Appendix F

TRAVERSABLE WORMHOLE SIGNATURES

To determine if the sparse learned Hamiltonian describes gravitational physics, we examine the Hamiltonian in Eq. 14.7,

$$\begin{aligned}\hat{H}_{L,R} = &-0.36\hat{\psi}^1\hat{\psi}^2\hat{\psi}^4\hat{\psi}^5 + 0.19\hat{\psi}^1\hat{\psi}^3\hat{\psi}^4\hat{\psi}^7 \\ &-0.71\hat{\psi}^1\hat{\psi}^3\hat{\psi}^5\hat{\psi}^6 + 0.22\hat{\psi}^2\hat{\psi}^3\hat{\psi}^4\hat{\psi}^6 \\ &+0.49\hat{\psi}^2\hat{\psi}^3\hat{\psi}^5\hat{\psi}^7,\end{aligned} \tag{F.1}$$

via two orthogonal approaches: first, we verify that it replicates relevant dynamics of the dense SYK Hamiltonian; and secondly, we evaluate if it satisfies necessary criteria of general holographic systems. These criteria are stricter than the similarity of dynamical observables: they include perfect size winding — the strongest form of size winding, which is sufficient to provide a geometric interpretation [1, 2, 3] — the causal time-ordering of teleported signals, which shows that the teleportation is not occurring due to random scrambling, and a time delay predicted by scattering in the bulk.

The learned Hamiltonian is consistent with gravitational dynamics of the dense SYK Hamiltonian beyond its training data. The mutual information $I_{PT}(t_1)$ for fixed t_0 shows behavior compatible with a qubit emerging from a traversable wormhole (Fig. 14.3a). The mutual information peak height and position strongly resemble the large-N SYK model computation in the double-scaled limit (Fig. F.1a). In the high-temperature limit, the mutual information asymmetry between couplings with $\mu < 0$ and $\mu > 0$ diminishes, corresponding to teleportation occurring via scrambling instead of through the wormhole, consistent with theoretical expectations [4]. Additionally, the learned Hamiltonian scrambles and thermalizes similarly to the original SYK model as characterized by the four-point and two-point correlators $-\langle[\hat{\psi}(0), \hat{\psi}(t)]^2\rangle$ and $\langle\hat{\psi}(0)\hat{\psi}(t)\rangle$ (Fig. F.1b). Since the scrambling time is approximately equal to the thermalization time, the gravitational interpretation suggests the boundary lies near the horizon.

Beyond comparison to the dense SYK model, we proceed to evaluate more general behavior predicted from gravity. The property of "perfect" size winding provides a necessary and sufficient "litmus test" to identify traversable wormhole behavior,

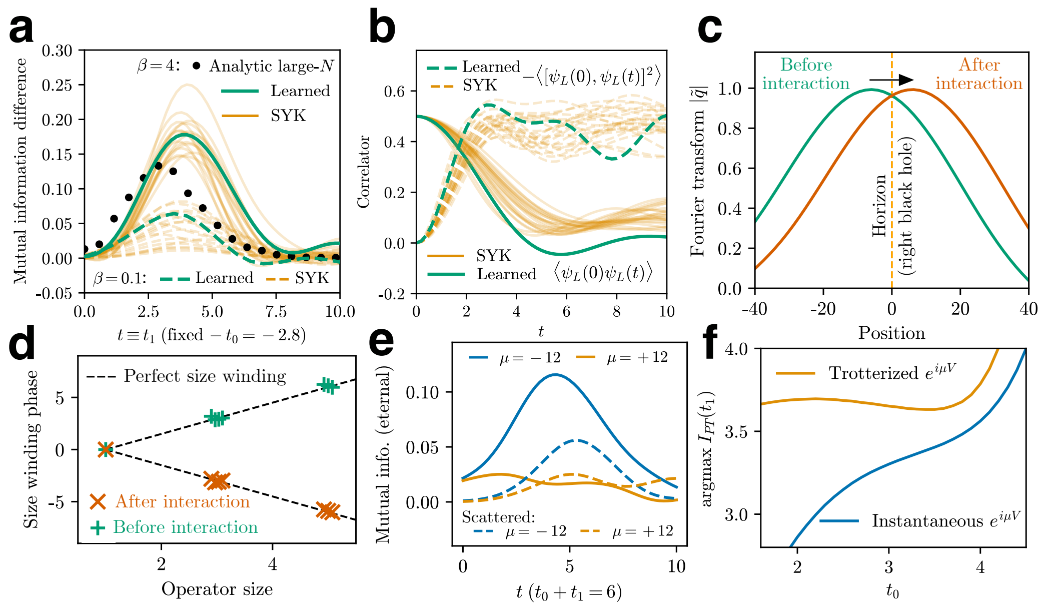

Figure F.1: Signatures of traversable wormhole dynamics for the learned sparse SYK Hamiltonian (Eq. F.1). a) Mutual information asymmetry $I_{\mu<0}(t) - I_{\mu>0}(t)$ for the learned (green) and SYK Hamiltonians (orange) at the low-temperature gravitational limit (solid) and high-temperature scrambling limit (dashed). An analytic computation in the large-N limit of the SYK model using chord diagrams (black) is shown for low temperatures, showing agreement with the peak position and height. b) Two-point function (solid) and four-point function (dashed), indicating thermalization time and scrambling time, respectively, of the SYK (orange) and learned (green) Hamiltonians. c) Bulk location of the infalling particle before and after the interaction with respect to the black hole horizon, as given by the Fourier transform $|\tilde{q}|$ of the winding size distribution. d) Perfect size winding before (green) and after (brown) the interaction; data at each operator size is horizontally staggered to make the different values visually distinct. The black dashed lines show a linear fit ($R^2 = 0.999$) with equal but opposite slopes, corresponding to the reversal of winding direction after the interaction. e) Shapiro time delay in the eternal traversable wormhole protocol caused by scattering in the bulk. The peak shifts right when an additional qubit is sent through the wormhole in the opposite direction (dashed) compared to sending a single qubit from left to right (solid). f) Causally time-ordered teleportation. The position of the mutual information peak is shown for an instantaneous at $t = 0$ (blue) and prolonged (orange) interaction over $t \in [-1.6, 1.6]$.

holding for quantum systems with a nearly AdS_2 bulk [1, 2, 3]. Perfect size winding is equivalent to a maximal Lyapunov exponent at large N, but unlike the Lyapunov exponent, size winding remains a meaningful quantity at small N. Non-gravitational systems, such as random non-local Hamiltonians, may teleport in the low-temperature limit with a weak asymmetry in μ; unlike gravitational systems, these have "imperfect" size winding. Systems that teleport in the high-temperature fully scrambled regime, such as random circuits [5] or chaotic spin chains, do not exhibit any size winding.

Given the thermal state $\rho_\beta \propto e^{-\beta \hat{H}_L}$, size winding describes the decomposition $\rho_\beta^{1/2} \hat{\psi}_L^1(t) = \sum_P c_P(t) \hat{\psi}_L^P$ over strings of $|P|$ fermions. The system exhibits perfect size winding at time t if the c_P^2 coefficients have a phase that linearly depends on $|P|$. For the Hamiltonian in Eq. F.1, an injected fermion is supported by operators of three sizes. We find that the learned Hamiltonian exemplifies perfect size winding (Fig. F.1c, d) at the time of teleportation, with the phases of the eight nonzero coefficients forming a line with $R^2 = 0.999$. This analysis shows that teleportation under the learned Hamiltonian is caused by the "teleportation by size" mechanism, not by scrambling or other non-gravitational dynamics. We visualize the resulting geometric interpretation of the learned Hamiltonian by taking the Fourier transform to obtain the bulk location of the infalling particle relative to the horizon.

The Hamiltonian is shown to adhere to the microscopic mechanism of wormhole teleportation via its perfect size winding description. To observe this at a macroscopic scale, we examine two phenomena: a Shapiro time delay and causal time-ordering of signals. For the time delay, we interrogate the learned Hamiltonian within the eternal traversable wormhole framework [6]. Besides sending a single qubit from left to right, we insert an additional qubit across the wormhole from right to left. From a gravitational perspective, this should cause the left-to-right signal to arrive later due to scattering in the bulk. We observe this in the learned Hamiltonian (Fig. F.1e). For causal time-ordering, we inspect the order in which infalling particles emerge from the wormhole. If a geometric interpretation is valid, infalling particles should arrive in a causally consistent order (Fig. 14.1b): signals must emerge in the same order they enter (time-ordered teleportation). In contrast, teleportation in the fully scrambled regime produces a time-inverted ordering of signals. Our learned Hamiltonian generates time-ordered teleportation (Fig. F.1f). The position of the mutual information peak is shown for an instantaneous at $t = 0$ (blue) and prolonged (orange) interaction over $t \in [-1.6, 1.6]$. A positive slope indicates time-inverted teleportation and a negative slope indicates time-ordered teleportation. When the coupling is applied over a window of time, the time-ordering of signals confirms through-the-wormhole behavior. When the coupling is instantaneous, the decreased slope suggests a combination of teleportation by scrambling and by traversing the wormhole.

The above analyses demonstrate gravitational teleportation by the learned Hamiltonian via an emergent wormhole; additional analyses examining spectral characteristics, dynamics at different temperatures and interaction strengths, and further properties of size winding are provided in the Supplementary Information of Ref. [7].

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