Part IV

Appendices

Appendix A

QUANTUM PHASED ARRAY PROTOCOLS

Here I describe two illustrative examples of quantum state synthesis with QPAs.

A.1 Reconfigurable beamsplitter

With two QPAs engineered to have the input and output relations of a beamsplitter unitary, a reconfigurable beamsplitter can realized for applications requiring quantum interference. The QPAs are configured to have array factors,

$$\mathrm{AF}_N^{\pm} = \frac{1}{\sqrt{2}}\left(\mathrm{AF}_N(\varphi^A) + e^{i(\delta\pm\pi)}\mathrm{AF}_N(\varphi^B)\right), \tag{A.1}$$

one with (+) and the other with (−). The fields interfere in free space to realize quantum interference with the beamsplitter unitary. A hallmark quantum interference effect is Hong-Ou-Mandel effect [1]. The Hong-Ou-Mandel effect arises from the interference of indistinguishable single photons incident to a beamsplitter.

$$\begin{aligned}|1\rangle_{\mathrm{in}_1}|1\rangle_{\mathrm{in}_2} &= \hat{a}^\dagger_{\mathrm{in}_1}\hat{a}^\dagger_{\mathrm{in}_2}|0\rangle_{\mathrm{in}_1}|0\rangle_{\mathrm{in}_2}\\ &\to \frac{1}{2}(\hat{a}^\dagger_{\mathrm{out}_1} + i\hat{a}^\dagger_{\mathrm{out}_2})(i\hat{a}^\dagger_{\mathrm{out}_1} + \hat{a}^\dagger_{\mathrm{out}_2})|0\rangle_{\mathrm{out}_1}|0\rangle_{\mathrm{out}_2}\\ &= \frac{i}{\sqrt{2}}\left(|2\rangle_{\mathrm{out}_1}|0\rangle_{\mathrm{out}_2} + |0\rangle_{\mathrm{out}_1}|2\rangle_{\mathrm{out}_2}\right).\end{aligned} \tag{A.2}$$

A.2 N00N state generation

HOM interference can be generalized to create N00N states for any $N = 2^k$. N00N states are path entangled states of the form $(|N\rangle|0\rangle + |0\rangle|N\rangle)/\sqrt{2}$ that enable quantum-enhanced phase measurements at the Heisenberg limit, with applications in quantum metrology, imaging, and lithography [2, 3, 4]. The creation operator representation of a N00N state can be factorized as,

$$\frac{1}{\sqrt{2}}(\hat{a}_1^\dagger)^N + e^{i\theta}(\hat{a}_2^\dagger)^N = \frac{1}{2}\prod_{\pm}(\hat{a}_1^\dagger)^{N/2} + e^{i\left(\frac{\theta}{2}+\pi\pm\frac{\pi}{2}\right)}(\hat{a}_2^\dagger)^{N/2}. \tag{A.3}$$

The goal is to factorize the N00N creation operator representation into products of N single creation operator superpositions,

$$\frac{1}{\sqrt{2}}\left((\hat{a}_1^\dagger)^N + e^{i\theta}(\hat{a}_2^\dagger)^N\right) = \frac{1}{2^{N/2}}\prod_{\varphi}(\hat{a}_1^\dagger + e^{i\varphi}\hat{a}_2^\dagger). \tag{A.4}$$

We can determine what the set of φ are from recursively applying Eq. A.3. For $N = 2^k$, we find:

$$\varphi = \frac{\theta}{2^k} + \sum_{j=1}^{k} \frac{\pi}{2^{k-j}} \pm \frac{\pi}{2} \pm \frac{\pi}{4} \pm \ldots \pm \frac{\pi}{2^k} \tag{A.5}$$

$$= \left(\frac{\theta}{2^k} + 2\pi - \frac{\pi}{2^{k-1}}\right) \pm \frac{\pi}{2} \pm \frac{\pi}{4} \pm \ldots \pm \frac{\pi}{2^k}. \tag{A.6}$$

Let Φ be the set of φ. Note that the k terms with $\pm$ mean that there are a total of 2^k φ's (i.e. $|\Phi| = 2^k$) as expected. Therefore, a N00N state can be created using a multimode interferometer with N inputs and 2 outputs, where the each input sends a single photon into $\frac{1}{\sqrt{2}}(a^\dagger_{\text{out}_1} + e^{i\phi}\hat{a}^\dagger_{\text{out}_2})$ for each $\phi \in \Phi$.

This protocol can be flexibly performed this protocol[1] using N QPA's, where each QPA sends a single photon into $\frac{1}{\sqrt{2}}(\hat{a}^\dagger_{\theta_1} + e^{i\phi}\hat{a}^\dagger_{\theta_2})$, and then align the θ_1, θ_2's of all the QPA's on the detector plane.

References

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[1] This protocol was independently derived by Ref. [5]. Here I considered the even case of $N = 2^k$.