ON THE ARRIVAL ANGLE OF MICROWAVES

Thesis by

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Abstract

This research describes a preliminary effort to measure the effect of the atmosphere on the refraction of microwaves. Variations in the structure of the lower atmosphere cause small changes in the index of refraction, and these small changes might alter the angle at which the wave front arrives at the receiver.

To measure the minute variations in the arrival angle, a transmitter, radiating at a frequency of 9520 megacycles was placed on Mt. Wilson. The radiated energy was focused by means of a metallic lens, and the shift of the diffraction pattern of the lens as measured in its focal plane, was taken as a measure of the change in the angle of arrival. This shift was determined by detecting the position of the upper null of the diffraction pattern. There were changes in the position of this null and these changes were, in general, consistent with the meager meteorological information available.

Variations of the order of one tenth of a degree were noticed. These changes are larger, in all probability, than can be accounted for by calculations based on ray theory. Reflections from nearby buildings and trees could conceivably influence the nature of the diffraction pattern, but the exact nature of their effect would be extremely difficult to calculate.

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I. Introduction

From the time that it was first proved practical to transmit intelligence by means of electromagnetic waves, the question of the propagation of these waves has been the subject of a vast amount of theoretical and experimental research.

In 1909, by means of function theory, Sommerfeld succeeded in obtaining a solution to the problem of a dipole radiating over a plane of finite conductivity (1). Even today his original solution is the subject of critical investigation (2) (3).

Until rather recent years, the frequencies used in wireless transmission were relatively low, and the associated wave lengths were, as a consequence, quite long. Though the transmission of longer wave lengths was influenced by factors such as reflections from the ground, diffraction around obstacles, and the constitution of the ionsphere - to mention but a few - it was insensitive to variations in the structure of the lower atmosphere. Consequently little work involving the effect of meteorological variations in the troposphere on the propagation of radio waves was done.

It had been realized early that anything which could change the value of the dielectric constant of the medium through which the waves propagated would affect their velocity in the medium and would, therefore, alter the path of the rays associated with the wave fronts. In 1913, Eccles (4) published a solution dealing with the path of the rays of an electromagnetic wave traveling "in a medium disposed in concentric spherical shells of continuously varying refractive index". He brought out clearly the analogy between this situation and a familiar problem in optics. He showed that the ray would be curved and he obtained the polar equation for it.

In 1927 Baker (5) published an extensive treatment on the tracing of rays of an electromagnetic wave through an atmosphere whose index of refraction followed certain variations in its vertical structure. (It is remarkable to note that the work of Baker could be applied without essential change to present day theory.) Baker pointed out the vast difficulty involved in the computation of the track of the ray, and he showed that to calculate it accurately, it would be necessary to know the refractive index of each level the ray would reach with an accuracy to the fifth decimal (6).

Thus, while thought had been given to the question of the propagation of radio waves through media with a variable index of refraction, no particular occasion arose to consider in detail the nature of the changes in the refractive index of air - other than those changes due to the number and nature of the ionized particles found in it.

As progress in radio transmission developed, useful wave lengths became increasingly shorter, and it was the accepted opinion that the horizon was the limit of their practical utility. The occurrence of phenomena associated with the reception of signals beyond the horizon was attributed to the effects of diffraction.

The first realization that effects other than diffraction might play an important role in the propagation of very short waves beyond the horizon seems to have occurred to Jouaust (7). He noted that, in the course of hot days, there was a very strong transmission during the night between two short wave stations about 205 km apart. One of these stations was in France, near Nice, the other was on the island of Corsica. The straight line joining the two stations passed more than 100 meters below the sea. The usual explanation of the longer range by diffraction effects seemed to him to be unlikely, especially in view of the excellent correlation between the intervals of strong transmission and certain characteristic variations in the daily weather. He stated that the occurrence of this phenomenon could be explained only by the effect of the lower atmosphere on the range of very short radio waves. So far as could be found, this was the first direct reference to the effect of the troposphere on the range of very short radio waves.

In 1933 Schelling, Burrows, and Ferrell published a paper on the propagation of ultrashort waves (8). In the course of their analysis of the effect of the atmosphere on the ray path, they established and introduced a technique by means of which the refractive effect of the atmosphere was taken into account by increasing the radius of the earth to such a degree that the rays, though actually curved, appeared as straight lines. This was an important contribution because in the subsequent development of the complicated mathematical methods applicable to the solution of the wave equation in a medium of varying index of refraction some such coordinate system is generally introduced. Often the transformation is one in which the surface of the earth becomes a plane and the rays have some curvature relative to it (9) (10) (11).

Roughly then, though some work had been done on the effect of the lower atmosphere on short wave propagation, no impressive instances of any extreme effect had been noted.

In 1934 the situation changed abruptly. Ross Hull, an associate editor of Q.S.T., in the course of transmitting with a new directive antenna established contact (using a wave length of 5 meters) with stations 100 miles away (12). This was an unbelievable happening. To his credit it must be stated that, though he was enthusiastic about the antenna, he realized that other factors must play the dominant role.

He instigated what seems to be the first intensive, large scale investigation into the meteorological conditions in the lower atmosphere. He was especially interested in correlating the effect of these conditions on the propagation of short waves beyond the horizon. It was established that periods of long range communication were always associated with certain fairly definite meteorological conditions - in particular, with temperature inversions that existed in the regions up to about 2500 meters above the surface of the earth (13) (14).

Unfortunately, Hull was accidentally electrocuted in the course of his experiments and was unable to summarize and interpret the greater part of the data that had been accumulated. An account of the results of Hull's work was recently given by Friend (15).

With the development of extremely efficient ultra-short-wave transmitters and receivers, and with the extended use of such devices in World War II, it soon became amply evident that there waw still much to be learned about the propagation of these waves.

On numerous occasions "radar vision" extended for several hundred miles along the surface of the earth, and in at least one instance, on the Indian Ocean objects were visible at a distance of about 1500 miles (16). Such unorthodox effects and their importance to military tactics stimulated a new interest in the study of propagation at very high frequencies. Eminent scientists of all nations took part in a broad program of intensive investigation; and, as a consequence, great progress has been made, during the last five years, in understanding the meteorological factors in the propagation of extremely short waves (17).

In spite of the fact that a vast amount of data has been accumulated relating very short wave propagation to the weather, the whole problem is of such complexity it cannot yet be said to be solved (18).

II. Radio Meteorology

If the earth were devoid of an atmosphere the path of the rays associated with a plane electromagnetic wave front would be straight lines; and, barring the small effects due to diffraction, the practical limit of useful short wave reception would be the horizon.

Because the atmosphere is present, and because its structure changes in the vertical direction, the effect is to cause the index of refraction to decrease slightly with increasing height. This decrease in the index of refraction is attributed to the fact that temperature, pressure, and water vapor density vary with height.

It is generally assumed that, under "standard" conditions the decrease of the refractive index is linear with height and is given by (19)

$$\frac{dn}{dh} = -0.039 \times 10^{-6} \text{ units per meter} \quad (2.1)$$

Because of this decrease in the index of refraction the upper portion of the wave front travels slightly faster than the lower portion with the result that it is tilted forward by a small amount as it progresses; hence, the associated rays are bent slightly downward. The bending of the rays under normal conditions is extremely small.

The expression for the refractive index of dry air is given by (20)

$$(n_1 - 1) = \frac{79 \times p}{T} \times 10^{-6} \qquad (2.2)$$

n_1 = index of refraction of dry air

p = pressure in millibars

T = temperature in absolute degrees

The atmosphere normally contains some water vapor and the water vapor molecule has a permanent dipole moment which reacts with the electric field. The refractive index of water vapor is given by (21)

$$(n_2 - 1) = \left(\frac{68e}{T} + \frac{3.8 \times 10^5 e}{T^2}\right) \times 10^{-6} \qquad (2.3)$$

n_2 refractive index of water vapor

e partial pressure of water vapor in millibars

T absolute temperature

Since the right sides of (2.2) and (2.3) are additive the final result for the index of refraction for a combination of dry air and water vapor is given by

$$(n-1) = \left(\frac{79P}{T} - \frac{11e}{T} + \frac{3.8 \times 10^5}{T^2} \times e\right) \times 10^{-6} \qquad (2.4)$$

n = refractive index of atmosphere

P = total atmospheric pressure in millibars

T = absolute temperature

e = partial pressure of water vapor in millibars

The above formula is claimed to be fairly accurate up to frequencies in the region of 10,000 mc/sec (22). Extensive discussions of this formula appear in the literature (23).

In the actual problem of ultra-short wave propagation, it is not customary to use the index of refraction directly but to use instead a quantity known as the "modified index of refraction". This quantity is designated by the symbol M and is formed by adding to the left of (2.4) the term h/a and multiplying by 10^6. Therefore,

$$M = \left(n-1+\frac{h}{a}\right) \times 10^6 \qquad (2.5)$$

n = index of refraction of the atmosphere

h = height above the earth in meters

a = mean radius of the earth in meters

In the actual evaluation of n-1 the second term in the right member of (2.4) is very small in comparison with the remaining terms and it is usually neglected. Since $\frac{1}{a}$ is approximately equal to 0.157×10^{-6} when a is measured in meters

$$M = \frac{79P}{T} + \frac{3.8 \times 10^5}{T^2} e + 0.157h \qquad (2.6)$$

By means of the above formula it is possible to calculate M in terms of meteorological measurements made at a given height.

Many measurements of M made throughout the world show that, when M is plotted against the height, certain

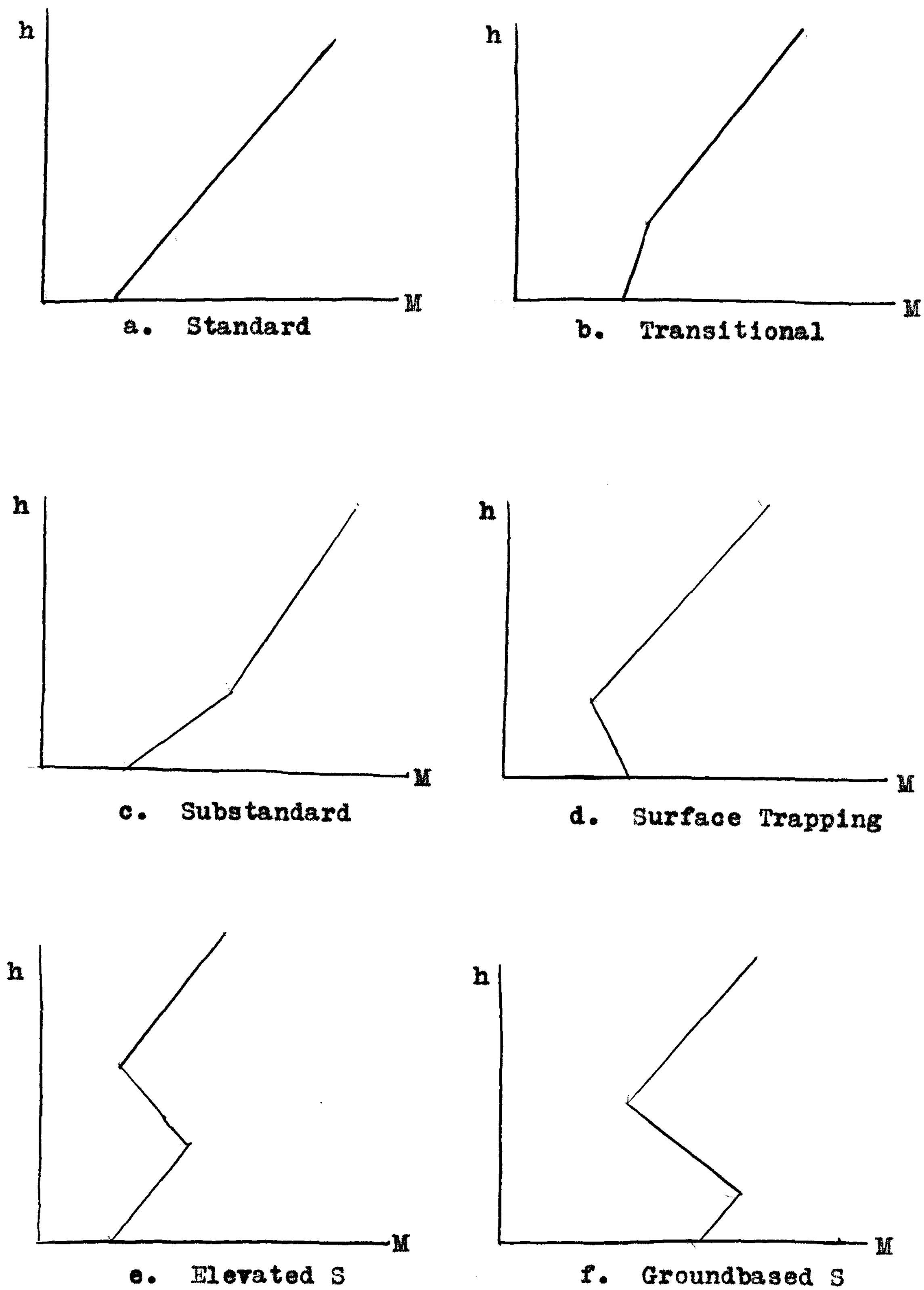


Figure 1. Types of M Curves

characteristic curves are the usual result. These are outlined and classified in Figure 1 (24).

The expression for the M curve contains a term involving h, and under standard conditions the effect of this term is to cause M to increase with height. Under some conditions, however, the change in n with increasing height is sufficiently negative to overcome the increase due to h, with the result that there is an inversion in the slope of the M curve.

The type of M curve in which this inversion occurs at a considerable height above the ground (curve e, Fig. 1) is fairly common in the coastal regions of southern California and the structure of these curves has been quite extensively investigated in the vicinity of San Diego (23). The base of the inversion layer may occur up to heights of 5,000 feet and the inversion may be 1,000 feet in thickness. The physical reason for the large negative change in n is attributed to two conditions (26):

1. The condition of increasing temperature with increasing height - temperature inversion
2. The variations of humidity, or vapor pressure, with height

The individual effects of each of these conditions, together with numerical values for the contribution of each to the lapse rate of the index of refraction have been discussed in detail by Sheppard (27) (28). While each of the two causes mentioned is an important factor, it is stated that the lapse rate of vapor pressure is of the greater significance.

The proper conditions for a temperature inversion occur frequently in southern California in the coastal regions. For a few hours after midnight the breeze is from the land to the sea - the cold air flowing down from the canyons where it has accumulated during the day. In the early morning the breeze may die down and leave a stationary mass of cool air in the lower regions with a layer of warm air overhead. The resultant temperature inversion generally has its base at a height varying from 800 to 2,000 feet above the earth, and it will persist only so long as the atmosphere is very calm. It is normally dispersed with the occurrence of even a slight breeze (29).

Another cause of the elevated temperature inversions in southern California is the phenomenon known as subsidence (30) - a slow vertical sinking of air over a very large area. The air in a subsiding layer is usually very dry, and when such a mass overlies a moister layer near the ground a sharp moisture gradient is produced which is favorable to the formation of an elevated inversion in the M curve.

Though abnormal short wave reception is in general agreement with the inversions in the M curve, there is evidence to indicate that much can yet be learned about the phenomenon of anomalous propagation. Some experiments made near San Diego seem to show that the reception of a strong signal over a non-optical path is more in agreement with reflection from a layer than with refraction through it (31).

Such a theory had once been proposed by Englund, Crawford, and Mumford in 1938, (32) but Appleton and others believed it impossible to account for anomalous reception by means of an atmospheric discontinuity (33). Experiments performed on the Arizona desert seem to confirm the conclusion that strong fields are not always associated with a decreasing rate of change in the M curve (34).

Other experiments indicate that signal strengths computed from the observed M curve do not agree with observations, and that the temperature gradient has been greatly underestimated (35). Hoyle, in fact, found that the correlation between radio signal strength and the rate of change of partial vapor pressure with height was very poor, while the correlation with the rate of change in T with height was very good (35). The opposite situation is ordinarily presumed to be true.

Further study of the situation appears, therefore, to be warranted.

III. Theory

It is difficult, if not impossible, to obtain an exact solution for the wave equation when the medium through which the wave propagates has an index of refraction whose value depends in some functional manner upon the height.

Much mathematical labor has been expended upon this problem and many useful results have been obtained. An exact solution involving propagation through a stratified medium was given by Epstein (36) and his results were frequently utilized by workers in the field of short wave propagation during the war (37). It is conceivable that the Epstein solution might be applied to research of the type to be described.

When the M curve is of Type d, Figure 1, valid solutions of the wave equation can be obtained under the assumption that the radiator is a dipole over a perfectly conducting earth (38) (39) (40). The technique of obtaining a solution involves a perturbation method analagous to the mathematical device so frequently useful in quantum mechanics; the work is, however, of considerable complexity. Solutions for other types of M profiles can be obtained by integration in the complex plane (41).

For the purposes of this investigation, the solution of the wave equation is unnecessary because ray tracing methods give the desired results. It can be shown that ray tracing methods are valid in media in which the change in the index of refraction is small within the space of one wave length (42) (43), provided the neighborhood of a caustic or a focus is avoided. Theoretically, also, the conductivity of the atmosphere and reflections from atmospheric layers affect the geometry of the ray, but Epstein (42) (44) has shown that under usual conditions both of these effects are negligible, and that the energy follows the ray. The ray is essentially normal to the wave front.

Specifically the conditions under which ray theory is applicable may be written as (45)

1. The radius of curvature of the surfaces of equal phase must be larger than the local wave length,

and

2. The variation of refractive index with height must satisfy the relation

$$\frac{dn}{dh} \times \frac{\lambda}{2\pi} \times \sin\alpha \ll 1$$

where α is the angle the ray makes with the local horizontal.

For wave lengths in the 3 cm. region both conditions are normally well satisfied. Assuming conditions 1 and 2 to be satisfied, the differential equation of the ray is

$$\frac{d^2h}{dx^2} = \left[\frac{2}{a+h} + \frac{1}{n}\frac{dn}{dh}\right]\left(\frac{dh}{dx}\right)^2 - \left[\frac{1}{a} + \frac{1}{n}\left(1+\frac{h}{a}\right)\frac{dn}{dh}\right]\left(1+\frac{h}{a}\right) \quad (3.1)$$

If the "flat earth" coordinate system is used, equation (3.1) may be derived either by utilizing a method developed by Epstein (46), or by an application of Fermat's Principle (47).

The equation is not readily integrable in the form given above, but, if the angle the ray makes with the horizontal is not too large, the first term on the right is negligible compared with the second. It is also true that h $\ll$ a, and that n $\doteq$ 1. Making use of these assumptions, the simplified equation may be integrated (48). If α is written for $\frac{dh}{dx}$ the result is

$$\alpha^2 - \alpha_0^2 = 2(M - M_0) \times 10^{-6} \qquad (3.2)$$

The significance of the symbols is shown in Figure 2.

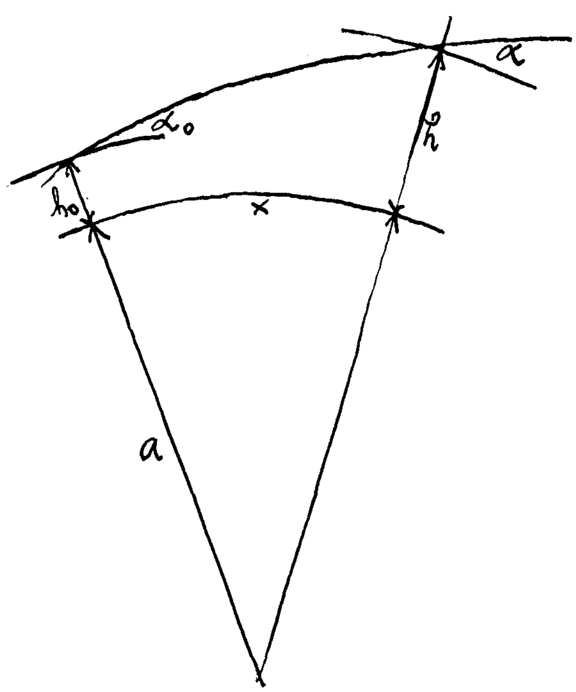


Figure 2 Ray Geometry

To apply ray tracing methods, it is necessary to have some check on the horizontal distance traversed by the ray. Assuming a linear variation of refractive index with height, the proper relation is given to a close approximation by

$$x = 2 \times \frac{(h - h_0)}{\alpha + \alpha_0} \qquad (3.3)$$

By applying equations (3.2) and (3.3) a ray may be traced through a medium stratified in layers, provided the following are known:

(1) the height of the layer boundaries above the earth

(2) the values of M at the boundaries of the layers

(3) the arrival angle of the ray at the receiver or the departure angle of the ray at the transmitter.

Since the M values were known at only the receiver and transmitter locations, certain logical assumptions concerning the variation of M in the intermediate region must be made. The assumptions are that the variation of M in a layer is linear with the height and that M is continuous at the boundary of two layers.

Ray tracing enables one to compare the observed arrival angles with those theoretically possible for given M distributions. The method is further illustrated in section V.

IV. The Equipment and Its Characteristics

To make angle-of-arrival measurements, a transmitter (a Sperry Klystron, type 2K39) oscillating at a frequency of 9520 megacycles was installed in the Snow telescope on the Mt. Wilson observatory grounds. The transmitter was modulated to produce an audio note whose frequency was approximately 850 cycles a second. The transmitting antenna was a parabola, 30 inches in diameter, with a beam width of about 3 degrees between half power points. The power output of the transmitter, as measured by a varistor bridge, was 300 milliwatts. The wave was transmitted with the E field vertical. Choice of the Mt. Wilson site was dictated, in part, by the desire to avoid a strong ground reflected ray. Figure 3 shows the transmitting equipment. A profile of the propagation path is shown in Figure 4.

The receiving antenna was a metallic lens constructed according to the formulas published by W. E. Kock (49). Designed to have an effective index of refraction of one half and a focal length of 400 centimeters, the lens was 73.5 wavelengths long and it was "stepped" at appropriate intervals to avoid excessive thickness. By means of bakelite separators the lens plates were held 1.85 cm apart and the entire structure was mounted, in vertical position, to a long "A" frame. A horn, which could be moved vertically by an accurate screw drive was attached to the "A" frame at the focal plane of the lens. The position of the horn apex

Figure 3 The Transmitter

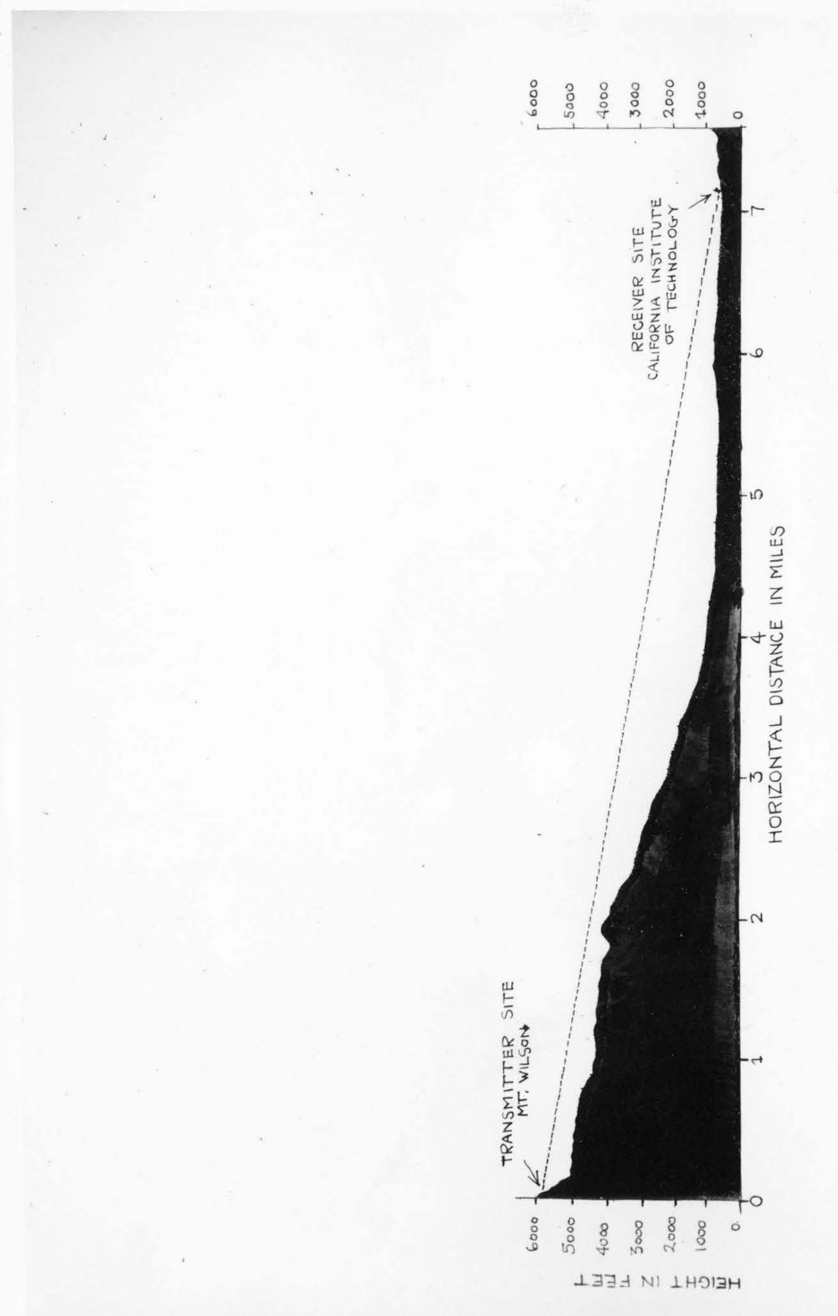


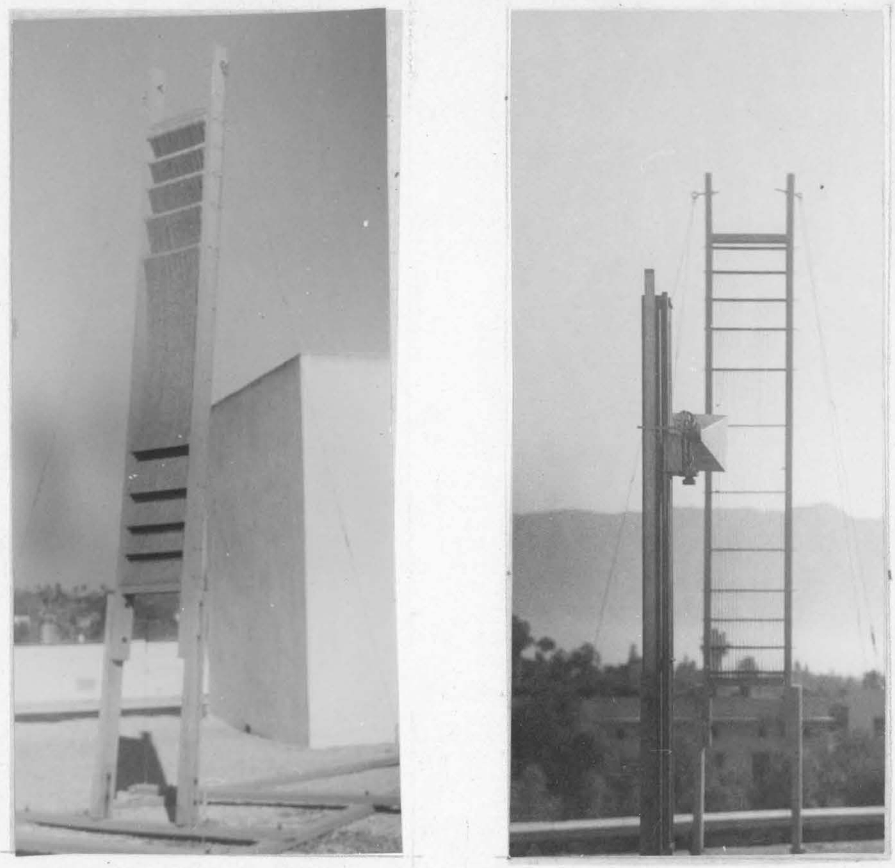
FIGURE 4. THE PROPAGATION PATH

could be read by means of a scale attached to the horn mount. The lens assembly is shown in Figures 5a and 5b.

Incoming radiation was focused by the lens, intercepted by the horn, and transmitted to the receiver by means of coaxial cable. The incoming signal was mixed with the output of a local oscillator, a reflex klystron (type 723 A/B), and the 30 megacycle beat frequency was amplified, detected, and the audio note further amplified. The audio output could be connected either to earphones or to an indicating amplifier. A circuit diagram of the receiver is shown in Fig. 6. The drawing of the IF amplifier is reproduced in part from page 465 of Radio System Engineering, volume 1 of the MIT Radiation Laboratory Series with the permission of the McGraw-Hill Book Company.

Calculations made at the time the lens was designed indicated the width between the principal nulls, when measured in the focal plane of the lens should be about 11 cm. Numerous tests showed the actual separation between them to be 13.6 cm, and that the focal length of the lens was closer to 390 cm. Tests also showed that the diffraction pattern had uniform characteristics so far as the separation of the principal minima was concerned, but the structure of the minor lobes showed some variations.

For purposes of illustration, two of the diffraction patterns are exhibited in Figures 7 and 8. When the position of the test oscillator was raised vertically by a small distance, the diffraction pattern shifted slightly



a. b.

Figure 5 The Receiving Lens

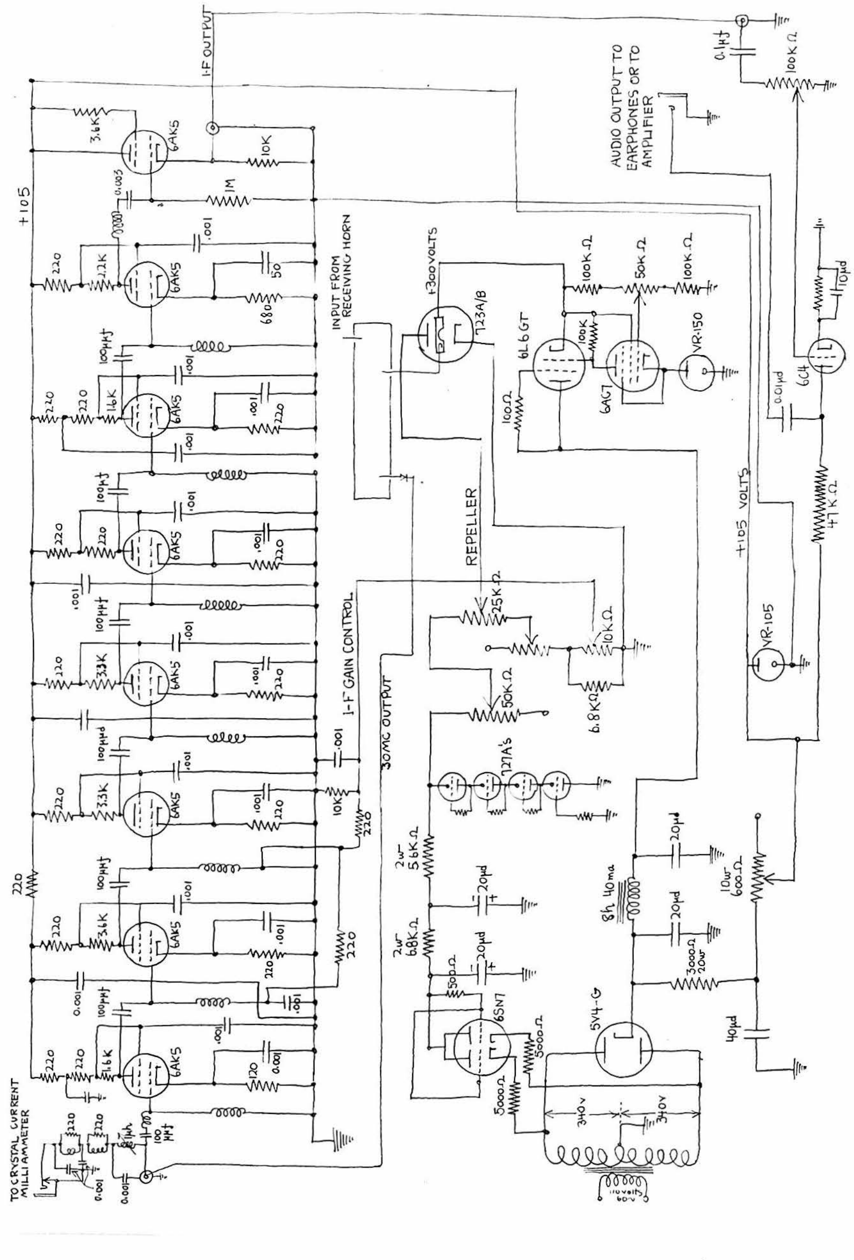


FIGURE 6, CIRCUIT DIAGRAM OF THE RECEIVER

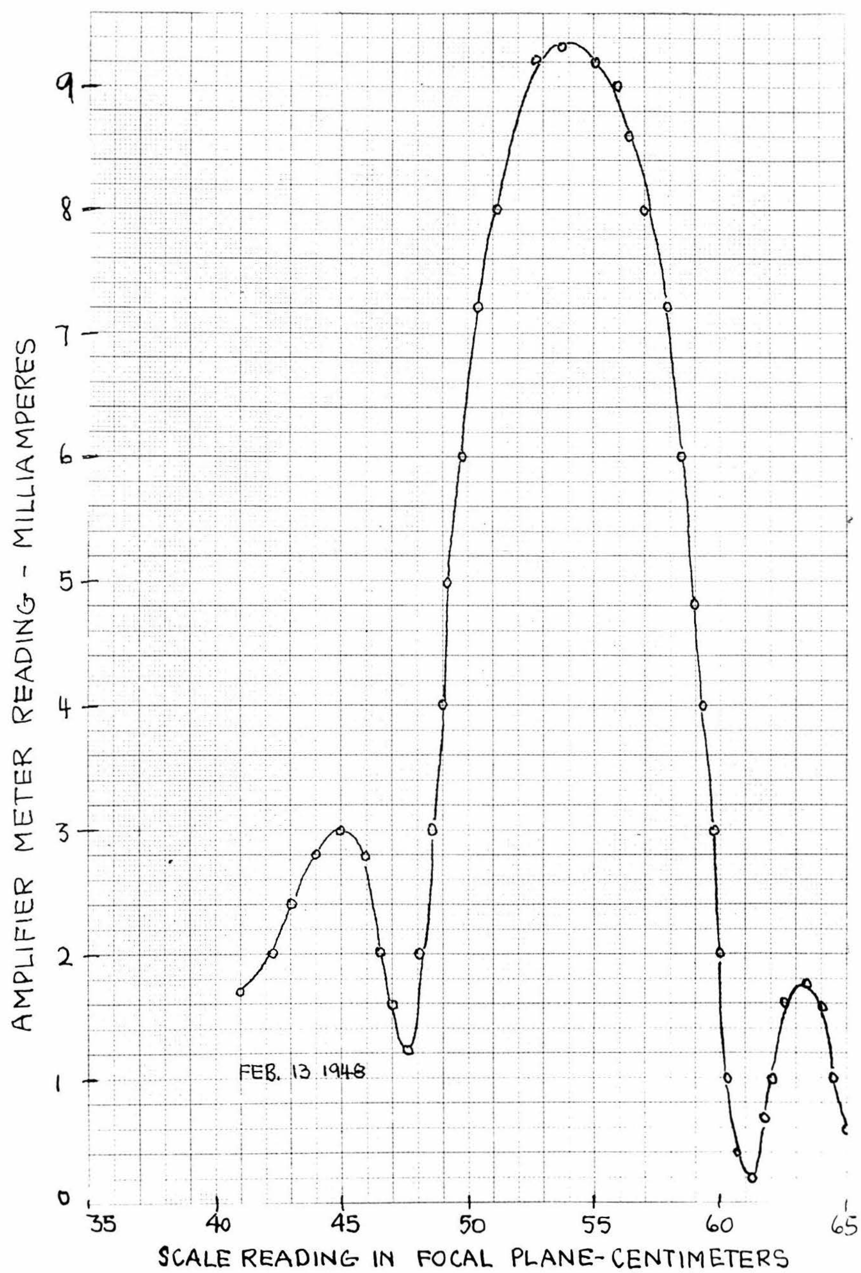


Figure 7 Diffraction Pattern of the Lens

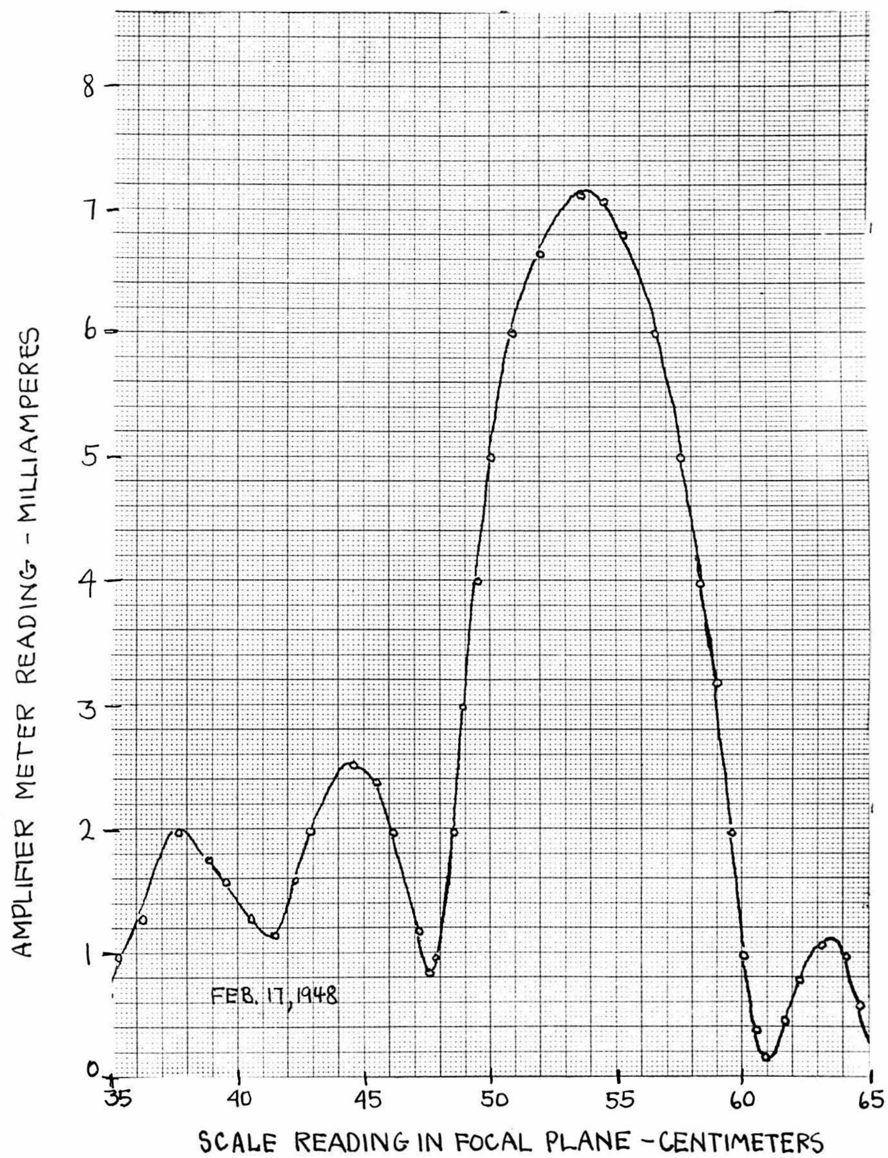


Figure 8 Diffraction Pattern of the Lens

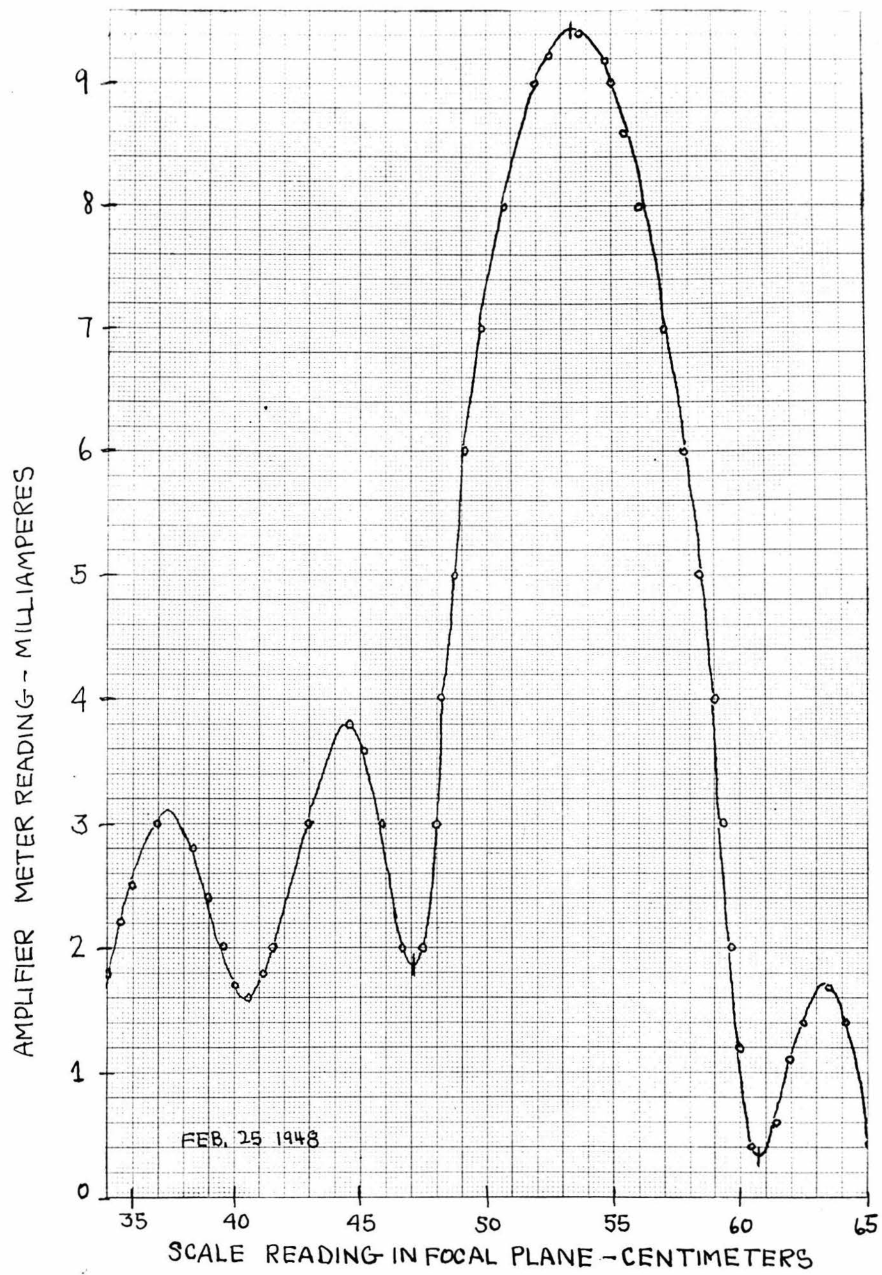


Figure 9 Diffraction Pattern of the Lens with the Transmitter Elevated

downward. This is shown in Fig. 9.

The distance between the two principal minima, subtended an angle of 2° as measured in the focal plane from the center of the lens. This is equivalent to a change of 0.015° per millimeter of distance on the vertical measuring scale.

Due possibly to a defect in the construction of the lens the first upper minimum was much sharper than the lower one. It was decided to utilize this minimum as a means of locating the position of the diffraction pattern in the focal plane. Under controlled conditions its location could be determined with great accuracy, and it was seldom that different observers would make measurements showing discrepancies of more than 2 mm in its location on the scale. This would correspond to an accuracy of 0.03°.

An effort was made to determine the center of the lens. A small transmitter was placed on the roof of the east wing of Norman Bridge Laboratory. The receiving equipment was placed on the roof of the west wing. The receiving horn was set on the maximum position of the diffraction pattern of the lens (using the amplifier meter as indicator) and the intersection of the line between the transmitting and receiving horns was marked on the lens frame.

When the lens was moved to the position to receive from Mt. Wilson, it was first tilted back to make an angle of 7.5° with the vertical. A theodolite was placed behind the lens in such a position that the axis of its telescope coincided with the straight line from the transmitting parabola through the mark on the lens frame. The point on the scale determined by the intersection of the horizontal cross-hair of the theodolite was taken as the maximum point of the diffraction pattern. This point was the 59.6 cm mark on the scale. A study of numerous lens patterns showed that the upper minimum was 7.1 cm above the maximum. The "line of sight" position of this minimum was, therefore, placed at the 66.7 cm mark on the scale.

When the lens was in position to receive from Mt. Wilson, certain difficulties became apparent. Frequent and rather violent fluctuations of the incoming signal precluded effective use of the amplifier. These fluctuations might have been due to rapid frequency drifts in the transmitting oscillator, since it was only partially shielded from the breeze, or they might have been due to atmospheric conditions, since what is known vaguely as "patchiness" of the atmosphere has been given as a probable cause of such fluctuations (50). Police calls on the 30 mc band were also bothersome.

A sensitive pair of earphones was, therefore, used to locate the null position because the fluctuations were not noticeable on them. The police calls could be separated readily from the desired audio tone. It is, in fact, well known that the ear recognizes pure tones in the presence of noise, even when the note is so weak that it can no longer be perceived on an oscilloscope. In null detection at audio frequencies earphones are also superior to meters and amplifiers, and it is believed that in the present case, this type of detection offered no serious disadvantages.

V. Method of Calculating Angle-of-Arrival

To compare the observed arrival angle with values obtained by ray tracing methods, it was necessary to fix the values of certain quantities: (1) the height of the transmitter and the receiver, (2) the horizontal distance between them, and (3) the angle formed by the local horizontal and the line-of-sight from the transmitter to the receiver. The first two quantities were determined from the official contour maps of the United States Coast and Geodetic Survey (Mt. Wilson, Altadena, and Sierra Madre quadrangles, edition of 1941, reprint of 1947). Elevation of the receiver was fixed at 790 feet, that of the transmitter at 5800 feet, both measured from sea level. The horizontal distance was found to be 7.23 miles. Slight discrepancies in the above values were noted when compared with other maps of the region. The visual angle to the transmitter was measured with a theodolite and found to be 7.5°. Because of the curvature of the earth, the local horizontals at the receiver and the transmitter are not to be considered parallel planes. The dihedral angle between these planes is about 0.1°.

To trace the ray it was necessary to determine the value of M at the location of the transmitter and the receiver and to postulate a possible M curve for the vertical region between these two points. Tables for calculating M were available (51) provided that the centigrade temperature, the relative humidity, and the height of the location at which M was desired were known. The required information to the extent it existed was made available by the United States Weather Bureau. The M values could also be found at the receiver location from a psychrometric nomogram prepared by the M.I.T. Radiation Laboratory (52). It should be stated that there were always some discrepancies in the M values determined by those two methods.

To construct the M distribution required postulating an M curve that experimental invextigation had shown to be quite typical of southern California (53). Inversions in the M curve are of the elevated type. Lower and upper segments are linear and vary at the rate for a "standard" atmosphere, that is, there is an increase of about 36 M units for each increase of 1000 feet in elevation. The inverted segment is also linear but it has a variable negative slope. The base of these inversions is often found to be between 2800 and 4200 feet above sea level. The thickness of the inversion layer is most frequently of the order of 500 feet.

After the M curve was postulated the ray associated with the observed arrival angle was traced back to the transmitter. By comparing the total distance traversed by ray, and its angle of departure from the transmitter, with known values, it was possible to check the observed angles with theoretical ones.

Temperature inversions between the receiver and transmitter were much less common than had been anticipated. In fact, such inversions occurred on only two days. On May 25 at 8 A.M. PST the temperature at Mt. Wilson was 61° F the dew point 31° F. At Pasadena the temperature was 56° F, the dew point 52°. These values give M to be 524.7 and 369.0 at Mt. Wilson and Pasadena respectively. Had the M change been "standard" its value at Mt. Wilson would be 549. A small inversion could therefore be placed at some point above the ground. A logical M curve would seem to be the one shown in Fig. 10.

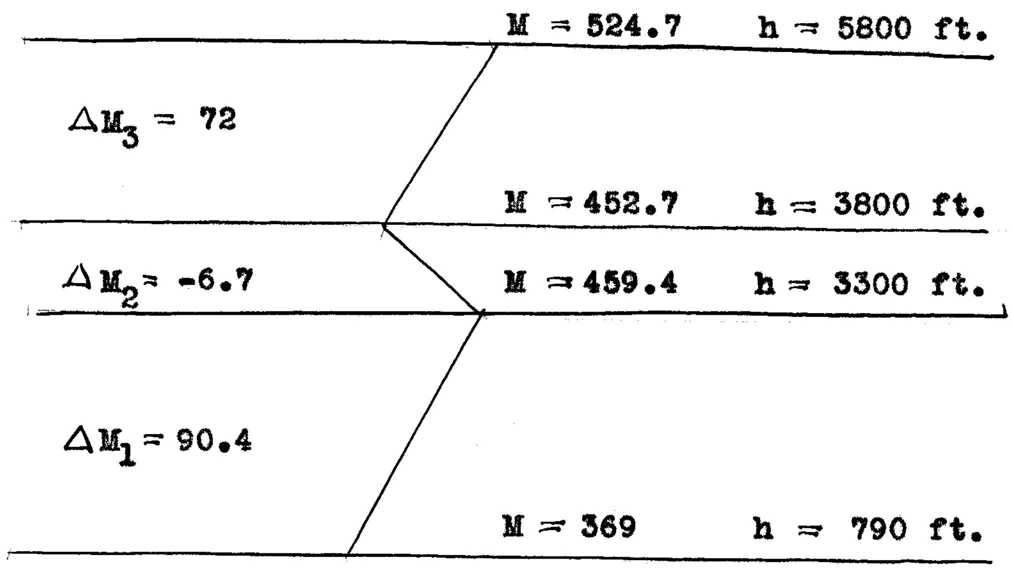


Fig. 10 Postulated M Curve 8 A.M. (PST) May 25, 1948

At 8 A.M. the observed position of the upper null read 66.3 cm. On a line of sight it would read 66.7 cm, corresponding to a depression of 0.06°. The incoming ray appeared to be arriving at an angle of 7.56° above the local horizontal at the receiver.

From 3.2 with $\alpha_0 = 7.56^\circ$ (0.13194 radians)

$$\alpha_1 = \sqrt{0.017410 + .000181} = 0.017591$$

$$\alpha_1 = 0.13263$$

By (3.3)

$$x_1 = \frac{5020}{(.2647)\ (5280)} = 3.59 \text{ miles}$$

Through the second layer

$$\alpha_2 = \sqrt{0.017591 - .000015} = \sqrt{0.017576}$$

$$\alpha_2 = 0.13258$$

$$x_2 = 0.714 \text{ mi}$$

Through the third layer

$$\alpha_3 = \sqrt{0.017576 + .000144}$$

$$\alpha_3 = \sqrt{0.017720}$$

$$\alpha_3 = 0.13312 = 7.63^\circ$$

$$x_3 = 2.86 \text{ mi}$$

$$\sum x = 3.59 + 0.71 + 2.86 = 7.16 \text{ mi}$$

It seems evident from the above that a ray arriving at 7.56° would not likely have had the transmitter as its point of origin, if the M distribution were that assumed.

Further calculations are summarized for the purpose of comparison in Table V - 1. This table shows that the inversion has slight effect upon the calculation of possible angles of arrival and that substantially the same calculated angles are obtained if the M curve is taken as a straight line connecting the two known values.

Table V - 1

Calculations of Arrival Angle

8 A.M. (PST) May 25, 1948

Type of M Curve	Assumed Arrival Angle at Receiver	Calculated Angle of Departure from Transmitter	Distance Spanned Miles
3 Linear Segments	7.56°	7.62°	7.17
Straight Line	7.56	7.63	7.16
Standard	7.56	7.64	7.17
3 Linear Segments	7.53	7.60	7.19
Straight Line	7.53	7.60	7.19
Standard	7.53	7.61	7.19
3 Linear Segments	7.52	7.59	7.21
Straight Line	7.52	7.59	7.22
Standard	7.52	7.60	7.19
3 Linear Segments	7.51	7.58	7.21
Straight Line	7.51	7.59	7.22
Standard	7.51	7.59	7.26

Computations made with variations in the form of the M curve between the two known points result in no significant change from the values of Table V - 1.

On the morning of May 25 the temperature inversion persisted. At 10 A.M. the readingsat Mt. Wilson were 65° F with a dew point of 38° F; at Pasadena the readings were

58° F and 52° F, respectively. These values yield 532.6 for M at Mt. Wilson and 364.9 for M at Pasadena. M at Mt. Wilson was still below the value of 545 based on the "standard" rate of increase. Again a theoretical angle of 7.51° was possible.

Weather observations were made at Mt. Wilson at three hour intervals, hence, the theoretical calculations could be compared with the observed values only at scattered intervals.

The results for the period of observation on May 25 are summarized in Table V - 2, page

A comparison of the actual readings shows a reasonably constant value for the arrival angle during the early period of the observations with a tendency for the angle to approach more closely the line-of-sight value as the morning passed.

Neither of the above facts is inconsistent with known meteorological data. Such variations might be expected, for, as the day progressed, conditions causing the inversion would likely disappear and the ray would approach the line-of-sight path.

The foregoing example illustrates the difficulties, discussed further in the conclusion, to be overcome in research of this type. The variations are very small, and there is a sizable discrepancy between the observed arrival angles and those calculated on the basis of a logical M distribution between two known points.

Table V - 2

Arrival Angles for the Morning of May 25, 1948

PST (AM)	Scale Reading	Departure from Line of Sight	Observed Angle of Arrival	Calculated Possible Angle of Arrival
7:10	66.3	-0.4	7.56°	
7:12	66.4	-0.3	7.54	
7:15	66.4	-0.3	7.54	
7:20	66.4	-0.3	7.54	
7:27	66.4	-0.3	7.54	
7:30	66.4	-0.3	7.54	
7:35	66.2	-0.45	7.57	
7:40	66.2	-0.45	7.57	
7:45	66.3	-0.4	7.56	
7:50	66.2	-0.5	7.57	
7:53	66.3	-0.4	7.56	
8:00	66.3	-0.4	7.56	
8:05	66.3	-0.4	7.56	7.52
8:20	66.3	-0.4	7.56	7.51
8:25	66.1	-0.6	7.59	
8:30	66.4	-0.3	7.54	
8:37	66.4	-0.3	7.54	
8:45	66.3	-0.4	7.56	
8:55	66.3	-0.4	7.56	
9:02	66.3	-0.4	7.56	
9:10	66.5	-0.2	7.53	
9:15	66.6	-0.1	7.51	
9:25	66.6	-0.1	7.51	
9:30	66.3	-0.4	7.56	
9:40	66.5	-0.2	7.53	
9:45	66.4	-0.3	7.54	
10:00	66.5	-0.2	7.53	7.51
10:05	66.6	-0.1	7.51	

Here the transmitter was turned off

VI. Further Observations

Daily observations of the angle-of-arrival, under a variety of weather conditions were made between May 5, 1948 and July 8, 1948. Weather conditions seemed to have no noticeable effect on the quality of the audio note heard through the earphones, and no difficulty was ever experienced with its reception.

On the vast majority of days, the wave front arrived at the receiver at an angle which would indicate the atmosphere had little, if any, influence upon its inclination. Two notable exceptions occurred and they will be tabulated below.

Observations made on June 2, 1948, between the hours of 8:45 and 11:00 A.M. are typical of days when the null point remained essentially at its line-of-sight location. Variations in its position were very small and might well be due either to a change in direction of the ray or to the limitations of the equipment. Results of the observation were independent of the mode of oscillation of the local oscillator.

On June 2 rain during the early hours had stopped by the time observations began. The ground and roof were still wet and a thick fog obscured the foothills. Mt. Wilson was not visible. The signal was very strong and the null was well defined. At 8:30 A.M. the temperature at Pasadena was 56° F and the dew point was 54° F. M at the receiver was calculated to be 372. At Mt. Wilson the dry bulb temperature was 42° F and the dew point was 42° F. These values gave M at Mt. Wilson to be 551. From these M values it is evident that the change in M corresponds closely to the change which would take place in a "standard" atmosphere. By 11 A.M. the M value at Pasadena had become 366 while at Mt. Wilson it had changed to 555, a difference of 189 M units. The M values are tabulated in Table VI - 1 and the observations are summarized briefly in Table VI - 2. It can be seen that the observed angles are not inconsistent with the meteorological data.

Table VI - 1

M Values for June 2, 1948

	Mt. Wilson			Pasadena		
Time (PST)	T° C	RH%	M	T° C	RH%	M
7:00 A.M.	5	100	550			
7:30				13.4	98.0	375
8:30				13.4	94.5	372
9:30				14.4	83.5	366
10:00	7.23	100	554			
10:30				15.0	78.0	366
11:30				15.0	80.0	368
12:30 P.M.				15.0	78.0	366
1:00	8.9	100	558			

Table VI - 2

Angle-of-Arrival Data June 2, 1948

Time (PST) A. M.	Position of null	Departure from line of sight	Observed Angle of Arrival	Calculated Angle of Arrival
8:45	66.6	-0.1	7.51°	7.51°
8:55	66.9	+0.2	7.47	
8:57	66.9	+0.2	7.47	
9:02	66.8	+0.1	7.49	
9:08	66.9	+0.2	7.47	
9:13	66.9	+0.2	7.47	
9:25	66.9	+0.2	7.47	
9:30	66.9	+0.2	7.47	
9:45	69.9	+0.2	7.47	
10:00	66.8	+0.1	7.49	7.49
10:27	66.8	+0.1	7.49	
10:32	66.8	+0.1	7.49	
10:35	66.8	+0.1	7.49	
10:45	66.9	+0.2	7.47	
10:58	66.8	+0.1	7.47	
11:00	66.8	+0.1	7.47	7.49

In this table, and in those to follow, a plus sign indicates that the null point is above its line-of-sight value; and a minus sign indicates that the null point is below its line-of-sight value.

Observations on June 10 and 11, 1948, were of particular interest. On each afternoon a pronounced depression in the position of the null point occurred and no clue for such a radical change could be found in the calculated M value.

On June 10 at 7:45 A.M. the sky was clear. Mt. Wilson was visible but there were isolated clouds at the level of about 4500 feet and a haze covered the valley. The M values as calculated for the day are given in Table VI - 3.

Table VI - 3

M Values for June 10, 1948

	Mt. Wilson			Pasadena		
Time (PST)	T° C	RH%	M	T° C	RH%	M
7:00 A.M.	12.8	43	529			
7:30				16.7	78	369
9:30				20.6	63	366
10:00	15.5	52	541			
10:30				24.4	49	359
11:30				24.4	49	359
1:00 P.M.	16.1	67	554			
1:30				25.0	49	361
4:00	15.5	69	549			
4:30				25.6	42	354

The above values, though scattered, tend to show that M at Mt. Wilson increased continually until about 1:00 P.M. and then decreased.

During the first few hours of observation an inversion could be placed in the M curve and a slight bending of the rays might be expected. From 10 A.M. the M variation appeared to be essentially normal; the arrival angle might be expected to be normal. This day was characterized by an intense "smog" which began to blow in about 10 A.M. The data tabulated in Table VI - 4 show the observed angles for this day, and it will be noticed that while there is a qualitative agreement with the expected results during the early morning hours, there is a violent departure from the expected results during the early afternoon.

Table VI - 4

Angle-of-Arrival Measurements June 10, 1948

Time (PST)	Scale Reading	Deviation from Line-of-sight Position	Observed Angle of Arrival
7:28 A.M.	66.6	-0.1	7.51°
7:32	66.6	-0.1	7.51
7:40	66.7	0.0	7.50
7:55	66.5	-0.2	7.53
8:00	66.7	0.0	7.50
8:05	66.6	-0.1	7.51
8:30	66.8	+0.1	7.49
8:45	66.8	+0.1	7.49
9:56	66.8	+0.1	7.49
10:00	66.9	+0.2	7.47
10:11	66.7	0.0	7.50
10:15	66.4	-0.3	7.54
10:15	66.4	-0.3	7.54
10:20	66.4	-0.3	7.54
10:30	66.6	-0.1	7.51
10:45	65.9	-0.8	7.62
10:50	66.0	-0.7	7.61
10:55*	66.3	-0.4	7.56
11:03	66.4	-0.3	7.54
11:07	66.4	-0.3	7.54
11:12	66.5	-0.2	7.53
11:20	66.7	-0.0	7.5
11:25**	66.8	+0.1	7.49
12:05 P.M.	65.9	-0.8	7.62
12:10	65.9	-0.8	7.62
1:00	65.9	-0.8	7.62
1:05	65.7	-1.0	7.65
1:10	65.7	-1.0	7.65
1:12	65.7	-1.0	7.65
1:15	65.9	-0.8	7.62
1:30	65.6	-1.1	7.66
2:00	66.0	-0.7	7.61
2:05	66.0	-0.7	7.61
2:10	66.0	-0.7	7.61
2:15	66.0	-0.7	7.61
2:20	66.1	-0.6	7.59
2:30	66.3	-0.5	7.58
2:45***	66.4	-0.3	7.54

* Smog now blowing in thick
** Smog now extremely thick
*** Transmitter turned off

A breeze was beginning to thin the smog and the null point was returning slowly to its normal value. Results of June 10 (together with the next day) mark the greatest bending of the ray observed, the magnitude being about 0.1^{o}. This is a much greater bending than any expectation on the basis of ray tracing through an M layer.

To lend support to the fact that bending to the extent indicated was possible, observations for June 11, 1948, are summarized. Again the morning was clear and quiet, with a blue sky and a haze in the valley. A dense smog drifted over Pasadena during the late morning and lifted in the mid afternoon. The tabulation of the M values appears below.

Table VI - 5

M Values for June 11, 1948

	Mt. Wilson			Pasadena		
Time (PST)	T^{o} C	RH%	M	T^{o} C	RH%	M
7:00 A.M.	13.3	58	542			
7:30				20.0	56.0	357
8:30				21.6	52.0	357
9:30				22.6	49.0	356
10:00	18.4	48.5	540			
10:30				23.8	46.0	355
11:30				25.6	45.0	358
12:30				26.6	43.6	358
1:00 P.M.	17.8	61.0	554			
1:30				26.6	43.0	357
2:30				26.0	44.0	354
3:30				26.0	40.0	352
4:00	18.9	50.0	546			
4:30				23.8	50.5	360

The above table shows that the M values of June 11 bear no noticeable similarity to the values listed in Table VI - 3.

Table VI - 6

Angle-of-Arrival Measurements June 11, 1948

Time (PST)	Scale Reading	Deviation from Line-of-sight Position	Observed Angle
7:45 A.M.	66.3	-0.4	7.56°
7:55	66.9	+0.2	7.53
7:57	66.9	+0.2	7.53
8:06	66.9	+0.2	7.53
8:07	66.8	+0.1	7.52
8:35*	66.8	+0.1	7.52
10:15**	66.6	+0.2	
10:18	66.7	0.0	7.50
10:20	66.7	0.0	7.50
10:25	66.7	0.0	7.50
10:30	66.7	0.0	7.50
10:57	66.5	-0.2	7.53
11:00	66.4	-0.3	7.54
11:02	66.5	-0.2	7.53
11:05	66.5	-0.2	7.53
11:10	65.9	-0.8	7.62
11:13	65.9	-0.8	7.62
11:15	66.0	-0.7	7.60
11:17	66.0	-0.7	7.60
11:20	66.0	-0.7	7.60
11:25***	66.1	-0.6	7.59
12:37 P.M.	65.8	-0.9	7.64
12:40	65.8	-0.9	7.64
12:45	65.9	-0.8	7.62
12:52	65.8	-0.8	7.62
1:00	65.9	-0.8	7.62
1:05	65.7	-1.0	7.65
1:07	65.8	-0.9	7.64
1:20	65.6	-1.1	7.66
2:32	65.6	-1.1	7.66
2:35	65.9	-0.8	7.62
2:45	66.0	-0.7	7.60
2:47	66.1	-0.6	7.59
2:50	65.7	-1.0	7.65
2:52	65.9	-0.8	7.62
3:00	66.0	-0.7	7.60
3:05	65.8	-0.9	7.64
3:11	65.9	-0.8	7.62
3:22	65.9	-0.8	7.62
3:50	65.8	-0.9	7.64
4:00	65.9	-0.8	7.62
4:10	66.0	-0.7	7.60

(Here the transmitter was turned off)

*At this time some measurements were made with the theodolite for the Snow Telescope was visible

** Smog is now beginning to make the mountains invisible

***By now the smog is extremely thick

Table VI - 7

Angle-of-Arrival Measurements May 27, 1948

Time (PST)	Scale Reading	Deviation from Line-of-sight Position	Observed Angle
7:20 A.M.	66.3	-0.4	7.56°
7:30	66.3	-0.4	7.56
7:35	66.2	-0.5	7.58
7:45	66.2	-0.5	7.58
7:50	66.3	-0.4	7.56
7:56	66.3	-0.4	7.56
8:00	66.4	-0.3	7.54
8:05	66.3	-0.4	7.56
8:35	66.4	-0.3	7.54
8:40	66.4	-0.3	7.54
8:50	66.3	-0.4	7.56
9:00	66.3	-0.4	7.56
9:10	66.2	-0.5	7.58
9:20	66.3	-0.4	7.56
9:30	66.3	-0.4	7.56
9:35	66.4	-0.3	7.54
9:40	66.2	-0.5	7.58
9:50	66.2	-0.5	7.58
10:00	66.3	-0.4	7.56
10:10	66.3	-0.4	7.56
10:20	66.3	-0.4	7.56
10:25	66.4	-0.3	7.54
10:36	66.3	-0.4	7.56
10:42	66.3	-0.4	7.56
10:47	66.4	-0.3	7.54
10:50	66.5	-0.2	7.53
10:55	66.4	-0.3	7.54

A comparison of Table VI - 4 with Table VI - 6 shows the general similarity of the observations on these two days. Though smog was present on other days, it seemed to cause no noticeable effect on the null position.

The measurements of May 27, 1948, were made in a drizzle of rain with a heavy fog blanket all around. The rain stopped about 10 A.M. but the fog continued to be very heavy. The value of M at the receiver throughout the morning was approximately 370, but unfortunately the weather readings from Mt. Wilson were not available. Since there appeared to be a significant departure of the null point from its line-of-sight value, the readings are displayed in Table VI - 7. (This set of readings was continually checked by Lt. E. F. Barker, USN)

VII. Conclusion

No ready explanation appears to exist for the marked refraction observed on the afternoons of June 10 and 11, 1948. The values of M recorded at these times display no characteristics which would permit the placing of an inversion in the M curve. Bending of the order of 0.1° could be accounted for qualitatively if a sharp inversion of 50 M units at an elevation of 3000 feet above sea level is assumed.

If the inversion took place within the space of 100 feet and if the M curve were standard in the lower and upper layers an arrival angle of 7.60° is feasible. M variations of this kind are not impossible in southern California (54) but appear inconsistent with the conditions observed on the above days at the receiver and transmitter.

It is also possible to postulate two regions consisting of media, with different indices of refraction, separated by a plane surface. If the upper region is dry air application of (2.2) gives n equal to 1.00025. This value is obtained by taking the surface of discontinuity at 3000 feet above sea level, $p = 898$ millibars, and $T = 12^\circ$ C.

The value of n thus obtained is in very good agreement with that usually given for dry air. Application of Snell's law indicates a ray could be bent by 0.1° if it entered a region whose refractive index was equal to 1.00046. This represents a change in $n - 1$ of 21×10^{-5} a much greater increase than any that has been recorded in the available literature. Rather sharp changes in n do occur at heights of about 1 km, but these changes appear at most to be of the order of 50×10^{-6}.

The ordinary partial pressure due to water vapor in the air is given as 1% of the total pressure (55). This would mean the vapor pressure at 3000 feet might be expected to be 9 mb.

If this value is doubled to 18 mb and an abrupt temperature inversion of 5° C is assumed application of (2.4), with $P = 915$ mb, gives $n = 1.00034$. It would seem necessary to assume beneath the dry air a layer of damp air with a partial vapor pressure of about 40 mb and an abrupt temperature inversion of 5° C to account for so large a change in n. The temperature inversion might be possible, but it is not known whether such large vapor pressures exist. It seems highly unlikely.

It is possible that the size of small spherical moisture drops suspended in the air is a factor in the value of the index of refraction of a moist air. An effort to relate the drop size to the index of refraction is, at present, only partially successful.

It is seen from the foregoing that it is difficult to account for the results of the observations on certain days. Reflection from buildings and trees and radiation scattered from nearby objects could have had some influence on the null position. Strong reflections were present in the region of the receiver. A small hand carried horn was used to probe the vicinity around the receiver and marked echoes were picked up from all directions. Because it was thought that some of this radiation might enter the side of the receiving horn, thereby changing the diffraction pattern, the lens was covered with a large sheet of aluminum. When so covered no audible tone was received. This would indicate that reflections from the side were not influencing the pattern but that the energy received was passing through the lens.

This would still leave the possibility that the pattern could be influenced by ground reflections from points between the transmitter and the receiver. However, this does not appear to be too likely. The line of sight clears by at least 400 feet all obstructions lying directly beneath it. A ridge, about 1.7 miles from the transmitter rises to a height of 4,100 feet. This ridge might be a point of reflection. It lies 3° below the line of sight and, since the angular width of the major lobe of the transmitting parabola is 6°, the radiation striking this point would not be intense. No strong reflections should, therefore, arise.

About three quarters of a mile from the transmitter, the beam passes through a depression, the left side of the depression being the slope of Mt. Harvard. The line of sight clears the ground below by about 600 feet, and the ridge along the slope of Mt. Harvard by an equal amount. It would not appear that this vicinity could be the origin of a strong reflected ray.

Furthermore, the slope is covered irregularly with trees of various heights and the rocky ground is quite rough. At a frequency of 10,000 mc and a grazing angle of 2°, an application of the Rayleigh criterion (56) for roughness shows that diffuse reflection results if the irregularities of the terrain exceed heights of the order of 7 cm. A view from the transmitter toward the receiver is shown in Figure 11.

Figure 11. View From Transmitter

So far as is known, this is the only attempt to measure atmospheric refraction from an elevated site. Two other efforts, both from low elevations, to measure arrival angles are described in the literature (57) (58).

The results tend to show that the effect of the atmosphere is ordinarily negligible but that at times effects are noted which are difficult to explain since they would involve a more abrupt reversal of the lapse rate of the index of refraction than is, at present, believed possible.

The importance of making simultaneous meteorological measurements must be emphasized, since the location of the inversions is at a height at which no information was available. Meteorological conditions are, unfortunately, beyond control and only repeated observations can indicate whether bending of the magnitude given above again occurs.

Some disadvantages of an elevated site are apparent. The path of the radiation through an inversion is too short; and the approximations made in integrating the ray equation are less valid for increasing values of the slope of the ray. Observations are, at present, being made with improved equipment and the results of these observations will, no doubt, provide further information on the atmospheric refraction of microwaves.

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