ECONOMIC POSSIBILITIES OF
LONG RANGE COMMERCIAL ROCKET TRANSPORTS

Thesis by

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SUMMARY

In recent publications the economic aspects of both subsonic and supersonic jet transports have been discussed in some detail. Though the high subsonic speed turbojet transport, with some restrictions on range, was found to be of interest economically, no reasonable economic case was found for the supersonic jet transport utilizing either turbojet or ramjet engines. These calculations were made, however, with the assumption of a conventional level flight path. The purpose of this study is to investigate the possibility of using a rocket motor for propulsion of a supersonic transport flying a ballistic trajectory, with the hope that the direct operating expense can be reduced to a value comparable to that of subsonic transports, and thus to indicate the economic feasibility of such high speed transports.

It is shown that direct operating costs of rocket motored transports can be less than those indicated for the turbojet and ramjet transports, and in some cases, even approach the cost of operating currently proposed subsonic turbojet engine powered transports. Also the operating ranges can be extended beyond those possible with the supersonic jet transports postulated.

The flight technique used is one which allows the high thrust of the rocket motor to produce sufficient kinetic energy in the vehicle to permit it to coast and glide the desired range.

Performance is calculated with consideration given to endurance to acceleration limits imposed by human occupants. It is shown that

payload optimization decreases design acceleration to a point that the human endurance is not the limiting factor. Weight breakdowns for the major components of the airplane are made, and costs computed are based on Air Transportation Association formulas given for the purpose of comparing operating costs of various proposed aircraft.

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SYMBOLS

A - burnout point

c - effective exhaust velocity $= I_{sp}\, g$

C - constant

d - diameter of rocket

D - drag

e - base of natural logarithms

F - thrust

g - acceleration of gravity

h - altitude

I_{sp} - specific impulse

L - lift

$\ln$ - natural logarithm

m - instantaneous mass

M - Mach number

n - acceleration in terms of units of g

P - power

P_{ℓ} - payload in tons

Q - rate of flow of blood

r - radius of curvature of pullout path

R - radius of earth (3956 miles)

S - range in miles

t - time

T_s - surface temperature

$T_{(n)}$ - time to blackout

u - velocity during glide

SYMBOLS (CONTINUED)

v - velocity during elliptic trajectory

V - average flight velocity

w - velocity during pullout

W - weight

x - surface length

y - altitude during burning

$\dot{y}$ - velocity during burning

$\frac{¢}{TM}$ - direct operating cost in cents per ton mile

ϵ - specific resistance

ζ_o - flow parameter

θ_o - central angle measured from center of earth, between initial point and summit of elliptic trajectory

λ - lift to drag ratio

μ - viscosity of air

ξ - propellant loading ratio

ρ - density of air

ψ^* - optimum angle of pitch upon entry into elliptic trajectory

Subscripts

o - conditions at beginning of any phase

1 - conditions at end of phase

all - allowable

ave - average

f - free stream conditions

s - surface conditions

SYMBOLS (CONTINUED)

Superscripts

1 - elliptic trajectory conditions

2 - pullout conditions

3 - glide conditions

I. INTRODUCTION

Recent studies of both subsonic (Reference 1) and supersonic (Reference 2) jet transports point out the tremendous cost of speed. Hage (Reference 1) shows that subsonic turbojet transports can be made to compete costwise with piston-engine powered transports of today, within certain limitations on range. However, it is concluded in a later publication by Hage (Reference 2) that a reasonable case cannot be made for the supersonic turbojet and ramjet powered transports, due not only to their high operating costs, but due also to their very limited operating range.

In the latter investigation consideration is given the rocket motor as the power plant for the transport, but its use is quickly ruled out because of its high specific fuel consumption. For steady level flight, as was considered in this reference, this result is logical. In this study, however, a flight technique somewhat different is used, as proposed by several writers (References 3, 4, 5). This technique will allow the high thrust of the rocket motor, burning over a relatively short time, to produce sufficient kinetic energy for the airplane to coast and glide over the desired range.

Figure 1 shows a schematic diagram of this postulated flight trajectory which can be broken down into the following four phases: (a) With its motor producing constant thrust, the airplane takes off vertically and climbs on a controlled curved path which brings the airplane to the burnout point with the desired angle of climb, ψ^* . A takeoff of this nature minimizes drag losses, and eliminates the need for launching ramps. Since takeoff is performed without boost, the

PROPOSED TRAJECTORY

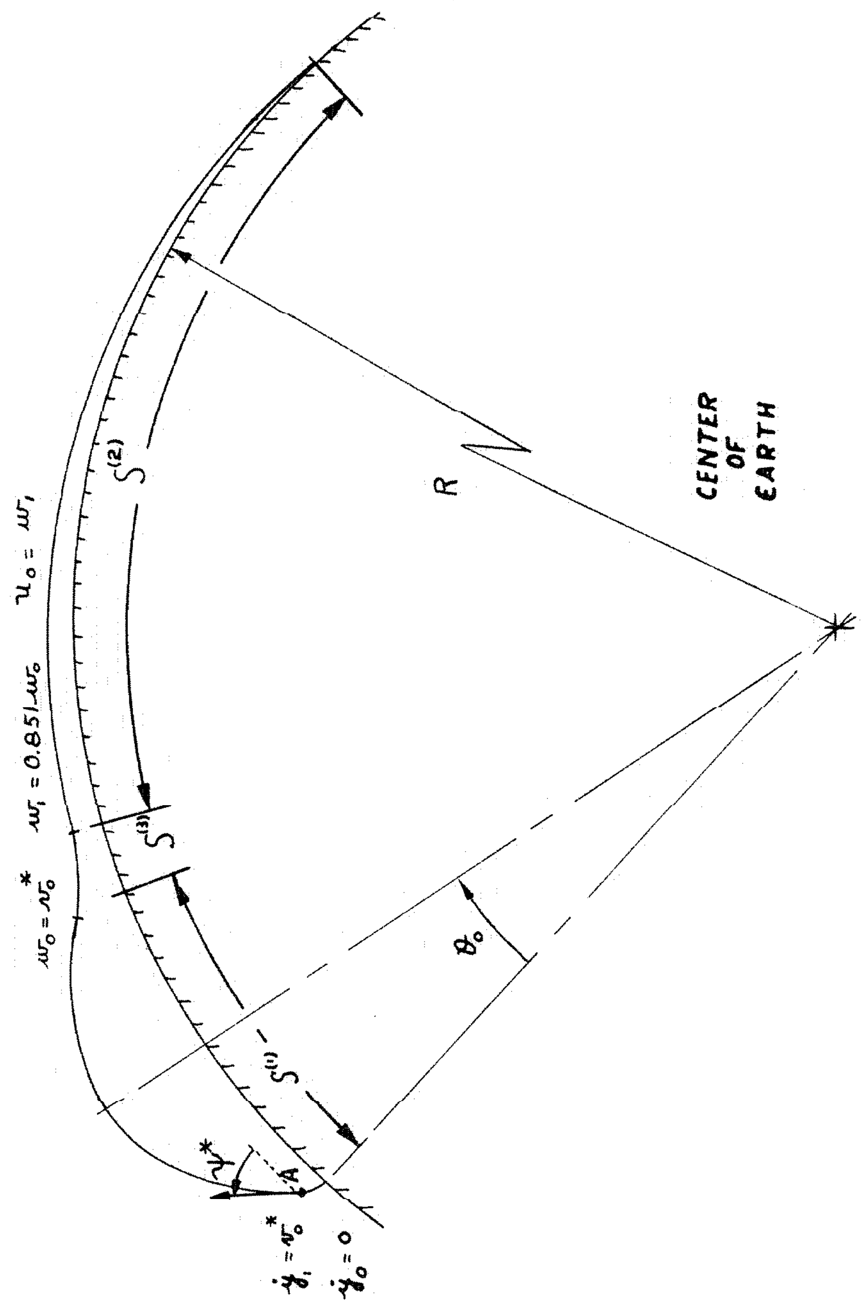


FIGURE 1

initial velocity, $\dot{y}_o$, is zero, and the resulting burnout velocity is $\dot{y}_1$. Both the altitude gained and the range covered during this part of the trajectory are neglected as being small compared to the values of these quantities in other parts of the trajectory. (b) From the burnout point, A, the airplane follows an elliptical ballistic trajectory determined by the burnout velocity, and the angle of climb. Since drag in this phase is neglected, the velocity upon re-entry into the atmosphere of the earth is just equal to $\dot{y}_1$, the burnout velocity. (c) Upon reaching an altitude where sufficient aerodynamic forces can be produced, the airplane is pulled out of the dive, and set up on a course to the destination. (d) By use of its wings throughout a long shallow glide, the airplane extends its range materially, and reduces the fuel requirements for a given range considerably.

The performance of the transport is calculated, using simplifying assumptions only where they do not affect the accuracy of the analysis greatly. During takeoff, for instance, it is assumed that the velocity at burnout is closely approximated by the velocity of the vertical ascent trajectory. Drag corrections have been shown to be small (Reference 3), and therefore, since the accuracy of this analysis is not sufficient to warrant their inclusion, these corrections have not been made.

Since it may turn out that the minimum value of the required fuel load compared to gross weight is determined by the limits of endurance of the human occupants, a method for checking a specific acceleration program is given.

Weight estimates are made on the basis of best available data on components of rocket missiles. Other than the V-2, little is known, so dependence upon its design is heavy. For purposes of calculation, a gross weight of 200,000 pounds is selected, which results in landing configurations of a size comparable to present day transports. Consideration is given the aerodynamic heating problem upon re-entry into the denser air of the lower atmosphere, and a weight penalty is taken in the form of a coolant for the airplane's surface material. In order to provide for better control and flexibility during landing, the rocket aircraft is assumed to be equipped with a turbojet of sufficient power to fly the vehicle at speeds near the landing speed. Accordingly, fuel for one-half hour's operation is provided. It is felt that conservative estimates have been made throughout the analysis.

Cost analysis is based on standard Air Transport Association comparison formulas, which have been modified for the new power plant where necessary. For purposes of comparison, a fuel cost approximately equal to the present day gasoline price is plotted.

Comparison is made with other modes of transportation, on the basis of specific resistance; i.e., the amount of power required to move a pound of the vehicle. Also a possible configuration is given to illustrate the size of the vehicle proposed.

II. PERFORMANCE ANALYSIS

A. Takeoff

It is shown in Reference 3 that if m is the instantaneous mass of the rocket at time t, y the altitude, c the effective exhaust velocity, D the drag, and g the gravitational attraction, then the equation of motion of the rocket during vertical ascent is given by:

$$c\frac{dm}{dt} + m\left(\frac{d\dot{y}}{dt} + g\right) = -D(\dot{y}, y) \tag{1}$$

Herein we neglect, for a first approximation, the change of both c and g with altitude. Then, we can write Equation (1) as:

$$e^{\left(-\frac{\dot{y}+gt}{c}\right)}\frac{d}{dt}\left[e^{\left(\frac{\dot{y}+gt}{c}\right)}m\right] = -\frac{D}{c}$$

If the subscript 1 is used to denote conditions at burnout, and the subscript 0 is used for conditions at time t = 0, then it can be shown that:

$$\frac{m_1}{m_0} = e^{-\left[\frac{(\dot{y}_1-\dot{y}_0)+gt_1}{c}\right]} - e^{-\left[\frac{\dot{y}+gt_1}{c}\right]}\int_0^{t_1}\frac{D}{cm_0}e^{\left[\frac{\dot{y}+gt}{c}\right]}dt \tag{2}$$

It will be seen from this equation that since D_2 is proportional to the cross sectional area of the rocket, or d^2, whereas m_o is proportional to its volume, or d^3, the importance of drag decreases with the size of the rocket. In the case studied in this paper, namely, when the drag is negligible, Equation (2) reduces to:

$$\dot{y}_1 - \dot{y}_0 = c \ln\left(\frac{m_0}{m_1}\right) - g t_1 \qquad (3)$$

Since we will consider the case of takeoff without boost, $\dot{y}_0 = 0$. Because of the weak dependence of the performance on drag, it is noted that a better approximation to actual performance can be obtained by using a small perturbation procedure around the trajectory when the drag is negligible. However, it is felt that in this analysis the closer approximation is not necessary.

In Equation (3) it will be seen that for given values of the initial velocity and burnout velocity, the ratio of initial mass to the final mass is the least, or the expenditure of propellant is smallest if $t_1 = 0$; i.e., if the propellant is used all at once as an impulse. Also it is seen that for a fixed t_1, the mass ratio is not influenced in any way by the manner in which either the thrust or acceleration varies with time or altitude.

From the above observations, the designer of the rocket vehicle would naturally tend to reduce the burning time to a minimum, to keep the fuel consumption to a minimum. However, with the consideration that a human crew and/or passengers will be carried in the transport, another design parameter must be considered in the programming of thrust. This parameter is the ability of the human body to withstand prolonged acceleration. Figure 2 gives a recent estimate (Reference 6) of this human tolerance in the form of allowable acceleration as a function of time. These curves are based on tests which indicate the limits of useful consciousness.

HUMAN TOLERANCE

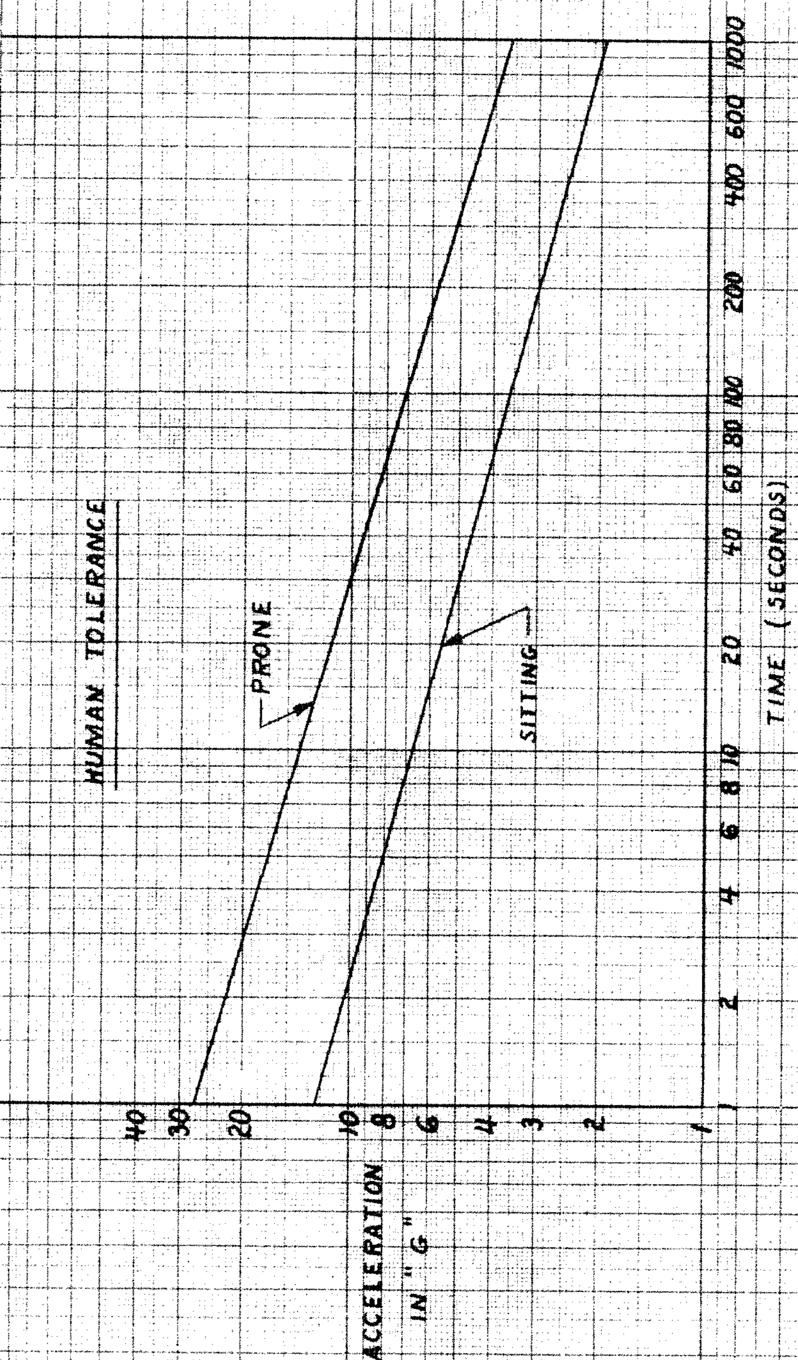


FIGURE 2

For the first calculations of performance, the curve representing the allowable acceleration imposed transversely is selected. To insure that the resulting design conforms with this limitation, the curve is used to determine the permissible value of burnout velocity, $\dot{y}_1$, that can be obtained in a given burning time, t_1, since this velocity will depend on the allowable accelerations. The assumption is made that the allowable acceleration as given by the curve in Figure 2 is equal to the design average acceleration during the burning time. That is:

$$n_{ALL}\, g\, t_1 = n_{AVE}\, g\, t_1 = \dot{y}_1 \tag{4}$$

where $(n_{all}\, t_1)$, as taken from Figure 2, is plotted as a function of t_1 in Figure 3.

Also if we define the propellant loading ratio, ξ , as:

$$\xi = \frac{m_0 - m_1}{m_0} \tag{5}$$

we can write Equation (3) as:

$$\dot{y} = c \ln\left(\frac{1}{1-\xi}\right) - g\, t_1 \tag{6}$$

Thus by selecting a value for t_1, one can find the value of $(n_{all}\, t_1)$ from Figure 3; and from the value of $\dot{y}_1$ found from Equation (4), the corresponding value of ξ can then be found from Equation (6).

A plot of this relation in the form of ξ as a function of the burnout velocity for various values of I_{sp} is given in Figure 4.

Before proceeding further with the performance analysis to find the total range, S, that can be realized as a function of the burnout

ALLOWABLE ACCELERATION × BURNING TIME vs. BURNING TIME

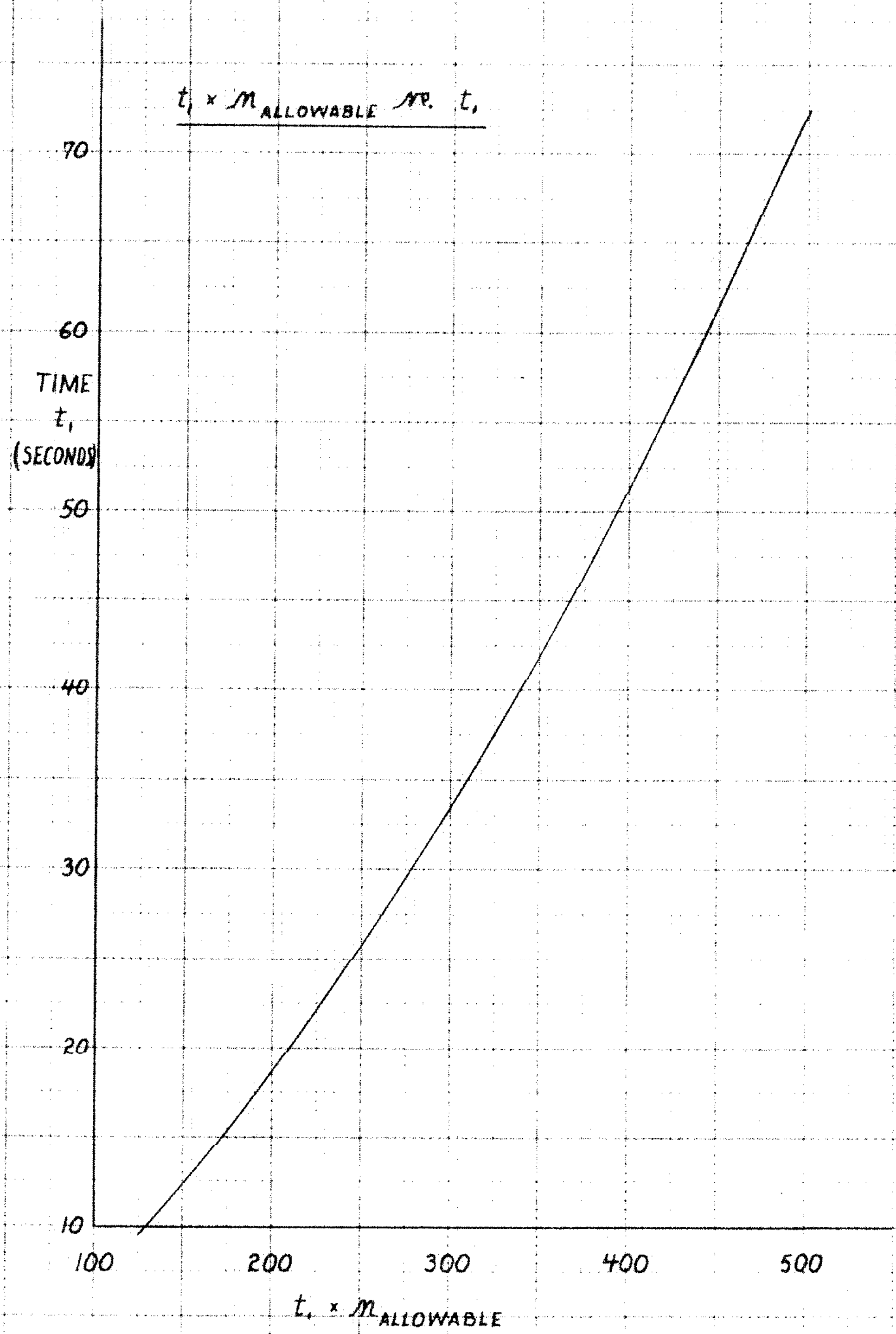


FIGURE 3

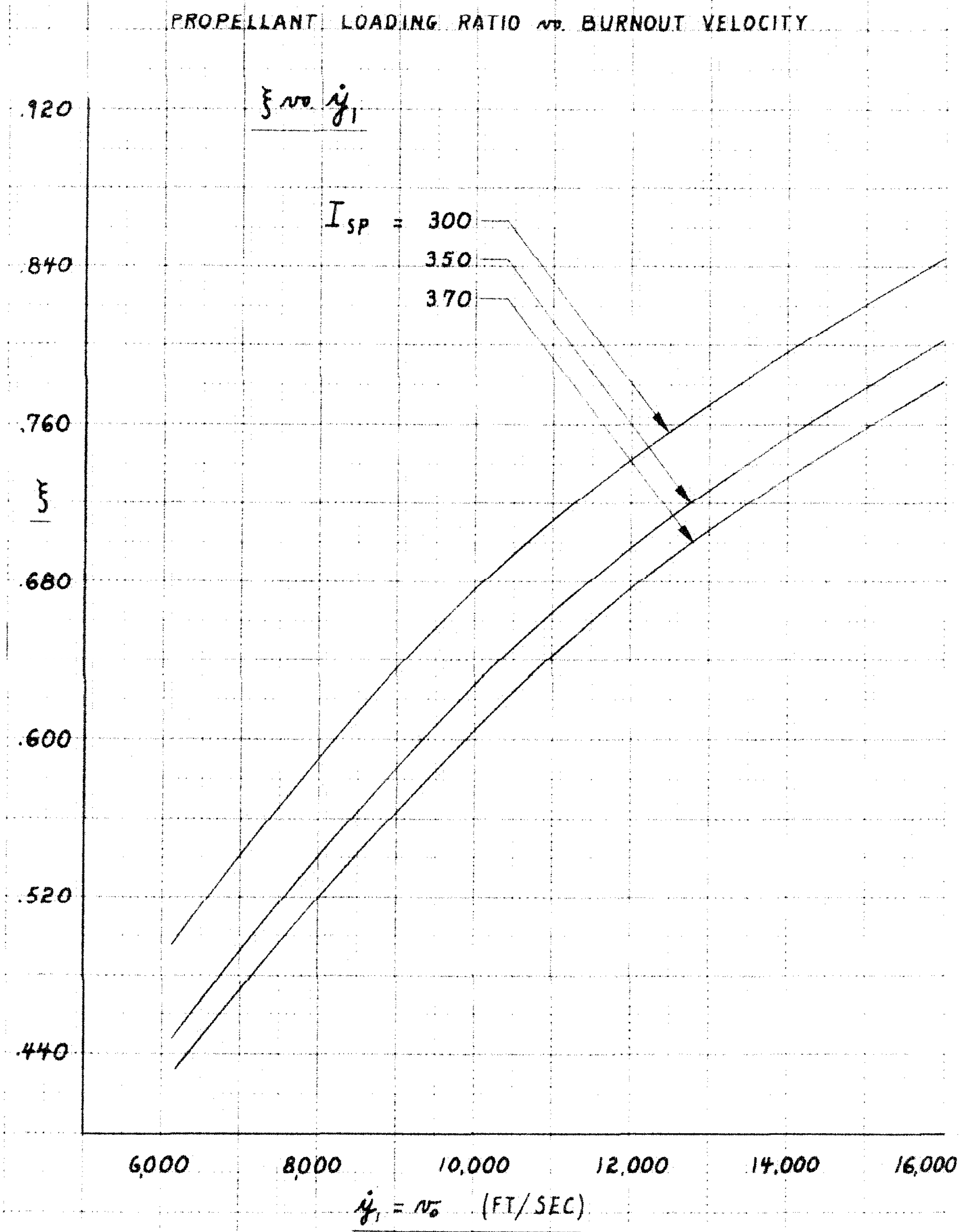

PROPELLANT LOADING RATIO vs. BURNOUT VELOCITY
ξ vs. $\dot{y}_1$
I_{SP} = 300
350
370
.920
.840
.760
ξ
.680
.600
.520
.440
6,000
8,000
10,000
12,000
14,000
16,000
$\dot{y}_1 = v_0$ (FT/SEC)

FIGURE 4

velocity, we propose a further check on the blackout problem.

Let us assume that blackout is determined by the differential flow of a quantity of blood, Q, out of the brain. Q, then, is a measure of how much more blood flows out in a given time, than is pumped into the brain by the heart. Let T(n) denote the time until blackout at an acceleration of ng. Then the rate of flow of blood at ng is:

$$\frac{Q}{T(n)}$$

Therefore if t_1 is the burnout time, the condition for the absence of blackout is:

$$Q > \int_0^{t_1} \frac{Q}{T(n)}\, dt$$

or

$$1 > \int_0^{t_1} \frac{dt}{T(n)} \qquad (7)$$

Using the curve $T(n) = \frac{C}{n^4}$, as found in Reference 6 for the prone position, and noting that in the constant thrust rocket

$$(n+1)\,g = \frac{F}{m_0 - \left(\frac{m_0 - m_1}{t_1}\right)t}, \text{ then}$$

$$\int_0^{t_1} \frac{dt}{T(n)} = \frac{1}{C}\int_0^{t_1} n^4\, dt = \frac{1}{C}\int_0^{t_1} \left\{ \frac{F}{g\left[m_0 - \left(\frac{m_0 - m_1}{t_1}\right)t\right]} - 1 \right\}^4 dt$$

but

$$\frac{m_0 - m_1}{t_1} = \frac{m_0 - m_1}{m_0}\,\frac{m_0}{t_1} = \xi\,\frac{m_0}{t_1}$$

$$\therefore \int_0^{t_1} \frac{dt}{T(n)} = \frac{1}{C}\int_0^{t_1} \left\{ \frac{F^4}{g^4\left[1 - \xi\frac{t}{t_1}\right]^4 m_0^4} - 4\,\frac{F^3}{m_0^3 g^3\left[1 - \xi\frac{t}{t_1}\right]^3} + 6\,\frac{F^2}{m_0^2 g^2\left[1 - \xi\frac{t}{t_1}\right]^2} - 4\,\frac{F}{m_0 g\left[1 - \xi\frac{t}{t_1}\right]} + 1 \right\} dt$$

now
$$\frac{F}{m_o g} = n_o + 1 \qquad \& \qquad \frac{F}{m_o g(1-\xi)} = n_1 + 1$$

So:
$$\int_0^{t_1} \frac{dt}{T(n)} = \frac{t_1}{C}\left\{\frac{F}{m_o g \xi}\left[\frac{1}{3}\left\{(n_1+1)^3-(n_o+1)^3\right\} - 2\left\{(n_1+1)^2-(n_o+1)^2\right\} + 6\left\{(n_1+1)-(n_o+1)\right\} - 4 \ln\frac{n_1+1}{n_o+1}\right] + 1\right\}$$

or since
$$(1-\xi) = \frac{n_o+1}{n_1+1} \qquad \& \qquad \xi = \frac{n_1-n_o}{n_1+1}$$

$$\therefore \quad \frac{F}{m_o g \xi} = \frac{F}{m_o g}\,\frac{1}{\xi} = \frac{n_o+1}{n_1+1}\,(n_1-n_o)$$

Therefore:
$$\int_0^{t_1} \frac{dt}{T(n)} = \frac{t_1}{C}\left\{\frac{n_o+1}{n_1+1}(n_1-n_o)\left[(n_1-n_o)\left\{\frac{1}{3}(n_1+1)^2 + \frac{1}{3}(n_1+1)(n_o+1) + \frac{1}{3}(n_o+1)^2 - 2(n_1+1) + 2(n_o+1) + 6\right\} - 4 \ln\frac{n_1+1}{n_o+1}\right] + 1\right\}$$

or

$$\int_0^{t_1} \frac{dt}{T(n)} = \frac{t_1}{T(n)}\left\{\left(\frac{n_o+1}{n_1+1}\right)\left(1-\frac{n_o}{n_1}\right)\left[\left(1-\frac{n_o}{n_1}\right)\left\{\frac{1}{3}\left(1+\frac{1}{n_1}\right)^2 + \frac{1}{3}\left(1+\frac{1}{n_1}\right)\left(\frac{n_o}{n_1}+\frac{1}{n_1}\right) + \frac{1}{3}\left(\frac{n_o}{n_1}+\frac{1}{n_1}\right)^2 - 2\left(\frac{1+\frac{1}{n_1}}{n_1}\right) + 2\left(\frac{\frac{n_o}{n_1}+\frac{1}{n_1}}{n_1}\right) + \frac{6}{n_1^2}\right\} - \frac{4}{n_1^3}\ln\frac{n_1+1}{n_o+1}\right] + \frac{1}{n_1^4}\right\} \tag{8}$$

A check on the proposed minimum propellant loading ratio trajectories shows that, in general, they are very close to the limit of acceleration the human body can withstand. As t_1 increases, the linear approximation proposed earlier becomes worse. However, as will be shown in the section entitled Weight Breakdowns, the acceleration programming for a minimum amount of rocket fuel does not give the configuration for maximum payload, due to the assumptions made on the variance of component weights with thrust and burning time. A further check on the resulting accelerations will be made in that section.

B. Elliptic Path

Returning to the performance problem, we attempt to find a relation between ξ and S to enable us to find the fuel that is needed for a given range. First we must determine what range can be obtained with a given burnout velocity, and then from Figure 4, a cross plot can be made to give ξ as a function of S.

If we neglect the rotation of the earth, it will be seen from Figure 1 that the range, $S^{(1)}$, covered during the elliptic part of the trajectory is given by:

$$S^{(1)} = 2 R \theta_0 \tag{9}$$

where R is the radius of the earth, and θ_0 is the angle, measured from the center of the earth, between the initial point and the summit. Tsien (Reference 3) shows that for the maximum range trajectory, the relation between initial velocity, v_o, and the central angle, θ_o, is given by:

$$\frac{v_o^{*2}}{gR} = 2 \tan\theta_o \left(\frac{1 - \sin\theta_o}{\cos\theta_o}\right) \qquad (10)$$

where v_o^* refers to minimum possible velocity. A plot of this relation combined with Equation (9) to give v_o^* as a function of $S^{(1)}$ is also given.

C. Pullout

In this same reference, to find relations between the initial velocity, w_o, and final velocity, w_1, and the initial velocity, w_o, and the range, $S^{(3)}$, it is assumed that the earth is flat and the gravity field is parallel. The initial acceleration during the pullout is taken to be:

$$\frac{w^2}{r} = 3g \qquad (11)$$

where r is the radius of curvature of the pullout path. Then the velocity after pullout is:

$$w_1^2 = 0.728\, w_o^2 \qquad (12)$$

and the range covered during pullout is:

$$S^{(3)} = 0.2355 \frac{w_o^2}{g} \qquad (13)$$

D. Glide

The glide part of the trajectory is assumed to be carried out at a constant radius, R, equal to the radius of the earth, since the altitude variation is small compared to both the range and the radius of the earth. Now lift has only to counterbalance the resultant of gravitational attraction and centrifugal force. Therefore, the equation of motion is given by:

$$-m\frac{du}{dt} = -\frac{m}{2}\frac{du^2}{dx} = \frac{m}{\lambda}\left(g - \frac{u^2}{R}\right) \tag{14}$$

where m is the mass of the vehicle, u is its velocity, t the time, λ the average L/D ratio during glide, and x the distance along the earth's surface. By reducing this equation, and substituting the boundary conditions that at $x = S^{(2)}$, $u = 0$, we find:

$$S^{(2)} = -\frac{\lambda R}{2}\ln\left(1 - \frac{u_o^2}{gR}\right) \tag{15}$$

Then the overall velocity picture is this: At time $t = 0$, $\dot{y}_o = 0$. After the burning time, t_1, the burnout velocity, $\dot{y}_1$, is reached. It is assumed that this velocity for the vertical ascent trajectory is a good approximation for the burnout velocity of the optimum curved trajectory. Since energy is conserved during the elliptic trajectory, (no drag), when the rocket returns to the atmosphere it again has the velocity $\dot{y}_1 = v_o^*$, which is taken for the initial value of the velocity for the pullout, w_o. Then from Equation (12), w_1, the velocity after pullout, is found, and this value is taken for the initial velocity for the glide, u_o. Correspondingly it will be seen that from Equations

(9), (10), (13), and (15) the total range of the flight can be found for a given burnout velocity.

Thompson (Reference 7) gives values of the specific impulse, I_{sp}, for several high energy propellants, as indicated in Table I. With this table in mind, curves are plotted for various parameters as functions of the range using values of I_{sp} equal to 300, 350, 370.

Theoretical and experimental studies, (References 4, 8, and 9) indicate that values of the lift to drag ratio, for the Mach numbers considered, may be as high as 6. Since the glide trajectory is planned to make use of the maximum L/D value at all times, this is taken as the average value to obtain a plot of $\dot{y}_1 = v_o^*$ as a function of the total range, S, as shown in Figure 5. A similar plot for L/D = 8 is given to indicate the increase in performance to be gained by improvements in this L/D ratio.

To obtain a plot of ξ as a function of S is then merely a job of cross plotting Figures 4 and 5 to obtain Figures 6 and 7 for L/D = 6 and 8 respectively. Figure 8 shows the plot of total flight time as a function of range as found from equations and plots given in Reference 3.

The results of our performance analysis, and the basis of the weight breakdowns to follow, namely, the plots of minimum propellant loading ratio, ξ , are determined by the limitations of the human occupants.

TABLE I

Specific Impulse (sec.) for Various Propellant Combinations

Oxidant	Fuel: Liquid Hydrogen	Liquid Ammonia	Hydrazine	Methane	Lithium	Beryllium
Liquid Oxygen	370	260	290	270	340	360
Liquid Ozone	400	280	300	290	360	380
Fluorine	380	270	320	280	370	390
Chlorine Trifluoride	320	220	260	240	320	340
Fluorine Monoxide	390	280	320	280	370	370
Nitrogen Tetroxide	360	250	270	260	330	350

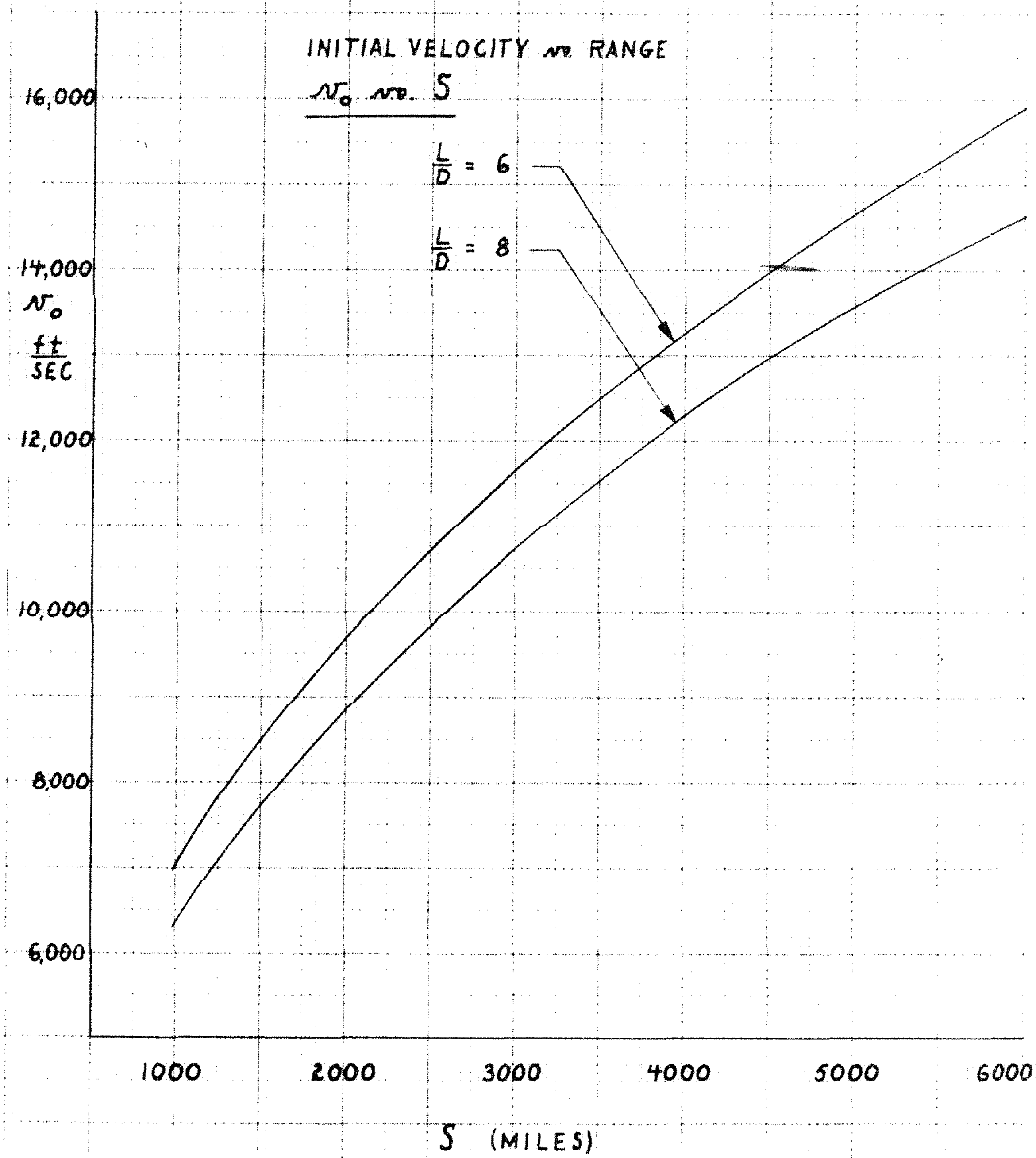

INITIAL VELOCITY vs. RANGE
v_0 vs. S
L/D = 6
L/D = 8
16,000
14,000
12,000
10,000
8,000
6,000
v_0
ft/SEC
1000
2000
3000
4000
5000
6000
S (MILES)

FIGURE 5

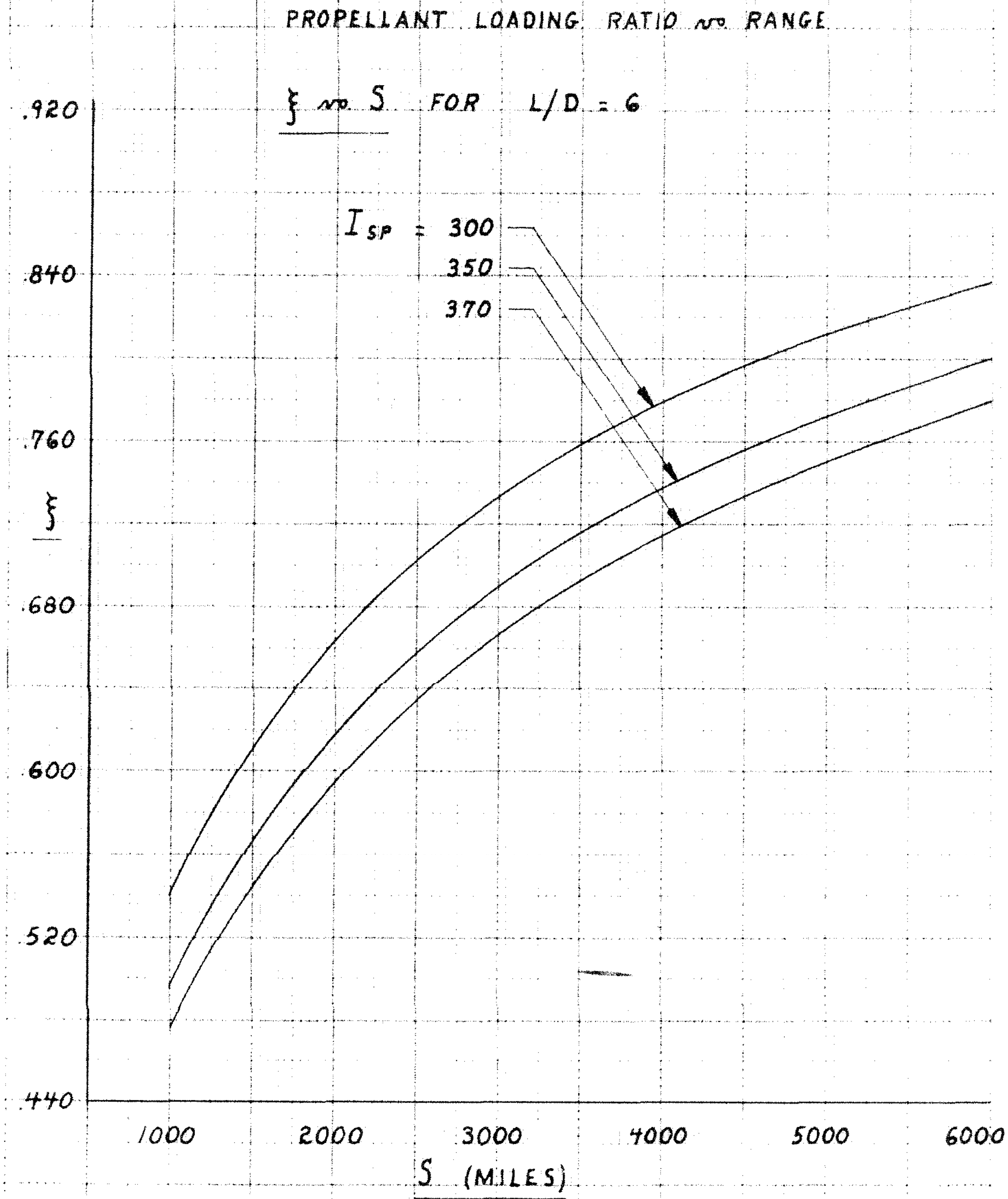

PROPELLANT LOADING RATIO vs RANGE
ξ vs S FOR L/D = 6
I_{SP} = 300
350
370
.920
.840
.760
ξ
.680
.600
.520
.440
1000
2000
3000
4000
5000
6000
S (MILES)

FIGURE 6

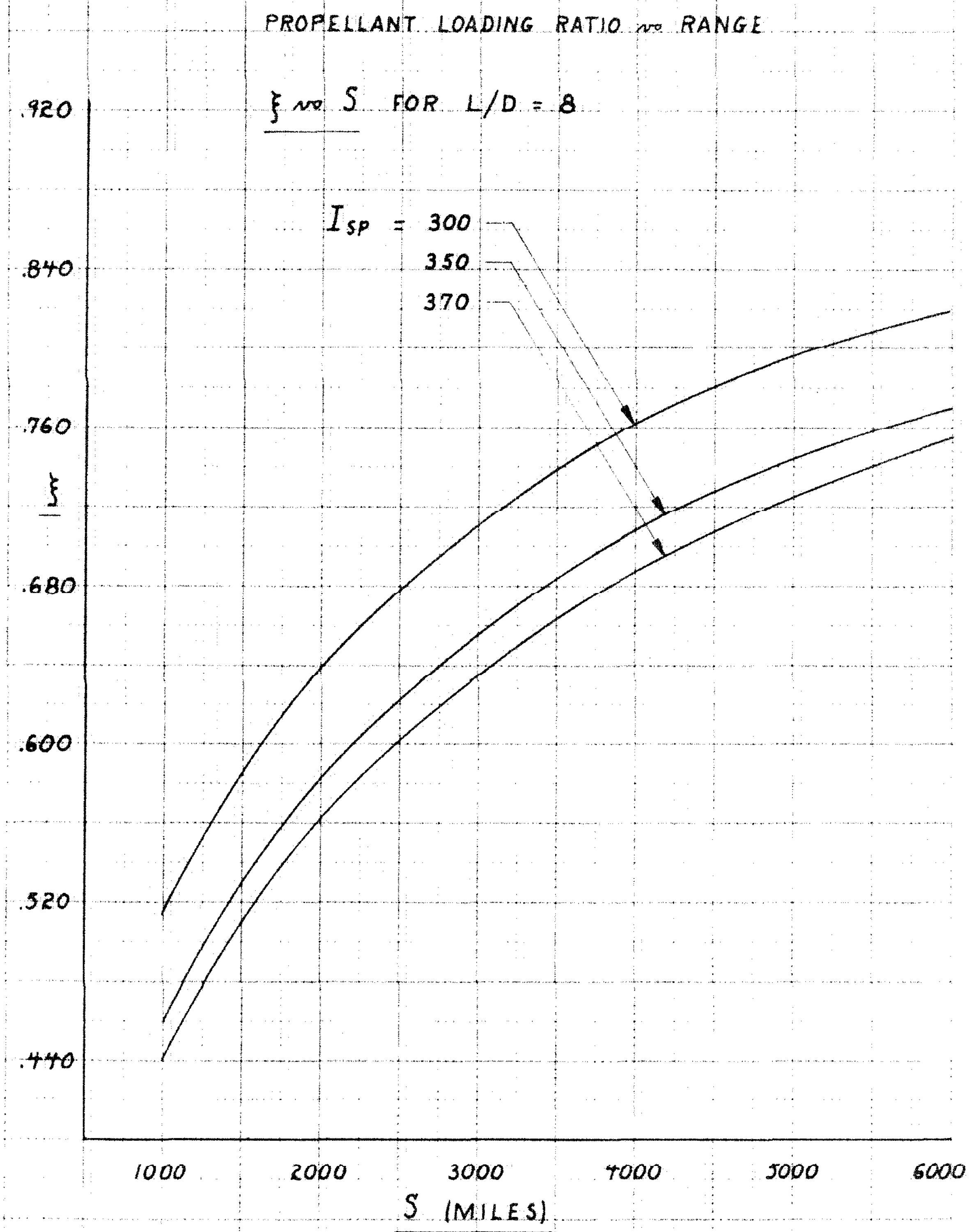
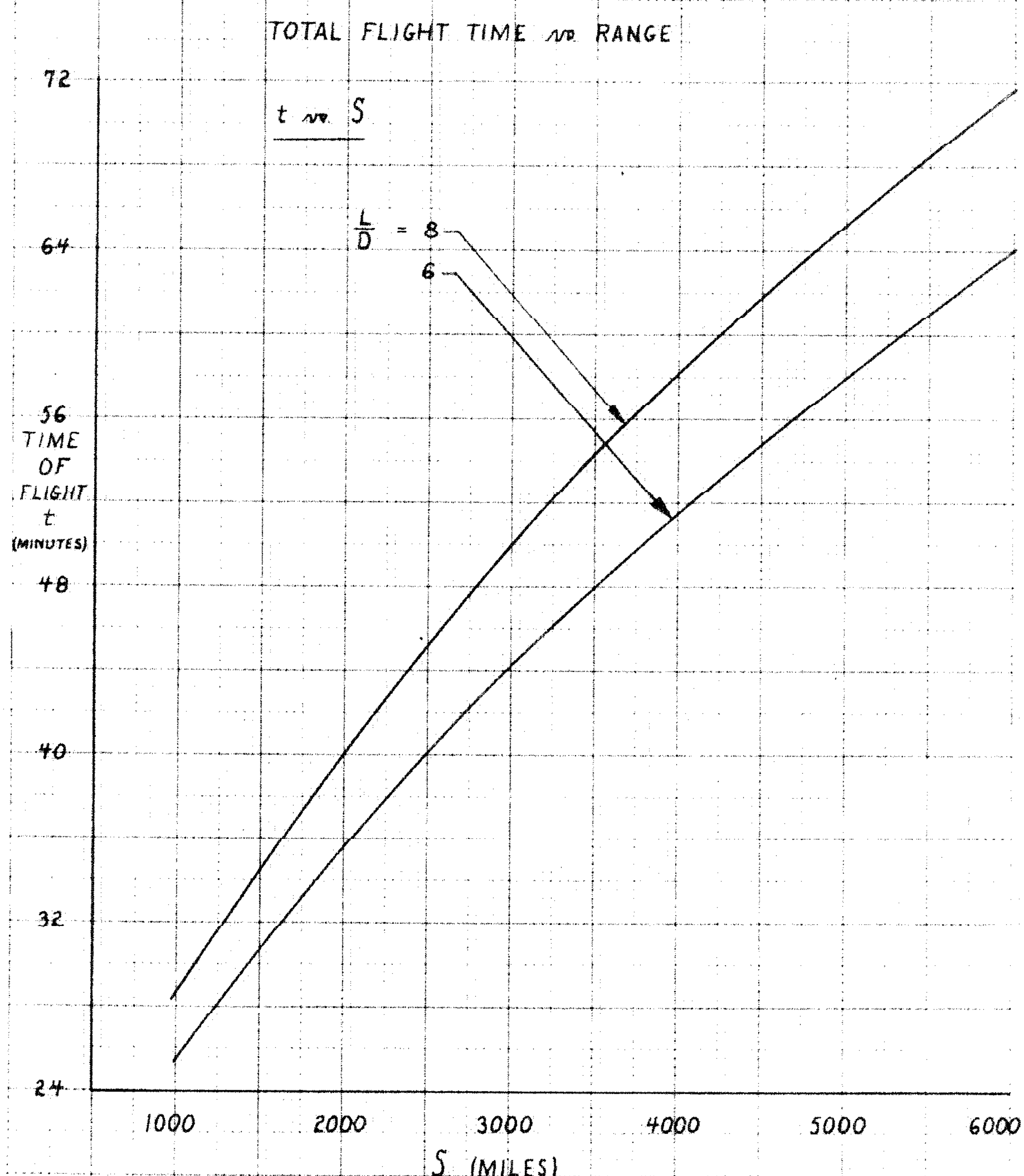

PROPELLANT LOADING RATIO ~ RANGE
ξ ~ S FOR L/D = 8
I SP = 300
350
370
.920
.840
.760
ξ
.680
.600
.520
.440
1000
2000
3000
4000
5000
6000
S (MILES)

FIGURE 7



TOTAL FLIGHT TIME vs RANGE
t vs S
L/D = 8
6
TIME OF FLIGHT t (MINUTES)
72
64
56
48
40
32
24
1000
2000
3000
4000
5000
6000
S (MILES)

FIGURE 8

III. WEIGHT BREAKDOWNS

Following the calculation of the required rocket propellant loading ratios for a given range, a determination of component weights for the transport itself must be made. Significant components are: structure, rocket power plant, pumping turbine, turbine fuel, landing system (composed of a turbojet for holding, turbojet fuel, crew, equipment, and the landing gear) and coolant to maintain a given surface temperature upon re-entry into the denser air of the lower atmosphere. What weight remains can then be designated as payload, and used as a basis for cost analysis.

A. Structural Weight

In deriving a method that can be used for estimating the required structural weight, a statistical study of present day aircraft was made. Now, the design load of a given airframe is equal to the design load factor times the design gross weight. A plot of a series of points representing the structure weight as a function of gross weight for current airframes is given in Figure 9. (Reference 10.) For the purpose of weight estimation in this paper then, a best straight line is passed through these points to give a relation between structure weight and design load.

Finally the design load for the rocket transport is selected as the thrust of the rocket motor, since this is the largest force to which the airframe will be subjected. This is seen to be the case, since maneuvering and gust loads imposed upon the airframe will occur at

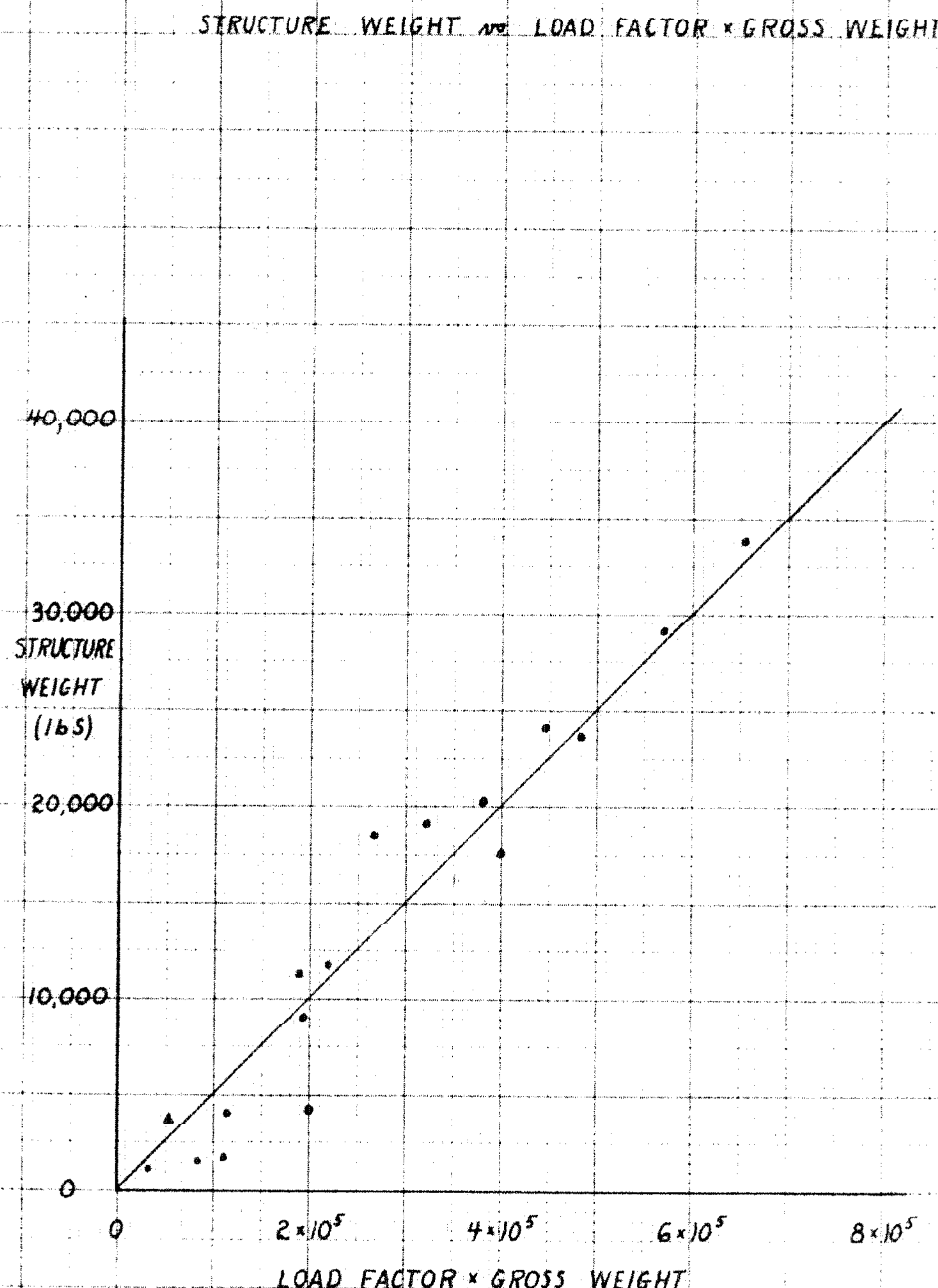

STRUCTURE WEIGHT vs LOAD FACTOR × GROSS WEIGHT
40,000
30,000
STRUCTURE WEIGHT (lbs)
20,000
10,000
0
0
2×10^5
4×10^5
6×10^5
8×10^5
LOAD FACTOR × GROSS WEIGHT

FIGURE 9

a time when the gross weight is greatly reduced; i.e., after the rocket fuel is expended. In view of the fact that the applied load in the case of the rocket transport is a compression load, whereas the applied load in the case of the conventional airplane is a bending load, use of the curve given in Figure 9 to estimate the structural weight of a particular rocket transport is felt to be conservative.

B. Power Plant Weight

A study as conducted for the structural weight cannot be made for components such as the rocket motor weight, the turbine pump weight and the turbine fuel weight, due to the lack of published information on rocket missiles using these components. Therefore the best approximation for weight estimation is made by linearly scaling up values given for the V-2 missile in Reference 5.

The rocket motor weight is assumed to depend only on its thrust, as is the turbine pump weight. However, it is reasoned that the weight of the turbine fuel would more nearly depend on the product of the thrust times the burning time. Plots of the assumed weight estimation curves for these components are shown as Figures 10 and 11.

C. Weight of Other Components

Further assumptions for weight estimation are as follows: Using an L/D ratio of ten to find the required thrust during the holding operation, the weight of the turbojet was estimated by setting

TURBINE + POWER PLANT WEIGHT vs THRUST

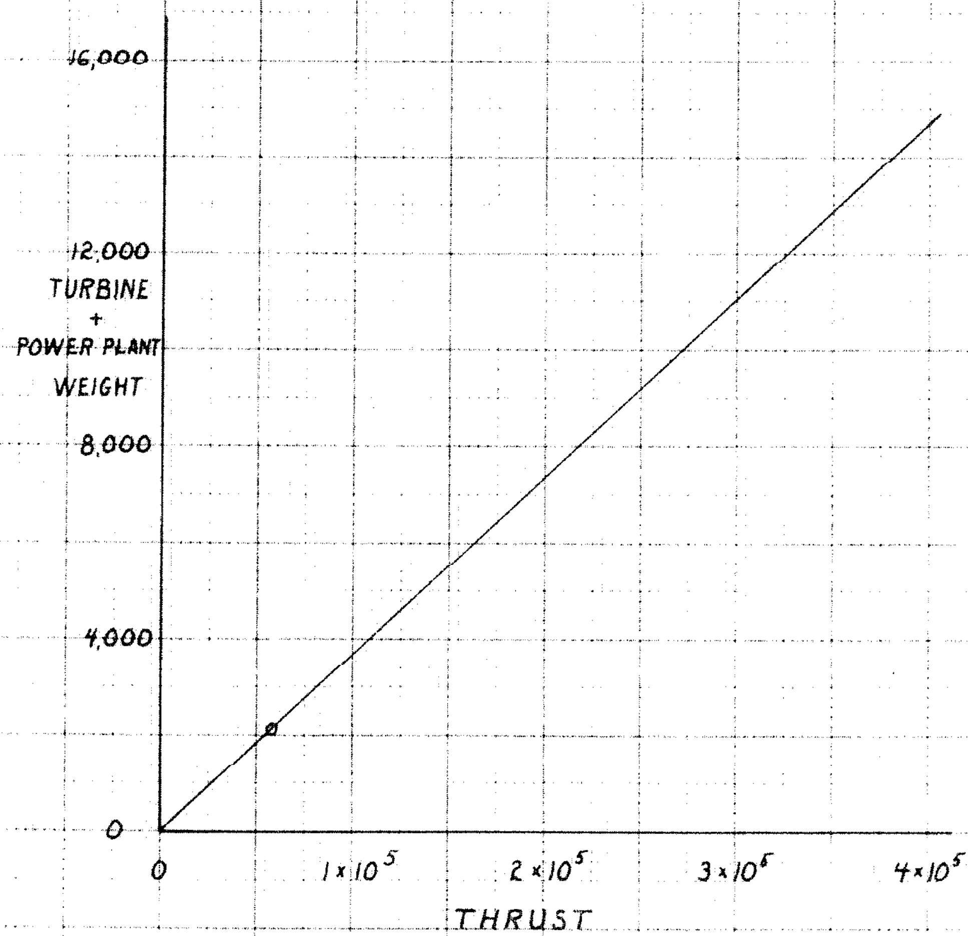
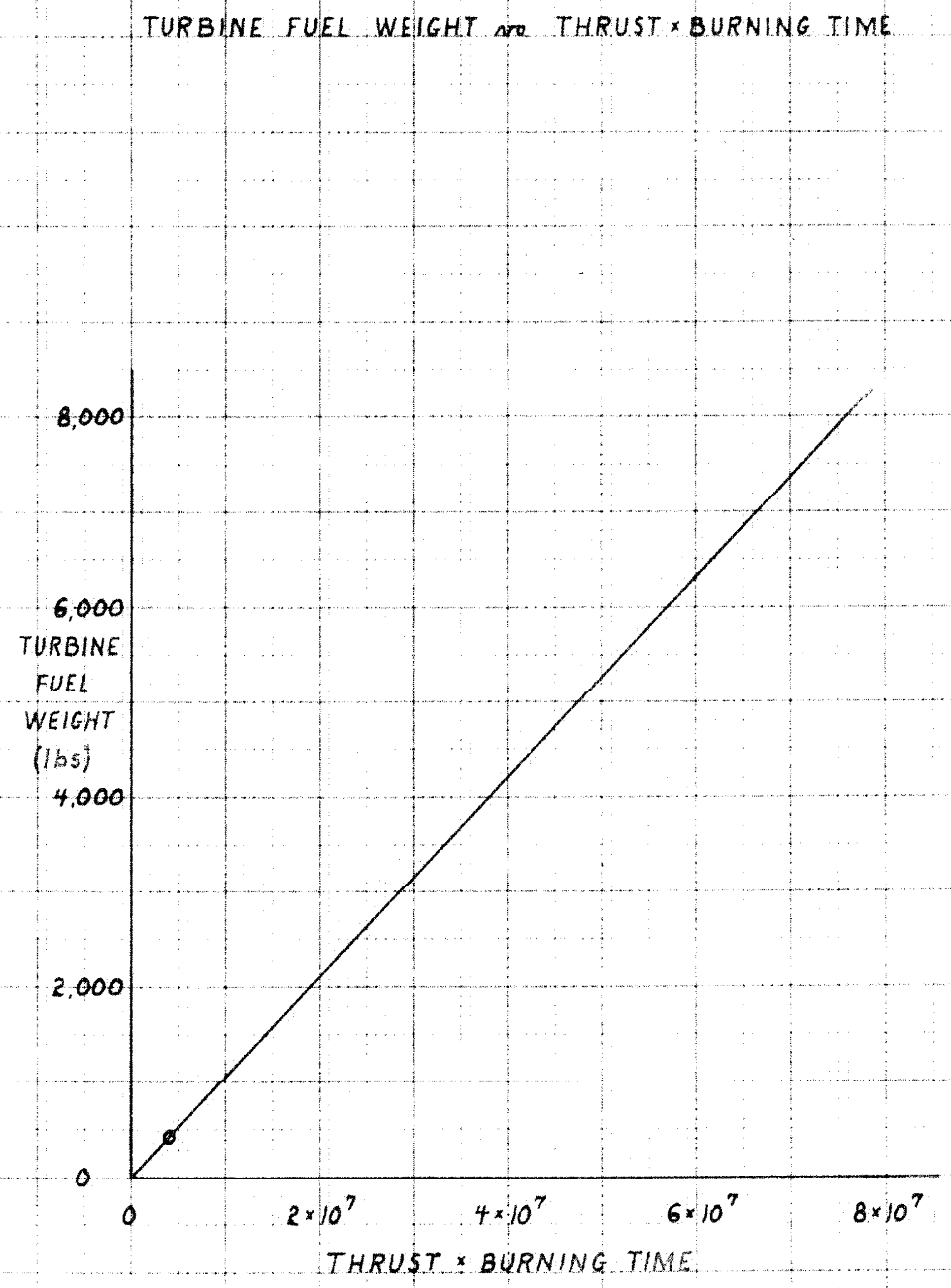


FIGURE 10



TURBINE FUEL WEIGHT vs THRUST × BURNING TIME
8,000
6,000
TURBINE FUEL WEIGHT (lbs)
4,000
2,000
0
0
2×10⁷
4×10⁷
6×10⁷
8×10⁷
THRUST × BURNING TIME

FIGURE 11

the value of its installed weight equal to 30 percent of the value of its thrust. Fuel for the turbojet was computed on the basis of a specific fuel consumption of one pound of fuel per pound of thrust per hour, with a duration of one half hour. Following a statistical study as in the case of structural weight, the weight of the landing gear was taken to be five percent of the landing weight. Weights for the crew and equipment were taken arbitrarily as 400 and 200 pounds respectively.

D. Weight of Coolant for the Airplane's Surface

Klunker and Ivey (Reference 11) give a method of computing the amount of coolant needed to maintain a given surface temperature as a function of the flight Mach number and flight altitude, assuming laminar boundary layer. In this work the cooling liquid is assumed to be liquid air, and the process of cooling is taken as the vaporization of the liquid air at 147^{o}F absolute followed by a heating of the gaseous air from this vaporization temperature to the temperature of the surface. Night time operation only is assumed in order to eliminate consideration of solar and atmospheric radiation.

To use the curves given in this report, three design quantities pertaining to the vehicle must be known, namely: operating altitude, h, Mach number, M , and required surface temperature, T_s . An operating altitude to be used was found by the following procedure: A value of the density, ρ , needed to sustain level flight, disregarding centrifugal force, was calculated from knowledge of the coefficient of lift for maximum L/D, (Reference 7), the design wing loading, and the velocity. Then from a table such as Reference 12, the altitude was read as a function of ρ . From this same table a value of the speed of sound was found for calculation of the Mach number. The surface equilibrium temperature, T_s, was chosen from a chart such as given in Reference 13, which indicates the loss of strength with temperature of various metals. Figure 12 indicates the curve for steel as given in this reference.

From these curves a value of the flow parameter, ζ_o , defined as:

$$-\zeta_o = 2\frac{\rho_s v_s}{\rho_f v_f}\sqrt{\frac{\rho_f v_f x}{\mu_f}}$$

is found, where the subscript f refers to free stream conditions, the subscript s to coolant flow at the surface, and the x is the surface length in question. The rate of coolant flow for one second for each square foot of surface area is easily found from:

$$\rho_s v_s g = -\frac{\zeta_o}{2}\rho_f v_f\sqrt{\frac{\mu_f}{\rho_f v_f x}}\, g \qquad (16)$$

The assumptions for calculation of the coolant weight in this paper are taken as: Equilibrium surface temperature equals 600°F (1060°F absolute), the Mach number used was that corresponding to w_o, the velocity at the start of pullout, and allowance was made for the reduction in speed by halving the flow rate thus found, and using this value as the average value throughout the glide. However to find the operating altitude, the more conservative value of $\frac{w_o}{2}$ was used,

EFFECT OF TEMPERATURE ON STRENGTH OF AIRCRAFT MATERIALS

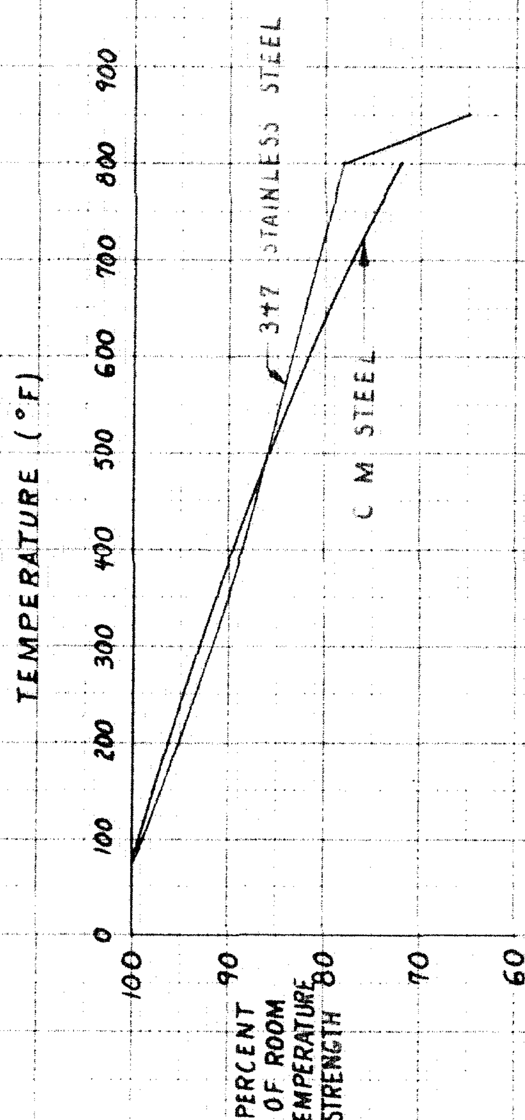


FIGURE 12

since this allows considerable altitude for pullout, and requires that the airplane operate in much denser air.

Now this reference considers only a laminar boundary layer, so a more conservative estimate is made by multiplying the result by ten, which corresponds to the ratio of heat transfers between turbulent boundary layers and laminar boundary layers. A total time equal to the pullout time plus the glide time was used in the calculations.

Now it is quickly seen by comparing the heats of vaporization of water and liquid air, that much lower weights of water should do the same job of cooling. By using a similar cooling process; i.e., heating of the water from storage temperature to the vaporization temperature, evaporating the water, and then heating the steam to the surface temperature, it is found that one pound of water will do the same job as five pounds of liquid air.

Summing up the conversion from liquid air cooling to water cooling yields the following result. Use of water allows a reduction of the flow rate by a factor of five, due to the increased heat of vaporization. The flow rate is also halved because the initial velocity of the glide is used in its calculation, and an average value is deemed more realistic. However, since we consider the possibility of turbulent boundary layer, and therefore multiply our result again by ten, the factors all cancel, and the flow parameter as computed gives the required flow of water under the listed assumptions. With these conservative assumptions it is felt that adequate consideration has been given the cooling problem. Sample calculations of required coolant weights for $I_{sp} = 350$, $L/D = 6$ are given in Appendix I.

E. Optimum Acceleration

Observation of the component weight estimation curves suggests that more favorable payloads might be obtained by decreasing the thrust of the rocket motor, while extending the burning time. This is indeed the case, since the reduction of component weights is greater than the increase in propellant weight. By plotting the resulting payload as a function of the average acceleration, it was found that, in general, a comfortable average acceleration of $n_{ave} \cong 2.0$ gave the largest value of the payload. A check on the acceleration endurance limit as proposed in Equation (7) in the previous section indicated that these trajectories utilized about ten percent of the endurance limit as compared to 95 to 100 percent in the minimum propellant loading ratio trajectories. Thus the payload optimizing decreases both the average acceleration loads that must be withstood by the occupants, and the maximum design accelerations to which components will be subjected. All the following calculations were made using this optimum acceleration for maximum payload.

Appendix II shows a sample breakdown of weights for a rocket transport which utilizes an $I_{sp} = 350$ seconds, an L/D in the glide equal to 6, and a motor weight linearly scaled up from V-2 figures. Another sample computation given in Appendix III shows a similar breakdown for $I_{sp} = 350$, $L/D = 8$, and a motor weight of 60 percent of the linearly extrapolated value of the V-2. This is given as an indication of what increased cost reduction will be gained by these improvements in component weights.

IV. COST ANALYSIS

Reference 1 gives cost formulas as offered by the Air Transport Association for the purpose of presenting data on proposed aircraft. These formulas are written in terms of block speed of the aircraft. A transformation can easily be made to find an effective block speed for the rocket transport in the following manner. If one assumes that the overhaul period for the turbojet engine is 500 hours and the life of the rocket motors used can be judged by the number of times they will be started, say 100; the block speed can be replaced by its equivalent, the range of each rocket flight times the number of flights (100) divided by the overhaul period of the turbojet. Thus we arrive at the following formulas:

$$\left.\frac{¢}{TM}\right)_{AIRF} = 0.585 \frac{W_{AIRFRAME}}{S P_a}$$

$$\left.\frac{¢}{TM}\right)_{ENG} = 2.50 \frac{W_{ENGINES}}{S P_a}$$

$$\left.\frac{¢}{TM}\right)_{INS} = 0.148 \frac{W_{EMPTY}}{S P_a}$$

$$\left.\frac{¢}{TM}\right)_{CREW} = \frac{4990 + 4.80 S}{S P_a}$$

$$\left.\frac{¢}{TM}\right)_{FUEL} = 2.00 \frac{W_{FUEL}}{S P_a}$$

Using two values for the L/D ratio during the glide, one can obtain the curves shown in Figures 13 and 14 for the direct operating cost per ton mile as a function of range. In comparison with Figure 15, as given in Reference 2, it is seen that for moderately long ranges, 1000 to 2500 miles, there is no competition for the rocket transport. Even in the lower ranges; i. e., under 1000 miles, there is the indication that the rocket can do the job as cheaply as any other form of propulsion, compared only on the basis of direct operating cost.

Appendices IV and V give the cost analysis calculations for the transports for which a weight breakdown was made in Appendices II and III.

These curves in Figures 13 and 14 are based on a propellant cost of $40.00 per ton, which compares with the figure used for fuel cost in the above reference. According to Reference 7, this is a rather optimistic value for present day rocket propellants, especially those of high energy content. However, it is felt that due to the limited production of most rocket propellants, it would be unfair to use present costs in a comparison. A further comparison of the three figures shows that for the rocket transport with high energy propellants, and an L/D ratio during the glide of 8, the operating cost drops down to a figure very near that of currently proposed subsonic turbojet transports.

DIRECT OPERATING COST ~ RANGE $\frac{L}{D} = 6$

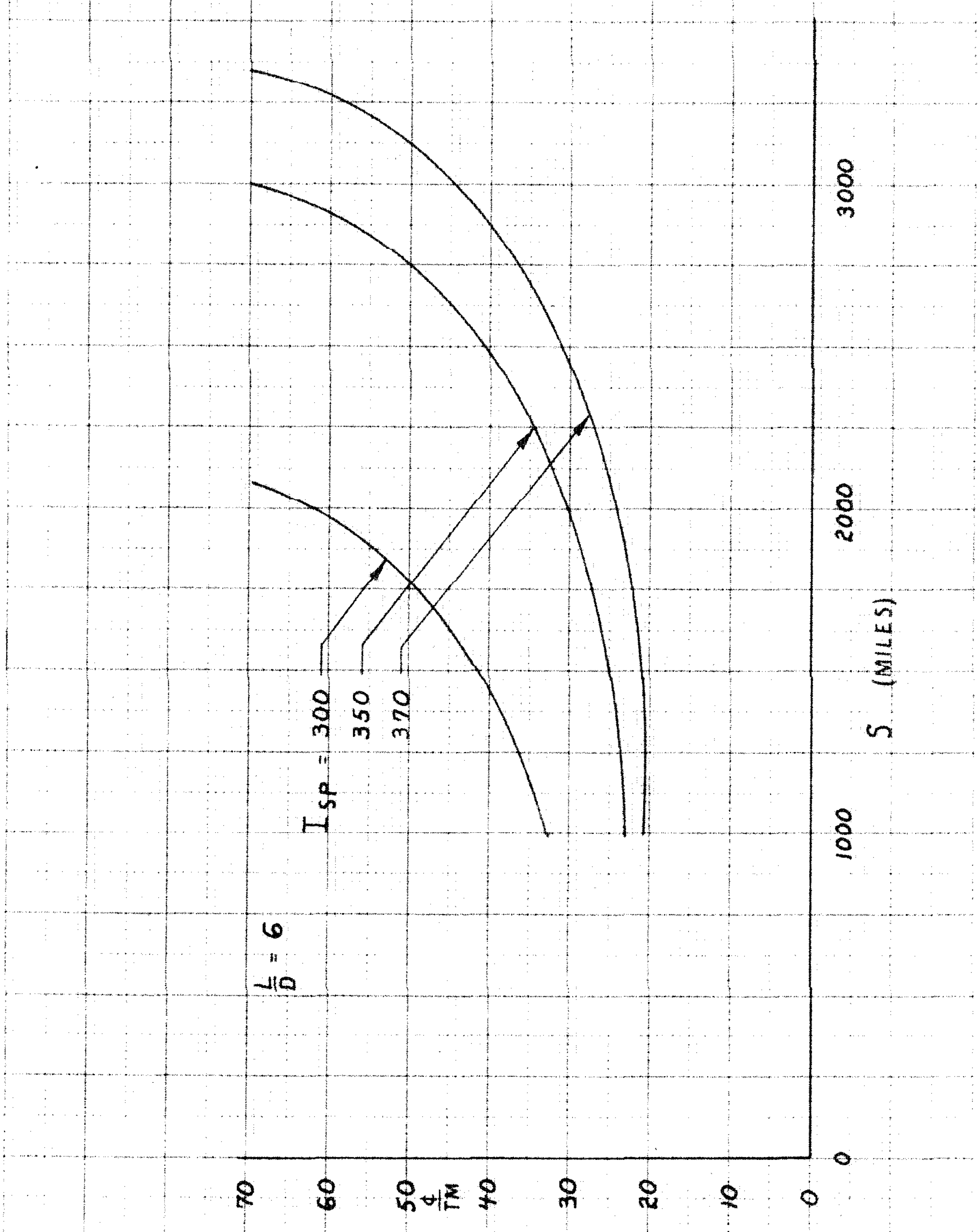


FIGURE 13

DIRECT OPERATING COST vs RANGE $\frac{L}{D} = 8$

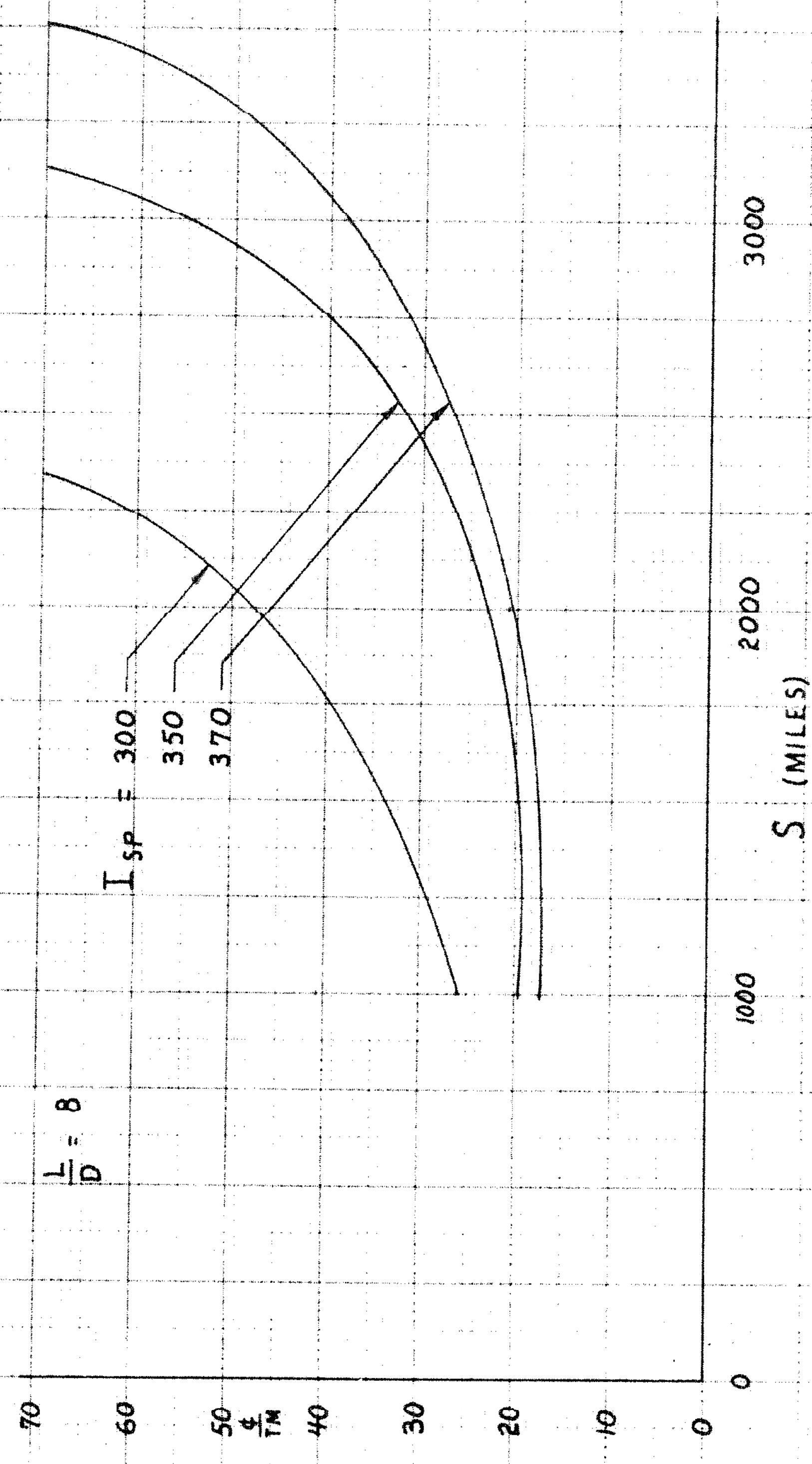


FIGURE 14

DIRECT OPERATING COST vs RANGE

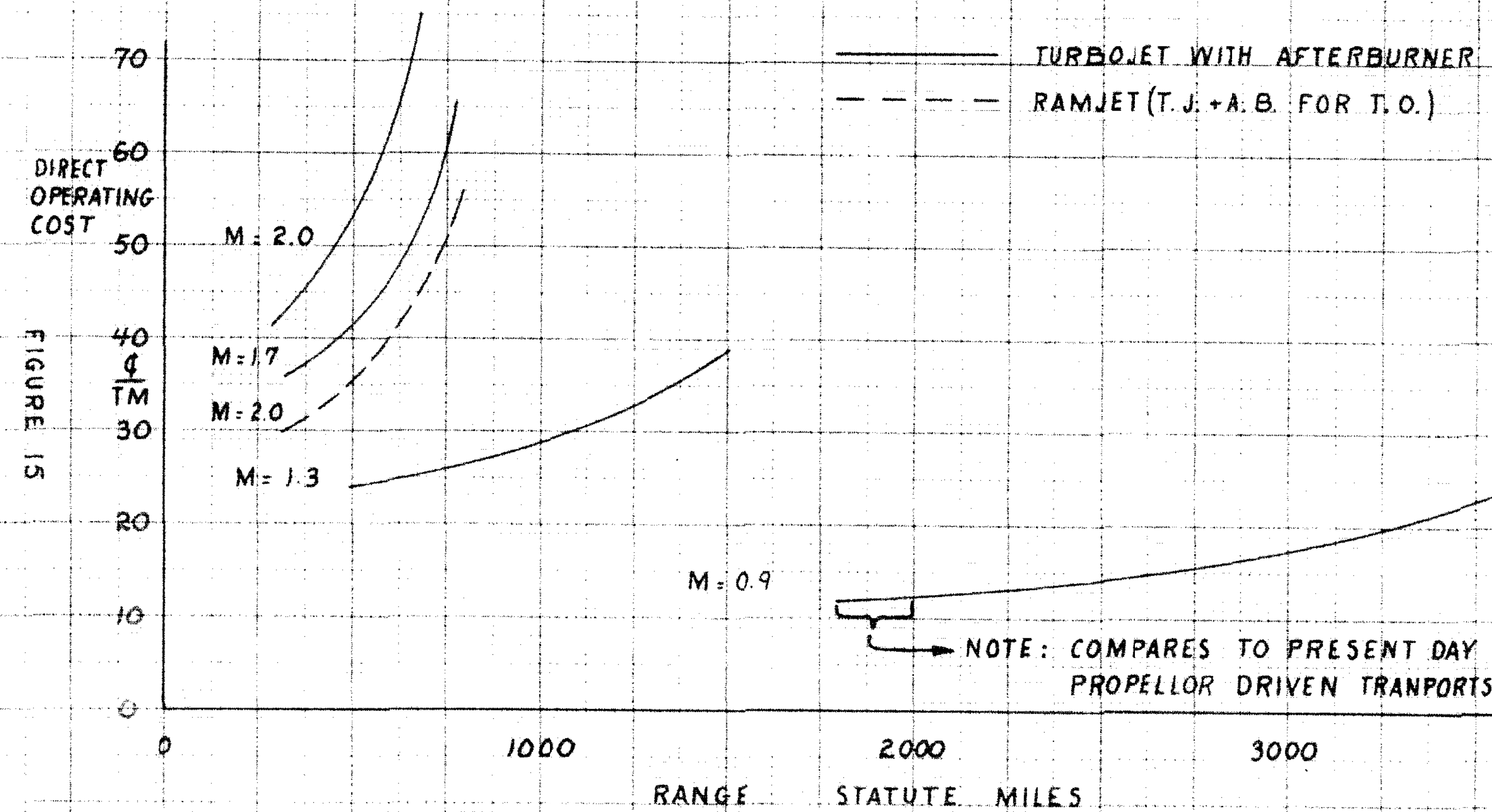


FIGURE 15

V. DISCUSSION

Gabrielli and von Karman (Reference 14) in their comparison study of power required to transport unit loads, introduce the parameter, $\epsilon = \frac{P}{WV}$, defined as the specific resistance. This is used in comparing many modes of transportation, in an effort to show what price, in terms of increased power required, must be paid for increases in speed. Figure 16 shows a plot given in this reference, with the above defined specific resistance as a function of the maximum speed attainable, for various vehicles. A limiting line, supposedly enclosing all vehicles of all types, is given to point out this rapid rise in required power with speed.

In another portion of the above paper, mention is made of the possibility that rockets using the type of trajectory outlined in this work, might fall well below the limiting line indicated. This would mean the rocket transport is an efficient means of travel. As will be seen in Figure 16, the rocket transports do fall below the limiting line. These values of ϵ are calculated by using the average values of the velocity over the total range, the landing weights, and the average values of the thrust; i.e., actual thrust times burning time, divided by total flight time. This then, does indicate that advances in speed can be made without paying the full cost in power installed.

Figure 17 shows a possible configuration for the rocket transport as postulated in this paper. Four rocket motors and two turbojets make up the power plant installation. A semi-circular cylinder is used for the fuselage, in consideration of the higher lift

SPECIFIC RESISTANCE OF SINGLE VEHICLES

P = POWER IN lb ft/S.; W = WEIGHT IN lb; V = SPEED IN ft/S.

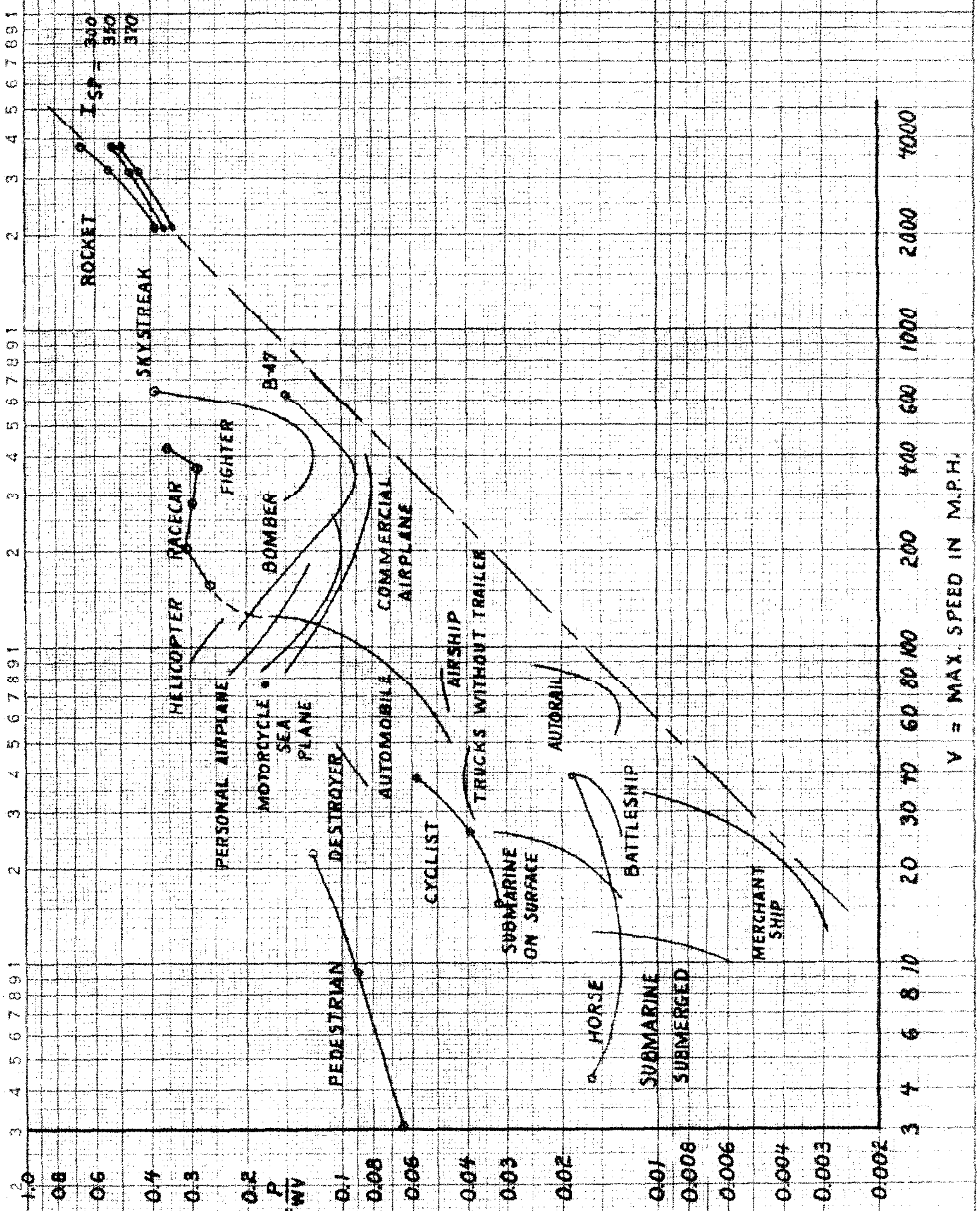


FIGURE 16

3 VIEW DRAWING

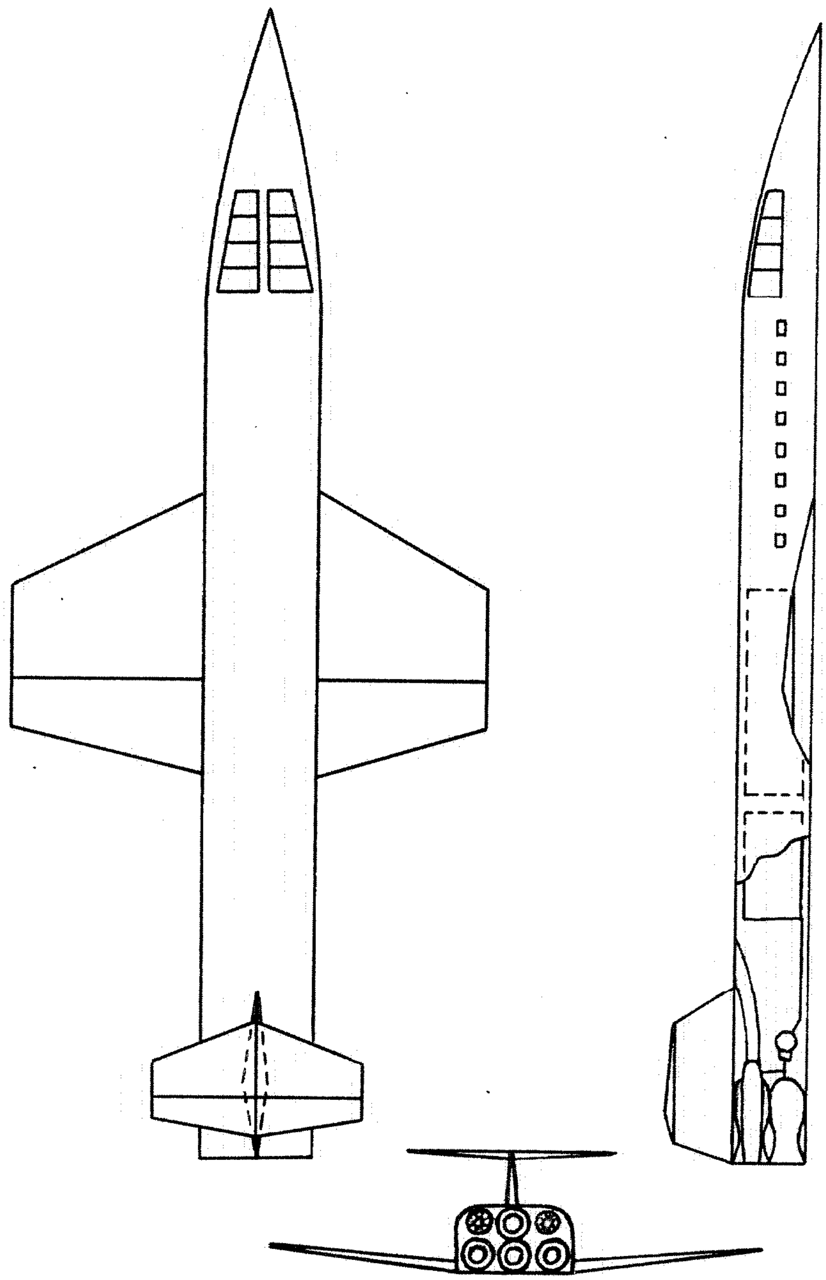


FIGURE 17

to drag ratios of this shape over the circular cylinder (Reference 8). Simple wedge airfoils of straight tapered planform are shown to be good in the Mach number ranges at which the rocket transport operates, and interference effects will aid lift to drag ratios most for planforms of small aspect ratio.

As yet no comparison of flight time has been made between the rocket transport, and the supersonic jet transports postulated in Reference 2. Average Mach numbers for the rocket transports range from about three for the 1000 mile range to about five for the 3000 mile range. This is seen to be a substantial improvement in speed over the Mach two jet transports which are at the same time limited in range to 1000 miles.

VI. CONCLUSIONS

In reviewing the assumptions made for the performance analysis of the rocket transport it is found that in some cases the simplifications tend to counteract each other. Neglect of the change of the effective exhaust velocity, c, and the gravitational attraction, g, with altitude, plus the neglect of the range and altitude covered during burning tend, in part, to account for the error made in setting the drag during the burning time equal to zero.

Though design of the rocket transport to the limitations of the human body in acceleration is used as a basis for the performance calculations, it is found that this practice would not only be undesirable for the passengers, but also it would be uneconomical in terms of possible payload that could be carried. Resulting accelerations were between one and six times the acceleration of gravity, with the higher accelerations occurring for a relatively short time just previous to burnout.

Fuel costs as given in the formulas of the Air Transport Association are quite optimistic compared to present day rocket propellant costs as given in Reference 7. However it was felt that a true comparison of operating costs could not be made on these values, since for the most part these fuels are in such limited production.

Night time operation, as specified in the coolant weight assumptions, is seen to be a drawback, but should it be possible to maintain a laminar boundary layer over most of the surface, the reduction in coolant weight needed should help to overcome the increase needed for daytime operation. Also conservative values were used for both operating altitude and flight speed, since during some of the latter part of the glide, no coolant would be needed at all because of the low flight speed.

Also noted from Reference 7, rather hopeful figures for the value of the specific impulse, I_{sp}, are taken for the curves. The present day cost of liquid oxygen and liquid hydrogen would prevent the use of these propellants to attain a specific impulse of 375 seconds. However, new processes, and increased demand may bring their costs down to a figure that would allow the rocket transport to compete in cost of operation to ranges as high as 4000 miles.

Considering then, the assumptions made and conditions imposed on the rocket transport, it must be assumed that one is unable to rule out this mode of transportation merely on operating cost alone. Many problems indeed remain to be solved, but the outlook should not be so dark as has been offered by other writers without a more complete consideration of the possibilities of this type of transportation.

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APPENDIX I

COOLANT WEIGHT (Water, with Turbulent Boundary Layer)

I_{sp} = 350; L/D_{glide} = 6; Regular motor weight

S	1,000		2,000		3,000	
$\dot{y}_1 = v_o = w_o$	7,010		9,750		11,700	
$t^{(2)} + t^{(3)}$	1,140		1,650		2,030	
v_{glide}	3,500		4,875		5,850	
ρ_{glide}	7.22×10^{-5}		3.74×10^{-5}		2.60×10^{-5}	
μ_{glide}	2.96×10^{-7}		3.42×10^{-7}		3.63×10^{-7}	
h	83,500		123,300		132,000	
a	971		1,059		1,097	
M	7.2		9.2		10.7	
	Fuselage	Wing	Fuselage	Wing	Fuselage	Wing
Length	70	40	70	30	70	20
Area	2,500	4,000	2,500	2,600	2,500	2,000
$-\zeta_o$ ($T_s = 600^o$)	-.20	-.25	0	-.15	0	-.20
$-\zeta_o$ ($T_s = 800^o$)	-.10	-.15	0	0	0	0
$\rho_s v_s g \times 10^4$ (600°)	1.5	2.5	0	1.5	0	2.5
$\rho_s v_s g \times 10^4$ (800°)	.75	1.5	0	0	0	0
$W_{coolant_{600}}$	425	1,125	0	675	0	980
TOTAL	1,550		675		980	
$W_{coolant_{800}}$	210	750	0	0	0	0
TOTAL	960		0		0	

APPENDIX II

WEIGHT BREAKDOWN $I_{sp} = 350$; $L/D_{Glide} = 6$; Regular motor weight

S	1,000	2,000	3,000
$\dot{y}_1$	7,010	9,750	11,700
n_{ave}	1.9	1.9	2.0
t_1	115	159	181
ξ	0.614	0.733	0.790
F	374,000	322,400	304,600
$F \times t_1$	42.9×10^6	51.3×10^6	55.3×10^6
n_o	0.870	0.612	0.523
n_1	3.90	5.04	6.25
$W_{fuel_{rocket}}$	122,800	146,600	158,000
$W_{power\ plant\ +\ turbine}$	13,780	11,880	11,210
$W_{fuel_{turbine}}$	4,500	5,390	5,810
$W_{structure}$	18,700	16,120	15,230
$W_{turbojet}$	2,180	1,440	1,085
$W_{fuel_{turbojet}}$	3,640	2,400	1,810
$W_{land.\ gear}$	3,640	2,400	1,810
$W_{equip + crew}$	600	600	600
$W_{coolant}$	1,550	675	980
$W_{payload}$	28,710	12,495	3,425
W_{gross}	200,000	200,000	200,000
$W_{landing}$	72,700	48,010	36,190

APPENDIX III

WEIGHT BREAKDOWN $I_{sp} = 350$; $L/D_{glide} = 8$; 60°/o motor weight

S	1,000	2,000	3,000
$\dot{y}_1$	6,380	8,880	10,750
n_{ave}	1.8	1.9	1.9
t_1	110	145	176
ξ	0.585	0.696	0.767
F	372,000	335,800	306,000
$F \times t_1$	40.9×10^6	48.7×10^6	53.8×10^6
n_o	0.860	0.679	0.530
n_1	3.48	4.52	5.57
$W_{fuel_{rocket}}$	117,000	139,200	153,400
$W_{power\ plant\ +\ turbine}$	8,200	7,390	6,730
$W_{fuel_{turbine}}$	4,500	5,360	5,910
$W_{structure}$	18,600	16,790	15,300
$W_{turbojet}$	2,360	1,660	1,220
$W_{fuel_{turbojet}}$	3,930	2,770	2,040
$W_{land.\ gear}$	3,930	2,770	2,040
$W_{equip.\ +\ crew}$	600	600	600
$W_{coolant}$	1,200	2,300	2,800
$W_{payload}$	39,680	20,960	9,960
W_{gross}	200,000	200,000	200,000
$W_{landing}$	78,500	55,440	40,690

APPENDIX IV

COST CALCULATIONS I_{sp} = 350; L/D_{glide} = 6; Regular motor weight

S	1,000	2,000	3,000
$W_{payload}$	28,710	12,495	3,425
$W_{airframe}$	22,340	18,520	17,040
W_{empty}	38,490	32,040	29,530
$W_{engines}$	15,960	13,320	12,300
W_{fuel}	130,940	154,390	165,620
$V_{B_{effective}}$	200	400	600
S x P	15,080	13,160	6,660
$\frac{\text{Cents}}{\text{Ton-Mile}}$			
engines	2.78	2.66	6.00
airframe	0.91	0.87	1.95
insurance	0.40	0.38	0.86
crew	0.68	1.17	3.78
fuel	18.30	24.70	64.50
TOTAL	23.1	29.8	77.1

APPENDIX V

COST CALCULATIONS $I_{sp} = 350$; $L/D_{glide} = 8$; 60 °/o motor weight

S	1,000	2,000	3,000
$W_{payload}$	39,680	20,960	9,960
$W_{airframe}$	22,530	19,560	17,340
W_{empty}	33,090	28,810	25,290
$W_{engines}$	12,130	10,160	8,770
W_{fuel}	125,430	147,330	161,350
$V_{B_{effective}}$	200	400	600
S x P	19,800	20,960	14,950

$\frac{\text{Cents}}{\text{Ton-Mile}}$

engines	1.52	1.21	1.47
airframe	0.66	0.55	0.68
insurance	0.25	0.20	0.26
crew	0.49	0.70	1.30
fuel	12.60	14.13	21.60
TOTAL	15.5	16.9	25.3