3743.578	45	71700	7970	C	19
	3-4	3748.614	12	13700	1960	C	
	2-3	3757.174	18	6700	1340	C	
(46)*	a^{5}G-v^{5}D$^\circ$5-4	†3656.261	50	65400	5940	C	21
	4-3	3663.206	40	18800	2090	C	
	3-2	3666.642	25	12000	1830	D	
	4-4	†3656.261	50	65400	7150	C	21
	2-2	3666.19	8	7450	1490	D	

TABLE 5 - Continued

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(47)*	$a^5G-u^5F^o$6-5	3639.802	100	86600	6660	B	
	5-4	†3636.590	50	41400	3760	B	22
	4-3	3641.830	50	42600	4720	B	
	3-2	3648.997	50	18800	2670	D	
	2-1	3653.912	45	34500	6900	C	
	5-5	†3640.39	20	24000	2180	B	23
	4-4	†3636.590	50	41400	4590	B	22
	3-3	3641.470	15	17800	2540	B	
	2-2	3648.534	15	9500	1900	C	
	4-5	†3640.39	20	24000	2650	B	23
	2-3	3641.01	3	6600	1320	M	
(48)*	$a^5G-x^5H^o$5-6	3646.161	12	8350	759	D	
(49)*	$a^5G-u^5F^o$5-4	†3632.839	40	20500	1860	B	24
	3-2	3609.479	18	11300	1610	B	
	2-1	3610.052	10	9270	1850	B	
	4-4	†3632.839	40	20500	2280	B	24
(51)*	$a^5G-w^3G^o$5-4	†3455.602	35	17800	1620	B	25
	4-3	3465.57	5	2150	239	M	
	5-5	†3445.618	40	63600	5780	B	26
	4-4	†3455.602	35	17800	1980	B	25
	3-3	3465.245	15	6700	958	M	
	4-5	†3445.618	40	63600	7080	B	26
	3-4	3455.281	10	3650	521	C	
(52)*	$a^5G-w^5G^o$6-6	3433.598	50	84200	6470	B	
	5-5	†3436.187	50	44100	4100	B	27
	4-4	†3441.439	35	34400	3820	B	28
	3-3	3447.430	30	24100	3450	B	
	2-2	3453.328	35	25800	5160	B	
	6-5	3435.679	10	5940	466	C	
	5-4	†3441.439	35	34400	3120	B	28
	4-3	3447.760	20	11000	1220	B	
	3-2	3453.743	12	3760	538	C	
	5-6	3434.112	20	8760	796	C	
	4-5	†3436.187	50	44100	4900	B	27
	2-3	3447.015	12	7810	1560	B	
(53)*	$a^5G-x^3F^o$5-4	†3435.819	6	4850	440	C	29
	4-4	†3435.819	6	4850	539	C	29
	3-3	3431.995	7	4210	602	M	
	2-2	3431.284	10	4800	960	M	
	2-3	3431.59	3	3920	789	M	
(54)*	$a^5G-t^5F^o$4-3	3379.825	8	15500	1720	D	
	2-2	3384.24	3	8700	1740	D	

TABLE 5 - Continued

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(58)*	a^{5}P-y^{5}F^{o}3-2	5225.821	50	1400	200	D	
	1-2	5230.228	40	660	220	D	
(59)*	a^{5}P-x^{5}P^{o}3-2	5222.676	30	752	108	M	
	2-1	5238.971	65	1340	268	D	
(60)*	a^{5}P-y^{5}D^{o}3-4	5013.316	100	2730	390	C	
	2-3	5067.714	75	1720	344	D	
	3-3	5065.910	50	91.0	1.30	M	
	2-2	5110.751	40	2300	460	D	
	1-1	5144.672	50	807	269	M	
(62)	a^{5}P-z^{5}S^{o}3-2	4697.062	40	2700	385	D	30
	2-2	†4698.615	50	4200	840	D	31
	1-2	4700.608	40	2530	843	C	
(63)*	a^{5}P-w^{5}P^{o}3-3	4459.34	18w	1790	256	M	
	3-2	4473.782	40	2860	407	D	
	2-5	4460.769	18	3420	684	M	
	1-2	4477.02	35w	1980	660	M	
(64)*	a^{5}P-v^{5}P^{o}3-3	4295.757	25	11900	1700	B	
	1-1	4382.853	20	1090	363	M	
	3-2	4340.130	18	5760	822	B	
	2-1	4381.112	35	4750	950	D	
	2-3	4297.050	15	2960	592	B	
	1-2	4343.163	18	2100	700	C	
(65)*	a^{5}P-y^{3}D^{o}3-3	4108.400	6	3240	462	D	
	2-2	4120.613	12	6510	1320	B	
	1-1	4127.643	8	4360	1453	B	
	2-1	4126.099	6	1800	360	M	
	2-3	4109.584	8	2950	590	B	
	1-2	4122.162	8	1670	557	C	
(66)*	a^{5}P-w^{5}D^{o}3-4	4066.938	6	6500	928	D	
	2-3	4077.089	12	3540	708	D	
	2-2	4080.221	5	3900	780	D	
(67)	a^{5}P-u^{5}P^{o}3-3	3992.845	30	16300	2330	Br	
	2-2	3979.798	10	4060	812	B	
	1-1	3972.688	7	1800	600	M	
	3-2	3978.677	18	6750	963	B	
	2-1	3971.255	20	6750	1350	B	
	2-3	3993.968	15	1370	274	B	
	1-2	3981.233	15	7030	234	B	

TABLE 5 - Continued

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(69)*	$a^5P-v^5D^o$3-4	3841.277	50	72600	10400	C	
	2-3	3850.042	50	49900	9980	D	
	3-3	3848.983	40	37800	5360	C	
	2-2	3854.220	50	26900	5380	D	
	1-1	3857.631	20	19000	6330	D	
	1-0	3855.286	12	11000	3670	D	
(70)*	$a^5P-u^5F^o$3-4	3819.564	40	6400	913	D	
	2-3	3826.425	40	12900	2580	D	
	1-2	3836.070	12	9040	3013	M	
	2-2	3834.735	15	9000	1800	M	
(71)*	$a^5P-u^5F^o$3-4	3815.433	30	21000	3000	D	
(74)*	$a^5P-t^5P^o$3-3	3574.039	15	22600	3230	D	
	2-2	3602.574	12	11100	2220	C	
	3-2	3601.666	40	47600	6810	B	
	2-1	†3603.745	12	14200	2840	C	32
	1-2	†3603.745	12	14200	4730	C	32
(75)	$a^5P-y^5S^o$3-2	3572.748	12	10700	1530	D	
	2-2	3573.643	18	19400	3880	D	
	1-2	3574.805	12	46000	15300	D	
(81)	$a^3P-y^3P^o$2-2	4619.551	40	10300	2060	D	
	1-1	4501.788	25	4400	1470	C	
	2-1	4622.761	25	4940	988	C	
	1-0	4501.112	35	6550	2180	C	
	1-2	4598.730	35	6830	3380	C	
	0-1	4432.175	40	5870	5870	B	
(82)	$a^3P-y^3D^o$2-3	†4527.339	40	3860	792	D	33
	1-2	4424.075	10	2500	833	M	
	0-1	4362.95	7	1500	1500	M	
(83)	$a^3P-w^5D^o$2-2	4491.858	35	6760	1520	M	
	1-2	4377.549	30	6600	2200	D	
	0-1	4321.238	20	4640	4640	D	
(84)*	$a^3P-u^5P^o$2-3	4387.380	10	3660	732	M	
(93)	$z^7P^o-e^7S$ 4-3	7462.37	100	2720	302	C	
	3-3	7400.23	150	1880	269	C	
	2-3	7355.94	200	2100	420	C	

TABLE 5 - Continued

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(94)	$z^7P^o-e^7D$ 4-5	5328.339	200w	22400	2490	Ar	
	3-4	5297.360	60w	15400	2200	Ar	
	2-3	5275.171	75w	7150	1430	B	
	4-4	5329.12	65w	10500	1170	B	
	3-3	5297.976	40w	11800	1690	Ar	
	2-2	5275.689	50w	9640	1930	B	
	4-3	5329.719	25w	3290	366	B	
	3-2	5298.44		4800	686	C	
	2-1	5276.03	75w	8700	1740	B	
(95)	$z^7P^o-f^7S$ 4-3	4514.531	40	8270	920	C	
	3-3	4491.678	30	6520	931	D	
	2-3	4475.345	50	6140	1230	C	
(96)	$z^7P^o-f^7D$ 4-5	4261.354	25	9650	1070	C	
	3-4	4272.910	12	5780	827	C	
	2-3	4284.725	12	6260	1250	D	
	4-4	4293.565	20	3360	373	D	
	3-3	4299.718	20	5160	739	C	
	2-2	4305.453	30	4350	870	M	
	4-3	†4320.592	30	1940	216	M	34
	3-2	†4320.592	30	1940	277	M	34
	2-1	4319.641	40	3000	600	M	
(97)*	z^7P-g^7D 4-5	4129.21	(20)	31200	3470	B	
	3-4	4110.87	(8)	13000	1860	B	
	2-3	4097.65	(5)	5570	1110	C	
	4-4	4129.96		10900	1210	B	
	3-3	4111.36	(6)	11000	1570	B	
	2-2	4097.96	(7)	7950	1590	B	
	3-2	4111.67	(3)	7200	1030	C	
	2-1	4098.18	(7)	7130	1430	C	
(99)*	$a^3H-z^3H^o$6-6	4727.153	40	6400	492	D	
	5-5	4693.949	45	7280	661	D	
	4-4	4666.215	25	12700	1410	D	35
	4-5	4667.181	30	4560	517	M	
	5-5	4693.949	45	6500	591	D	
(100)*	$a^3H-y^3F^o$5-4	4543.74	20	10600	964	D	
(102)*	$a^3H-w^5F^o$6-5	4442.268	30	7400	570	M	
	5-4	4410.967	25	5250	477	M	
(103)	$a^3H-z^3G^o$6-5	4387.496	30	8900	685	M	
	5-4	4375.333	30	10200	928	D	
	4-3	4363.134	12	11000	1220	M	

TABLE 5 - Continued

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(104)	$a^{3}H-y^{3}G^{o}6-5$	4374.158	40	21600	1660	D	
	5-4	4346.833	30	12300	1120	C	
	4-3	4325.075	40	12100	1340	C	
(105)*	$a^{3}H-z^{3}I^{o}6-7$	4255.502	25	9300	716	C	
	5-6	†4240.705	30	8350	759	C	36
(106)*	$a^{3}H-x^{5}H^{o}6-7$	4175.945	15	24600	1890	D	
(108)*	$a^{3}H-y^{3}H^{o}6-6$	4123.387	10	8370	644	B	
	5-5	4121.817	10	6000	545	M	
	4-4	4104.867	10	5140	670	C	
	5-6	4099.016	6	4100	372	M	
	4-5	4101.163	8	4700	522	M	
(125)*	$b^{5}D-y^{3}D^{o}3-3$	4571.105	25	8550	1220	D	
(126)*	$b^{5}D-w^{5}D^{o}4-4$	4515.440	25	6850	761	C	
(127)*	$b^{5}D-w^{5}F^{o}4-5$	4458.538	45	13700	1520	C	
	2-3	4465.357	35	6600	1320	M	
(128)*	$b^{5}D-z^{3}G^{o}4-5$	4403.372	35	4850	539	M	
	3-4	4423.318	12	6200	886	M	
(129)*	$b^{5}D-u^{5}P^{o}4-3$	4424.281	40	26300	2920	D	
	3-2	4411.093	40	11000	1570	D	
	2-1	4399.823	30	6280	1260	M	
	3-3	4428.501	35	11400	1630	C	
	2-2	4410.304	40	10000	2000	D	
	1-1	4397.251	30	11600	3870	D	
(131)*	$b^{5}D-v^{5}D^{o}4-4$	4238.957	35	7950	884	C	
(132)*	$b^{5}D-u^{5}F^{o}4-5$	4217.626	30	17000	1880	C	
	3-4	4216.365	15	9400	1230	M	
	2-3	4222.732	20	8600	1720	D	
(133)*	$b^{5}D-u^{6}F^{o}3-4$	4211.349	15	6300	900	M	
(134)	$b^{5}D-w^{5}G^{o}4-5$	3945.968	10	5200	578	M	
(135)*	$b^{5}D-x^{3}F^{o}4-4$	3945.495	9	4500	500	M	
	3-3	3944.25	2	4120	588	M	
(136)*	$b^{5}D-t^{5}P^{o}4-3$	3915.843	40	2550	284	M	
	3-2	3952.399	15	10400	1490	B	
	2-1	3953.163	18	8250	1650	B	
	2-2	3951.765	8	5850	1170	C	

TABLE 5 - Continued

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(138)*	$b^5D-t^5F^o$4-5	3849.365	50	83500	9270	D	
(143)*	$a^3G-z^3H^o$5-6	4922.267	300	30800	2800	Br	
	4-5	4887.013	150	21600	2400	C	
	3-4	4870.796	100	18600	2660	C	
(145)*	$a^3G-y^3F^o$5-4	4756.113	100	54500	4950	C	
	4-3	4737.350	75	31000	3440	D	
	3-2	4730.711	50	24500	3500	D	
	4-4	4724.416	35	3310	368	D	
(146)*	$a^3G-y^3D^o$4-3	†4698.615	50	4600	510	D	37
(147)*	$a^3G-w^5D^o$4-3	4656.189	30	187	20.8	M	
(150)*	$a^3G-y^3G^o$5-5	4540.719	50	48100	4360	C	
	4-4	4511.903	60	22600	2510	C	
	3-3	4500.295	40	13500	1930	C	
(154)*	$a^3G-y^3H^o$5-6	4271.061	15	9400	855	M	
(155)*	$a^3G-x^3G^o$5-5	4209.756	15	6850	622	M	
(166)*	$a^3F-z^3H^o$4-5	4954.811	80	12700	1410	C	
	3-4	4936.334	150	12700	1810	D	
(171)*	$a^3F-w^5F^o$3-3	4632.180	35	12500	1780	D	
(179)*	$a^3F-x^3D^o$3-2	4179.257	30	15000	2140	D	
(186)*	$z^7F^o-f^7D$ 6-5	4718.429	75	48100	3700	D	
	5-4	4708.040	60	48000	4360	M	
	3-2	4689.374	65	22900	3260	D	
	2-1	4680.49	50	9900	1980	M	
	5-5	4669.336	50	10300	936	M	
	4-4	4666.512	55	16600	1850	D	
	3-3	4664.798	60	23100	3300	C	
	2-2	4663.832	55	21600	4320	D	
	1-1	4663.328	40	14800	4900	D	
	3-4	4633.286	30	8600	1230	M	
	1-2	4646.808	20	14100	4700	D	
	0-1	4654.736	35	10300	10300	D	
(191)*	$b^3P-y^3P^o$2-2	5400.608	50	6850	1370	M	
(193)*	$b^3P-w^5D^o$1-2	5214.127	30	8900	2970	M	

TABLE 5 - Continued

Mult. No.	Designation	λ	Arc Int.	gf	f	Wt.	Notes
(197)*	b^3P-y^3S^o2-1	4492.312	40	32000	6400	M	
(201)*	z^7D^o-f^7D 3-2	5254.918	100	74500	10600	M	
	2-1	5248.395	75	80200	16500	M	
	4-5	5177.430	75	12900	1430	M	
	3-4	5184.590	100	12800	1830	M	
	2-3	5192.00	100	13100	2620	M	
	1-2	5200.188	50	8560	2850	M	
(207)*	b^3G-y^3G^o5-5	†5196.443	100	24500	2230	M	38
	3-3	5139.654	100	10700	1530	M	
	5-4	†5196.57	100	24500	2230	M	38
(225)*	y^7P^o-f^7D 4-5	5255.132	125	22700	2520	D	
	2-1	5340.437	35	7630	1530	M	

NOTES TO TABLE 5

1. Blend of transitions (5-4) and (4-4). Values are for blend.
2. Blend of transitions (5-5) and (4-5). Values are for blend.
3. Blend with line in multiplet (244). All (32) at furnace temperatures.
4. Blend of transitions (4-4) and (5-4). Values are for blend.
5. Blend of transitions (5-5) and (4-5). Values are for blend.
6. Blend of transitions (4-4) and (5-4). Values are for blend.
7. Blend with line in multiplet (82).
8. Wave length is incorrectly given in RMT as 4530.688. The value given here is from MIT Tables[32].
9. Blend of transitions (5-5) and (4-5). Values are for blend.
10. Blend of transitions (4-4) and (5-4). Values are for blend.
11. Blend of transitions (5-4) and (4-4). Values are for blend.
12. Blend of transitions (5-5) and (4-5). Values are for blend.
13. Blend of transitions (4-5) and (5-5). Values are for blend.

Notes to Table 5 - Continued

14. Blend of transitions (4-4) and (5-4). Values are for blend.
15. Blend of transitions (4-5) and (5-5). Values are for blend.
16. Blend of transitions (4-4) and (5-4). Values are for blend.
17. Blend of transitions (5-4) and (4-4). Values are for blend.
18. Blend of transitions (5-5) and (4-5). Values are for blend.
19. Blend of transitions (5-5) and (4-5). Values are for blend.
20. Blend of transitions (4-4) and (5-4). Values are for blend.
21. Blend of transitions (5-4) and (4-4). Values are for blend.
22. Blend of transitions (5-4) and (4-4). Values are for blend.
23. Blend of transitions (5-5) and (4-5). Values are for blend.
24. Blend of transitions (5-4) and (4-4). Values are for blend.
25. Blend of transitions (5-4) and (4-4). Values are for blend.
26. Blend of transitions (5-5) and(4-5). Values are for blend.
27. Blend of transitions (5-5) and (4-5). Values are for blend.
28. Blend of transitions (4-4) and (5-4). Values are for blend.
29. Blend of transitions (5-4) and (4-4). Values are for blend.
30. This line is in band head so value is very uncertain.
31. Blend with line in multiplet (146).
32. Blend of transitions (2-1) and (1-2). Values are for blend.
33. Blend with line in multiplet (33).
34. Blend of transitions (4-3) and (3-2). Values are for blend.
35. This may be blended with an impurity line.
36. Blend with line in multiplet (178).
37. Blend with line in multiplet (62).
38. Blend of transitions (5-5) and (5-4). Values are for blend.

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