THE STATISTIC STUDY OF TEMPERATURE VARIATION IN CHICAGO

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C O N T E N T S

INTRODUCTION

Every change in weather is produced by a certain number of physical causes. For instance, Foehn effect brings a rise of temperature; a clear and calm night shows a larger nocturnal cooling, and so on, but besides those known physical factors, there are other factors influencing the change of the weather which are completely inaccessible to our knowledge. Moreover, the complexity of the combination of those factors increases the difficulty of forecasting the future weather. By theoretical and observed results, we know that in an anticyclone the rainy weather is much less than fair, the reverse is true for a cyclone. Thus in a statistical point of view we can tell what is the probability of certain kinds of weather due to a certain weather factor. Statistics is the study of aggregates of observations or measurements. The most important thing in the study is the frequency distribution of the probability of variates. In general for an infinite universe we often assume the frequency is normally distributed.

Ordinarily the data at hand are finite and are regarded as a random sample drawn from a well defined class of variates called the universe, and from the random sample we draw inference about the universe. The whole theory of sampling is based on frequency distribution and probability. From sampling method we can estimate the mean and standard deviation of the universe. Conversely if we already get the estimates of parameters of the universe, then we can test the significance of the deviation of the parameters of a sample. If the error is not significant it means the observed criterion is not exceptional and vice versa.

The scope of this subject is to study the temperature variation of Chicago in winter. The randum samples are drawn under some specified weather factor, and test the significance of the deviation from the universe.

THE ESTIMATION OF THE POPULATION MEAN AND STANDARD DEVIATION

The mean of a population is the average value of the variates, while the standard deviation measures the extent to which the data are spread out on the average on either side of the mean. In the present case the values are calculated from the data beginning 1936 to 1941.

(a) Maximum temperature:

Table 1

Class Interval		Class Mark	f	u	uf	u^2	fu^2
-5	-1	-3	2	-7	-14	49	98
0	4	2	5	-6	-30	36	180
5	9	7	6	-5	-30	25	150
10	14	12	18	-4	-72	16	288
15	19	17	24	-3	-72	9	216
20	24	22	44	-2	-88	4	176
25	29	27	54	-1	-54	1	54
30	34	32	122	0	0	0	0
35	39	37	123	1	123	1	123
40	44	42	56	2	112	4	224
45	49	47	38	3	114	9	342
50	54	52	33	4	132	16	528
55	59	57	15	5	75	25	375
60	64	62	2	6	12	36	72
			$\Sigma f = 542$		$\Sigma uf = 208$		$\Sigma fu^2 = 2829$

Mean $= \bar{X} = 33.90$

Standard Deviation $= \sigma_x = 10.75$

Where f is the frequency distribution and u is the new variates referred to an arbitrary new origin and expressed in units of class interval.

(b) Minimum temperature

Table 2

Class Interval		Class Mark	f	u	uf	u^2	fu^2
-20	-16	-18	2	-9	-18	81	162
-15	-11	-13	5	-8	-40	64	320
-10	- 6	- 8	8	-7	-56	49	392
- 5	- 1	- 3	10	-6	-60	36	360
0	- 4	2	29	-5	-145	25	725
5	9	7	40	-4	-160	16	640
10	14	12	42	-3	-126	9	378
15	19	17	51	-2	-102	4	204
20	24	22	93	-1	- 93	1	93
25	29	27	108	0	0	0	0
30	34	32	102	1	102	1	102
35	39	37	36	2	72	4	144
40	44	42	7	3	21	9	63
45	49	47	8	4	32	16	128
50	54	52	1	5	5	25	25
			$\Sigma f = 542$		$\Sigma uf = -546$		$\Sigma fu^2 = 3736$

mean $= \tilde{x} = 21.75$

Standard Deviation $= \sigma_x = 12.05$

THE RELATION BETWEEN
TEMPERATURE AND PC HIGH

In winter whenever a Pc High appears in the west part of Canada there is a good probability of break of Pc air and thus causes the drop of temperature. The following samples are drawn from the data of 1936-1941.

Table 3

Year	Month	Date	Date #1	Max °F	Min °F
1936	Dec.	3	4	29	19
1936	Jan.	4	5	30	11
1936	Feb.	4	5	0	-13
1936	Feb.	10	11	10	- 6
1936	Feb.	14	15	10	- 2
1937	Dec.	6	7	22	9
1937	Jan.	4	5	35	17
1937	Jan.	21	22	14	7
1937	Feb.	1	2	23	8
1938	Dec.	26	27	16	3
1938	Jan.	24	25	28	11
1938	Feb.	9	10	35	29
1938	Feb.	19	20	30	28
1939	Jan.	14	15	34	30
1939	Jan.	28	29	36	29
1939	Feb.	2	3	21	14
1940	Dec.	2	3	16	- 5
1940	Dec.	11	12	37	32
1940	Jan.	23	24	17	1

Table 3 (Continued)

Year	Month	Date	Date +1	Max ^oF	Min ^oF
1940	Feb.	21	22	29	21
1941	Dec.	9	10	32	12
1941	Jan.	3	4	23	6
1941	Jan.	18	19	17	4
1941	Feb.	1	2	37	30

A. Test of significance of the deviation of the mean maximum temperature.

a. First Method

Table 4 $\tilde{X} = 33.90$ $\sigma_x = 10.75$

X_i ^oF	$(X_i - \tilde{X})$	$(X_i - \tilde{X})^2$	$\frac{(X_i - \tilde{X})^2}{\sigma_x^2} = \chi^2$
29	4.90	24.01	.20
30	3.90	15.21	.13
0	33.90	1149.21	9.95
110	23.90	571.21	4.95
10	23.90	571.21	4.95
22	11.90	141.61	1.23
35	1.10	1.21	.01
14	19.90	396.01	3.43
23	10.90	118.81	1.03
16	17.90	320.41	2.77
28	5.90	34.81	.30
35	1.10	1.21	.01
30	3.90	15.21	.13
34	.10	1.00	.01
36	2.10	4.41	.04
21	12.90	166.41	1.44
16	17.90	320.41	2.77
37	3.10	9.61	.08
17	16.90	285.61	2.47
29	4.90	24.01	.20
32	1.90	3.61	.03
23	10.90	118.81	1.03
17	16.90	285.61	2.47
37	3.10	9.61	.08

$\Sigma \chi^2 = 39.71$

Degree of freedom = 22

Chance of exceeding a given $\chi^2 = p$ $< .01$

This means that there is a close relation between the temperature at Chicago and the Pc High.

b. Second Method

In the following table pj is the probability of the variates and is estimated from Table 1, f is the frequency distribution of the sample, and N is equal Σf.

Table 5

Class Mark	f	pj	pjN	$(f-p_jN)$	$(f-p_jN)^2$	$\frac{(f-p_jN)^2}{p_jN} = \chi^2$
-3	0	0	0	0	0	0
2	1	.01	.22	.78	.62	2.85
7	0	.01	.26	- .26	.07	.27
12	3	.03	.79	2.21	4.88	6.16
17	4	.04	1.03	2.97	8.81	8.54
22	4	.08	1.94	2.07	4.23	2.17
27	3	.10	2.40	.60	.36	.15
32	4	.23	5.40	-1.40	1.96	.36
37	5	.23	5.45	- .45	.20	.04
42	0	.10	2.47	-2.47	6.11	2.47
47	0	.07	1.68	-1.68	2.82	1.68
52	0	.06	1.46	-1.46	2.14	1.46
57	0	.03	.67	- .67	.45	.67
52	0	0	0	0	0	0

$\Sigma \chi^2 = 26.81$

Degree of freedom = 14

p $\doteq$ .02

This also shows that there is a close relation between Pc High and the temperature at Chicago.

c. Third Method

Table 6

Class Mark	f	u	fu
2	1	-3	-3
7	0	-2	0
12	3	-1	-3
17	4	0	0
22	4	1	4
27	3	2	6
32	4	3	12
37	5	4	20

$\Sigma u = 24$ Σ fu = 36

$u \equiv 1.5$

Mean of the sample $= \bar{X} = 24.5$

$$\text{Deviation} = t_0 \quad \frac{\bar{X} - \tilde{X}}{\frac{\sigma_x}{\sqrt{N}}} = 4.3$$

c. Third Method (Continued)

$$2\int_0^{t_0} \phi(t)\,dt \doteq 1$$

$\therefore$ Probability for X will not be within

$|t_0|$ of $\tilde{X}$ $= P_{t_0}$
$= 0$

This also shows that there is a relation between Pc High and temperature at Chicago.

B. Test of Minimum temperature

a. First Method

$\tilde{X} = 21.75$ $\sigma_x = 12.05$

Table 7

X_i	$X_i - \tilde{X}$	$(X_i - \tilde{X})^2$	$\frac{(X_i - \tilde{X})^2}{\sigma_x^2} = \chi^2$
19	2.75	7.56	.06
11	10.75	115.56	.80
-13	34.75	1207.56	8.32
- 6	27.75	770.06	5.30
- 2	23.75	564.06	3.88
9	12.75	162.56	1.12
17	4.75	22.56	.16
7	14.75	217.56	1.50
8	13.75	189.06	1.30
3	18.75	351.56	2.42
11	10.75	115.56	.80
29	7.25	52.56	.36
28	6.25	39.06	.27
30	8.25	68.06	.47
29	7.25	52.56	.80
14	7.75	60.06	.41
- 5	26.75	715.56	4.93
32	10.25	105.06	.72
1	20.75	430.56	2.97
21	.75	5.62	.04
12	9.75	95.06	.65
6	15.75	248.06	1.71
4	17.75	315.06	2.17
30	8.25	68.06	.47

$\Sigma\chi^2 = 41.63$

degree of freedom $=$ 24

$p < .01$

This means there is a close relation between Pc High and temperature at Chicago.

b. Second Method:

In the following table p_j is estimated from Table (2).

Table 8

Class Interval	f	p_j	p_jN	$f - p_jN$	$(f - p_jN)^2$	$\frac{(f-Np_j)^2}{Np_j} = \chi^2$
-18	0	0	0	0	0	0
-13	1	.009	.22	.78	.62	2.85
- 8	1	.014	.34	.66	.44	1.31
- 3	2	.018	.43	1.57	2.46	5.69
2	3	.053	1.27	1.73	2.99	2.35
7	4	.073	1.75	2.25	5.05	2.88
12	4	.077	1.85	2.15	4.63	2.51
17	2	.094	2.26	- .26	.07	.03
22	1	.171	4.10	-3.10	9.64	2.35
27	3	.199	4.78	-3.78	14.26	2.99
32	3	.188	4.51	-4.51	20.36	4.51
37	0	.066	1.58	-1.58	2.51	1.58
42	0	.012	.29	- .29	.08	.29
47	0	.014	.34	- .34	.11	.34
52	0	0	0	0	0	0

$\Sigma \chi^2 = 29.67$

Degree of freedom $= 15$

$p < .01$

This means that there is a close relation between the Pc High and the temperature at Chicago.

c. Third Method

Table 9

Class Interval	f	u	fu
-13	1	-5	-5
8	1	-4	-4
3	2	-3	-6
2	3	-2	-6
7	4	-1	-4
12	4	C	0
17	2	1	2
22	1	2	2
27	3	3	9
32	3	4	12

$\Sigma f = 24$

$\Sigma fu = 0$

$\overline{X} = 12$

$t_0 = 5.2$

$p_{t_0} < .01$

This means there is a close relation between Pc High and the temperature at Chicago.

THE RELATION BETWEEN RETURN Pc AIR MASS
AND THE TEMPERATURE OF CHICAGO

Whenever there is a High of Pc air mass located at the southern part of middle United States there is a good chance for return flow of Pc air to the station of Chicago. The following sample is drawn from the data of 1936-1941.

Table 10

Year	Month	Date	Date +1	Max.	Min.
1936	Dec.	4	5	36	25
1936	Dec.	21	22	37	24
1941	Jan.	6	7	31	23
1941	Jan.	20	21	45	25
1941	Feb.	4	5	35	21
1941	Feb.	23	24	38	24
1937	Dec.	2	3	36	30
1937	Dec.	30	31	52	33
1940	Jan.	9	10	32	22
1940	Feb.	3	4	34	29
1940	Feb.	22	23	32	20
1939	Jan.	22	23	24	9
1939	Jan.	27	28	35	31
1939	Feb.	22	23	32	6
1938	Dec.	22	23	35	29
1939	Dec.	14	15	47	31
1938	Jan.	8	9	23	8
1938	Feb.	4	5	55	43
1938	Feb.	16	17	48	33
1940	Dec.	8	9	52	23
1940	Dec.	23	24	53	32
1937	Jan.	23	24	28	17
1937	Feb.	2	3	33	21
1941	Dec.	6	7	38	26
1941	Dec.	30	31	35	30
1936	Jan.	7	8	34	28
1936	Feb.	5	6	15	3
1936	Feb.	19	20	20	3

A. Test of significance of deviation of the mean of the maximum temperature.

a. First Method

Table 11 $\tilde{X}$ 33.90 $\sigma_x = 10.75$

X_i	$X_i - \tilde{X}$	$(X_i - \tilde{X})^2$	$\frac{(X_i - \tilde{X})^2}{\sigma_x^2} = \chi^2$
36	2.10	4.41	.04
37	3.10	9.61	.08
31	2.90	8.41	.07
45	11.10	123.21	1.07
35	1.10	1.21	.01
38	4.10	16.81	.15
36	2.10	4.41	.04
52	18.10	327.61	2.84
32	1.90	3.61	.03
34	.10	.01	0
32	1.90	3.61	.03
24	9.00	8 1.00	.70
35	1.10	1.21	.01
32	1.90	3.61	.03
35	1.10	1.21	.01
47	13.10	171.61	1.49
23	10.90	118.81	1.03
55	21.10	445.21	3.85
48	14.10	198.81	1.72
52	18.10	327.61	2.84
53	19.10	364.81	3.16
28	5.90	34.81	.30
33	.90	.81	.007
38	4.10	16.81	.15
35	1.10	1.21	.01
34	.10	.01	0
15	18.90	357.21	3.09
20	13.90	193.21	1.67

$$\Sigma \chi^2 = 24.42$$

Degree of freedom — 28 — 2 = 26

$p > .5$

This means that the return Pc air mass has no significant relation to the variation of temperature at Chicago.

b. Second Method

Table 12 $N = 28$

Class Mark	f	p_j	p_jN	$f - p_jN$	$(f - Np_j)$	$\frac{(f-Np_j)}{Np_j} = \chi^2$
-3	0	.004	.11	- .11	.01	.12
2	0	.009	.25	.25	.06	.25
7	0	.011	.31	.31	.09	.31
12	0	.033	.92	.92	.85	.92
17	1	.043	1.20	.20	.04	.03
22	3	.081	2.27	.73	.54	.24
27	1	.100	2.80	1.80	3.24	1.16
32	7	.225	6.30	.70	.49	.08
37	9	.227	6.36	2.64	6.99	1.10
42	0	.103	2.88	2.88	8.32	2.88
47	2	.070	1.96	.04	.00	.00
52	3	.061	1.71	1.29	1.67	.98
57	0	.026	.73	.73	.53	.73
62	0	.004	.11	.11	.01	.12

$\Sigma \chi^2 = 8.91$

Degree of freedom $= 14$ $p > .5$

This means that the return Pc air mas has no significant relation with the variation of temperature at Chicago.

c. Third Method

Table 13

Class Mark	f	u	fu
17	1	-4	-4
22	3	-3	-9
27	1	-2	-2
32	7	-1	-7
37	9	0	0
42	0	1	0
47	2	2	4
52	3	3	9
	$\Sigma f = 28$		$\Sigma fu = -9$

$\overline{X} = 35.9$ $t_0 = 1.5$

$p_{t_0} = .13$

This means the return flow of Pc has no significant relation with the variation of the temperature at Chicago.

B. Testing of minimum temperature

Table 14 $\widetilde{X} = 21.75$ $\sigma_x = 12.05$

X_i	$X_i - \widetilde{X}$	$(X_i - \widetilde{X})^2$	$\frac{(X_i - \widetilde{X})}{\sigma_x^2} = \chi^2$
25	3.25	10.56	.07
24	2.25	5.06	.03
23	1.25	1.56	.01
25	3.25	10.56	.07
21	.75	5.62	.04
24	2.25	5.06	.03
30	8.25	68.06	.47
33	11.25	126.56	.87
22	.25	.06	.0
29	7.25	52.56	.80
20	1.75	3.06	.02
9	12.75	162.56	1.12
31	9.25	85.56	.59
6	15.75	248.06	1.71
29	7.25	52.56	.36
31	9.25	85.56	.59
8	13.75	189.06	1.30
43	21.25	451.56	3.11
33	11.25	126.56	.87
23	1.25	1.56	.01
32	10.25	105.06	.72
17	4.75	22.56	.19
21	.75	5.62	.04
26	4.25	18.06	.12
30	8.25	68.06	.47
28	6.25	39.06	.27
3	18.75	351.56	2.42
3	18.75	351.56	2.42

$\Sigma \chi^2 = 18.72$

Degree of freedom = 28 — 2 = 26

$p > .5$

This means the return flow of Pc air mass has no significant relation with the variation of minimum temperature at Chicago.

b. Second Method

Table 15 N = 28

Class Mark	f	p_j	p_jN	$(f-p_jN)$	$(f-p_jN)^2$	$\frac{(f-p_jN)^2}{Np_j} = \chi^2$
-18	0	0	0	0	0	0
-13	0	.009	.25	.25	.06	.25
- 8	0	.014	.39	.39	.15	.39
- 3	0	.018	.50	.50	.25	.50
2	2	.053	1.48	.52	.27	.18
7	3	.073	2.04	.96	.91	.45
12	0	.077	2.16	2.16	4.68	2.16
17	1	.094	2.63	1.63	2.66	1.01
22	8	.171	4.79	3.21	10.32	2.15
27	6	.199	5.57	.43	.18	.03
32	7	.188	5.26	1.74	3.01	.57
37	0	.066	1.85	1.85	3.42	1.85
42	1	.012	.34	.66	.44	1.31
47	0	.014	.39	.39	.15	.39
52	0	0	0	0	0	0

$\Sigma\chi^2 = 11.249$

Degree of freedom = 15 $p > .5$

This means the return flow of Pc air mass has no significant relation with the variation of the minimum temperature at Chicago.

c. Third Method

Table 16 $\tilde{\chi} = 21.75$ $\sigma_X = 12.05$

Class Mark	f	u	fu
2	2	-4	-8
7	3	-3	-9
12	0	-2	0
17	1	-1	-1
22	8	0	0
27	6	1	6
32	7	2	14
37	0	3	0
42	1	4	4
	$\Sigma f = 28$		$\Sigma fu = 6$

$\bar{u} = .21$

$\bar{X} = 23.1$ $t_o = .62$

$p_{t_o} = .36$

This means the return flow of Pc air masses has no significant relation with the variation of minimum temperature at Chicago.

THE RELATION BETWEEN THE TEMPERATURE
AND BASIN HIGH

Whenever a High remained in the Great Basin of the Rocky Mountains there is a tendency of flow of Polar Basin air mass, and hence there may be some variation of the temperature at Chicago. The following sample is drawn from the data of 1936 to 1941.

Table 17

Year	Month	Date	Date + 1	Max.	Min.
1937	Jan.	2	3	22	13
1938	Jan.	3	4	37	20
1938	Jan.	21	22	38	34
1939	Jan.	9	10	53	34
1939	Jan.	18	19	31	25
1939	Jan.	29	30	30	23
1941	Jan.	11	12	43	29
1941	Jan.	29	30	37	25
1936	Dec.	1	2	37	32
1936	Dec.	13	14	48	32
1937	Dec.	17	18	33	28
1938	Dec.	2	3	49	36
1938	Dec.	11	12	32	23
1939	Dec.	3	4	36	30
1939	Dec.	12	13	38	26
1940	Dec.	6	7	42	27
1940	Dec.	29	30	37	33
1941	Dec.	12	13	33	23
1937	Feb.	10	11	40	17
1937	Feb.	15	16	33	27
1938	Feb.	21	22	38	34
1939	Feb.	17	18	47	31
1940	Feb.	9	10	32	19
1941	Feb.	4	5	35	21

A. Testing of maximum temperature

a. First Method

Table 18 $\tilde{X} = 33.90$ $\sigma_x = 10.75$

X_i	$X_i - \tilde{X}$	$(X_i - \tilde{X})^2$	$\frac{(X_i - \tilde{X})^2}{\sigma_x^2} = \chi^2$
22	11.90	141.61	1.23
37	3.10	9.61	.08
38	4.10	16.81	.15
53	19.10	364.81	3.16
31	2.90	8.41	.07
30	3.90	15.21	.13
43	9.10	82.81	.72
37	3.10	9.61	.08
37	3.10	9.61	.08
48	14.10	198.81	1.72
33	.90	.81	.007
49	15.10	228.01	1.97
32	1.90	3.61	.03
36	2.10	4.41	.04
38	4.10	16.81	.15
42	8.10	65.61	.57
37	3.10	9.61	.08
33	.90	.81	.007
40	6.10	37.21	.32
33	.90	.81	.007
38	4.10	16.81	.15
47	13.10	171.61	1.49
32	1.90	3.61	.03
35	1.10	1.21	.01

$\Sigma \chi^2 = 12.26$

Degree of freedom = 24 — 2 = 22

$p > .5$

This shows that there is no significant relation between the Basin High and the variation of the temperature at Chicago.

b. Second Method

Table 19 N 24

Class Mark	f	p_j	p_jN	$f-p_jN$	$(f-p_jN)^2$	$\frac{(f-p_jN)^2}{p_jN} = \chi^2$
-3	0	.004	.096	- .096	.009	.093
2	0	.009	.216	- .216	.047	.217
7	0	.011	.264	- .264	.070	.265
12	0	.033	.792	- .792	.627	.791
17	0	.043	1.032	-1.032	1.065	1.031
22	1	.081	1.944	- .944	.891	.458
27	0	.100	2.400	-2.400	5.760	2.400
32	7	.225	5.400	1.600	2.560	.474
37	9	.227	5.448	3.552	12.617	2.315
42	3	.103	2.472	.528	.279	.112
47	3	.070	1.680	1.320	1.742	1.036
52	1	.061	1.464	- .464	.215	.146
57	0	.028	.672	- .672	.452	.672
62	0	.004	.096	- .096	.009	.093

$\Sigma \chi^2 =$ 10.103

Degree of freedom = 14 $p > .5$

This means that Basin High has no significant relation with the variation of temperature at Chicago.

c. Third Method

Table 20 $\tilde{X} =$ 33.9 $\sigma_X = 10.75$

Class Mark	f	u	fu
22	1	-3	-3
27	0	-2	0
32	7	-1	-7
37	9	0	0
42	3	1	3
47	3	2	6
52	1	3	3

$\Sigma f = 24$ $\Sigma fu = 2$

$\bar{u} = .08$

$\bar{X} = 37.4$

$t_0 = 1.6$

$p_{t_0} = .11$

This means that the Basin High has no significant relation with the variation of temperature at Chicago.

B. Testing of minimum temperature

A. First Method

Table 21 $\tilde{X}=21.75$ $\sigma_x = 12.05$

X_i	$(X_i - X)$	$(X_i - \tilde{X})^2$	$\frac{(X_i - \tilde{X})^2}{\sigma_x^2} = \chi^2$
13	8.75	76.56	.53
20	1.75	3.06	.02
34	12.25	150.06	1.03
34	12.25	150.06	1.03
25	3.25	10.56	.07
23	1.25	1.56	.01
29	7.25	52.56	.80
25	3.25	10.56	.07
32	10.25	105.06	.72
32	10.25	105.06	.72
28	6.25	39.06	.27
36	14.25	203.06	1.40
23	1.25	1.56	.01
30	8.25	68.06	.47
26	4.25	18.06	.12
27	5.25	27.56	.19
33	11.25	126.56	.87
23	1.25	1.56	.01
17	4.75	22.56	.16
27	5.25	27.56	.19
34	12.25	150.06	1.03
31	9.25	85.56	.59
19	2.75	7.56	.06
21	.75	5.62	.04

Degree of freedom = N-2
= 24-2
= 22

$P > .5$

This means there is no significance.

b. Second Method

Table 22 N = 24

Class Mark	f	p_j	p_jN	$f-p_jN$	$(f-p_jN)^2$	$\frac{(f-p_jN)^2}{p_jN} = \chi^2$
-18	0	0	0	0	0	0
-13	0	.009	.22	- .22	.05	.22
- 8	0	.014	.34	- .34	.11	.34
- 3	0	.018	.43	- .43	.19	.43
2	0	.053	1.27	-1.27	1.62	1.27
7	0	.073	1.75	-1.75	3.07	1.75
12	1	.077	1.85	- .85	.72	.39
17	2	.094	2.26	.26	.07	.03
22	5	.171	4.11	.90	.80	.20
27	7	.199	4.78	2.25	4.95	1.04
32	8	.188	4.51	3.49	12.17	2.70
37	1	.066	1.58	- .58	.34	.22
42	0	.012	.29	- .29	.08	.29
47	0	.014	.34	- .34	.11	.34
52	0	0	0	0	0	0

$\Sigma \chi^2 = 9.17$

Degree of freedom = 15

$p > .5$

This means the deviation is not significant.

c. Third Method

Table 23 $N = 21.75$ $= 12.05$

Class Mark	f	u	fu
12	1	-3	-3
17	2	-2	-4
22	5	-1	-5
27	7	0	0
32	8	1	8
37	1	2	2

$\Sigma f = 24$

$\Sigma fu = -2$
$\bar{u} = .08$
$\bar{x} = 26.6$
$t_o = 1.95$
$p_t = .052$

This shows the deviation is not significant.

THE RELATION BETWEEN ZONAL FLOW AT UPPER LEVELS AND THE TEMPERATURE VARIATION AT CHICAGO

If the circulation at the upper levels is zonal flow, then in general the circulation on the surface is also zonal. There are little chances for temperature variation during a zonal flow. The following sample is drawn from data at 3 km., 1932 to 1938.

Year	Month	Date	Date + 1	Max.	Min.
1937	Dec.	27	28	37	33
1937	Dec.	12	13	28	22
1936	Dec.	7	8	40	21
1934	Dec.	28	29	35	22
1933	Dec.	12	13	31	23
1933	Dec.	22	23	58	34
1932	Dec.	9	10	25	13
1936	Feb.	19	20	43	33
1936	Feb.	12	13	36	22
1936	Feb.	2	3	27	16
1938	Jan.	17	18	30	25
1938	Jan.	22	23	43	30
1933	Jan.	4	5	43	27
1937	Jan.	5	6	42	27
1937	Jan.	15	16	30	13
1937	Jan.	26	27	35	21
1936	Jan.	10	11	39	31
1937	Feb.	12	13	49	29
1937	Feb.	18	19	49	32
1936	Jan.	16	17	29	26
1933	Jan.	3	4	41	31
1933	Dec.	31	Jan. 1	46	24

A. Testing of maximum temperatures

a. First method

Table 27 $\tilde{X} = 33.90$ $\sigma_x = 10.75$

x_i	$(x_i - x)$	$(x_i - \tilde{x})^2$	$\frac{(x_i - \tilde{x})^2}{\sigma_x^2} = \chi^2$
37	3.10	9.61	.08
28	5.90	34.81	.30
40	6.10	37.21	.32
35	1.10	1.21	.01
31	2.90	8.41	.07
58	24.10	580.81	5.03
25	8.90	79.21	.69
43	9.10	82.81	.72
36	2.10	4.41	.04
27	6.90	47.61	.41
30	3.90	15.21	.13
43	9.10	82.81	.72
43	9.10	82.81	.72
42	8.10	65.61	.57
30	3.90	15.21	.13
35	1.10	1.21	.01
39	5.10	26.01	.23
49	15.10	228.01	1.97
49	15.10	228.01	1.97
29	4.90	24.01	.20
41	7.10	50.41	.44
46	12.10	146.41	1.27

$\Sigma\chi^2 = 16.03$

Degree of Freedom $= 24 - 2 = 22$

$p > .5$

b. Second method

Table 28 N = 22

Class Mark	f	p_j	p_jN	$f - p_jN$	$(f - p_jN)^2$	$\frac{(f - p_jN)^2}{p_jN} = \chi^2$
-3	0	.004	.088	.088	.007	.079
2	0	.009	.198	-.198	.039	.196
7	0	.011	.242	-.242	.058	.239
12	0	.033	.726	-.726	.527	.725
17	0	.043	.946	-.946	.895	.946
22	0	.081	1.782	-1.782	3.176	1.782
27	4	.100	2.200	1.800	3.240	1.472
32	3	.225	4.950	1.950	3.802	.768
37	5	.227	4.994	.006	.000	.000
42	6	.103	2.266	3.734	13.943	6.153
47	3	.070	1.540	1.460	2.132	1.348
52	0	.061	1.342	-1.342	1.801	1.342
57	1	.026	.572	.428	.183	.319
62	0	.004	.088	-.088	.007	.079

$\Sigma \chi^2 = 15.584$

Degree of freedom = 14

p = .33

This means the deviation is not significant.

c. Third Method

Table 29 $\tilde{x} = 33.90$

Class Mark	f	u	fu
27	4	-3	-12
32	3	-2	- 6
37	5	-1	- 5
42	6	0	0
47	3	1	3
52	0	2	0
57	1	3	3

$\Sigma f = 22$ $\Sigma fu = -17$

$\bar{u} = -.77$

$\bar{x} = 38.2$

$t_o = 1.89$

$p_{t_o} = .059$

This means the deviation is not significant.

B. Testing of significance of minimum temperature.

a. First Method

Table 30 $\tilde{x} = 21.75$ $\sigma_x = 12.05$

x_i	$(x_i - x)$	$(x_i - x)^2$	$\frac{(x_i - x)}{\sigma_x} = \chi^2$
33	11.25	126.56	.87
22	.25	.06	.00
21	.75	5.62	.04
22	.25	.06	.00
23	1.25	1.56	.01
34	12.25	150.06	1.03
13	11.25	126.56	.87
33	11.25	126.56	.87
22	.25	.06	.00
16	5.75	53.06	.37
25	3.25	10.56	.07
30	8.25	68.06	.47
27	5.25	27.56	.19
27	5.25	27.56	.19
13	8.75	76.56	.53
21	.75	5.62	.04
31	9.25	85.56	.59
29	7.25	52.56	.36
32	10.25	105.06	.72
26	4.25	18.06	.12
31	9.25	85.56	.59
24	2.25	5.06	.03

$\Sigma \chi^2 = 7.96$

Degree of freedom = 22

$p > .5$

This means the deviation is not significant.

b. Second Method

Table 31 $N = 22$

Class Mark	F	p_j	p_jN	$(f-p_jN)$	$(f-p_jN)^2$	$\frac{(f-p_jN)^2}{p_jN} = \chi^2$
-18	0	0	0	0	0	0
-13	0	.009	.20	-.20	-.04	.20
-8	0	.014	.31	-.31	-.10	.31
-3	0	.018	.40	-.40	.16	.40
2	0	.053	1.17	-1.17	1.36	1.17
7	0	.073	1.61	-1.61	2.58	1.61
12	2	.077	1.69	- .31	.09	.06
17	1	.094	2.07	1.07	1.14	.55
22	7	.171	3.76	-3.24	10.49	2.79
27	5	.199	4.38	- .62	.39	.09
32	7	.188	4.14	-2.86	8.20	1.98
37	0	.066	1.45	-1.45	2.11	1.45
42	0	.012	.26	- .26	-.07	.27
47	0	.014	.31	- .31	.10	.31
52	0	0	0	0	0	0

$\Sigma \chi^2 = 11.16$

Degree of Freedom = 15

p 7.5

This means the deviation is not significant.

c. Third Method

Table 32 $\tilde{x} = 21.75$ $\sigma_x = 12.05$

Class Mark	f	u	fu
12	2	-2	-4
17	1	-1	-1
22	7	0	0
27	5	1	5
32	7	2	14

$\Sigma f = 22$ $\Sigma fu = 14$

$\bar{u} = 64$

$\bar{x} = 25.2$

$t_0 = 1.36$

$p_{t_0} = .16$

This means the deviation is not significant

THE RELATION BETWEEN MERIDIANAL FLOW AT UPPER LEVEL AND THE TEMPERATURE VARIATION AT CHICAGO

If the circulation at the upper levels is a meridianal flow, then there is a good chance for the variation of temperature on the surface. The following sample is drawn from the data at 300 km. level 1932-1938.

Table 33

Year	Month	Date	Date + 1	Max	Min
1937	Dec.	20	21	37	22
1936	Dec.	11	12	37	16
1937	Dec.	5	6	23	11
1936	Dec.	19	20	36	20
1932	Dec.	15	16	11	-4
1933	Feb.	8	9	-6	-19
1935	Feb.	25	26	21	10
1935	Feb.	17	18	40	31
1936	Feb.	10	11	10	-6
1937	Feb.	21	22	20	12
1938	Jan.	25	26	17	10
1938	Jan.	12	13	31	14
1938	Jan.	7	8	12	0
1936	Jan.	30	31	6	-6
1936	Jan.	21	22	13	-17
1937	Jan.	2	3	22	13
1933	Jan.	13	14	34	20

A. Testing for maximum temperature.

a. First Method

Table 34 $\tilde{X} = 33.9$ $\sigma_x = 10.75$

X_i	$X_i - \tilde{X}$	$(X_i - \tilde{X})^2$	$\frac{(X_i - \tilde{X})^2}{\sigma_x^2} = \chi^2$
37	3.1	9.61	.08
37	3.1	9.61	.08
23	10.9	118.81	1.03
36	2.1	4.41	.04
11	22.9	524.41	4.54
-6	39.9	1149.21	9.95
21	12.9	166.41	1.44
40	6.1	37.21	.32
10	23.9	571.21	4.94
20	13.9	193.21	1.67
17	16.9	285.61	2.47
31	2.9	8.41	.07
12	21.9	479.61	4.15
6	27.9	778.41	6.74
13	20.9	436.81	3.78
22	11.9	141.61	1.23
34	.1	.01	.00

$\Sigma \chi^2 = 42.53$

Degree of freedom = 17-2 = 15

$p < .01$

This means the deviation is significant.

b. Second Method

Table 35

Class Mark	f	p_j	p_jN	$f-p_jN$	$(f-p_jN)^2$	$\frac{(f-p_jN)^2}{p_jN} = \chi^2$
-8	1	0	0	0	0	0
-3	0	.004	.07	- .07	.01	.07
2	0	.007	.15	-.153	.02	.02
7	1	.011	.19	.813	.66	3.53
12	4	.033	.56	3.44	11.84	21.08
17	1	.043	.73	.27	.07	.1
22	4	.081	1.35	2.65	7.04	5.23
27	0	.100	1.70	1.70	2.89	1.70
32	2	.225	3.83	-1.83	3.34	.87
37	3	.227	3.87	-0.87	.76	.25
42	1	.103	1.75	.75	.56	.32
47	0	.070	1.23	1.23	1.51	1.23
52	0	.061	1.07	1.07	1.14	1.07
57	0	.028	.49	.49	.24	.49
62	0	.000	.07	.07	.01	0

b. Second Method (Continued)

$$\Sigma \chi^2 = 35.86$$

Degree of freedom = 15

$$p < .01$$

This means the deviation is significant.

c. Third Method

Table 36 $\bar{X} = 33.9$ $\sigma_x = 10.75$

Class Interval		Class Mark	freq	u	uf
-10	-6	-8	1	-5	-5
-5	-1	-3	0	-4	0
0	4	2	0	-3	0
5	9	7	1	-2	-2
10	19	12	4	-1	-4
15	19	17	1	0	0
20	24	22	4	1	4
25	29	27	0	2	0
30	34	32	2	3	6
35	39	37	3	4	12
40	44	42	1	5	6

$\Sigma f = 17$ $\Sigma uf = 17$

$\bar{u} = 1.00$

$\bar{X} = 22.0$

$t_0 = 4.45$

$p_{t_0} < .01$

This means the deviation is significant.

B. Testing of minimum temperature.

a. First Method

Table 37 $\tilde{X} = 21.75$ $\sigma_x = 12.05$

X_i	$(X_i - \tilde{X})$	$(X_i - \tilde{X})^2$	$\frac{(X_i - \tilde{X})^2}{\sigma_x^2} = \chi^2$
22	.25	.06	.00
16	5.75	33.06	.23
11	10.75	115.56	.80
20	1.75	3.06	.02
-4	25.75	663.06	4.57
-19	40.75	1660.56	11.44
10	11.75	138.06	.95
31	9.25	85.56	.59
-6	27.75	770.06	5.30
12	9.75	95.06	.65
10	11.75	138.06	.95
14	7.75	60.06	.41
0	21.75	473.06	3.26
-6	27.75	770.06	5.30
-17	38.75	1501.56	10.34
13	8.75	76.56	.53
20	1.75	3.06	.02

$\Sigma \chi^2 = 45.36$

Degree of freedom = 17-2 = 15

$p < .01$

This means the deviation is significant.

b. Second Method

Table 38 N = 17

Class Mark	f	pj	$p_j N$	$f - p_j N$	$(f - p_j N)^2$	$\frac{(f - p_j N)^2}{p_j N} = \chi^2$
-18	2	0	0	0	0	0
-13	0	.009	.15	-.15	.02	.15
- 8	2	.014	.24	1.76	3.11	13.05
- 3	1	.018	.31	.69	.48	1.58
2	1	.053	.90	.09	.01	.01
7	0	.073	1.24	-1.24	1.54	1.24
12	6	.077	1.31	4.69	22.01	16.81
17	1	.094	1.60	-.60	.36	.22
22	3	.171	2.91	.09	.01	0
27	0	.199	3.38	-3.38	11.45	3.38
32	1	.188	3.20	-2.20	4.82	1.51
37	0	.066	1.12	-1.12	1.26	1.12
42	0	.012	.20	- .20	.04	.21
47	0	.014	.24	- .24	.06	.24
52	0	0	0	0	0	0

$\Sigma \chi^2 = 29.51$

Degree of freedom = 15

b. Second Method (Continued)

$p < .01$

This means the deviation is significant.

c. Third Method

Table 39 $\tilde{X} = 21.75$ $\sigma_x = 12.05$

Class Interval		Class Mark	f	u	uf
-20	-16	-18	2	-6	-12
-15	-11	-13	0	-5	0
-10	- 6	- 8	2	-4	- 8
- 5	- 1	- 3	1	-3	- 3
0	4	2	1	-2	- 2
5	9	7	0	-1	0
10	14	12	6	0	0
15	19	17	1	+1	1
20	24	22	3	+2	6
25	29	27	0	+3	0
30	34	32	1	+4	4

$\Sigma f = 17$ $\Sigma uf = 14$

$\bar{u} = -.825$

$\bar{X} = 12 - 4.1 = 7.9$

$t_0 = 4.75$

$p_{t_0} < .01$

This means the deviation is significant.

CONCLUSION

In the foregoing test we find that the variation of temperature at Chicago is effected by the break of pc air mass and meridianal flow only. There is no significant deviation in temperature due to the return Pc air mass, the Polar Basin air mass and the zonal flow, however. The mean value of the temperatures of all the five samples could be used as a reference in forecasting. For example:

a. When a PC High located in west Canada the meane maximum temperature 24.5° F. shown in page 5, mean temperature 12.0° F. shown in page 7 can be used as a reference in 24 hour forecast.

b. When a High of Pc air mass located at the southern part of middle United States the mean maximum temperatures 35.9° F. shown in page 10, mean minimum temperature 23.1° F. shown in page 12, can be used as a reference in 24 hour forecast.

c. When a High remained in the Great Basin the mean maximum temperature 37.4° F. shown in page 15, the mean minimum temperature 26.6° F. shown in page 18, can be used as a reference in 24 hour forecast.

d. When the circulation at the upper level is zonal the mean maximum temperature 38.2° F. shown in page 22, the mean minimum temperature 25.2° F. shown in page 24 can be used as a reference.

e. When the circulation at the upper level is meridianal the mean maximum temperature 22° F. shown in page 27, the mean minimum temperature 8° F. shown in page 29, can be used as a reference.