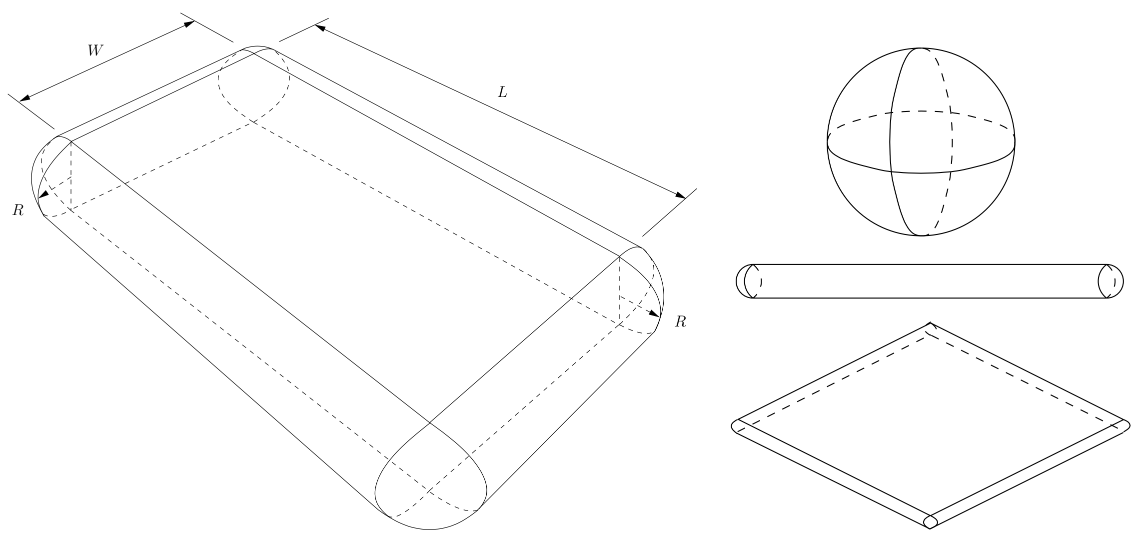
Appendix G

Analytic geometric characterization of limiting surfaces.

Consider the generic surface in Figure G.1(a). It consists of two planar parallel sheets of area LW separated a distance of $2R$; four halves of circular cylinders of radii R and lengths L and W by pairs, tangent to the planar sheets that they connect; and four quarters of a sphere of radius R tangent to the circular cylinders. The resulting surface is closed. The surface is $\mathcal{C}^1$ along the curves of tangency among its parts (across which curvature is discontinuous) and $\mathcal{C}^2$ everywhere else. The area-based jpdf of S and C is thus still applicable.



(a) (b)

Figure G.1: Generic structure (a) and limiting cases (b)

Define $\xi \equiv L/R$, $\eta \equiv W/R$. Note that for $\xi = \eta = 0$ the surface is a sphere, for $\xi \gg 1$ and $\eta = 0$ (and vice versa) the surface is a circular tube with spherical caps (more stretched as ξ increases), and for $\xi, \eta \gg 1$ the surface is predominantly sheet-like (see Figure G.1(b)).

The area and volume of this surface are:

$$A = 4\pi R^2 \left[1 + \frac{1}{2}(\xi + \eta) + \frac{1}{2\pi}\xi\eta\right], \qquad V = \frac{4}{3}\pi R^3 \left[1 + \frac{3}{4}(\xi + \eta) + \frac{3}{2\pi}\xi\eta\right]. \tag{G.1}$$

Therefore

$$\mu \equiv \frac{3V}{A} = R\,\frac{1 + \frac{3}{4}(\xi + \eta) + \frac{3}{2\pi}\xi\eta}{1 + \frac{1}{2}(\xi + \eta) + \frac{1}{2\pi}\xi\eta}, \qquad \lambda \equiv \sqrt[3]{36\pi}\,\frac{V^{2/3}}{A} = \frac{\left[1 + \frac{3}{4}(\xi + \eta) + \frac{3}{2\pi}\xi\eta\right]^{2/3}}{1 + \frac{1}{2}(\xi + \eta) + \frac{1}{2\pi}\xi\eta}. \tag{G.2}$$

The principal curvatures, κ_1 and κ_2, are both $1/R$ in the spherical regions, $1/R$ and 0 respectively in the circular cylindrical regions, and both nil in the planar regions of such surface. Thus the dimensionless curvedness associated with each region is $C_{sph} = \mu/R$, $C_{cyl} = \mu/\sqrt{2}R$, $C_{pla} = 0$, respectively. The absolute value of the shape index is $S_{sph} = 1$ for the spherical regions and $S_{cyl} = 1/2$ for the circular cylindrical regions, while its value is undefined for the planar regions. For the purpose of this illustrative example, define such a value as $\gamma \in [0, 1]$.

The mean values of S and C for the surface, in terms of the dimensionless parameters ξ and η, result:

$$\bar{S} = \frac{1}{A}\left[S_{sph}A_{sph} + S_{cyl}A_{cyl} + S_{pla}A_{pla}\right] = \frac{1 + \frac{1}{4}(\xi + \eta) + \frac{1}{2\pi}\xi\eta\gamma}{1 + \frac{1}{2}(\xi + \eta) + \frac{1}{2\pi}\xi\eta}, \tag{G.3}$$

$$\bar{C} = \frac{1}{A}\left[C_{sph}A_{sph} + C_{cyl}A_{cyl} + C_{pla}A_{pla}\right] = \frac{\left[1 + \frac{3}{4}(\xi + \eta) + \frac{3}{2\pi}\xi\eta\right]\left[1 + \frac{1}{2\sqrt{2}}(\xi + \eta)\right]}{1 + \frac{1}{2}(\xi + \eta) + \frac{1}{2\pi}\xi\eta}. \tag{G.4}$$

In the limiting cases:

(i) for a sphere ($\xi = \eta = 0$): $\bar{S} = \bar{C} = 1$;

(ii) for a predominantly tube-like surface ($\xi \gg 1$, $\eta = 0$): $\bar{S} \approx 1/2$, $\bar{C} \approx 3/2\sqrt{2} \approx 1.06$;

(iii) for a predominantly sheet-like surface ($\xi = \eta \gg 1$): $\bar{S} \approx \gamma$, $\bar{C} \approx 0$.

Figure G.2 shows the dependence on ξ of $\bar{S}$, $\bar{C}$, and λ for the two last cases (surfaces becoming, as ξ increases, tube-like ($\eta = 0$) and sheet-like (with $\eta = \xi$ for simplicity)), starting from the sphere limit ($\xi = 0$). A particular value of γ has been chosen, without loss of generality, in order to represent the limit $\bar{S}_{sheet}$ graphically. In a general sheet-like surface, γ can take any value between 0 and 1, depending on its particular configuration. In the limiting cases ($\xi = 0$ and $\eta = 0$; $\xi \gg 1$ and $\eta = 0$; $\xi = \eta \gg 1$) $\hat{S} \approx \bar{S}$, $\hat{C} \approx \bar{C}$. Thus, a surface predominantly blob-, tube- or sheet-like can be distinguished based on its values of $\hat{S}, \hat{C}, \lambda$.

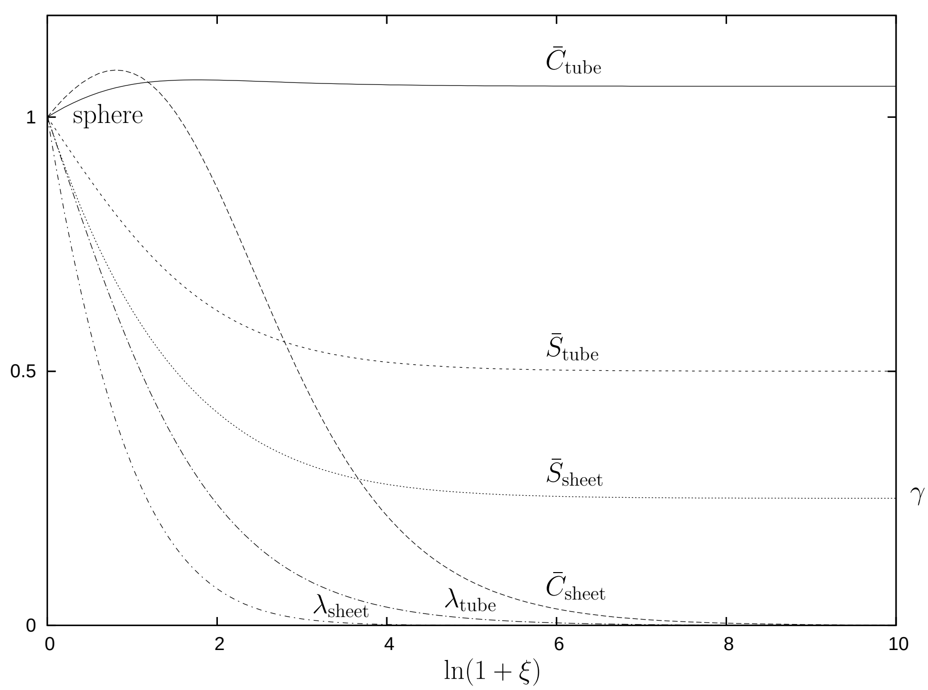


Figure G.2: $\bar{S}$, $\bar{C}$, and λ as a function of ξ for the tube-like and sheet-like limits, evolving from the sphere limit ($\xi = 0$). Note that the abscissa has been rescaled as $\ln(1 + \xi)$ to show more clearly the transition region