TWO ESSAYS ON THE ECONOMICS OF ELECTRICITY SUPPLY:

1. HAS THE AVERCH-JOHNSON EFFECT BEEN EMPIRICALLY VERIFIED?

2. ELECTRICITY PRICING

Thesis by

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ABSTRACT

This thesis reports on investigations in two areas of the economics of electricity supply. The first chapter examines empirical evidence to determine whether rate of return regulation has produced detectable overcapitalisation in this industry. The second chapter studies the determination of optimal pricing mechanisms for electricity, particularly in the presence of uncertainty.

Chapter 1:

Three studies which claim to confirm, and one which claims to reject, the existence of the Averch-Johnson effect in the electric power industry, have recently been published. This paper examines the general problem of what the nature of the A-J effect might be and what sort of data would be required in order to confirm its presence. The other studies are then critically examined on the basis of this discussion. A modification of the method used in one previous study is then used to test a suitably restricted form of the A-J hypothesis, and no evidence of capital bias is found. The principal conclusion of this study is that if the A-J effect is significant in distorting input choices in the electric utility industry, very different sorts of data than those that have been used thus far are going to be required in order to verify its presence. Mechanical usage of gross input and output numbers, without understanding

of the technological processes involved, leads only to erroneous conclusions.

Chapter 2:

In this chapter we examine the issues involved in setting electricity prices. The existing literature on peak load pricing and pricing under uncertainty is reviewed. Recent technological developments applicable to metering and load management of electricity are examined. The pricing problem is then reformulated so that its solution may take advantage of these innovations. New technologies such as wind power promise to be economically attractive within this new framework. This formulation of the problem suggests a method of investment planning which would better distribute risk. In addition it provides a way in which the generation sector of the industry can be made competitive, thus reducing the need for regulation.

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Chapter 1

HAS THE AVERCH-JOHNSON EFFECT BEEN EMPIRICALLY VERIFIED?

I. INTRODUCTION

A. Electricity and the Averch-Johnson Effect

The Averch-Johnson model is a simple one, and before any tests of its empirical validity are possible it is necessary to reconcile the realities of the particular industry with the assumptions of the model. The model in its basic form predicts that a profit maximizing firm will substitute capital inputs for fuel (labor, maintenance) inputs when subjected to effective rate of return regulation. This substitution will be such that for a given level of output the capital-fuel ratio will be greater than that chosen by an unregulated cost minimizer. There have been four attempts to empirically study this hypothesis as it applies to the regulated electric utility industry in the United States.

All of these papers fail to relate the models they use to the particular features of the electric power industry. This is surprising since both Courville and Petersen cite Galatin (7) whose analysis of the technology of the industry is sound.

This current work will examine this question in detail as well as exploring some previously untouched areas of the more general econometric investigations of this industry. Correct representation of the technological possibilities is required before any tests of actual input choices can be made. This requires a precise understanding of the processes involved and the measurement of the outputs and inputs. The apparent homogeneity of kilowatt-hours has perhaps been the main reason other investigators have used incorrect or incomplete formulations of the production relationship. While this is the major reason that these other studies have produced spurious results, a number of other issues, such as taxation and fuel choice, are also of importance.

Whether the actual A-J model is in any way a realistic enough representation of the regulatory process to justify empirical investigation is certainly an open question. This study avoids such motivational issues by limiting itself to the correction of previously accepted results. The difficulties involved in this task seem to indicate that further empirical work on this particular narrow area of electricity regulation would be misdirected effort.

B. Validity of Neoclassical Production Relations

Regulatory commissions require that instantaneous demand be met and in general a single price for energy is charged. Since the time-varying demand generally cannot be met by changes in inventory, this requirement amounts to a constraint specifying the time-path of energy production. Thus, two plants may supply the same energy but, if they are faced with different time-paths of demand, they will generally choose different input combinations, even if they face the same prices. Fortunately, it is possible to identify the way in which the attributes of capital equipment contribute to energy production and satisfaction of the constraint. The use of either total cost or capacity as the measure of capital employed completely obscures this.

A useful way of viewing this problem is to consider production through time as a multiproduct operation. The first product is annual energy, the output of which is determined by price. The second product is the continuity of instantaneous power supply, the output of which is determined by regulatory decree. These two products have joint costs, but they are not pure joint products. While both are produced using the same capital inputs, the different attributes of the capital that each requires are distinguishable. To fashion a production function for annual energy output simply from the factors of production without considering the constraints (which are not necessarily the same for each utility) that are operating on the other output is clearly wrong. Using a single measure of the capital involved, without a clear description of how it contributes to the output being considered, leads to erroneous results.

The single most important objection to these other studies is that they neglect to take into account one of the basic assumptions which is made when production (or cost) functions are used to represent technological possibilities. This is the assumption that engineering suboptimizations have taken place so that the function gives the maximum output attainable with any given inputs. The use of annual energy as the output and either total plant cost or capacity as the measure of capital contradicts this assumption for the reasons given in the first paragraph. Failure to take account of the other active constraints invalidates any further optimization procedures using the production function, such as the derivation of the A-J hypothesis.

C. Characterization of Generating Equipment

This section attempts to find more suitable measures of the capital input and an alternative production relationship. A brief description of the engineering processes will be given and the points at which choices are made that have an economic significance will be pointed out. The generation of electricity by the combustion of fossil fuels is a well-understood technology. The details of the technology will be avoided and the discussion limited to how usefully to describe the technological possibilities.

The equipment used can be characterized by two main attributes:

1. The maximum power output it will produce without risk of catastrophic failure (e.g., the point at which the short-run cost curve becomes vertical); i.e., the capacity.
2. The efficiency of its operation, i.e., the ratio of energy output to energy input.

The capacity of the equipment is measured in kilowatts (kw) and, unfortunately, does not have a completely unambiguous definition. The nameplate rating of the equipment, which is often used as the measure of capacity, is usually the a priori design specification of the equipment. The actual peak output is also often used as a capacity measure, but this is usually only sustainable for short periods. The peak output rating is almost always larger than the nameplate rating, and it is common practice for the capacity rating to be increased after a plant has been in service for some time.

Conservatism in the original rating of design capacity is possibly a response to the penalties in contracts that are enforced when specifications are not achieved. The capacity of equipment is determined by, among other things, its size, the strength of materials used in its construction, and the quality of its cooling system.

The quality or efficiency of the capital equipment is expressible as the amount of fuel required to produce a particular time integral of power output. The input of fossil fuel can be measured in BTU/time (flow of energy) and, apart from minor differences, such as moisture content, the heat energy made available in the boiler is independent of fuel type. The output is usually measured in kw's, which are merely different units of energy flow more suited to electricity.[1] One gross measure of efficiency is the heat rate of the plant, which is the amount of fuel energy in BTU that is required to produce one kwh of electrical energy (a power of 1 kw for an hour). Steam stations range in annual average values of this parameter from 8,000 to 14,000.

The efficiency of equipment can be increased by several possible modifications to the thermal cycle, all of which are costly. The maximum possible efficiency is theoretically limited by the maximum temperature and pressure which can be achieved, and the conditions of the heat sink. The temperature and pressure are limited by the quality of the metal boiler tubing used, and the heat sink usually by the temperature and quantity of cooling water available.

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1. 3412 BTU/hr = kw.

The preceding discussion suggests a fairly simple way of characterizing capital equipment by the capacity and efficiency of a unit. There are, however, additional decisions involved with the purchase of equipment that have a bearing on the A-J effect. First, we will look at the choice of capacity and efficiency to see what form the A-J effect would take in these decisions. The other decisions on fuel type, location and whether the plant should have been replaced with purchased power are then examined. From this discussion it is clear that only very limited types of A-J effects can hope to be detected with the typical data used.

D. Capacity Choice

The choice of what capacity an electric generating unit should have is influenced by a number of factors -- the dominant one being the regulatory requirement that instantaneous demand be met at all times. Because a single price is charged through time, and storage of electricity is prohibitively expensive, generated power varies significantly with time. The installed capacity clearly has to be at least equal to the maximum power demand. In fact, it has to be significantly greater than this in order that periodic maintenance can be undertaken as well as providing a reserve in case of equipment failure. There are a variety of ways that have evolved for economically handling this demand variation (with a fixed price and a regulatory stipulation that demand be met with high reliability). Those utilities lucky enough to have hydroelectric power available generally make use of the flexibility it allows to handle peaks. Some others, where geography is suitable, construct pumped storage facilities for peak service. In general, the trend has been towards installing low capital cost but high running cost gas turbine units specifically for peaks and interconnecting with another system whose peaks occur at other times. In small systems it is still common to use general purpose "cycling" steam plants which operate over a very large range of output.

A related problem is caused by the uncertainty in demand and the random nature of a particular unit's availability. The amount and composition of the reserve capacity required is a difficult engineering question. Rules such as fifteen percent of peak load, the sum of the two largest units in the system, etc., have all been used. With the advances in computer capability and the accumulation of data, more refined methods are now used. From an economic point of view they are still rather arbitrary as the objective is now to reduce the probability of having to shed load below some arbitrary amount. This probability level is not actually set by the regulators, but is discussed by them. An informal agreement apparently is reached which at least does not appear to cause much vocal consumer dissatisfaction. The capital inputs to maintain an adequate or excessive reliability of supply can at worst be viewed as gold plating. In general, they cause no substitution for fuel in the production of energy output; overcapacity is not a manifestation of the A-J effect.

E. Choice of Thermodynamic Efficiency

The choice of efficiency is determined by the expected cost of fuel; when fuel is expensive, more efficient capital equipment is justified for a given energy output. If the unit is going to run at a high load factor (near capacity for much of the time) a higher efficiency is justified than in a plant which runs at a low load factor. As there are increasing returns in efficiency with unit size, the choice of efficiency is connected with the choice of unit size.[2] We must, therefore, conclude that before any predictions of the effect of regulation on unit size can be made, complete engineering information on the system must be examined.

F. A-J Choices

Two main points emerge from this recitation of the technology of electricity production. First, as long as the capacity constraint is not active for a large proportion of the time, capacity does not influence the total energy produced. Second, the thermodynamic efficiency of the capital equipment is the input that can be substituted for fuel in the production of energy.

The empirical work has considered data at two levels, the plant and the firm. All authors place much more confidence in the plant data than the firm data. At the plant level the points above concerning the contributions of capacity and efficiency are strictly

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2. Peck (14) has covered some of these problems, especially those of investment timing.

true. It is only if excess efficiency is built into a plant (in relation to the energy output produced) that the A-J effect can be detected. The size of the units and the number of units in a plant must be considered as externally determined, as the decisions regarding these are based on considerably more information about the rest of the system, including the transmission network, than is normally available. They are of the same nature as questions such as whether the plant was built in the correct location, whether the plant was strictly necessary and should perhaps be replaced by a transmission line for delivery of purchased power. There is possibility in all of these decisions for A-J effects, but to detect them requires knowledge of the particular alternatives available to the firm. For example, a "mine-mouth" coal burning plant could be evidence of the A-J effect since the coal-handling equipment and transmission lines expand the rate base and lower fuel costs. A mine-mouth plant could, of course, also be the least expensive way of delivering energy to a particular locality, so that it is impossible to generalize about such choices.

At the plant level, the appropriate cost-minimizing marginal condition that can be tested is whether the cost of improving thermodynamic efficiency equals the suitably discounted annual savings in fuel cost over the life of the plant. As data on instantaneous output and the transformation curve are not available, the best we can hope to measure is whether the annual average heat rate chosen was correct in relation to the price of fuel and capital for the actual annual energy output produced. There is a problem even with this approach, for after the plant is constructed it is obviously in the firm's interest to adopt the cost-minimizing operating procedure. This means that generating sets will be loaded so as to equalize the marginal generating costs (adjusted for transmission losses). If the newer units are of higher efficiency than the system average, the effect will be to load these more heavily and unload some older, less efficient plants. Thus, unless the overall system efficiency is too great, we will not detect overcapitalization by observing the new plants, but instead should be examining the lightly loaded or retired old plants. Even this is hazardous as the planned life of equipment is dependent on expectations regarding technological change. Pessimism in this respect that is revealed only by retrospective analysis is not particularly strong evidence of the A-J effect. By examining new plants we are unlikely to observe overcapitalization unless it is of large magnitude and spread over the whole system. Detection at the firm level is even more difficult, considering the problem of determining what is the correct amount of reserve capacity and hence which old plants are correctly included.

Choice of fuel is a potential source of the A-J effect, as coal requires greater investment in structures and equipment than either gas or oil. But to determine whether coal was chosen in order to get the extra equipment into the rate base, or because there was a shortage of gas not reflected in its price, requires that the availability and expected future costs of the alternative fuels be known.

The choice between purchased power (usually requiring investment in transmission and switching facilities), or a new plant, is dependent on the availability and cost of purchasable power.

All four empirical studies have used the FPC reports (6) as their basic source of data. These plant data are limited to output, amount of fuel and its price, and the total cost of the plant. From these data the only meaningful test of overcapitalization is to specifically test whether the efficiency of the plant is too great in relation to its output and the prices it faces. An attempt to do this is reported in a later section.

G. A Practical Production Relationship

For almost all boiler-generator units in use in the United States the instantaneous thermodynamic efficiency depends on the level of power output. Typical input-output curves are shown in figure I-1.[3] The method of constructing both the boiler and the turbines contributes to the nonlinearity of this relationship. In the United States, a multi-valve turbine is typical while, for example, Britain typically uses single-valve turbines. The difference is important, since the choice of turbine affects the cost minimizing operating procedures. The almost linear transformation curves of single valve turbines mean that, when run as part of a system, generating sets are fully loaded or not loaded at all. This is so-called merit-order loading, and requires a "strong" transmission system, usually a feature of geographically compact

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3. Reproduced from (16).

STATION PERFORMANCE AND OPERATION CHARACTERISTICS

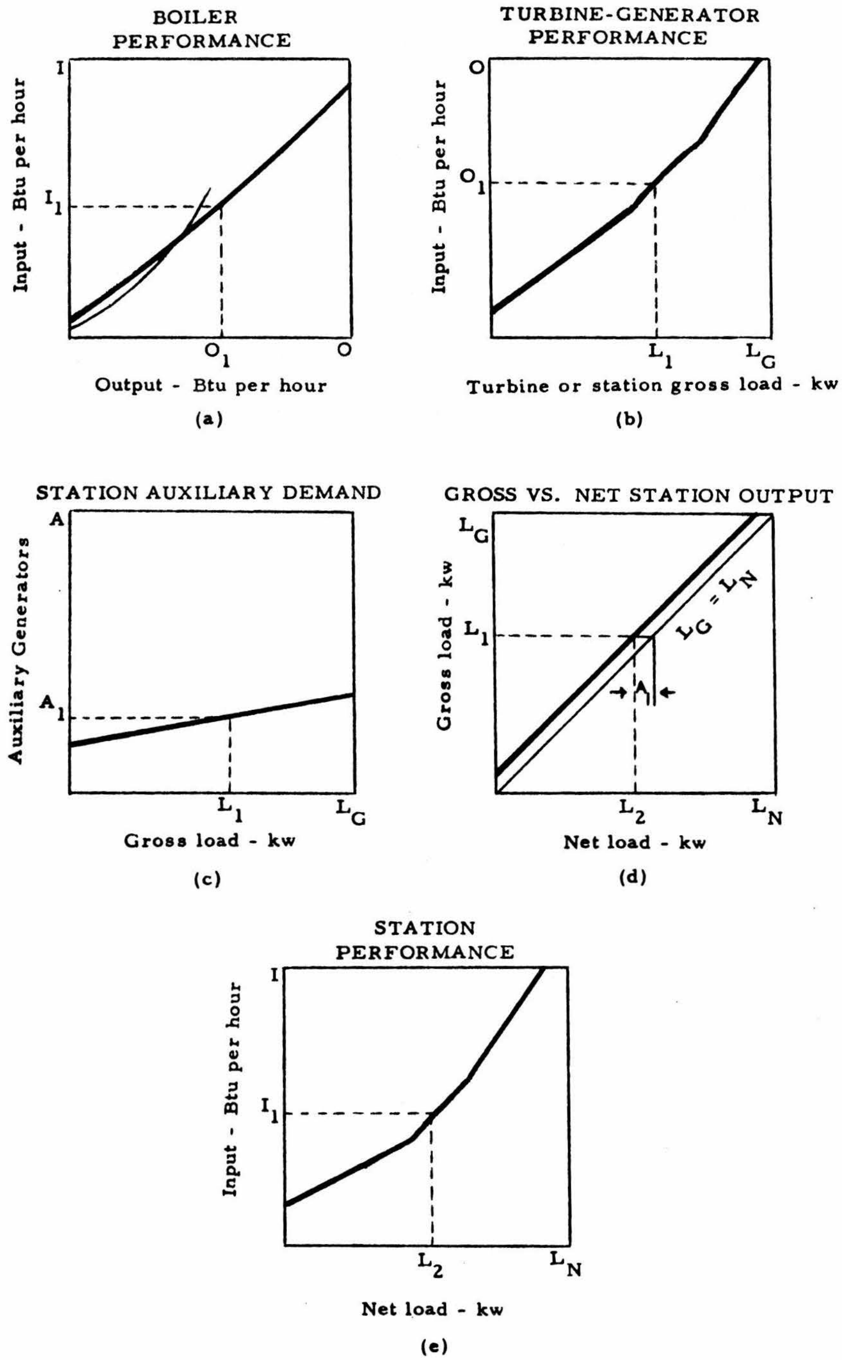

Figure I-1. Input-output curves of component station equipment and derivation of station input-output curve.

systems. The cost-minimizing operating conditions for American utilities that employ multi-valve turbines are then to partially load most sets by equating all marginal costs (adjusted for transmission losses).[4] This is a much more complex procedure and, unfortunately, introduces some extra difficulties for formulating production functions. Because of the curvature in the transformation curve, average heat rate will depend crucially on the time path of output. To illustrate,

$$\text{average heat rate} = \frac{\int_0^T f(t)dt}{\int_0^T q(t)dt}$$

where f and q are the instantaneous inputs and outputs, respectively. Hence, annual observations on energy input and output do not allow us to estimate the underlying transformation curve. The parameters of this curve are sufficient for most purposes to characterize a particular unit. This function is essentially what Nordin (3) was estimating and, as Galatin points out, he appears to be the only author of the econometric studies to appreciate fully the instantaneous nature of the output.

While we have just described a way in which the capital equipment can be fully characterized, for economic decisions the

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4. This is one possible criticism of Galatin's (7) rather thorough study, as he assumes merit order loading rather than the equating of marginal operating costs (a very difficult task) as his rule for partially loading a plant.

cost as a function of these characteristics is required.[5] If the data on the transformation curve were available (it is known to the operating engineers), a cost function could be estimated.

Unfortunately it would not be of any immediate use as far as testing the A-J effect unless instantaneous fuel and output information were also available. We will thus consider the validity and uses of annual heat rate as a measure of the quality of the equipment.

First we will consider the magnitude of actual output variation to be expected from the base load plants which form the sample used later. The maximum and minimum annual load factors occurring here are 99% and 58%. This minimum value probably exaggerates the deviation that occurs in actual output when the plant is producing any output (i.e., not shut down for maintenance and repairs).

We can thus deduce (see (16) chapter 32) some bounds on the transformation function by assuming it to be concave. The annual heat rate is clearly equal to the marginal heat rate occurring somewhere in the operating region. Whether this chord is in fact a good approximation to the curve in this region depends on the curvature of the transformation function. A highly concave function will mean that from year to year the annual heat rate will show large

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5. This is rather an overstatement, as other qualities are needed for engineering purposes. Features such as power factor, behavior to electrical and mechanical transients and maintenance requirements, etc., are all important but can safely be neglected for the purposes of this study.

variations with the year to year output variations. An examination of the heat rates and outputs for this sample over a period of several years leads to the conclusion that annual heat rate is a relatively stable measure of the plant's normal thermodynamic efficiency.

We can thus write the production relationship as

annual energy output = fuel used / heat rate

and can expect this to produce consistent results as long as the plant is operated within its design output range.

II. THE AVERCH-JOHNSON MODEL AND TAXATION EFFECTS

The distorting effects of both rate of return regulation and taxation are well known to economists. It is therefore somewhat surprising to find a total preoccupation with the Averch-Johnson effect in the literature dealing with public utilities. The proposition that a profit maximizing firm will tend to overcapitalize when subject to effective rate of return regulation has been rigorously explored -- the most complete treatments being Baumol and Klevorick (2) and McNicol (10). The effects of property taxation and corporate income taxation have not been addressed apart from Spann (18)(19) who does not appear to appreciate the implications of his partial result. The purpose of this section is to show that the central result of the A-J literature concerning inefficient input proportions no longer holds in general and depends on the relative magnitudes of the tax rates and the allowed rate of return. The traditional approach has been to contrast the choices made under rate of return legislation with those made by a cost minimizer. The inclusion of taxation brings the relevance of this comparison into question. Taxation would be barely worth considering if it were neutral among industries in its effect upon the apparent price of capital. The major problem caused would be a savings-consumption misallocation, but this is hardly the realm of regulatory economics. There is evidence, however, that it is not neutral; utilities face higher property tax rates than industry in general. During the period studied, the investment tax credit was also discriminatory concerning the electric utility industry, being 3.5% compared with 7% for industry in general. The differential effect of corporate income taxation is not clear cut, as it depends on the relative risk characteristics of the industries.

Neglecting the effect of taxation is understandable in that it simplifies the presentation of the theoretical arguments. To continue this neglect when attempting quantitative evaluation of the A-J effect clearly makes the results of such investigations suspect. Of the recent papers (3)(4)(15)(18) which have attempted empirical investigation, only Spann includes a limited consideration of income taxation. A clear distinction must be made between detecting relative distortion due to rate of return regulation and absolute input inefficiency. In fact, rate of return regulation could conceivably improve the firm's choice of inputs if taxation had induced a bias in favor of labor. As it seems that it is the possible welfare losses due to inefficient input combinations which has motivated this empirical work, this distinction appears to be worth exploring.

The traditional A-J model is reformulated to include taxation, and the first order conditions are derived and it is confirmed that the lemma bounding the Lagrange multiplier is not changed. Hence we demonstrate that it is no longer possible to sign the bias (if any) in input choices. For completeness the principal comparative statics results are derived in Appendix E and shown to be unaffected by the inclusion of taxation.

A. Model

The model used is basically that of Baumol and Klevorick (2) with corporate income tax and property tax included. To handle this complication it is necessary to refine the price of capital by using an interest rate for the debt portion and an opportunity cost for the equity portion of the capital. This is because interest charges are an allowable cost from an income tax point of view.

The symbols used are defined as:-

R = revenue function (assume diminishing revenue returns to labor)
K = capital input
L = labor input
i = interest rate on debt
r = opportunity cost of equity
b = ratio of debt to total capital (<1)
q = $ib+r(1-b)$ the average cost of capital
s = allowed rate of return ($s>q$)
w = wage rate
a = depreciation (or maintenance charge on capital)
p = property tax rate
c = corporate income tax rate (<1)

The subscripts k and l denote partial derivatives with respect to capital and labor, respectively. The subscripts c and m, which are introduced later in this section, have a different meaning, which is defined then.

We wish to maximize after tax profits

$$\text{Profit} = [R-wL-(p+a+ib)K][1-c]-r[1-b]K$$

subject to the rate of return constraint

$$[R-wL-(p+a)K]-c[R-wL-(p+a+ib)K]-sK \leq 0$$

This definition of rate of return computes earnings after all taxes have been allowed as costs, and follows that given by the Federal Power Commission in (5). In common with most previous analyses we shall assume that this constraint holds with equality and that the allowed rate of return is strictly less than that which the unregulated monopolist could earn. Also "the appropriate second-order maximum (concavity-convexity) conditions are taken to hold throughout" (2).

This formulation of the problem does simplify somewhat the depreciation and tax accounting policies of the firm but to handle a more realistic model would require the complication of dynamic regulatory considerations. Baumol and Klevorick have demonstrated some of the difficulties that are encountered when this path is taken.

B. Optimality Conditions

We adjoin the constraint to the objective function with the Lagrange multiplier λ and then derive the first order conditions for a maximum.

$$H=[R-wL-(p+a+ib)K][1-c]-r[1-b]K$$
$$-\lambda[(R-wL-(p+a)K)-c(R-wL-(p+a+ib)K)-sK]$$

$$H_k=0=(1-c)(1-\lambda)(R_k-p-a)-(ib+r(1-b)-ibc)$$
$$+\lambda(s-ibc)$$
$$=(1-c)(1-\lambda)(R_k-p-a)-(1-\lambda)(ib+r(1-b)-ibc)$$
$$-\lambda(ib+r(1-b)-s)$$

substitute for q and as $\lambda\neq 1$ (for this would imply $q=s$) divide through by $(1-c)(1-\lambda)$ to get

$$0=R_k-p-a-\frac{q-ibc}{1-c}-\frac{\lambda}{1-\lambda}\left(\frac{q-s}{1-c}\right)$$

This is essentially the relationship Spann gives in his footnote but without some of the extensions which are included in this model.

The conditions on labor are essentially unchanged.

$$H_1=0=(1-c)(1-\lambda)(R_1-w)$$

Thus we have the marginal efficiency conditions for labor and capital. It is of interest to note that the income tax rate directly enters the capital equation and only indirectly the labor equation.

C. The Lambda Lemma [$0 < \lambda < 1$]

The lemma as stated by Baumol and Klevorick is still true with the proof which follows the same path sketched below.

$\lambda \neq 1$ from first order condition

$\lambda > 0$ as constraint is active (i.e., s < monopoly rate of return)

The second order conditions for constrained maximization lead to the requirement that

$$-[(s-q)/(1-\lambda]^2 (1-\lambda)(1-c)R_{11} > 0$$

and as we have assumed $c < 1$ and $R_{11} < 0$ this implies

$$\lambda < 1$$

which gives us the result

$$0 < \lambda < 1$$

D. Inefficiency?

In the standard analysis of the A-J effect this lemma is sufficient to show that

$$R_k / R_1 < (a+q)/w$$

which implies that the firm is using excess amounts of capital compared with the cost minimizing firm which would choose

$$R_k / R_1 = (a+q)/w$$

This comparison is often used to imply an associated welfare loss, but usually without the necessary assumptions for the rest of the economy being explicitly stated. We shall continue this partial equilibrium approach in this section and will show that the regulated monopolist is not necessarily more or less efficient in choice of inputs than the unregulated and untaxed monopolist. In the next section a slightly more realistic comparison will be made in a general equilibrium framework.

From the first order conditions we get that

$$\frac{R_k}{R_1} = \frac{p+a+\frac{q-ibc}{1-c} + \frac{\lambda}{1-\lambda}\cdot\frac{q-s}{1-c}}{w}$$

As $0 < \lambda < 1$, $s > q$ and $0 < c < 1$ we get

$$\frac{\lambda}{1-\lambda}\cdot\frac{q-s}{1-c} < 0$$

and as $0 < b < 1$

$$p + \frac{q-ibc}{1-c} > q$$

Thus the ratio of the marginal revenue products is no longer unambiguously less than (a+q)/w. In fact the possibility now exists for choosing the tax rates and allowed rate of return so as to equate the ratio to (a+q)/w. This is not necessarily an advisable policy but does provide motivation for looking at the relationship between these policy variables.

The determination of the second best value of λ is an extremely difficult problem and is discussed in the following section.

E. Optimal Tax Rate

Klevorick (8) has examined the question of what degree of regulation is optimal from a social welfare point of view. He took the allowed rate of return as his policy variable and by considering its effect on output and inputs, maximized a welfare function.

Here we shall examine a different sort of problem. A slightly more general model will be used to study the nature of the efficient input conditions. A two sector economy will be used with both sectors being taxed, but with only one being regulated. The unregulated industry is assumed to act competitively and the regulated industry as a monopolist. The model of the monopolist's behavior remains as above and a competitive industry producing another good is introduced. The total amounts of the factors of production are considered fixed and the factor markets are taken to be competitive. By considering the stock of capital to be a fundamental resource, the price does not affect the amount provided. This is clearly a limiting assumption but does avoid the complications of a multi-sector growth model. The competitive industry is also subject to income taxation and property taxation, though the latter is at a different rate than the monopolist. This particular formulation raises a difficulty which does not occur in the partial equilibrium framework previously used. As it is the presence of risk which necessitates the consideration of debt and equity financing, it would seem appropriate that the general equilibrium framework introduced should reflect this. Unfortunately if the two industries are considered to have dissimilar risk characteristics, then the limited results presented below are no longer always true. As this discussion is mainly illustrative, we shall simply consider that both industries face the same cost of capital and have the same debt to total capital ratios. The following additional notation is introduced with the subscripts c and m denoting competitive and monopolistic respectively.

G = production function of competitive industry

y = price of competitive good

The first order conditions for profit maximization are thus:-

$$0 = (1-c)(yG_k - p_c - a - ib) - r(1-b)$$

$$0 - yG_1 - w$$

At this point we could introduce the rest of the equations characterizing the equilibrium solution and attempt a second best type of analysis. This is similar to the approach of Klevorick but with more policy variables. We do not follow this path because it adds little in the way of policy implications. Instead, the input and output Pareto conditions are examined separately and it is shown that the tax rates and rate of return are sufficient, if properly set, to maintain efficient input choices. Correction of the monopolistic output conditions requires some other measure such as a differential sales tax. The counteracting effect of taxation and rate or return regulation allow the taxing authorities to raise revenue from the monopolist without distorting his choices. It does, however, alter the price of capital and this would need consideration in the real world, where the supply of capital is not inelastic.

The Pareto conditions for input efficiency are that the ratio of the marginal productivities of the factors should be equal in both industries.

This gives us

$$(1-c)(p_c - p_m) = \lambda(q-s)/(1-\lambda)$$

which implies that

$$p_m > p_c$$

It is of interest to note that Netzer (11) has found that the average property tax leveled on utilities is 1.3% of total assets while that on industry in general is 1.0%. More recently, Manvel (9) has recompiled data in this area and concludes an even larger difference exists. These are very aggregated figures, but they do suggest that the property tax is at least qualitatively in the direction required to offset the effect that rate of return regulation may have on input choices.

If the property tax rate on the competitive industry is considered as an acceptable status quo and the rate of return and property tax for the monopolist are set so as to remove input inefficiency, then the monopolist behaves as if he were unregulated and taxed identically to the competitive industry.

i.e. if $p_m = p_c + \lambda(s-q)/(1-\lambda)(1-c)$

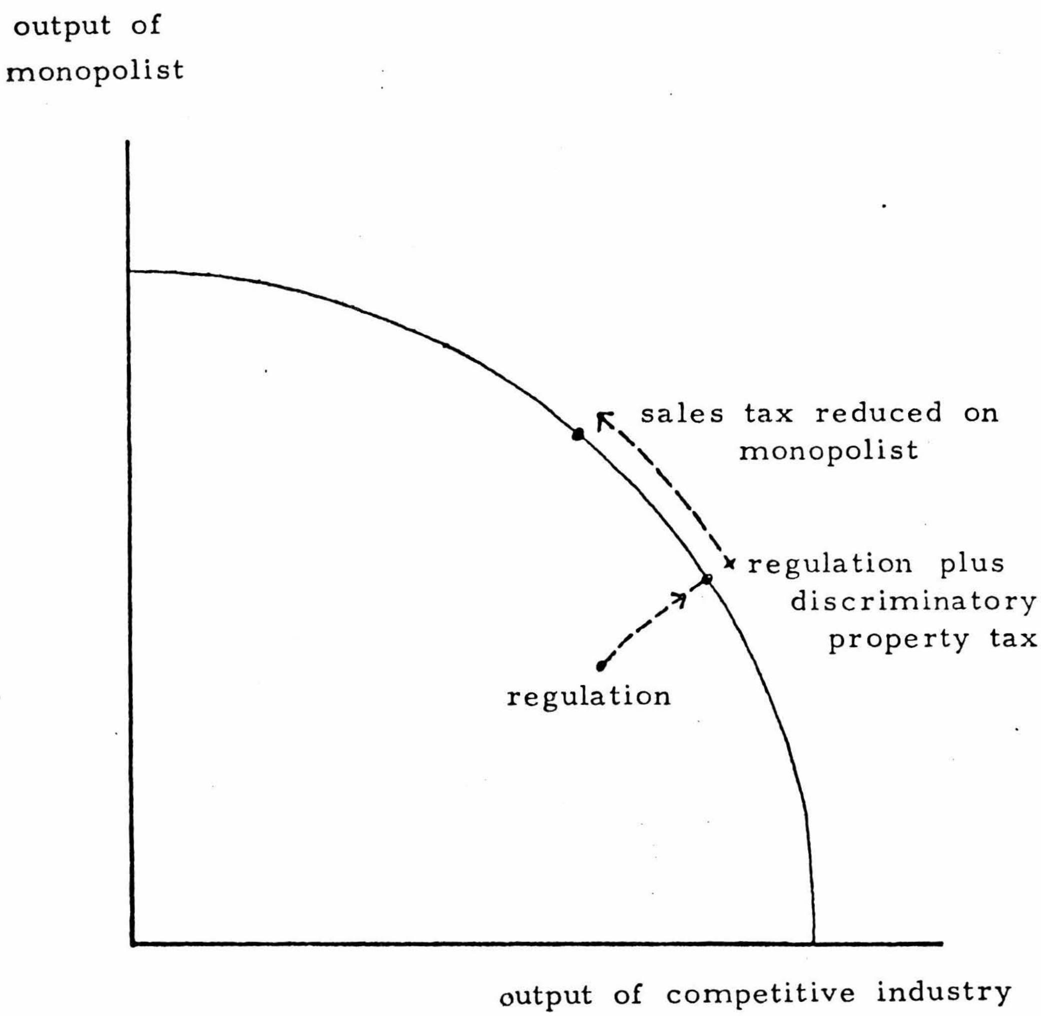
The monopolist's first order conditions then reduce to

$$0 = (1-\lambda)[(1-c)(R_k - p_c - a)-(q-ibc)]$$

$$0 = (1-\lambda)(1-c)(R_1 - w)$$

which are identical to those of the unregulated monopolist. Graphically (II-1) this can be seen as moving from a point inside the production possibility set to the frontier. We still need to move along the frontier to satisfy the output Pareto conditions.

II-1


output of
monopolist
sales tax reduced on
monopolist
regulation plus
discriminatory
property tax
regulation
output of competitive industry

III. OTHER STUDIES

A. Courville

Courville attempts to verify the existence of the A-J effect in a very direct manner. He estimates a Cobb-Douglas production function of annual energy produced, and then tests whether the ratio of the factor marginal productivities derived from this is different from the price ratio. He concludes that overcapitalization has been confirmed.

His data consist of observations on new steam electric plants in the period 1948-1966, which are split into four subgroups according to vintage. The 1956-1959 group had to be discarded as it gave results which were drastically different from the other three periods. This was explained on the basis of the "electric conspiracy."

The major fault with this study is the totally inappropriate formulation of the technological possibilities. A Cobb-Douglas production function relating total plant cost and the fuel used to annual energy production is not relevant. It is surprising that Courville does not realize this, as he specifically gives the two reasons why it is inappropriate. On page 63 he notes that plants facing higher fuel costs will tend to be more efficient for a given capacity and in a footnote on the same page he notes that as peak loads must be met "overcapitalization can be inferred only if excess capacity is present at all points in time" (when using capacity as a capital measure). As these are the points which are being made here, it is indeed hard to understand why Courville failed to realize that, by using total cost as his measure of capital, he was confusing the choice of capacity with the choice of efficiency.

It was shown earlier that, to a reasonable approximation, the annual energy output of a plant is simply proportional to the energy input. Courville's results certainly bear this out as his estimated values of β (the coefficient of log F) are definitely not significantly different from unity. This certainly explains why such impressive values of R-squared were achieved, though when there are such strong a priori reasons to believe in linearity, these R-squared values are somewhat misleading. His procedure, therefore, actually tries to estimate the coefficient of proportionality in this linear process by a simple power of the total cost.[6] To illustrate this, the following equation was estimated using the 1960-1966 vintage plants.

$$\frac{1}{\text{heat rate}} = \frac{\text{annual output } (10^6\text{kwh})}{\text{fuel input (MBTU)}} = A \cdot [\text{total cost}]^{\alpha}$$

The results were

$$\hat{\log} A = -10.009$$

$$\alpha = \underset{(0.0167)}{0.0889} \quad \Rightarrow \quad t = 5.327$$

$$R^2 = 0.434$$

$$\text{D-F} = 37$$

This shows that as total cost increases, so does the efficiency with which the plant converts fuel to electrical energy. Two factors cause

6. His inclusion of utilization is quite a reasonable way of handling peak loads.

this: (a) for the same unit expenditure ($/kw of capacity) a larger plant will be more efficient; and (b) for a given size of plant a larger unit expenditure will purchase a thermodynamically more efficient plant. We have previously shown that at the plant level the first factor cannot be considered as a choice variable when considering the A-J effect, so we are thus interested in pinpointing the second factor. The previous result, like Courville's procedure, did not distinguish between these two factors. Indeed, by using a very simplistic approach the first effect can be shown to have contributed almost all the explanation to the total cost term.

This is done by estimating the following equation:

$$\frac{10^6}{\text{heat rate}} = A + \alpha \left[\frac{\text{total cost (\$1,000)}}{\text{capacity (MW)}} \right]$$

The following results were obtained.

$$\hat{\alpha} = -0.017 \Rightarrow \quad t = 0.39$$

$$(0.044) \qquad R^2 = 0.004$$

$$\hat{A} = 105.4 \qquad \text{D-F} = 37$$

The above indicates that unit costs explained none of the observed variation in heat rate. On this basis it appears that the coefficient of capital which Courville has estimated is totally determined by the size of the plant, and this cannot be considered as a choice variable within the context that he is considering. It should be noted that in both the above estimations, plant rather than unit data are used, not because this is the correct level to look for these effects but to make the results more directly comparable to Courville's.

There are some other perturbing factors about this functional form when used with his measure of capital. Using Courville's description of his data set (i.e., new plants built during 1960-1966, under 800 Mw, capacity, and in first full year of operation) an attempt to reproduce his results was made (table III-1). They are designed as data set A^*. The coefficients differ significantly from those reported by Courville. They are also in the direction to make detection of the A-J effect less likely, and in fact when the t-test for overcapitalization was done with these coefficients, indecisive results were obtained. Clearly this was not the data set used by Courville. The same plants were then examined and the "best" year of operation before 1969 was selected. The criterion for "best" was the highest output for what appeared to be a typical plant factor and after as many capacity additions as possible (provided they seemed to be part of the original design). The coefficients for this data set are shown as data set A (which is listed in entirety in appendix B) and these appear to agree well with those obtained by Courville. This is certainly a rather perverse behavior of the data, as the second set would appear to be less likely to show overcapitalization, because output in general was greater and in many cases expenditure per unit of capacity was less. The feeling that this measure of capital has such serious flaws so as to make it totally unreliable was further reinforced when Courville's equation was reestimated using a sample of twenty-nine municipal, federal and Texas plants from the same period. The coefficient of capital was indistinguishable from zero.

TABLE III-1

Attempted Replication of Courville's Results

(1960-1966)

$$\log[\text{output}] = \log A + \alpha \log[\text{total cost}] + \beta \log[\text{fuel used}] + \delta\ \text{utilization} + b\ \text{capacity}$$

Data Set	$\log \hat{A}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{b}$	R^2
A*	-9.392	0.2146 (2.9708)	0.8623 (10.746)	0.003 (1.791)	0.00066 (0.247)	0.9856
A	-9.8709	0.0925 (2.158)	0.9718 (16.645)	0.002 (2.146)	0.0000 (0.0341)	0.9937
Courville	-1.2602	0.1036 (3.10)	0.9705 (17.36)	0.3361 (3.04)	0.00002 (0.96)	0.994

The values in parenthesis are t ratios in contrast to the rest of this paper where they are standard errors.

While the above shows that Courville's estimated production function does not enable us to detect the A-J effect, his general approach is more promising than the two other methods. He gives many valid reasons why looking at a firm as a whole can lead to serious problems. He is also the only author to consider peak load effects, and within the context of his production function, his method of allowing for them seems reasonable.

He is in error on one point, however, when he postulates a MIN formulation as the appropriate joint production function for distribution, transmission and generation. He appears to be confused by the fact that the peak power which can be delivered is limited by the rating of the weakest link in the system. The annual energy is not subject to any such limitation, unless equipment is running at capacity for most of the year. The capacity of distribution systems is very seldom a limiting factor, as overload merely produces degradation of performance (lower line voltage) with little risk of equipment failure. Transmission systems (AC) do have a point of maximum capacity which is determined by the reactive characteristics of the load and the line.[7] When considering the delivery of energy, it is usually possible to use quite a simple model, as the transmission losses are simply proportional to the square of the power level.[8] If the location of a plant is fixed and its cost of production determined, the choice of a transmission system can easily be stated in marginal terms. The value of

7. Description of the relative phase of the voltage and current in A-C circuits.

8. See V. L. Smith (17) for discussion of this classical problem in engineering economics.

the marginal energy lost as heat should be equal to the annual marginal expenditure on the transmission system.

There are possibilities (as outlined earlier) for A-J type interactions between transmission and generation showing up in fuel choice and location decisions. These are clearly not detectable unless the cost characteristics of all the alternatives facing the firm are known.

B. Spann

Spann uses an indirect method in his attempts to confirm the A-J thesis in the electric power industry. He tests two related hypotheses:

1. The regulatory constraint is inactive given that the firm is a profit maximizer (i.e., Lagrange multiplier = 0), or
2. If the constraint is active, the firms do not maximize profits.

To test these hypotheses, Spann assumes a trans-log production function for annual energy output and, from the normal Averch-Johnson first order conditions, derives two equations giving the factor shares of total revenue. By assuming that all firms face the same constant elasticity of demand he obtains a restriction on a coefficient in each equation. For the first hypothesis he jointly estimates these equations subject to this constraint. The estimated value of the Lagrange multiplier is significantly different from zero. From this he concludes that inefficient input choices have been made.

Unfortunately evidence which shows that the constraint is active does not necessarily infer inefficiency. By considering taxation, Spann introduces a very good reason why the constraint may be active and efficient input combinations chosen. As explained earlier, the effect of property taxation is to make capital appear more expensive and hence introduce a bias which favors the substitution of fuel for capital. Corporate income tax works in a similar manner, as debt expenses are excluded from profit when computing taxes and it is the after tax rate of return that the regulators consider. Thus, if utilities face higher capital taxes than other business, taxation and rate of return regulation are offsetting effects.

Even assuming no problems with his theoretical model, Spann's results do not demonstrate a capital bias. Spann's plant data set was reconstructed and is listed in the appendix B. The two equations (for simplicity these have the labor terms neglected[9]) are, using Spann's equation numbers:

S (8) $$\frac{rK}{PQ} = \lambda \frac{sK}{PQ} + b_1 + b_2 \log K + b_3 \log F$$

S (10) $$\frac{p_f F}{PQ} = b_4 + b_5 \log F + b_6 \log K$$

and these are subject to

$$b_3 = b_6 (1-\lambda).$$

9. It should be noted that in the relevant equations Spann did not find the coefficients of the labor term to be significant. Their omission thus seems justified, considering that no difference in either the value or significance of the Lagrange multiplier resulted.

To check Spann's results, these equations were reestimated separately using two different methods. The linear regression program produced superior printout[10] with regard to errors and gave good agreement with the nonlinear program which was to be used for the joint estimation. The inverse of the standard error estimates were used to weight the observations when doing the joint estimation, as was done by Spann. The joint estimation was done with λ fixed at variety of values later be considered as alternative hypotheses. To test the significance of the estimated value of λ, the following chi-square statistic was computed.

$$\chi_1^2 = -T \log \frac{\text{error sum of squares with } \lambda \text{ free}}{\text{error sum of squares with } \lambda = \lambda_o}$$

where λ_o is the alternative hypothesis. The results are displayed table III-2 and figure III-1.

Spann concluded from the fact the λ was significantly different from zero that there was overcapitalization. As has been shown, property and income taxation can produce a bias against capital inputs. The appropriate comparison is thus with the value of λ which implies the same input combinations chosen by an untaxed and unregulated cost minimizer. This value of λ_o is given by

$$\lambda_o = \frac{p - q + (q - ibc)/(1 - c)}{p - q + (q - ibc)/(1 - c) + (s - q)\ (1 - c)}$$

where

p - property tax rate

c - corporate income tax rate

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10. See Appendix F for these results.

TABLE III-2

λ	SSE	$\left[\frac{\text{SSE } \lambda_{free}}{\text{SSE } \lambda = \lambda_o}\right]$	log [ ]	χ_1^2
0.5847	69.87	1.000	0.0000	0.00
0.000	99.35	0.703	-0.3524	26.70
0.050	94.30	0.741	-0.2997	22.08
0.100	90.14	0.775	-0.2549	18.85
0.300	76.93	0.908	-0.0965	7.14

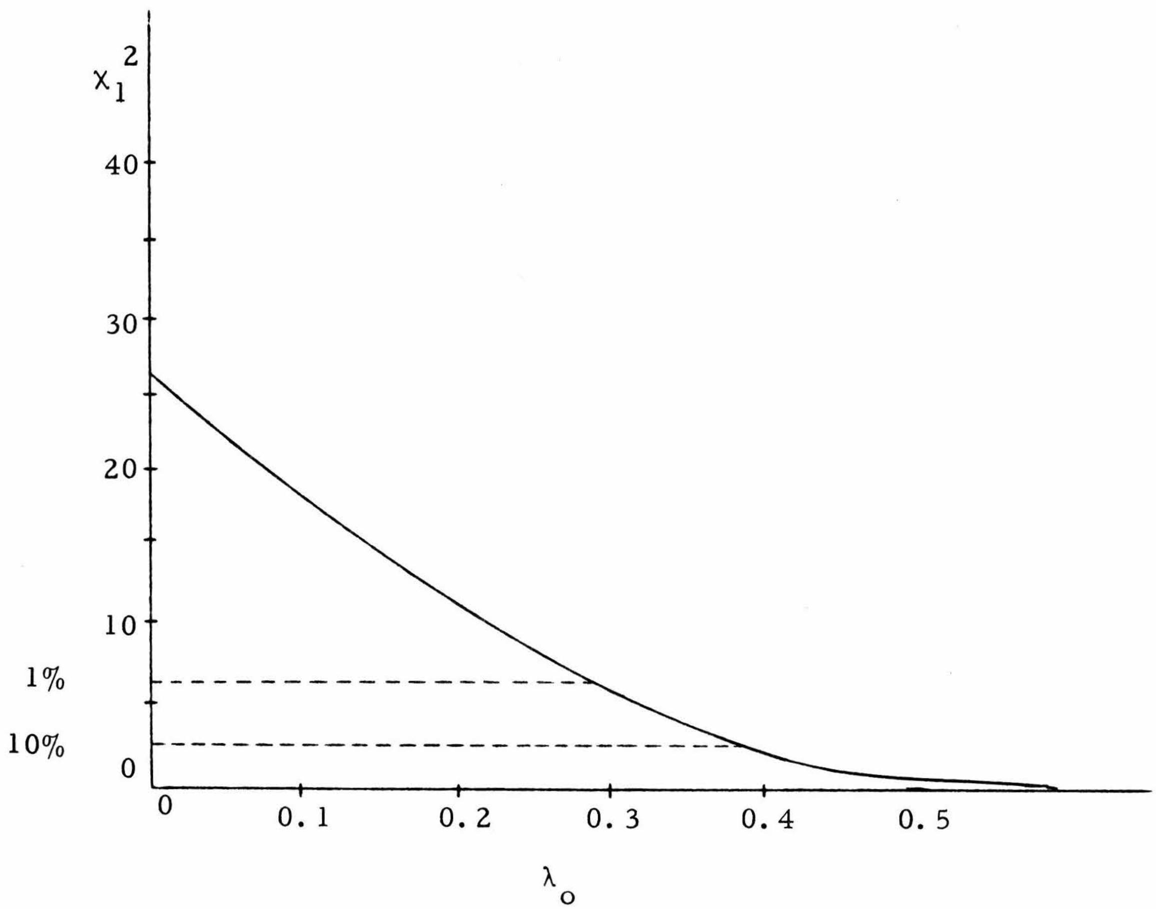
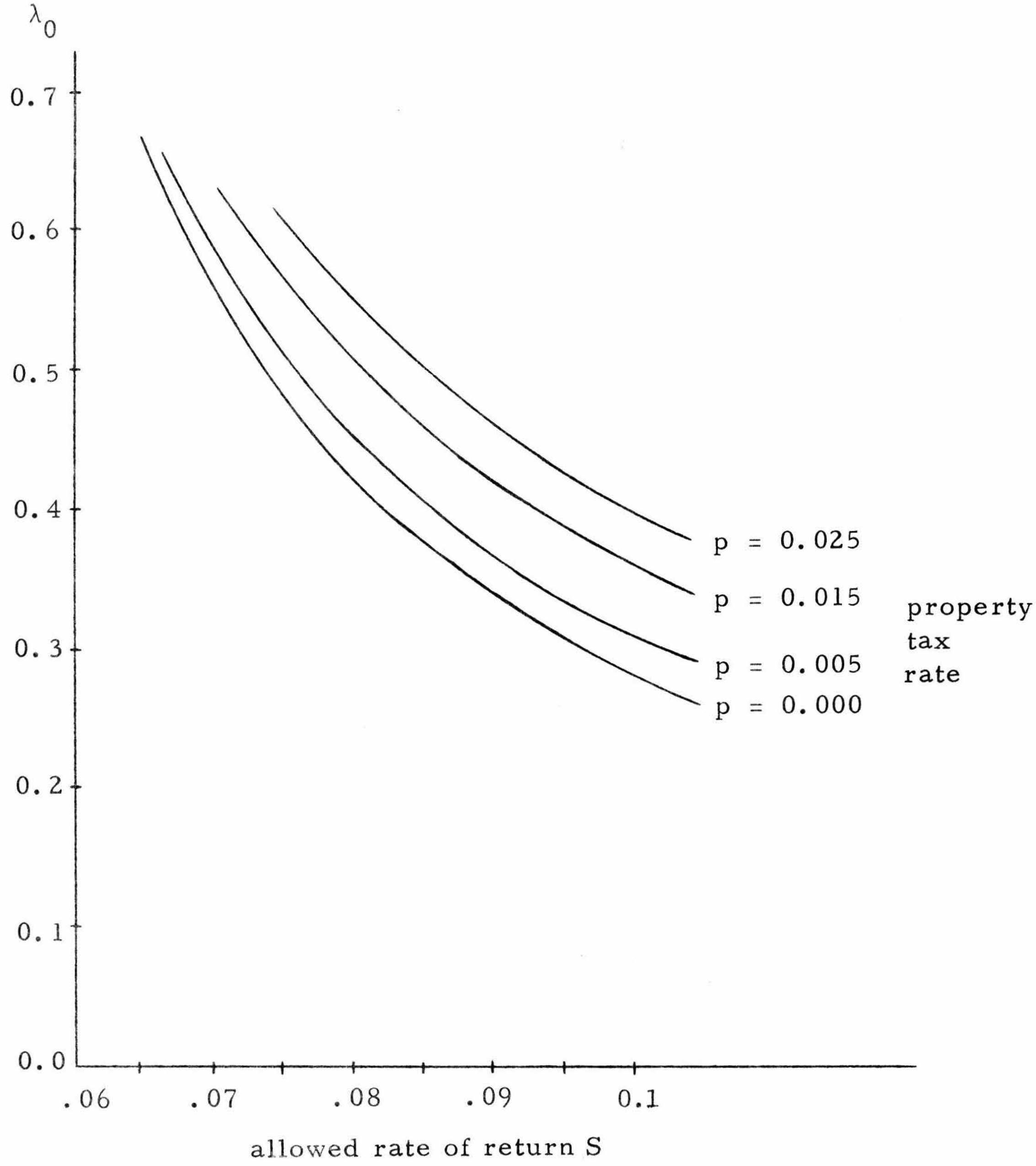


Figure III-1



λ_0
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
.06
.07
.08
.09
0.1
allowed rate of return S
p = 0.025
p = 0.015
p = 0.005
p = 0.000
property tax rate

Figure III-2

i - interest cost of capital

b - fraction of debt capital to total

r - opportunity cost of equity capital

q - cost of capital = ib + r (1 - b)

s - allowed rate of return

These values are displayed graphically as a function of allowed rate of return and the property tax rate (figure III-2). The parameters used were

$$\left.\begin{array}{l} q = 0.056 \\ b = 0.52 \\ c = 0.52 \end{array}\right\} \text{Spann}$$

and

i = 0.043 Moody's AAA bond yield 1963

These last two graphs show that even if the maximum rate of return occurring in Spann's data (s = 0.0912) is used and a property tax of zero is assumed, the hypothesis that there is no distortion is accepted at the 1% confidence level.

So far, Spann's results have been considered under the assumption that his methodology is otherwise acceptable. This, unfortunately, is not the case. Earlier it was shown that it is incorrect to represent technological possibilities by a neoclassical production function for annual energy output without considering the constraints imposed by the instantaneous nature of the output. This particular method requires that such a function be appropriate and in particular that there by a non-zero coefficient for a capital-fuel interaction term. The reasons why annual output can be expected to be linear with fuel inputs were discussed previously. Small departures from linearity are caused by variation in output through the year, and not by interaction between some measure of capital and fuel used.

The joint equation method as used by Spann should be consistent and more efficient than an estimation based solely on the capital share equation. The surprising feature that emerges is that when equation S (8) is estimated by itself, the coefficients b_2 and b_3 are not significantly different from zero.

$$\frac{rK}{PQ} = \underset{(0.1088)}{0.5992}\,\frac{sk}{PQ} + 0.0119$$

$$+ \underset{(0.0179)}{0.000293}\,\log K + \underset{(0.0197)}{0.006159}\,\log F$$

$$R = 0.0197$$

Spann does not discuss this result at all, though this is probably an editorial error as a nonexistent discussion is mentioned in footnote 14. What in fact appears to be occurring is that the λ is merely the average ratio of the cost of capital to the allowed rate of return. While this interpretation of λ does not necessarily contradict the Lagrangian one it does seem that a measurement which has absolutely nothing to do with the Averch-Johnson effect is possible. The inequality which is assumed between allowed rate of return and cost of capital does not by itself imply any particular factor choice behavior.

Joint estimation of the equations produced widely differing numerical estimates compared to those obtained by Spann, which is because of the different units used to measure capital and fuel. The coupling of the equations means that the scaling effects no longer influence only the constant but the actual coefficients themselves.

$$\frac{rK}{PQ} = \overbrace{0.5847}^{\lambda}\frac{sK}{PQ} + 0.2819 + 0.0192 \text{ logK} - 0.0215 \text{ log F}$$

$$\frac{p_f F}{PQ} = \quad - 0.1130 + 0.0632 \text{ logF} - 0.0518 \text{ logK}$$

The signs are consistent with those obtained by Spann. No error estimates are included due to the nonlinear program used.

One difficulty with Spann's approach is that it gives absolutely no consideration to peak load effects. These effects are not the same for each plant. Furthermore, the assumption of constant elasticity of demand is also open to criticism, as different utilities certainly face different demand curves (due to, for example, regional climatic differences). There is also an econometric problem which Spann does not address. By jointly estimating the factor share equations using ordinary least squares, he ignores the fact that there is strong reason to believe that the pairs of disturbances are not uncorrelated. This would seem to require that generalized least squares is needed. In practice,[11] however, the correlation is small and, compared to the other faults with this approach, a minor problem. The major error, though, is that the production process is incorrectly represented by the

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11. See Appendix F.

function chosen. Since this function is necessary in order for the method to be successful, the meaning of the results is dubious.

C. Petersen

Petersen attempts to detect the A-J effect by using the comparative static results that show a rise in unit production costs and a rise in the share of costs going to capital as regulation tightens. He uses three measures to quantify tightness of regulation. The first is a dummy variable to distinguish states that evaluate the rate base on an original cost basis from those that use a replacement cost or fair value. The second measure is a dummy variable to distinguish between those states with statewide regulatory commissions and those without; the contention being that original cost and statewide regulation are "tighter" forms of regulation. His third measure is an adjusted return to equity capital which leads, after some assumptions, to a continuous variable measuring regulatory tightness. By starting with a generalized cost function he attempts to explain unit costs and capital's share of costs. He includes his measures of regulatory tightness as additive shifts in the cost estimations.

Petersen's results lead him to conclude that regulation induces higher costs and thus that the Averch-Johnson thesis has been confirmed. Specifically, his results are that while the coefficient of his fair-value dummy is of the predicted sign, it is not significantly different from zero in either the unit cost or share of cost estimation. His variable distinguishing statewide regulation has the expected sign and is significantly different from zero in both equations. The continuous measure of regulatory tightness is also of the correct sign and significant. Unfortunately, Petersen failed to examine any other explanations for these observed results. A particularly simple explanation does exist which has absolutely nothing to do with the Averch-Johnson effect. There are differences in the fuels used which happen to vary systematically with the statewide regulation and return to equity variables.

Petersen's data set was reconstructed from the FPC reports and is listed in the appendix. While a more precise measure of this effect would be to weight each plant by output or capacity, a simple counting test suffices to show the correlations between fuel use and Petersen's measures of regulatory tightness.

TABLE III-3

Fuel Used

	Number of Plants						
	C	O	G	CG	CO	OG	COG
Statewide Reg.	15	3	4	7	3	12	2
Texas	0	0	7	0	0	0	0
Iowa	0	0	0	1	0	0	0
Minnesota	1	0	0	0	0	0	0

C - coal O - oil G - gas

Plants using only gas have significantly lower capital costs as well as production costs, compared with those plants which can use coal.[12]

12. Appendix C; 'Bi-annual Steam Station Cost Survey', Electrical World.

Because the Texas plants dominate the unregulated part of the sample, both in number and capacity (table III-3), it is not at all surprising that the costs were less, and the share of costs going to capital less, than in the regulated states, which contained a majority of coal-burning plants. The other, though less important effect, is that completely outdoor construction is more prevalent in Texas than the nation as a whole due to climatic differences. The failure to control for these systematic technological differences means that it is incorrect to attribute cost differences to regulation. When using the return to equity measure, Petersen uses the same sample (excluding some for lack of data) which means that, if Texas firms do enjoy a higher rate of return (which they appear to do), the results of this measure are also thrown into doubt by the fuel differences.

The unanimous choice of gas by the Texas utilities compared with the choice of other fuels by regulated utilities could itself be taken as an indication of an A-J distortion. If so, regulated states with ample gas supplies might be observed to choose plants that burn other fuels. But this is not borne out by examination of choices in Louisiana, a state with regulation and readily-available natural gas, which also shows an overwhelming preference for gas-fired plants.

The above discussions would seem to be strong enough reasons for considering the Petersen case far from convincing, but there are other points which would need consideration if this approach were to be reattempted. The question of load factor has not been considered, and while it is unlikely that it systematically varies with regulatory activity, this would need to be confirmed, as firms with poorer load factors do have both higher unit costs and a greater share of costs going to capital. The other possible effect which could show up as higher costs and share of costs is if regulatory institutions set higher reliability standards. This is not an A-J type of capital-fuel substitution, but would be indistinguishable from conventional A-J distortions using this method.

Petersen's choice of a sample is, in many ways, arbitrary. He includes a plant if it expands its capacity at least 50%, claiming that this is a marginal decision and thus suitable for the theory. This is generally not true, as most of the expansion included in his sample was simply part of the original construction schedule as another identical unit was brought on line. Totally new plants are more nearly marginal decisions by the firms. Petersen also errs in believing multiple observations on the same plant constitute independent observations. While the desire to accumulate a large number of observations is understandable, the practice of using three annual observations on each plant without examining the data for possible heteroskedasticity leaves much to be desired. If the plants are operating under normal conditions the yearly observations will be almost identical, and if not (either due to breakdown or scheduled maintenance), the observation is spurious with regard to the A-J effect.

One additional point worth noting about this particular approach has been noted by Noll (12). This is that increased costs are consistent with the A-J effect but not necessarily caused by it.

D. Boyes[13]

Boyes uses a similar approach to Spann but does take considerably more care with the econometric problems which arise from the simultaneous equations generated. His results are that the constraint does not appear to be active and he gives a number of reasons why this is not surprising.

His representation of the technological possibilities suffers from all the same problems which were previously mentioned when discussing Spann's work. The disagreement of these results with those of Spann is somewhat surprising and very likely due to the different sample used. Boyes uses plants from 1957-64, while Spann uses 1961-63 and the inclusion of the 'electric conspiracy' years by Boyes may well be introducing the perverse behavior noted by Courville for this subsample.

Even though the results of Boyes' study are in agreement with those reported in the next section of this current work there appear to be good reasons for not considering them convincing.

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13. This work appeared after the current work was completed and consequently no reconstruction and detailed study of the data has been attempted.

IV. REVISED TEST OF THE AVERCH-JOHNSON HYPOTHESIS

Because of its directness and lack of restrictive assumptions, a test of the equality of the ratios of marginal productivities and of prices, as attempted by Courville, is attractive.[14] To do this in light of the points made previously about characterization of the technological possibilities, a new formulation of the profit maximizing model is needed. As outlined earlier, when constructing a new plant, the choice variable, as far as the A-J hypothesis is concerned, is the efficiency with which that plant converts fuel to electrical energy. The decisions concerning the location, capacity, number of units and fuel are constrained by exogenous technical and economic factors. These may be subject to A-J effects, but much more information on the alternatives available is required before this can be determined. The efficiency of a plant is not a unique quantity, but depends on the level and the time distribution of output. Because instantaneous output data are not available, this efficiency cannot be captured more finely than by the simple annual heat rate. This section presents a reformulation of the classic A-J model with the heat rate as the capital input variable.

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14. It is worth noting once again that this is merely a test of whether an individual plant has excess capital in relation to its output. Finding no overcapitalization here does not rule it out as it may be manifesting itself as early unloading of old plants. The only way to attack this problem is tedious and may be totally impractical as it necessitates checking whether the marginal capital

A. <u>Model</u>

Notation:
- q - instantaneous output (KW)
- f - instantaneous rate of fuel input (BTU/hr)
- S - capacity of unit (KW)
- H - annual heat rate (BTU/KWH)[15]
- Q - annual energy output (KWH)
- F - annual fuel consumption (BTU)
- r - cost of capital ($/$)
- g - instantaneous transformation function (KW)
- s - allowed rate of return ($/$)
- p_f - price of fuel ($/BTU)
- p - price of output ($/KWH)
- C - cost of capital equipment (unit). Assumed to be a well behaved function of S and H.

The production conditions are

$$q = g(f)$$

and

$$q \leq S$$

---

costs did in fact equal the marginal fuel saved over the complete life of the plant. While this point is of extreme importance, it was not recognized by any of the other authors; in this section we will continue to look for the more limited effect they were seeking.

15. $\frac{3412}{H} \times 100$ = thermodynamic efficiency (%)

but as we only have annual data we must use

$$H\ Q = F.$$

For a given output the regulated monopolist will attempt to maximize

$$\text{Profit} = pQ - p_f F - rC\ (S,H)$$

subject to the rate of return constraint

$$pQ - p_f F - sC(S,H) \leq 0.$$

The standard assumptions that s > r and that the constraint holds with equality are made. To obtain the conditions for maximization, adjoin the constraint to the objective function with the multiplier λ.

$$L = (p - Hp_f)Q - rC(S,H) - \lambda((p - Hp_f)Q - sC(S,H)).$$

The first order conditions for a regular maximum (considering output fixed) are:

$$\frac{\partial L}{\partial H} = 0 = - p_f Q - r \frac{\partial C}{\partial H} + \lambda p_f Q + \lambda s \frac{\partial C}{\partial H}$$

which rearranges to the following:

$$p_f Q + r \frac{\partial C}{\partial H} = \frac{\lambda}{1 - \lambda} (s - r) \frac{\partial C}{\partial H}\ .$$

The second order conditions for regular constrained maximization require that:

$$\frac{\partial^2 L}{\partial H^2} \leq 0$$

and hence:

$$(-r + \lambda s) \frac{\partial^2 C}{\partial H^2} \leq 0.$$

Assuming that thermodynamic efficiency is subject to diminishing returns:

i.e.,

$$\frac{\partial^2 C}{\partial H^2} > 0 \text{ and } \frac{\partial C}{\partial H} < 0 \text{ it follows that}$$

it follows that

$$-r + \lambda s \leq 0$$

Hence

$$\lambda \leq 1 \text{ since } s > r.$$

Using the same arguments as Baumol and Klevorick (2), the relation $0 < \lambda < 1$ can be obtained. Using this information in the rearranged form of the first order condition gives the marginal productivity condition if the A-J effect is active:

$$p_f Q + r \frac{\partial C}{\partial H}$$

The condition for a cost minimizer is:

$$p_f Q + r \frac{\partial C}{\partial H} = 0$$

B. The Cost of Capital Equipment

To test this form of the A-J hypothesis, an estimable cost function must be derived from which we can extract the marginal cost of thermodynamic efficiency. Some preliminary investigations of the determinants of this equipment cost are reported in Appendix C. In addition we can deduce the following restriction on the functional form. The cost, C, tends to infinity as the heat rate approaches some minimum achievable level (determined by the temperatures and pressures of the cycle). An absolutely perfect, but impractical cycle could at best have a heat rate of 3412 BTU/kwh, which corresponds to 100% thermodynamic efficiency. Considering current metallurgical limitations, in practice this asymptotic heat rate is likely to be in the vicinity of 6,000 BTU/kwh (approximately 57% efficiency). The reasonableness of this assertion is evident in figure IV-1.[16]

The preliminary work in Appendix C with this additional restriction leads us to use:

log [equipment cost/unit] = $A + \alpha C_D + \beta \log$ [unit size] + $\gamma \log$ [number of units in plant] + $\delta \log$ [heat rate − B].

B = asymptotic heat rate C_D = coal dummy

16. Reproduced from (16).

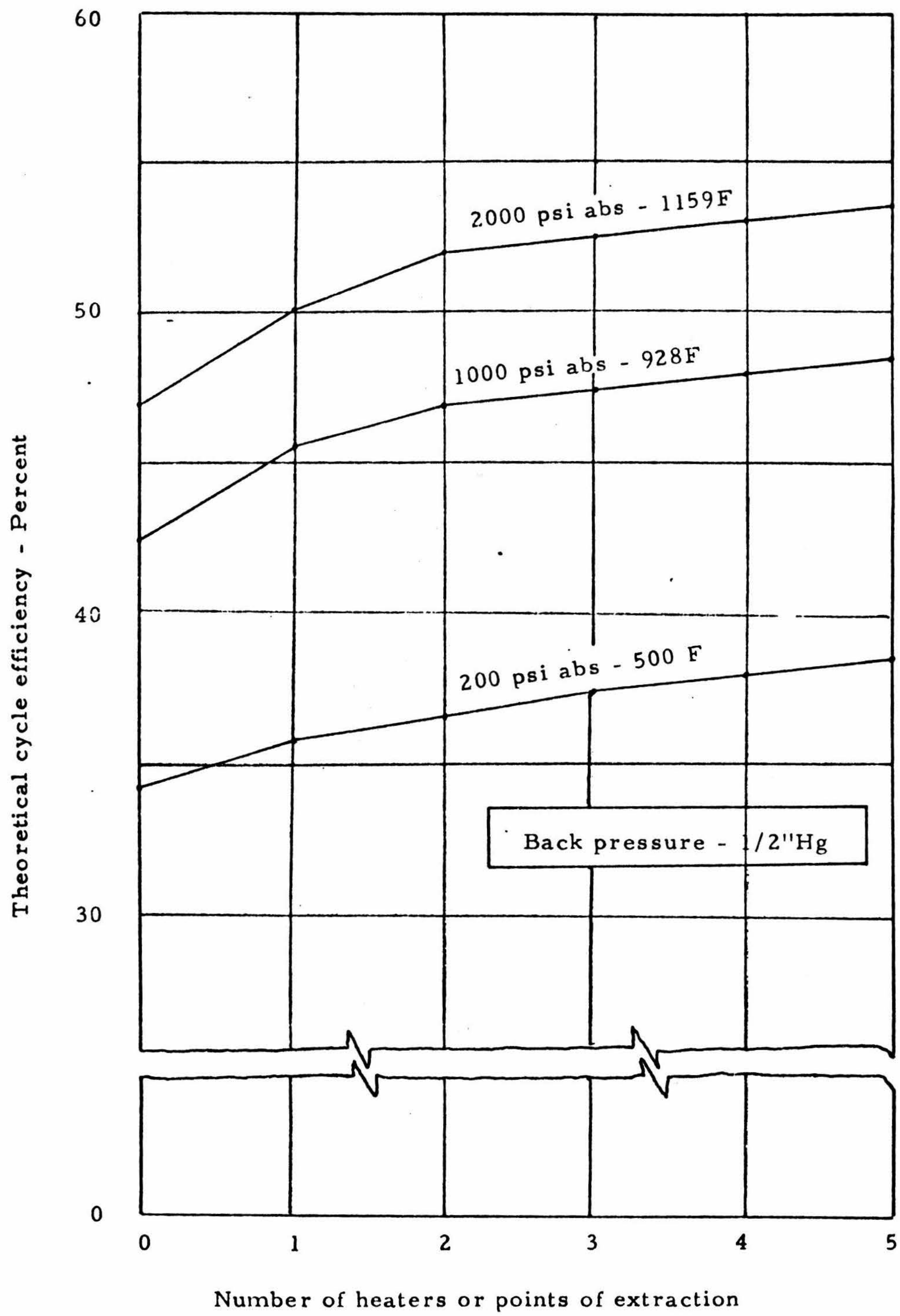

Figure IV-1. Regenerative-cycle efficiency variation with number of heaters. Curves are for ideal cycle.

In terms of these parameters the marginal cost is:

$$\frac{\partial C}{\partial H} = \frac{\delta C}{H - B}$$

If the estimate of δ is used in the above relation, a bias is introduced due to the stochastic nature of C. This is discussed in detail in Appendix A where such a bias is shown not to affect the result of the tests reported in this section.

The data set described as A in the section on Courville's work was used to estimate this equation. Six different values of B were used to demonstrate the insensitivity of the hypothesis tests to this assumption. The results of the estimation are tabulated in table IV-7. A variety of nonlinear search methods were initially used in an attempt to simultaneously estimate B. The lack of variation in the standard error shows why these were not successful.

C. Hypothesis Testing I

We are interested in performing a test to determine whether

$$p_f Q + r \frac{\partial C}{\partial H} < 0$$

as predicted by the model. Unfortunately a direct T test of each observation is not particularly helpful. This is because the

$$T_i = \frac{P_{fi} Q_i + \dfrac{r\hat{\delta} C_i}{H_i - B}}{r\hat{\sigma}_\delta \, C_i/(H_i - B)}$$

tend towards $\frac{\hat{\delta}}{\hat{}}$ as r is increased. We can reasonably expect the

TABLE IV-1

Estimation of Cost Function for Capital Equipment

B	A	α	β	γ	δ	Std. Error	R
3000	11.947	0.178 (0.081)	0.719 (0.080)	-0.096 (0.090)	-0.666 (0.509)	0.21070	0.933024
4000	10.993	0.178 (0.081)	0.719 (0.080)	-0.096 (0.090)	-0.568 (0.435)	0.21071	0.933016
5000	10.054	0.178 (0.081)	0.719 (0.079)	-0.097 (0.090)	-0.470 (0.360)	0.21072	0.933012
6000	9.113	0.178 (0.081)	0.720 (0.079)	-0.097 (0.090)	-0.370 (0.284	0.21075	0.932994
7000	8.169	0.179 (0.081)	0.722 (0.078)	-0.098 (0.090)	-0.267 (0.206)	0.21080	0.932958
8000	7.173	0.181 (0.081)	0.728 (0.090)	-0.101 (0.090)	-0.155 (0.122)	0.21098	0.932839

Numbers in parentheses are standard errors of coefficients.

$$\log[\text{cost/unit}] = A + \alpha C_D + \beta \log[\text{unit size}] + \gamma \log[\text{number of units}] + \delta \log[\text{heat rate} - B]$$

A-J hypothesis to be confirmed for some sufficiently large value of cost of capital, but as $\frac{\hat{\delta}}{\sigma_\delta}$ is not significant at normal levels this is not particularly informative. This poor estimate of δ arises from the lack of variation in heat rate with capacity within the sample. However some useful information can still be extracted from this result. The alternative hypothesis that the plants are constructed with too low a thermodynamic efficiency, can be tested. The results are presented in table IV-2, which shows the percentage of the sample for which undercapitalization could be inferred (in a one-tailed test) as a function of the cost of capital. (Cost of capital is taken to include the effects of income and property taxation.)

For costs of capital used by others (6 to 8%) it does not appear that a case for the A-J effect can be made.

D. Hypothesis Testing II

Another approach was also used to test for the A-J effect. If there were no A-J bias, a particular power plant would have equal probability of the quantity

$$p_{f_i} Q_i + \frac{r\delta C_i}{H_i - B}$$

being positive or negative. Hence, a counting procedure can be used to test whether the probability of this term being positive is indeed 0·5. As before, the problem of bias in $\hat{\delta} C_i$ must be considered. Here the statistic of interest in the median, and it can be shown that the bias in this quantity is indeed negligible (appendix A).

TABLE IV-2

t Test of Undercapitalization

Cost of Capital	Percent of Sample which H1 Rejected		
	1%	5%	10%
.0100	100.	100.	100.
.0200	100.	100.	100.
.0300	97.	100.	100.
.0400	87.	92.	97.
.0500	77.	87.	90.
.0600	62.	79.	85.
.0700	28.	69.	77.
.0800	18.	36.	67.
.0900	10.	23.	38.
.1000	5.	18.	23.
.1100	0.	10.	18.
.1200	0.	5.	13.
.1300	0.	3.	10.
.1400	0.	0.	5.
.1500	0.	0.	3.
.1600	0.	0.	0.
.1700	0.	0.	0.
.1800	0.	0.	0.
.1900	0.	0.	0.
.2000	0.	0.	0.
.2100	0.	0.	0.
.2200	0.	0.	0.
.2300	0.	0.	0.
.2400	0.	0.	0.
.2500	0.	0.	0.

$$\text{H0:} \quad \frac{p_f Q(H - B)/r + \hat{\delta} C}{\hat{\sigma}_{\delta} C} > 0 \qquad B = 6000$$

$$\hat{\delta} = -.370$$

$$\text{H1:} \quad \frac{p_f Q(H - B)/r + \hat{\delta} C}{\hat{\sigma}_{\delta} C} \leq 0 \qquad \hat{\sigma}_{\delta} = .284$$

The percentage of the sample for which

$$p_{f_i} Q_i + \frac{r\hat{\delta}C_i}{H_i - B}$$

is negative for a range of r and B is displayed in table IV-3. As the results are insensitive to B, the case B = 6,000 will be examined in detail. These results are graphed on figure IV-2 as is a cumulative binomial distribution with $\mu = .5$ and $n = 39$. The 5% and 10% confidence limits can then be transferred and read as limits on the cost of capital. The cost minimizing hypothesis can only be accepted at the 5% confidence level when the cost of capital lies between 16.4% and 18.6%. This tends to contradict the A-J prediction and even suggest the opposite may be true (i.e., plants have been built with less than the optimum efficiency).

Rather than use the point estimate for δ, in view of its relatively large standard error, the above test was repeated using its 95% lower confidence limit (table IV-4). This was plotted over the central range and changes the 5% confidence limits on cost of capital to 7.2% and 8.0% respectively. This is still larger than the apparent cost of capital in this period that was used by Courville, Spann and Petersen, but with the inclusion of a small property tax it appears that the null hypothesis may be accepted.

Thus, within the limits of the data that are readily available, there seems to be no evidence to support the more carefully formulated version of the A-J hypothesis.

TABLE IV-3

B

Cost of Capital	3000	4000	5000	6000	7000	8000
.01	0	0	0	0	0	0
.02	0	0	0	0	0	0
.03	0	0	0	0	0	0
.04	0	0	0	0	0	0
.05	0	0	0	0	0	0
.06	0	0	0	0	0	0.00
.07	0	0	0	0	0.00	5.13
.08	0.00	0.00	0.00	0.00	2.56	7.69
.09	5.13	5.13	5.13	7.69	7.69	10.26
.10	7.69	7.69	7.69	10.26	10.26	10.26
.11	10.26	10.26	12.82	12.82	12.82	17.95
.12	15.38	15.38	15.38	15.38	15.38	17.95
.13	17.95	17.95	20.51	20.51	20.51	25.64
.14	20.51	20.51	20.51	23.08	25.64	28.21
.15	23.08	20.51	20.51	25.64	25.64	30.77
.16	33.33	35.90	33.33	30.77	30.77	33.33
.17	43.59	43.59	46.15	43.59	43.59	35.90
.18	61.54	61.54	58.97	61.54	58.97	38.46
.19	66.67	66.67	71.79	71.79	66.67	48.72
.20	71.79	74.36	74.36	76.92	76.92	64.10
.21	74.36	74.36	74.36	79.49	79.49	71.79
.22	76.92	79.49	82.05	82.05	82.05	71.79
.23	79.49	82.05	82.05	82.05	84.62	82.05
.24	84.62	84.62	84.62	87.18	87.18	82.05
.25	87.18	84.62	87.18	89.74	89.74	82.05

Percentage
of
Sample
Negative

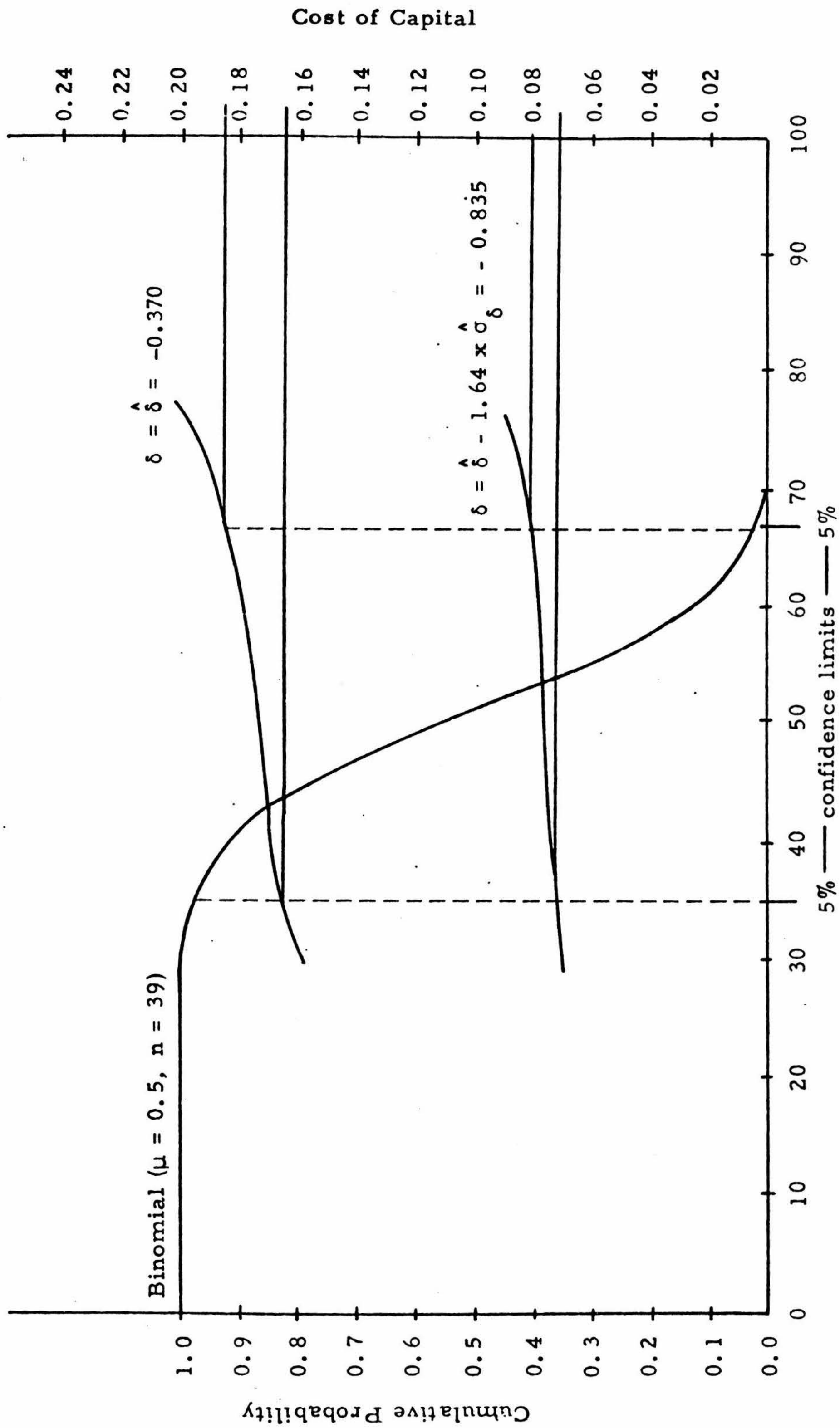

Figure IV-2. % of Sample Negative

TABLE IV-4

Using 95% lower confidence limit for $\hat{\delta}$ i.e., for

B = 6000

δ = -0.835

Cost of Capital	% of Sample Negative
.030	0.00
.032	0.00
.034	0.00
.036	2.56
.038	2.56
.040	7.69
.042	7.69
.044	10.26
.046	10.26
.048	12.82
.050	12.82
.052	12.82
.054	15.38
.056	15.38
.058	20.51
.060	20.51
.062	23.08
.064	23.08
.066	25.64
.068	25.64
.070	28.21
.072	35.90
.074	38.46
.076	43.59
.078	61.54
.080	61.54
.082	66.67
.084	69.23
.086	71.79
.088	74.36

V. CONCLUSIONS

This study has examined the regulated electric utility industry in relation to the well known Averch-Johnson model which predicts overcapitalization. It has been shown that great care is necessary when making the transition from the simplified economic model to the realities of the industry. In particular, close attention must be paid to the measurement of the capital input in order that it may be appropriate to the output being priced. The other studies which attempt to examine this question have all made serious errors in this area. Input choices are not always assessible in quantitative terms as often discrete decisions such as fuel, location and number of units must be made. While there are possibilities in these choices for regulation-induced substitutions, these cannot be evaluated unless the discrete alternatives available to the firm can be obtained. A more serious problem which is encountered when doing empirical work is the separation of unexpected technological change from the A-J effect. This would occur when a plant is unloaded (retired) before its planned service life is up.

The previous empirical literature has been examined in some detail and a number of errors uncovered. The most important of these is the totally inappropriate representation of the technological possibilities facing the firm. It is shown that controlling for the effects of unit size, fuel used and taxation are necessary, as these can all produce spurious results.

The results obtained when a more careful formulation of the hypothesis is tested do not support the Averch-Johnson effect. Specifically, there is no evidence to show that the thermodynamic efficiency has been substituted for fuel in order to increase regulated profits. As this work is carried out on a single cross section of relatively new base load plants, it still leaves the question of early unloading of older, less efficient plants unexamined. It seems unlikely that a definitive answer to this question can ever be obtained.

APPENDIX A

Bias in Mean, Variance and Median of $\hat{\delta C_i}$

As noted in the text, the quantity $\hat{\delta C_i}$ is a biased estimate of δC_i^* (C_i^* being the systematic part of C_i). For the purposes of the two tests used in this paper, of principal interest are the biases in the mean, variance and median of this quantity.

The model that is used to estimate δ is:

$$C_i = e^A e^{\alpha C_D} S_i^{\beta} N_i^{\gamma} (H_i - B)^{\delta} e^{U_i}$$

in which U_i is assumed to be $N(0,\sigma^2)$ and $E(U_i U_j) = 0$ for $i \neq j$. The least squares estimates of the coefficients (after taking logarithms) will differ from the actual value by a linear combination of the U_i.

$$\begin{pmatrix} \hat{A} - A \\ \hat{\alpha} - \alpha \\ \hat{\beta} - \beta \\ \hat{\gamma} - \gamma \\ \hat{\delta} - \delta \end{pmatrix} = (X^T X)^{-1} X^T U$$

where X is the matrix of observations such that the system of equations is

$$C = X \begin{pmatrix} A \\ \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} + U$$

As only $\hat{\delta}$ is of interest, the last row of $(X^TX)^{-1}X^T$ can be designated as a vector with elements K_j. These were evaluated and are listed in table A-1; note that none exceed + 0.39.

i.e., $\hat{\delta} - \delta = \sum_j K_j U_j$

The quantity of interest is $\hat{\delta C_i}$, which is equivalent to

$$C_i^* \left\{ \delta + \sum_{j=1}^{N} K_j U_j \right\} e^{U_i}$$

where C_i^* is the nonstochastic part of C_i.

<u>Mean</u>

The mean of $\hat{\delta C_i}$ (because of the independence of the disturbance terms) is

$$C_i^* \left\{ \delta \int_{-\infty}^{\infty} e^{U_i} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{U_i}{\sigma}\right)^2} dU_i \right.$$

$$\left. + K_i \int_{-\infty}^{\infty} U_i e^{U_i} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{U_i}{\sigma}\right)^2} dU_i \right\}$$

which, after completing the square, gives

$$C_i^* \left\{ \delta + K_i \sigma^2 \right\} e^{\sigma^2/2}$$

The bias is thus

$$E(\hat{\delta C_i}) - C_i^* \delta = C_i^* \left\{ \delta \left(e^{\sigma^2/2} - 1 \right) + K_i \sigma^2 e^{\sigma^2/2} \right\}$$

TABLE A-1

Table of K_i for Data Set A

1	0.03379
2	- 0.13099
3	0.17476
4	- 0.18241
5	- 0.36034
6	0.03084
7	0.15815
8	0.23329
9	0.02782
10	0.28841
11	0.38155
12	- 0.02776
13	0.17115
14	0.39065
15	- 0.10335
16	- 0.21188
17	0.11988
18	- 0.01551
19	- 0.29955
20	- 0.24847
21	- 0.07543
22	- 0.36981
23	- 0.24541
24	0.29296
25	- 0.01419
26	0.14316
27	- 0.28871
28	- 0.04934
29	0.19731
30	- 0.22138
31	0.00128
32	0.32888
33	0.08438
34	0.12424
35	- 0.14388
36	- 0.32003
37	0.23961
38	0.15963
39	- 0.27332

The worst case of the bias, from the point of view of the test using this statistic, occurs for large positive K_i where the bias works against finding the A-J effect. In the actual test it works in favor of accepting the undercapitalization hypothesis. To see what the approximate magnitude of this bias is we substitute the estimates δ, $\hat{\sigma}^2$ and the worst K_i.

$$E(\hat{\delta}C_i) - C_i^* \delta = C_i^* \Big\{ \big(-0.370\big)\big(1.0224 - 1.00\big) + \big(0.3906\big)\big(0.04441\big)\big(1.0224\big) \Big\} = C_i^* \Big\{ 0.00943 \Big\}$$

The effect of this can best be judged by expressing this as a fraction; i.e.,

$$\frac{E(\hat{\delta}C_i) - C_i^* \delta}{C_i^* \delta} = + 0.0254$$

This worst case bias is only approximately 2.5% and, in fact, as only 8 out of the 39 observations have a bias in this direction, we can safely conclude that this does not materially influence the results.

Variance

We are interested in the variance of

$$C_i^* \, e^{Ui} \left\{ \delta + \sum^{N} K_j U_j \right\}$$

about

$$C_i^* \, e^{\sigma^2/2} \left\{ \delta + K_i \sigma^2 \right\}$$

The variance about the origin is

$$= C_i^{*2} \left\{ \left(\sqrt{2\pi}\,\sigma \right)^{-N} \int_{-\infty}^{\infty} \cdots \cdots \int_{-\infty}^{\infty} \left[\delta + \sum^{N} K_j U_j \right]^2 e^{2U_i} \, e^{-\frac{1}{2} \sum^{N} \left(\frac{U_j}{\sigma} \right)^2} \right.$$

$$\left. dU_1 \cdots \cdots dU_N \right\}$$

which by completing the square, gives

$$= C_i^{*2} \left\{ \delta^2 e^{2\sigma^2} + 4\delta K_i \sigma^2 \; e^{2\sigma^2} \right.$$

$$+ \sum_{j \neq i}^{N} e^{2\sigma^2} \int_{-\infty}^{\infty} K_j^2 \, U_j^2 \, \frac{1}{\sqrt{2\pi}\,\sigma} \, e^{-\frac{1}{2} \left(\frac{U_j}{\sigma} \right)^2} dU_j$$

$$\left. + K_i^2 \, e^{2\sigma^2} \left(\sigma^2 + 4\sigma^4 \right) \right\}$$

$$= C_i^{*2} \left\{ e^{2\sigma^2} \sigma^2 \sum_{j=1}^{N} K_j^2 + K_i^2 \, e^{2\sigma^2} \, 4\sigma^4 \right.$$

$$\left. + \delta^2 e^{2\sigma^2} + 4\delta K_i \sigma^2 e^{2\sigma^2} \right\}$$

to find the variance about the mean we subtract

$$C_i{*}^2 \, e^{\sigma^2} \left\{ \delta + K_i\sigma^2 \right\}^2$$

Thus we have

$$C_i{*}^2 \left\{ e^{2\sigma^2} \sigma^2 \sum_{j=1}^{N} K_j^{\,2} + \delta^2 \left(e^{2\sigma^2} - e^{\sigma^2}\right) + \delta K_i \sigma^2 \left(4e^{2\sigma^2} - 2e^{\sigma^2}\right) \right.$$

$$\left. + \; 3\sigma^4 \, K_i^{\,2} \, e^{2\sigma^2} \right\}$$

The first term is simply $e^{2\sigma^2} \, \sigma_\delta^2$. The last term is totally insignificant when worst case K_i are substituted. We thus have after substitution

$$= C_i{*}^2 \left\{ (1.0929)(0.0806) + (0.1369)(1.0929 - 1.0454) \right.$$

$$\left. - \; (0.370) \; K_i \; (0.044415)(4.3715 - 2.0908) \right\}$$

$$= C_i{*}^2 \left\{ (0.0946) - (0.0374) \; K_i \right\}$$

The bias in the standard deviation is approximately

$$C_i^* \left\{ [(0.0946) - (0.0374) \; K_i]^{1/2} - 0.284 \right\}$$

We have only 2 out of the 39 observations where this bias is negative and its maximum magnitude is - 0.3%. The maximum positive bias is only + 16% and as this is in the direction favoring acceptance of this cost minimization hypothesis (against that of undercapitalization) we conclude that the bias in the variance can safely be neglected.

Unfortunately it is difficult to give the exact combined effect of the mean and median as this depends on the individual C_i. We do notice however that these effects work in opposing directions and that this in general should tend to reduce the net bias.

Median

We are interested in the median of

$$C_i^* \left\{ \delta + \sum_{j=1}^{N} K_j U_j \right\} e^{U_i}$$

The median of e^{U_i} is unity, so the first term remains unaffected. The second term depends on the median of $U_i e^{U_i}$, which is not easily computed analytically. This was simulated by drawing a sample of size 1,000 from a $N(0,\sigma^2)$ distribution and computing the variable $U_i e^{U_i}$. The median of this variable, with $\sigma = 0.2107$, was 0.0034. The worst case, again when the largest K_i is used, yields, on replacement of δ by its estimate, C_i *(-0.370 + .00132), which is a bias against finding the A-J effect, but of negligible magnitude.

APPENDIX B

The Data Used

1. The data set which is identified as A in the text was constructed by replicating Courville's 1960-1966 vintage plants. As noted in the text, a criterion of best year of operation was used. The sample is listed in the following table (table B-1) where the following notation is used.

name - the name as listed in the FPC index

a - the year of observation

b - the capacity of the plant in megawatts

c - the number of units in the plant

d - the plant factor in per cent

e - the heat rate in BTU/kwh

f - the cost of structures in \$1,000

g - the cost of equipment in \$1,000

h - the annual output in million kwh

i - the total fuel used in 10^6 MBTU

j - the price of fuel in ¢/MBTU

k - type of construction (C = conventional, S = semioutdoor, 0 = outdoor) l - coal used as fuel

m - oil used

n - gas

2. The data used to replicate Spann's plant results are listed in the following table (table B-2). The columns contain the following:

a - the page in the FPC reports from which data come

b - the year of the observation

c - the total cost of the plant in $1,000

d - the fuel cost in $1,000

e - the other production costs in $1,000

f - the total fuel used in 10^6 MBTU

g - the plant load factor

h - the 3 year average rate of return

3. The data set used in discussing Petersen's work is listed in the following table (table B-3). The plant name, state, capacity in megawatts and fuels used are shown.

TABLE B-1

Data Set A

Name	a	b	c	d	e	f	g	h	i	j	k	l	m	n
Greene County	68	568	2	73	9401	9461	45603	3634.7	34.169	22.66	C	x		
Cholla	64	115	1	92	9632	1833	18390	931.4	8.946	23.66	O	x		
Helena	66	325	1	69	9902	4170	38653	2166.7	21.424	26.34	O			x
South Bay	68	474	3	74	9788	9219	40451	3082.0	30.169	33.85	C		x	x
Cool-Water	62	65	1	80	9963	797	10333	457.0	4.554	34.15	O		x	x
Cape Kennedy	66	402	1	58	9461	4683	23670	2055.4	19.446	32.30	O		x	
Lansing Smith	68	340	2	75	10019	7022	38808	2231.9	22.342	25.67	C	x		
Harllee Branch	66	299	1	59	9692	6350	25709	1557.8	15.077	30.61	C	x		
Bailly	63	194	1	75	9584	8474	25904	1270.6	12.178	27.14	C	x		x
McDonough	68	598	2	75	9873	7912	43721	3963.8	39.115	25.20	C	x		
Coffeen	66	330	1	78	9930	11066	31948	2258.2	22.408	17.20	C	x		
Breed	66	450	1	80	8957	12824	59884	3155.7	28.256	19.01	S	x		
Neal	69	147	1	72	10090	2962	17024	930.5	9.381	30.48	C	x		x
Cimmaron River	66	59	1	66	12099	917	6304	339.5	4.108	34.55	S			x
Gordon Evans	66	150	1	83	9886	2286	14467	1092.1	10.795	21.65	O			x
Big Sandy	65	265	1	93	8959	6119	31642	2165.0	19.388	16.16	C	x		
Little Gypsy	68	669	2	78	9832	4399	46194	4602.0	45.246	19.25	O			x
Crane	69	400	2	77	9541	16256	48810	2695.2	25.711	31.90	C	x	x	
Chalk Point	67	728	2	76	8700	12225	77220	4823.0	41.958	30.59	C	x		
New Boston	66	359	1	79	9034	4571	27937	2487.1	22.468	32.01	C		x	
Mt. Tom	61	125	1	95	9685	5186	18273	1036.1	10.043	33.60	S	x		
Brayton Point	65	483	2	91	8811	12130	57361	3822.1	33.650	34.77	C	x		
Campbell, J. H.	65	265	1	84	8905	13037	33686	1953.9	17.389	32.10	C	x		
Sibley	64	100	2	78	11476	3180	14423	687.1	7.883	24.74	S	x		
Reid Gardiner	69	228	2	81	9948	3276	27982	1617.5	16.062	28.32	S	x		
Tracey	68	133	2	63	11295	2054	13668	739.0	8.347	36.84	S		x	x
Sunrise	67	82	1	74	9942	2111	9277	525.4	5.222	36.07	S			x
Merrimack	67	114	1	84	9805	5188	17039	833.7	8.167	34.30	S	x		
Hudson	66	455	1	64	9339	5334	66522	2543.8	23.755	31.14	O	x	x	
Mercer	64	653	2	79	8878	11210	100612	4532.4	40.241	27.63	S	x		x
England, B. L.	68	300	2	77	9777	5783	37648	2036.5	19.949	31.20	S	x		
Four Corners	65	634	3	76	10277	3600	75481	4226.0	43.386	13.32	O	x		x
Raveswood	64	800	2	65	9631	20477	90954	4588.8	44.201	34.81	C		x	x
Roxboro	67	411	1	76	9271	8186	27449	2739.8	25.373	27.72	O	x		
Ashville	69	207	1	79	9221	3034	18716	1438.2	13.249	30.93	O	x		
Marshall	68	700	2	99	8690	13751	60949	6359.6	55.263	25.92	C	x		
Northeastern	69	170	1	84	10580	4061	17765	1253.4	13.257	19.33	O			x
Brunner Island	68	768	2	76	9439	11119	68039	5156.7	48.551	25.79	O	x		
Canadys	66	272	2	71	9310	2642	29686	1698.9	15.809	30.86	S	x		x

TABLE B-2

a	b	c	d	e	f	g	h
4	63	20223	1970	570	8.2711	85	6.503
8	63	43221	3854	422	14.9033	53	6.670
13	61	21690	3056	540	9.1147	78	6.333
11	62	12226	1555	326	48.3030	80	5.900
67	63	34520	3307	721	12.1782	79	8.447
43	61	72787	4866	1149	26.0382	67	6.503
85	63	16884	2085	326	9.7927	76	7.087
92	63	37946	2671	809	16.6877	81	9.123
107	62	38001	3857	842	11.6870	75	6.783
100	62	27756	3479	416	16.0419	77	7.320
112	62	23616	3420	569	10.5271	94	8.313
116	63	48469	4577	1029	14.1276	69	6.843
139	63	17871	1878	218	14.4444	69	6.043
86	61	22383	2768	405	6.9575	74	6.023
150	63	115241	11475	2386	38.4321	75	7.123
145	63	23710	2476	673	7.8589	68	6.813
97	64	113680	15393	2354	44.2010	65	5.313
193	62	22385	2109	452	10.4102	69	7.537
201	63	45674	5812	999	21.6205	73	6.537
213	63	19449	2521	266	8.4961	79	7.667
59	61	43682	4497	527	20.6165	57	7.313
15	61	50452	10782	964	32.1942	92	6.653
3	63	116856	17232	1357	66.9311	78	7.860
5	62	25238	5263	445	15.7863	80	6.503
12	62	11425	1718	461	10.6044	65	5.900
31	61	30175	3861	577	11.6941	94	6.773
44	63	47072	9090	768	27.7750	75	8.123
71	63	87250	7187	1673	37.4764	73	6.773
99	63	23195	2531	380	11.5098	78	7.313
147	62	115462	12674	2584	37.5460	70	7.123
153	63	21544	2809	330	12.7305	77	8.050
168	63	27914	3265	400	11.4047	65	6.863
186	63	34889	2238	886	11.0153	50	7.203
207	62	157426	14338	2342	41.9304	79	6.420
265	63	29128	2906	415	10.6659	54	7.270
268	63	35313	1164	699	14.5545	76	6.343
117	63	77324	9904	991	31.1856	75	6.843

TABLE B-3

Plant Name	State	Capacity	Coal	Oil	Gas
Pittsburg	California	951		x	x
Huntington Beach	California	653		x	x
Cameo	Colorado	66	x		
Meredosia	Illinois	300	x		
New Albany	Indiana	450	x		
Lawrence	Kansas	211	x		x
Clay Boswell	Minnesota	128	x		
Gulf Coast	Mississippi	296			x
Bergen	New Jersey	580	x		x
Reeves	New Mexico	175			x
Port Jefferson	New York	400	x	x	
Dunkirk	New York	560	x		
Elrama	Pennsylvania	447			
Bates	Texas	166			x
T. H. Wharton	Texas	323			x
Willow Island	West Virginia	215			
Morro Bay	California	1056		x	x
South Bay	California	474		x	x
Etiwanda	California	911		x	x
Norwalk Harbor	Connecticut	326	x	x	x
P. L. Bartow	Florida	494		x	x
Riviera	Florida	738		x	x
Will County	Illinois	1268	x		
State Line	Indiana	972	x		x
Tecumseh	Kansas	346	x		x
Arthur Mullergren	Kansas	133		x	x
Sibley	Missouri	100	x		
Sewaren	New Jersey	841	x		
Barret E. F.	New York	374	x	x	x
Lake Shore	Ohio	512	x		
Conesville	Ohio	434	x		
Portland	Pennsylvania	383	x		
Handley	Texas	523			x
Nelson Dewey	Wisconsin	228	x		
Four Corners	Arizona	634	x		x
Contra Costa	California	1276		x	x
El Segundo	California	1017		x	x
Cool-Water	California	147			x
Valmont	Colorado	274	x		x
Middletown	Connecticut	422	x	x	
Port Everglades	Florida	1255		x	
Wood River	Illinois	650	x		x
Des Moines	Iowa	325	x		x
Graham	Maine	58		x	
England B. L.	New Jersey	299	x		
North Lake	Texas	708			x
Port Wentworth	Georgia	208	x		
Hutchison	Kansas	252		x	x
Little Gypsy	Louisiana	669			x
Wyman Walter F.	Maine	214		x	
Ravenswood	New York	1827		x	x
Brunner Island	Pennsylvania	768	x		
Nueces Bay	Texas	258			x
Webster	Texas	614			x
Stryker Creek	Texas	703			x

APPENDIX C

Some Determinants of the Cost of Capital Equipment

While many of the assertions made previously about the relationships between the various attributes of capital equipment and its cost are deducible from physical principles, some empirical verification is desirable. This appendix reports the results of a number of simple regressions which were run in an attempt to explain the component costs of capital equipment. Two different data sets were used, each of which has its own inadequacies.

The data set earlier described as A (Courville's 1960-1966 group) is used, though the capital costs are disaggregated into structures and equipment. The costs are expressed per unit (rather than per plant), as a unit is the fundamental piece of equipment. This assumes plants composed of identical units, which was in general the case with this sample. No deflator was used on the costs.

The second data set was constructed from the biannual surveys of construction costs published in Electrical World. This survey provides much more disaggregated cost figures than does the FPC reports, but only identifies the plants by number (thus making very tedious the task of combining data sets). A plant appears in the survey each time a unit is added, but the figures reported are plant aggregates. In an attempt to isolate unit characteristics, only those plants in which the units were in the same capacity range were included. As capacity was only reported as an ordinal number, the capacity was computed from the peak output deflated by the utilization factor. Three measures of heat rate are given: the actual net, the actual gross and the design value. None of these proved useful in explaining costs. The reasons are, most probably, that the actual figures suffer from being measured in the first year of operation and that the design data appear to have been very loosely collected (on examination it appears that the questionnaire did not make clear whether net or gross design values were desired). The data spanned the years 1956-1965, and as this included the years of the "electric conspiracy," the Handy-Whitman regional deflators for each account were applied, converting costs to 1949 dollars.

For both data sets log-linear equations were fitted. The restrictions imposed by the functional form are recognized, but the small number of data points, the qualitative nature of the investigation, and the simplicity of estimation are the justification for its use. In these regressions, use was made of qualitative data on fuel type, construction type and geographical location by introducing dummy variables. The results are presented in tables C-1 and C-2, with the absence of an entry meaning that the particular variable was not included in the list of regressors. The standard errors of the estimates are shown in parentheses.

Data Set A:

The results using data set A support the intuitive notions about the determinants of cost. The structural cost is apparently not influenced by the efficiency of the plant. There appear to be both economies of scale with size and number of units housed in the structure. Fully outdoor construction requires significantly less expensive structures, and the use of coal requires significantly more expensive. The cost of equipment has significant economies of scale but is influenced far less than the structural cost by the number of units in the plant. The more efficient a unit, the higher is its cost, but the estimates are not significantly different from zero. The equipment costs increase, as expected, with outdoor construction and the use of coal as a fuel.

Electrical World Data Set:

With the greater disaggregation of costs in this data set, it was hoped that better estimates could be achieved. The results were disappointing in one major respect: no satisfactory variation in costs with efficiency could be obtained. This is apparently due to two factors: the actual heat rates given are plant averages during the startup year of a new unit, and the design values given seem not to be consistently gross or net values. Nevertheless, the results agree well with the previous data set in several respects. It appears that all cost components are subject to increasing returns with unit size, though electrical accessories only slightly. Only structural cost and miscellaneous equipment have increasing returns with the number of units in a plant. Boiler and turbogenerator costs actually seem to increase with the number of units, a result that is evidence of crowding diseconomies. The regional dummies capture two distinct effects: that due to climate and that due to different costs of local labor and materials. Turbogenerators vary little, as these are usually shipped in a preassembled condition. Boiler costs are significantly more expensive in the North Atlantic region as is miscellaneous equipment. Unfortunately, little can be said concerning the effect of heat rate; not even consistency of sign was obtained. It is mainly for this reason that no further use was made of this data set.

TABLE C-1

DATA SET A

Estimated Coefficients

Dependent Variable	C	LUS	LNU	LHR	SOD	FOD	CD	GD	R
LSC	6.67	0.867	-0.581	-0.298	-0.159	-0.659	0.229	0.040	0.887
		(0.161)	(0.170)	(1.352)	(0.176)	(0.159)	(0.176)	(0.238)	
LEC	14.98	0.755	-0.092	-0.995	0.111	0.037	0.171	0.001	0.936
		(0.092)	(0.097)	(0.776)	(0.101)	(0.091)	(0.101)	(0.136)	

39 observations

Variables:
- LSC - log [structural cost in $1000]
- LEC - log [equipment cost in $1000]
- C - constant
- LUS - log [unit size in megawatts]
- LNU - log [number of units in plant]
- LHR - log [heat rate in BTU/kwh]
- SOD - dummy equal to one if semi-outdoor construction
- FOD - dummy equal to one if full outdoor construction
- CD - dummy equal to one if any coal used as fuel
- GD - dummy equal to one if only gas used as fuel

TABLE C-2

ELECTRICAL WORLD DATA SET

Estimate of Coefficient

Dependent Variable	C	LUS	LNU	LDH	LAH	CD	GD	PAC	PLT	NCN	SCN	NAT	R
LBC	2.336	0.906	-	-	0.155	0.437	-0.220	-	-	-	-	-	0.944
		(0.063)	-	-	(0.468)	(0.061)	(0.083)	-	-	-	-	-	
LBC	8.242	0.846	-	-0.454	-	0.422	-0.207	-	-	-	-	-	0.944
		(0.067)	-	(0.526)	-	(0.060)	(0.084)	-	-	-	-	-	
LBC	1.771	0.860	0.122	0.228	-	0.405	-0.131	-0.114	0.063	0.092	-0.027	0.282	0.961
		(0.065)	(0.051)	(0.502)	-	(0.066)	(0.092)	(0.112)	(0.101)	(0.069)	(0.089)	(0.067)	
LSC	4.467	0.711	-0.174	-	-	-0.081	-0.642	-0.693	-0.667	-0.386	-0.281	0.231	0.799
		(0.114)	(0.133)	-	-	(0.171)	(0.241)	(0.295)	(0.258)	(0.182)	(0.231)	(0.176)	
LTC	- 0.443	0.806	0.144	-	0.519	-0.037	-0.131	-0.101	0.022	-0.019	-0.098	0.035	0.937
		(0.058)	(0.049)	-	(0.412)	(0.062)	(0.087)	(0.108)	(0.094)	(0.066)	(0.084)	(0.064)	
LEC	-11.444	0.989	-0.009	1.447	-	0.200	-0.158	-0.206	0.050	-0.231	-0.199	0.191	0.866
		(0.115)	(0.089)	(0.882)	-	(0.115)	(0.161)	(0.197)	(0.178)	(0.122)	(0.156)	(0.119)	
LMC	-19.664	0.857	-0.306	-	2.160	0.502	-0.123	1.143	0.184	0.416	0.432	0.732	0.699
		(0.222)	(0.172)	-	(1.701)	(0.[illegible]22)	(0.311)	(0.380)	(0.343)	(0.235)	(0.302)	(0.230)	

86 observations

TABLE C-2 (Continued)

Variables:

LSC - log [structural cost/unit in $1,000, FPC acct. no. 311]

LBC - log [boiler cost/unit in $1,000, FPC acct. no. 312]

LTC - log [turbogenerator cost/unit in $1,000, FPC acct. no. 314]

LEC - log [electrical accessories cost/unit in $1,000, FPC acct. no. 315]

LMC - log [misc. equip. cost/unit in $1,000, FPC acct. no. 316]

All deflated to 1949 dollars by the appropriate Handy-Whitman regional index.

C - constant

LUS - log [unit size in megawatts]

LNU - log [number of units]

LDH - log [design heat rate in BTU/kwh]

LAH - log [actual net heat rate in BTU/kwh]

CD - dummy equal to one if any coal used

GD - dummy equal to one if only gas used

PAC - dummy equal to one for Pacific region

PLT - dummy equal to one for Plateau region

NCN - dummy equal to one for North Central region

SCN - dummy equal to one for South Central region

NAT - dummy equal to one for North Atlantic region

The reference dummy for fuel type is oil and oil-gas stations.

The reference dummy for region is the South Atlantic region.

APPENDIX D

Cumulative Binomial Distribution (see Fig. IV-2)

n = 39, p = 0.5

% of n	Cumulative Probability
97.43	0.
94.87	0.
92.30	0.000
89.74	0.000
87.17	0.000
84.61	0.000
82.05	0.000
79.48	0.000
76.92	0.000
74.35	0.001
71.79	0.004
69.23	0.011
66.66	0.026
64.10	0.054
61.53	0.099
58.97	0.168
56.41	0.261
53.84	0.374
51.28	0.500
48.71	0.625
46.15	0.738
43.58	0.831
41.02	0.900
38.46	0.945
35.89	0.973
33.33	0.988
30.76	0.995
28.20	0.998
25.64	0.999
23.07	0.999
20.51	0.999
17.94	0.999
15.38	0.999
12.82	0.999
10.25	1.000
7.69	1.000
5.12	1.000
2.56	1.000

APPENDIX E

Comparative Statics

The inclusion of taxation does not change the standard comparative statics results concerning the effect of tightness of regulation in the total amount of capital used. In addition to this we also get unambiguous results concerning the effect of changing the tax rates. As before, the sign of the changes in labor with these parameters is not fixed unless more stringent assumptions are made on the revenue function. By totally differentiating the first order conditions and the constraint we get the following matrix equation of variations.

$$\begin{bmatrix} (1 - c)(1 - \lambda)R_{kk} & (1 - c)(1 - \lambda)R_{lk} & (s - q)/(1 - \lambda) \\ (1 - c)(1 - \lambda)R_{kl} & (1 - c)(1 - \lambda)R_{ll} & 0 \\ (s - q)/(1 - \lambda) & 0 & 0 \end{bmatrix} \begin{bmatrix} dK \\ dL \\ d\lambda \end{bmatrix} =$$

$$\begin{bmatrix} -\lambda & (1-\lambda)(R_k-p-a-ib) & (1-c)(1-\lambda) \\ 0 & 0 & 0 \\ -K & (p+a+ib)(K+wL-R) & -(1-c)K \end{bmatrix} \begin{bmatrix} ds \\ dc \\ dp \end{bmatrix}$$

This leads immediately to

$$\frac{dK}{ds} = \frac{-(1-\lambda)K}{s-q} < 0$$

which is the standard result showing that the use of capital increases as the allowed rate of return is reduced. The following results are also simply derived.

$$\frac{dK}{dc} = -\frac{(1-\lambda)(R-wL-(p+a+ib)K)}{s-q} < 0$$

and

$$\frac{dK}{dp} = -\frac{(1-\lambda)(1-c)K}{s-q} < 0$$

These last two conditions give the expected results that the use of capital declines with an increase in either of the tax rates. This unfortunately is the most that can be extracted from the analysis unless stronger conditions are placed on the allowed form of R.

APPENDIX F

Separate Estimations of Spann's Equations

Results

$$\frac{rK}{PQ} = \underset{(0.1088)}{0.5992}\,\frac{sK}{PQ} + 0.0119 + \underset{(0.0179)}{0.00029}\,\log K + \underset{(0.0197)}{0.00616}\,\log F$$

Std. error = 0.0441

R = 0.7234

$$\frac{P_f F}{PQ} = -\,0.1590 + \underset{(0.0311)}{0.0717}\,\log F - \underset{(.0280)}{0.0610}\,\log K$$

Std. error = 0.0754

R = 0.3814

Independence of Disturbance Terms in these Estimations

The correlation between the appropriate paired disturbance terms is

$r = -0.1185$

If the disturbance terms are normally distributed then this correlation coefficient can be transformed into an approximately normal variate with mean

$$= 1/2 \log_e \frac{1 + p}{1 - p}$$

and variance

$$= \frac{1}{n-3}$$

where n is the sample size and p is the population correlation coefficient. This leads to a transformed value of 2.03 when testing the above statistic against the no correlation hypothesis. Thus the pairwise correlation is barely significant at the 5% level.

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Chapter 2

ELECTRICITY PRICING

I Introduction

There are two reasons why a special theory* of electricity pricing has arisen. The first is that electricity belongs to a class of commodities that are extremely expensive to store. Consequently short run demand variations cannot be handled by inventory adjustment. The second reason is that in general electricity supply is in the hands of public or private monopolies. We would therefore like to know socially optimal pricing rules in order to operate the public monopolies efficiently and to regulate the private monopolies correctly. These problems are common to other public utilities but as the technologies are sufficiently specific we will limit the scope of this study to the case of electricity.

The non-storable nature of electricity and the fact that productive capacity cannot be rapidly varied, has led to what is known as the peak load problem. The instantaneous demand for electricity varies cyclically within a day and year. Price schedules and capacity choices must be made in advance, and it is this problem that the economic theory addresses. The conclusion is the familiar one of marginal cost pricing, but great care must be taken in correctly defining the marginal costs. The capacity should be adjusted so that the sum of its marginal value to society over all periods equals the marginal cost of capacity. The distinction between expansion and contraction costs is important and must be carefully handled.

*For an extensive bibliography see (1)

In addition to the regular variation in demand there are random contributions to demand due to things like weather conditions. This has motivated attempts to extend results of the deterministic case to conditions of uncertainty. The major difficulty encountered within this framework is the lack of a satisfactory method for handling the occasional occurrence of excess demand. Excess demand requires that there be some mechanism to allocate available output among consumers. Blackouts and brownouts are not particularly attractive methods of doing this as they impose costs which are difficult to evaluate. In fact, if consumers have convex preferences, it can be shown that in general blackouts are an inefficient method of rationing.

Recent innovations in electronics appear to provide a solution to this difficulty. The application of microprocessors to the metering and load management tasks introduces the possibilities of improving the economic transaction when electricity is sold. This technology makes individual rationing in real time possible, thus suggesting a new class of mechanisms for pricing electricity.

These mechanisms involve the introduction of capacity rights which consumers must purchase (rent) in advance. These options on the output of a plant are ordered in priority of service, with the level of priority being controlled by the plant manager. We show that this scheme is economically efficient and provides opportunities for randomly available generation facilities to be integrated into the system. This new control equipment allows us to set up a limited spot market in electricity which produces a more satisfactory allocation of resources than if no response to the random event were possible.

The major advantage of this arrangement is the way it handles uncertainty. In addition it does provide a mechanism with which it is possible to introduce competition into the generation sector of the industry. This is achieved by specifying that all capital must be raised through the sale of capacity rights. A large number of investors, each attempting to maximise the return on his investment, will produce a competitive rental market in these rights.

II Pricing Under Certainty

The published work dealing specifically with pricing problems applicable to electricity supply is somewhat repetitive. Most of these articles add little of practical significance to the results contained in Houthakker's(2) and Boiteux's(3) original contributions. There are of course interesting methodological variations, and the following discussion presents the essential features of this work.

As is common in this kind of welfare economics the general approach is to maximise a welfare function with respect to prices and capacity, subject to the particular constraints. Two models are presented in this section which demonstrate the major results of the literature and the dependence of these results on the common assumptions. Two main classes of assumptions are made; the first characterises the welfare criterion and the second characterises the production technology.

The most commonly used welfare criterion is consumers' surplus minus production costs. The demands in different periods are generally assumed to be independent. Steiner(4) provides a brief analysis of the dependent case, and Pressman(5) examines it somewhat more rigorously but as will become clear from the second model, none of the results concerning efficiency depend in a crucial way on this assumption. A different welfare criterion was introduced to this problem by Mohring(6) (consisting of an arbitrary differentiable function of the individual consumers' utilities). It is reassuring to know that none of the results are sensitive to the welfare specification.

Production technologies are represented by two distinct approaches. The first is to assume that for each plant running costs are linear with output but become infinite at capacity and that capacity costs are linear with capacity. This method has often been used with the further assumption that there is only a single type of plant. It will be shown that this particular simplification has often led to conclusions which are of little practical significance. The second has been to use a neoclassical production or cost function.

Model I

In this model a simple linear technology and a welfare criterion of consumers' surplus are assumed. The results are widely known but have often been advocated as a policy prescription in situations where the simple assumptions are clearly not true.

Assume:

1/ The welfare criterion is: maximise consumers' surplus minus production costs.

2/ The period of time considered (one demand cycle) is divided into T equal length subperiods during which demand conditions are constant.

3/ Aggregate demand in each sub period can be represented by a simple invertible demand function of the price in that period (no interperiod substitution).

4/ The marginal production cost b is constant with output up to the point where output equals capacity, and infinite in excess of capacity.

5/ The marginal cost of capacity c is also constant and is the rental cost of the generating equipment over all T subperiods.

The notation used is:

$p_j(x_j)$ - inverse demand function in period j

x_j - output in period j

K - capacity of plant

The optimal prices and capacity are determined by the solution of the following programming problem.

$$\max \quad \sum_j \int_0^{x_j} p_j(s)\, ds \; - \; \sum_j b x_j \; - \; cK$$

subject to

$$K - x_j \geq 0 \qquad \text{for all } j \text{ (capacity constraint)}$$

Form the Lagrangian

$$L = \sum_j \int_0^{x_j} p_j(s)\, ds \; - \sum_j bx_j \; - cK + \sum_j w_j(K - x_j)$$

First order conditions for constrained maximum are

$K - x_j = 0$ and $w_j > 0$ (for active constraint)

$p_j(x_j) = b + w_j$ for $x_j > 0$ (interior max)

$c = \sum_j w_j$ for $K > 0$ (optimal capacity)

These conditions are easily interpreted as prescribing that the price should be set equal to the marginal operating costs during any subperiod when the capacity constraint is not active. The price should be raised sufficiently to reduce demand to capacity in subperiods when capacity is scarce. Capacity is optimal when the sum of its marginal value over all periods equals its marginal costs. Under this linear technology revenues equal costs.

These are equilbrium conditions and thus the periods which are peak under uniform pricing should not necessarily be charged all of the capacity charges. Even with independent demands it is quite possible for an off peak period to become the peak period should this be done. As will be shown the same conditions can be deduced when the independence assumption is relaxed. The only difference is that the calculation of the capacity charges must also take into account the substitution effects.

Earlier we noted that the above representation of the technology was quite restrictive. In the next model we shall present a generalisation where it is assumed that there are many plant types. These are assumed to have a variety of running and capacity costs. We do make the simplifying assumption that all dominated* technologies are removed from consideration.

Model II also demonstrates the welfare approach taken by Mohring who relaxed the independence of demand assumption and the restrictions of consumers' surplus by utilising a social welfare function. We will employ a linear heterogeneous model of the technology which contrasts with the neoclassical cost function employed by Mohring.

Model II

This model relies on a two stage maximisation procedure to find the optimal prices and capacities. First, individuals are assumed to maximise utility subject to their budget constraints. Second, by considering the variations in utility caused by varying the prices an arbitrary welfare function is maximised. Lump sum taxes are required to achieve a satisfactory income distribution.

* a dominated plant is one which has zero capacity in the optimal solution.

Each consumer is assumed to maximise

$$U_i(x_{i1}, , , , ,x_{ij}, , ,x_{im},y_i)$$

subject to the individual budget constraint

$$I_i - h_i - \sum_j p_j x_{ij} t_j - y_i = 0$$

where

I_i - is the ith individual's income

h_i - is the ith individual's head tax(transfer payment)

p_j - is the price of electricity in period j

x_{ij} - is the ith individual's power consumption in jth period

t_j - is the length of subperiod j (x is measured in kilowatts)

y_i - is the ith individual's consumption of other goods

The use of power rather than energy in the utility function is arbitrary. As it only involves a simple transformation, none of the results are changed if it is deemed more plausible to use energy.

let

$$H_i = U_i + v_i(I_i - h_i - \sum_j p_j x_{ij} t_j - y_i)$$

then the first order conditions are:*

$$0 = \frac{\partial U_i}{\partial y} - v_i$$

* Note that for simplicity the individual subscript has been omitted in the partial derivatives where the meaning is obvious.

$$0 = \frac{\partial U_i}{\partial x_j} - v_i p_j t_j$$

$$0 = I_i - h_i - \sum_j p_j x_{ij} t_j - y_i$$

We assume that there is a welfare maximiser who sets the prices p_j, the transfer payments h_i, chooses the output q_{jk} and also chooses the the capacity of each type of generation equipment K_k.

The problem is thus to

$$\max Z(U_1,,,,U_i,,,,,U_N)$$

subject to

$$M - \sum_k c_k K_k - \sum_j t_j \sum_k b_k q_{jk} - \sum_i y_i = 0$$

(transformation function)

$$\sum_i x_{ij} - \sum_k q_{jk} = 0 \quad \text{for all } j$$

(no storage condition)

$$K_k - q_{jk} \geq 0 \quad \text{for all } j,k$$

(capacity constraint for each plant)

where

M - total resources of economy in units of y

c_k - the constant marginal capacity cost of type k equipment

b_k - the constant marginal running cost of type k equipment

K_k - the capacity of plant type k

q_{jk} - the output of plant k in the jth period

Z - differentiable welfare function of individual utilities

We form the Lagrangian L by adjoining the constraints to the objective function:.

$$L = Z + z(M - \sum_k c_k K_k - \sum_j t_j \sum_k b_k q_{jk} - \sum_i y_i)$$

$$+ \sum_j \sum_k w_{jk}(K_k - q_{jk}) + \sum_j a_j(\sum_i x_{ij} - \sum_k q_{jk})$$

The resulting first order conditions are

$$(1) \quad \frac{\partial L}{\partial h_i} = \frac{\partial Z}{\partial U_i}(\sum_j (\frac{\partial U_i}{\partial x_j}\frac{\partial x_{ij}}{\partial h_i}) + \frac{\partial U_i}{\partial y}\frac{\partial y_i}{\partial h_i})$$

$$+ \sum_j a_j \frac{\partial x_{ij}}{\partial h_i} - z\frac{\partial y_i}{\partial h_i} = 0 \quad \text{for all } i$$

$$(2) \quad \frac{\partial L}{\partial p_n} = \sum_i \{\frac{\partial Z}{\partial U_i}(\sum_j (\frac{\partial U_i}{\partial x_j}\frac{\partial x_{ij}}{\partial p_n}) + \frac{\partial U_i}{\partial y}\frac{\partial y_i}{\partial p_n})$$

$$+ \sum_j a_j \frac{\partial x_{ij}}{\partial p_n} - z\frac{\partial y_i}{\partial p_n}\} = 0 \quad \text{for all } n$$

$$(3) \quad \frac{\partial L}{\partial K_k} = -zc_k + \sum_j w_{jk} = 0 \quad \text{for all } k$$

(4) $$\frac{\partial L}{\partial q_{jk}} = -zt_j b_k - w_{jk} - a_j = 0 \quad \text{for} \quad q_{jk} > 0$$

From the individual's budget constraints, derive allowable variations in y_i.

$$\frac{\partial y_i}{\partial p_n} = -x_{in}t_n - \sum_j p_j t_j \frac{\partial x_{ij}}{\partial p_n}$$

$$\frac{\partial y_i}{\partial h_i} = -1 - \sum_j p_j t_j \frac{\partial x_{ij}}{\partial h_i}$$

Substituting these variations and the individual first order conditions to eliminate

$\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$ we get

(1a) $$0 = \frac{\partial Z}{\partial U_i}(-v_i) + \sum_j a_j \frac{\partial x_{ij}}{\partial h_i} + z(1 + \sum_j p_j y_j \frac{\partial x_{ij}}{\partial h_i}) \quad \text{for all } i$$

(2a) $$0 = \sum_i \{\frac{\partial Z}{\partial U_i}(-v_i t_n x_{in}) + \sum_j a_j \frac{\partial x_{ij}}{\partial p_n}$$

$$+ z(t_n x_{in} + \sum_j p_j t_j \frac{\partial x_{ij}}{\partial p_n})\} \quad \text{for all } n$$

Now multiply (1a) by $t_n x_{in}$ and sum over all individuals and then subtract from (2a)

$$(5) \quad 0 = \sum_i \sum_j (zp_j t_j + a_j)\left(\frac{\partial x_{ij}}{\partial p_n} - t_n x_{in} \frac{\partial x_{ij}}{\partial h_i}\right) \quad \text{for all n}$$

The second parenthesised expression is simply the rate at which the ith individual's consumption of electricity in period j changes with changes in the price of electricity in period n when his income is simultaneously adjusted so as to leave him on the same indifference surface.(Hicks-Slutsky pure substitution effect).

Define an aggregate Slutsky term:

$$S_{jn} = \sum_i \left(\frac{\partial x_{ij}}{\partial p_n} - t_n x_{in} \frac{\partial x_{ij}}{\partial h_i}\right)$$

Rewrite (5) as a matrix equation

$$S.f = 0$$

where f is a vector whose jth element is $(zp_j t_j + a_j)$.

S is negative definite, (by the second order conditions for individual utility maximisation), so it has an inverse, and hence $f_j = 0$ for all j.

This condition when interpreted in conjunction with equations (3) and (4) leads to two possible situations:.

(a) If in period j all plants which are producing any output are running at capacity then we get the following optimal price

$$p_j = b_k + w_{jk}/(zt_j)$$

where k is any plant which is operating. This occurs where all operating plants are running at capacity but the optimal price is still less than the running costs of the next most expensive to run plant. This is identical to the peak period case derived in the previous model when only one type of plant was available. Note once again that the summation over all periods of the value of the plants capacity w_{jk} should be equal to the marginal capacity cost. This condition now yields the optimal plant mix as well as the total capacity. The above pricing rule contrasts with the usual conclusions reached from the single technology model in that even when system capacity is not being completely utilised there should be capacity charges in some off peak periods.

(b) If in period j some plant is operating but not producing at capacity then the optimal price should be set equal to the running cost of that plant. i.e.

$$p_j = b_{k'}$$

where $q_{jk'} > 0$ and $q_{jk'} < K_{k'}$

These rules are obtained by assuming that the cost minimising operating procedure of merit order loading is being followed. Thus no plant will be turned on until all plants with lower running costs are producing at capacity. If capacity is optimal there will be at least one period when case (a) holds. A special case of these conditions arises if the operating times of the plants coincide with the pricing periods.

III Pricing Under Uncertainty

The results of the previous section provide us with optimal pricing rules for electricity if the assumptions of the models are satisfied. One major objection to these results is that the demand conditions are unlikely to be deterministic as has been assumed. Random rather than purely periodic variations are to be expected and this has motivated the attempts to extend the theory. The uncertainty covered in this section is limited to short term random fluctuations about a known mean. This is important and represents quite accurately the effect of weather conditions. There is however another source of uncertainty which needs to be considered for this industry, and this is the uncertainty in long term demand estimates. Investment decisions are required well in advance of actual production, and if future demand conditions are not accurately known, the poor decisions which result can be very costly. We go some way to providing solutions to this problem in Section V where new mechanisms are considered.

In this section we present two models which introduce uncertainty into electricity pricing in totally different ways. The first model describes Boiteux's approach* which differs quite drastically from the

*Boiteux's(7) works on uncertainty are not available in English but Dreze(8) has produced an excellent summary of them.

models of section II. He attempts to identify the stochastic elements which are costly and then evaluates their marginal costs. The pricing rule then combines these marginal costs into a charge which depends on the consumer's stochastic parameters. The second model is based on the recent work of Crew and Kliendorfer(9) and is similar to the approaches demonstrated in Section II. Crew's and Kliendorfer's model contains as a special case the less interesting model of Brown and Johnson(10).

Model III

This model assumes the same technology as Model I. Rather than being based on a direct optimisation of welfare the method relies on the more general result that marginal cost pricing is efficient. It merely remains to determine the relevant marginal costs. The notation is:

b - marginal running cost of plant

c - marginal capacity cost

k - capacity of plant

f(k,q) - total cost function

e - probability of not meeting demand

i - subscript to distinguish individual quantities from aggregate

q' - mean value of aggregate electricity supplied

s - standard deviation

The total cost function of a plant of capacity k, is of the form:

$$f(k,q) = ck + bq \qquad \text{for } 0 < q < k$$

$$= \text{infinity} \qquad \text{for } q > k$$

Let there be a large number of individual consumers with demand q_i which is a random variable with mean q'_i and variance s_i^2. Boiteux assumes that these individual distributions are sufficiently independent to enable the Central Limit Theorem to imply that aggregate demand is an approximately normal variable.

$$q' = \sum_i q'_i \text{ and } s^2 = \sum_i s_i^2$$

Thus if the utility wishes to operate so that the probability of not meeting aggregate demand is 1 - e it should build a plant of capacity equal to

$$k(e) = q' + g(e)s$$

where g is the inverse cumulative normal function. The total costs for outputs not exceeding capcity are thus:

$$f(k(e),q) = ck(e) + bq$$

$$= c \{ \sum_i q'_i + g(e)(\sum_i s_i^2)^{1/2} \} + b \sum_i q^*_i$$

where q^*_i is the amount actually delivered to the ith individual. It will equal his demand unless there is a shortage. The rationing rule chosen by Dreze assumed supply in proportion to demand should excess demand exist. The mechanism is not elaborated except that brownouts are mentioned. It should be pointed out that brownouts do not constitute proportional rationing unless the consumer has an ohmic load. In general a voltage decrease does not change individual's power consumption in the same proportions.

Boiteux suggests that this cost function be used to find the marginal cost of sales, expected demand and the individual standard deviation of demand. The consumer would then pay a three part tariff based on these costs. Under the linear technology this leads to total revenue equalling total costs. Dreze proceeds to point out that the above argument is incorrect and variance should be priced in place of standard deviation. This is because the marginal rate of substitution of mean for standard deviation will not equal the marginal rate of transformation. By using variance this error is corrected though it now means that revenues and costs are not equal; uncertainty has converted constant returns to scale into increasing returns.

The value of the reliability level e has not been considered within the above model. If this choice is not to be arbitrary, then it should be based on marginal arguments. The marginal expected welfare losses due to rationing should be balanced against the marginal costs of maintaining a particular reliability level. Boiteux suggested that each consumer could nominate the probability level he desired and would then pay the marginal cost of maintaining it. Dreze sees little value in this suggestion as he feels that an electric utility should sell electricity rather than insurance.

The whole approach outlined in this model does lend itself to relatively simple implementation and has been used as a basis for rate setting by Electricite de France. It is interesting as an example of applied marginal cost principles, but a more complete solution specifying e could have been obtained from an explicit maximizing model. The major contrast with the next model is, however, the attempt to charge individuals for their contribution to the uncertainty.

Model IV

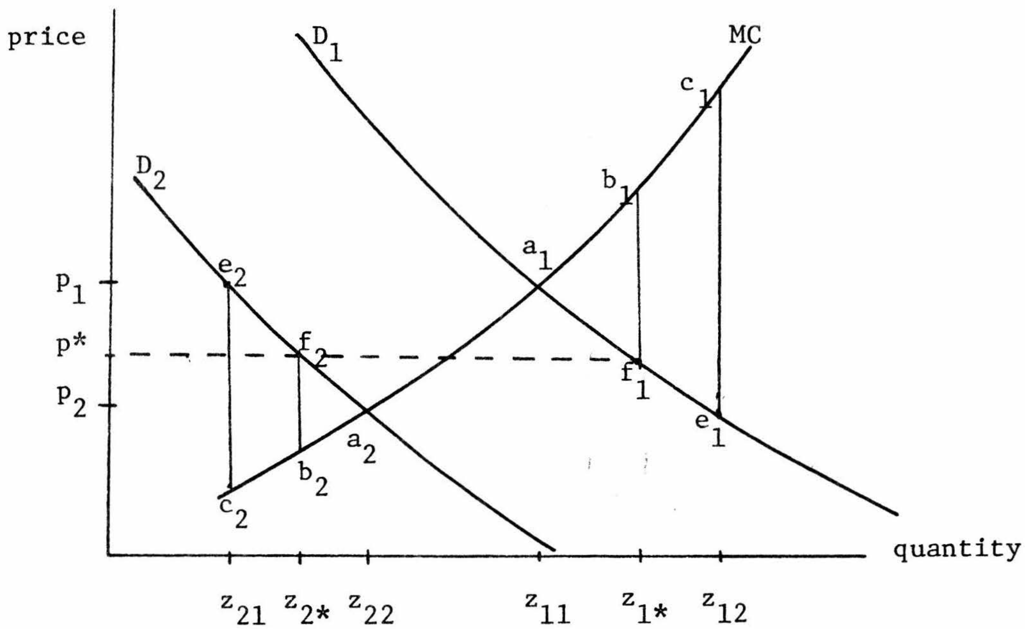
Crew and Kliendorfer present a model which has many plant types, many periods and rationing costs. They maximise expected consumers' surplus minus expected production and rationing costs by setting an ex ante price for each period and choosing the capacity of each plant type. When excess demand occurs they assume that rationing causes the least possible decrease in surplus by curtailing those consumers who are least willing to pay.

The details of this model are tedious and the results are not particularly obvious from the mathematics. However the arguments and conclusions are quite easy to understand. For expository convenience we will consider a single period model with a technology represented by an increasing short run marginal cost function. The slope and curvature of this function can be altered by the expenditures on fixed costs. The demand function is random, and output will therefore be determined by the ex ante price and the realisation of the demand function. With a given marginal cost function and price, the consumers' surplus minus production costs can be evaluated for each demand function. The welfare is merely the correctly weighted sum of these values. Given reasonable probability distributions and demand functions, the welfare function has a maximum with respect to price.

For example, consider two possible demand functions w probability of occurrence q and 1 - q respectively. The following figure illustrates the situation.

The prices p_1 and p_2 shown are the prices that would be chosen if we knew with certainty which demand function would occur. The quantities z_{ij} are the outputs if demand i occurs with ex ante price j. The areas (aec) represent the loss in welfare should we pick the wrong price. Every price p^* between p_1 and p_2 has loss triangles (abf) associated with each demand function. The maximum welfare occurs when the areas of the loss triangles weighted by the appropriate probabilities are equal.

$$\text{area}(abf)_1 . q = \text{area}(abf)_2 . (1 - q)$$



In addition to choosing the optimal price we have to choose the optimal marginal cost curve by the fixed cost decision. The choice of marginal cost curve is optimal when the marginal increase in fixed cost equals the expected marginal increase in consumer surplus minus running costs. The increased expenditure on fixed costs will in general lower the marginal cost in the region of the demand curves.

In the Crew and Kliendorfer model, instead of a continuous marginal cost function there is a staircase function. The choice of the steplengths is equivalent to the choice of slope and curvature of the marginal cost function above. While they used a general function to represent rationing costs the same effect can be visualised in our graphical example by adding a final step to the staircase. This step is of infinite length, contributes nothing to fixed costs and is sufficiently high to be a suitable penalty function.

Brown and Johnson use a model which is a very special case of the Crew and Kliendorfer model. It has a single linear technology and no rationing costs. Their result is that price should always be equal to marginal operating cost. This is a somewhat unappealing solution as the prescription suddenly changes to operating plus capacity cost when the variance in the demand becomes zero. This strange result occurs because there was only a single level to their marginal operating cost staircase. There is no real equivalent in their model to the choice of a marginal cost function.

This general approach to the problem of uncertainty would be useful if an adequate choice of marginal cost is available and the probability distribution is suitably bounded. If however, the probabilty distribution admits large fluctuations in demand then a rationing mechanism becomes important.

IV Metering and Loan Management Technologies

The sale of electricity requires instrumentation, and the more complex the conditions of sale are, the more sophisticated the metering equipment must be. In this section we discuss the metering equipment of the past and future. We will also discuss schemes for directly managing the consumer's load in real time by using remote control techniques. Recent innovations in both these areas are largely the result of the remarkable cost reductions in electronics due to large scale integration. The devices that are now available at extremely low cost make it feasible to consider complex arrangements for selling electricity.

Metering:

The integrating watt meter is the standard device which is used to record energy usage. It is an electromechanical device which has remained virtually unchanged for many decades. It consists of an electric motor whose speed of rotation is closely proportional to the power applied. The shaft of this motor is connected to a gear train which performs the display and memory function. Properly adjusted, this instrument is capable of accurate operation over a wide range of power inputs. It is also rugged and reliable, though somewhat bulky.

Implementation of a peak period pricing scheme would require time differentiated measurements of energy use. The common suggestion is a modification of the integrating watt meter. This modification often appears to be quite elaborate and expensive, involving replication of the mechanism for each pricing period. Such an instrument does allow detailed accounting and permits prices to be changes at will without alteration or adjustment of the device. The cost of these instruments has been a common argument used in opposition to peak load pricing. However a far simpler, though less versatile alternative does exist and this is a simple electrical modification of existing meters. The period selection mechanism would alter the calibration of the meter so that peak period energy was recorded at a higher rate. As long as the ratio of peak to off peak prices stays relatively constant this will achieve the desired results.

Recently there have been developments which indicate that the whole metering environment is likely to change drastically. The cost of reading meters is not insubstantial and this has led to the development of a number of systems for remote meter reading. In such a system each consumer's meter can be remotely interrogated by a central computer (either over power lines, telephone lines, or by radio). If this communication system were established, it does appear that other utility services as well as electricity could use it. This would mean substantial savings in the billing costs but could also cause enormous regulatory headaches. Postal charges are significant in the evaluation of these metering systems and have caused the introduction of portable billing computers for conventional meter readers. This preparation and delivery of the bill on the same visit appears to be quite attractive.

Even if the meters are not to be remotely read, there have been enormous technological changes in what arrangements are economically feasible. The major reason that meters have been so mechanically intricate, is because there was no other type of memory which retained its contents without power. The introduction of magnetic core memories did not change this appreciably as the cost and complexity of the surrounding components was high. Semiconductor memories in general lose their contents when power is removed. However, the recent developments of nitride memories which are being considered as automobile odometer replacements, indicate that this reliance on mechanical memories may now be ending. Meters which can be produced in an integrated form through large scale integration will mean enormous cost decreases for future metering technologies.

The major cost element in the remotely read metering systems is the requirement that individual meters be able to transmit data on request. If this requirement is relaxed so that only communication from the central computer to each meter is needed, costs decrease dramatically. This is especially so if all information is common and hence no addressability is required. This is exactly the same type of system which can be used for remote load control and can be simple or complex depending on the information requirements. One interesting possibility that such a system suggests is that the restriction, that price needs to be announced in advance, can be relaxed. It is quite technologically feasible to broadcast and display a continous price for electricity. With this assumption the peak load problem under uncertainty vanishes as we now have a continuous market clearing price. Unfortunately this system of a floating price suggests that a peculiar kind of monopoly behavior will occur. There would appear to be strong incentives to construct too little capacity so that monopoly profits can be made. A price setting rate of return mechanism could not easily be made to work in this framework. In addition, the continous monitoring of price would be of inconvenience to consumers. The meters could however easily incorporate a sophisticated microprocessor which would enable the consumer to program his automatic responses to price changes (e.g. activate warnings and control appliances).

Most new pricing schemes for electricity will involve the necessity for eventually changing large numbers of meters. It seems likely that any new meters will be totally solid-state and consequently their cost will not be influenced greatly by their logical complexity. This same reasoning applies to load control technology which is discussed next.

Load Control:

The direct control of a consumer's instantaneous load has been used in numerous instances as a non-price response to the peak load problem. The justification for this approach is that the lowering of information costs to the consumer are sufficient to offset any other inefficiencies. The widely held belief that the elasticity of power demand was small led to the additional argument that acceptable price differentials would not influence demand sufficiently. The least interesting method of implementing such direct control has been the use of time clocks to control specific appliances such as space and water heaters. More interesting are the methods where some form of remote control has been used. These allow electricity suppliers to respond to non-regular conditions of peak load as well as the more common cyclic variations in demand.

One other method known as interlocking has also been used. This consists of a group of appliances which are connected so that only one may be operated at a time. This for example might allow only, either the stove or, the hot water heater to be operated, but not both simultaneously.

In the past, remote load control by the supplier has been accomplished using quite simple equipment. For example, by the early nineteen fifties electric water heaters in New Zealand were controlled by audio frequency pulses transmitted over the power wires. This so called "ripple control" was sensed by a tuned relay in each consumers house. The pulse train caused the relay (which controlled the water heater) to latch or unlatch depending on the sequences used. These relays were relatively simple devices but because of their mechanical nature needed careful adjustment. Additional equipment of this type was later introduced to control street lighting and domestic space heating. The binary nature of the information transmitted did limit its use to appliances which were easy to equip with thermal storage. This avoided inconvenience during periods of interruption. It has proved an extremely useful tool with which suppliers can react to random changes in demand (e.g. a sudden thundershower which causes massive increases in heating and lighting load). It has also proved useful in a less conventional way by enabling substantial control of annual energy consumption during years of poor rainfall.

In the next section we will look at more sophisticated applications of load control. These rely not on switching a particular appliance on and off, (though this could be a response) but in actually allowing the supplier to control the instantaneous rating of the consumer's circuit breaker. The consumer would be free to choose the way in which he met this constraint and could nominate a programmed response such as modifications of the interlock scheme. The complexity of the arrangement which the consumer uses will largely depend on the value he places on manual intervention. Many such arrangements come to mind, but here we will limit the discussion to ways in which the overall rationing might be implemented. It would seem that digital control is the most likely to be used as the information can then be utilized for a wider range of tasks than a simple frequency or pulse width modulation arrangement. The medium by which the information is sent is somewhat immaterial and as long as coded signals representing the level of rationing and the energy charges can be transmitted reliably then little else is required. Each consumer would be equipped with a microprocessor which handles the details of supervising the programmable circuit breakers, displaying any information required, doing accounting, giving warning of a current overload condition, etc. If each processor could also identify messages addressed to it, billing may even become unnecessary as consumers could read their current charges and the payments could be remotely subtracted when received by the suppliers.

As it seems increasingly likely that new metering techniques will eventually be introduced for electricity sales it means that theoretical models of this problem should not be constrained to the technology of twenty years ago. Load control can be incorporated in any peak pricing scheme which uses solid state metering at negligible marginal cost. In the next section we will show in fact that if uncertainty is to be handled in an efficient manner then load management is at least as important as conventional pricing methods.

V Optimal Rationing Schemes for Electricity

Previous authors, proposing pricing mechanisms under uncertainty have assumed the existence of a technology for performing rationing in the event of instantaneous excess demand. The crudest method is local blackouts and the most refined is selective rationing of the individuals least willing to pay. Since provision must be made to implement rationing, more efficient transactions and prices can be devised if the rationing mechanism is explicitly considered in the analysis. The models presented in this section assume that the supplier will use load control equipment to individually constrain the demand of each consumer in accordance with some prior aggreement.

A heuristic discussion of one arrangement incorporating rationing has been presented by this author(11). Recently Panzar and Sibley(12) have analysed a similar but more restricted scheme. Their results are contained as a special case of the next model. Panzar and Sibley postulate a self-rationing scheme whereby each consumer purchases a fuse (circuit breaker) at a fixed cost per kilowatt (or ampere). Since the consumer also pays a charge for the electrical energy actually consumed, their method is merely a particular two part tariff. The scheme is only efficient if consumers are not statistically diverse. To be efficient the consumers must all hit their individual rationing constraints at the same value of the random variable. The random variable might, for instance be temperature. If this is not true then there exists a possible reallocation of fuses or an increase in the fuse size which allows an increase in welfare. Brown and Johnson suggest in their conclusions that a spot market in such fuses would lead to an optimal solution, but unfortunately this does not make much sense. If such a spot market in capacity rights could exist, the difficulties due to uncertainty would vanish. Panzar and Sibley have captured a far more realistic aspect of the problem and it would certainly be feasible to use their suggestion to improve the allocation of risk bearing in electricity supply.

In the next model a simple system is presented which serves as an introduction to arrangements characterised by consumers communicating their risk preferences before the random event has occurred. This class of arrangements can be characterised by the following:

1/ Capacity must be determined and constructed before the random variable is observed. (ex ante)

2/ Price schedules can not be established ex post.

3/ Income transfers can not be made ex post.

4/Individual physical rationing can be accomplished ex post.

There are two distinct decision periods which require consideration and hence the choice of a welfare criterion must be made carefully. A welfare criterion allows us to compare arrangements which belong to the above class. These decisions require that we do an optimisation before the random variable is known to choose the capacity and the rationing agreement. After the random variable is known we must optimally allocate the available commodities subject to the necessity to ration. Both these optimisations are in effect welfare decisions and stipulation 3 above constrains us to use the same implicit welfare function in each decision. Another way of looking at this is to consider the ex ante decision to be based on a Pareto condition which uses expected utilities. The weightings in the equivalent linear welfare function must also be used in the ex post decision.

Model V

Assume that the producer has a device that can instantaneously and continuously set an upper constraint on each individual's power consumption. The device enables the supplier to broadcast a real variable l and each consumer is constrained to consume no more power than l times r_i units of power, where r_i is the amount of ration coupons the ith consumer holds. This variable l can be viewed as a rationing level and the supplier will endeavour to set it as high as possible without overloading the plant. The ration coupons can be viewed as scalable fuses which determine the severity of rationing for an individual should rationing become necessary.

The rest of the notation is as follows:

$U_i(x_i, y_i, w)$ - is the ith consumer's measurable utility function

x_i - is the amount of electricity consumed

y_i - is the amount of other commodity consumed w - is a global random variable such as temperature

b - is marginal operation costs (as before)

c - is marginal capacity costs (as before)

The Pareto maximisation is performed in two stages: first to find a decision rule to allocate x and y after the random variable is known, and then using this decision rule to allocate the ration coupons. This process is basically that of stochastic dynamic programming. We know that the first order conditions arising from the maximisation of an arbitrary linear social welfare function are identical to those derived by operating on the complete pareto programming problem. We use this in finding our optimal decision rule because can always find a set of weights for the welfare function that are consistent with the wealth assumptions in the pareto problem.

Specifically we wish to maximise, after observing the realisation of w, (with respect to x and y)

$$\sum_i a_i U_i$$

subject to

$$lr_j - x_j \geq 0 \quad \text{for all } j \text{ (rationing)}$$

and

$$M - \sum_i y_i - b\sum_i x_i - c\sum_i r_i = 0 \quad \text{(transformation)}$$

where

a_i - is the social weighting of the ith individual

M - the total resources of society in units of y

The level of rationing l will be set as large as possible so as to maximise the electricity consumption subject to the total capacity side condition

$$\sum_i r_i - \sum_i x_i \geq 0$$

Note that this condition holds with equality except when no individual is being rationed. Increasing l will relax individual rationing constraints and thus will not decrease the value of the objective function. As total consumption of electricity need never be restricted to be below the total capacity, we know that l is always at least one. At the point where no individual's rationing constraint is effective the value of l need only be greater than some critical value. Note that we do not include the total capacity constraint in the actual mechanics of the above problem because in a large number of situations it would violate the constraint qualification, i.e., the sum of the individual constraints imply the total capacity constraint. We have instead used it to define the value of l.

$$l = (\sum_i r_i - \sum_g x_g) / \sum_q r_q$$

Where the indices q and g denote individuals whose constraints are active and those whose are not respectively.

We form the Lagrangian:-

$$L = \sum_i (a_i U_i + z_i(lr_i - x_i)) + v(M - \sum_i y_i - b\sum_i x_i - c\sum_i r_i)$$

the first order conditions are thus:-

$$a_j \frac{\partial U_j}{\partial x} - z_j - vb = 0$$

$$a_j \frac{\partial U_j}{\partial y} - v = 0$$

$$z_j(lr_j - x_j) = 0$$

$$M - \sum_i y_i - b\sum_i x_i - c\sum_i r_i = 0$$

where the first three conditions hold for any individual j

These conditions imply an optimal allocation x' and y' (decision rule) for any particular distribution of the ration coupons and value of the random variable. We now wish to look at the second stage of our optimisation procedure and determine the optimal allocation of the ration coupons and hence the capacity. The ex ante problem is thus to maximise with respect to r_j the following objective:-

$$J = \sum_i a_i E\{U_i\}$$

where the utilities are evaluated at x' and y'.

The first order conditions are then:-

$$\frac{\partial J}{\partial r_j} = 0 = \sum_i e_i E\{\frac{\partial U_i}{\partial x}\frac{\partial x_i}{\partial r_j} + \frac{\partial U_i}{\partial y}\frac{\partial y_i}{\partial r_j}\}$$

Substituting from previous individual first order conditions we get:-

$$0 = E\{\sum_i ((z_i + vb)\frac{\partial x_i}{\partial r_j} + v\frac{\partial y_i}{\partial r_j})\}$$

From the transformation constraint obtain:-

$$\sum_i \frac{\partial y_i}{\partial r_j} = -b\sum_i \frac{\partial x_i}{\partial r_j} - c$$

which leads to

$$0 = E\{\sum_i z_i\frac{\partial x_i}{\partial r_j} - vc\}$$

By differentiating each constraint $z_i(lr_i - x_i) = 0$ and summing over all individuals, we get

$$\sum_i z_i\frac{\partial x_i}{\partial r_j} = \sum_i z_i r_i\frac{\partial l}{\partial r_j} + lz_j$$

In order to evaluate the derivative of l it helps to rewrite the total capacity constraint as

$$\sum_i r_i - \sum_q x_q - \sum_g x_g = 0$$

where the index q is used to sum over all individuals whose rationing constraint is currently binding, and the index g is used for those for whom the constraint is ineffective. Differentiating this with respect to r_j we obtain:-

$$1 - \sum_q \frac{\partial x_q}{\partial r_j} = 0$$

By differentiating $\sum_q (lr_q - x_q) = 0$ we obtain:-

$$\sum_q \frac{\partial x_q}{\partial r_j} = l_j + \frac{\partial l}{\partial r_j} \sum_q r_q$$

where

$l_j = 1$ if $z_j > 0$

and

$l_j = 0$ if $z_j = 0$

Combining these results we get

$$0 = E\{ z_j l + (1-l_j)(\sum_q z_q r_q)/(\sum_q r_q) - vc\} \text{ for all } j$$

This condition is easily interpreted and compared with the result of Panzar and Sibley. The first term is the expected private value of ration coupons to individual j and as will be shown shortly, it is the quantity that the consumer equates to the price he is charged for ration coupons. The last term is the expected marginal cost of capacity. Selling ration coupons at a price equal to the marginal cost of capacity will not lead to a correct result unless the middle term is zero for all j. The only situation when the middle term is zero for each j is if all individuals' rationing constraints become effective at the same value of the random variable. This is exactly the condition which Panzar and Sibley require in order that their scheme be efficient. In all other cases this middle term can be viewed as an externality in consumption due to the diversity of consumers. The externality is caused by the ration coupons one individual buys expanding capacity which becomes available to others for certain values of the random variable. This term will also in general be different for each consumer, implying that different prices should be charged different people for ration coupons. However, groups of consumers may have similar values of this term. Statistical classes of consumers (possibly closely related to current classification) may be identifiable and consequently this scheme may have practical value.

We will now demonstrate the behaviour of a consumer within this model. Following the same steps as previously we wish to maximise:-

$$U(x,y,w)$$

subject to

$$lr - x \geq 0 \qquad \text{(rationing)}$$

and

$$I - px - y - fr = 0 \qquad \text{(budget)}$$

where

I is the individuals income

p is the price of electricity

f is the price of ration coupons

For any given r and w we can find the optimal values of x and y. Form the Lagrangian:-

$$H = U + z(lr - x) + v(I - px - y - fr)$$

The first order conditions are:-

$$\frac{\partial U}{\partial x} - z - vp = 0$$

$$\frac{\partial U}{\partial y} - v = 0$$

$$z(lr - x) = 0$$

$$I - px - y - fr = 0$$

The consumer uses x' and y', derived above, to choose r.

$$\max \qquad E\{U(x',y',w)\}$$

The first order conditions are:-

$$\frac{\partial E\{U\}}{\partial r} = 0 = E\{\frac{\partial U \partial x}{\partial x \partial r} + \frac{\partial U \partial y}{\partial y \partial r}\}$$

Substitute from the decision functions' first order conditions:-

$$0 = E\{(z + vp)\frac{\partial x}{\partial r} + v\frac{\partial y}{\partial r}\}$$

and from budget constraint

$$0 = E\{z\frac{\partial x}{\partial r} - vf\}$$

Since $\frac{\partial x}{\partial r} = 1$ for $z > 0$

$= 0$ for $z = 0$

thus

$$0 = E\{zl - vf\}$$

A consumer will therefore equate the expected marginal personal value of ration coupons to their cost. Diversity implies that in general ration coupons should be priced differently than the marginal cost of capacity and that the price should vary according to the response of the consumer to the random variable. This result is somewhat analogous to Boiteux's prescription except that here it is the covariance that is the important statistical parameter.

The above analysis implicitly assumes that the consumers know the distribution of the random variable and the rationing function l. This is a plausible assumption in steady state situations but there is no guarantee of convergence in a learning situation. Later, when looking at practical implementations, we discuss the types of damping that could operate to ensure stability.

The results of the previous model are not particularly surprising. This is equivalent to running a securities market with only a single security. Optimal risk bearing is impossible with a single price for the security unless all individuals have similar risk preferences. A variety of types of ration coupons should be introduced in order that consumers may choose their optimal portfolio. This is the approach taken in the next model.

<u>Model VI</u>

For this model the random variable is discrete and each value is associated with some particular state of the world s. We introduce a separate type of ration coupon for each state of the world. After the random variable becomes known, the supplier uses load control equipment to ensure that each consumer consumes no more than the amount allowed by his holdings of that type of ration coupon. Otherwise the model is similar to model V.

Let:

$U_i(x_i, y_i, w_s)$ - the ith consumers utility

x_i - electricity consumed by ith individual

y_i - other commodity consumed by ith individual

w_s - discrete random variable associated with state of the world s

Each consumer i, holds an option (ration coupon) r_{is} to consume up to r_{is} units of electricity should state of the world s occur.
That is,

$$r_{is} - x_i \geq 0 \qquad \text{for all i and s}$$

Using the same two stage welfare maximisation procedure as previously we determine the decision rule and then the option allocation.
We thus have the familiar optimisation problem:-

$$\max \quad \sum_i a_i U_i$$

subject to

$$r_{is} - x_i \geq 0 \qquad \text{for each } i \text{ (rationing)}$$

$$M - \sum_i y_i - b\sum_i x_i - c\sum_i r_{is} = 0 \qquad \text{(transformation)}$$

The sum over i of the r_{is} is constant for any s, since the total capacity is determined in advance and we constrain the sum of the ration coupons in each state to equal capacity.
Form the Lagrangian:-

$$L = \sum_i (a_i U_i + z_{is}(r_{is} - x_i))$$

$$+ v_s(M - \sum_i y_i - b\sum_i x_i - c\sum_i r_{is})$$

The following first order conditions for the decision rules are:-

$$a_j\frac{\partial U_j}{\partial x} - z_{js} - v_s b = 0$$

$$a_j\frac{\partial U_j}{\partial y} - v_s = 0$$

$$z_{js}(r_{js} - x_j) = 0$$

$$M - \sum_i y_i - b\sum_i x_i - c\sum_i r_{is} = 0$$

The optimal allocation of the options is derived by finding the

$$\max \qquad J = \sum_i a_i E\{U_i\}$$

with respect to r_{js}. This leads after substitution to the first order condition:-

$$\frac{\partial J}{\partial r_{js}} = 0 = E\{\sum_i z_{js}\frac{\partial x_i}{\partial r_{js}} - v_s c\}$$

Since

$$\frac{\partial x_i}{\partial r_{js}} = 1 \text{ if } \qquad i=j \text{ and } z_{is} > 0$$

$$= 0 \qquad \text{if} \qquad i \neq j \text{ or } z_{is} = 0$$

we have

$$0 = E\{z_{js} - v_s c\}$$

The marginal expected private value of an option should equal its marginal expected cost. This implies that a decentralised market in ration coupons will allocate these options efficiently. The similarity of this result to the standard Arrow contingent claims market should be noted. The schemes are in fact identical in the case when consumers have correct subjective probability distributions. If however the consumers do not share the same probabilistic view of the world this scheme basically introduces enforcibility into the Arrow contracts. Clearly, the consumer will purchase exactly the amount of coupons that will be used. In other words, all the individual constraints will hold with equality. In some states of the world however the market price for options is likely to be zero, implying that capacity is a free good.

It is not difficult to produce practical objections to this arrangement. For example, the possibility of identifying all the states of the world and communicating their specifications seems remote. Furthermore imperfect consumer foresight can cause mismatched supply and demand ex poste. The transformations that follow simplify transactions and decrease the harm done by imperfect foresight, while preserving the attractive theoretical properties of the model.

Instead of separate ration coupons for each state of the world we introduce the idea of priority rights. After the random variable is observed, the supplier will no longer proceed by imposing the rationing levels previously purchased, but will instead nominate a priority level.

Instead of ration coupons for each state the consumer will have a priority right for each priority level. The priority rights are defined in terms of the ration coupons as follows:

$$r_{is'} = \sum_{s=1}^{s'} p_{is}$$

where p_{is} is the amount of priority rights held by individual i at level s. The states of the world s have been reindixed in the above identity so that the market prices for the options now decrease with the index. Thus state of the world s=1 corresponds to our highest priority level. The supplier can now react in a similar manner to model V where the rationing level was set. The priority level allows some adjustment ex post to improve the situation caused by imperfect consumer foresight. The consumer will have the right to consume electricity in amount up to the algebraic sum of all priority rights held at this level and higher. We have thus introduced an ordering of our ration coupons and allowed them to be cumulative. The following matrices illustrate this transormation:

	option prices ->		lower priorities ->
states	100000000000	priorities	100000000000
	010000000000		110000000000
	001000000000		111000000000
	000100000000		111100000000
	000010000000		111111000000
	etc		etc

where a one indicates that an option or priority may be used.

This ordering and compounding of the options into priorities allows the consumers to make far simpler decisions. It also makes it much easier to visualise the replacement of a very large number of different type of ration coupons into a relatively small number of types. The actual contraction will naturally depend on the transaction and information costs associated with maintaining a market in an extra type of security. While the choice of the optimal number of types of options can conceptually be solved by enumeration, it does not lend itself to analytical mathematical programming.

The transformation into priorities requires that consumers are able to buy negative rights (sell) at lower priorities in order for a complete risk market to exist. Unless a consumer has some generation facility, the amount of rights which can be sold are limited to the algebraic sum of the higher priority rights held. Whether this theoretical completeness of the market is important in a practical sense will only be determined by comparing the transaction costs to the potential benefits. Incompleteness will probably result in a relatively small distortion of the efficient scheme.

The above system yields a suggestion for assisting in the investment decisions of the supplier. Instead of raising capital to build new generation facilities by selling conventional financial instruments, the supplier could enter the futures market in priority rights. By selling options on the capacity of the plant he would raise the capital to build the new facility. The owners of these options would then earn the return on their capital by renting and leasing the rights to consumers. The investor would thus be forced to evaluate the risk characteristics of each investment of the firm without the guarantees that the current regulatory climate and massive assets of the firm now provide. However, because of the existence of this futures market much more information about consumers' preferences will be available. This more widespread bearing of the risk should allow more efficient economic decisions to be made.

So far we have only considered rationing under the simple technological assumption of a single available type of plant. As other investigators have run into difficulties through using this simplification it is worthwhile to investigate the generalisation to multiple plant types.

Model VII

The assumptions and notation are the same as in the previous model except for the following.

b_k - marginal operating cost of plant k

c_k - marginal capacity cost of plant k

x_{ik} - the ith individuals consumption of plant k output

r_{iks} - the ith individuals option on the output of plant k in state of the world s

Using identical procedures as before, the problem is thus to initially determine the decision functions x' and y'.

$$\max \quad \sum_i a_i U_i(X_i, y_i, w_s)$$

where

$$X_i = \sum_k x_{ik}$$

and subject to

$$r_{jks} - x_{jk} \geq 0 \qquad \text{for all } j,k \text{ and } s$$

$$M - \sum_i y_i - \sum_k b_k \sum_i x_{ik} - \sum_k c_k \sum_i r_{iks'} = 0$$

Let

$$L = \sum (a_i U_i + \sum z_{iks}(r_{iks} - x_{ik}))$$

$$+ v_s(M - \sum_i y_i - \sum_k b_k \sum_i x_{ik} - \sum_k c_k \sum_i r_{iks'})$$

The first order conditions are:-

$$\frac{\partial L}{\partial x_{jk}} = a_j \frac{\partial U_j}{\partial X} - z_{jks} - v_s b_k = 0 \text{ for } x_{jk} > 0$$

$$< 0 \qquad \text{for } x_{jk} = 0$$

$$\frac{\partial L}{\partial y_j} = a_j \frac{\partial U_j}{\partial y} - v_s = 0$$

and

$$z_{jks}(r_{jks} - x_{jk}) = 0$$

As in the previous analysis we now wish to find the optimal allocations of the ration coupons and hence the capacities.

$$\max \sum_i a_i E\{U_i\}$$

The first order condition is:-

$$\frac{\partial H}{\partial r_{jks}} = E\{\sum_i a_i (\frac{\partial U_i}{\partial X}\frac{\partial x_{ik}}{\partial r_{jks}} + \frac{\partial U_i}{\partial y}\frac{\partial y_i}{\partial r_{jks}})\}$$

$$= 0 \text{ for } r_{jks} > 0$$

This case must hold whenever an individual uses output from the kth plant in state s. If we limit ourselves to these cases we can substitute from the individual first order conditions.

$$0 = E\{z_{jks} - v_s c_k\}$$

This familiar condition implies that there is a simple decentralised scheme to allocate the options on the output of each plant. An interesting feature of the technology assumed in this model is that it gives rise to the possibility of charging a non linear price for energy. The marginal units are being priced at marginal operating cost but earlier units are charged only for operating costs. Thus economic efficiency can be achieved without creating the usual surplus revenue.

This non-constant price for energy still satisfies the restriction which specified that the price schedule could not be altered after the random variable occurred. Each consumer knows in advance what energy from a particular plant will cost. Thus the purchase of a particular mix of ration coupons determines for each state of the world what the price as a function of consumption will be.

We can perform the transformation of options into priorities for each plant. This means that each plant broadcasts its own priority signal to consumers. Each consumer's load control equipment will automatically ensure that energy is charged the appropriate price and naturally will utilise the lowest priced energy available. A special case simplification can be considered. We assume that all plants contract priorities in the same way (not necessarily optimal). All plants are considered to be available in all states of the world. Then clearly the priority level in effect becomes the same for all plants, though some high running cost plants may have zero prices on their lower priorities. This may be acceptable in a practical implementation.

We now explore the implications of supply uncertainty caused for example by either breakdowns or the introduction of novel technologies such as wind power.

Model VIII

So far we have only considered uncertainty entering into the consumption side of the models. However if model VII is slightly modified we can introduce uncertainty in supply. This takes the form of a plant's capacity varying with states of the world, though its maximum value remains the same. We compound the demand and supply states of the world and ensure that the sum of the ration coupons for a plant in any state of the world corresponds to its available capacity. The state of the world s'_k is a state where the rated capacity if plant k is available. We define availability q_{ks} as the fraction of the kth plant's rated capacity available in state s, i.e.

$$q_{ks} \sum_i r_{iks'} = \sum_i r_{iks}$$

The first order condition of model VII thus becomes

$$0 = E\{z_{jks} - v_s c_k / q_{ks}\}$$

where s is a state with $q_{ks} > 0$.

This result implies the possibility of incorporating technologies with random availability into the capacity rights scheme without losing economic efficiency. Unfortunately, we can no longer simplify to a single priority level common to all plants. The random supply means that the ordering of states of the world (according to the prices of the ration coupons) is in general different for each plant. The transformation to priorities preserves this ordering so each consumer's equipment needs to be informed of the availability of each class of energy.

To preserve efficiency and still have the system of priorities we need a separate set of priorities for each plant. This does appear to be a very unwieldy scheme which would have high transaction costs. The introduction of diverse technologies does however suggest that model V might still have practical value. The choice of many plants appears to allow at least some more optimal adjustment to risk. This scheme would certainly be much more practicably feasible, as each plant need only issue a single class of scalable rights.

The introduction of multiple technologies to model V is straightforward and yields the first order condition:

$$0 = E\{z_{jn}l_n + (1 - l_{jn}) \sum_k ((\sum_q z_{qk}r_{qk})/(\sum_q r_{qk})) - v_n c_n\}$$

for all plants n and individuals j.

The middle term which characterises the diversity externality will decrease in importance the greater range of plant types we have available. This suggests that in practical applications the original model may appear attractive in terms of transaction costs even though it does imply some inefficiency.

VI A Practical Scheme

In this section we will briefly sketch a way in which the electricity supply industry in the U.S. could be restructured. The ideas which were developed in the previous section are used as the basis for this suggestion. It is not however intended that the following outline should imply that this is the only method of implementation. When turning theoretical results into practical prescription there is always an element of judgment as to exactly where the bounds of practicality lie. This is particularly true when technology is being assessed and there are undoubtedly simpler and less costly methods of achieving the objectives than the ways presented here. However, it is hoped that the following will stimulate such innovative thought.

The Scheme

The industry is separated vertically into generation, transmission and distribution activities. The generation sector is not subject to rate of return regulation. It is, however, broken up horizontally into separate plants which will compete. The ownership of generation is plant specific rather than firm specific. Ownership consists of holding options on the capacity of the plant. These options are leased to distributors and consumers on an open option market. All capital for plant construction must be raised with these instruments. Whether these options are the simple scaled variety of model V or the priorities of model VI will depend on implementation costs. In either case the operator of the plant (via the system dispatcher) broadcasts a rationing level control signal that prevents excess demand.

The transmission systems are operated as common carriers but may still be regulated in some conventional manner. Extensive computer-based coordination and protection techniques will be used but in general operation will not be noticeably different from current practice.

Distribution systems may be organized on any suitable local scale. They are responsible for reselling capacity options to small consumers and will probably aggregate options into standard packages for consumers. The ownership of this sector is not critical but it is likely that municipalities will elect to take control.

The consumer rents capacity options which are under the continuous control of the distributor. These options are aggregated parcels of the more fundamental generator options held by the distributor. In general there will be only a few types of option available to the consumer in order to simplify his decision. These will range from high capacity cost but low energy cost options through to low capacity cost but high energy cost options. The distributor will broadcast a signal which scales these options in real time in order that he may respond to supply and demand fluctuations.

The electricity bill will itemise the energy usage in such a way that the consumer may verify that his current option configuration is optimal. Each bill may also present a computation showing what the ex post optimal option holding would be. If the consumer wishes to change his options a space could be included on the return portion of the bill for this request.

Initially there are likely to be stability problems as the consumers attempt to learn the probabilities associated with the system. In order to alleviate these it is likely that a transaction charge should be made when an individual wishes to alter his option mix. This charge would cover the actual transaction costs as well as introducing damping into the system. The optimal level of this charge will undoubtedly decrease through time as consumers gain knowledge.

The consumer has equipment that ensures that the energy he consumes at any particular time is charged at the cheapest rates available to him. This equipment would also probably display the marginal and average energy costs at all times. The load control equipment prevents the consumption of more power than the options held. It would no doubt notify the consumer by means of some alarm system of an impending overload before taking any action to curtail supply. This equipment may also provide the consumer with the ability to preprogram the operation of electrical appliances according to the instantaneous prices and time of day. One particularly simple way to implement this type of arrangement which does not require rewiring of the household circuits would use these circuits to also carry the local control signals. This would allow portable plug-in regulators to be used in any receptacle. These special plugs could be coded to correspond to a particular option type or could uniquely identify an appliance. If they were to be coded as to option then they could be viewed as a physical form of the option and could with suitable safeguards be traded as options. Naturally the user's central black-box would need to be programmed to be compatible with the particular plugs used.

A reorganization of the industry along these lines promises many benefits. Reserves of capacity will be determined by prices rather than arbitrary reliability standards. Resource allocation will certainly improve, as true marginal costs are charged. Regulatory activity can concentrate on more straightforward technical supervision of the system because the monopoly problem in generation has been eliminated. This technology has in fact allowed true competition to operate in the generation sector. Even if there are only a few plants in a particular region, each plant will have numerous owners. The owners will earn a return from their investment by leasing the options they hold to consumers. Barring collusion between owners, each will be forced to rent at the market price which will be determined competitively. In operation of the plants competition will also prevail (if there are sufficient plants), as each consumer will continually be able to purchase the cheapest energy available which he holds rights to. If a plant prices energy too high, the rental value of its options will decrease and hence the owners will have an incentive to monitor the management. In fact, the owners will maximise their return by forcing the plant to operate at cost. The investors are faced with considerably more risk than currently because they are no longer guaranteed a rate of return by the regulatory process. In addition, the large assets of the monopolist are no longer available to insure this new investment. This absence of internal risk-spreading will force investors to examine the particular investment more carefully. The consumers, however, will be providing much more information on future demand because the prices of the capacity option futures will reflect the expected demand. Planning will be simplified because the supplier will no longer have the sole responsibility for predicting the future in an uncertain world.

VII Conclusions

In this study we first examined the traditional pricing mechanisms which have been advocated for electricity supply. When correctly interpreted, the conventional peak load pricing models provide conditions for efficient pricing which are useful guides to public policy. However, the technological and institutional framework necessary to implement peak load pricing has largely been ignored. The results of the models addressing the uncertainty issue are not as useful because they ignore the details of how the necessary rationing will be implemented.

Recent advances in electronics make complex transactional arrangements feasible. These advances in metering and load control devices allow the incorporation of individual rationing into the design of electricity allocation methods. By utilising the capability of these new tecnologies we have shown that an arrangement similar to an Arrow-Debreu contingent claims market will operate efficiently. Since our scheme allows for uncertainty in production as well as in consumption, technologies which have random availability such as wind and solar power, can be incorporated into supply systems without requiring (economically crippling) back-up capacity.

We concluded this study with a sketch of one particular way in which these ideas might be implemented. These suggestions may or may not be judged to be politically feasible. We can predict with certainty, however, that in the future, information will become less expensive and energy will not. This implies that there will be strong incentives to utilize energy more efficiently (in an economic sense) by increased information flows. This study has demonstrated some ways in which this can be done.

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