A NEW GYROMAGNETIC EFFECT

Thesis by

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The existence of this effect was suggested by Professor Albert Einstein to Professor S. J. Barnett in private conversations. Soon afterward Professor Barnett developed the quantitative theory and began investigations to test its validity. Circumstances, however, made it necessary to stop the work temporarily, before it could be completed.

The author takes great pleasure in expressing his deep appreciation to Professor Barnett for introducing him to this problem, and for the many suggestions and valuable assistance which he rendered.

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ABSTRACT

A circular cylinder of powdered ferromagnetic substance was magnetized to saturation along its axis by a strong magnetic field. A comparatively weak oscillating field, with a frequency of either 21.75 KC or 30.50 KC, was applied normal to the axis. An intensity of magnetization due to a new gyromagnetic effect appeared in a direction normal to the directions of the two applied magnetic fields. A quantitative theory for this effect was developed. Measurements of the magnitude of this magnetization indicated a value for the gyromagnetic ratio of $\rho = (1.04 \pm 0.06)\frac{m}{e}$ for permalloy and $\rho = (1.09 \pm 0.10)\frac{m}{e}$ for iron.

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I. Review of Related Research

A. Introduction

In some elementary descriptions of the magnetic properties of matter, it is assumed that the fundamental magnetic particle consists of a magnetic dipole. This description has been found inadequate to describe many magnetic phenomena, so other models have been devised.

Ampere suggested that paramagnetic and ferromagnetic properties arise from intra-molecular currents which flow in closed paths without resistance. In accordance with a theory of Weber, supported now by experimental evidence, electricity has inertia or mass. As a consequence of these two hypotheses, it can be expected that there will be properties analagous to angular momentum associated with these whirling Ampere molecular currents.

Gyromagnetic experiments demonstrate the existence of this angular momentum by measuring the numerical ratio between the angular momentum and the magnetic moment (each expressed in proper units) of macroscopic aggregates of Ampere molecular currents. These studies have led to greater understanding of the origin and nature of magnetic properties.[1]

B. Maxwell's Experiment

The first experiment designed to detect the angular momentum associated with magnetization was made by Clerk Maxwell[2] about 1861. He predicted that if a cylinder of iron was permanently magnetized along its axis, angular momentum associated with the intensity of magnetization, would lie along the same direction. If the rod was then mechanically rotated about an axis perpendicular to the direction of magnetization, the rod would, in accordance with the fundamental principles of mechanics, attempt to move about an axis perpendicular to both the direction of the magnetization and to the direction of rotation. Maxwell rotated a magnetized rod, and unsuccessfully tried to detect movement normal to the axis of rotation.[2] We know to-day that, even under the most favorable circumstances, one could hardly hope to detect any angular momentum associated with magnetization by such an experiment, because this angular momentum is so small.[3] More sensitive means were developed before the existence of this concealed angular momentum could be revealed.

C. Barnett Effect

The first successful experiments in the entire field of gyromagnetism were performed by Professor S.J. Barnett[4] His report to both the Ohio Physical Society and to the American Physical Society in 1914 contained the first gyromagnetic measurements ever presented. His results and conclusions have been fully substantiated by subsequent work.

If a previously demagnetized ferromagnetic cylinder is rotated about its axis of symmetry, the Ampere currents will, in accordance with the laws of mechanics, tend to precess. If the motion continues for an extended period of time, the elementary magnets will experience many interactions with neighboring particles. In the presence of these interactions the magnetic elements will behave similarly to macroscopic gyroscopes with friction. The elementary magnets will tend to turn so that the direction of their spins will be more closely aligned with the axis of rotation.

In general, the elementary magnets in a solid substance are not free to rotate, for they are acted upon by forces coupling them with neighboring particles. The current whirls, therefore, will not all align completely in the same direction, but will show a tendency to do so.

When the elementary magnets in a ferromagnetic substance orient themselves in a preferred direction as described above, the substance is said to be magnetized. The phenomenon of magnetization which appears during rotation is known as the Barnett effect.

In Professor Barnett's experiment, a bar of ferromagnetic material was rotated in a region of space in which the Earth's magnetic field was neutralized, and the intensity of magnetization acquired by rotation was compared with that produced in the same rod by a known magnetic field.

Professor Barnett defined the gyromagnetic ratio* ρ as the ratio of the angular momentum of the magnetic elements to the magnetic moment of the elements. He measured the gyromagnetic ratio for various substances obtaining values between 1.00 m/e where m is the electron mass and e the electron charge in EMU (here e is a negative number).

Theoretical considerations showed that if the magnetic properties were due to the orbital motion of the electrons, the gyromagnetic ratio would be 2.00 m/e ; and if the properties were due to electron spin, the ratio would be 1.00 m/e . Professor Barnett's results were interpreted to show that the greater part of ferromagnetism is associated with the spinning electron, with the orbital motion contribution at most a few per cent.

This experiment gave the first direct proof of the existence of Ampere's molecular currents. It proved that these currents were composed of negative charges and that the charges had mass. The Barnett effect is sometimes called "magnetization by rotation" because it revealed this new method of magnetizing a rod.

D. Einstein and de Haas Effect

Soon after Professor Barnett presented his results, another experiment, closely related to the Barnett effect, was reported by Einstein and de Haas.(7) Their experiment measured in effect, the change in angular motion associated with a change in magnetization.

They obtained for ρ about 2 m/e, and concluded that the sign was negative. Soon afterward Lorentz(8)

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* Some authors use a quantity g as the gyromagnetic ratio. The ratio between ρ and g is $\rho = 2/g$

demonstrated that their determination of the sign was inconclusive. In 1916 another determination of the sign was made by Einstein[9] and separately by de Haas[10], at which time they demonstrated that the sign of ρ was negative.

Professor Barnett has studied and removed many sources of error that appear in this type of experiment[11], and obtained results in close agreement with those obtained from the Barnett effect.

E. Resonance Effects

In recent years a number of experiments using resonance phenomena have been devised to measure gyromagnetic ratios of the atomic nuclei and of the electrons. The variety of these experiments has greatly increased with the advancement of microwave techniques.

Rabi[12] and others have developed atomic beam resonance methods to measure the gyromagnetic ratio of the nuclei and other elementary particles. Bloch and others[13] have measured gyromagnetic ratios of the nuclei and of paramagnetic substances by applying oscillating fields normal to steady fields.

Griffiths and others[14][15][16] have used resonance methods to measure the gyromagnetic ratios for electrons in ferromagnetic substances. In one of their experiments, one plane wall of a resonant cavity was constructed of the substance to be studied. The cavity was excited in a mode of oscillation such that the oscillating magnetic fields were in planes parallel to this wall. A steady magnetic field was applied in a direction such that the lines of force lay in planes parallel to the plane surface of the cavity wall. Under the influence of this field, the electrons in the substance precessed as described by Kittel.[17] When the frequency of the cavity was equal to the precessional frequency, the energy absorbed by the wall was increased. By measuring the "Q" of the cavity, this absorption could be detected, and the gyromagnetic ratio calculated from Kittel's formula.[18]

The numerical results obtained by these resonance methods differed from those obtained from the Barnett effect and the Einstein-de Haas effect.* The differences lay outside the experimental errors involved. Charles Kittel[19] has offered a theoretical explanation of these differences.

F. Experiments with Rotating Fields

In another type of experiment, closely related to this present study, a cylinder of magnetic substance was held stationary, and a rotating magnetic field was applied in such a manner as always to be perpendicular to and rotate about the axis of the cylinder. Since the vector intensity of magnetization rotated with this magnetic field, a gyromagnetic intensity of magnetization along the axis might be expected.

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* For table of values, see page (85).

J. W. Fisher[20] performed experiments by this method between the years 1922 to 1925. He had expected to find a longitudinal intensity of magnetization equal to that which would have resulted from mechanically rotating the rods, initially unmagnetized, at the same high frequency. He had expected an effect of this magnitude even when he employed weak magnetic fields.

Fisher employed frequencies from 20,000 to 50,000 cycles per second, and field strengths of less than 100 gausses. He obtained magnetometer deflections of 1 mm or less in the wrong direction. If the rod had been mechanically rotated at the same frequency as is done to obtain the Barnett effect, deflections of about 350 mm would have resulted.[21]

Soon afterward Professor Barnett[22] showed that Fisher's expected deflections, under the same favorable hypothesis, should be multiplied by the small factor $3J_1/2J_\infty$, where J_1 is the transverse intensity of magnetization, and J_∞ is the saturation intensity of the magnetization of the rod. The actual deflection was much smaller than possible under the most favorable hypothesis.

In a series of investigations, more extensive and more precise than the previous ones, Professor Barnett[21] used frequencies of 14,650 and 21,000 cycles per second. He obtained, as was expected, results far smaller than the values calculated on the most favorable hypothesis.

In considering these results, Professor Einstein suggested that the magnetic elements did not rotate with the intensity of magnetization vector, but had their moments periodically reversed by the rotating magnetic field.

Consider a cylinder in a rotating magnetic field, with a cartesian coordinate system placed so that the Z coordinate axis lies along the axis of the cylinder. The rotating magnetic field is assumed always to lie in the XY plane. For simplicity one may imagine that one-half of the magnetic elements have their magnetic moments oriented in the X direction, and one-half in the Y direction, and none in the Z direction. Further, one can assume that these elementary magnets change their direction of orientation only by abrupt jumps of 180°.

As the X and Y components of the rotating field change, the elements in the X and Y groups will independently jump in such numbers that the intensity of magnetization, which is the vector sum of all the magnetic moments per unit volume, will rotate and follow the magnetic field. Under these conditions, no gyromagnetic effect would be expected.

We can extend Einstein's explanation and obtain a simple classical explanation for this null effect.

With respect to the axis of the cylinder, some of the elements would revolve clockwise and some counterclockwise, as they jumped from one position to another. There is no a priori reason to expect either clockwise or counterclockwise rotations to predominate. The clockwise rotating elements would give rise to a gyromagnetic effect in one direction along the axis, while the counterclockwise rotating ones would give rise to an effect in the opposite direction. In a polycrystalline rod, the macroscopic gyromagnetic intensity of magnetization resulting from the jumps of many magnetic elements would be close to zero.

Professor Einstein's explanation is consistent with evidence on the process of magnetization in weak fields, where it is known that most of the change in intensity of magnetization is associated with Barkhausen jumps, involving simultaneous jumps of the magnetic elements in regions inclosing as many as 10^{15} atoms.(23)

The present study describes a successful attempt, using a procedure suggested in 1931 by Professor Einstein and developed by Professor Barnett and the author to eliminate jumps of the magnetic elements. Under the conditions that were obtained in this study, elementary magnets could be made to rotate, considered classically, under the influence of applied external fields, and gyromagnetic measurements could be made.

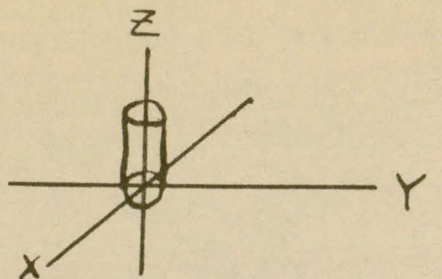
The fundamental principle involved in Bloch's recent experiments in nuclear induction is idential with that of this experiment, but the latter experiments are much more complicated because resonance and near resonance frequencies are involved and the material is not saturated. Bloch's experiments deal with the magnetic moments of the nucleus, while this experiment deals with the elements associated with ferromagnetism.

II. Derivation of Equations

It is well known that in the region of near saturation after the completion of most of the Barkhausen jumps, changes in the direction or magnitude of the intensity of magnetization occur as a result of rotation of the elementary magnets out of the direction of easiest magnetization (along the 100 planes of the crystal) toward the direction of the applied magnetic field.[23] It was suggested that a cylinder be magnetized to near saturation along its axis, so that practically all of the magnetic elements point in the same direction. Then a small oscillating magnetic field, applied normal to the axis of the cylinder, would cause the in tensity of magnetization to rotate through a small angle about the axis of the cylinder. Since the rod was magnetized to near saturation in one direction, one can assume that all of the change in the direction of the intensity of magnetization would be due to rotations of the individual elements, and would not be due to quantized jumps.

If these elements could be made to rotate, then they would process, and give rise to gyromagnetic effects. This experiment employs procedures designed to detect this precession.

In this work it is assumed that the results obtained with the Barnett effect[4] and the Einstein de Haas effect demonstrate the validity of the concept of the gyromagnetic ratio, and verify equation (3) page 13. This equation is used as a basis to develope the relations needed for this experiment.



A long circular cylinder of the substance to be studied is placed with its axis in the Z direction, in a homogeneous time independent magnetic field H_z. This field is sufficiently strong to produce the axis along an intensity of magnetization J_z which is close to the saturation value, so that one can assume that all of the magnetic elements point essentially in the Z direction.

In a direction X, normal to Z, a comparatively weak oscillating magnetic field is applied. When the rod is absent, the magnetic intensity in the space the rod is to occupy is uniform and can be described by the formula*

$$H_x = H_{0x} \sin \omega t \tag{1}$$

This oscillating magnetic field causes the total intensity of magnetization of the rod always practically parallel to and equal to J_z to oscillate through the small angle θ in the XZ plane, thus producing a small intensity of magnetization J_x, synchronous with H_x, and proportional to it. The angle θ is given by the relation**

$$\theta = \frac{J_x}{J_z} \tag{2}$$

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* Here, $\omega = 2\pi f$ when f is the frequency.

** Eddy currents are neglected in this derivation.

The rotation of these elements gives rise to a gyromagnetic intensity of magnetization in the Y direction (normal to X and Z) equivalent to that which would be produced by a real magnetic intensity H_y of strength [4] [22]

$$H_y = \rho \frac{d\theta}{dt} \qquad (3)$$

In this equation H_y is the magnetic intensity measured in gausses, ρ is the gyromagnetic ratio, and $\frac{d\theta}{dt}$ is the angular velocity of the magnetic elements.

Substituting in equation (3) the value for Θ from equation (2), one obtains

$$H_y = \frac{\rho}{J_z} \frac{dJ_x}{dt} \qquad (4)$$

An exact calculation of the intensity of magnetization in a cylinder of finite length is difficult. However, a close approximation for a long, narrow rod can be obtained by assuming that the intensity of magnetization is uniform inside the rod along its entire length. It is, however, important to include the demagnetizing factor in the calculations.

A well known formula for the intensity of magnetization inside a uniformly magnetized medium is (See Appendix I, page 88).

$$J_s = \left[\frac{\mu_s - 1}{4\pi + (\mu_s - 1)D_s}\right] H_s \qquad (5)$$

where J_s is the intensity of magnetization, μ_s the permeability, D_s the demagnetism factor depending upon the geometrical shape of the medium, and H_s the applied magnetic intensity.

On substituting the value for J_x from equation (5) into equation (4), one obtains

$$H_y = \frac{\mu_x - 1}{4\pi + (\mu_x - 1)D_x} \frac{e}{J_z} \frac{dH_x}{dt} \qquad (6)$$

Substituting the expression for H_x from equation (1), one obtains

$$H_y = \frac{(\mu_x - 1)\, e\, \omega H_{ox} \cos \omega t}{[4\pi + (\mu_x - 1)D_x]\, J_z} \qquad (7)$$

Now, from equation (5) again, one obtains for the value of J_y the expression

$$J_y = \frac{\mu_y - 1}{4\pi + (\mu_y - 1)D_y} H_y \qquad (8)$$

Substituting for H_y from equation (7) into equation (8), one obtains *

$$J_y = J_{yo} \cos \omega t = \left[\frac{\mu - 1}{4\pi + (\mu - 1)D}\right]^2 \frac{\omega e H_{ox} \cos \omega t}{J_z} \qquad (9)$$

This is the formula for the intensity of magnetization due to the gyromagnetic effect.

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* Here we assume $\mu_x = \mu_y$ and $D_x = D_y$. From this point on the subscripts are eliminated.

It is of interest at this point to deviate from the main trend of the derivation, to estimate the order of magnitude of the effect. From equation (9), after setting D equal to its approximate value 2π*, one obtains

$$J = \frac{1}{4\pi^2}\left[\frac{\mu-1}{\mu+1}\right]^2 \frac{e\,\omega H_{ox}\cos\omega t}{J_z} \qquad (10)$$

Now, the magnetic induction B_y is related to the intensity of magnetization, in the case of a cylinder of infinite length, by the formula

$$B_y = 2\pi J_y \qquad (11)$$

Approximate values for the quantities concerned, as obtained in the experiment, were

$$\mu = 10$$
$$f = 3\times10^4$$
$$H = 10 \text{ gauss}$$
$$e = 5\times10^{-8}$$
$$J_z = 10^3$$

With these numerical values for the quantities in equation (10), one obtains for B_y from equations (10) and (11)

$$B_y = 10^{-5} \text{ gausses} \qquad (12)$$

The problem of detecting an oscillating magnetic field of a strength of 10^{-5} gauss in the Y direction, with an oscillating magnetic field of a strength of 10 gauss in the X direction proved to be difficult.

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* See page (41) for an experimentally determined value of D.

From equations (1) and (10) it can be seen that the gyromagnetic field would be in quadrature phase with the applied field. Hence the EMF induced in a coil by B_y would be in quadrature phase with the EMF induced by H_x. This phase difference proved to be of importance in detecting and identifying the gyromagnetic effect.

We now return to the derivation of the formula for the voltage induced in a secondary coil due to the gyromagnetic induction. From equation (9), one obtains for J_{oy}

$$J_{oy} = \left[\frac{\mu - 1}{4\pi + (\mu - 1)D}\right]^2 \frac{\omega \rho}{J_z} \tag{13}$$

Now, the secondary coil, used to measure the intensity of magnetization, was wound on a frame symmetrically surrounding the cylindrical sample. (See figure on page (95)).

This coil, of length L, was wound parallel to and at a distance c from the axis of the cylinder. The cylindrical sample was of length l and radius b, where $L > l$ and $c > b$.

From Appendix IV, the expression for the RMS voltage E (practical units) in this secondary coil is

$$E = 2\sqrt{2}\,\pi N b l A \omega \left(1 + \delta - \frac{\Delta}{b}\right) J_{oy} \times 10^{-8} \tag{14}$$

The meanings of the symbols in equation (14) are as follows:

E is the RMS voltage from the gyromagnetic effect, measured in practical units.

μ is the transverse permeability of the ferromagnetic substance.

D is the demagnetizing factor measured across the rod.

b is the radius of the rod.

N is the number of turns in the secondary coil.

H is the amplitude of the applied oscillating magnetic field.

ω is related to the frequency of the oscillating field $\omega = 2\pi f$

ρ is the gyromagnetic ratio.

J_z is the intensity of magnetization (near saturation) along the axis of the rod.

l is the length of the rod.

A is an experimentally determined constant of the secondary coil.

Δ is equal to c-b.

D is the transverse demagnetizing factor. It is equal to 2π for an infinitely long cylinder. For a long, narrow rod it will differ from 2π by a small amount as indicated by the relation

$$D = 2\pi(1-\delta) \qquad (16)$$

where δ is a small number. This quantity δ was measured experimentally, as was described on page (41), and found to be 0.01.

Upon substituting the expression for J_{oy} from equation (13) into equation (14), one obtains

$$E = 2\sqrt{2}\,\pi N b l A \omega^2 \left(1+\delta-\frac{\Delta}{b}\right) \frac{(\mu-1)^2 \rho H_{ox} 10^{-8}}{(4\pi+(\mu-1)D)^2 J_z} \qquad (15)$$

Substituting this expression for D into equation (15), one obtains

$$E = \frac{\sqrt{2}}{2\pi}\left(\frac{\mu-1}{\mu+1}\right)^2\left(1+3\delta-\frac{\Delta}{b}\right)\frac{N b H_{ox} \ell \rho \omega^2 A \times 10^{-8}}{J_z} \qquad (17)$$

Solving equation (17) for ρ, one obtains

$$\rho = \sqrt{2}\,\pi\left(\frac{\mu+1}{\mu-1}\right)^2\left(1-3\delta+\frac{\Delta}{b}\right)\frac{J_z E \times 10^{8}}{N b \ell H_{ox} \omega^2 A} \qquad (18)$$

In developing these formulas, the discussion was based upon macroscopic arguments. A derivation based upon microscopic arguments would produce similar results. If the elementary magnets are aligned in the Z direction, by the application of a strong external magnetic field, and if a small disturbing magnetic field is suddenly applied in the X direction, the elementary magnets at first respond diamagnetically to this change and start to precess. In the solid state, this precession is dampened out by interactions between particles, leaving the magnetic elements in a new orientation such that the direction of their magnetic moments is more closely aligned with the applied field. If, instead of a single, abrupt change, one applies an oscillating field with a frequency small compared with the relaxation time, and small compared with the precessional frequency, then the magnetic elements will continually readjust their positions so as to follow the applied field. In this way the magnetic elements will rotate through small angles, always synchronized with the applied fields.

In this discussion it was not necessary to use quantum mechanics to obtain a valid formula, because the quantum numbers concerned are so large. Experiments indicate that, in ferromagnetic substances, the magnetic elements in volumes containing approximately 10^{15} atoms are coupled together and turn together. Classical methods will apply in such cases involving large quantum numbers.

III. Apparatus

A. General Discussion of Experiment

The apparatus could, for convenience, be divided into three groups:

1. The first group included all of the electronic equipment that aided in producing strong oscillating currents. These currents were used to energize the primary coils and produce the oscillating magnetic field.

2. The second group included the primary coils which aided in the production of the magnetic field H_x; a large 300 lb. coil, which produced the steady magnetic field H_z; the secondary coil, which was used to detect the gyromagnetic induction B_y; the substance to be tested; and thermal devices to control the temperature of these coils.

3. The third group included the amplifier used to increase the strength of the gyromagnetic signal appearing in the secondary coil, and devices to measure the magnitude and phase of this signal.

In selecting the proper frequencies, it was desirable to employ frequencies in a range such that the permeability of the compressed powders would be modified only slightly by eddy currents. It was also desired to use frequencies such that the time of oscillation was long, compared with the relaxation time of the material. On the other hand, since the voltage in the secondary coil would increase with the square of the frequency, (See equation (17)),

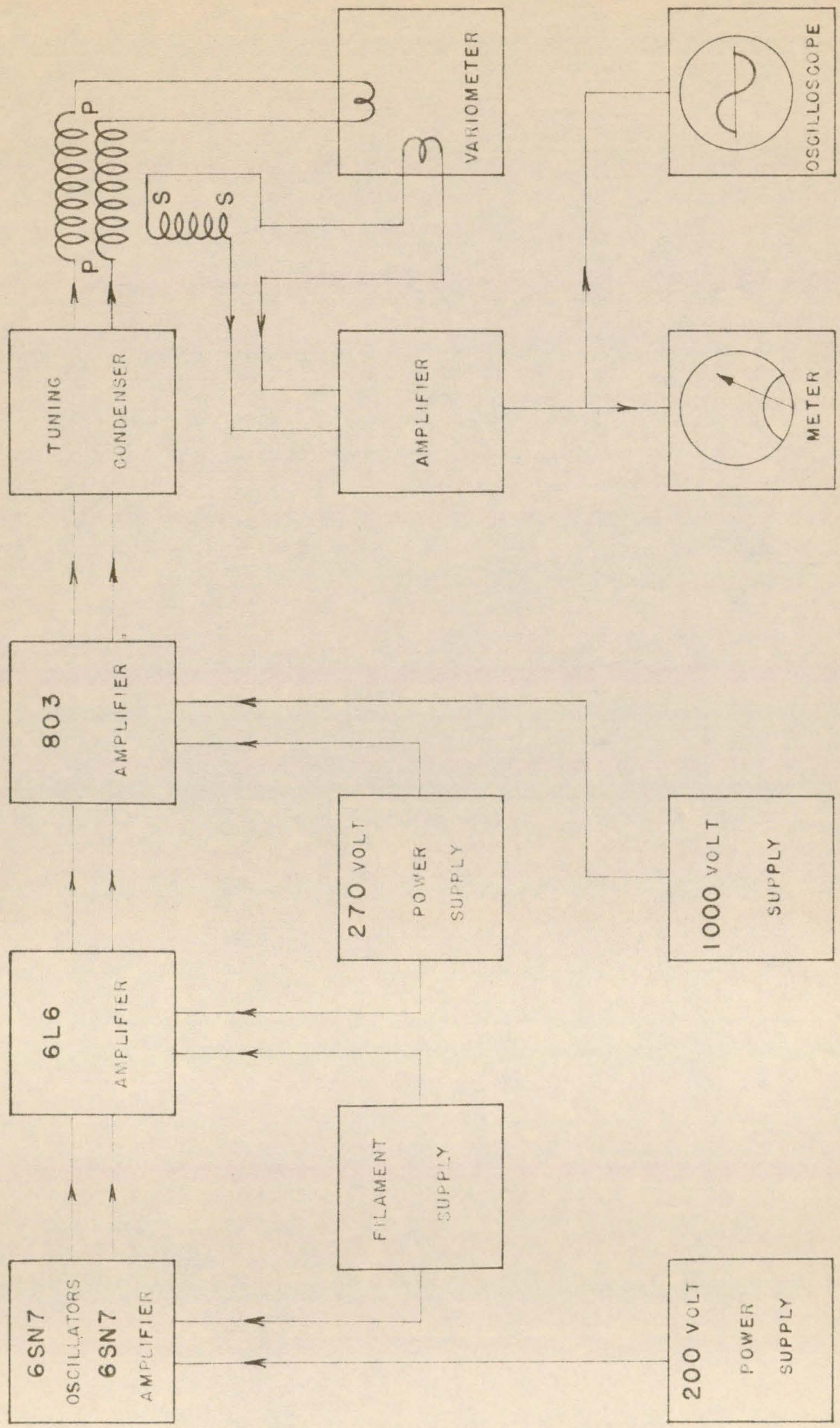

FIGURE 1 - BLOCK DIAGRAM

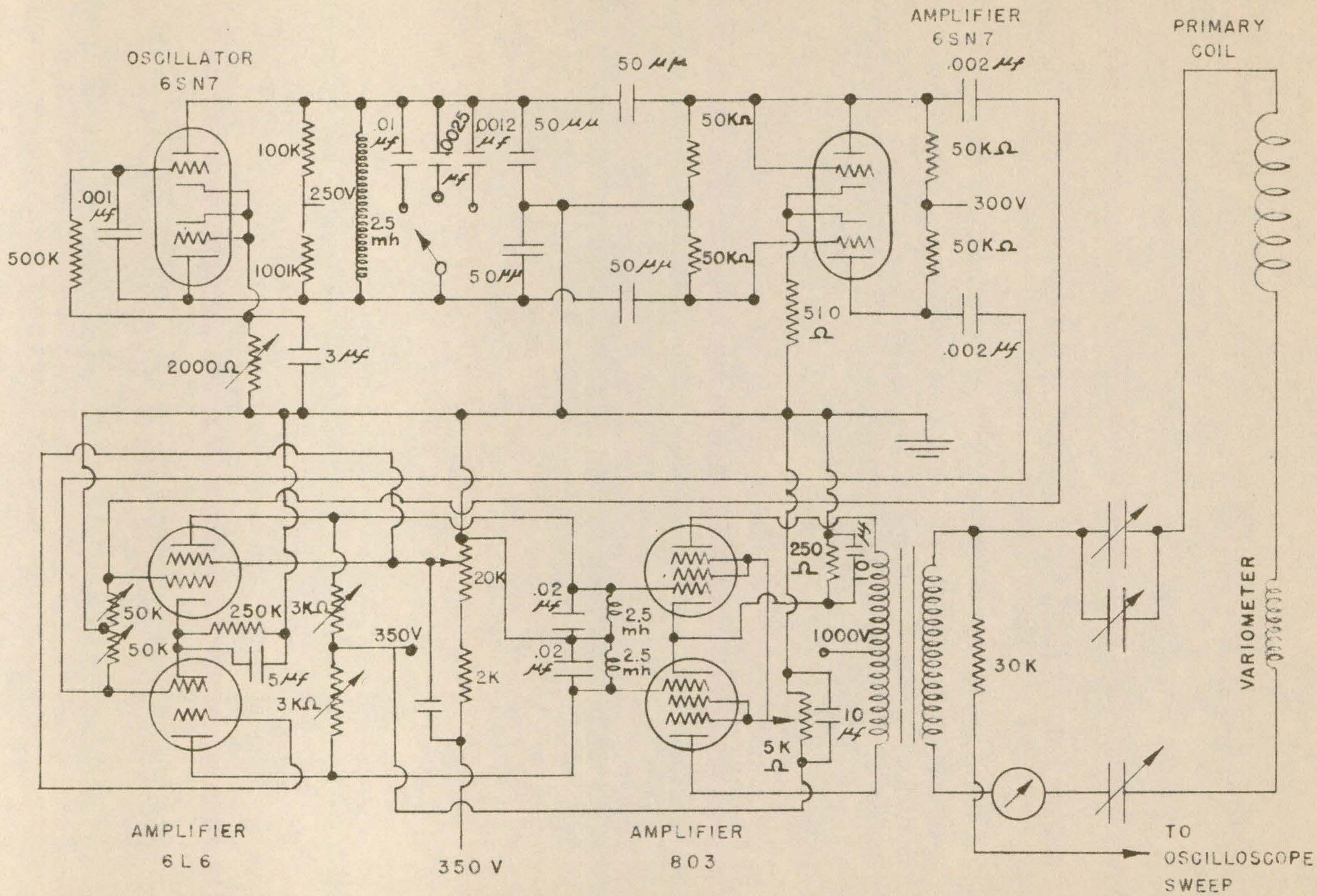

FIGURE 2- CIRCUIT DIAGRAM OF PRIMARY CIRCUIT

it was desirable to use a high frequency so as to obtain large voltages. From these considerations, frequencies in the range from 20 KC to 32 KC were selected.

To make the apparatus as symmetric as possible, push-pull circuits were used where practicable.

B. Primary Circuits

The circuit adopted produces a steady output voltage at a constant frequency, and provides an easy means to change the frequency if desired (Figure 2). This oscillator consisted essentially of a 6SN7 vacuum tube in a multivibrator circuit, with an attached tank coil. Specially selected mica condensers and a high "Q" inductance were used in the tank circuit to insure favorable resonance characteristics. The output leads were symmetrically placed on the tank circuit.

The oscillator was followed by a 6SN7 push-pull, buffer amplifier. The coupling between the oscillator and the amplifier was made small in accordance with accepted practices.

The plate power for these tubes was furnished by a regulated power supply, using 5U4, 6B4, and 6SJ7 tubes in standard circuits, furnishing a plate voltage of about 200 volts. A common 60 cycle filament voltage supply was provided for these and other tubes.

The oscillator and buffer amplifier were inclosed together in a grounded iron box that provided magnetic shielding. The tank circuit was inclosed in a separate grounded copper box, with additional thermal insulating material, which was, in turn, placed inside the iron box.

Measurements indicated that the oscillator could be adjusted so that its frequency would change only by 1/10 of 1% when the plate voltage changed from 150 to 300 volts. Experiments indicated that the voltage output of the regulated power supply changed by an amount less than 1% during normal operation. Hence, changes in the frequency due to changes in the plate voltage could be neglected. Experiments indicated that the changes in frequency that occurred during actual experiments, resulting from all causes, were less than 1/10 of 1%.

The output of the buffer stage was fed into two 6L6 vacuum tubes in a resistor-condenser coupled, push-pull amplifier (Figure 2). Variable resistors were placed in the grid and screen grid circuits to provide a choice of output signal strengths, while variable tap plate resistors were used to balance the gain of the two channels. An iron box inclosed the 6L6's and the accompanying circuit elements.

A separate regulated power supply, employing 5Z4, 2A3, and 6SJ7 tubes in standard circuits, furnished the 270 volt plate voltage. This power supply was inclosed in a separate shielded box.

In the next stage, the final power amplifier, a pair of 803 vacuum tubes in push-pull were used. Two high "Q" parallel resonant circuits, connecting the grids with the ground, removed the higher harmonics and insured that a relatively pure sine wave was applied to the input.

The plates were connected to the terminals of a center tap transformer. This transformer was designed* in such a manner as to provide excellent insulation and screening between primary and secondary coils. The transformer functioned excellently at these relatively high frequencies. The 803 tubes and their circuits were contained in a grounded iron box.

The 270 watt power supply that furnished the plate current for the 6L6 tubes in the previous stage also furnished the current for the screen grid circuits of these tubes.

The plate voltage was supplied from a separate rectified power source. A continuously variable transformer furnished voltages between zero and 1200 volts. The output of the transformer was connected to two 866-A mercury-arc tubes, used in full wave rectifier circuit.

The output of the push-pull 803's amplifier was fed into a tuned series resonant circuit, the inductance consisting of the Helmholtz coil used to furnish the

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* This transformer was designed by Professor S. J. Barnett.

oscillating magnetic field, and the capacity consisting of two variable condensers. The two condensers were so placed as to provide symmetry and balance. A small trimmer condenser was placed in parallel with one of the large variable condensers for a fine tuning control. These condensers were inclosed in another iron box.

C. Coils and Magnetic Samples

The second group of apparatus consisted of several coils, the sample to be tested, and the thermal control devices. Some of these components showed microphonic behavior and were placed on special tables, separate from the rest of the apparatus.

The output of the 803 amplifier was used to excite the primary coil (P-P in Figure (5)) and provide the oscillating magnetic field H_x. The primary coils finally adopted were constructed on a rectangular linen base bakelite frame, 16½ inches long and 3 and 6/10 inches wide, and one inch thick. Two sets of grooves (Figure (3)), ½ inch by 8/10 inch were milled in the bakelite, 3/10 inch from the outside edge. In each groove 300 turns of #29 silk, enamel covered copper wire were wound in 15 layers of 20 turns per layer. Each turn was separated from the next by a single layer of empire cloth, and was liberally covered with General Electric electrical varnish, to increase the mechanical strength and electrical insulation. The ends of the wire from each coil were brought to separate terminals on the side of the frame.

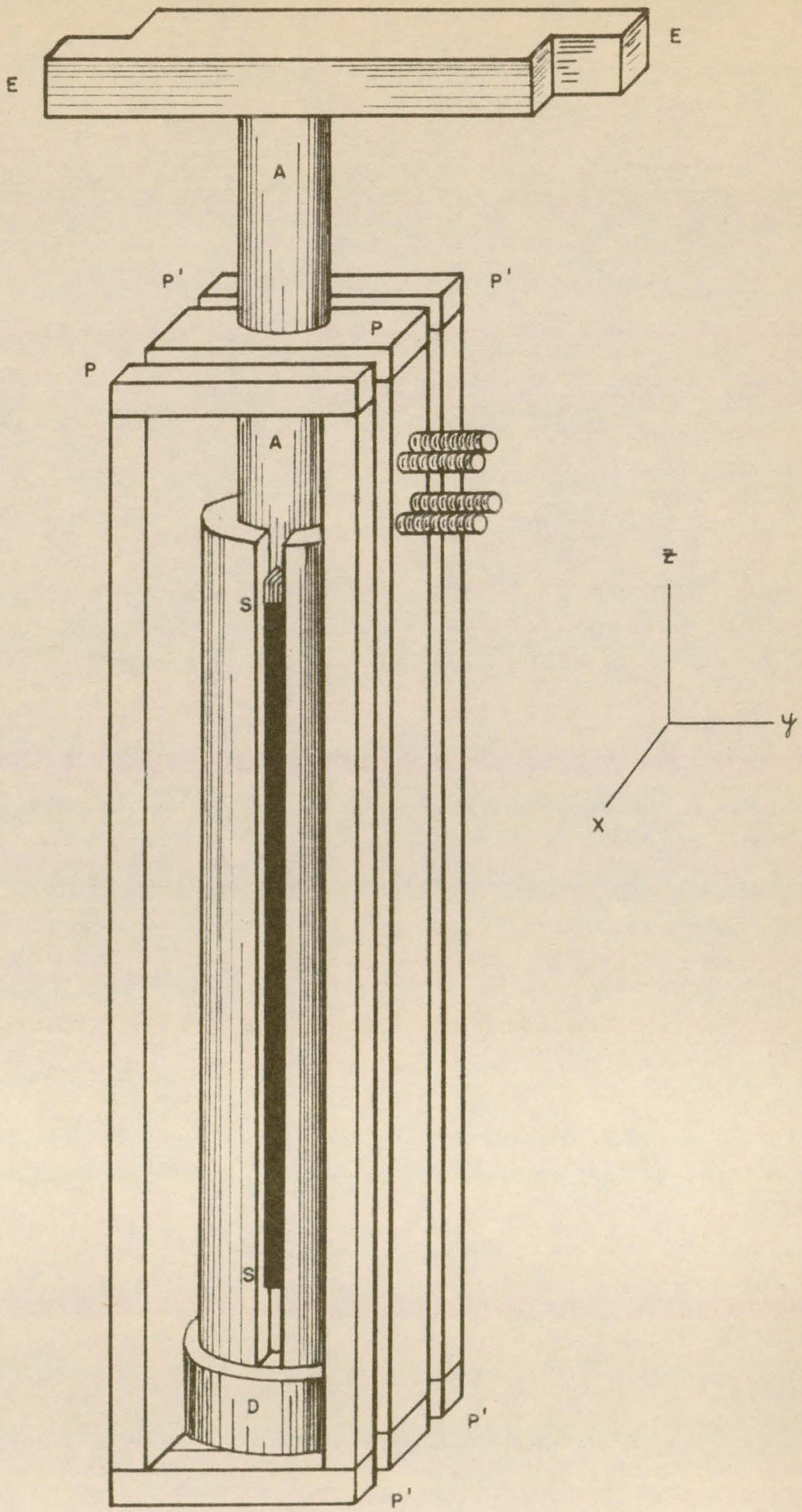

FIGURE 3— PRIMARY AND SECONDARY COIL

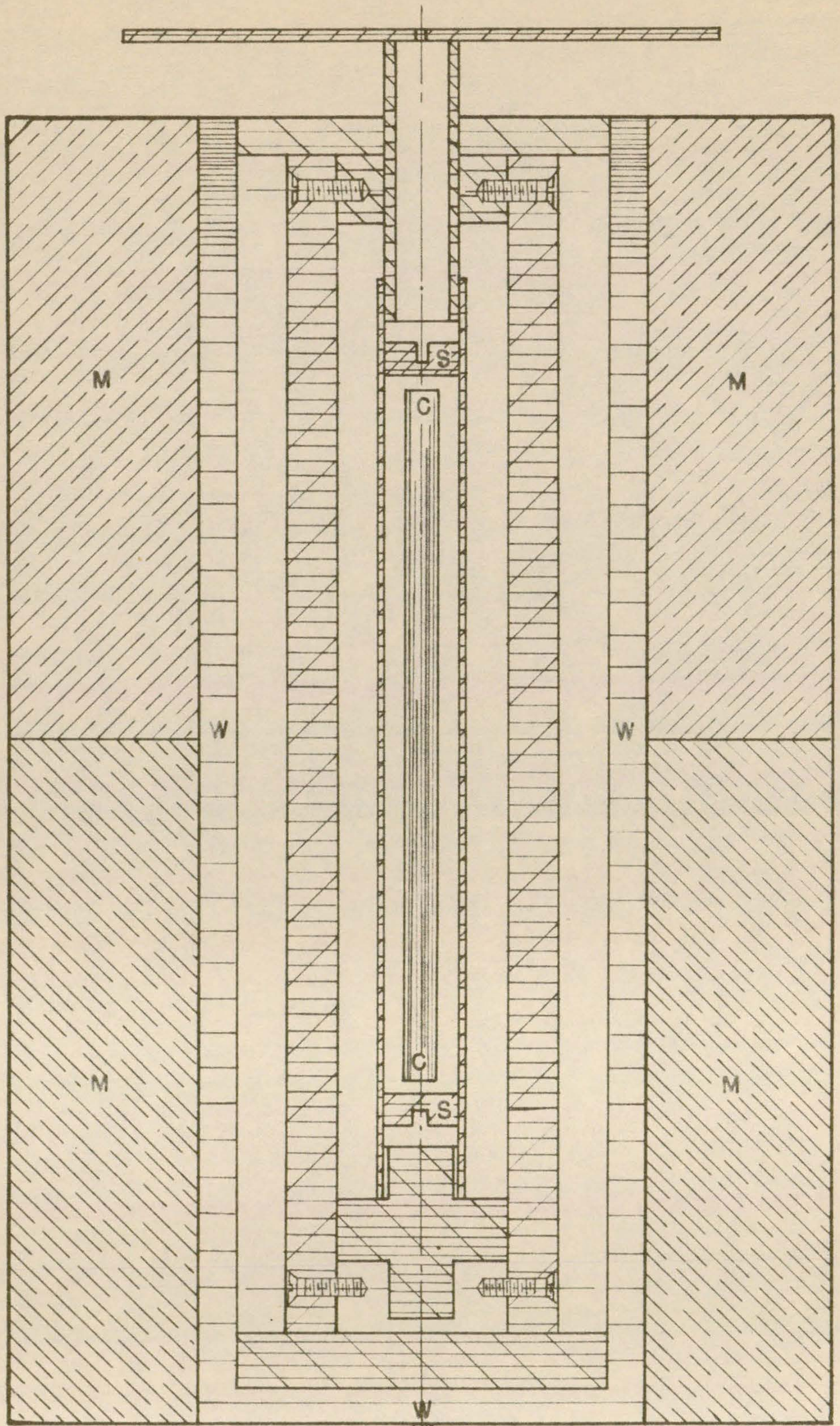

FIGURE 4- CROSS SECTION OF 300 LB COIL M, COOLING COIL W, ROD C, AND SECONDARY COIL S

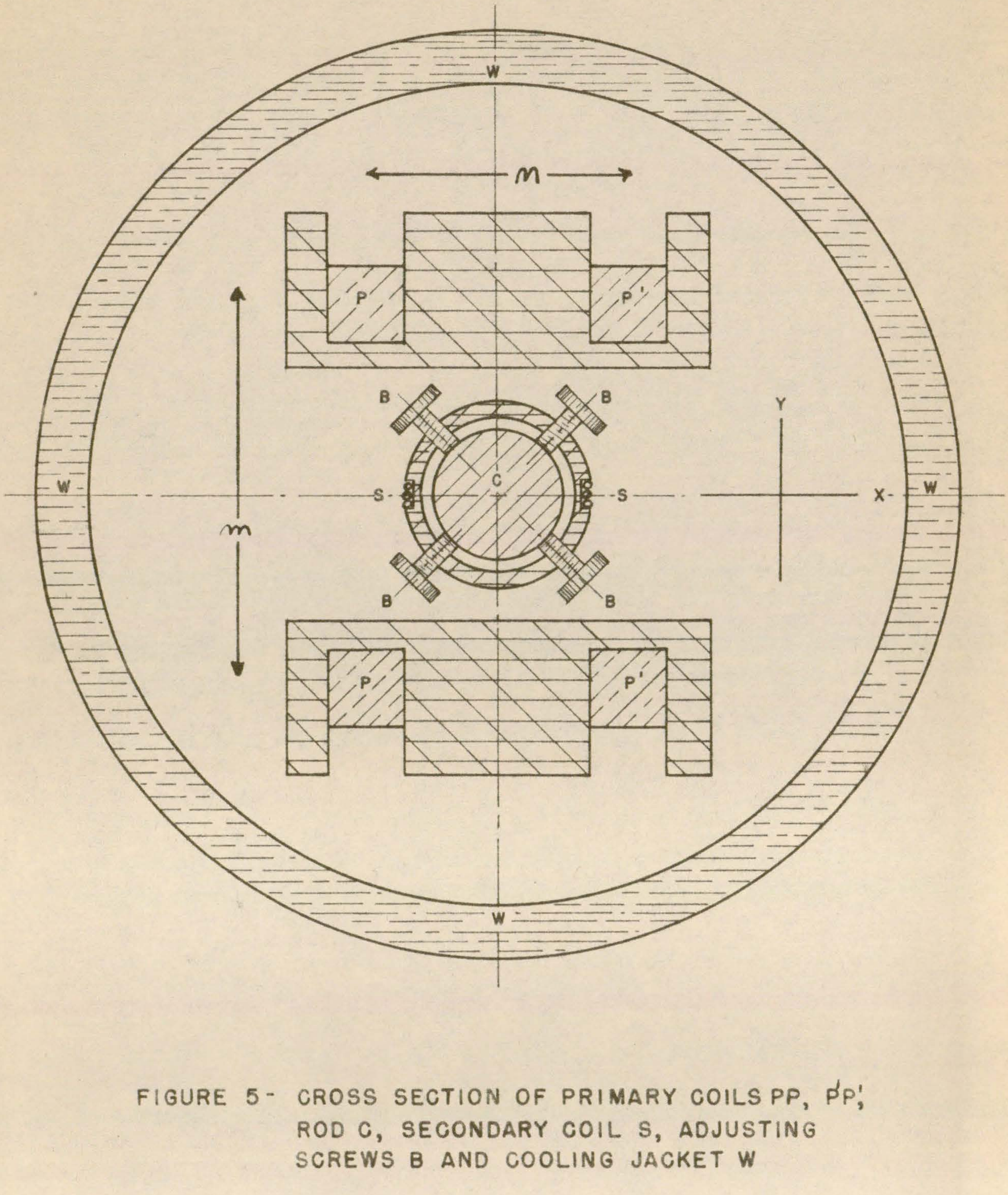

FIGURE 5 - CROSS SECTION OF PRIMARY COILS PP, P'P',
ROD C, SECONDARY COIL S, ADJUSTING
SCREWS B AND COOLING JACKET W

The ratio of the width m of each coil to the distance n separating the coils was (See figure 5) chosen so as to approximate the conditions for maximum uniformity of the magnetic intensity at the center of the coils.

The coil was placed in an electrical oven and baked at 75° C. for 18 hours, and at 100° C. for eight hours. This process dried the varnish. The resistance of the two halves of the coil in series, measured with direct currents, was 52 ohms.

At each end of the coil frame, a hole, one inch in diameter, was drilled so as to be centered accurately between the two coils. This hole was used to emplace and remove the sample to be tested, and to provide an opening through which the sample and secondary coil could be rotated.

Three rods of compressed powdered ferromagnetic material were used in the experiment.* Powdered materials, rather than solid materials, were chosen to reduce the effects of eddy currents. These rods were cylindrical in shape, about 25 cm. in length, and averaged 1.411 ± 0.003 cm. in diameter. The rods were formed from small disks of compressed iron and permalloy dust. Disks 1.411 cm. in diameter, cut from rings of the compressed material, and packed closely, with cement between them, in bakelite tubes 5/8 inch outside diameter and 1/32 inch thick. The disks were squeezed together in a vise.

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* These same rods, from the material supplied by the Bell Telephone laboratories, were used by Professor S. J. Barnett in experiments with rotating magnetic fields. (See Reference (21)).

while the cement dried. The length of each rod is listed below:

#1	Permalloy	25.07 cm
#2	Iron	24.40 cm
#3	Permalloy	24.00 cm

The secondary coil used in the final experiment (Figure 3) was constructed upon a linen base bakelite tube 12 inches long, $1\frac{1}{4}$ inches outside diameter, and 1 inch inside diameter. Narrow grooves (S-S in Figure 3) were milled in the tube, and 100 turns of #40 silk wire wound in the grooves.

Two groups of four holes were drilled in the bakelite tube, the groups being about 9 inches apart. In each group, the four holes were accurately spaced at 90° intervals around the cylinder, and threaded. The cylindrical sample of the substance to be tested was placed inside the tube. Since the diameter of the tube was greater than the diameter of the rod, a small space remained between the sample and the walls of the tube. Eight bakelite screws (B-B , Figure 5) penetrated the cylinder through the holes, and clamped the rod in the center of the tube. With these screws, the rod could be aligned and centered with respect to the tube.

The secondary coil, with the sample inside, was placed in the center of the primary Helmholtz coil. (Figure 5), in such a position that the mutual inductance between primary and secondary was close to zero. A bakelite bearing (D, Figure (3)) fastened the bottom of the secondary coil to the bottom of the frame of the primary coil. At the top of the secondary coil, a piece of one inch outside diameter bakelite tube (A-A, Figure (3)) was fastened to the secondary coil with bakelite screws. This tube extended through a hole in the primary coil frame, and several inches beyond. A brass bar (E - E, Figure (3)) was fastened to the top of the tube, so as to provide a lever to rotate the tube and secondary coil. Screw clamps, mounted on fixed supports, provided means of rigidly holding the bar, and hence, the secondary coil, at any selected angular position.

D. Secondary Circuits

In order to detect the small gyromagnetic signal it was important to reduce to a small value all extraneous voltages in the secondary coil that might mask the effect. Since the secondary coil was placed in the oscillating field of the primary coil, precautions were taken to reduce the induced voltages in the secondary coil. To do this, first the secondary coil was mechanically rotated in azimuth into a position of approximately minimum mutual inductance and clamped in this position. To reduce the coupling between the two coils further, a variable mutual inductance, coupling the primary and secondary circuits was used. (Figure (2)).

The stationary coil of the variometer was placed in series with the primary coil. Two rotating coils, one with a large effective area for coarse adjustment, and the other with a small area for fine adjustments, were placed in series with the secondary circuit. By adjusting the two rotating coils, the coupling between the primary and secondary circuits, and, hence the induced voltages, could be reduced to a small quantity.

The large coil,(24) used to produce the steady magnetic field, H_z , was constructed of about 300 pounds of copper wire, wound in 3900 turns, giving a D.C. resistance of 10.4 ohms at room temperature. (Figure 4). It produced a magnetic field of approximately 90.2 gauss per ampere, averaged over the region in which the sample was placed.

The circular opening in the center of the coil, into which the apparatus was placed, was about 6 3/16 inches in diameter and 20 inches long. The current for this coil, from zero to 30 amperes, was furnished at first by storage batteries, and later by a generator. Large variable resistors and a reversing switch were placed in series with this coil to control the magnitude and direction of the current.

On top of the large magnet two screw adjustments were bolted. These adjustments engaged the brass bar (E E, Figure 4) and rotated the sample and secondary coil through small angles as described above.

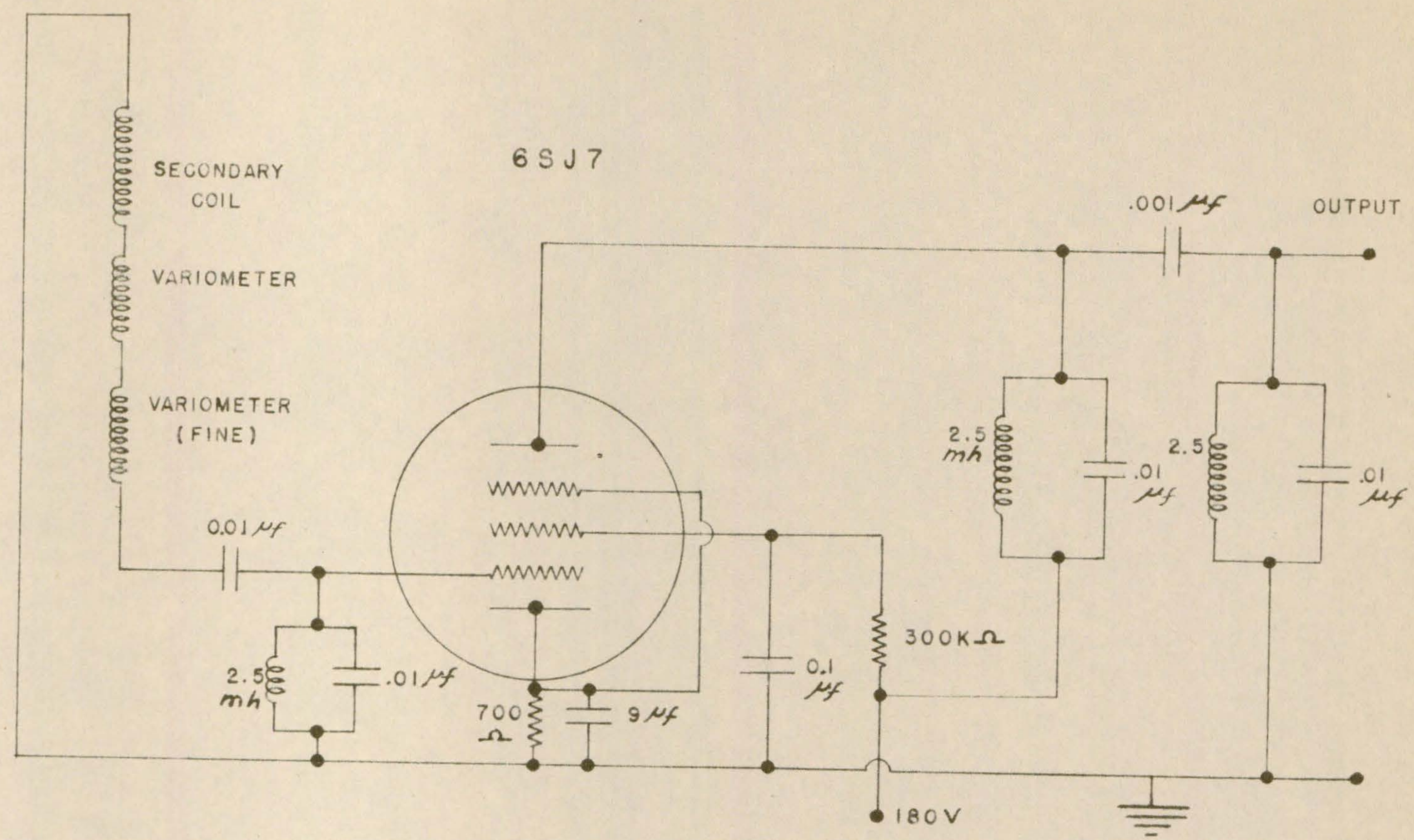

FIGURE 6 - CIRCUIT DIAGRAM OF SECONDARY CIRCUIT

The large coil, as well as the primary coil, at first became excessively hot during the experiment, disturbing the stability of the apparatus and making it necessary to provide a cooling coil to stabilize the temperature. For this, a coil was wound in the form of a helix from 3/8 inch copper tubing, on the curved surface and bottom of a brass tube 4½ inches in diameter and 20 inches long, with large quantities of soft solder filling the space between turns. The cooling unit assembly fitted tightly in the circular hole of the 300 lb coil. Connections to the water mains were provided so that tap water could be forced through the coils from bottom to top. The primary coil and assembly were placed inside this tube and held there by machined bakelite discs bolted to the primary coil, and fitting closely to the brass tube. The remainder of the space inside the brass tube was filled with transformer oil. In this way, temperature changes in the region inside the cylinder could be kept to less than a tenth of a degree during all experiments.

The third group of apparatus included an amplifier (Figure (6)), used to increase the strength of the gyromagnetic signal of a few tenths of a millivolt, to a magnitude most convenient to measure by standard instruments. The circuit finally adopted employed a 6SJ7 vacuum tube with a tuned grid, tuned plate circuit. The gain was 10.6 at 22KC and 17.5 at 30 KC.

The amplifier was provided with its own filament transformer and with a bank of batteries for its plate supply. Special shielded leads were used for input and output connections for all power leads. The amplifier was inclosed in a grounded box.

High "Q" parallel resonant circuits were used in the grid and plate circuits to peak the amplifier and allow only a narrow band of frequencies to be amplified. In this way, background noise, always troublesome, was greatly reduced.

An oscilloscope and an electron volt meter were attached to the output of the amplifier (Figure (1)) to measure the phase and magnitude of the signal. The oscilloscope was operated on external sweep, with a synchronizing signal tapped from the output of the 803 amplifier.

In all, four different meters were used in the course of the experiment. Two were manufactured by the Ballantine Corporation, and two by the Hewlett Packard Company. One of the Hewlett Packard meters was calibrated by comparison with a thermoammeter. The two Ballantine meters were also separately calibrated by another laboratory. The meters were then compared among themselves for all scale readings. In the final calculations, data taken with each of the four meters were used, properly weighted, to calculate the gyromagnetic ratio.

IV. Measurement of Experimental Quantities.

A. General Discussion

From equation (18), the gyromagnetic ratio ρ, is related to other experimentally determined quantities by the equation

$$\rho = \sqrt{2}\,\pi \left[\frac{\mu+1}{\mu-1}\right]^2 \left(1 + \frac{\Delta}{b} - 3\delta\right) \cdot \frac{J_z E \times 10^8}{N b l H_{ox} \omega^2 A} \qquad (19)$$

In order to calculate the value of ρ, each quantity on the right hand side of this equation had to be determined experimentally. The measurement of these quantities will be discussed in the order ω, H_{ox}, $\frac{\mu+1}{\mu-1}$, b, l, N, A, and E.

The frequency "f" appears explicitly raised to the second power in formula (19),* and appears implicitly in the measurement of several of the other quantities. So, it was desirable to determine the frequency to an accuracy of about one part in a thousand, in order to obtain satisfactory precision in the final value for ρ.

B. Frequency

The frequency f of the 6SN7 oscillator was measured by comparing its frequency with that of a Western Electric oscillator, Model #13-A. The Western Electric oscillator was first calibrated by means of the vibrating reeds furnished with the instrument. Then the calibration was

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* The frequency f is related to the quantity ω of equation (19) by the relation $\omega = 2\pi f$

checked at the dial settings 60, 120, 180 and 240 cycles per second by observing Lissajou's figures on an oscilloscope synchronized with a 60 cycle line voltage. The 60 cycle frequency was checked with a frequency meter manufactured by the Weston Electrical Instrument Company, Model #214, and found to vary less than 1/10 of 1%. In this way the dial readings of the Western Electric oscillator were checked and found to be correct.

Then the signal from the 6SN7 oscillator (tapped from the amplifier of the secondary circuit) was applied to the horizontal sweep, and a signal from the Western Electric oscillator applied to the vertical sweep of an oscilloscope. The Western Electric oscillator was adjusted until Lissajou's figures were obtained.

Next, the amplifier leads were disconnected from the horizontal sweep, and a Hewlett Packard oscillator, which had been previously turned on for a sufficiently long time to reach a steady state, was connected to the oscilloscope. New Lissajou's figures were obtained when the Hewlett Packard oscillator was adjusted to exactly 1/5 of the original frequency. Next, the Western Electric oscillator was readjusted until its frequency was 1/25 the original frequency, and new Lissajou's figures were obtained. This process was continued until the setting on the Western Electric oscillator fell within the range of from 60 to 240 cycles per second. Then the frequency could be computed from the reading on the calibrated part of the scale of the Western Electric oscillator.

By these procedures the two frequencies used in the experiment were found to be 21.75 KC and 30.50 KC.

C. Oscillating Magnetic Field

The amplitude of the oscillating magnetic field H_{ox} was determined during the experiment from a knowledge of the current in the primary coil, and an empirically determined constant K (gauss per ampere) of the primary coil.

To determine the constant K, a test coil was constructed on a bakelite frame, shaped to the approximate dimensions of the samples of powdered iron and powdered permalloy. On this frame 40 turns of #40 silk covered wire were wound in a single layer, pressed close to the frame, and varnished into place, The dimensions before and after winding were measured, and averaged to give the mean dimensions of the coil.

The test coil was placed in the region ordinarily occupied by the material being tested, and a meter connected across the terminals. The test coil was then rotated in azimuth until a maximum voltage appeared across the terminals, and then the coil was locked in place. Knowing the frequency of the magnetic field in which the coil was placed, and the dimensions of the coil, one could calculate the magnetic field strength from the induced voltage.

To measure the primary current, a thermoammeter was connected in series with the primary circuit. Numerous readings of the primary current I_p were recorded along with the voltage induced in the secondary coil. In all, four different voltmeters were used to reduce possible systematic errors arising from the meters.

An average value of the voltage was calculated for each separate value of I_p. From this information it was possible to determine K.

In order to determine H_x at any time during the experiment it was necessary only to measure I_p with the thermoammeter, and calculate H_x from the formula

$$H_x = 55.4\, I_p \tag{20}$$

It should be noted that, in the method adopted here, the absolute calibration of the thermoammeter need not be known. It was, of course, impossible to proceed in the usual manner to determine the constant of the coil on account of the presence of the surrounding conducting tube. It was found that the constant of the coil was reduced by some 10% by the presence of the tube.

D. Demagnetizing Factor

The value of δ was determined by direct experiment. From equation (52), Appendix II, one has the relation

$$B_x^i = \frac{1 + \frac{\mu-1}{\mu+1}}{1 - \frac{\mu-1}{\mu+1}\delta} H_x^o \qquad (21)$$

If δ is small, compared to unity, and μ is large compared to unity, this equation can be arranged to the approximate form

$$\delta = \frac{1}{2}\left(\frac{\mu+1}{\mu}\right)\frac{B_x^i}{H_x^o} - 1 \qquad (22)$$

A number of experiments were made to determine the quantities on the right hand side of equation (22). In order to evaluate the factor $\frac{\mu+1}{\mu}$, the longitudinal permeability μ (measured in weak fields) was used. An expression for was obtained from equation (46) Appendix I

$$J_z = \frac{\mu_z - 1}{4\pi + (\mu_z - 1)D_z} H_x^o \qquad (23)$$

Solving for μ_z, one obtains

$$\mu_z = \frac{4\pi J_z}{H_z^o - D_z J_z} + 1 \qquad (24)$$

To determine the quantity μ_z in equation (24) it was necessary only to measure J_z with the solenoid when the rod was placed in a known magnetic field H_x^o. The value of D_z, the longitudinal demagnetizing factor, was obtained from the tables and was found to be 0.07. The measurements for J_z and H_z^o were made with both direct currents and oscillating currents with a frequency of 22 KC. The longitudinal permeability for permalloy was found to be 26.

Next, to determine B_x in equation (22), a rectangular coil was wound closely on the rod as indicated in the figure on page (27). The rod and coil were placed inside the primary coil. The voltage induced in this coil by the oscillating magnetic field of the primary coil was measured by each of several voltmeters. The value of B_x^i was calculated from the formula (64), Appendix III. The value of B_x^o in equation (22) was determined by measuring the current in the primary coil with a thermoammeter, and using the procedures on page (40).

These experiments were repeated, using direct currents in the primary coil, and a galvanometer connected to the coil.

Then the results from both alternating current and direct current measurements were used in equation (2) independently to calculate δ. The results indicated that δ was equal to 0.01.

This experimentally determined value was checked with an approximate value obtained from calculations for an ellipse with a ratio of major to minor axis equal to the ratio of length to diameter of the rod. This calculation gave a value of 0.005 in reasonable agreement with the experimentally determined value. In this experiment the value 0.01 was used throughout.

E. Dimensions of the Magnetic Material

The radius of the cylindrical sample, b, was determined by measuring with a micrometer the diameters of a large number of disks of the compressed powder, and calculating one-half the mean diameter. The length l of each rod was measured with micrometer calipers. The dimensions for each sample are listed on pages (30) and (31).

F. Transverse Permeability

The quantity $\frac{\mu+1}{\mu-1}$ was determined experimentally for each sample. Since the transverse permeability is a function of the longitudinal magnetic field strength H_z^o repeated measurements were made for various values of H_z^o, and a curve plotted through the mean of a large number of readings. (23)

Now, a well known equation relating the magnetic induction B_x^i in an infinitely long cylinder with the applied magnetic field H_x is

$$B_x^i = \frac{2\mu}{\mu+1} H_x^o \tag{25}$$

Dividing through by H_x, and then subtracting unity from each side, it is found that (25) reduces to

$$\frac{\mu-1}{\mu+1} = \frac{B_x^i}{H_x^o} - 1 \tag{26}$$

It is shown in Appendix II that, if the demagnetizing factor due to the finite length of the rod is included in the calculations, formula (26) must be modified to

$$\frac{\mu-1}{\mu+1}(1+2\delta) = \frac{B_x^i}{H_x^o} - 1 \qquad (27)$$

where δ is related to the demagnetizing factor in the X direction, as discussed on page (17).

In measuring the quantities appearing on the right hand side of equation (27), the externally applied magnetic field H_x^o was determined from a knowledge of the current in the primary coil, as explained above. To measure the magnetic induction B_x^i, a coil of 20 turns of #40 silk covered copper wire was wound close to the powdered core, so that each turn lay parallel to one element of the curved surface, and across the flat ends (See figure on page (95)). The mean dimensions of this coil were measured with a micrometer. From these dimensions a formula relating the magnetic induction B_x and the RMS voltage induced in the coil could be computed. This formula was found to be

$$B_x = \frac{\sqrt{2}\, V \times 10^{-8}}{2N\omega\left[b + \frac{\gamma}{\mu}\right]} \qquad (28)$$

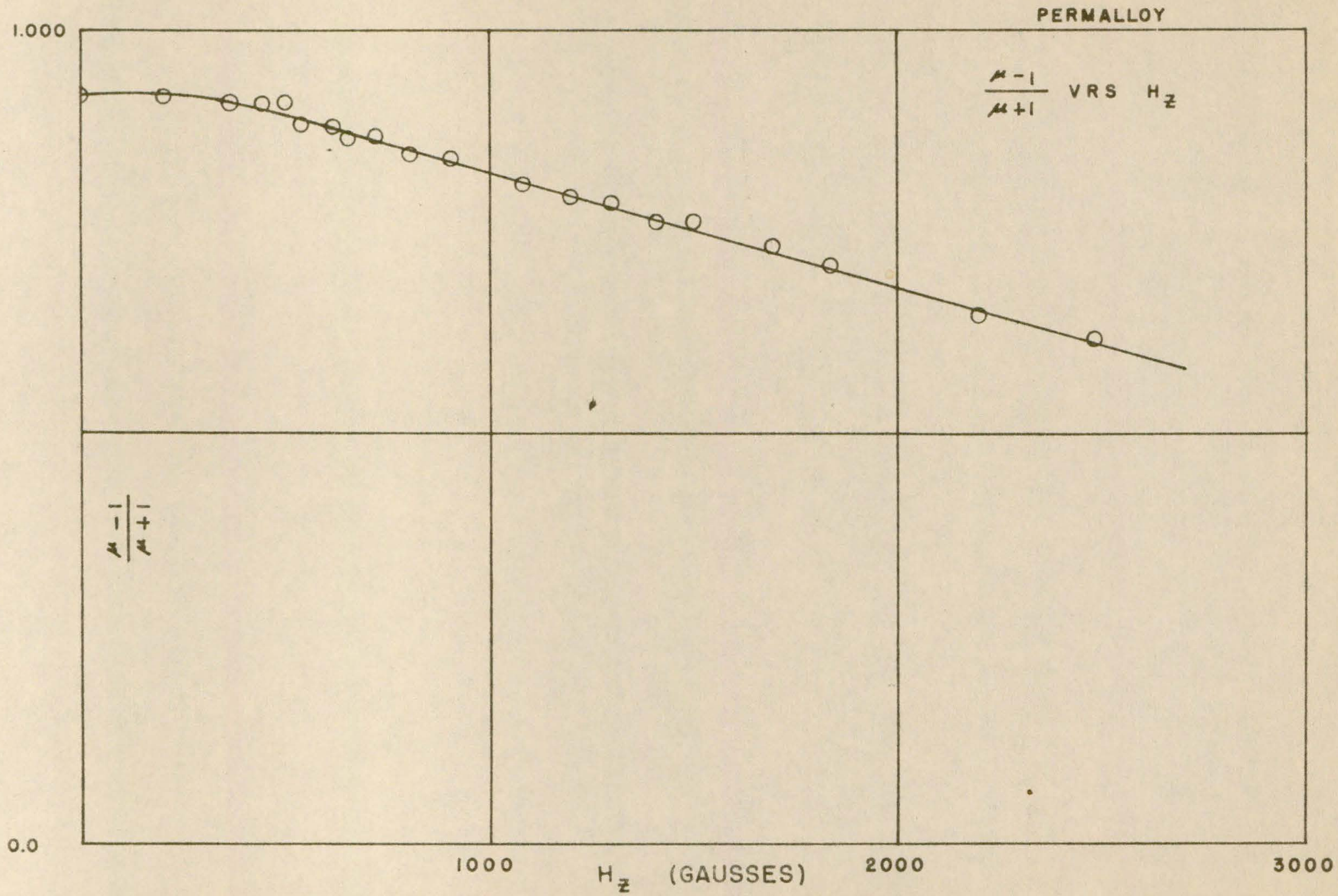


FIGURE 7– $\frac{\mu-1}{\mu+1}$ AS A FUNCTION OF H_z

The core and coil were placed in the oscillating homogeneous magnetic field of the primary coil, and the rod rotated until a maximum flux threaded the coil. The voltage V was then measured from a meter connected to the terminal of the coil. A mean of the value of V was substituted into equation (28) to determine B_x^i. The values for B_x^i and H_x^i were substituted into equation (27) to determine $\frac{\mu-1}{\mu+1}$.

The value of δ was determined by previous experiments. From this information the quantity $\frac{\mu-1}{\mu+1}$ was computed.

Since the quantity $\frac{\mu-1}{\mu+1}$ was not a constant, but a function of H_z measurements were made for various values of H_z, and a curve of $\frac{\mu-1}{\mu+1}$ against H_z was plotted. (Figure (7)).

G. Longitudinal Magnetization

To measure J_z, each cylindrical sample was placed in a solenoid 44.8 cm long and 1.60 cm in diameter. The solenoid was wound with silk covered wire, 18.1 turns per cm. The solenoid and sample were accurately centered in the large 300 lb magnet, so that the magnetic field H_z was in the direction of the axis of the sample. The terminals of the solenoid were connected to a ballistic galvanometer.

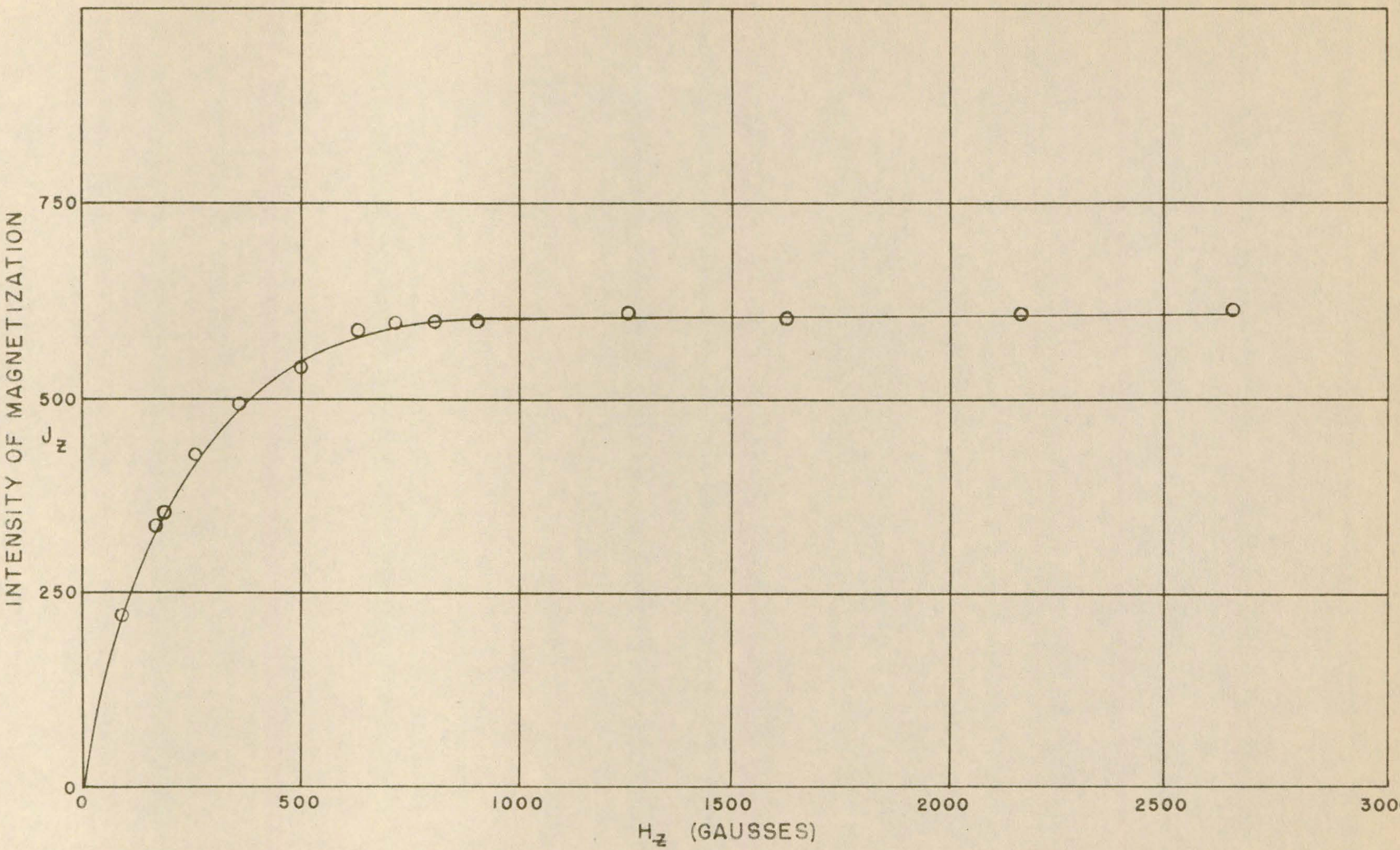


FIGURE 8 - MAGNETIZATION CURVE FOR PERMALLOY

By a well known theorem, the flux through the turns of the solenoid, due to the magnetization of the rod, is

$$\Phi = M\gamma = 4\pi NM \tag{29}$$

where Φ is the flux, N the turns per centimeter, and M the magnetic moment of the rod. Now, if the rod is uniformly magnetized, then

$$M = VJ \tag{30}$$

where J is the intensity of magnetization and V the volume of the rod. Combining equation (29) with equation (30), one obtains

$$\Phi = 4\pi NVJ_z \tag{31}$$

With the sample in the solenoid, and the galvanometer connected to the terminal of the solenoid, the current in the 300 ft coil was abruptly changed from zero to a value I_p. The resultant deflection d_1 of the galvanometer was recorded.

Then the core was removed from the solenoid, and the deflection d_2 of the galvanometer recorded with the same change in current I_p. Next, the solenoid was disconnected, and a Hibbert Standard substituted in its place. The deflection d_3 of the galvanometer was noted when the Hibbert Standard produced a known flux change Φ.

By using the formula

$$\frac{d_1 - d_2}{d_3} = \frac{4\pi N v J_z}{\phi} \tag{32}$$

the value of J_z could be computed. This procedure was repeated for various values of the current in the 300 lb coil, and a curve was derived, plotting J_z against H_z, drawn through the mean of a large number of readings. A curve of J_z against H_z for permalloy is reproduced in Figure (8).

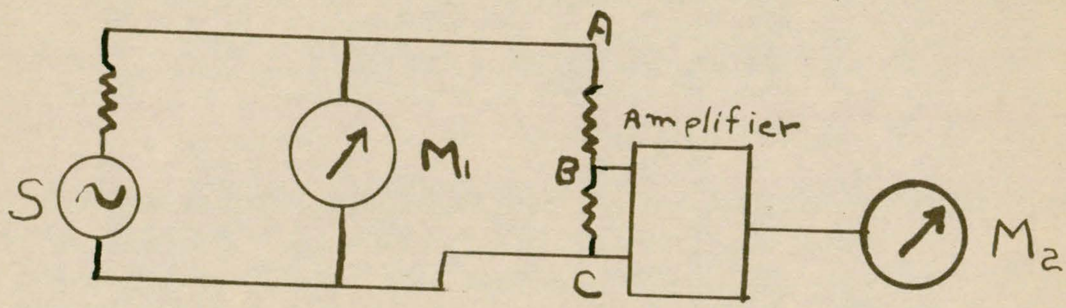
H. Dimensions of Rods

The length l of the rods was measured by micrometers. Measurements were taken before and after winding, and a mean value of each dimension computed. The quantity N, the number of turns, was obtained by counting them as the coil was wound.

I. Gyromagnetic Voltage

The strength of the gyromagnetic signal E appearing across the terminal of the secondary coil was increased in strength by the amplifier in the secondary circuit, and then measured by an electron voltmeter. To calculate E from the meter reading, it was necessary to determine the gain G of the amplifier.

The approximate gain was equal to the ratio of the output voltage to the input voltage as measured by an electron voltmeter.



To measure the gain more exactly, a voltage dividing network, A-B-C, was constructed of non-inductive resistors. The attenuation, BC/AC, which was accurately known, was approximately equal to the gain G. A signal generator S and a meter M_1 were connected across the input AC, and the amplifier connected to the output BC of the attenuator. Another meter, M_2, was connected across the output of the amplifier.

With this arrangement, the readings of the two meters were recorded. Then the meters M_1 and M_2 were interchanged from input to output and the readings again recorded. With the attenuator in the circuit, the readings of either meter would be approximately the same in the output position as they would be in the input position. This verification reduced errors arising from the non-linearity of the electron voltmeters.

The gain G was calculated from the formula

$$G = \frac{V_{out}}{V_{in}} \times \text{Attenuation}$$

The values found were:

Frequency	G
22.4	10.6
30.0	17.5

To measure E, the RMS gyromagnetic voltage, it was necessary to reduce to a sufficiently small value all electromotive forces in the secondary circuit (such as induced voltages), that might mask the small voltage E. In a preliminary experiment, methods of adjusting the apparatus were developed to reduce these masking voltages.

In these preliminary experiments a secondary coil, wound on a bakelite frame, was placed inside the primary coil. This coil assembly was, in turn, placed inside the hole in the bobbin of the 300 lb coil. The secondary coil was rotated in azimuth, varying the mutual inductance between the primary and secondary coils, until an induced voltage of convenient amplitude was obtained. This induced voltage was in quadrature phase with the gyromagnetic effect.

An amplifier was connected to the terminals of this coil. An electron voltmeter and the vertical deflection terminals (Y deflection) of an oscilloscope were connected to the output of the amplifier. In order to synchronize the sweep (X deflection) of the oscilloscope to the current flowing in the primary circuit, the oscilloscope was operated on "external sweep". The synchronizing signal, obtained from the terminals of the transformer in the primary circuit, was attenuated by a resistance network, and applied to the oscilloscope.

The positions (X coordinate) of the maximum and minimum of the induced voltages were noted on the oscilloscope screen. These extrema furnished a standard with which to compare phase relations of any other sine wave appearing across the output of the amplifier.

For convenience in this discussion, alternating voltages with the same phase as the gyromagnetic effect will be called "in phase voltages", and alternating voltages with the same phase as the induced voltages will be called "quadrature voltages".

The bakelite coil was then removed from the apparatus, and the ferromagnetic sample, with a surrounding secondary coil, was placed inside the hole in the bobbin of the 300 lb coil. The amplifier was connected to the terminals of this secondary coil. The electron voltmeter and synchronized oscilloscope were again connected to the amplifier output.

The secondary coil and magnetic sample were rotated by means of the controls provided until an approximate minimum mutual inductance was obtained between the primary and secondary coils. With the screw adjustments previously described, this coil could be positioned to within ten minutes of arc of any desired position.

The residual voltage in the secondary circuit contained chiefly a quadrature component arising from the remaining mutual inductance, and an in phase component arising from the gyromagnetic effect, and other causes as described below. To reduce the voltage still further, the variometer coupling the primary and secondary circuits was adjusted, until the output voltage was reduced to a minimum value. Adjusting the variometer changed the mutual inductance between the primary and secondary circuits, and changed the quadrature component of the voltage.

When this minimum was reached, it was found that the remaining voltage in the amplifier output was an in phase voltage, and could not be removed by readjusting the variometer. This phase relation could be established with the synchronized oscilloscope. This remaining in phase voltage contained both the gyromagnetic effect and an in phase disturbing effect.

The presence of this in phase disturbance can be understood when one considers the currents flowing in the coils and the eddy currents flowing in the conducting walls of the containers surrounding the coils. The coils of the variometer were placed in a large iron box, in such a manner that the coils were comparatively remote from the walls. In contrast, the primary and secondary coils were placed inside a brass tube, the walls of which were comparatively close to the coils.

Because of this difference in the geometry, the effects of eddy currents flowing in the walls of the containers would modify the two magnetic fields by different amounts. If the walls were perfect conductors, this alteration would affect the magnitude but not the phase of the magnetic fields. Since the walls were constructed of material with a resistance different from zero, a phase difference would be produced.

This phase difference, although so small that it could not be detected with an oscilloscope, produced a small in phase residual voltage when the voltage induced in a coil by one magnetic field was used to oppose the voltage induced in a second coil by the other magnetic field.

Next, a method was found to remove this residual in phase voltage. It was found that detuning one of the condensers in the primary circuit altered symmetry conditions, and produced a large voltage, consisting of both a quadrature component and a small in phase component (less than one-twentieth as large). The quadrature component could be eliminated by readjusting the variometer, leaving a small in phase component added to or subtracted from the residual voltage.

By a series of adjustments, first turning the condenser one way or the other, and then removing the quadrature voltage, by adjusting the variometer, the magnitude of the in phase voltage in the output of the amplifier could be set to any desired small value E_1, and the quadrature component eliminated.

This concluded the preliminary experiments designed to eliminate voltages that might mask the gyromagnetic effect.

The experimental method finally adopted to measure E enabled one to measure E in the presence of certain disturbing effects. The method will be described first, and then the disturbances will be discussed.

In measuring E_1, the sample and secondary coil were emplaced in the apparatus, as described above. The current in the 300 lb coil was set to some predetermined value, producing a steady magnetic field, H_z, and an intensity of magnetization, J_z, along the axis of the rod. The variometer and condenser were adjusted, as described above, until the output meter indicated a small in phase voltage E_1. The current in the 300 lb coil was then reversed, changing H_z to $-H_z$, and also changing J_z to $-J_z$. With this change in J_z, the gyromagnetic voltage changed from EG to -EG. The output meter, which was originally reading E_1, would then change its reading to E_2, where $E_2 - E_1 = 2$ EG.

In order to use this procedure to measure 2 EG, certain precautions were necessary to eliminate possible errors. These precautions can be explained with the help of the diagram on page (56). In these diagrams, voltages in phase with the gyromagnetic effect are plotted along the Y axis, and the quadrature voltages are plotted along the X axis.

Figure (9_a) shows the relations between E_1 and E_2 that were desired, with the phase angle between E_1 and E_2 equal to zero. With this phase relation it is proper to write $E_2 - E_1 = 2$ EG.

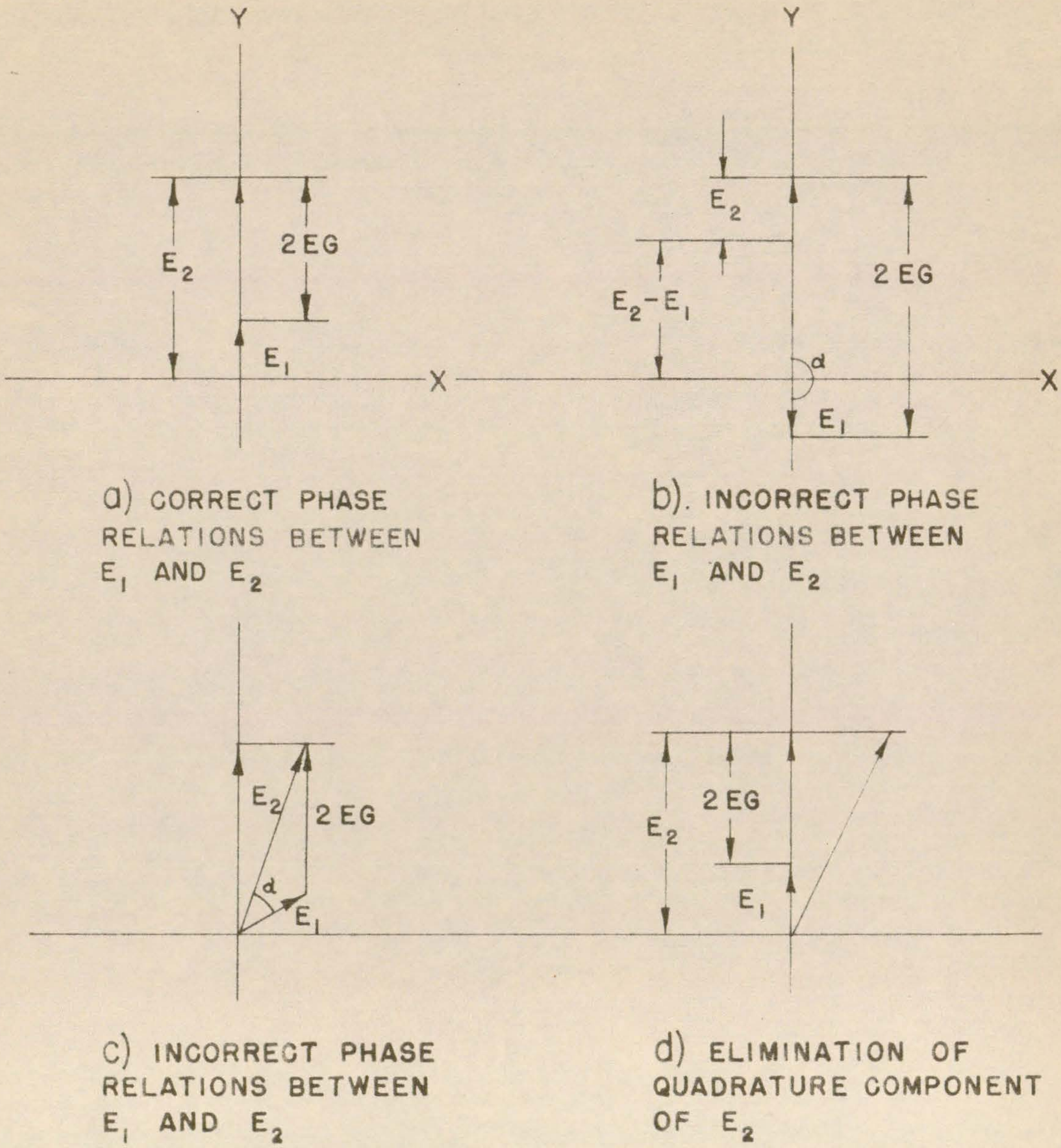

Y
E2
2 EG
E1
X
Y
E2
E2-E1
2 EG
d
E1
X
a) CORRECT PHASE RELATIONS BETWEEN E1 AND E2
b). INCORRECT PHASE RELATIONS BETWEEN E1 AND E2
E2
2 EG
d
E1
2 EG
E2
E1
c) INCORRECT PHASE RELATIONS BETWEEN E1 AND E2
d) ELIMINATION OF QUADRATURE COMPONENT OF E2

FIGURE 9

Figure (9b) displays an incorrect phase relation between E_1 and E_2. Here E_1 and E_2 are 180° out of phase, and $E_2 - E_1 \neq 2EG$.

Figure (9c) also displays another incorrect phase relation between E_1 and E_2, where the phase angle between them is an arbitrary value α. Here again $E_2 - E_1 \neq 2EG$.

In order to insure that the phase angle between E_1 and E_2 was zero, it was necessary to check continually this phase relation with the oscilloscope during the experiment. The residual voltage E_1 was not reduced to zero, but was made large enough so that its phase could be determined on the oscilloscope. To eliminate any possible quadrature component of E_1, the variometer was adjusted until a minimum voltage appeared in the output meter. It was found, by observing the oscilloscope and meter simultaneously, that this minimum occurred when all of the quadrature component was eliminated, leaving only an in phase component of E_1.

In general, a quadrature disturbance modified the procedure. If E_1 was in phase with the gyromagnetic effect, the voltage E_2 appearing upon reversing the magnetic field H_z would, in general, not be exactly in phase, but would have a quadrature component (See figure (9d)). The appearance of this quadrature component can be understood if one considers the effect of the reversal of H_z upon the apparatus.

Since the magnetic field H_z was actually somewhat inhomogeneous, translational magnetic forces were exerted upon the rod. Rotational torques also appeared. It was noticed that both the magnitude and the direction of this quadrature voltage could be altered by slight changes in the angle between the magnetic field H_z and the axis of the ferromagnetic cylinder. This suggested that torques were exerted on the cylinder because it was not perfectly aligned with the magnetic field. By trial and error, the cylinder was aligned as accurately as possible, and this disturbance was made as small as practicable.

These forces and torques would distort the bakelite frames supporting the rod, and modify the mutual inductance between the primary and secondary coils.

Upon reversing the magnetic field H_z, an impulse would be imparted to the rod, and hence to the frames supporting the rod and coils. It was reasonable to assume that the apparatus would not return to exactly the same configuration. Any change in the configuration of the apparatus would change the mutual inductance, and produce a voltage change in quadrature with the gyromagnetic effect.

It was noticed that the magnitude of the quadrature voltage, while not exactly reproducible, in general increased with the increasing strength of H_z. The magnitude of this quadrature voltage was smaller than the gyromagnetic voltage, except for very intense magnetic fields, when it sometimes became larger than the gyromagnetic effect.

In order to remove this quadrature component from E_2 and measure only the in phase component of E_2, the variometer was readjusted each time before E_2 was recorded. This removed the quadrature component and left the in phase voltage. By observing the phase on the oscilloscope, it was possible to determine the proper adjustment of the variometer to eliminate the quadrature component. It was always found that the output meter indicated a minimum reading when the oscilloscope indicated that all quadrature voltage had been removed.

It was suggested that the practice of continually readjusting the variometer would introduce an error in the results. As described on page (54), the change in voltage in the secondary circuit, produced by adjusting the variometer, contained a quadrature component and a small in phase component. If this small in phase component is added to E_2, then $E_2 - E_1 \neq 2EC$.

A study was made to see if this small inaccuracy appreciably altered the results. First, it was noticed that the difference between the magnet fields in the primary coil and the variometer coil was less than the smallest value that could be detected on the oscilloscope (less than one part in twenty). So, the error was expected to be small.

In order to obtain an estimate of the magnitude of the error introduced by adjusting the variometer, the data were examined to see if there were any systematic trends in the results that could be correlated with this disturbance. The data were divided into four groups. The first group included results obtained with the most intense values of the steady magnetic field H_z. Each of the other groups included data taken in progressively weaker magnetic field strengths. Since the magnitude of the quadrature disturbance varied with the strength of H_z, the mean gyromagnetic ratios computed for each of the different groups were studied. No systematic trend was found.

Then it was noticed that, over a period of days, the quadrature effect changed magnitude and, occasionally, direction. This change arose, possibly, from slight changes in the orientation of the rod and the coils. The mean gyromagnetic ratio was computed for each day's results. Again, no systematic deviation could be found that could be correlated with the quadrature effect.

From these studies, it was concluded that errors introduced by this quadrature voltage could be neglected.

In order conveniently to take several readings of 2EG in a short space of time, the current in the 300 lb coil was set to some value, and the in phase voltage E_1 was recorded. Next, the reversing switch in the circuit of the 300 lb coil was thrown, and the meter reading, E_2, recorded. Next, the reversing switch was thrown back to its original position and another reading for E_1 recorded. Four or more readings of E_1 and E_2 were recorded within a minute's time. The switch was thrown at approximately equally spaced time intervals so as to minimize the effects of systematic drifts.

For the single value of the primary current, an average value for E_1, and an average value for E_2 were calculated. The differences between these averages was equal to 2 EG.

For each set of five or more readings, the value of the primary current I_p was recorded so that H_z could be computed as described on page (40). Also, the value of the current in the 300 lb coil was recorded so that the proper value of $\frac{\mu+1}{\mu-1}$ could be selected.

Many repeated readings of 2EG were taken for the values of H_z between zero and 2700 gausses. Figures (10) and (11) display graphs of $2EG/H_{ox}$ plotted against H_z for permalloy and iron, respectively. Also, on these pages is shown the curve expected for $2EG/H_{ox}$ in a heavy black line, as predicted by equation (15). To calculate this curve, the value $\rho = 1.04 \frac{w}{e}$ for permalloy and $\rho = 1.09 \frac{w}{e}$ for iron were used in equation (15), with other quantities appearing on the right hand side of the equation determined as described earlier in this chapter.

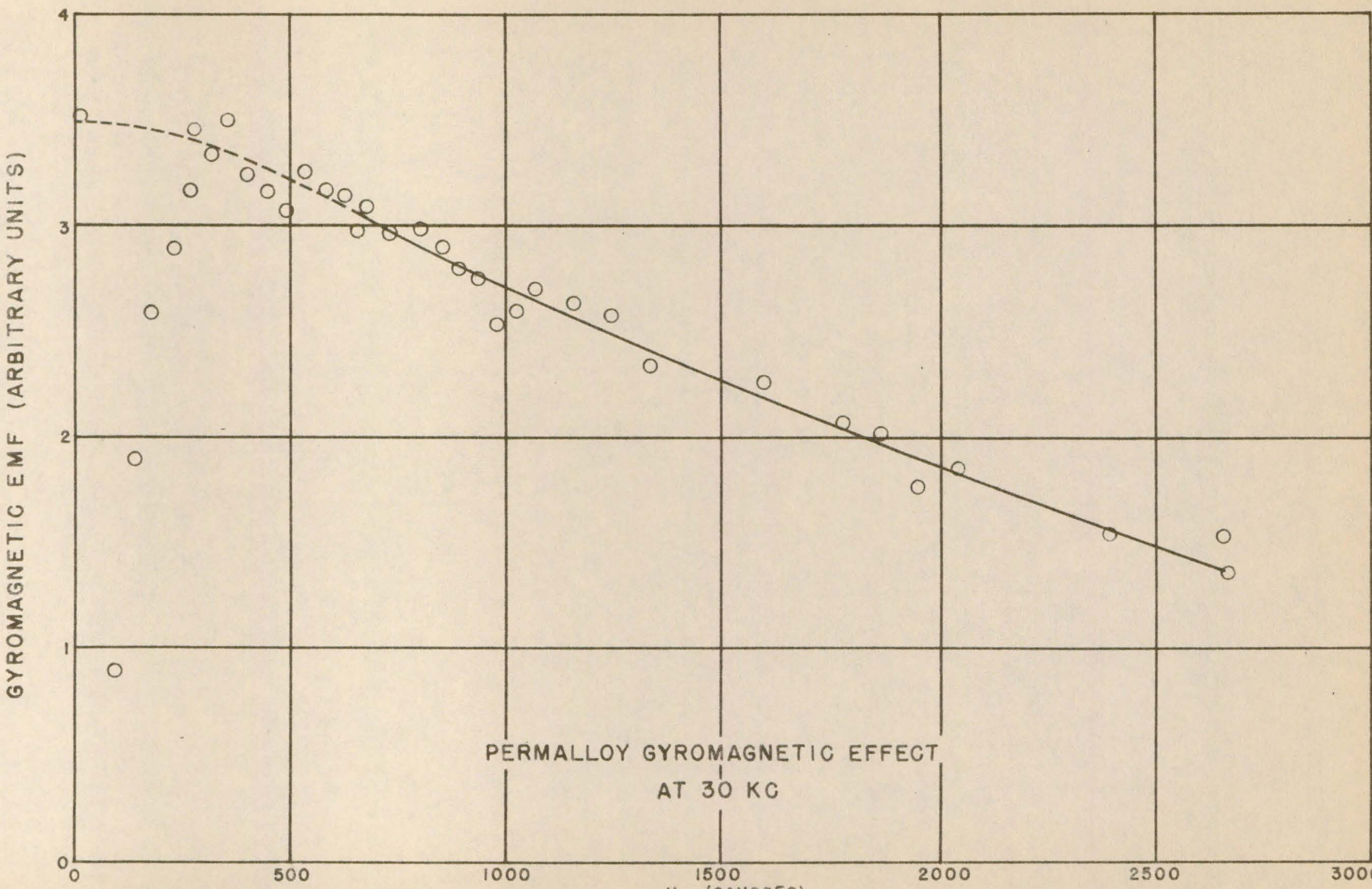


FIGURE 10- GYROMAGNETIC EMF FOR PERMALLOY

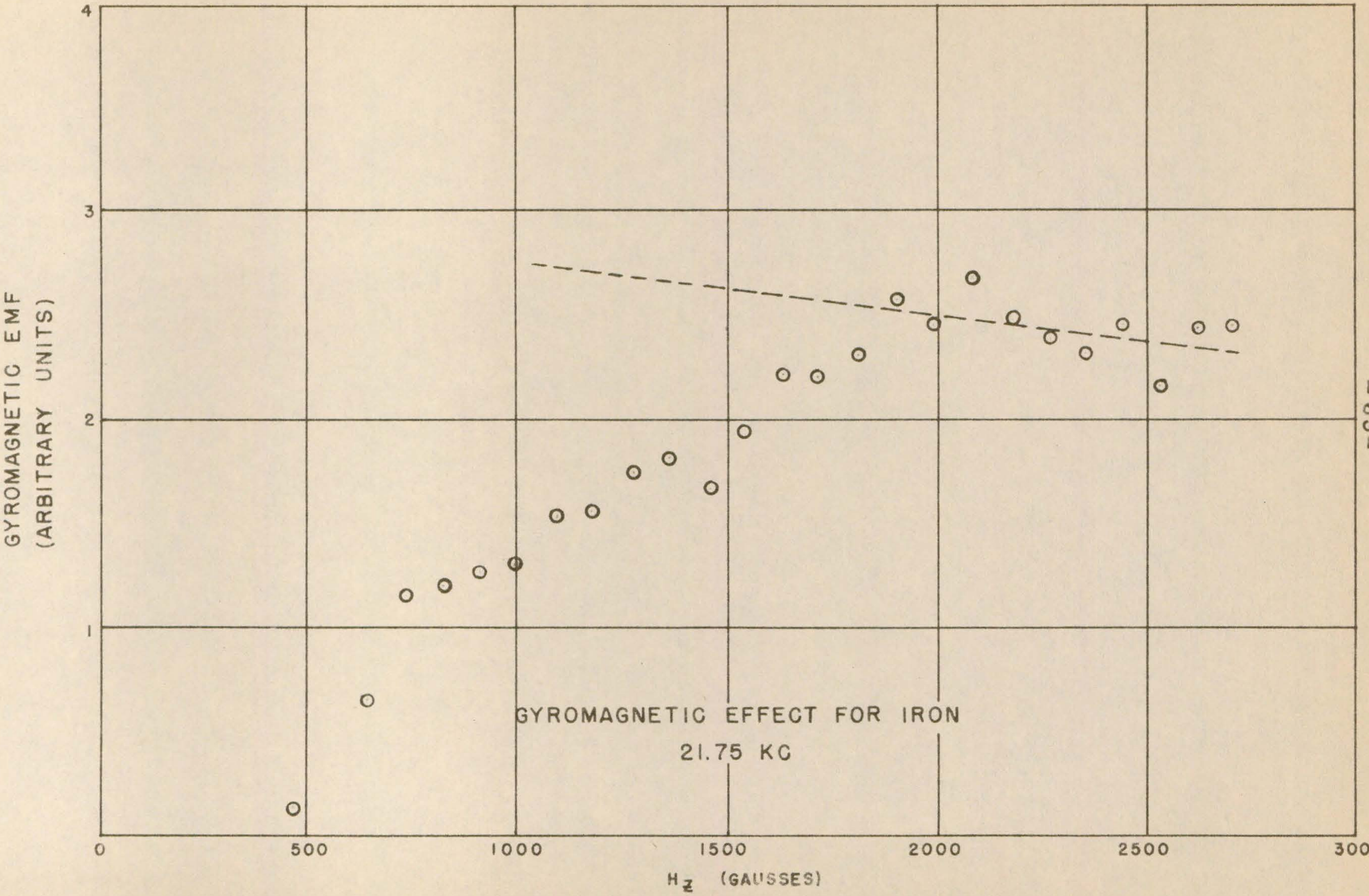


FIGURE 11- GYROMAGNETIC EMF FOR IRON

J. Quadrature Voltages

A study of the results of experiments with rotating magnetic fields indicated that, if the axial magnetic intensity was zero, the gyromagnetic voltage appearing in the secondary coil would also be zero. A study of the theory for this new effect indicated that a measurable gyromagnetic voltage would appear if H_z was not zero.

When the experiments were first begun, it was hoped that the gyromagnetic effect could be detected by observing the change in the reading of the output meter that occurred when the axial magnetic field H_z was changed from zero to some definite value. In actual practice it was found that the voltage appearing in the secondary circuit with the application of H_z contained a large quadrature voltage, as well as the gyromagnetic voltage.

Measures with the synchronized oscilloscope indicated that this disturbing effect contained only a quadrature component. The magnitude of the disturbance could be varied between large limits, in some cases being over 100 times as large as the gyromagnetic effect; and, in other cases, being several times as large.

First, the origin of this disturbance will be discussed, then the experiments that were conducted to investigate it, and, last, the procedures that were used to eliminate it.

As has been stated in the previous discussion, the rod and sample were not rigidly fixed with respect to one another. The apparatus was carefully built so as to be as rigid as possible. The coil frames and devices used to hold the rods were made of thick bakelite. The various parts were formed on machine tools to close tolerances, and rigidly fastened together whenever possible. When not rigidly fastened, bakelite bearings were used. With this arrangement, only small motions of the parts with respect to one another was possible, due to the flexibility of the bakelite and slight imperfections in machine parts.

The relative motions of the various parts would give rise to quadrature voltages, because such motions would slightly alter the mutual inductance between the primary and secondary coils. As will be described below, small motions of the apparatus could be detected with a microscope.

A qualitative formula for the effect of certain modes of motion can be developed with the aid of Figures (12a, 12b, 12c and 12d). It is well to recall at this point that the strong steady magnetic field is applied in the Z direction, the oscillating magnetic field in the X direction. The secondary coil was used to detect the magnetic induction appearing in the Y direction.

Figure (12a) indicates these three directions by the three coordinate axes. The relative position of the rod is also shown.

Figure (12b) shows the projection of the rod in the XZ plane. The small angle θ_1 indicates the angle between the axis of the cylinder and the Z axis of the coordinate system.

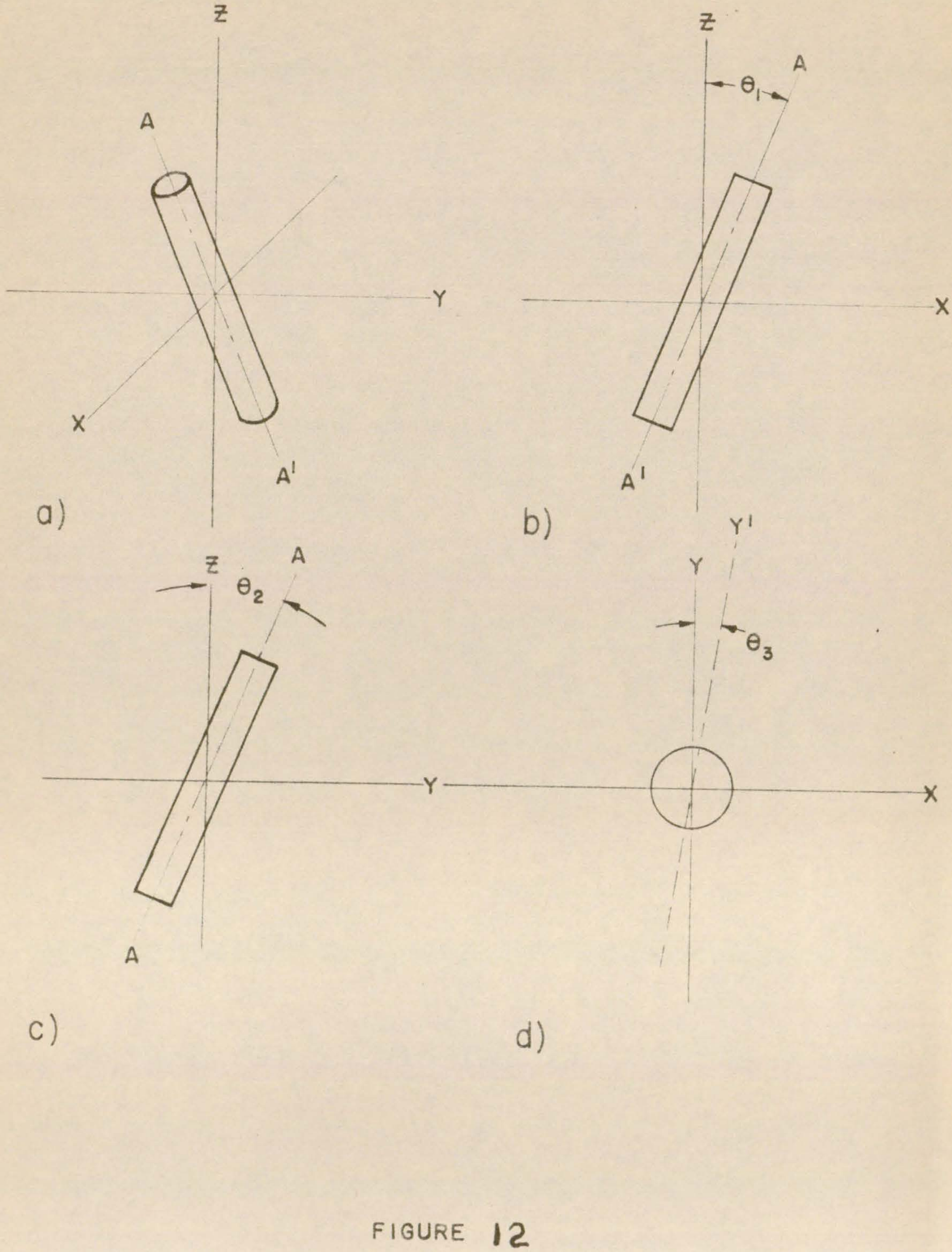


FIGURE 12
POSSIBLE ANGULAR MISALIGNMENT OF APPARATUS

If the rod were aligned along the Z axis, as was desired in this experiment, then Θ_1 would be zero.

Figure (12c) illustrates the projection of the rod in the YZ plane. Here, angle Θ_2 shows the error in aligning the rod along the Z axis.

Figure (12d) indicates the mis-alignment that might exist if the directions (X) of the applied oscillating magnetic field and (Y), the direction of sensitivity of the secondary coil, were not exactly perpendicular. The angle Θ_3 measures the deviation from the perpendicular.

First, the effects of the changes in the angle Θ_3 will be considered. From equation (77), Appendix IV, one has, for the voltage induced in a coil by an oscillating magnet flux ϕ, the relation

$$E = \frac{\sqrt{2}}{2} \omega \phi \times 10^{-8} \qquad (33)$$

For simplicity, the contributions to ϕ_o, arising from the magnetic moment of the rod, will be neglected. This simplification will make this estimate conservative.

Now, the flux ϕ_o is related to the magnetic induction B, the cross-section of the coil A, and the number of turns by the relation

$$\phi = NAB \sin\theta_3 \qquad (34)$$

Combining equations (33) and (34), one obtains

$$E = \frac{\sqrt{2}}{2} \omega NAB \sin\theta_3 \times 10^{-8} \qquad (35)$$

For a small change in the angle Θ_3, one obtains for the change in E

$$\delta E = \frac{\sqrt{2}}{2} \omega NAB \cos \Theta_3 \times 10^{-8} \delta \Theta_3 \quad (36)$$

If the numerical values for B, A, f, N (as determined in Chapter IV) are substituted into the above formula, one obtains

$$\delta E \cong 100 \delta \Theta_3 \quad (37)$$

The disturbance voltage, even under the most unfavorable conditions, was not greater than 0.1 volt. As seen from equation (37), a change in Θ_3 of only four minutes of arc could produce a voltage of 0.1 volt. Since this estimate is conservative, probably less displacement was needed to produce a voltage of this size.

This motion, which has been previously described, is not the only possible mode. As is seen from Figure (12b), if Θ_1 is not zero there will be a component of H_{xo} directed along the axis A-A of the rod. This alternating magnetic intensity will produce an oscillating magnetic moment along the axis of the rod. If Θ_2 is not zero, this oscillating magnetic moment will produce an oscillating magnetic flux in the Y direction.

It can be shown that the magnetic moment M_c produced in the rod in this manner is given by the approximate relation

$$M_c = \frac{1}{4\pi}(\mu_L - 1) V H_{ox} \sin\theta_1 \sin\omega t \quad (38)$$

where V is the volume of the rod and μ_L is the longitudinal permeability.

This oscillating magnetic moment will produce a flux in the secondary coil if the angle θ_2 is not zero. The voltage induced in the secondary coil in this manner will be appreciable in the region where μ_L is different from unity.

There exist other types of motion, such as twisting or distortions of the rod and coil. All of these motions would produce quadrature disturbances which would be superimposed upon one another in a complicated manner. (See Figure (15)).

The first arrangement used to measure the disturbance employed two sets of Helmholtz coils, one set, the primary, and the other, the secondary, wound in grooves cut in a single piece of ebonite. The grooves were cut in such a manner as to place the primary and secondary coils at right angles to one another. A hole was drilled in the center of the ebonite, and a permalloy rod fastened in the hole. A variable mutual inductance was used to reduce the residual mutual inductance of the primary and secondary coils.

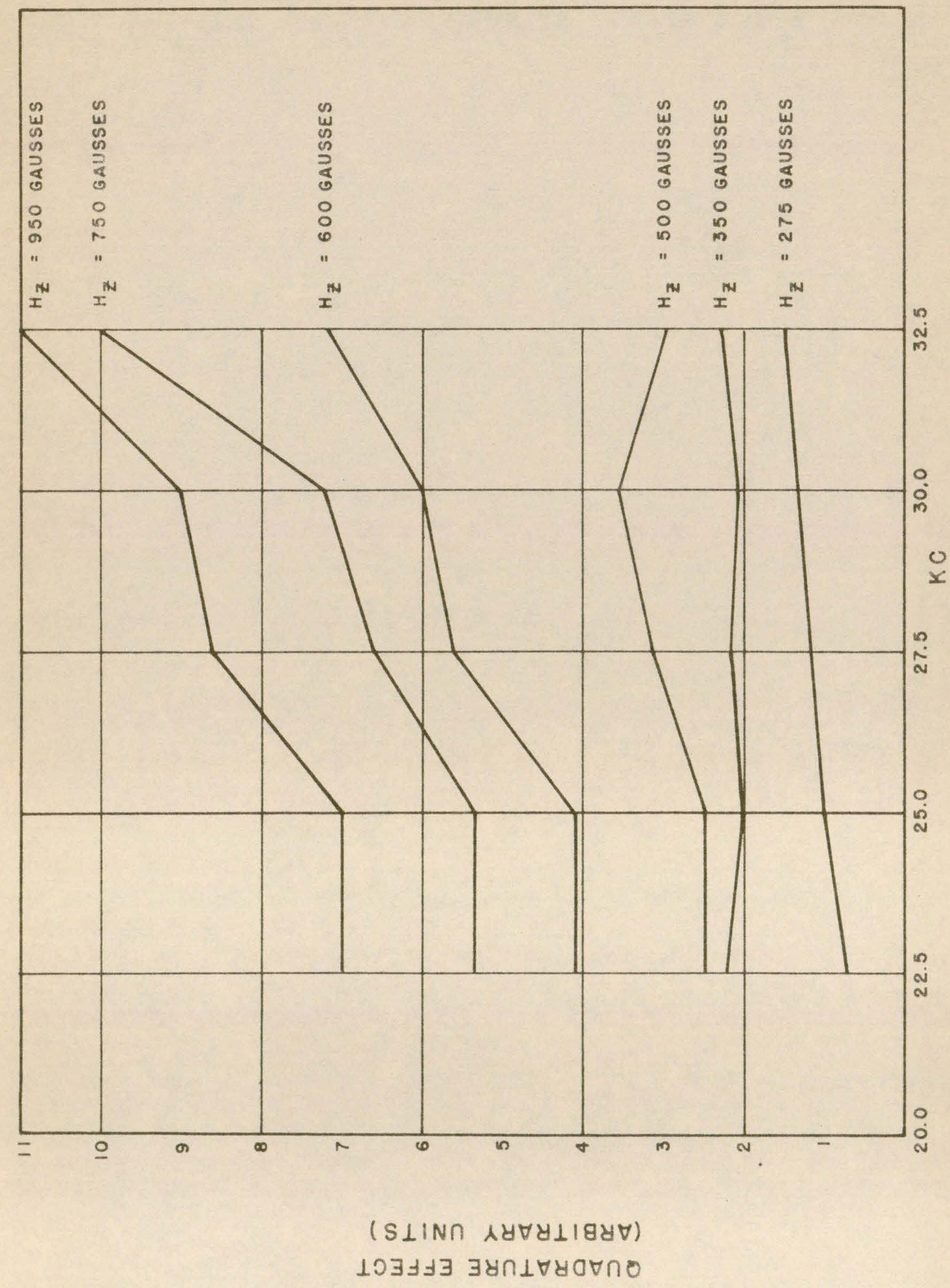


FIGURE 13- QUADRATURE EFFECT VS FREQUENCY

The disturbance was soon noticed with this arrangement, and experiments made to determine its magnitude. Measurements were taken at various frequencies, and for various values of H_z. (See Figure (13)).

It was suggested that this quadrature voltage might originate as a result of changes in the electrostatic coefficient of induction between the primary and secondary circuits, due to the effect of strong magnetic forces on the rod, or due to some other cause. The distributed capacity between the primary and secondary coils was measured on a General Radio Corporation Impedance Bridge, Model #650-A, in conjunction with a Du Mont Oscilloscope and a Hewlett Packard Oscillator, M 200-A. These measurements indicated that the change, if any, in the coefficients of inductance between the two coils must have been less than a few microfarads.

In order to pursue the study, new Helmholtz coils were wound on two separate linen base bakelite frames. The coils wound on each of these rectangular frames (one of which was used for the primary coil in the final gyromagnetic experiments, and is described in detail on page (23)), were wound with 600 turns of silk enamel covered wire. The two frames were tightly bolted together in such a position that the mutual inductance between them was close to zero. Provisions were made to emplace or remove, through a hole in one end of the frames, the material to be tested. When in position, the material was held by a one inch outside diameter bakelite tube that extended from one end of the frames to the other.

In order to reduce the electrostatic coefficients of induction between the primary and secondary coils, an earthed wire helix was wound around the secondary coil, almost completely shielding the secondary coil and sample from the primary. Tests with this arrangement revealed an effect similar in magnitude and phase to that observed with the other arrangement. With this apparatus it was found that the disturbing effect appeared with both the permalloy and the iron samples.

Still another arrangement was tried, to eliminate the undesired disturbance. It was soon suspected that the core might be mis-aligned with respect to the axial magnetic field H_z. If this was the case, torques would be exerted on the rod when the magnetic induction was applied. In this way, changes in the angles θ_1 , θ_2 , and θ_3 might appear, and produce effects as described on page (68).

To investigate this, apparatus was built that enabled the rod to be tipped with respect to the secondary coil. This device is described in detail on page (31). Tipping the coil enabled one to change the angles θ_1 and θ_2.

When measurements of the disturbance were made with this apparatus, it was found that, as the rod was tipped with respect to the coils, the magnitude of the effect changed. As the rod was rotated toward a position of symmetry with respect to the secondary coil (that is, θ_1 and θ_2 approached zero), the effect diminished to a minimum value; and then increased again as the rod was rotated farther from the central position (See Figure (14)), reaching a maximum of about 100 times the gyromagnetic effect. Thus the effect reached a minimum, but was not entirely eliminated. A variety of differently shaped curves could be obtained with small changes in the orientation of the rod in the neighborhood of the minimum. Several examples of these curves are displayed in Figure (15)) as they were obtained with the permalloy rod.

Still further efforts were made to study this effect. A microscope was mounted above the apparatus in such a position that the motions of the secondary coil could be observed. A movement of about 0.001 cm was observed upon application of intense fields H_z. The magnitude of this motion was recorded, along with the strength of the effect that also appeared. In this way it was found that about 110 volts per centimeter displacement appeared.

Then a micrometer screw was mounted on the 300 lb coil so that the screw pressed against the tube that held the secondary coil. By turning the micrometer screw, the tube was displaced to one side. The amount of the displacement could be read from the calibrated head of the micrometer. Once again the magnitude of the displacement was recorded, along with the voltage appearing in the secondary coil. From this information it was found that the voltage in the secondary coil increased at the rate of from 80 to 160 volts per centimeter displacement.

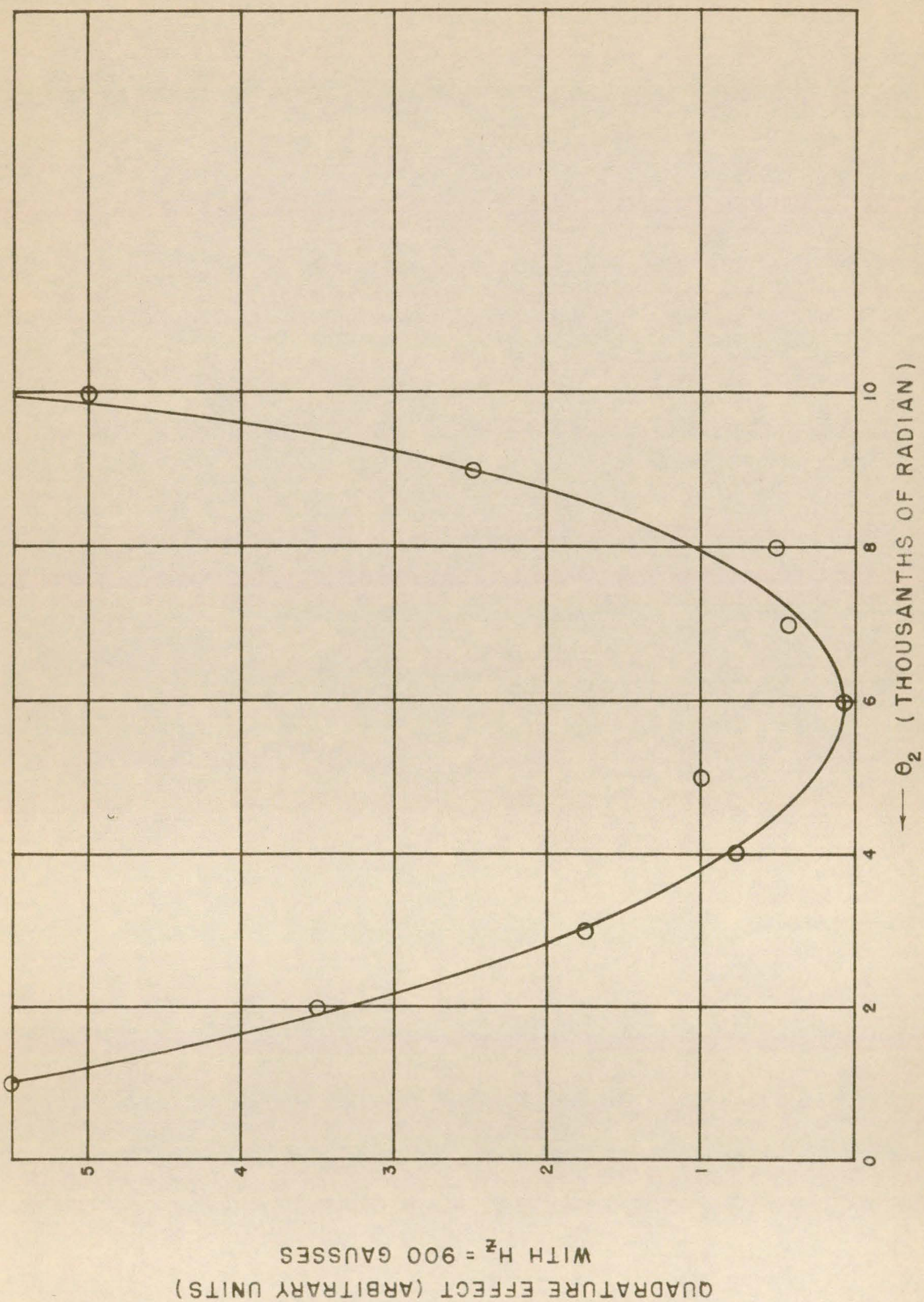


FIGURE 14 QUADRATURE EFFECT AS FUNCTION OF θ_2

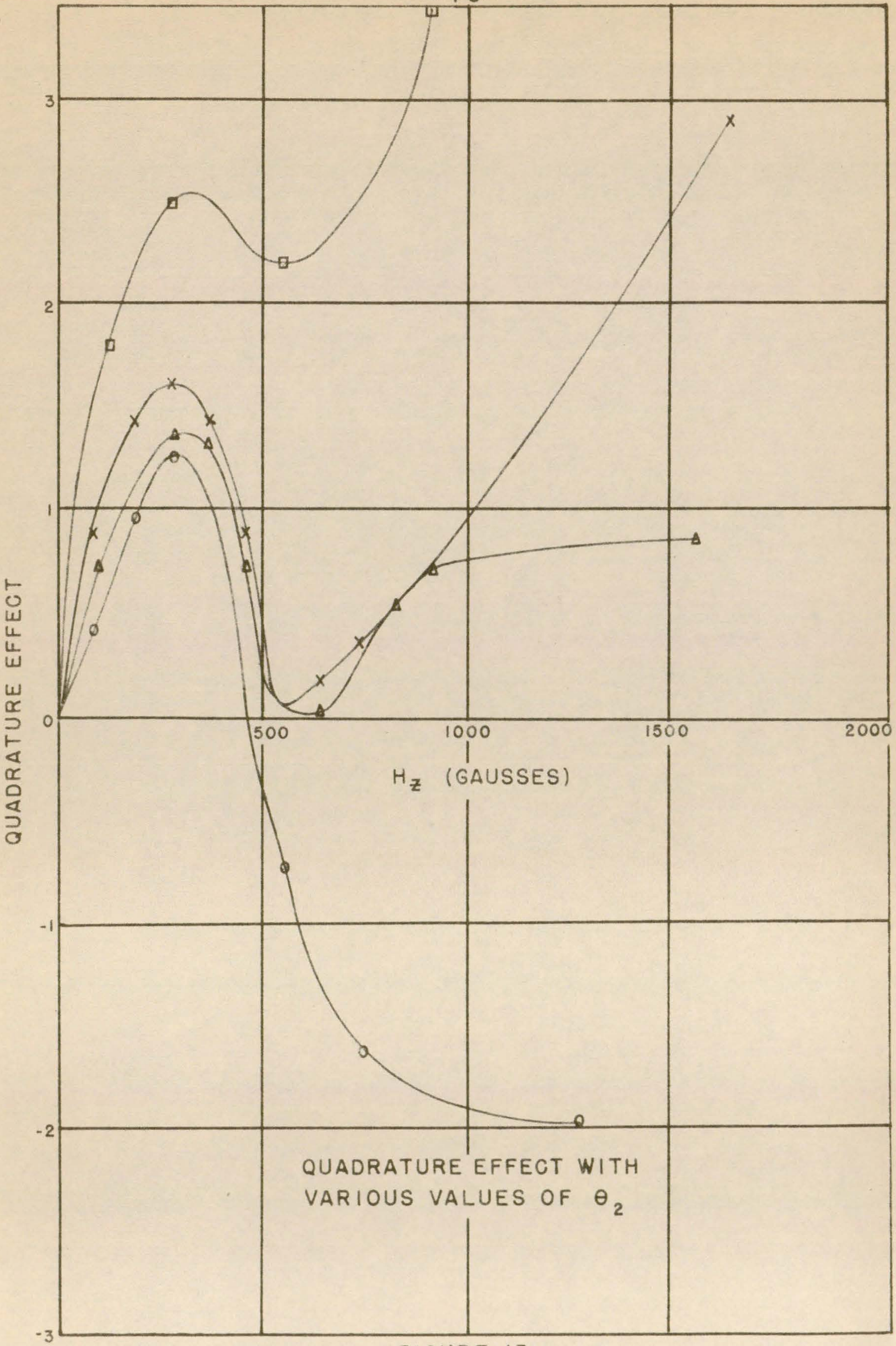

3
2
1
0
-1
-2
-3
500
1000
1500
2000
QUADRATURE EFFECT
H_z (GAUSSES)
QUADRATURE EFFECT WITH
VARIOUS VALUES OF θ_2

FIGURE 15

Since these numbers seemed to be of the same order of magnitude, it was concluded that at least part of this disturbing effect originated from the relative motions of the various parts of the apparatus.

Attempts were made to prevent the relative motions of the various parts. One such attempt is described here. The transformer oil was removed from the tube inside the cooling coils. The apparatus was adjusted so that the disturbance was close to a minimum. Then the entire apparatus was heated to a temperature of 74° C. Molten wax, also heated to a temperature of 75° C., was then poured into the large tube of the cooling coils so as to surround the Helmholtz coil and the secondary coil. Holes were drilled in the bakelite tube supporting the secondary coil, so that the wax flowed inside the tube and came in contact with the permalloy rod. The entire apparatus was allowed to cool slowly to room temperature. It was hoped, in this way, to hold the apparatus firmly in a solid block paraffine, thus preventing any relative motion of the critical parts.

However, trials indicated that the effect was still present with about the same magnitude and phase. It was noted that, when the wax solidified, it changed its volume. As a result of this, the alignment of the apparatus was slightly disturbed. If the coils and rod were aligned before the wax was poured in, they were no longer aligned after the wax had solidified. As there were many possible modes of motion, such as the twisting of the rod, it was not certain that the wax prevented all types of motion, especially with the apparatus mis-aligned. Because of this, the wax did not remove the disturbing effect. Thermal disturbances became greatly aggravated, because the wax furnished a thermal insulating medium around the coil. So, the wax was not used during the final experiments.

It should be mentioned that still further attempts were made to eliminate this disturbance. Other arrangements of coils and rod were tried. For one reason or another, no other arrangement proved to be as satisfactory as the one used.

As mentioned on page (54), detuning the primary circuit by detuning the condensers, unbalanced the equipment, and produced voltages in the secondary circuit. Now, the inductance of the series resonant circuit in the primary was the Helmholtz coil. The rod was in the magnetic field of this coil. The self-inductance of this coil must have changed upon the application of the magnetic field H_z, since the cross permeability of the rod changed. It was suspected that this might detune the primary circuit, and give rise to voltages in the secondary.

To investigate this possible effect, a special thermocouple was placed in series with the primary circuit. The output of the thermocouple was connected to a sensitive voltmeter. The voltmeter was observed to see whether there was any small change in the primary current when the magnetic intensity H_z was applied.

Careful investigation showed that there was a small change in the primary current, resulting from the detuning of the primary current, but that this change was only 1/100 th as large as would be needed to account for the disturbing effect.

All of these experiments indicated that this disturbing effect originated largely from the magnetic forces exerted on the ferromagnetic rod. As the rod was not homogeneous, and the magnetic field, H_z, was not homogeneous, translational forces were certainly exerted on the rod. In addition, the rod departed from the shape of a right circular cylinder, due to small imperfections, and altered its shape slightly with the application of the H_z. Hence it was impossible to align the rod in such a manner that the magnetic intensity would never exert rotational torques or translational forces.

These many serious complications made it necessary to adopt experimental procedures which would enable the gyromagnetic measurements to be made, even though these motions occurred. It was known that the torques and forces exerted on a previously demagnetized rod by an externally applied magnetic field would remain unchanged upon reversing the applied magnetic intensity. If the geometry of the apparatus was distorted upon the application of the magnetic intensity H_z, it was suspected that the same distortions would appear upon the application of $(-H_z)$. This hypothesis was tested by experiment in the following manner. The apparatus was adjusted so that the disturbing effect was over 100 times larger than the gyromagnetic effect. It was then found that the disturbing voltage was unaltered by reversing H_z.

In contrast to the disturbing effect, the gyromagnetic effect reversed sign (i.e., changed phase by 180°) when H_z was reversed.

The experimental procedure used was to measure the change in output meter reading that accompanied the reversal of H_z. The disturbing voltages remained unchanged, while the gyromagnetic voltage alone changed with this reversal. The details of the procedure used have been described above.

V. Conclusions

A. Identification of the Gyromagnetic Effect

Since a disturbing effect appeared under certain conditions, it was necessary to compare carefully the experimental results with theoretical conditions, in order to identify the gyromagnetic effect. The following evidence supported the conclusion that a real gyromagnetic effect had been found:

1. By varying the angles θ_1 and θ_2, as described in Chapter IV, it was possible to vary the magnitude of the disturbing effect. Careful measurements of the gyromagnetic effect were taken with the ferromagnetic sample tipped at various angles with respect to the secondary coil. It was found in every case that the magnitude, phase, and other characteristics of the gyromagnetic effect were independent of such changes.

2. The shape of the curve of E, plotted from the experimental data followed the form expected. The gyromagnetic effect was expected to be small when the intensity of magnetization, J_z, was close to zero, and was expected to increase as H_z increased, and to reach a maximum in the neighborhood of near saturation, and then slowly to decrease in intense fields along with the decrease in transverse permeability. The curve of E (Figures (10) and (11)) displayed these characteristics.

3. The magnitude of the gyromagnetic effect was predicted by formula (17). The results obtained were in agreement, within experimental error, with measurements made by other methods.

4. The gyromagnetic effect was expected to be in quadrature phase. (see page (16)) with induced voltages. The phase of the observed effect was measured, and found to be as predicted.

5. The gyromagnetic ratio, ρ , as defined here, is a negative number. The sign of ρ was experimentally found by comparing the phase relationship of the gyromagnetic voltage with the phase of the induced voltages produced by small rotations of the secondary coil. This experiment showed that ρ , as computed from the data, was a negative number.

6. From equation (17) it can be seen that the gyromagnetic voltage varied as the square of the frequency. Measurements were made at two different frequencies, and the magnitude was found to vary as the square of the frequency.

7. The formula (16) was found to apply to two different ferromagnetic substances with different permeabilities, as well as with two samples of the same alloy (permalloy).

8. The phase of the gyromagnetic effect, as seen in equation (16), was expected to change by 180° when the intensity of magnetization J_z along the axis of the cylinder changed from J_z to $-J_z$. Experiments indicated that the effect observed reversed phase when J_z was reversed in direction.

As no other known effect would satisfy all eight of these characteristics, it seemed reasonable to conclude that a real gyromagnetic effect had been observed.

B. Calculations

From equation (17) one has, for the gyromagnetic ratio ρ the equation

$$\rho\frac{e}{m} = \sqrt{2}\,\pi\left[\frac{\mu+1}{\mu-1}\right]^2\left(1-3\delta+\frac{\Delta}{b}\right)\frac{JE\frac{e}{m}\times 10^8}{NbA\ell\omega^2} \quad (39)$$

where e is the charge in EMU and m the mass of the electron.

When the experimentally determined values of the constant quantities for the permalloy rod #1 were substituted into the right hand side of equation (26), one obtained for 30.50 KC.

$$\rho\frac{e}{m} = 4.85\times 10^3\,\frac{2EG}{H_{ox}}\left[\frac{\mu+1}{\mu-1}\right]^2 \quad (40)$$

The numerical values for $\frac{\mu+1}{\mu-1}$, E, and H_{ox} were obtained from measurements as described in the previous section. All of these quantities depended upon the strengths of currents in different parts of the apparatus, and could be varied at will over limited ranges. A weighted value of ρ as calculated from equation (27) and (28) was, for permalloy:

$$\rho = (1.04 \pm 0.06)\frac{m}{e}$$

A few examples of the data and calculations are given on page 102.

Only the data obtained in the region of near saturation ($H_z > 800$ gausses) are used in the calculations. The gyromagnetic voltage seemed to rise to a maximum before approximation saturation of J_z was reached. (See figures (8) and (10)). The values of Q computed from the data obtained between the maximum of E and the region of near saturation of J_z, that is, the region of H_z between 400 and 800 gausses, were in close agreement with the results obtained from the remainder of the data. However, these additional data were not included in the calculations of the above value, for the equations have not been proved to hold in this region.

The experimental error was computed from the square root of the sum of the squares of errors arising from various sources. The statistical mean deviation of the data was combined with instrumental errors to obtain the probable error, in the following manner:

From formula (18) one has

$$\rho = \sqrt{2}\,\pi \left[\frac{\mu+1}{\mu-1}\right]^2 \left(1-3\delta+\frac{\Delta}{b}\right)\frac{J_z E\times 10^8}{N b H_{ox}\, l\, \omega^2 A}$$

The error in ρ is, then

$$\frac{\delta\rho}{\rho} = \sqrt{\left[\frac{\delta\left(\frac{\mu+1}{\mu-1}\right)^2}{\left(\frac{\mu+1}{\mu-1}\right)^2}\right]^2 + \left[\frac{\delta\left(1-3\delta+\frac{\Delta}{b}\right)}{\left(1-3\delta+\frac{\Delta}{b}\right)}\right]^2 + \left[\frac{\delta J_z}{J_z}\right]^2 + \left[\frac{\delta E}{E}\right]^2 + \left[\frac{\delta b}{b}\right]^2 + \left[\frac{\delta H}{H_{ox}}\right]^2 + \left[\frac{\delta l}{l}\right]^2 + \left[\frac{\delta A}{A}\right]^2 + \left[\frac{\delta\omega^2}{\omega^2}\right]^2}$$

On substituting the values for the quantities concerned, one obtains

$$\frac{\delta e}{e} = \sqrt{\left(\frac{4}{100}\right)^2 + \left(\frac{1}{200}\right)^2 + \left(\frac{1}{500}\right)^2 + \left(\frac{4.5}{100}\right)^2 + \left(\frac{1}{1000}\right)^2 + \left(\frac{1}{100}\right)^2 + \left(\frac{1}{1000}\right)^2 + \left(\frac{1}{100}\right)^2 + \left(\frac{1}{100}\right)^2}$$

$$\frac{\delta e}{e} = 0.062$$

It was not possible to saturate the iron sample with the magnetic fields obtainable, so the formulas could not be applied with certainty. However, it was possible to obtain a measurable gyromagnetic effect for iron in the region available. (See Figure (11)). As H_z increased, this effect started from zero, reached a maximum, and then decreased in magnitude in much the same way as did the effect for permalloy.

Since saturation of J_z could not be obtained, it was not possible to apply, with certainty, equation (26) to iron. By assuming that the gyromagnetic ratio for iron could be calculated from the values of E obtained after the maximum value of E had been reached (as was observed for permalloy), and by also assuming that the correct saturation value of J_z could be obtained by extrapolating the data obtainable for J_z, one obtained for iron

$$\rho = (1.09 \pm 0.10)\frac{m}{e}$$

The error for the measurements on iron was computed in the same way as for permalloy.

In view of this large experimental error, there is no certain disagreement between these results obtained with very intense fields and the results obtained from the Barnett effect(5) with rotational frequencies equivalent to exceedingly minute magnetic intensities, and from the Einstein and de Haas effect(5) with weak and moderate intensities.

The following table lists the results obtained from the various methods:

	Permalloy e/m	Iron e/m
Present study	1.04 ± 0.06	1.09 ± 0.10
Barnett effect(5&6)	1.048 ± 0.006	1.038 ± 0.010
Einstein & de Haas effect(5&6)	1.046 ± 0.003	1.032 ± 0.004
Resonant methods(18)	0.935 to 0.955	0.923 to 0.945

C. Discussion of Results

Kittel(12) differentiated between two different gyromagnetic ratios. In the Barnett effect, one determines the change of magnetic moment resulting from a change in angular momentum. In the Einstein and de Haas effect, one determines the change of angular momentum resulting from a change of magnetic moment. In both of these experiments Kittel defines the gyromagnetic ratio as

$$\rho = \frac{\Delta J}{\Delta M}$$

In resonance experiments, the resonance frequency f is related to the gyromagnetic ratio ρ', and the effective magnetic field strength H_{eff} by the formula

$$\rho' = \frac{2\mu_B}{\pi\hbar} H_{eff}$$

By means of this equation Kittel defines the "spectroscopic ratio" ρ'. The numerical values of these two gyromagnetic ratios, as determined from experimental data, are different.

The probable errors in this present study are large. However, the results obtained here are in reasonable agreement with those of the Barnett effect and the Einstein-de-Haas effect.

D. Suggestions for Further Research

In order to increase the precision of this experiment, several improvements should be made in the apparatus. First, the electronic circuits should be improved so as to increase the stability in both frequency and amplitude. It would be advisable to use battery operated tubes to increase stability and reduce ripple fluctuations.

Secondly, the geometry of the coils and sample could be modified so as to eliminate end effects and inhomogeneities arising from the demagnetizing fields. A torroidal shaped sample might be advantageously used for this reason, as in some of the experiments on rotary fields.

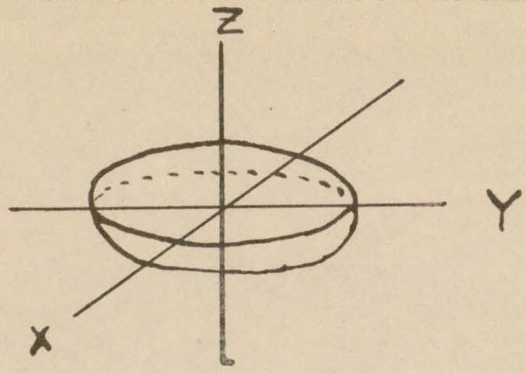
Next, it would be important to investigate further the disturbing effects and eliminate them by improving the design of the apparatus. A different geometrical arrangement might aid in this respect.

The most difficult quantity to measure in this experiment is E, the gyromagnetic voltage. Better measurements could be made of this quantity with thermocouples and D.C. voltmeters.

The quantity $\frac{\mu+1}{\mu-1}$ is also difficult to determine because it must be measured by a method employing oscillating magnetic fields, with intense steady cross fields. Because of the large transverse demagnetizing factor, accurate measurements with a cylinder are difficult. More precise data might be obtained with the procedure used by Bozorth and Dillinger.(16)

In passing, it is worth noting that little information concerning transverse permeabilities has been published. Bozorth and Dillinger(24) have published data on the transverse permeability in longitudinal fields varying in strength from zero to several times that needed to saturate the material. In this experiment, information was obtained on the transverse permeability with longitudinal magnetic fields as strong as five times the saturation field strength. Further information on this phenomenon might be obtained by experimenters quite easily.

APPENDIX I. Intensity of Magnetization in a Uniform Magnetic Field.



If a homogeneous isotopic ferromagnetic ellipsoid is oriented in space so that its principal axes lie along the X, Y, and Z directions, and if B_x^1 and H_x^1 are the X components, inside the ellipsoid, of the magnetic induction and the magnetic intensity, respectively, and if μ is the permeability, then a well known relation between B_x^1 and H_x^1 is

$$B_x^1 = \mu H_x^1 \tag{41}$$

Now, if D_x, D_y, and D_z are the demagnetizing factors in the X, Y, and Z directions, if H_x^o is the magnetic intensity of the applied uniform magnetic field measured at infinity, and if J_x is the intensity of the magnetization, then the relation between these quantities is [25]

$$H_x^1 = H_x^o - D_x J_x \tag{42}$$

On substituting the expression for H_x^1 from equation (42) into equation (41), one obtains

$$B_x^1 = \mu \left(H_x^o - D_x J_x \right) \tag{43}$$

Another equation that applies to these quantities inside the medium is

$$B_x^i = H_x^i + 4\pi J_x \tag{44}$$

Substituting for H_x from equation (42) into equation (44), one obtains

$$B_x^i = H_x^o + (4\pi - D_x) J_x \tag{45}$$

Equating the right hand side of the equation (43) and the right hand side of equation (45), and solving for J_x, one obtains

$$J_x = \frac{\mu_x - 1}{4\pi + (\mu_x - 1) D_x} H_x^o$$

APPENDIX II. Formula for Transverse Permeability.

From equation (41), Appendix I, one obtains

$$J_x = \frac{\mu_x - 1}{4\pi + (\mu_x - 1) D_x} H_x^{\circ} \tag{47}$$

and from equation (16), page (17),

$$D_x = 2\pi(1 - \delta) \tag{48}$$

On substituting for D_x from equation (48) into equation (47), one obtains

$$J_x = \frac{1}{2\pi} \left(\frac{\mu - 1}{\mu + 1}\right)^2 \frac{1}{1 - \frac{\mu - 1}{\mu + 1}\delta} H_x^{\circ} \tag{49}$$

From equation (45), Appendix I, one obtains

$$B_x^i = H_x^{\circ} + (4\pi - D_x) J_x \tag{50}$$

Substituting for D_x, from equation (48) into equation (50), one obtains

$$B_x^i = H_x^{\circ} + 2\pi(1 + \delta) J_x \tag{51}$$

On substituting for J_x from equation (49) into equation (51) and simplifying the results, it is found that

$$B_x^i = \frac{1 + \frac{\mu-1}{\mu+1}}{1 - \frac{\mu-1}{\mu+1}\delta} H_x^o \qquad (52)$$

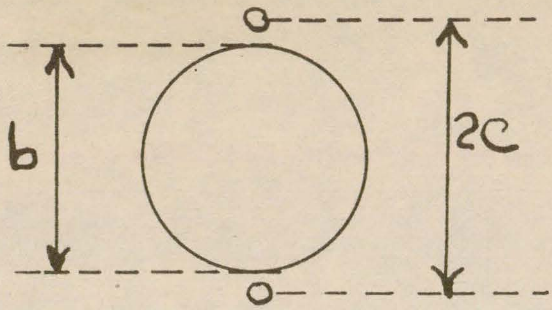
Now, if δ is small, and if μ is greater than one (as is the case in this experiment), then equation (52) reduces approximately to

$$B_x^i = \frac{1 + \frac{\mu-1}{\mu+1}}{1-\delta} H_x^o \qquad (53)$$

Dividing through by H_x and further simplifying, this reduces approximately to

$$\left[\frac{\mu-1}{\mu+1}\right](1+2\delta) = \frac{B_x^i}{H_x^o} - 1 \qquad (54)$$

APPENDIX III. Formula for Voltage Induced in a Test Coil.



Consider a cylinder placed in a real uniform magnetic field

$$H_x^o = H \sin \omega t \qquad (55)$$

The radius of the cylinder is b, and its length is l. A long, narrow rectangular coil of length l and width 2c is wound on the cylinder. A cross section of the coil and cylinder is shown above.

From equation (53), Appendix II, one obtains for B_x^i, the magnetic induction inside the cylinder, the expression

$$B_x^i = \frac{1 + \frac{\mu - 1}{\mu + 1}}{1 - \delta} H_x^o \qquad (56)$$

If μ is large, compared to one, and if δ is small, then equation (1) reduces approximately to

$$B_x^i = \frac{2\mu}{\mu + 1}(1 + \delta) H_x^o \qquad (57)$$

By considering boundary conditions, it can be shown that the magnetic induction, B_x^o, outside the cylinder, is approximately

$$B_x^o = \left[1 - \frac{\mu-1}{\mu+1}\,\frac{b^2}{r^2}\right] H_x^o (1+\delta) \qquad (58)$$

Now, the total flux Φ through the coil, is related to B_x^o and B_x^i by the expression

$$\Phi = 2N\ell \int_0^b B_x^i \, dr + 2N\ell \int_b^c B_x^o \, dr \qquad (59)$$

where N is the number of turns in the coil.

Substituting the values for the magnetic induction found from equations (57) and (58), into equation (59), and performing the indicated integration, one obtains

$$\Phi = 2\left[c + \frac{\mu-1}{\mu+1}\,\frac{b^2}{c}\right] N H_x^o \ell (1+\delta) \qquad (60)$$

Now, the RMS voltage (practical units) induced in this coil is given by

$$E = \frac{\sqrt{2}}{2}\,\Phi\,\omega \times 10^{-8} \qquad (61)$$

Substituting the expression for ϕ from equation (60) into equation (61), one obtains

$$E = \sqrt{2}\, c\,\omega \left[1 + \frac{\mu - 1}{\mu + 1}\,\frac{b^2}{c^2}\right] N H_x^o \ell (1 + \delta) \qquad (62)$$

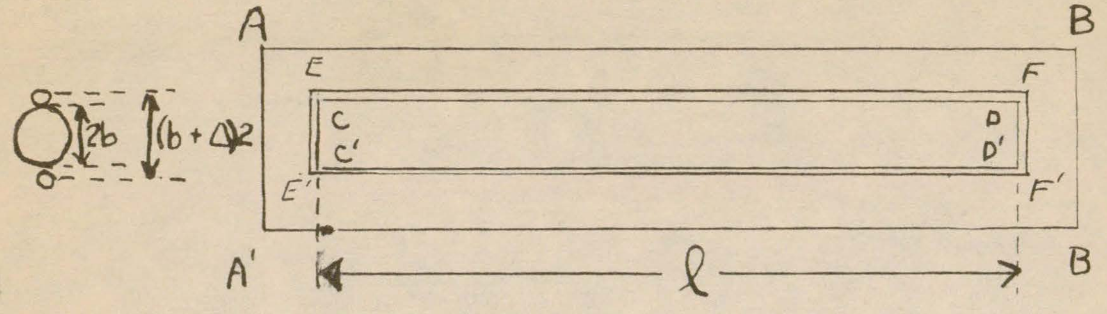
Now, solving equation (57) for H_x^o, and substituting the resulting expression into equation (62), and then solving for B_x^i, one obtains

$$B_x^i = \frac{\sqrt{2}\, E \times 10^{-8}}{N \omega \ell c \left[\left(1 + \frac{b^2}{c^2}\right) + \frac{1}{\mu}\left(1 - \frac{b^2}{c^2}\right)\right]} \qquad (63)$$

If one defines γ so that $c = b + \gamma$ where γ is small, then equation (63) simplifies to

$$B_x^i = \frac{\sqrt{2}\, E}{2 N \ell\, \omega \left[b + \frac{1}{\mu}\gamma\right]} \qquad (64)$$

APPENDIX IV. Formula for Voltage Induced in Secondary Coil .



In this appendix a semi-empirical formula is developed for the gyromagnetic voltage appearing in the secondary coil . This voltage is a function of the intensity of magnetization arising from the gyromagnetic effect, and a function of certain dimensions of the apparatus.

The secondary coil AA'B'B (see above figure) was wound on a bakelite frame that was longer and wider than the rod CC'D'D. Since a direct calculation of the flux threading the coil AA'B'B due to the magnetization of the rod was difficult, a semi-empirical method was used to determine the flux.

First, the magnetic flux Φ_s threading the coil EE'F'F, wound closely on the rod, due to the gyromagnetic induction B^1_y, was calculated. The ratio of the flux Φ_s threading this closely wound coil, to the flux Φ threading the large coil AA'B'B was determined experimentally. From this information a formula was developed for the voltage induced in the large coil.

First, the flux Φ_s through the small coil FE'F'F was calculated. From Appendix IV, equation (60), one has for the magnetic flux Φ_s' inside a coil wound closely to a rod, when the rod is placed in a magnetic field, the relation, where we substitute for C in equation (60) the quantity $b+\Delta$

$$\Phi_s' = 2\left[b+\Delta+\frac{\mu-1}{\mu+1}\,\frac{b^2}{b+\Delta}\right]N\ell H_y^{\circ}(1+\delta) \qquad (65)$$

Here b is the radius of the rod, $2(b+\Delta)$ is the width of the coil, and l is the length of the coil (See Figure, page 95).

If this equation is used with the gyromagnetic effect, it must be modified. In the case of a real magnetic field $H_y^{\circ\prime}$ applied to a rod, as described by the above formula, contributions to the magnetic induction $B_y^{i\prime}$ arise from both the intensity of magnetization J_y and the applied external field H_y°, as indicated by the formula

$$B_y^{i} = H_y^{\circ\prime} + (4\pi - D)\,J_y \qquad (66)$$

With the gyromagnetic case, torques are exerted on the magnetic elements of the rod, producing an intensity of magnetization J_y without the presence of an external field $H_y^{\circ\prime}$. In this case, B_y^i is related to J_y by the formula

$$B_y^{i} = 0 + (4\pi - D)\,J_y \qquad (67)$$

From these relations it can be shown that the value of the flux Φ_s for gyromagnetic phenomena can be calculated from equation (1) if the quantity $\Psi = AREA \times H_y^{\circ\prime}$ is subtracted from equation (65).

One then obtains.

$$\Phi_s = \Phi_s' - \Psi$$

$$\Phi_s = 2\frac{\mu-1}{\mu+1}\frac{b^2}{b+\Delta} N H_y^\circ \ell (1+\delta) + 2(b+\Delta) N \ell H_y^\circ \delta \qquad (68)$$

From equation (49), Appendix II, if $\mu > 1$ one has the relation

$$J_y = \frac{1}{2\pi}\frac{\mu-1}{\mu+1}\frac{H_y^\circ}{1-\frac{\mu-1}{\mu+1}\delta} \qquad (69)$$

If $\mu > 1$ and δ is small, this reduces to

$$J_y = \frac{1}{2\pi}\frac{\mu-1}{\mu+1} H_y^\circ (1+\delta) \qquad (69)$$

Combining equations (68) and (69), one obtains

$$\Phi_s = 4\pi N \ell \frac{b^2}{b+\Delta} J_y + 4\pi(b+\Delta)\frac{\mu+1}{\mu-1}\frac{\delta}{1+\delta} \qquad (70)$$

Since Δ is small compared to b, δ is small compared to unity, and μ is large compared to unity, equation (70) can be simplified to the approximate form

$$\Phi_s = 4\pi N b \ell J_y \left(1 + \delta - \frac{\Delta}{b}\right) \qquad (71)$$

If ϕ is the flux threading the large secondary coil, and ϕ_s is the flux threading the coil wound closely to the sample, then the ratio of these two fluxes is a constant, that is

$$\phi = \phi_s A \qquad (72)$$

The constant A was determined experimentally as described in Appendix VI. On substituting the expression for ϕ_s from equation (71) into equation (72), one obtains

$$\phi = 4\pi N b \ell A \left(1 + \delta - \frac{\Delta}{b}\right) \qquad (73)$$

or

$$\phi = \phi_0 \cos \omega t \qquad (74)$$

where

$$\phi_0 = 4\pi N b \ell A J_{oy} \left(1 + \delta - \frac{\Delta}{b}\right) \qquad (75)$$

and

$$J_y = J_{oy} \cos \omega t \qquad (76)$$

The RMS voltage E, induced in a coil, is given by the formula

$$E = \frac{\sqrt{2}}{2} \omega \phi_0 \times 10^{-8} \qquad (77)$$

Combining equations (74) and (77) one obtains

$$E = 2\sqrt{2}\,\pi N b \ell A \omega \left(1 + \delta - \frac{\Delta}{b}\right) J_{oy} \times 10^{-8} \qquad (78)$$

APPENDIX V. Determination of the Constant A.

The constant A was defined in Appendix IV as the ratio between the flux ϕ threading the secondary coil, to the flux ϕ_s threading a small coil wound closely on the rod. This constant was defined so as to include only that flux arising from the magnetizing of the rod.

To experimentally determine A, it was necessary to wind a test coil on the rod. The rod and coil were placed inside the primary Helmholtz coil. The galvanometer deflection d_1 accompanying a known change in the primary current was recorded.

Then the rod and test coil were removed, and a bakelite coil, with dimensions similar to those of the core, was placed in the same position. The deflection d_2 of the galvanometer, associated with a known change of the primary current, was again recorded.

Next, the secondary coil was placed inside the primary coil. The deflections d_3 and d_4, associated with the reversal of the primary current, with the rod in place, and with the rod removed, were recorded.

The galvanometer was standardized with a Hibbert Standard. The dimensions of each coil were measured with a micrometer.

From this information, the changes in flux (per ampere) were determined in each case. Corrections were made for the small differences in area of the coil on the rod and the bakelite core. The constant A was then calculated from the relation

$$\frac{\Phi_4 - \Phi_3}{\Phi_2 - \Phi_1} = A \tag{79}$$

where each Φ_k is associated with the corresponding d_K. The value of A was found to be 0.440.

APPENDIX VI. Samples of Data.

1. H_z = 722 gausses H_{ox} = 9.81 gausses

Meter Reading Position 1	Meter Reading Position 2
10×10^{-4} volts	24×10^{-4} volts
11	26
09	26
12	30
14	
Avg. 11.20×10^{-4} volts	26.65×10^{-4} volts

Average $2EG = (29.75 - 14.00) \times 10^{-4}$
$2EG = 15.75 \times 10^{-4}$ volts

$$\rho = 4.85 \times 10^3 \frac{2EG}{H_{ox}} \left(\frac{\mu+1}{\mu-1}\right)^2 \frac{m}{e} \qquad (40)$$

$$\rho = 4.85 \times 10^3 \frac{15.75 \times 10^4}{10.08} \times (1.341)\frac{m}{e}$$

$$\rho = 1.002 \frac{m}{e}$$

2. H_z 770 gausses H_{ox} 10.70 gausses

Meter Reading Position 1	Meter Reading Position 2
14×10^{-4} volts	30×10^{-4} volts
18	36
18	37
20	40
21	
Avg. 18.20×10^{-4} volts	35.75×10^{-4} volts

Average $2EG = (35.75 - 18.20) \times 10^{-4}$
$2EG = 17.55 \times 10^{-4}$ volts

$$\rho = 4.85 \times 10^3 \times \frac{17.55 \times 10^{-4}}{10.70} \times (1360) \frac{m}{e}$$

$$\rho = 1.077 \frac{m}{e}$$

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