BOLTED TIMBER JOINTS

Presented as partial fulfillment of the requirements
for the degree of Master of Science in Civil Engineering.

Martin R. Gayer

June 19, 1944

CONTENTS

BOLTED TIMBER JOINTS

I. Introduction

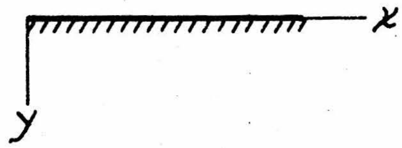
In a bolted timber joint the stress in the wood under the bolt is not uniform; on the contrary, high stress concentrations occur at the edges of the timbers. The more slender the bolt the greater is the stress concentration and the smaller the portion of the length of bolt over which the actual bearing is distributed. Tests conducted at the Forest Products Laboratory have revealed the existence of a relationship between the length to diameter ratio of the bolt and the proportional limit of the average bearing value of the wood. The tests indicated that the average proportional limit stress (stress at which the slip of the joint ceases to be proportional to the applied load) decreases with increasing length to diameter ratios. Therefore safe uniform design stresses, based on the average proportional limit stress, must decrease with increasing length to diameter ratios.

It is the purpose of the following discussion to determine analytically the relationship between the stress distribution in bolted joints and the elastic properties of the bolt and the wood.

II. Deflection Curve of a Beam on an Elastic Foundation

The action of a bolt in a timber joint is that of a beam on an elastic foundation. That is, both the bending strength of the bolt and the elastic properties of the wood must be considered.

Consider a beam on an elastic foundation, with coordinate axes as shown. The bending moment M is taken as positive when



it produces upward concavity, and for such bending $\frac{d^2y}{dx^2}$ is negative when the coordinate axes are directed as indicated above. Hence the differential equation of the deflection curve is $EI\frac{d^2y}{dx^2}=-M$. Differentiating both members of the equation twice, there results $EI\frac{d^4y}{dx^4}=-q$, in which q denotes the intensity of the load acting on the beam. Now consider the beam to be imbedded along its entire length in a material capable of exerting both downward and upward forces on it. It will be assumed that when the beam is deflected the intensity of the continuous reaction at every section is proportional to the deflection at that section. Hence the intensity of the load acting on the beam may be expressed as ky, where k is the modulus of the foundation. k denotes the magnitude of the continuous reaction per unit of length of the beam per unit of deflection. For an unloaded portion of the beam, the only force acting on the beam is the continuous reaction. Therefore the differential equation of the deflection curve of a beam on an elastic foundation is

$$EI\frac{d^4y}{dx^4}=-ky .$$

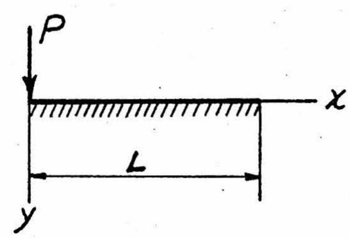
The general solution of this differential equation may be represented as follows:

$$y = \sin\beta x(C_1 \sinh\beta x + C_2 \cosh\beta x) + \cos\beta x(C_3 \sinh\beta x + C_4 \cosh\beta x)$$

where $\beta = \sqrt[4]{\frac{K}{4EI}}$, and C_1, C_2, C_3, C_4 are arbitrary constants to be determined from the known conditions at certain points of the beam.

The deflection curve will first be established for two fundamental types of loading: 1) a concentrated load applied at the end of a beam, and 2) a bending moment applied at the end of a beam. For other conditions of loading the deflection curve will be found by superposition.

Case 1:



$$a.\ \left.\frac{d^2y}{dx^2}\right)_{x=0} = 0 : \quad C_1 = 0$$

$$b.\ \left.\frac{d^2y}{dx^2}\right)_{x=L} = 0 : \quad C_2 \cos\beta L \sinh\beta L - C_3 \sin\beta L \cosh\beta L - C_4 \sin\beta L \sinh\beta L = 0$$

$$c.\ EI\left.\frac{d^3y}{dx^3}\right)_{x=0} = -V = P.: \quad K(C_2 - C_3) = 2\beta P$$

$$d.\ \left.\frac{d^3y}{dx^3}\right)_{x=L} = 0 : \quad C_2(\cos\beta L \cosh\beta L - \sin\beta L \sinh\beta L) - C_3(\sin\beta L \sinh\beta L + \cos\beta L \cosh\beta L) - C_4(\sin\beta L \cosh\beta L + \cos\beta L \sinh\beta L) = 0$$

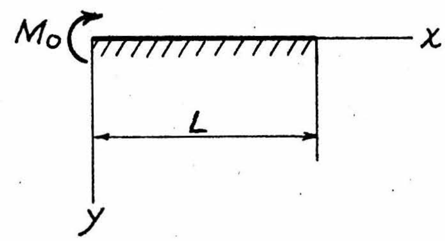
$$C_2 = -\frac{2P\beta}{K}\left(\frac{\sin^2\beta L}{\sinh^2\beta L - \sin^2\beta L}\right) ; \quad C_3 = -\frac{2P\beta}{K}\left(\frac{\sinh^2\beta L}{\sinh^2\beta L - \sin^2\beta L}\right) ;$$

$$C_4 = \frac{2P\beta}{K}\left(\frac{\sinh\beta L \cosh\beta L - \sin\beta L \cos\beta L}{\sinh^2\beta L - \sin^2\beta L}\right)$$

Substituting these constants in the original equation, there results the following expression for the deflection curve of the beam:

$$y = \frac{2\beta P}{K}\ \frac{\sinh\beta L \cos\beta x \cosh\beta(L-x) - \sin\beta L \cosh\beta x \cos\beta(L-x)}{\sinh^2\beta L - \sin^2\beta L}$$

Case 2:



$$a.\ EI\frac{d^2y}{dx^2}\Big)_{x=0} = -M = -M_o \quad : \quad C_1 = -\frac{2\beta^2}{K}M_o$$

$$b.\ \frac{d^2y}{dx^2}\Big)_{x=L} = 0 \quad : \quad C_1\cos\beta L\cosh\beta L + C_2\cos\beta L\sinh\beta L - C_3\sin\beta L\cosh\beta L - C_4\sin\beta L\sinh\beta L = 0$$

$$c.\ \frac{d^3y}{dx^3}\Big)_{x=0} = 0 \quad : \quad C_2 = C_3$$

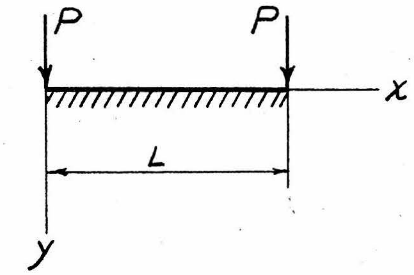
$$d.\ \frac{d^3y}{dx^3}\Big)_{x=L} = 0 \quad : \quad C_1(\cos\beta L\sinh\beta L - \sin\beta L\cosh\beta L) + C_2(\cos\beta L\cosh\beta L - \sin\beta L\sinh\beta L) - C_3(\sin\beta L\sinh\beta L + \cos\beta L\cosh\beta L) - C_4(\sin\beta L\cosh\beta L + \cos\beta L\sinh\beta L) = 0$$

$$C_2 = C_3 = \frac{2\beta^2 M_o}{K}\left(\frac{\sinh\beta L\cosh\beta L + \sin\beta L\cos\beta L}{\sinh^2\beta L - \sin^2\beta L}\right); \quad C_4 = -\frac{2\beta^2 M_o}{K}\left(\frac{\sinh^2\beta L + \sin^2\beta L}{\sinh^2\beta L - \sin^2\beta L}\right)$$

Substituting these constants in the original equation, the following expression is obtained:

$$y = -\frac{2\beta^2 M_o}{K(\sinh^2\beta L - \sin^2\beta L)}\Big[\sinh\beta L\cos\beta x\sinh\beta(L-x) - \sinh\beta L\sin\beta x\cosh\beta(L-x) + \sin\beta L\cosh\beta x\sin\beta(L-x) - \sin\beta L\sinh\beta x\cos\beta(L-x)\Big]$$

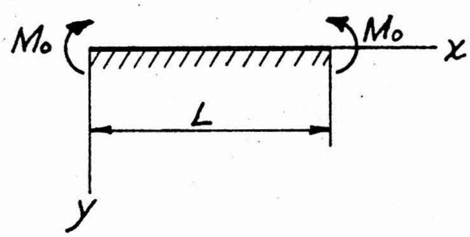
Case 3:



By superposition, using the equation for case 1, the following expression is obtained for the deflection curve of the beam:

$$y = \frac{2\beta P}{K} \frac{\cosh \beta x \cos \beta(L-x) + \cos \beta x \cosh \beta(L-x)}{\sinh \beta L + \sin \beta L}$$

Case 4:

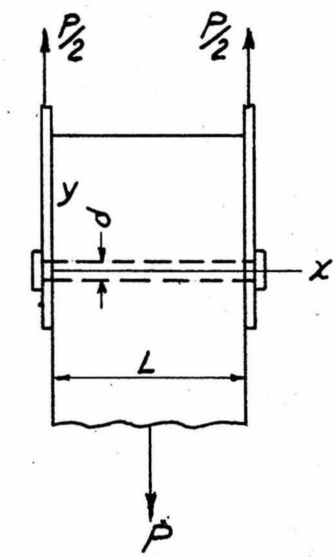


By superposition, using the equation for case 2, there results the following expression for the deflection curve of the beam:

$$y = -\frac{2\beta^2 M_o}{K(\sinh \beta L + \sin \beta L)} \Big[\sinh \beta x \cos \beta(L-x) - \cosh \beta x \sin \beta(L-x) + \cos \beta x \sinh \beta(L-x) - \sin \beta x \cosh \beta(L-x) \Big]$$

III. Bolted Joint with Steel Splice Plates

Assuming that the bolt fits tightly in the hole so that both an upward and a downward reaction may be developed, the above equations can be applied to the deflection curve of the bolt in a bolted timber joint.



Since the thickness of the splice plates is small relative to that of the main timber, and since the deformation of the splice plates under the bolt is negligible in comparison to that of the wood, this case may be considered to be that of a beam with concentrated loads applied at its extremities. If P denotes the tension force in the central member of the joint, a load of P/2 will be transmitted to the extremities of the bolt in the central member by the splice plates. The deflection of the bolt at any section at a distance x from the edge of the central member is

$$y = \frac{\beta P}{K} \frac{\cosh \beta x \cos \beta(L-x) + \cos \beta x \cosh \beta(L-x)}{\sinh \beta L + \sin \beta L}$$

The magnitude of the continuous reaction per unit of length at any section is equal to k times the deflection at that section. Hence the unit stress in bearing under the bolt is equal to ky/d, where d is the diameter of the bolt. If the load were uniformly distributed over the length of the bolt, the unit stress in bearing would be P/Ld. It follows that the ratio of the actual bearing stress at any section to the average bearing stress is equal to kLy/P.

$$\frac{\text{actual stress}}{\text{average stress}} = \frac{kLy}{P} = (\beta L)\frac{\cosh\beta x \cos\beta(L-x) + \cos\beta x \cosh\beta(L-x)}{\sinh\beta L + \sin\beta L}$$

Since $\beta = \sqrt[4]{\frac{k}{4EI}}$, where E and I are the modulus of elasticity and moment of inertia of the bolt, respectively, for a given length and diameter of bolt βL is a constant. In a following paragraph it is shown that k is a function of the modulus of elasticity of the wood and is independent of the diameter of the bolt. Furthermore, since $I = \frac{\pi d^4}{64}$, $\beta L = 1.50\sqrt[4]{\frac{k}{E}}\,\frac{L}{d}$ That is, βL is directly proportional to the L/d ratio of the bolt. Hence the following discussion will be in terms of βL, since this term is the more general and may be applied to any species of wood and to any strength bolt.

On sheet 10 are plotted curves showing the distribution of stress along the length of the bolt for various values of βL. The curves are symmetrical about the center line of the timber member. For small values of βL (hence small values of the L/d ratio) the stress is substantially uniform; for higher values of βL there are high stress concentrations at the edges of the timber. For values of βL 4 and above a reversal of stress occurs. Hence it may be seen that for the higher values of βL the actual bearing is distributed over only a small portion of the length of the bolt. These curves also represent the relative deflections of the bolt.

On sheet 11 is plotted a curve showing the variation of the maximum stress, occurring at the edge of the timber, with βL. For values of βL of 7 and higher, the maximum stress increases in direct proportion with βL.

It is of interest to note how the bending moment in the bolt varies with βL. An expression for the bending moment at any section is as follows:

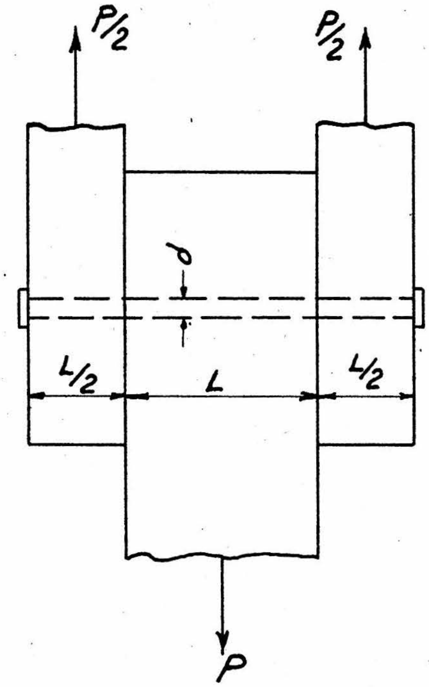
$$M = -EI \frac{d^2y}{dx^2} = -\frac{P}{2\beta} \frac{\sinh\beta x \sin\beta(L-x) + \sin\beta x \sinh\beta(L-x)}{\sinh\beta L + \sin\beta L}$$

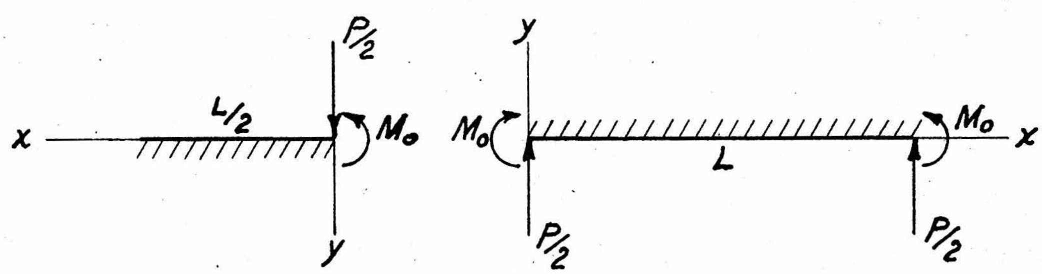
The minus sign results from the chosen direction of the coordinate axes. The ratio of the bending moment to PL, neglecting the minus sign, is expressed in terms of βL alone.

$$\frac{M}{PL} = \frac{1}{2\beta L} \frac{\sinh\beta x \sin\beta(L-x) + \sin\beta x \sinh\beta(L-x)}{\sinh\beta L + \sin\beta L}$$

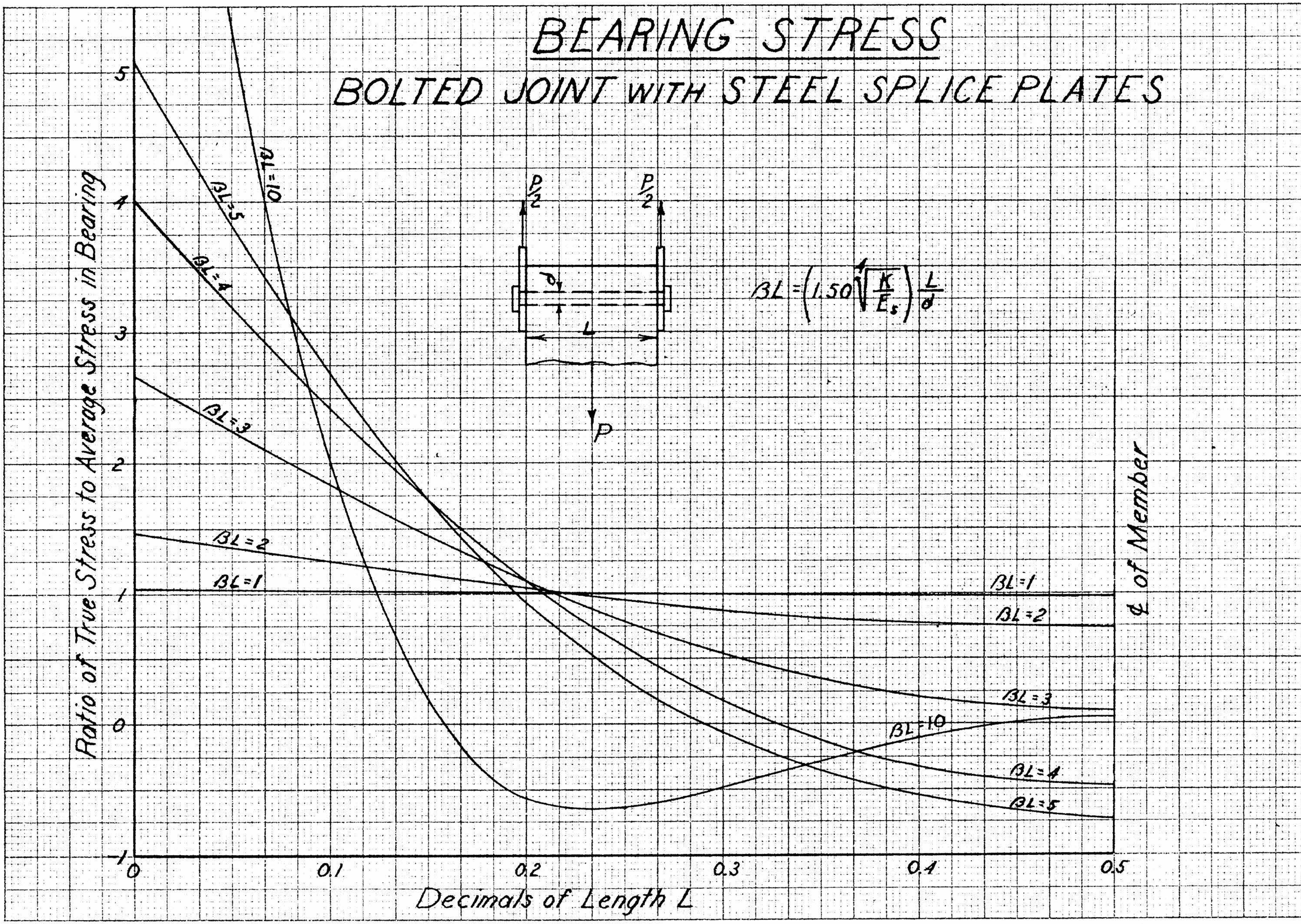
On sheet 12 are plotted curves showing the variation in bending moment along the length of the bolt for various values of βL. As βL increases, the point of maximum moment approaches the edge of the timber. For high values of βL (10 and above) a reversal of bending moment occurs, but the magnitude of the negative moment is small and cannot be shown due to the scale of the diagram. The curve on sheet 15 shows how the maximum bending moment decreases with increasing values of βL.

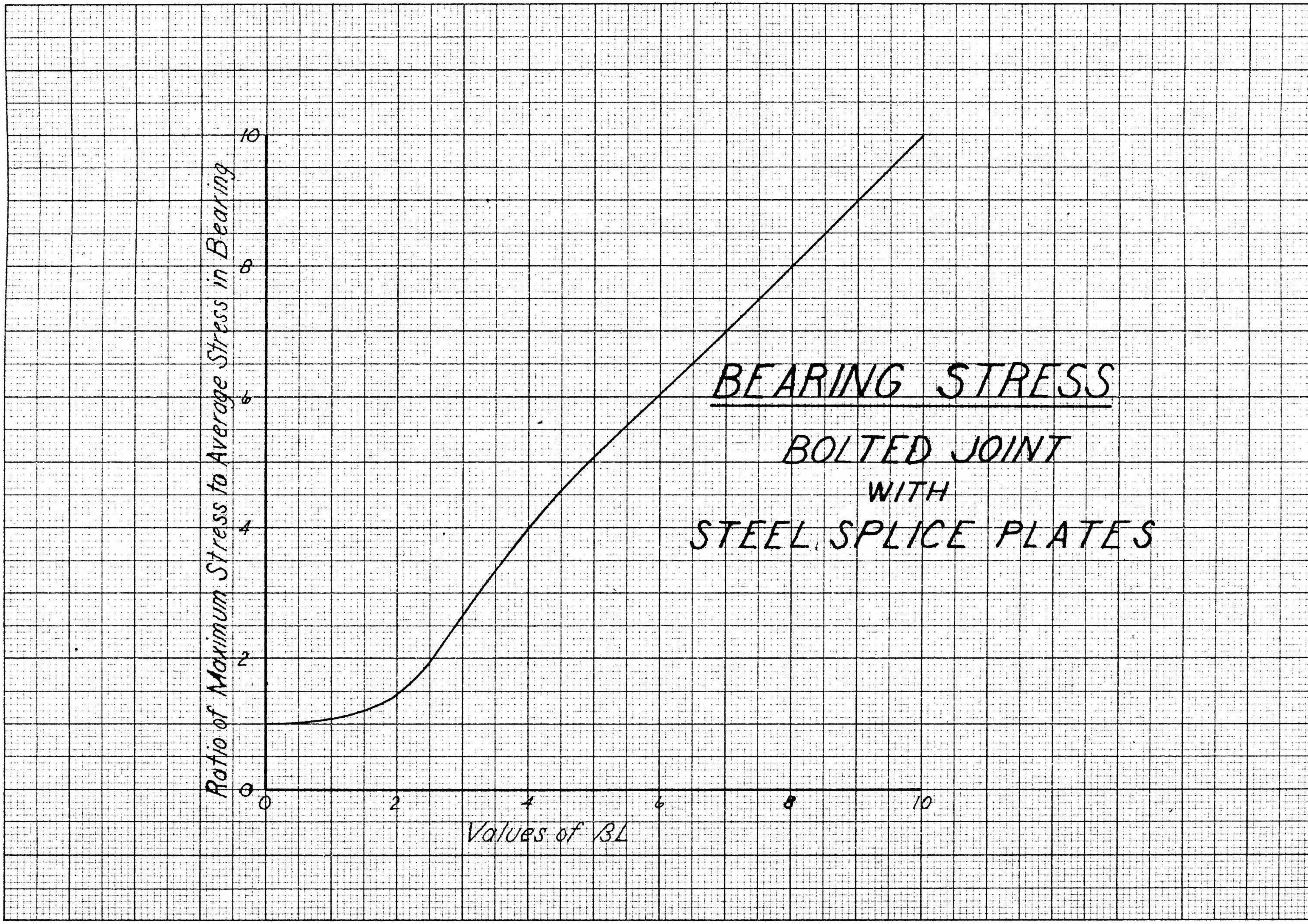
IV. Bolted Joint with Wood Splice Plates





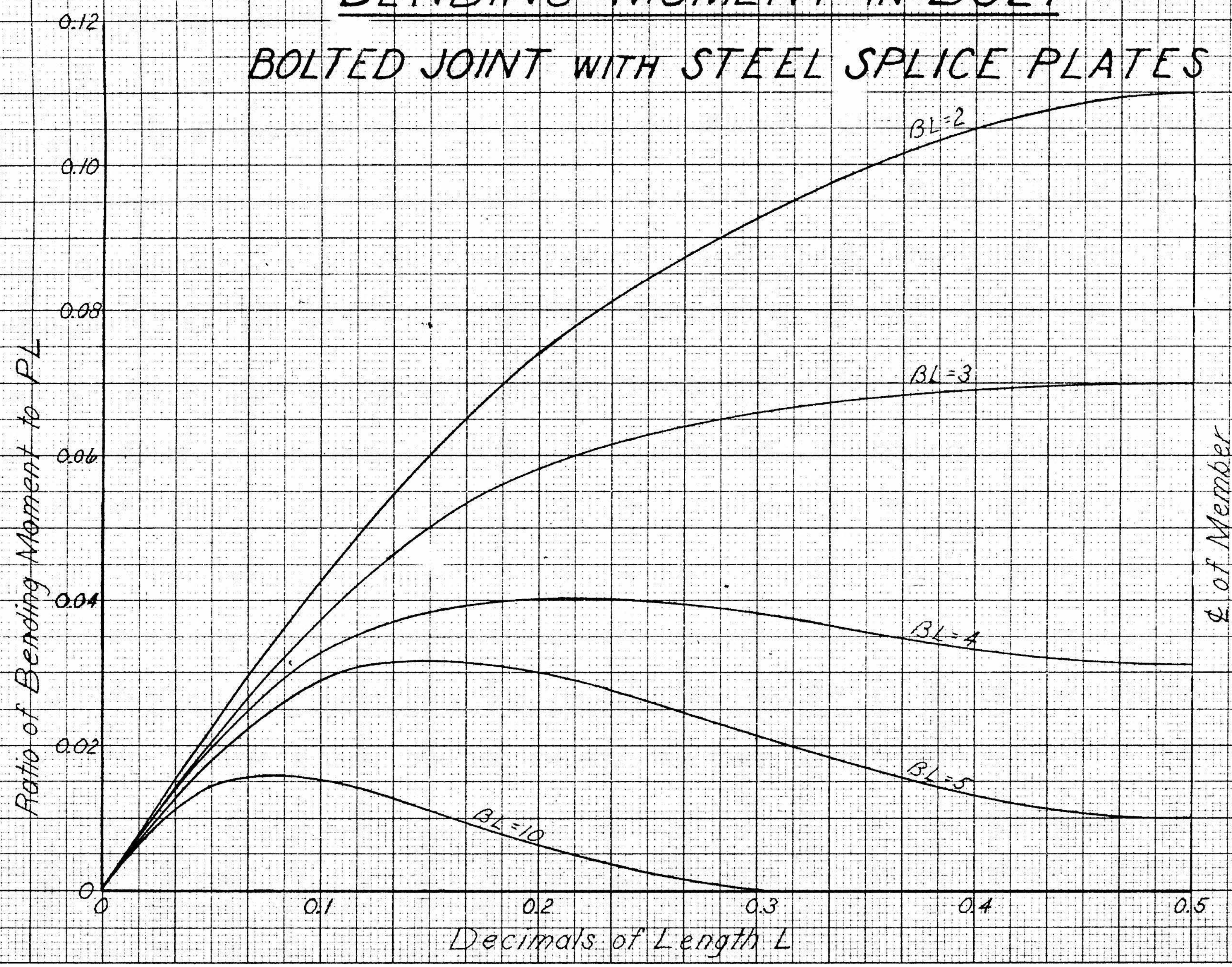
Since the bending of the bolt in the side member must be considered, and since the foundation is discontinuous at the

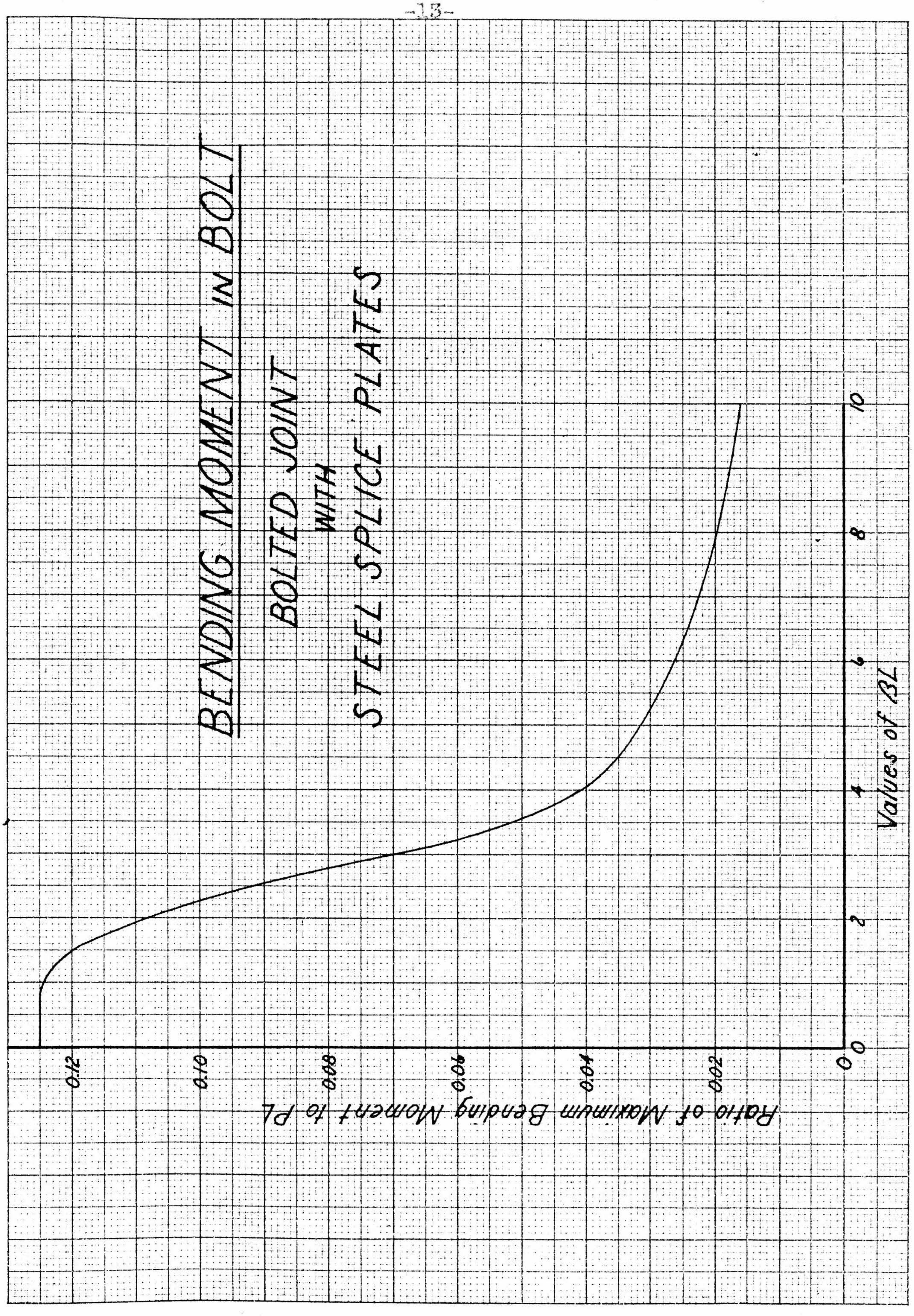

BEARING STRESS
BOLTED JOINT WITH STEEL SPLICE PLATES
$\beta L = \left(1.50 \sqrt[4]{\frac{K}{E_s}}\right) \frac{L}{d}$
P/2
P/2
d
L
P
βL=10
βL=5
βL=4
βL=3
βL=2
βL=1
Ratio of True Stress to Average Stress in Bearing
Decimals of Length L
℄ of Member
5
4
3
2
1
0
-1
0
0.1
0.2
0.3
0.4
0.5


BEARING STRESS
BOLTED JOINT
WITH
STEEL SPLICE PLATES
Ratio of Maximum Stress to Average Stress in Bearing
Values of βL
0
2
4
6
8
10

BENDING MOMENT IN BOLT

BOLTED JOINT WITH STEEL SPLICE PLATES




BENDING MOMENT IN BOLT
BOLTED JOINT
WITH
STEEL SPLICE PLATES
Values of βL
0
2
4
6
8
10
Ratio of Maximum Bending Moment to PL
0
0.02
0.04
0.06
0.08
0.10
0.12

edges of the central timber, this case must be broken down into that of two separate beams as shown above. The bending moment in the bolt, M_o, at the edge of the central timber, may be determined by the principle of continuity. That is, assuming rotation of the side members is prevented, the slope of the deflection curve of the bolt at the point of discontinuity must be the same for the two beams.

For the center timber,

$$y_{P/2} = \frac{\beta P}{K} \frac{\cosh\beta x \cos\beta(L-x) + \cos\beta x \cosh\beta(L-x)}{\sinh\beta L + \sin\beta L}$$

$$y_{M_o} = \frac{2\beta^2 M_o}{K} \frac{\sinh\beta x \cos\beta(L-x) - \cosh\beta x \sin\beta(L-x) + \cos\beta x \sinh\beta(L-x) - \sin\beta x \cosh\beta(L-x)}{\sinh\beta L + \sin\beta L}$$

$$\left.\frac{dy}{dx}\right)_{x=0} = \frac{\beta^2}{K(\sinh\beta L + \sin\beta L)}\left[P(\sin\beta L - \sinh\beta L) + 4\beta M_o(\cos\beta L - \cosh\beta L)\right]$$

For the side timbers,

$$y_{P/2} = \frac{\beta P}{K} \frac{\sinh\frac{\beta L}{2}\cos\beta x \cosh\beta(\frac{L}{2}-x) - \sin\frac{\beta L}{2}\cosh\beta x \cos\beta(\frac{L}{2}-x)}{\sinh^2\frac{\beta L}{2} - \sin^2\frac{\beta L}{2}}$$

$$y_{M_o} = -\frac{2\beta^2 M_o}{K\left(\sinh^2\frac{\beta L}{2} - \sin^2\frac{\beta L}{2}\right)}\left[\sinh\frac{\beta L}{2}\cos\beta x \sinh\beta(\tfrac{L}{2}-x) - \sinh\frac{\beta L}{2}\sin\beta x \cosh\beta(\tfrac{L}{2}-x) + \sin\frac{\beta L}{2}\cosh\beta x \sin\beta(\tfrac{L}{2}-x) - \sin\frac{\beta L}{2}\sinh\beta x \cos\beta(\tfrac{L}{2}-x)\right]$$

$$\left.\frac{dy}{dx}\right)_{x=0} = \frac{\beta^2}{K\left(\sinh^2\frac{\beta L}{2} - \sin^2\frac{\beta L}{2}\right)}\left[4\beta M_o\left(\sinh\frac{\beta L}{2}\cosh\frac{\beta L}{2} + \sin\frac{\beta L}{2}\cos\frac{\beta L}{2}\right) - P\left(\sinh^2\frac{\beta L}{2} + \sin^2\frac{\beta L}{2}\right)\right]$$

Equating these two expressions for the slope at the point of discontinuity, the value of M_o is as follows:

$$M_o = \frac{P}{\beta} \frac{\sinh \beta L \sin^2 \frac{\beta L}{2} + \sin \beta L \sinh^2 \frac{\beta L}{2}}{(\sinh \beta L + \sin \beta L)^2 + (\cosh \beta L - \cos \beta L)(\cosh \beta L + \cos \beta L - 2)}$$

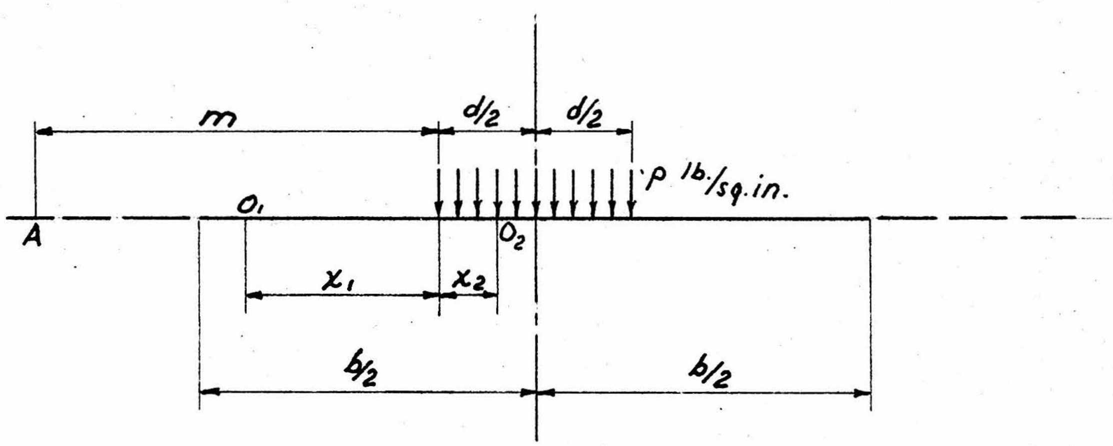
For small values of βL, the above equation should result in the same expression for M_o as for the case of uniform stress distribution. For very small angles the sine and sinh are equal to the angle, and the cosine and cosh are equal to 1. Making this substitution, the above equation reduces to $M_o = PL/8$, which is correct for uniform stress distribution.

On sheet 17 is plotted a curve showing the variation of M_o with values of βL. For values of βL above 5, M_o is equal to zero. Taking a free body diagram of the side member and the portion of the bolt within the side member, if the only forces acting on the free body are the tension force P/2, the shear in the bolt P/2, and M_o, then M_o must be equal to PL/8. Assuming no rotation of the side member, M_o is less than PL/8 for the higher values of βL; hence there must be other forces acting on the side member. This problem is discussed in a later paragraph.

As before, the ratio of the actual bearing stress to the average bearing stress is equal to kLy/P for both the center and side timber. This ratio for the two members is similar to that for the case of steel splice plates, being in terms of βL. The curves on sheet 18 show the variation of stress along the bolt for various values of βL. These curves clearly indicate the high stress concentration at the edges of the members and demonstrate that the actual bearing is distributed over only a small portion of the bolt length for higher values of βL.

V. Determination of k, the Modulus of the Wood

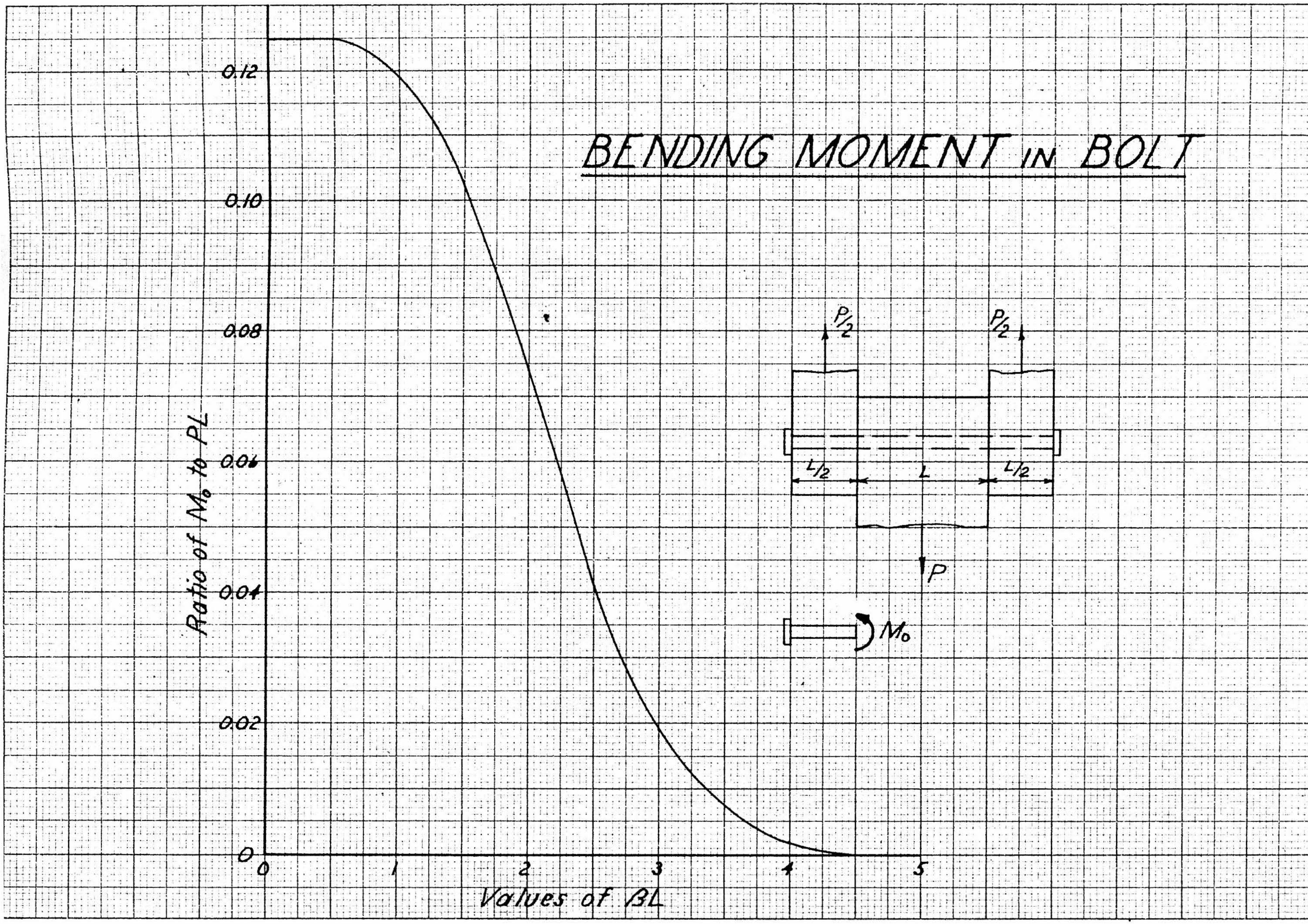
The modulus of the wood may be approximated analytically by considering the deformation of an elastic body under direct bearing stress. Consider the case in which a uniform pressure of p lbs. per sq. inch is applied over a width d across the surface of a semi-infinite plate, for which case an expression for the deflection of the plate has been developed.*



This case may be applied to the bearing of the bolt in a timber if it be considered that the bearing stress is distributed to the wood on a plane through the center of the bolt. Let d represent the diameter of the bolt and b the width of the timber through which the bolt is inserted.

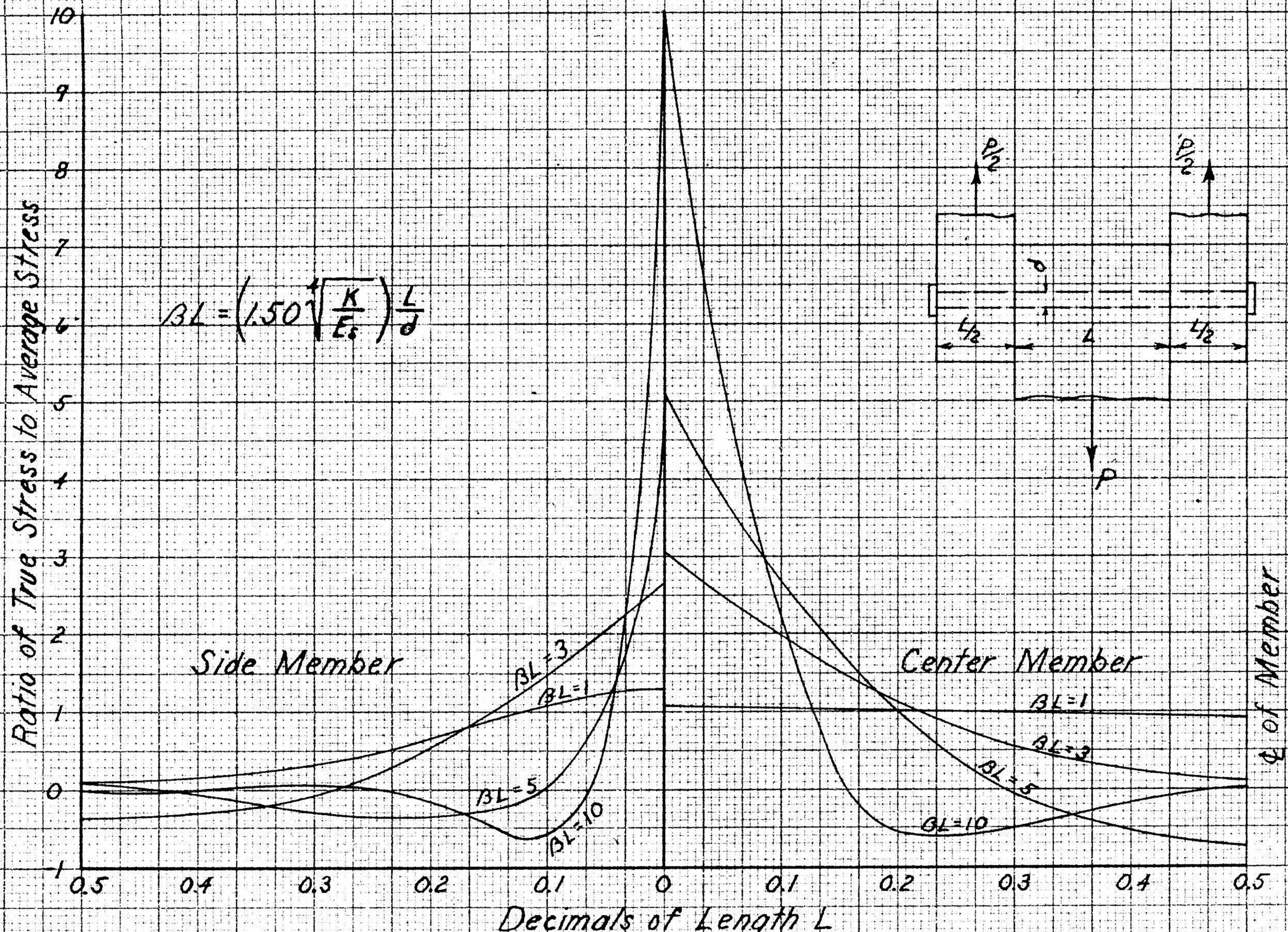
---

*Formulas for Stress and Strain, by Raymond J. Roark, p. 246


BENDING MOMENT IN BOLT
P/2
P/2
L/2
L
L/2
P
Mo
0.12
0.10
0.08
0.06
0.04
0.02
0
Ratio of Mo to PL
0
1
2
3
4
5
Values of βL

BEARING STRESS

BOLTED JOINT WITH WOOD SPLICE PLATES



y = deflection relative to a remote point A a distance m from the edge of the loaded area

ν = Poisson's ratio

a) At any point O_1 outside of loaded area,

$$y = \frac{2p}{\pi E}\left[(d+x_1)\log_e \frac{m}{d+x_1} - x_1 \log_e \frac{m}{x_1}\right] + pd\left(\frac{1-\nu}{\pi E}\right)$$

At the edge of the timber, $x_1 = \frac{b-d}{2}$, and

$$y = \frac{2p}{\pi E}\left[\left(\frac{b+d}{2}\right)\log_e \frac{2m}{b+d} - \left(\frac{b-d}{2}\right)\log_e \frac{2m}{b-d}\right] + pd\left(\frac{1-\nu}{\pi E}\right)$$

b) At any point inside the loaded area,

$$y = \frac{2p}{\pi E}\left[(d-x_2)\log_e \frac{m}{d-x_2} + x_2 \log_e \frac{m}{x_2}\right] + pd\left(\frac{1-\nu}{\pi E}\right)$$

At the center of the bolt, $x_2 = \frac{d}{2}$, and

$$y = \frac{2pd}{\pi E}\log_e \frac{2m}{d} + pd\left(\frac{1-\nu}{\pi E}\right)$$

c) y_1 = deflection at center of bolt relative to the edge of the member

$$= \frac{p}{\pi E}\left[(b+d)\log_e(b+d) - (b-d)\log_e(b-d) - 2d\log_e d\right]$$

Letting b = nd,

$$y_1 = \frac{pd}{\pi E}\left[(n+1)\log(n+1) - (n-1)\log(n-1)\right]$$

Over the ordinary range of values of n, from 5 to 15, the variation of y_1 is small, and the average value over this range is $y_1 = \frac{2pd}{E}$. From the definition of the modulus of the wood, it follows that

$$K = \frac{pd}{y_1} = \frac{E}{2}$$

Since the equations employed in this derivation are based upon an homogeneous, elastically isotropic body, and since wood is neither homogeneous nor isotropic, it is understood that the above expression for the modulus of the wood is merely an approximation.

VI. Determination of Allowable Average Bearing Stress

For the case of a bolted joint with steel splice plates, the maximum deflection, which occurs at the edge of the timber, is given by

$$y_{max.} = \frac{\beta P}{K} \frac{\cosh \beta L + \cos \beta L}{\sinh \beta L + \sin \beta L}.$$

Also, the maximum bearing stress under the bolt is given by

$$S_{max.} = \frac{K y_{max}}{d}.$$

Hence, if the maximum bearing stress be fixed, the allowable load on the joint is

$$P = (S_{max})\left(\frac{d}{\beta}\right) \frac{\sinh \beta L + \sin \beta L}{\cosh \beta L + \cos \beta L}$$

Since the average bearing stress is equal to P/Ld, it follows that the allowable average bearing stress, s, is

$$s = \frac{s_{max.}}{\beta L} \frac{\sinh \beta L + \sin \beta L}{\cosh \beta L + \cos \beta L}$$

For design purposes it is necessary to express the allowable bearing stress in terms of the L/d ratio of the bolt. Consider the case of a bolted joint with steel splice plates loaded parallel to the grain, using Douglas Fir (coast region). $E_W = 1.2 \times 10^6$ lbs. per sq. inch, $s_{max} = 1300$ lbs. per sq. inch, and $E_s = 29 \times 10^6$ lbs. per sq. inch. Hence k, the modulus of the wood, is equal to $E_W/2 = 0.6 \times 10^6$ lbs. per sq. inch.

$$\beta L = 1.50 \sqrt[4]{\frac{K}{E}} \left(\frac{L}{d}\right)$$

$$= 0.57 \frac{L}{d}$$

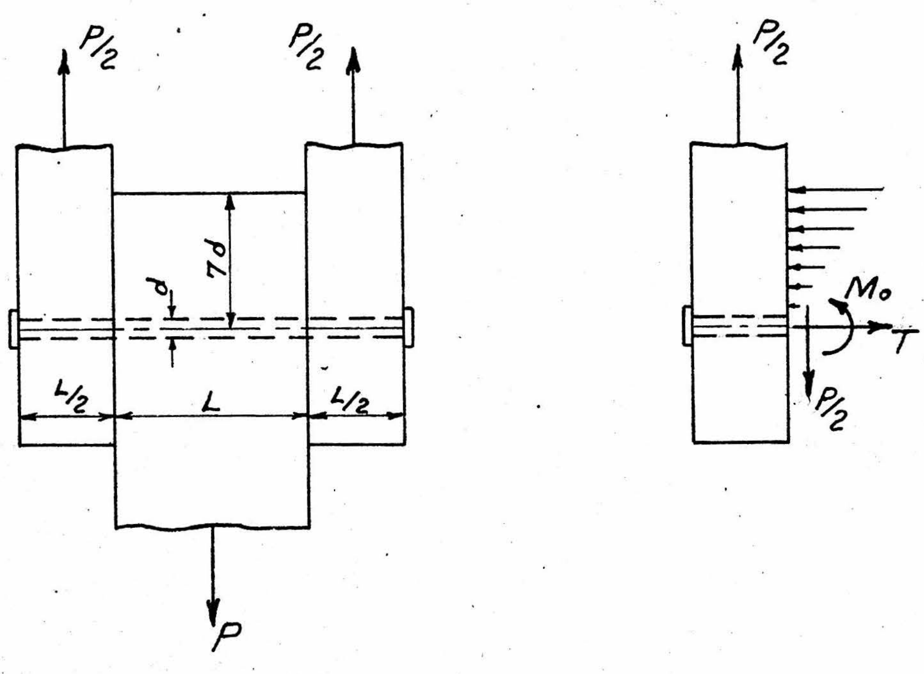
$$\therefore \quad \frac{s}{s_{max.}} = \frac{1}{.57\frac{L}{d}} \frac{\sinh .57\frac{L}{d} + \sin .57\frac{L}{d}}{\cosh .57\frac{L}{d} + \cos .57\frac{L}{d}}$$

This ratio of the allowable uniform stress to the basic stress is plotted against the L/d ratio on sheet 23. It may be seen that the allowable stress drops off rapidly with increasing L/d ratios. The reduction of the basic stress recommended by the Forest Products Laboratory is less than that indicated by this theoretical curve. This discrepancy may be due to three factors: 1) the bolt actually does not fit tightly in the timber; 2) the determined modulus of the wood is incorrect;

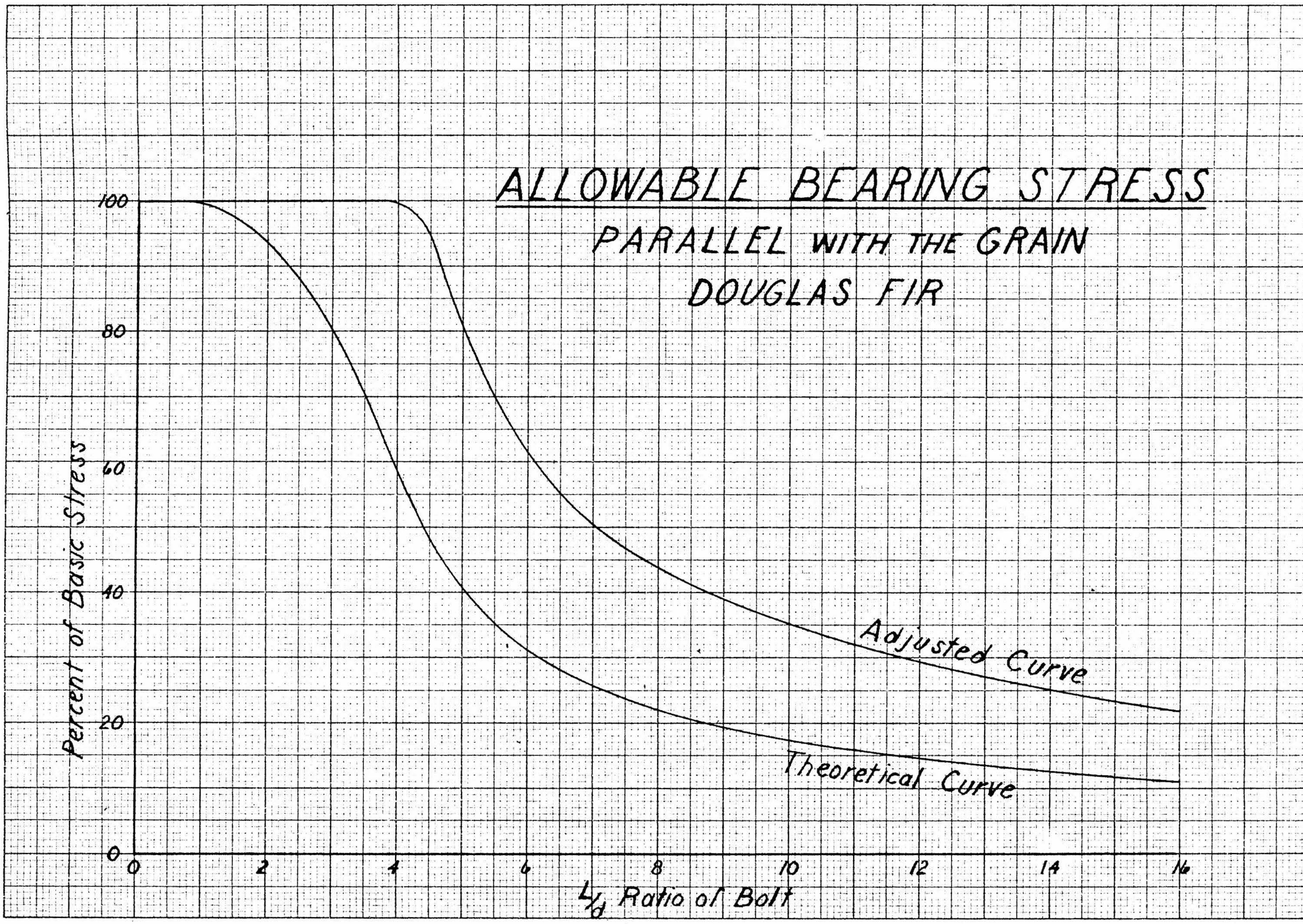
and 3) the maximum edge stress at which the slip of the joint ceases to be proportional to the applied load does not correspond to the proportional limit stress of the wood itself. However, if the theoretical curve be adjusted at one point, the remaining portion of the curve closely follows the Forest Products Laboratory recommendations.

If the joint is loaded perpendicular to the grain of the wood, this theoretical approach fails. In this case the wood fibers under the bolt are placed in tension, thus affording greater bearing capacity than normal. The bearing capacity is related to the width of the bolt, hence the modulus of the wood is not independent of the diameter of the bolt.

VII. Tension in the Bolt



It was stated that under the action of the tension in the in the member, the shear in the bolt, and M_0, the side member


ALLOWABLE BEARING STRESS
PARALLEL WITH THE GRAIN
DOUGLAS FIR
Adjusted Curve
Theoretical Curve
Percent of Basic Stress
100
80
60
40
20
0
0
2
4
6
8
10
12
14
16
L/d Ratio of Bolt

is not in equilibrium for high L/d ratios. This is due to the fact that for high L/d ratios M_o is not equal to PL/8. A portion of the excess moment is taken up by bending of the side member, and the remainder is taken by the tension in the bolt and compression between the side and central member. If it be assumed that bending of the side member is negligible, the magnitude of the tension in the bolt may be determined.

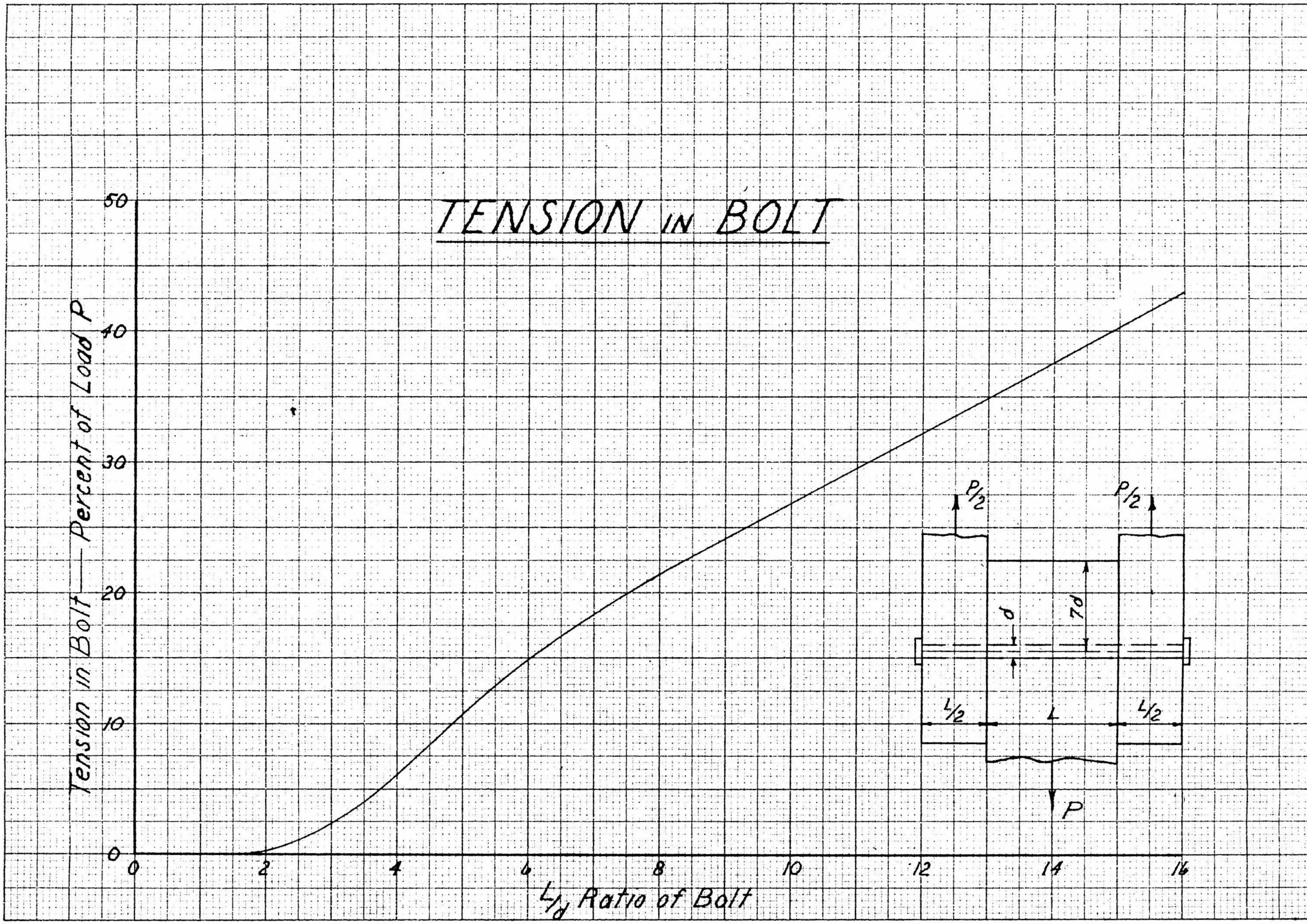
As is the design practice, let the distance from the bolt to the end of the central member be equal to 7d. Assume that the intensity of the compressive stress varies linearly from the bolt to the end of the central member. Let L/d = R, and M_o/PL = B.

$$T\left(\frac{14d}{3}\right) = \frac{P}{2}\left(\frac{L}{4}\right) - M_o$$

$$T\left(\frac{14L}{3R}\right) = \frac{PL}{8} - BPL$$

$$T = \frac{3}{14}\left(\frac{1}{8} - B\right)RP$$

The curve on sheet 25 shows how the tension in the bolt varies with the L/d ratio. It is understood that due to bending of the member itself the tension in the bolt is less than that indicated by the above expression.


TENSION IN BOLT
Tension in Bolt — Percent of Load P
L/d Ratio of Bolt
0
10
20
30
40
50
0
2
4
6
8
10
12
14
16
P/2
P/2
d
7d
L/2
L
L/2
P