THUNDERSTORM MECHANICS

by

Gerald Levin

Meteorology Cadet United States Army Air Corps

In partial fulfillment of the requirements for the degree of Master of Science in Meteorology

California Institute of Technology
Pasadena, California

1941

ACKNOWLEDGEMENT

The author would like to acknowledge the kind assistance and cooperation of Mr. Herbert Shapiro, Assistant Engineer, United States Army Engineering Corps, in the preparation of this thesis.

TABLE OF CONTENTS

PART I

Theromdynamical Structure of the Thunderstorm

PART II

Vertical Velocities in Thunderstorms as Found from the Motions of Hailstones and Raindrops

TABLE OF CONTENTS

List of Diagrams

PART I

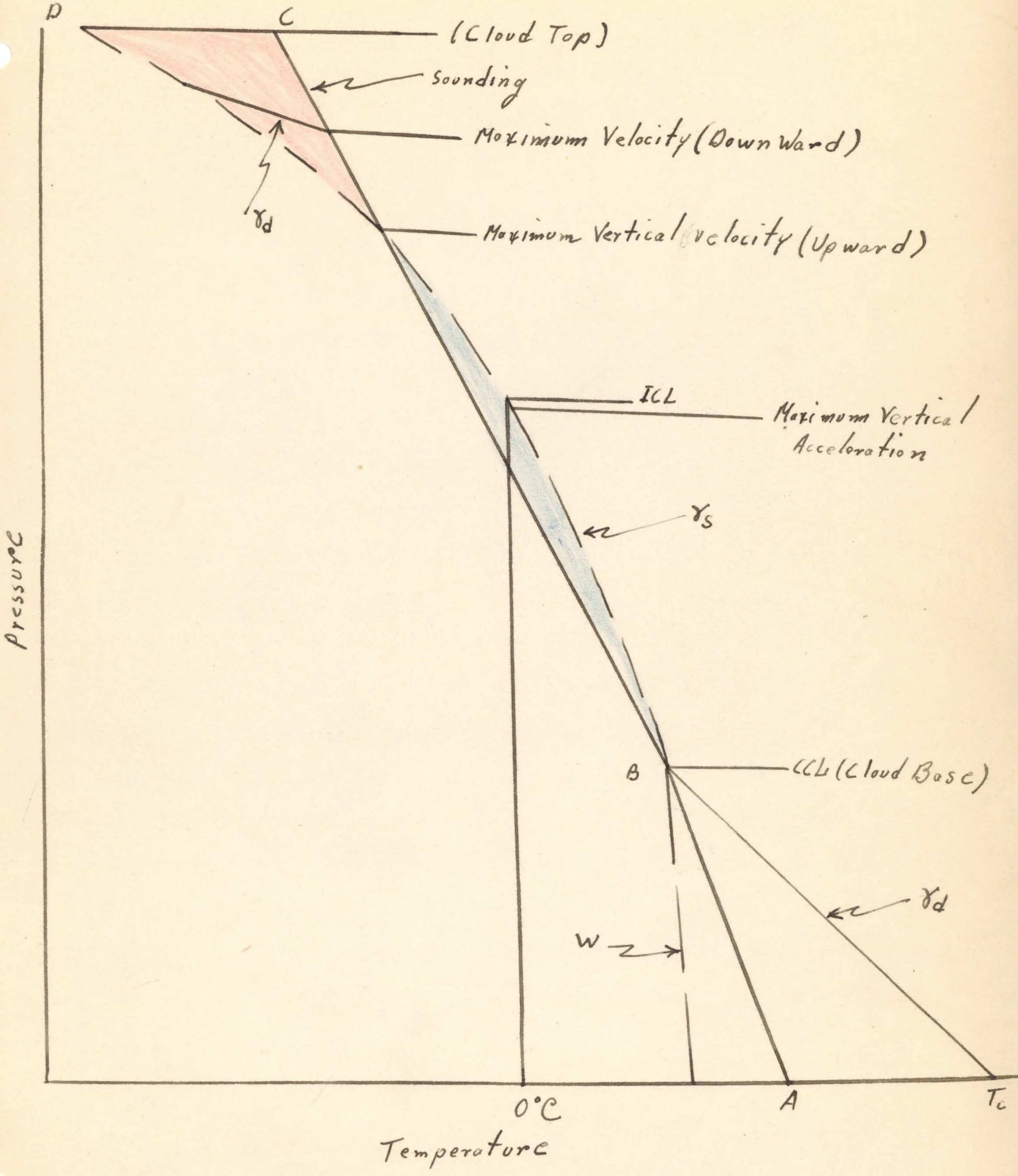
THERMODYNAMICAL STRUCTURE OF THE THUNDERSTORM

Thermodynamical Structure of the Convective Thunderstorm (General)

The pure air mass or convective thunderstorm is a direct result of insolational heating of the earth's surface. Its formation is most common in warm, moist air with a steep lapse rate; i.e., Tropical Gulf, Tropical Atlantic, or Polar Pacific air in spring. Although the convective thunderstorm occurs most often in summer due to the strong heating at this time of the year it may be found, but less frequently, at other times of the year.

If one refers to the diagram (1) on the thermodynamical structure of the thunderstorm, its formation can be easily followed. As the ground becomes heated the air layer near the ground becomes warmer partly due to conduction and partly to the absorption of the long terrestrial waves given off. This layer is then lighter than its surroundings and rises allowing a new and heavier layer to sink, which in turn is heated. If the ground continues to be heated the process goes on until the convectively mixed air reaches its saturation level. This level is called the convective condensation level (ccl). It is the base of the cloud and can be found as follows: Assuming that the specific humidity (w) at the ground remains constant and is the same at the ccl. we have merely to draw a constant (w) line till it intersects the original sounding curve to find the ccl. If (w) does not remain constant we must take an average value below the ccl. Convective mixing below the ccl. tends to establish a dry adiabatic lapse rate and therefore a particle arriving at the ccl. is in neutral equilibrium, that is, it has no tendency to either rise or fall.

Thermodynamical Structure of The Thunderstorm



W = gms. of H_2O vapor / Kgm. Dry Air

γ_s = Saturated Adiabatic

γ_d = Dry Adiabatic

Diagram (1)

The temperature that the ground must reach in order that the cloud formation may take place is called the Critical Temperature (Tc). Its position can be found by running a dry adaibetic from the ccl. to the ground. An answer as to whether the Tc. will be reached at a particular station under specified conditions can be found by referring to previously plotted diurnal temperature curves.

As soon as a cloud particle rises above the ccl. it follows a saturation adaibatic curve and is everywhere warmer than the surrounding air; this means that it is continually accelerated until the saturation adaibetic crosses the original sounding curve or the two temperatures, of the cloud particle and the surrounding air, are equal.

The point of maximum acceleration is where the temperature difference between the cloud particle and the surroundings is greatest. The largest vertical velocity is reached at the place where the saturation adaibetic along which the particle is ascending crosses the sounding curve, for the particle is continually accelerated to this point. If the particle's path lies to the right of the sounding curve, we call the area between the two curves the positive area. The greater this positive area is, the greater will be the maximum velocity reached by the ascending particle.

Above the point of maximum vertical velocity the ascent curve lies to the left of the sounding curve. The area between the two curves is then called negative area. The rising particle is everywhere colder than its surroundings and is therefore negatively accelerated. Our cloud particle's velocity thus decreases above the point of maximum vertical velocity.

At the cloud top vertical velocities are zero. We may roughly estimate the position of this top by saying that when the negative area equals the positive area the vertical velocity will be zero. The reason that this expression is not very accurate is that a small area at 1000 millibars (mbs.) is twice as effective in producing an acceleration as the same size area at 500 mbs. by the following:

From the equation of state $p = \rho R T$

p = pressure ρ = density R = gas constant

T = absolute temperature

At the 1000 mb. level let (1) denote the cloud particle and (2) the atmosphere.

$$\rho = \frac{p}{RT} \qquad \rho_2 - \rho_1 = \frac{p}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$ = density of particle density of air

At the 500 mb. level

$$\rho'_2 - \rho'_1 = \frac{p'}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

Since the lifting power is directly proportional to the density difference which is in turn directly proportional to the pressure we see that the rising particle will be accelerated twice as much at the 1000mb. level as at the 500 mb. level for the same positive area.

We can find the elevation of the maximum downward velocity of a cloud particle if we know how much liquid water is contained. If at the top of the cloud (w) the grams of water vapor per kilogram of air is equal to 0.5 and there is 0.5 grams of liquid water in a kilogram of air, the descending particle will follow a saturation adiabatic until the line of "W" equals 1 gm. intersects it. From this point on the descending particle follows a dry adaibetic, as it is no longer saturated, until the original sounding curve is intersected. This intersection is the point of maximum downward velocity.

Of especial interest is the position of the "Ice crystal level." This is the intersection of the line of $0^{\circ}C$ temperature and the ascent curve of the particle.

1. If the "ICL" lies well up near the top of the positive area and the maximum vertical velocity is greater than 8 meters per second the situation is well suited for a thunderstorm formation and no hail. This is true mainly because the rain drops are shattered before they reach the freezing level and if they do freeze there is not enough additional acceleration above the freezing level left to carry the frozen drops up and form hail.

2. If the "ICL" lies well down near the base of the positive area and there is a good deal of positive area above it, the conditions are favorable for hail and no thunderstorm, provided that the vertical velocity at the "ICL" is less than 8 meters per second. The rain drops are allowed to reach the freezing level without shattering and there is then plenty of area to carry the frozen drops aloft and form hail.

3. A combination of the two above cases is possible if the "ICC" lies near the center of a large positive area and the vertical velocity of the "ICL" is greater than 8 meters per second. In this situation it is possible to get a violent thunderstorm accompanied by heavy hail.

It has been found that an updraft of 8 meters per second is needed to support the large rain drops found in thunderstorms, and even greater velocities are needed to form hail. Also, rain drops of any size can not fall through air of normal density whose vertical velocity is 8 meters per second or more.

Calculation of Maximum Velocity in a Thunderstorm

For a rising cloud particle:

a = acceleration

ρ = density

g = gravity

p = pressure

T = absolute temperature

R = gas constant

The upward force on the cloud particle = $a\rho$

Buoyant force = $\rho_1 g - \rho g$ where ρ_1 = surrounding air density

These two forces are equal so: $a\rho = \rho_1 g - \rho g$

$$a = g\,\frac{\rho_1 - \rho}{\rho}$$

From gas law $p = \rho R T$ (for particle) $p_1 = \rho_1 R T_1$ (for surrounding air)

$$a = g\,\frac{\frac{p_1}{RT_1} - \frac{p}{RT}}{\frac{p}{RT}} \quad \text{but } p_1 \text{ (of outside air)} = p \text{ (of particle)}$$

$$a = g\,T\left(\frac{1}{T_1} - \frac{1}{T}\right) = g\,\frac{T - T_1}{T_1} = g\,\frac{\Delta T}{T_1}$$

The acceleration of the particle is therefore equal to the acceleration of gravity times the difference in temperature between the cloud and the surrounding environment divided by the temperature of the surroundings.

The above formula does not give an accurate value for the acceleration because it assumes the cloud to be a particle in a stationary atmosphere. In reality the surrounding atmosphere must be moving downward by the very nature of the upward movements in the thunderstorm. The downward motion along a dry adiabatic tends to heat the atmosphere and bring its temperature closer to the cloud's. The acceleration calculated for the cloud particle is too large because ΔT for a moving environment is smaller than the ΔT used.

We can find an approximate value for the maximum velocity in a thunderstorm by the following formula:

$$V = (2ah)^{\frac{1}{2}}$$

V = maximum velocity; a = average acceleration

h = height from base of cloud to level of maximum velocity

(See Diagram(1))

a at any point was found to be $g\frac{\Delta T}{T_1}$

If T_1' is the mean temperature of the surrounding air from the base of the cloud to the maximum velocity level, and $\Delta T'$ is the mean temperature difference between the cloud and its surroundings in this interval of height, the mean acceleration is

$$a = g\frac{\Delta T'}{T_1'}$$

The maximum velocity is then:

$$V = \left(2gh\frac{\Delta T'}{T_1'}\right)^{\frac{1}{2}}$$

This is an approximate method for finding the height that the cloud particles will rise above the level of maximum vertical velocity.

H = height cloud particles rise above maximum velocity level

ρ_1 = density of air that is rising

ρ_2 = density of surrounding air

T_1 = temperature of rising air

T_2 = temperature of surrounding air

R = gas constant

P = pressure

V = maximum velocity

ΔT = uniform change in temperature difference between rising air and surrounding air per centimeter of elevation.

g = acceleration of gravity

m = mass of air considered

The kinetic of mass (m) at the maximum velocity level is $\frac{1}{2} m V^2$

This kinetic energy must equal the work done against gravity in lifting the mass (m) a height of H

The weight of the mass (m) is effectively zero at the maximum velocity level as its density is the same as the surrounding air density.

At some other level its weight is greater due to the fact that the density of its environment is less.

Weight of mass (m) $= mg \frac{(\rho_1 - \rho_2)}{\rho_1}$ (at any level)

From gas law $\rho = \frac{p}{RT}$ $\quad \frac{\rho_2 - \rho_1}{\rho_2} = \frac{T_2 - T_1}{T_2}$

Weight of mass (m) at same level is then: $mg \frac{T_2 - T_1}{T_2}$

Weight of mass (m) at level "H" centimeters above maximum velocity level $= \frac{mg \, \Delta T \times H}{T}$

T is an average absolute temperature in the interval "H"

Average weight of mass (m) $= \frac{mg \, \Delta T \times H}{2 \times T}$

Work done in lifting average weight ht. "H" $= \frac{mg \, \Delta T \times H^2}{2 \times T}$

Equating above work to the kinetic energy

$$\frac{1}{2} m V^2 = \frac{mg \, \Delta T \times H^2}{2T}$$

$$H^2 = \frac{V^2 T}{g \, \Delta T} \qquad H = \left(\frac{V^2 T}{g \, \Delta T}\right)^{\frac{1}{2}}$$

For our limits of error T and ΔT can be found accurately enough from the sounding on the adiabatic chart. V is expressed in centimeters per second. g is expressed in centimeters per second per second. ΔT is expressed in degrees absolute per centimeter of elevation. The value of "H" found from the above equation will be greater than the "H" actually attained because of the effect of viscosity which was neglected.

PART II

VERTICAL VELOCITIES IN THUNDERSTORMS AS FOUND FROM THE MOTIONS OF HAILSTONES AND RAINDROPS

Raindrops

The following table shows the size of different particles in the atmosphere.

Phase	Size or Diameter (mm)		
Rain	0.1	-	6.6
Clouds and Fog	0.004	-	0.1
Snow-crystals fraction mm.	0.5	-	1.5
Dust and Smoke	0.0003	-	0.0017

Various formulas have been developed to find the descending velocities of the above particles. Among the most practical are those of Stokes, Oseen, Goldstein, and Zahm. The difficulty is that these formulas are applicable to the smaller drops while in thunderstorms the larger drops can easily be supported. Zahm's formula which can be applied to larger drops then any of the other formulas is not accurate for drops larger than 3 mm., but raindrops do not break up until they are from 5 to 7 mm. in diameter.

The observations of Schmidt and Lenard furnish the best information as to the size and velocity of falling raindrops. Their observations are included here.

Schmidt's Observations

Diameter (cm.)	Velocity (cm/sec.)
0.04	180
0.06	270
0.08	340
0.10	393
0.20	577
0.30	692
0.35	740

Lenard's Observations

Diameter (cm.)	Velocity (cm./sec.)
0.10	440
0.20	590
0.30	690
0.35	737
0.45	805
0.546	798
0.636	780

A curve showing Lenard's and Schmidt's observations is included in diagram (2). In the same diagram is a plot of Zahm's calculations. It must be remembered that these results are not accurate for rain drops over 3 mm. in diameter.

Looking at Lenard's table it is seen that the velocity decreases after a raindrop reaches a diameter of 0.45 cm. This decrease in velocity is probably due to deformation of the rain drop and the presentation of a larger cross sectional area to the direction of motion. The deformation takes place in the range of "Reynold's Number" from 740 to 3,950 or in rain drops whose diameters very from 1.7 mm. to 6.6 mm.

The following formula which determines whether a drop descending through a viscous fluid is a prolate or oblate spheroid has been developed by Saito.

$$205(1+m) - 200 - 638m - 444m^2 - 3m^3$$

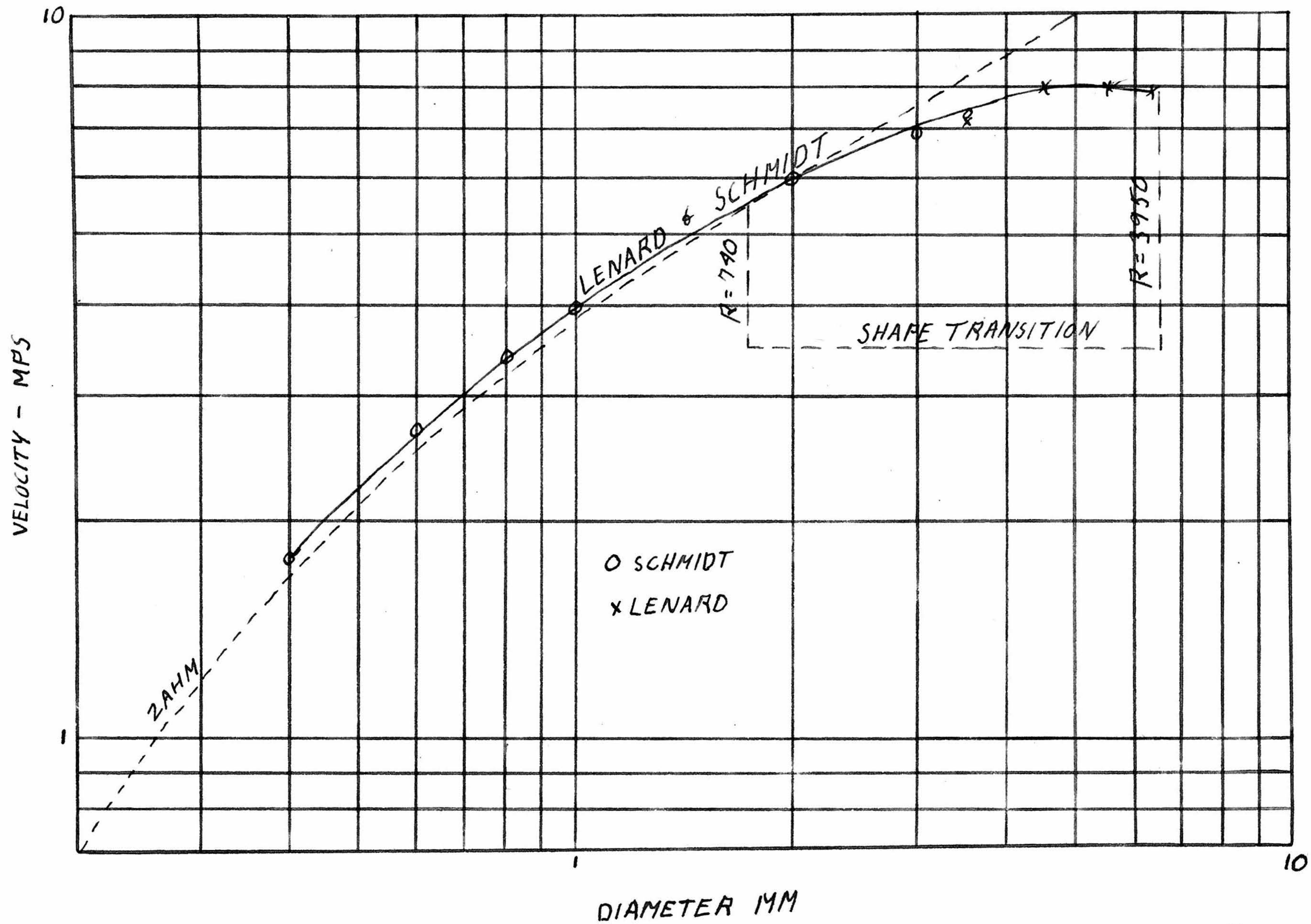
$$m = \frac{\mu_i}{\mu} \qquad S = \frac{\rho_i}{\rho}$$

μ_i = internal dynamic viscosity of the drop

ρ_i = internal density of the drop

μ = dynamic viscosity of the fluid

ρ = density of the fluid



10
1
VELOCITY - MPS
1
10
DIAMETER 1MM
LENARD & SCHMIDT
ZAHM
R=740
R=3950
SHAPE TRANSITION
O SCHMIDT
x LENARD

Diagram (2)

If the above expression is positive after substitution of correct values, the drop will assume the shape of a prolate spheroid. If the result comes out to be negative, the drop will have the form of an oblate spheroid. A rain drop falling through the atmosphere has been shown to take the form of an oblate spheroid, that is, a sphere flattened at both poles.

From diagram (2) the conclusions can be drawn that (1) raindrops can not fall through still air with a velocity greater than 8 meters per second, (2) rain drops can not fall through air whose upward velocity is 8 meters per second or more, (3) rain drops shatter when they reach a diameter of from 5 to 7 mm.

From an analysis of rain drops only it is known that velocities in thunderstorms exceed 8 meters per second. An idea of the larger velocities attained in thunderstorms can be found from the study of vertical currents needed to support hail stones. This will be taken up in the next section.

Hailstones

Development of formula to find the velocity of falling hailstones:

1. It is assumed for this development that the falling body is a smooth sphere.

2. It is also assumed that the resistance to motion is proportional to the relative velocity of the falling body.

v = relative velocity vo = initial velocity

Vt = terminal velocity

m = mass

w = weight

g = acceleration of gravity

c = a resistance coefficient

y = vertical velocity

t = time

According to the laws of motion by Newton, and if a downward direction is positive the equations of equilibrium give

$$m\frac{dv}{dt} = mg - cv^2 \quad \text{or} \quad mv\frac{dv}{ds} = mg - cv^2 \quad \left(\frac{dv}{dt} = v\frac{dv}{ds}\right)$$

Integrating: $s = -\frac{m}{2c}\ln\left(v^2 - \frac{mg}{c}\right) + A$

A is a constant of integration to be determined by the boundary condition $s = 0$ when $v = V_0$

This gives an $A = \frac{m}{2c}\ln\left(V_0^2 - \frac{mg}{c}\right)$

Then:

$$s = \frac{m}{2c}\ln\frac{cV_0^2 - mg}{cV^2 - mg} = \frac{m}{2c}\ln\frac{mg - cV_0^2}{mg - cV_2}$$

The above equation can be re-written:

Then: $$e^{\frac{2cs}{m}} = \frac{mg - cV_0^2}{mg - cV^2}$$

$$\downarrow V^2 = \frac{mg}{c} - \frac{mg - cV_0^2}{c}e^{-\frac{2cs}{m}}$$

For all practical purposes as $S \longrightarrow \infty$, $e^{-\frac{2cS}{m}} \longrightarrow 0$

Therefore $V_t^2 = \frac{mg}{c}$ and $cV_t^2 = mg$

V_t = terminal velocity or final velocity attained when S is large.

$CV_t^2 = D$ where D is the drag offered by the surrounding medium.

Since $mg = W$ we get $\underline{\underline{D = W}}$

The premise that D = W is a fundamental used in several formulas for the velocities of particles in a viscous medium.

Dynamic resistance is introduced by the equation $D = C_d q A$

C_D = absolute coefficient of drag (non-dimensional)

$q = \frac{1}{2} \rho V^2$ (lbs. ft.$^{-2}$)

A = cross sectional area of sphere normal to direction of motion = $\frac{\pi}{4} d^2$ (ft.^{2})

ρ = density of the air (slugs ft.$^{-3}$)

V = velocity relative to the air (ft. sec.$^{-1}$)

d = diameter (ft.)

$W = 1/6\, \pi d^3 (\lambda \gamma)$

where λ = weight of water per unit volume (lbs ft.$^{-3}$)

γ = specific gravity of hailstone

From $D = W = C_D q A$

$$1/2\, \rho V^2 C_D \pi \frac{d^2}{4} = 1/6\, \pi d^3 (\lambda \gamma)$$

Simplifying: $V^2 = \frac{4}{3} \frac{(\lambda \gamma)}{(\rho)} \frac{d}{C_D}$

Letting: $K^2 = \frac{4}{3} \frac{(\lambda \gamma)}{(\rho)}$

we get: $V^2 = K^2 \frac{d}{C_D}$ or $V = K \left(\frac{d}{C_D} \right)^{\frac{1}{2}}$

Reynold's Number is: $R = \frac{Vd}{\nu}$ ν = kinematic viscosity of air

$\nu = \frac{\mu}{\rho}$ where μ = dynamic viscosity

$V = \frac{R\nu}{d}$ putting into $V = K\left(\frac{d}{C_0}\right)^{\frac{1}{2}}$

$$d^3 = C_D \left(\frac{R\nu}{K}\right)^2$$

If we let $\left(\frac{\nu}{K}\right)^2 = N^3$ we get the equation

$$d = (R^2 C_D)^{\frac{1}{3}} N$$

In the above expression if values of R are assumed we can find C_D by its relation to R (diagram 3). N can be found if ρ is known as γ, λ, μ are constants once a value for γ (specific gravity of hailstone) has been chosen. The diameter of the hailstone can then be calculated. Its velocity is equal to: $V = \frac{R\nu}{d} = \frac{R\mu}{d\rho}$

Another form of the equation $V = K\left(\frac{d}{C_0}\right)^{\frac{1}{2}}$ is $V = \left(\frac{QR}{C_D}\right)^{\frac{1}{3}}$

where $Q = \frac{4}{3}\frac{\mu}{\rho^2}(\lambda\gamma)$

By using constants for the "Standard Atmosphere" set up by the National Advisory Committee for Aeronautics in 1925 the development can be extended. The following relations are good below the isothermal level (35,332 feet). The 0 subscript refers to sea-level.

$$\rho = \rho_0 \left(1 - \frac{a}{T_0}h\right)^{4.256}$$

a = temperature gradient 3.566×10^{-3} °F. ft.$^{-1}$

T_0 = ground temperature (absolute) = 518.40°

$\rho_0 = 2.378 \times 10^{-3}$ lbs. sec.^{2} ft.$^{-4}$

$\lambda = 62.4$

$\mu = 3.575 \times 10^{-7}$

$$V = \left[\frac{4}{3}\frac{\mu(\lambda\gamma)R}{C_D\,\rho_0^2\left(1 - \frac{a}{T_0}h\right)^{8.512}}\right]^{\frac{1}{3}}$$

The relation between R and C_D can be found from diagram (3).

γ (specific gravity of hailstone) varies from pure ice $\gamma = 0.915$ to $\gamma = 0.40$ $\gamma = 0.60$ is representative. The diameter can be found by Reynold's Number $d = \frac{R\nu}{V} = \frac{R\mu}{\rho V}$

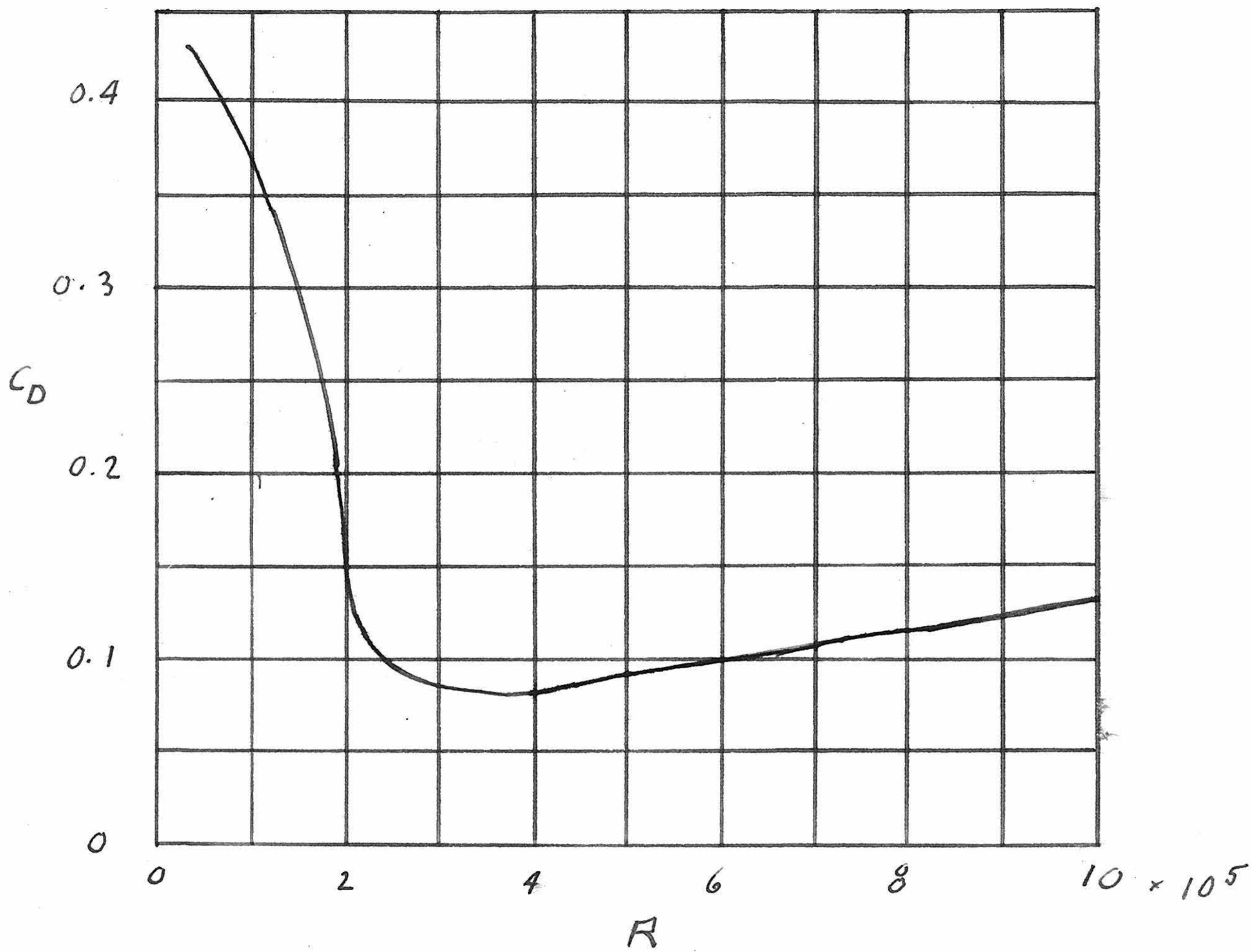


Diagram (3)

Three cases giving values of R, D and V for different values of the constants are given below. The values used are shown above the tables. The formula used was $d = (R^2 C_D)^{\frac{1}{3}} N$

Case I

Elevation = Sea Level $\gamma = 0.60$ $\frac{\mu}{\rho} = \nu = 1.50 \times 10^{-4}$ $K = 145$

$N = 1/9{,}750$ $\theta = \frac{R}{Vd} = 6.65 \times 10^{3}$

d (inches) =

$R \times 10^5$	$(R^2 C_D)^{\frac{1}{3}} \times 10^3$	$12 N (R^2 C_D)^{\frac{1}{3}} \times 10^3$	V (ft. sec^{-1}) $= R(\theta d)^{-1}$
0.50	1.01	1.25	72.3
1.00	1.55	1.91	94.3
1.50	1.89	2.34	116
1.75	1.97	2.42	130
2.00	1.88	2.32	156
2.25	1.76	2.16	188
2.50	1.81	2,23	203
3.00	1.97	2.43	223
3.50	2.14	2.64	240
4.00	2.37	2.92	247
5.00	2.83	3.49	259
6.00	3.29	4.06	267
7.00	3.76	4.64	272
8.00	4.18	5.15	280
9.00	4.63	5.71	285
10.00	5.10	6.29	287

For small diameters: $V = K\left(\frac{d}{C_D}\right)^{\frac{1}{2}} = 218\, d^{\frac{1}{2}}$; $C_D = 0.445$

C_D is considered constant

d (inches)	V (ft. sec^{-1})
0.25	31.4
0.50	44.4
0.75	54.4
1.00	62.8

Case 2

Elevation 10,000 $v = 0.60$ $\frac{\mu}{\rho} = \nu = 2.04 \times 10^{-4}$

$K = 169$ $N = \frac{1}{8,810}$ $\phi = \frac{R}{\nu d} = 4.90 \times 10^{3}$

$R \times 10^{5}$	$(R^{2} C_D)^{\frac{1}{3}} \times 10^{3}$	$12\,N(R^{3} C_D)^{\frac{1}{3}} \times 10^{3}$	$V(ft\ sec^{-1}) = R(\phi d)^{-1}$
0.50	1.01	1.38	88.4
1.00	1.55	2.11	116
1.50	1.89	2.57	143
1.75	1.97	2.68	160
2.00	1.88	2.56	191
2.25	1.76	2.39	230
2.50	1.81	2.46	248
3.00	1.97	2.69	273
3.50	2.14	2.92	293
4.00	2.37	3.23	303
5.00	2.83	3.85	317
6.00	3.29	4.49	326
7.00	3.76	5.12	334
8.00	4.18	5.69	344
9.00	4.63	6.31	347
10.00	5.10	6.95	352

For small diameters

$v = K\left(\frac{d}{C_D}\right)^{\frac{1}{2}} = 253 \times d^{\frac{1}{2}}$ $C_D = 0.445$

d (inches)	$V\ (ft.\ sec^{-1})$
0.25	36.5
0.50	51.6
0.75	63.2
1.00	73.0
1.25	81.9

Case 3

Elevation 20,000 $\gamma = 0.60$ $\frac{\mu}{\rho} = \nu = 2.82 \times 10^{-4}$ $K = 199$

$N = 1/7,910$ $\phi = \frac{R}{\nu d} = 3.54 \times 10^{3}$

d (inches) =

$R \times 10^3$	$(R^2 C_D)^{\frac{1}{2}} \times 10^3$	$12 N (R^2 C_D)^{\frac{1}{3}} \times 10^3$	$v(\text{ft. sec.}^{-1}) = R(\phi d)^{-1}$
0.50	1.01	1.54	110
1.00	1.55	2.35	144
1.50	1.89	2.87	177
1.75	1.97	2.99	198
2.00	1.88	2.85	238
2.25	1.76	2.66	296
2.50	1.81	2.74	308
3.00	1.97	2.99	340
3.50	2.14	3.25	365
4.00	2.37	3.60	377
5.00	2.83	4.29	394
6.00	3.29	4.99	407
7.00	3.76	5.70	416
8.00	4.18	6.34	427
9.00	4.63	7.02	434
10.00	5.10	7.74	438

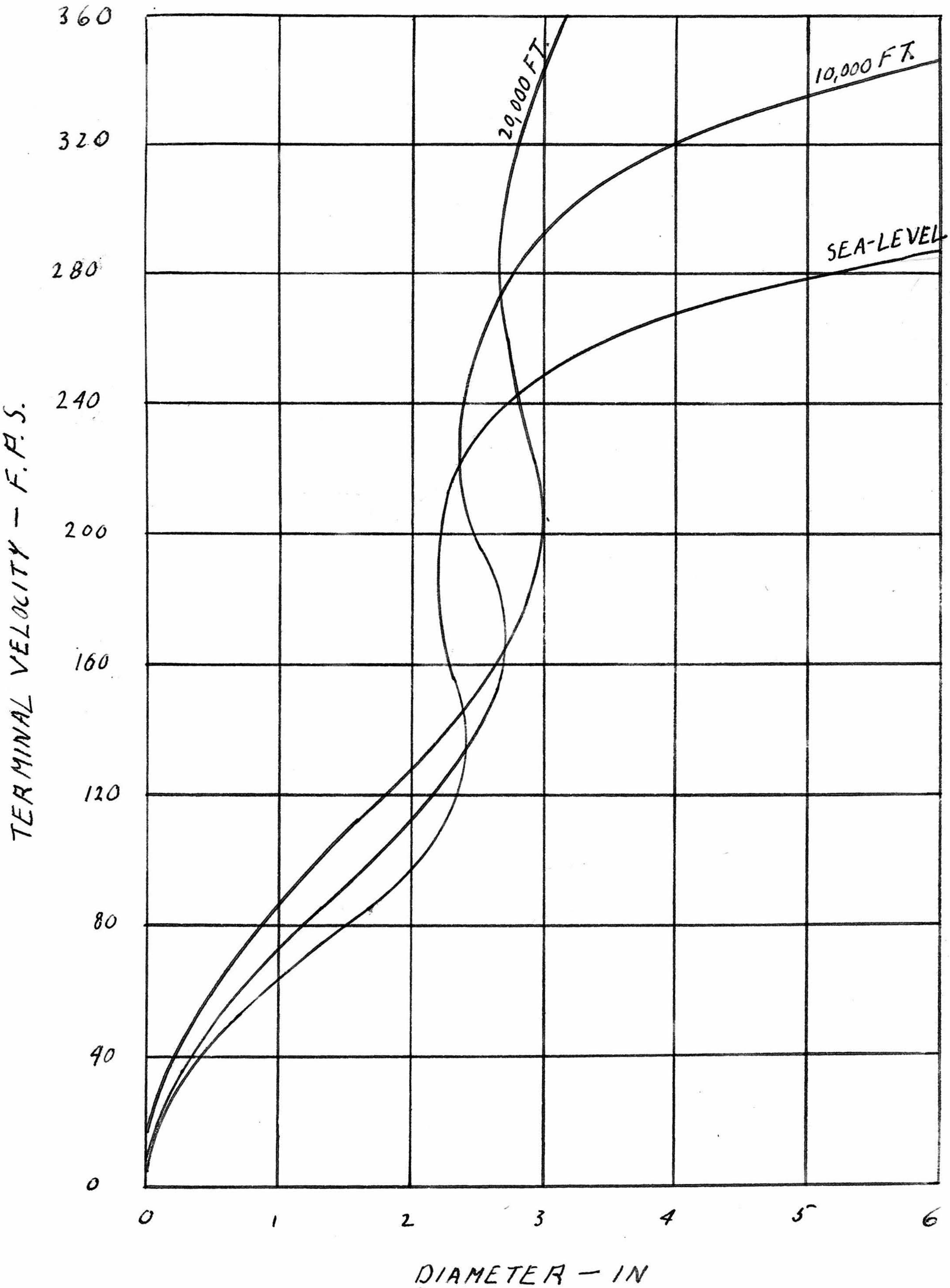
For small diameters

$V = K\left(\frac{d}{C_D}\right)^{\frac{1}{2}} = 298.\, d^{\frac{1}{2}}$ $C_D = 0.445$

d (inches)	$V(\text{ft. sec}^{-1})$
0.25	43.1
0.50	60.8
0.75	74.5
1.00	86.1
1.25	96.4
1.50	106

The plot of hailstone diameter against velocity is shown for the three cases tabulated in diagram (4).

In the region of Reynold's Number equals 0.5×10^{5} to 2×10^{5} there are three terminal velocities that a smooth hailstone may attain. This is shown by the interval on the three curves plotted in diagram (4)



360
320
280
240
200
160
120
80
40
0
TERMINAL VELOCITY — F.P.S.
20,000 FT.
10,000 FT.
SEA-LEVEL
0
1
2
3
4
5
6
DIAMETER — IN

Diagram (4)

where the slope is infinite. This transitional zone is caused by a change from laminar flow to turbulent flow. It is possible to pick the most probable maximum size for hailstones by choosing the point where the slope of the curves becomes infinite; this is the most stable condition.

Below are tabulated the results for the three curves plotted.

Elevation	γ	V (ft sec^{-1})	Optimum (d)"	weight lbs.
Sea level	0.60	133	2.42	0.154
10,000	0.60	160	2.70	0.220
20,000	0.60	204	3.00	0.310

From the above tabulation we can conclude that if hailstones are formed and sustained by vertical currents at an altitude of 20,000 , the maximum probable size would be 3.00 inches and the vertical current about 200 ft. sec^{-1}.

The size of hailstones and their frequency of occurrence as classified by W.J. Humphreys is as follows:

Diameter (inches)	Occurrence
1	very common
2	often reported
3	not extremely rare
4	doubtful

We can classify vertical velocities in thunderstorms accompanied by hail in the following manner assuming that the hailstone is supported at 10,000 feet and $\gamma = 0.60$

Diameter (inches)	Velocity (ft.sec.)	Occurrence
1	73	very common
2	112	often reported
3	296	not extremely rare
4	319	doubtful
5	332	very rare

I included diameters of 4 inches and 5 inches because at Dallas, Texas, stones of 4 inches were found during a storm and a stone of record size, measuring 5.4 inches in diameter and weighing one and one-half pounds was found at Potter, Nebraska, on July 6, 1928.

The highest conceivable vertical velocity in any hail storm should therefore be 332 ft.sec.$^{-1}$ = 102 meters sec.$^{-1}$ = 227 miles per hour.

Velocities of 200 miles per hour have been reported in some cases by pilots unfortunate enough to get caught in a thunderstorm and fortunate enough to get out.

The above discussion of velocities does not take into account turbulence except of the 2 per cent that was present when the relation between R and C_D was determined in the N.A.C.A. variable density wind tunnel for a smooth sphere. The turbulent factors are too many and too variable to be included here.

PART III

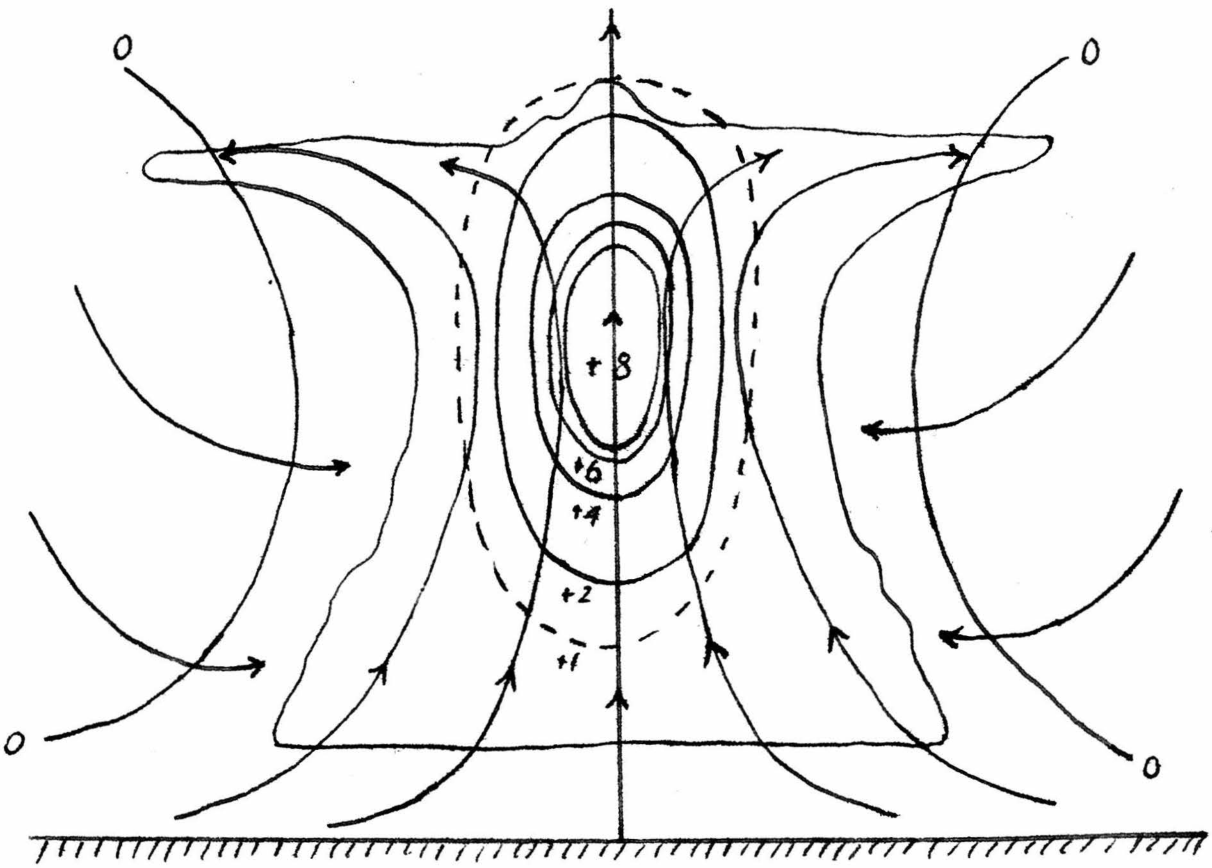
PHYSICAL STRUCTURE OF THUNDERSTORMS

Physical Structure

A thunderstorm is similar to a cumulus cloud with regard to the circulation of air but the motions and velocities are much more violent. In general there is an upward circulation of air inside the thunderstorm and a downward circulation in the air surrounding it. The cloud may flatten into an anvil shape on top if the tropopause is reached. Often a protrusion boils up in the center of the cloud. This is caused by a sudden release of latent heat when water condenses causing the air to expand and rise.

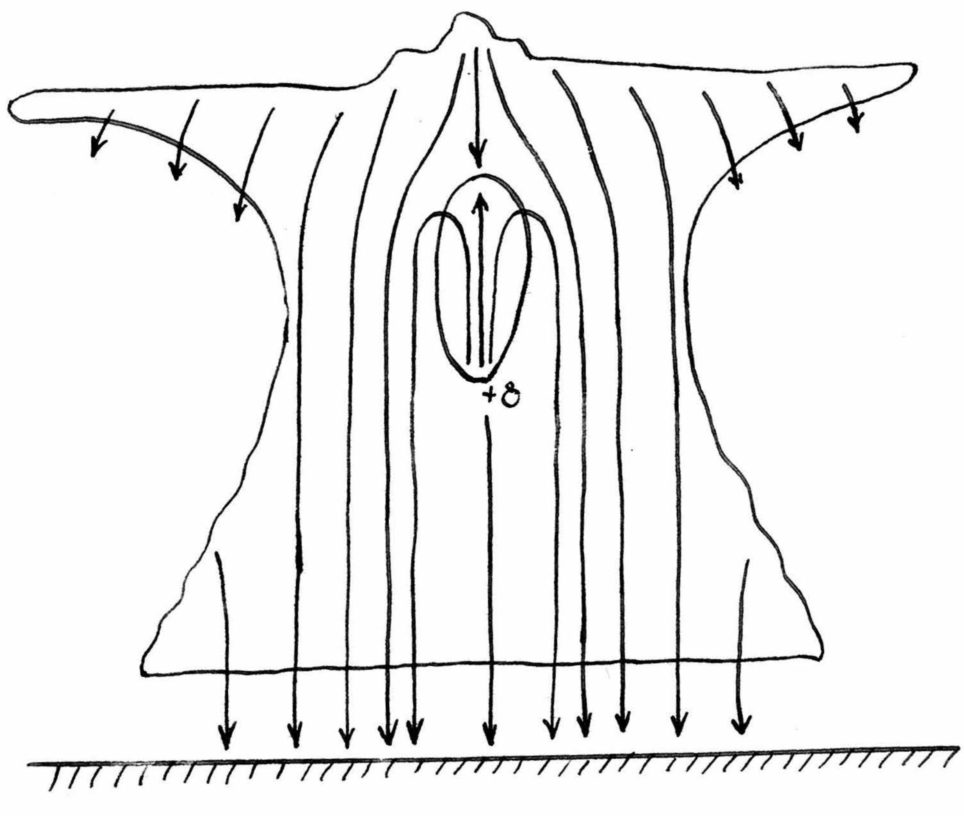
The precipitation in a stationary thunderstorm is usually heavy in the forward and rear sections but moderate in the center. The reason for the lighter precipitation in the center is the higher velocities that are experienced there than near the edges. The rain drops and smaller hailstones are blown upwards and allowed to descend in a ring around the center in areas where the vertical velocities are less. As was stated before no rain drops can descend through a vertical current greater than 8 meters per second. Diagrams (5) and (6) show respectively the schematic representation of the air circulation and velocities in an average thunderstorm, and the precipitation stream lines. It will be noted that very little of the precipitation from the anvil edges reaches the ground. It either sublimates or evaporates before doing so.

Diagrams (5) and (6) refer to a stationary thunderstorm. In reality a thunderstorm moves with the prevailing air currents. This has the effect of tilting the axis of the storm forward in the direction



Air Stream Lines And Vertical Velocities In A Stationary Thunderstorm

Diagram (5)



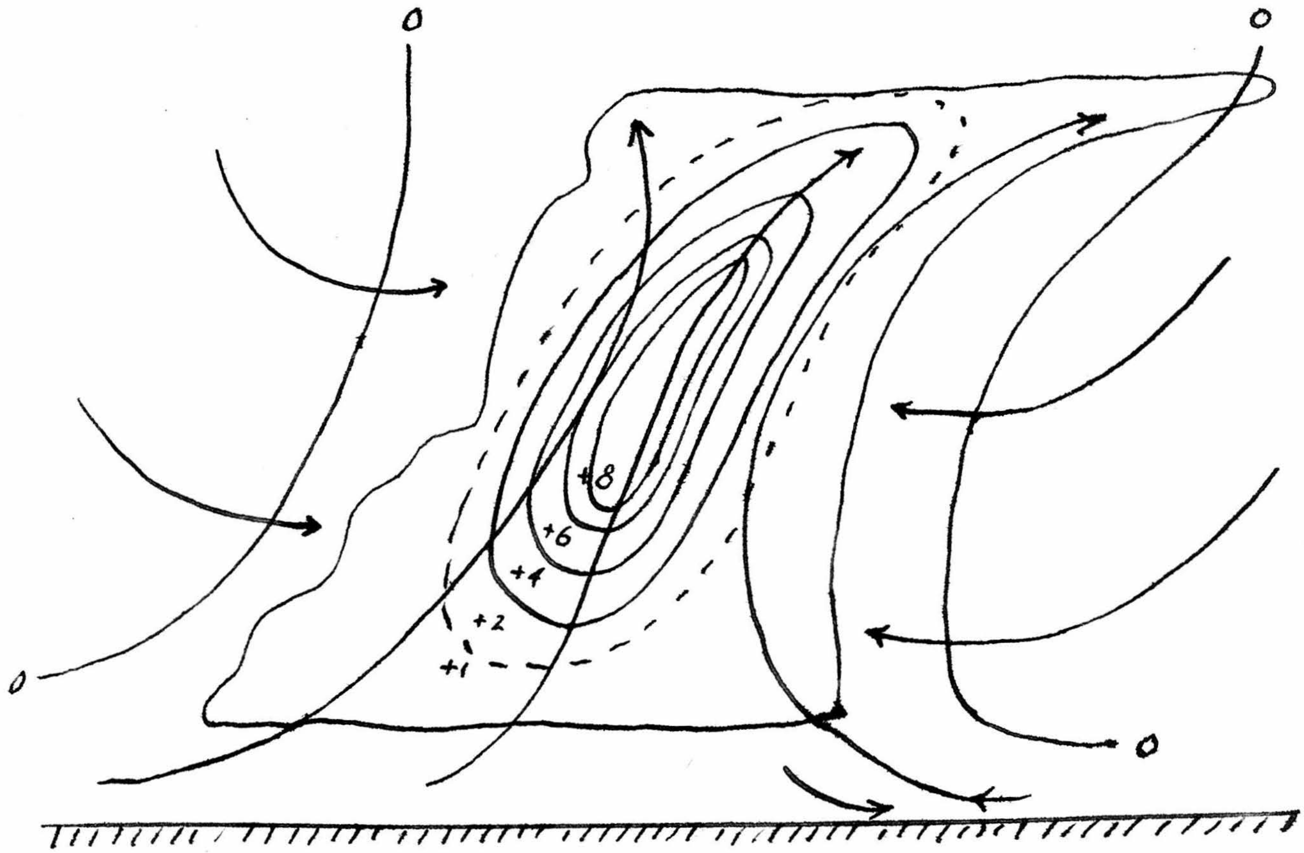
Precipitation Streamlines In A Stationary Thunderstorm

Diagram (6)

of motion and concentrating the precipitation products in a comparatively small area in the forward edge of the disturbance. The tilting is caused because the wind velocity increases with altitude. In diagram (7) the air stream lines and velocities are shown for a moving thunderstorm, while diagram (8) shows the precipitation stream lines in a moving thunderstorm.

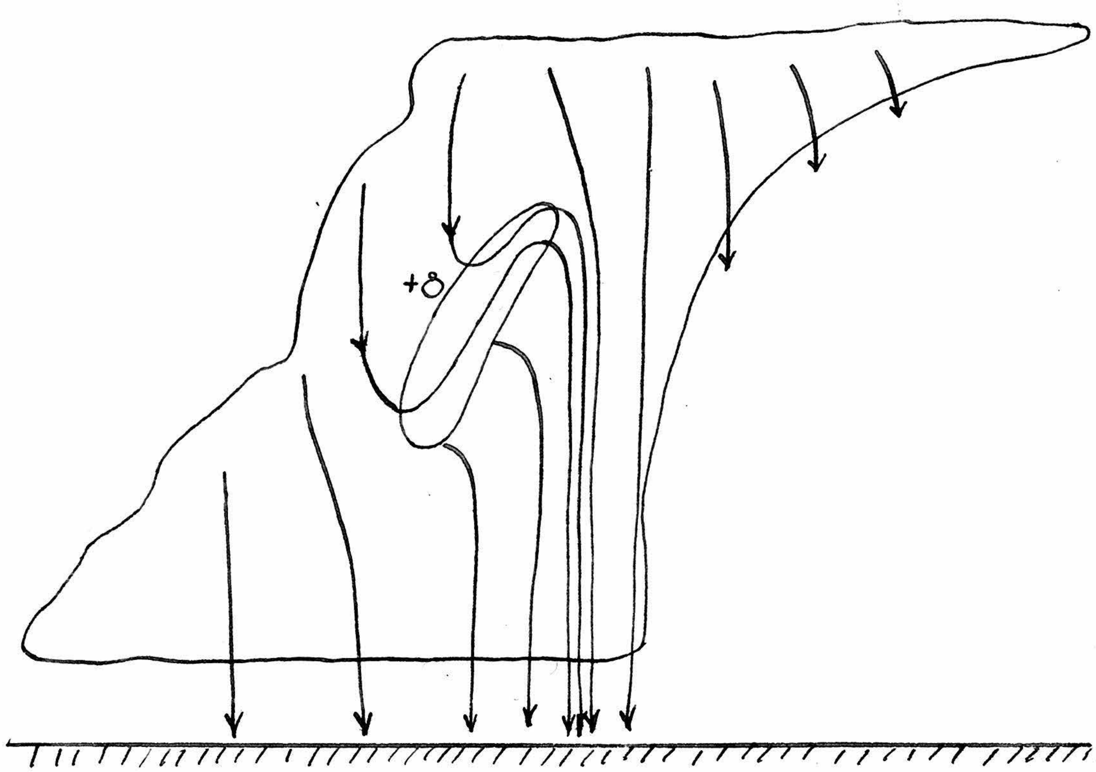
In diagram (7) a component of air is shown descending from the thunderstorm and running out in front of it. The origin of this air current can be explained as follows: Convective cooling of ascending air in the storm by evaporation causes precipitation products to form. These products in falling to the ground chill a column of air. The chilling is caused partly by the initial low temperatures of the precipitation products but mainly by their continued evaporation as they fall to the ground. The cool column of air is set in motion by the frictional drag of falling precipitation and the fact that the air being cooler and denser than its surroundings sinks. Just before a thunderstorm reaches a station, a cool current of air blowing from the storm is felt. The current owes its origin to the principles discussed above.

The low clouds, often called squall or curtain clouds, which are sighted near the forward edge of a thunderstorm are a by product of the cold outflow of air in front of the storm. The cool air under runs the warmer air in front of it and the lifting thus produced coupled with the cooling caused by the admixture of cool air with warm causes clouds to form in the warm air.



Air Streamlines And Vertical Velocities in A Moving Thunderstorm

Diagram (7)



Precipitation Streamlines In A Moving Thunderstorm

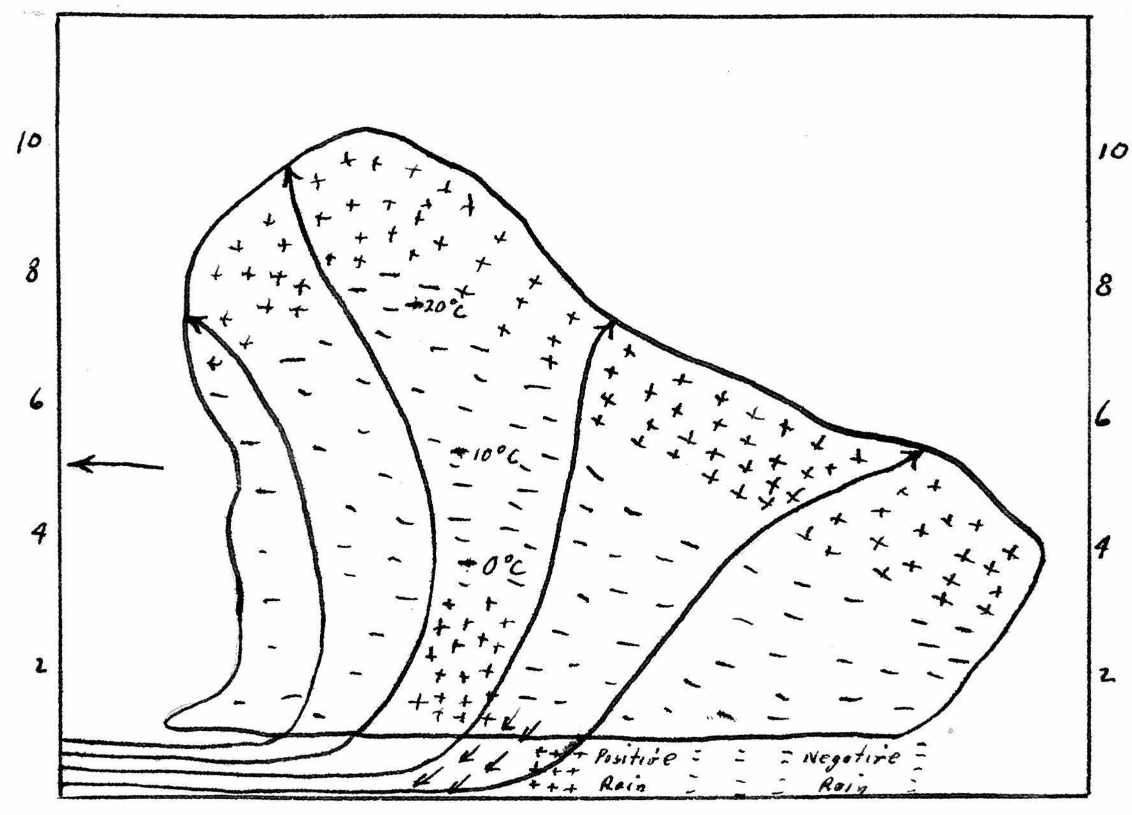
Diagram (8)

The circulation of a thunderstorm is not one typical of many encountered in meteorology; that is, a closed one whose energy is supplied by heating in one portion of the circuit and cooling in another portion; i.e., in land and sea breezes, mountain breezes, etc. The cool air that runs in front of the storm comes from high in the cloud and the warm air which forms the squall or curtain clouds from the surface in advance of the disturbance. These two air masses are not used over and over again but are continually replaced by new air as the storm advances. In other words a thunderstorm is an open convection,

Of especial interest is the origin of the electrical charges which cause the lightning flashes so often observed in thunderstorms. Many theories have been advanced, but the Simpson Rupture Theory and the Wilson Capture Theory seem to be the only plausible ones which stand up under careful scrutiny. While Simpson investigated the charge on rain drops in Simla, India, experiments were conducted which showed that if distilled water has spray torn from it by an air blast the breaking of the water drops is accompanied by the production of positive and negative ions. About three times as many negative as positive ions are released and thus the water drops retain a net positive charge.

In a thunderstorm it is not difficult for the large velocities present to break up innumerable water drops and release many negative ions. The water drops may coalesce and break up several times, but each successive rupture is accompanied by the production of a greater positive charge on the water drop. Light negative ions are blown up toward the cloud top where they unite with cloud particles. The heavier rain drops fall out as positive rain. The above explanation is the reason that heavy rain is usually positively charged and light rain negatively charged as observed by Simpson.

A thunderstorm should have a preponderance of negative charge above and a preponderance of positive charge below if Simpson's theory is correct. Actually it has been observed that a thunderstorm cloud is charged positively above and negatively below. This discrepancy is explained by Wilson in his "capture theory" as follows: A rain drop is charged positively on its lower half and negatively on its upper half due to the normal electric field of the earth. In falling through space filled with positive and negative ions the drop accumulates negative ions leaving the positive ions to be carried up to the cloud top. The negative charge predominates in the lower part of the cloud where it is carried by the rain drops. Diagram (9) shows the normal distribution of electric charge in a thundercloud (after Simpson).



Normal Distribution Of Electric Charge In A Thundercloud

Diagram (9)

PART IV

ANALYSIS OF ST. LOUIS STORM
(August 25, 1939)

Analysis of St. Louis Storm (August 25, 1939)

On August 25, 1939, St. Louis experienced the severest thunderstorm on record. At the Garrison and Lucas Streets station 5.02 inches of rain fell between 4:35 and 5:35 a.m. This was 1.4 inches greater the previous record of 3.6 inches. 3.92 inches was recorded in 30 minutes which exceeded any 30-minute or 60-minute previous rainfall. For the total storm 5.79 inches of rain fell at Garrison and Lucas streets.

I have chosen this storm to analyze, first, because it is a good example of a severe thunderstorm and, second, because a continuous record of rainfall was kept by automatic rain gauges during the entire storm. The Radio Sonde data is shown on the next page while the sounding is plotted in diagram (10). The most representative data that could be obtained as to the air that would be present st St.Louis during the storm was a sounding at Oklahoma City taken at 3:00 a.m. The storm commenced at St. Louis at 1:43 a.m. and continued intermittently until 7:05 a.m. The synoptic chart for August 29, 1939, showed a warm front just at St. Louis and since the thunderstorm occurred in the early morning it was suspected that the trigger action to set it off was supplied by lifting along the frontal surface. However, when the lifting condensation level was found and the particle's ascention curve plotted next to the sounding very little positive area was found. Certainly not enough to result in a thunderstorm. A plot of the ascention curve from the convective condensation level resulted in good positive areas. Although convective thunderstorms usually

ST. LOUIS STORM
Aug. 25,
1939

TIME SOUNDING 03 EST

COMPUTED BY ______

CALIFORNIA INSTITUTE OF TECHNOLOGY

AEROGRAPHIC DATA SHEET

STATION Oklahoma DESIG ______

DATE Aug. 25, 1939

ELEV HNDS M	PRESS MBS	TEMP DEGS C	RH %	W GRS./KG.	θ DEGS ABS.	θ_E DEGS. ABS	LIFTS WNDS M
4	970	23	86	14.8			
7	940	25	63	13.5			
14	870	24	48	10.5			
22	789	21	47	9.3			
30	714	16	46	7.6			
45	603	9	82	9.7			
50	566	5	90	8.7			
74	420	-8	49	2.4			
78	400	-9	59	1.8			
90	340	-18	46	1.2			
119	228	-40	46	0.2			
149	144	-59					
165	110	-61					
195	69	-53					

$\frac{d\theta_E}{dz}$ DEGS ABS/KM	CLOUDS	AIR MASS	REMARKS

ISENTROPIC DATA

θ	P	P_0	Ψ

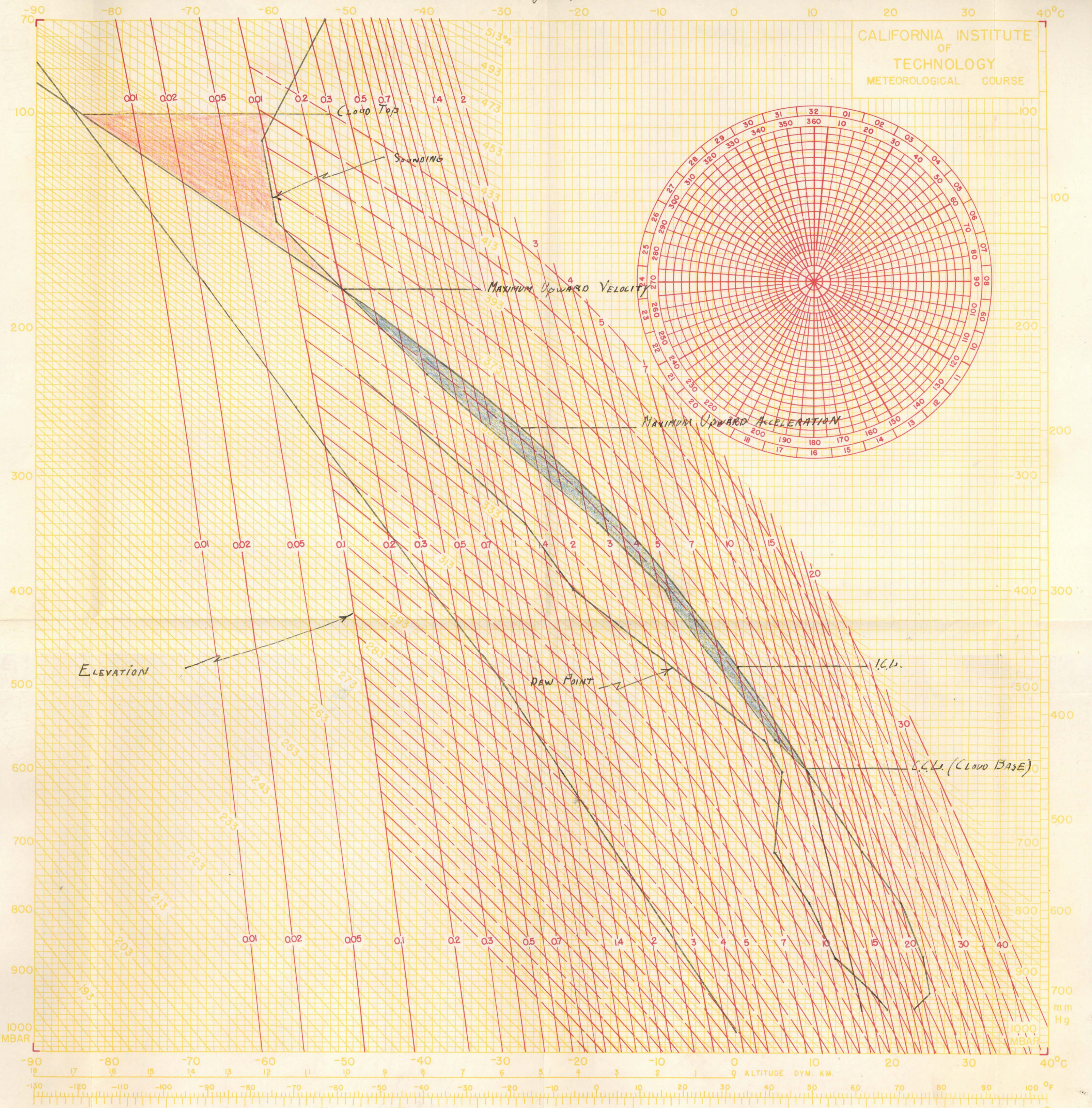
PRESSURE DATA

5,000 FT.	10,000 FT.	15,000 FT.	20,000 FT.

Form AL-3 30M 3-41

SOUNDING FOR ST. LOUIS THUNDERSTORM

St Louis Storm Aug. 25, 1939



* The Condensation Level Is Due To A Combination of Lifting & Convection

Diagram (10)

occur during the warmest part of the day it was assumed that this storm was a convective type one especially since there was a temperature of 77° F. present at the time. The storm actually was probably due to a combination of high humidity, steep temperature lapse rate, convective mixing and cooling at the cloud top.

The convective condensation level was found by averaging the lower four specific humidity points and running the constant value of specific humidity obtained straight up until the sounding curve was intersected. Very little change in the level of the convective condensation level was found if the lower three points were used instead of the four points in the calculation. The elevations of pertinent points in the cloud obtained from diagram (10) are listed below.

Convective condensation level (cloud base)	4,500 meters
Maximum acceleration level	10,900 meters
Maximum vertical velocity level	13,600 meters
Cloud top	17,000 meters
Ice crystal level	6,500 meters

In the following discussion I have tried to apply some of the formulas developed in Part I on "Thermodynamical Structure of the Thunderstorm." It should be remembered that the results will be correct only to the right magnitude because the formulas do not take account of certain factors and because only average values are used in the calculations.

Maximum acceleration

$a = g\frac{\Delta T}{T}$ g = acceleration of gravity in centimeters per second^2

ΔT = Temperature difference between the particle and the surrounding air

T = Temperature of the surrounding air

$$a' = \frac{980 \times 4.5}{241} = 18.2 \text{ cm/sec.}^2$$

Average acceleration

$a' = g\frac{\Delta T'}{T_1}$ The primes denote mean values.

$$a' = \frac{980 \times 3}{255} = 11.5 \text{ cm/sec.}^2$$

Maximum velocity

$V = \sqrt{2a'h}$ h = height that particle is accelerated

$$V = \sqrt{2 \times 11.5 \times 91 \times 10^4} = 4600 \text{ cm/sec.} = 46 \text{ meters/sec.}$$

Maximum Velocity using Height of Cloud Top

The formula developed in Part I for finding the height that the cloud rises above the maximum velocity level is:

$H = \left(\frac{V^2 T}{g\Delta T}\right)^{\frac{1}{2}}$ T is the average absolute temperature in the internal H

ΔT is uniform change in temperature difference between rising air and surrounding air per centimeter of elevation

If "H" is taken off of the sounding in diagram (10) as found by equating positive area and negative area, a value of V can be calculated from the above equation.

$$V = \frac{H}{\left(\frac{T}{g\Delta T}\right)^{\frac{1}{2}}} = \frac{340,000}{10 \times 5.46} = 62 \text{ meters/sec.}$$

This value differs from the previous value for the maximum velocity of 46 meters/sec partially because the previous formula considered the surrounding air as stationary partially because this formula does not consider viscosity of air, and partially because average values were used for temperature functions.

$$H = \left(\frac{V^2 T}{g \Delta T}\right)^{\frac{1}{2}} = \left(\frac{4600^2 \times 215}{980 \times 7.35 \times 10^{-5}}\right)^{\frac{1}{2}} = 2.5 \text{ Kilometers}$$

The height taken from the sounding by equating positive and negative areas was found to be 3.4 kilometers. It is to be expected that the value of "H" from the sounding should be less than "H" calculated because of viscosity effects. The only explanation is that the positive and negative areas are not equated very well or that too low a value was arrived at for a maximum velocity in the thunderstorm.

Calculations of Velocities in Lower Levels of Thunderstorms by Fulk's Diagram.

In the "Monthly Weather Review" of October, 1935 a discussion of "Fulk's Diagram of Approximate Precipitation" is given. Fulk has constructed a diagram from thermodynamical considerations which shows the precipitation received in millimeters per hour from a column of saturated air 100 meters thick ascending at the rate of one meter per second.

Some sources of error which make Fulk's diagram only approximate are as follows:

1. A pseudo-adaibetic process is assumed. That is all the products of precipitation are assumed to fall out as soon as they are formed.

2. It is assumed that the column of air does not change its thickness with height.

3. An approximate value of $\frac{de}{dh}$ (The rate of change of the saturation vapor pressure with height) is used. This may cause an error as much as 5%.

All that is required to find the rate of precipitation in millimeters per hour from the "Fulk's Diagram" is the temperature and pressure distribution, the column thickness, and its vertical velocity. If the precipitation rate is known and the vertical velocity is not known one can work backwards and find this velocity.

"Fulk's Diagram" is shown in diagram (11). The sounding curve for the St. Louis storm is given in diagram (10). Knowing the ratio of precipitation for this storm some average values of vertical velocities in the lower levels can be arrived at. The calculations are shown below.

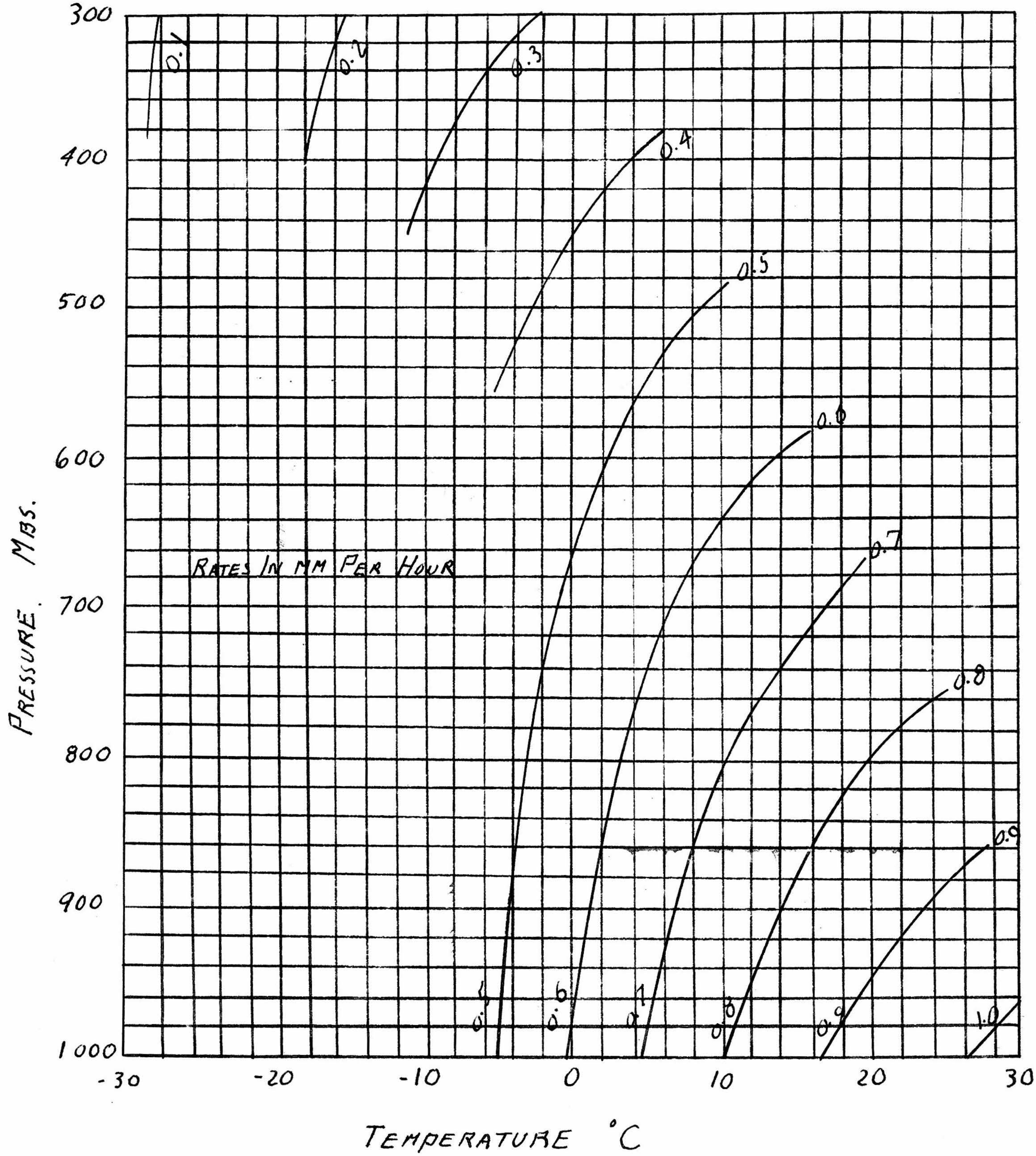
Maximum rainfall for one hour = 5.02" = 127 mm.

Average rainfall per hour for entire storm = 1.74" = 44.1 mm.

Let x = vertical velocity in meters per second.

Elevation			MM per hour
4.5	to 5 KM		2.7 x
5	6		4.9 x
6	7		4.2 x
7	8		3.3 x
8	9		2.7 x
9	10		1.8 x
10	11		1.3 x
11	12		0.7 x (Extrapolated)
12	13		0. 2 x (Extrapolated)
13	17		0.1 x (Extrapolated)
		Total	21.9 x mm per hour

FULK'S DIAGRAM



FOR COLUMN 100 METERS THICK ASCENDING AT 1 METER PER SECOND

Diagram (11)

For maximum rain fall: $21.9x = 127$ $x = 5.8$ meters per second

For average rain fall: $21.9x = 44.1$ $x = 2.0$ meters per second

If a value for the velocity below 10 KM. (where most of the rain fall takes place) is found by the formula previously used, the following results:

$$V = \sqrt{2a'h} \qquad a' = g\frac{\Delta T'}{T'} = \frac{980 \times 2.3}{267} = 8.2\ \text{cm/sec}^2$$

$$V = \sqrt{2 \times 8.2 \times 55 \times 10^4} = 30\ \text{meters/sec.}$$

A velocity of 30 meters per second is at the 10 KM. level, so an average value below this level is about 15 meters per second. This value is a good deal higher than that found by the method using "Fulk's Diagram." The fact that the surrounding atmosphere is considered stationary for the solution probably contributes to the large answer obtained.

Summary

Although a substantial part of the material presented in this thesis has been garnered from various references, it is difficult to write a thorough discussion of "Thunderstorm Mechanics" from a purely original point of view.

Upon commencing this paper the writer contemplated answering certain pertinent questions concerning thunderstorms. For example: What is the correct magnitude for minimum and maximum velocities in a thunderstorm, and how can these values be computed? How are the stream-lines of precipitation and velocity concentrated in a thunderstorm when moving and when stationary? What is the most plausible theory as to the production of the large electrical charge necessary in a thundercloud to produce the electrical phenomena observed? Is there a maximum size to the raindrops and hailstones that can be produced, and what shape does this precipitation take while falling? Can the height that a thundercloud rises above the maximum velocity level be computed by equating kinetic energy to potential energy, and can the maximum velocity be calculated if this height is found from the adiabatic chart sounding? What are the main physical factors causing the cold outflow of air that precedes a thunderstorm? All of these questions as well as others have been answered to the writer's satisfaction in this manuscript, and if only a little knowledge concerning "Thunderstorm Mechanics" is imparted to others reading the manuscript, then it has fulfilled its purpose.

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