

Phase Conjugate Optics
and
Nonlinear Optical Phenomena in Optical Fibers

Thesis by
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To my parents

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It has taken five years of learning and hard work to complete the work described in this thesis. However, I would not be at this stage, or I would have taken a much longer time to reach it, if throughout the course of this work I had not been receiving constant encouragement and the best professional guidance, which have both led and helped me to participate in the frontier research of Quantum Electronics. It is therefore with my deepest appreciation and utmost sincerity that I say thank you to my advisor, Professor Amnon Yariv. I will continue to benefit from his teaching and advice throughout the future years of my research career.

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ABSTRACT

Part I of this thesis describes the theoretical and experimental aspects of a new area in coherent optics known as phase conjugate optics. By using nonlinear optical mixing techniques, the complex conjugate of an arbitrary incident monochromatic wave can be generated. In particular, the theory of the degenerate four-wave mixing technique is presented. Experimental observation of phase conjugate wave front generation in a liquid crystal material (MBBA) is described. The ability of the conjugate wave front to correct for phase aberration is demonstrated. The quantum mechanical origin of the nonlinear third order susceptibility responsible for the four-wave mixing process is illustrated in an atomic system. Because of a finite memory time of such an atomic system, the electromagnetic waves taking part in the mixing process need not occur simultaneously. The applications of the phase conjugation process to real-time information processing and the characteristics of a phase conjugate resonator are discussed in detail.

The ability of an optical fiber to confine the propagation of electromagnetic waves over a long distance and within a small cross sectional area implies that large optical intensities can be maintained over a long interaction length even at moderate incident optical power. Part II of this thesis describes some third order nonlinear optical phenomena that have been observed under this condition. Especially, phase conjugation by degenerate four-wave mixing in a fiber has been shown to require significantly less pumping power. The theoretical and experimental studies of four-wave mixing in a liquid-filled fiber are presented.

Extensive experimental efforts have been devoted over the last few years to the study of stimulated Raman scattering and the oscillation of the Stokes radiation inside fibers. A detailed theoretical treatment will be given here to describe these effects. The analysis shows that pump depletion due to the nonlinear process is very important in describing the stimulated Raman scattering in low loss fibers and is the dominant saturation effect in a fiber Raman oscillator. Finally, the implications of nonlinear optical phenomena to fiber optic communication will be mentioned.

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Part I

PHASE CONJUGATE OPTICS

Chapter 1

INTRODUCTION

1.1 Outline of Thesis, Part I

In Chapter 2, the properties of a phase-conjugate wave front will be discussed, and a theory of phase conjugation using degenerate four-wave mixing will be presented. The latter analysis uses a coupled wave formalism and was first derived by Yariv and Pepper. Their treatment will be summarized in Chapter 2 as background information and to facilitate the discussions in the subsequent chapters of this thesis. The observation of phase-conjugate wave front generation by degenerate four-wave mixing in a nematic substance (MBBA) will be described in Chapter 3. The third order nonlinear susceptibility $\chi^{(3)}$ of the material is treated as a phenomenological constant in the discussions given in Chapters 2 and 3. In Chapter 4, it will be shown that because of a finite "memory" time of an atomic system, the electromagnetic waves illuminating such a medium need not be present simultaneously, and the conjugate wave front can be observed in the form of a photon echo. The analysis will use a perturbation approach, and will illustrate how the nonlinear susceptibility $\chi^{(3)}$ can be derived using quantum mechanical considerations. Finally, some applications of phase conjugation will be described in Chapter 5.

1.2 The Main Features of Phase Conjugate Optics

A new research area in coherent optics has emerged and has been receiving increasing attention from many scientists as its important applications are recognized. Phase conjugate optics is the name which seems to have been accepted to describe this new field. The main feature of phase conjugate optics is the generation of an electromagnetic wave with a phase distribution which is, at each point in space, the exact opposite of that of an arbitrary incoming monochromatic wave. The wave front, after being generated, proceeds to propagate in the opposite direction, retracing the original path of the incoming wave. Thus, the phase reversal or conjugation process results in what is frequently called a time-reversed replica of the incident wave. If we consider, as an example, an incoming spherical wave front diverging from a point with a radius of curvature R , its conjugate-replica will be an outgoing spherical wave front converging toward the same point with a radius of curvature $-R$. Phase conjugation techniques have been used in the past for imaging through phase distorting media; well known examples can be found in holography [1] and adaptive optical systems [2]. The new and attractive feature, which differentiates phase conjugate optics from the previous techniques, is the use of nonlinear optical mixing to generate in real time, without the need for intermediate electronics, and with amplification if desired, a time-reversed replica of an incident wave.

The first experiments in phase-conjugate optics were performed in 1970 both in the USSR [3] and the Netherlands [4]. The theoretical understanding of the effect was, however, only qualitative and the work seems

to have slowed down or shifted to more conventional holographic directions.

Using stimulated Brillouin scattering, Zeldovich and collaborators [5] demonstrated phase conjugation in the backward scattered beam. Yariv independently proposed [6,7] the use of three-wave mixing in crystals for real time holography and to compensate for the distortion that arises due to the modal dispersion in transmitting pictorial information through a single multimode optical fiber. Phase conjugation using this technique was subsequently demonstrated by Avizonis et al [8]. More recently, Hellwarth [9] showed that four-wave mixing [3,4] eliminates the phase matching restriction inherent in the three-wave process. Yariv and Pepper [10] applied the formalism of nonlinear optics to show that the four-wave process is capable of amplifying an incident wave, as well as rendering its complex conjugate version and, in the limit of sufficient pumping, of mirrorless oscillation. The mixing utilizes an optically induced polarization which is cubic in the electric field strength. This third order nonlinear polarization, though of a higher order than that involved in the second order (three wave) mixing, is not restricted to asymmetric crystals and can occur in any material as, for example, any crystal, liquid, or gas.

The first observations of phase conjugation by degenerate four-wave mixing were reported by Stepanov et al [3]. Amplified phase conjugate reflection and parametric oscillation were demonstrated by Bloom, Liao, and Economou [11] and by Pepper, Fekete, and Yariv [12]. The last experiment used CS_2 as the nonlinear medium and a Q-switched laser was required to provide the large optical intensities necessary to observe the effects.

However, the nonlinear index can be increased considerably if the optical frequency is chosen to be close to that of an atomic transition. This has led to the use of low power laser sources for performing similar experiments in sodium [11,13] and rubidium vapors [14].

Many physical phenomena can be used to perform four-wave phase conjugation. Fundamentally it is necessary that the local optical properties of the medium depend essentially instantaneously on the optical intensity. Physical effects used to date include the Kerr effect [15] (change of index of refraction proportional to the intensity), saturable optical absorption [16,17], and generation of free [4] and localized electrons [18].

More recently, phase conjugation via real-time holography in barium titanate [19] and BSO crystals [18] were reported. The same results have also been observed in the infrared (10.6 μm) in germanium [20], SF_6 , NH_3 , etc.

Four-wave mixing can be shown to be analogous to holography. Many applications normally associated with holography can thus be performed via nonlinear optical mixing in real time.

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Chapter 2

PHASE CONJUGATE OPTICS AND REAL TIME HOLOGRAPHY

2.1 Properties of a Phase-Conjugate Wave Front

Phase conjugate optics is an area in coherent optics which involves the use of nonlinear optical techniques for real-time processing of electromagnetic fields [1]. Specifically, it is concerned with the generation of a wave front which is the complex conjugate of an incident monochromatic wave. Before discussing in detail the techniques that can be used to produce such conjugate wave fronts, it is appropriate to define exactly what a conjugate wave front is, and to describe its properties.

Consider a component of an incident electromagnetic wave

$$E_i(\vec{r}, t) = \text{Re}\{A(\vec{r}) \exp[i(\omega t - \vec{k} \cdot \vec{r})]\} \quad (2.1)$$

which is propagating with a wave vector $\vec{k}$ and oscillating at an optical frequency ω . The complex amplitude $A(\vec{r})$ contains both amplitude and phase information of the field. The complex conjugate of this field is defined to be

$$E_r(\vec{r}, t) = \text{Re}\{A^*(\vec{r}) \exp[i(\omega t + \vec{k} \cdot \vec{r})]\} \quad (2.2)$$

It is clear that E_r is obtained by taking the complex conjugate of the spatial part of E_i , and it describes a wave propagating in the opposite direction to that of E_i . The field E_i , as it propagates inside a lossless dielectric medium with a real permittivity $\epsilon(\vec{r})$, obeys the scalar wave equation

$$\nabla^2 E_i + \omega^2 \mu \epsilon(\vec{r}) E_i = 0 \quad (2.3)$$

If the field is also assumed to be traveling in the positive z direction, equation (2.3) implies that

$$\nabla^2 A + [\omega^2 \mu \epsilon(\vec{r}) - k^2] A - 2ik \frac{\partial A}{\partial z} = 0 \quad (2.4)$$

The complex conjugate of equation (2.4) is

$$\nabla^2 A^* + [\omega^2 \mu \epsilon(\vec{r}) - k^2] A^* + 2ik \frac{\partial A^*}{\partial z} = 0 \quad (2.5)$$

and can be recognized as the wave equation describing the propagation of the conjugate wave E_r which is therefore a legitimate solution of Maxwell's equations. In general, in a lossless dielectric medium, there are two solutions to Maxwell's equations; they are two counter-propagating waves which are the complex conjugate of each other. When the conjugate wave is generated at a plane $z = z_0$, it propagates in the negative z direction and retraces the original path of the incident wave, evolving as the complex conjugate of the incident wave at all points for $z < z_0$. The phase fronts of the two waves are identical in shape except that they move in opposite directions.

A frequently quoted application of phase conjugation is its ability to compensate for phase distortions. For example, if an incident plane wave passes through a phase distorting medium such as a poor quality optical component, it will acquire an undesirable phase $\exp[i\phi(x,y)]$. The conjugate of this distorted wave, after being generated, will have a phase of $\exp[-i\phi(x,y)]$, which, upon retracing the original path through the phase distorting medium, will emerge as a plane wave with the phase aberration exactly cancelled.

2.2 Phase-Conjugate Wave Front Generation by Nonlinear Optical Mixing

The techniques proposed to date for phase conjugation include stimulated scattering and nonlinear optical mixing. In the case of three-wave mixing, first proposed by Yariv [2,3], two input waves: E_1 , at a frequency ω and a wave vector $\vec{k}_1(\omega)$, and an intense pump wave, E_2 , at a frequency 2ω and a wave vector $\vec{k}_2(2\omega)$ are incident simultaneously on a crystal. A nonlinear polarization proportional to $E_1^* E_2$ is formed in the medium in addition to the linear polarization. This second order nonlinear polarization, occurring only in crystals lacking inversion symmetry, acts as a source which radiates a third wave, E_3 , at the difference frequency $2\omega - \omega = \omega$ and with a spatial-amplitude distribution proportional to $E_1^* E_2$. As an example,

$$E_1 = \frac{1}{2} A_1 \exp\{i[\omega t - \vec{k}_1(\omega) \cdot \vec{r}]\} + \text{c.c.} \quad (2.6)$$

$$E_2 = \frac{1}{2} A_2 \exp\{i[2\omega t - \vec{k}_2(2\omega) \cdot \vec{r}]\} + \text{c.c.} \quad (2.7)$$

and

$$E_3 = \frac{1}{2} A_3 \exp\{i[\omega t - \vec{k}_3(\omega) \cdot \vec{r}]\} + \text{c.c.} \quad (2.8)$$

where $A_3 \propto A_1^* A_2$. In the event that E_2 is a plane wave, E_3 becomes the conjugate replica of E_1 and, in this case, travels in the forward direction. The field E_3 propagates in the crystal with a wave vector $\vec{k}_3(\omega)$, while the polarization source has a wave vector $\vec{k}_2(2\omega) - \vec{k}_1(\omega)$. Effective transfer of energy from E_2 to E_3 is possible only if $\vec{k}_3(\omega) \simeq \vec{k}_2(2\omega) - \vec{k}_1(\omega)$. This is known as the phase matching condition. If this condition is not fulfilled, the wave fronts of E_3 radiated at the different regions of the crystal do not add up in phase, and destructive interference will reduce

the amplitude of the E_3 so generated. If $\Delta k = |\vec{k}_2(2\omega) - \vec{k}_1(\omega) - \vec{k}_3(\omega)|$ is the mismatch in wave vectors, the crystal length that can be used such that there is minimal destructive interference effect is about $\pi/\Delta k$ [4], which typically is around 50 μm . Thus phase matching is an important consideration when a long interaction length is wanted to generate a strong E_3 . Usually exact phase matching can be satisfied along one direction only. This causes a limitation on the angular divergence of the input wave which can be tolerated, and hence on the amount of information or distortion that can be conjugated or corrected.

An alternative process, which overcomes the phase matching requirement, is what is now known as degenerate four-wave mixing [5-8]. All the optical fields involved in the mixing process are of the same frequency, ω , hence the name degenerate. The geometry of the interaction is shown in Figure 2.1 [8]. A nonlinear medium, for example, carbon disulphide (CS_2), is illuminated by two intense pump beams, A_1 and A_2 , both at frequency ω . These waves are chosen to be plane waves

$$E_1(\vec{r}, t) = \frac{1}{2} A_1 \exp[i(\omega t - \vec{k}_1 \cdot \vec{r})] + \text{c.c.} \quad (2.9)$$

and

$$E_2(\vec{r}, t) = \frac{1}{2} A_2 \exp[i(\omega t - \vec{k}_2 \cdot \vec{r})] + \text{c.c.} \quad (2.10)$$

which are counter-propagating such that the sum of their wave vectors $\vec{k}_1(\omega) + \vec{k}_2(\omega) = 0$. Upon incidence, along an arbitrary direction, of an arbitrary input wave

$$E_4(\vec{r}, t) = \frac{1}{2} A_4 \exp[i(\omega t - \vec{k}_4 \cdot \vec{r})] + \text{c.c.} \quad (2.11)$$

at the same frequency ω but with a wave vector $\vec{k}_4(\omega)$, a third order

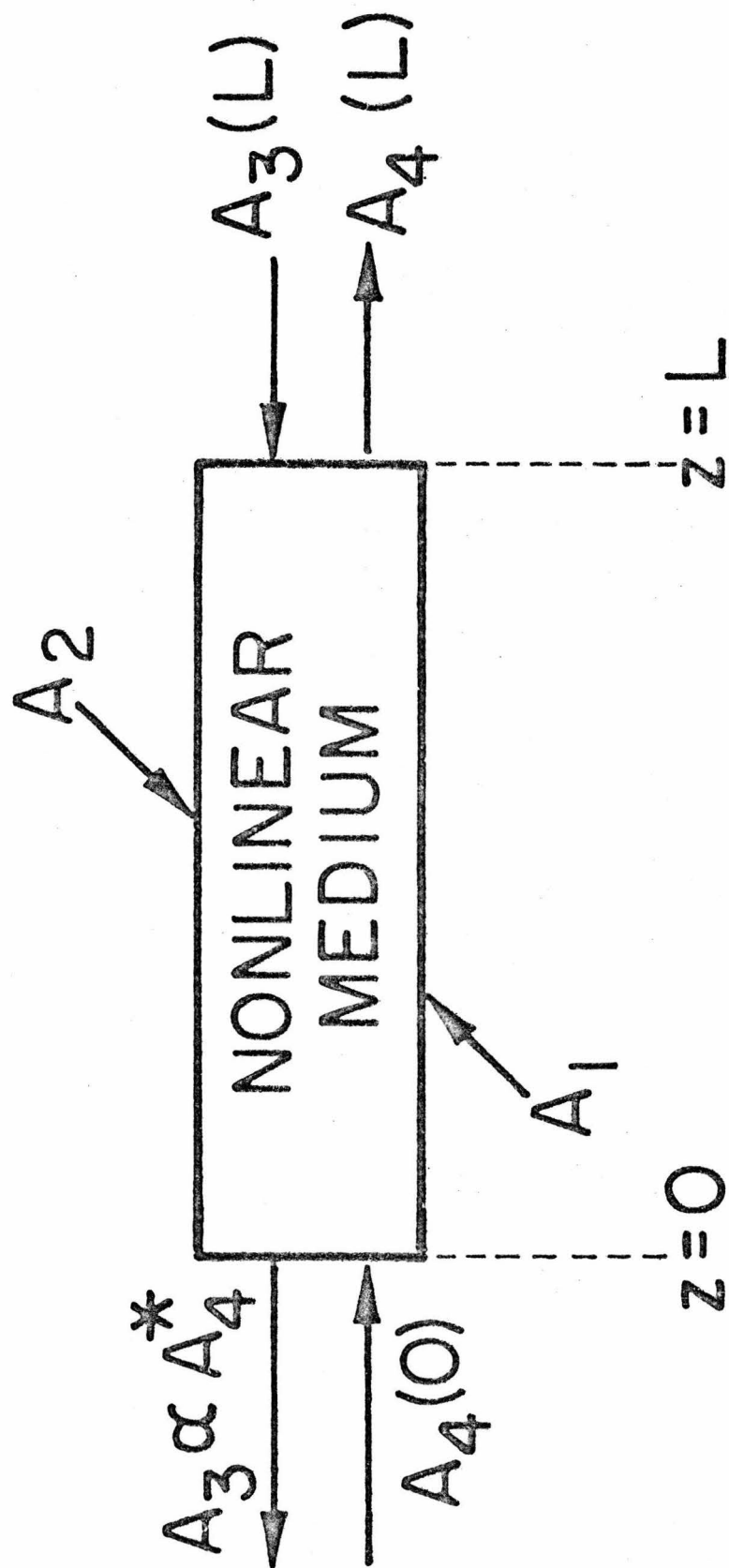


Fig. 2.1 Four-wave mixing geometry

nonlinear polarization

$$P_{NL}^{\omega+\omega-\omega} = \frac{1}{2} \chi^{(3)} A_1 A_2 A_4^* \exp\{i[(\omega+\omega-\omega)t - (\vec{k}_1 + \vec{k}_2 - \vec{k}_4) \cdot \vec{r}]\} + c.c. \quad (2.12)$$

is formed. It radiates to give a field

$$E_3(\vec{r}, t) = \frac{1}{2} A_3 \exp[i(\omega t - \vec{k}_3 \cdot \vec{r})] + c.c. \quad (2.13)$$

also at the frequency ω , but with a wave vector $\vec{k}_3(\omega) = \vec{k}_1 + \vec{k}_2 - \vec{k}_4$ that is equal to $-\vec{k}_4(\omega)$. $\chi^{(3)}$ is a third order nonlinear susceptibility tensor element of the medium. The backward going wave E_3 , with an amplitude proportional to A_4^* , in turn combines with the two pump waves to induce a third order nonlinear polarization,

$$P_{NL}^{\omega+\omega-\omega} = \frac{1}{2} \chi^{(3)} A_1 A_2 A_3^* \exp\{i[(\omega+\omega-\omega)t - (\vec{k}_1 + \vec{k}_2 - \vec{k}_3) \cdot \vec{r}]\} + c.c. \quad (2.14)$$

which radiates into E_4 . The phase matching condition is automatically satisfied in this process, since $\vec{k}_3 = -\vec{k}_4$. The two waves, without loss of generality, can be assumed to be counter-propagating along the z direction. The evolution of the amplitudes, $A_3(z)$ and $A_4(z)$, is obtained by solving the wave equation

$$\nabla^2 E - \frac{\epsilon}{c^2} \frac{\partial^2}{\partial t^2} E = \frac{4\pi}{c^2} \frac{\partial^2}{\partial t^2} P_{NL} \quad (2.15)$$

for each wave with its appropriate driving term given in equations (2.12) and (2.14), respectively. Neglecting the depletion of the pump beams E_1 and E_2 , and using the approximation

$$\left| \frac{d^2 A_j}{dz^2} \right| \ll \left| k_j \frac{dA_j}{dz} \right| \quad \text{for } j = 3, 4$$

two coupled equations can be obtained from the wave equation:

$$\frac{dA_3}{dz} = i\kappa^* A_4^* \quad (2.16)$$

and

$$\frac{dA_4^*}{dz} = i\kappa A_3 \quad (2.17)$$

where $\kappa^* = \frac{2\pi\omega}{cn} \chi^{(3)} A_1 A_2$ and n is the linear index of refraction of the nonlinear medium. The solutions, when expressed in terms of the boundary values $A_3(L)$ and $A_4(0)$ are

$$A_3(z) = \frac{\cos|\kappa|z}{\cos|\kappa|L} A_3(L) + \frac{i\kappa^* \sin|\kappa|(z-L)}{|\kappa| \cos|\kappa|L} A_4^*(0) \quad (2.18)$$

and

$$A_4^*(z) = \frac{i|\kappa| \sin|\kappa|z}{\kappa^* \cos|\kappa|L} A_3(L) + \frac{\cos|\kappa|(z-L)}{\cos|\kappa|L} A_4^*(0) \quad (2.19)$$

$z=0$ and $z=L$ specify the boundaries of the interaction region as shown in Figure 2.1. In particular, when there is only an incident wave $A_4(0)$ at $z=0$ with $A_3(L) = 0$, (2.18) yields

$$A_3(0) = -i\left(\frac{\kappa^*}{|\kappa|}\right) \tan|\kappa|L A_4^*(0) \quad (2.20)$$

while at the other end ($z=L$)

$$A_4(L) = \frac{A_4(0)}{\cos|\kappa|L} \quad (2.21)$$

The counter-propagating conjugate pair, E_3 and E_4 , coupled together through the nonlinear polarization, can be amplified simultaneously. The energy comes from the two pump beams. The coupling between E_3 and E_4 which is intermediated via the pump waves E_1 and E_2 serves as a feedback mechanism with gain, allowing the oscillation of this conjugate pair if sufficient

pump power is furnished.

The nonlinear medium, used under the condition prescribed above, can be viewed as a mirror, reflecting an input wave E_4 to give a backward wave $E_3 \propto E_4^*$. The difference between such a conjugate mirror and a conventional one lies in the fact that the reflected wave is a time-reversed replica and retraces the exact path of the incident wave; whereas the reflected wave from a conventional mirror obeys the conventional law of reflection. The intensity reflectivity of the conjugate mirror, as given in equation (2.20), is $\tan^2(|\kappa|L)$ which depends on the nonlinear properties of the material and the pump power. When $|\kappa|L > \frac{\pi}{4}$, $\tan|\kappa|L > 1$ and amplified reflection will occur; and at $|\kappa|L = \frac{\pi}{2}$, an infinite reflectivity, corresponding to self-oscillation, is predicted.

In the above analysis, E_3 and E_4 are plane waves. However, the result obtained is also applicable to waves of arbitrary phase front or transverse spatial dependence. This is because the solutions given in equations (2.18) to (2.21) are independent of the propagating direction and thus describe plane waves with any wave vector $\vec{k}$. An incident wave with a particular phase front and transverse spatial dependence can be expressed as a linear superposition of plane waves with different wave vectors. Each of these plane waves will give rise to its own conjugate plane wave with identical nonlinear reflectivity. A conjugate replica of the incident wave front is obtained when the individual conjugate plane waves are superposed together. It has been assumed implicitly in this argument that the individual plane waves traverse an identical

interaction length L . This is an approximation which is valid only when the transverse wave vector components are small. It therefore imposes a limitation on the angular divergence of the incident wave.

2.3 Real-Time Holography

An alternate viewpoint of four-wave mixing has a direct analogy in holography [5,6,9]. The incident (object) wave E_4 interferes with a (reference) pump beam, say E_1 , to form a grating which then diffracts the other counter-propagating (reconstruction) pump beam, E_2 , to give the conjugate image wave, E_3 . At the same time, the interference of E_4 and E_2 forms a hologram which diffracts E_1 to yield E_3 . The backward conjugate wave in turn forms a grating with either pump to diffract the other, resulting in the amplification of the incident object wave. Figure 2.2 shows the hologram recording and the reconstruction processes [9]. Four-wave mixing, however, differs from the conventional holographic experiments in the simultaneity of the grating formation and the reconstruction steps. This makes possible the processing of information in real time. The name, real-time holography, has been frequently used to refer to this kind of nonlinear optical means of wave front generation.

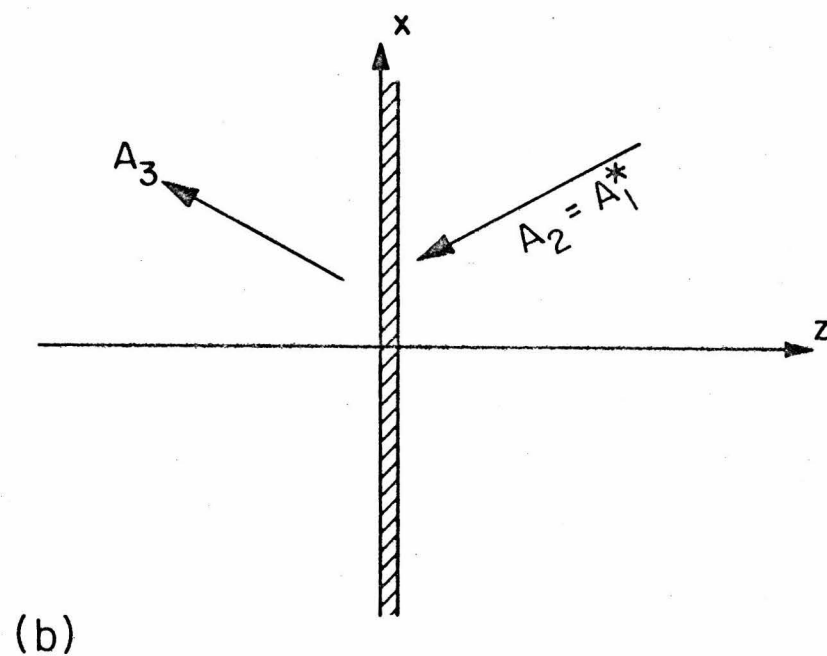
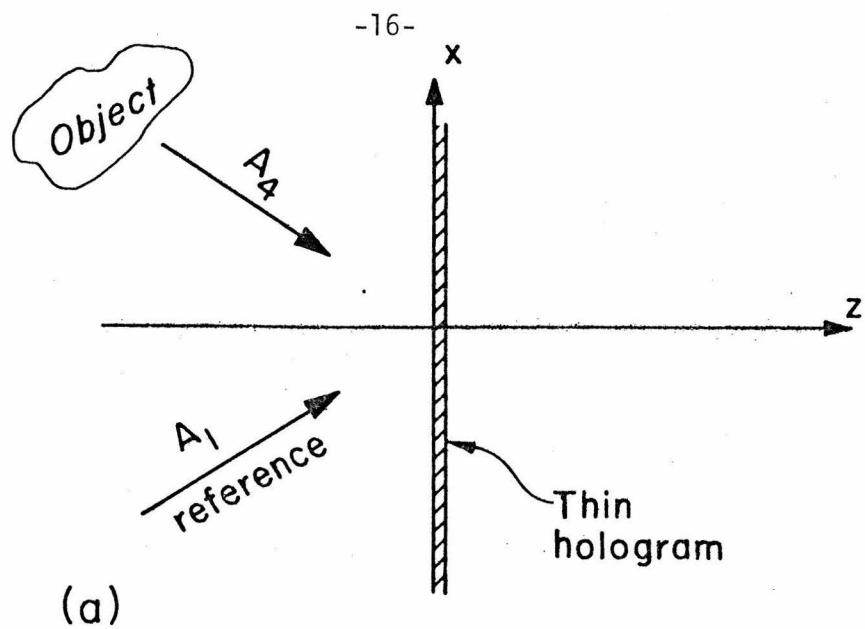


Fig. 2.2 (a) The recording and (b) the reconstruction of a hologram

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Chapter 3

PHASE CONJUGATION BY DEGENERATE FOUR-WAVE MIXING IN AN ISOTROPIC NEMATIC SUBSTANCE

In this chapter, an experiment which demonstrates the generation of a phase-conjugate wavefront by degenerate four-wave mixing is described. The ability of the conjugate wavefront to compensate for phase distortion is verified. The conjugation process takes place in a material, p-methoxybenzylidene p-n-butylaniline (MBBA) which is a nematic liquid crystal material at room temperature. The experiment was, however, done in the isotropic phase of this material which manifests some field-induced short range order above the transition temperature T_c . Extensive studies of nonlinear optical phenomena in this material have been reported elsewhere. [1-6]. Their findings, which are relevant to the phase-conjugation experiment, will be described later in this chapter. The main emphasis here will be on the verification of the theory presented in Chapter 2. The liquid crystal (MBBA) merely provides us with one of the many nonlinear media in which the interaction can take place; and thus the specific material properties of the medium will be discussed only when necessary.

3.1 The Experimental Arrangement

The experimental set-up is shown in Figure 3.1. The laser source was a Q-switched ruby laser operated in a single longitudinal and transverse mode. The linearly polarized (s-polarization) laser pulse had a duration of about 20 nsec, and a typical energy of 10 millijoules. The intensity

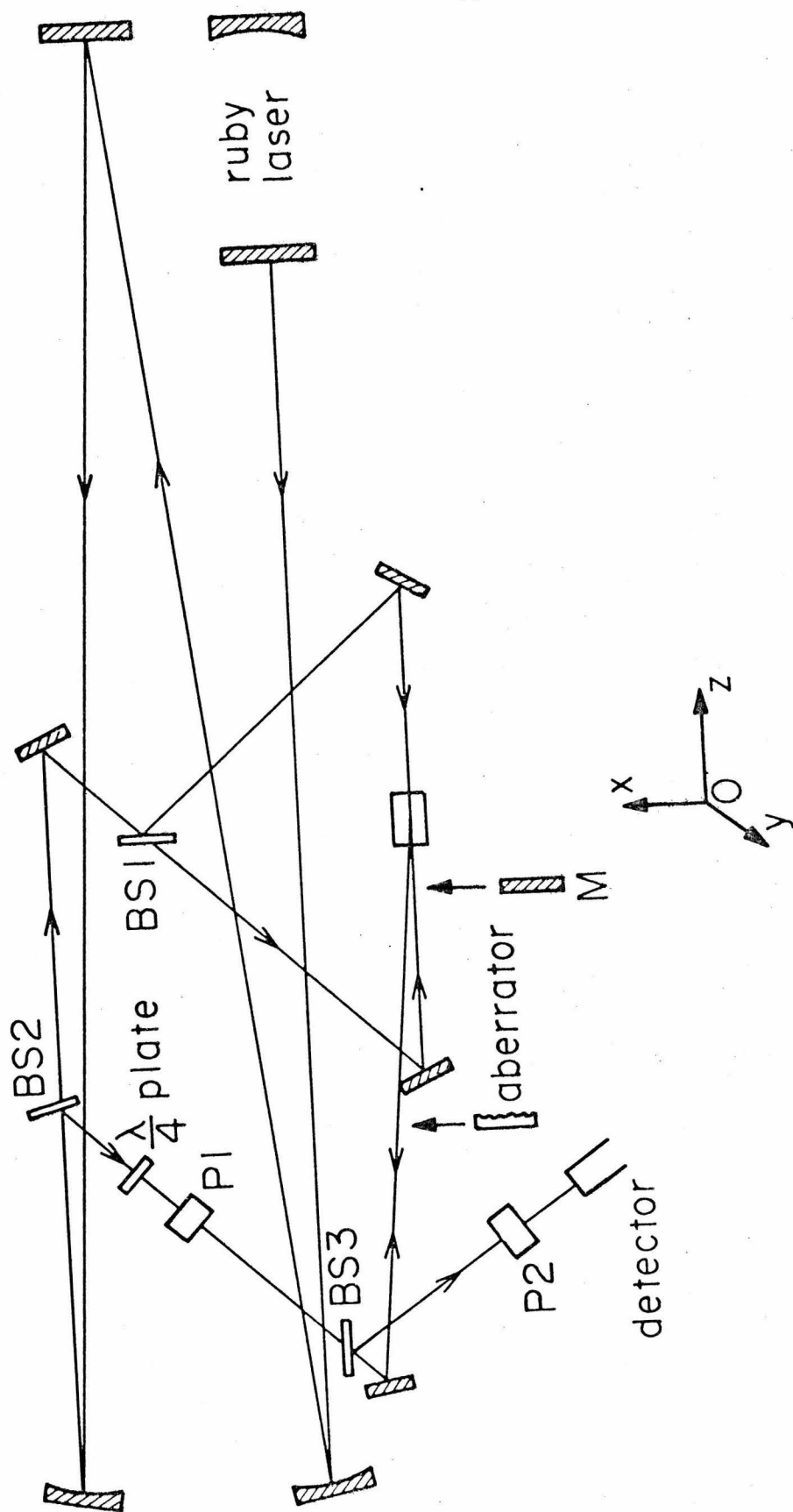


Fig. 3.1 Experimental set-up

of the beam varied transversely as $\exp(-r^2/w_0^2)$ with a beam radius w_0 of 0.33 mm. The laser pulse was divided by a beam splitter (BS1) into two beams which passed through the sample cell in opposite directions, to provide the two pump beams required for the four-wave mixing interaction. The field at the plane of the laser output mirror was imaged by two 6m radius-of-curvature mirrors onto a plane at the center of the sample cell. The path length between the laser and the cell was therefore 12 m, and this provided a round trip length of 24 m, or equivalently, 80 nsec delay to prevent feedback of the pulse back to the laser. However, it was discovered that when the two pump beams were so aligned as to propagate exactly opposite to each other in the sample cell, the laser was still perturbed and its output energy was reduced. A small portion of the laser energy was diverted by a beam splitter (BS2) to serve as the signal wave. Its polarization was converted by a quarter-wave plate and a polarizer (P1) to be orthogonal (p-polarized) to those of the two pump waves. The signal beam entered the cell almost parallel to the pump waves. The angle was limited to about ten milliradians to allow for a large overlap of the beams, and hence a large interaction region.

The nonlinear medium was p-methoxy-benzylidene p-n-butylaniline (MBBA) contained in a 5.7 cm long quartz cell. This is an opaque nematic liquid crystal at room temperature. It turns transparent at temperatures above $T_c = 42.5^\circ\text{C}$, at which point the material passes into the isotropic phase. Its properties at temperatures near and above the transition point, T_c , have been investigated elsewhere [1-6], and it has been found to possess a very high nonlinear susceptibility at these temperatures,

thus making it a suitable material for phase-conjugate wavefront generation.

The two s-polarized counterpropagating pump waves and the p-polarized signal wave induced a nonlinear polarization inside the MBBA which radiated a p-polarized conjugate wave. The conjugate beam propagated backward along the path of the signal wave, and was directed by a beam splitter (BS3) to a detector. The polarizer P2 in Figure 3.1 passed only the p-polarized conjugate wave and rejected any s-polarized light that was caused by scattering of the intense pump waves.

3.2 Theoretical Background

The specific material properties of the MBBA and the pulsed nature of the experiment make it necessary to modify the analysis given in the last chapter which is intended for steady state CW four-wave mixing.

The polarization P of a material is, in general, a function of the applied optical fields E and is expressed conventionally as a power series of the applied fields [7],

$$P = \chi \cdot E + \chi^{(2)} : EE + \chi^{(3)} : EEE + \dots \quad (3.1)$$

The first term describes the familiar linear susceptibility. The second term describes the lowest order nonlinear effects which include second harmonic generation and sum or difference frequency wave generation. Due to the presence of an inversion symmetry, the lowest order nonlinear effect in an isotropic material is the third term in the above series, which describes effects such as four-wave mixing, Raman scattering, and self-focusing of waves. The properties of the third order nonlinear polarization

in an isotropic medium and, in particular, of MBBA, will be given below.

The third order nonlinear polarization in an isotropic medium is [6]

$$P_i^{(3)}(\omega) = \sum_j 6[\chi_{ijjj}^{(3)} E_i(\omega) E_j(\omega) E_j^*(\omega) + \chi_{ijji}^{(3)} E_j(\omega) E_j(\omega) E_i^*(\omega)] \quad (3.2)$$

The nonlinear susceptibility tensor χ_{ijkl} is made up of two parts; for example, $\chi_{ijji}^{(3)}$ is typically written as [6]

$$\chi_{ijji}^{(3)} = \frac{1}{24} (\sigma + 2\beta) \quad (3.3)$$

σ is an electronic contribution which is due to the optical field induced deformation of the electron cloud around the molecules; β is the nuclear part which is due to the molecular response to the applied fields. It has been found by Shen et al. [6] that the electronic contribution to the third order nonlinear susceptibility in MBBA is negligible, and the only nonvanishing susceptibility tensor element is $\chi_{ijji}^{(3)}$ which, in this case, is equal to $\frac{1}{12} \beta$. Unlike the electrons which can respond instantaneously to the applied optical fields, the nuclear response is slower and has longer relaxation time. The relaxation time of the nuclear response for MBBA varies inversely as temperature, ranging from 800 nsec near the transition temperature T_c to about 40 nsec at 10°C. above T_c [6]. Since the laser pulse in the experiment was only 20 nsec, the actual nonlinear susceptibility due to the transient effect would be smaller than that of the steady state case.

The conjugate field will now be derived with the assumption that both the pump pulses are not depleted and the signal pulse remains unchanged. Because of the pulsed nature and the Gaussian profiles of the waves, a more complete analysis, which can deal with the amplification of the signal and conjugate pulses similar to the coupled wave formalism in the last chapter, has not been found. In this small signal analysis, the two pump pulses inside the nonlinear medium are written as

$$\begin{aligned} \vec{E}_1(\vec{x}, t) = E_1(\vec{x}, t) \hat{x} = \frac{1}{2} A_1 \frac{w_0}{w(z)} \exp[i(\omega t - kz + \eta(z) - \frac{kr^2}{2R(z)})] \\ \times \exp[-\frac{r^2}{2w^2(z)}] \exp[-\frac{\alpha}{2} (z + \frac{1}{2} L)] \exp[-\frac{1}{2} \frac{1}{\tau_p} (t - \frac{z}{c})^2] \hat{x} + \text{c.c.} \quad (3.4) \end{aligned}$$

$$\begin{aligned} \vec{E}_2(\vec{x}, t) = E_2(\vec{x}, t) \hat{x} = \frac{1}{2} A_2 \frac{w_0}{w(z)} \exp[i(\omega t + kz - \eta(z) + \frac{kr^2}{2R(z)})] \\ \times \exp[-\frac{r^2}{2w^2(z)}] \exp[\frac{\alpha}{2} (z - \frac{1}{2} L)] \exp[-\frac{1}{2} \frac{1}{\tau_p} (t + \frac{z}{c})^2] \hat{x} + \text{c.c.} \quad (3.5) \end{aligned}$$

and the signal pulse is

$$\begin{aligned} \vec{E}_4(\vec{x}, t) = E_4(\vec{x}, t) \hat{y} = \frac{1}{2} A_4 \frac{w_0}{w(z)} \exp[i(\omega t - kz + \eta(z) - \frac{kr^2}{2R(z)})] \exp[-\frac{r^2}{2w^2(z)}] \\ \times \exp[-\frac{\alpha}{2} (z + \frac{1}{2} L)] \exp[-\frac{1}{2} \frac{1}{\tau_p} (t - \frac{z}{c})^2] \hat{y} + \text{c.c.} \quad (3.6) \end{aligned}$$

$\hat{x}$ and $\hat{y}$ are unit vectors. For simplicity, all the pulses are assumed to be propagating along the z axis which passes through the sample cell. $z = 0$ is located at the center of the cell; $w(z)$ and $R(z)$ are the usual beam radius and the radius of curvature of a Gaussian beam. Their definitions

are [8]

$$w(z) = w_0 \left[1 + \left(\frac{z\lambda}{\pi w_0^2} \right)^2 \right]^{1/2} \approx \frac{z\lambda}{\pi w_0} , \quad \text{for large } z \quad (3.7)$$

and

$$R(z) = z \left[1 + \left(\frac{\pi w_0^2}{\lambda z} \right)^2 \right] \approx z , \quad \text{for large } z \quad (3.8)$$

w_0 is the beam radius at $z=0$, $n(z)$ in equations (3.4)-(3.6) is $\tan^{-1}(\frac{z\lambda}{\pi w_0^2})$, α is the attenuation of the wave intensity per unit length, and L is the length of the cell. The pulse is taken to be traveling at the speed of light in free space, c , with a $1/e$ intensity pulse width of τ_p . The third order nonlinear polarization induced in the MBBA is [9]

$$\begin{aligned} \vec{P}_{NL}(\vec{x}, t) = & \frac{1}{2} \frac{\beta}{\tau} \left\{ E_2(\vec{x}, t) \int_{-\infty}^t \exp[-(t-t')/\tau] E_1(\vec{x}, t') E_4^*(\vec{x}, t') dt' \right. \\ & \left. + E_1(\vec{x}, t) \int_{-\infty}^t \exp[-(t-t')/\tau] E_2(\vec{x}, t') E_4^*(\vec{x}, t') dt' \right\} \hat{y} \quad (3.9) \end{aligned}$$

where τ is the relaxation time of the nuclear response, τ is 2.2×10^{-7} sec at 44.5°C at which the experiment was performed [6]. The radiated field due to the induced polarization source is formally given as [10]

$$\vec{E}_3(\vec{x}, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} -\frac{1}{c^2} \left[\frac{\partial^2}{\partial t'^2} \vec{P}_{NL}(\vec{x}', t') \right] \frac{\delta(t' + \frac{1}{c}|\vec{x} - \vec{x}'| - t)}{|\vec{x} - \vec{x}'|} dt' d^3\vec{x}' \quad (3.10)$$

After much tedious but straightforward calculation, we find that the radiated field outside the nonlinear medium at the detector shown in Figure 3.1 is approximately

$$\vec{E}_3(\vec{x}, t) = E_3(\vec{x}, t) \hat{y} = \frac{1}{2} A_3 \frac{w'_0}{w'(z)} \exp[i(\omega t + kz + \frac{kr^2}{2z})] \exp\left\{\frac{-r^2}{[w'(z)]^2}\right\} \exp\left[-\frac{1}{\tau} \left(t + \frac{z}{c}\right) - \frac{1}{2\tau_p^2} \left(t + \frac{z}{c}\right)^2\right] + c.c. \quad (3.11)$$

where $w'_0 = w_0/\sqrt{3}$ and $w'(z) = |z|/\pi w'_0$. Hence, the radiated pulse is also a Gaussian beam, but it has a smaller beam waist. Due to the self-focusing effect, the spot size w_0 of the input pulses inside the medium is generally smaller than that of the laser output outside the medium. The amplitude A_3 in equation (3.11) is calculated to be

$$A_3 = \frac{16\pi^2}{\lambda} \frac{\beta}{\tau} \exp\left[\left(\frac{\tau_p}{2\tau}\right)^2 - \frac{3}{4} \alpha L\right] \frac{\epsilon_p}{cnw_0^2} L_{eff} A_4^* \quad (3.12)$$

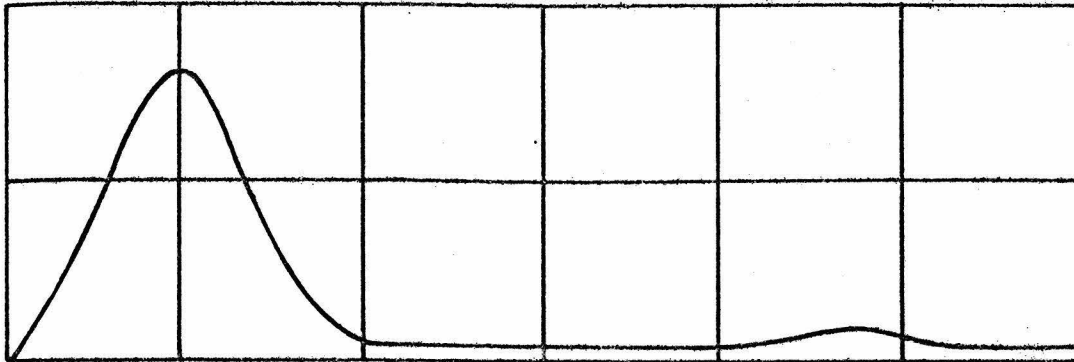
where n is the index of refraction of the medium and ϵ_p is the energy in each pump pulse. L_{eff} is the length of the interaction region, which is smaller than the length of the sample cell because of misalignment of the beams. In the experiment, it was found that if the two beams were exactly counterpropagating to each other, the ruby laser would be perturbed, and thus the pump beams were intentionally misaligned slightly. Also, in the calculation of $\vec{E}_3(\vec{x}, t)$, the signal pulse was assumed to overlap spatially with the two pump pulses. In the experiment, the signal pulse came in at an angle of 10 milliradians with respect to the pump pulses, and therefore further reduced the effective interaction volume. A_3 is shown to be proportional to A_4^* . The derivation can be carried out in the same way if A_4 is spatially dependent.

3.3 Experimental Results

In this section, the temporal and spatial behaviors and the energy of the conjugate beam observed in the experiment will be discussed.

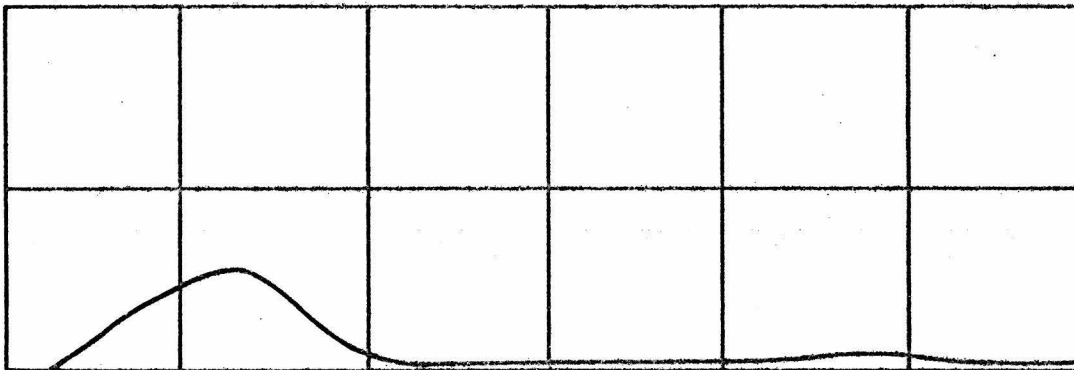
A typical pump pulse, shown in Figure 3.2a had a Gaussian profile with a $1/e$ width of about 25 nsec. The conjugate beam, detected only when all three input pulses were present, was shown in Figure 3.2b to have a similar pulse width as predicted in the last section. In a medium such as CS_2 , in which the response time is shorter, the conjugate pulse observed was typically shorter than the pump pulse because it is generated by the multiplication of three Gaussian pulses. In MBBA "the nonlinearity is mainly due to a consequence of electric field induced changes in the motions of nuclei whose electrons then respond linearly to the applied fields." [11]. The long relaxation time of the nuclear response integrates out the envelopes of the modulation pulses as shown in equation (3.9), thus giving rise to a conjugate pulse that is similar to the pump pulse.

To verify the conjugate nature of the radiated field and its ability to compensate for phase distortions, the following tests were carried out. First, a mirror M, as shown in Figure 3.1, was placed next to the sample cell to retroreflect the signal beam, and Figure 3.3a shows the spatial quality of the beam. Then a phase aberrator was inserted in the path of the signal beam and the picture of the reflected beam from the mirror M was taken. The beam had therefore passed through the aberrator twice. Figure 3.3b shows such a picture in which the aberrator was a piece of transparent double-stick Scotch tape. The mirror was then removed to allow the four-wave mixing to take place. Figure 3.3c shows the



20 nsec/div
pump pulse

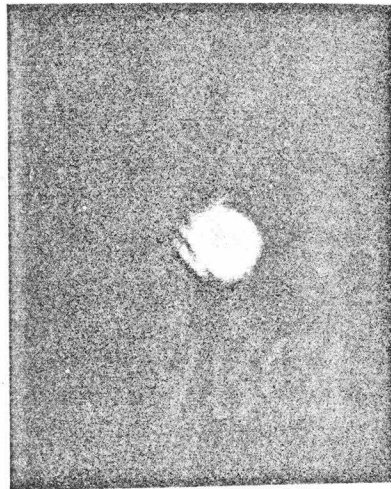
(a)



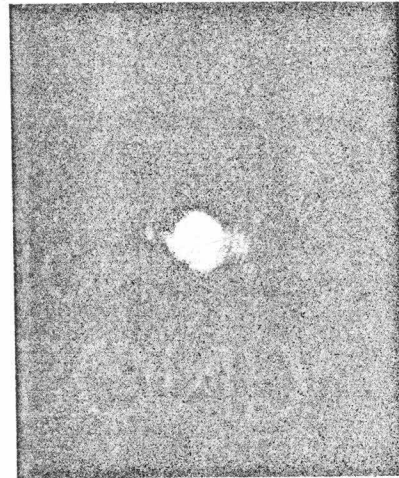
20nsec/div
conjugate pulse

(b)

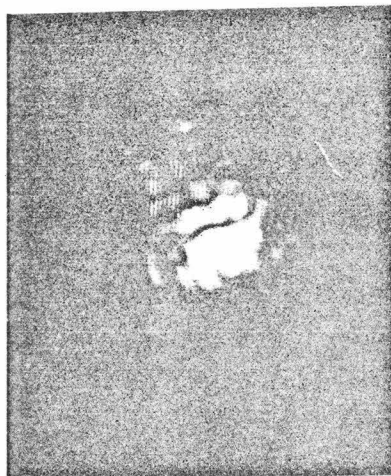
Fig. 3.2 The temporal profiles of a typical (a) pump pulse and (b) conjugate pulse



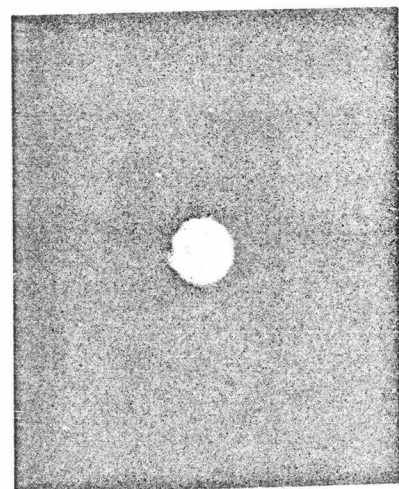
(a)
Beam reflected
from a mirror



(c)
Conjugate beam with
the aberrator in its path



(b)
Beam reflected from a
mirror with an
aberrator in its path

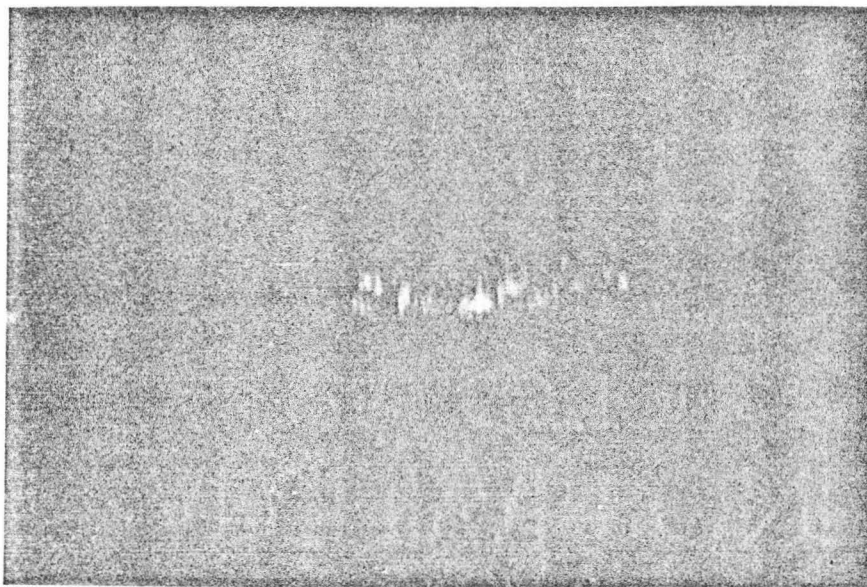


(d)
Conjugate beam

Fig. 3.3

generated conjugate beam which passed through the same aberrator before incidence onto the film. Figure 3.3d finally shows the picture of the conjugate beam with the aberrator removed. Since the conjugate beam had a different intensity than the mirror-reflected signal beam, Figure 3.3a and Figure 3.3b were taken with the appropriate neutral density filter in front of the film to simulate the intensity of the conjugate beam. The correction for phase distortion by the conjugate beam is quite evident. The ability of the conjugate beam to correct for different degrees of phase distortion was illustrated in Figures 3.4 and 3.5. Both Figures 3.4a and 3.5a show the mirror-reflected signal beams which had passed through their corresponding aberrators twice. Figure 3.4b and Figure 3.5b show the corrections by the conjugate beams.

The energy of the conjugate beam was measured and expressed as a percentage of the input signal energy. The result was plotted in Figure 3.6 as a function of pump pulse energy. The data points fit very well to the solid curve $\tan^2(0.209 \epsilon_p)$, suggesting the coupled wave formalism of the previous chapter might still be applicable to describe the present situation, because it does predict this kind of functional dependence on the pump energy. The nonlinear coefficient β was, however, much smaller than what was expected. It was shown by Shen et al. [5] that the spot size of an intense laser pulse has a very complicated nature due to the transient self-focusing effect inside the MBBA; and self-focusing of the laser beam was indeed detected in this experiment. Therefore the effective interaction volume was very small, giving rise to a weak conjugate signal from which the nonlinear coefficient was deduced.



(a)

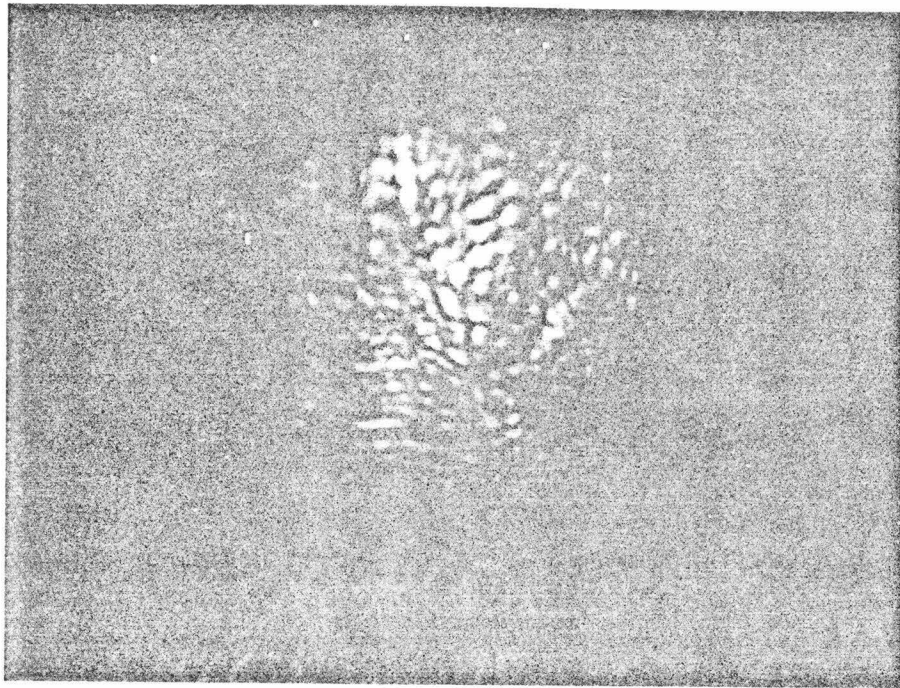
Reflection from a mirror with an aberrator
in the path of the beam



(b)

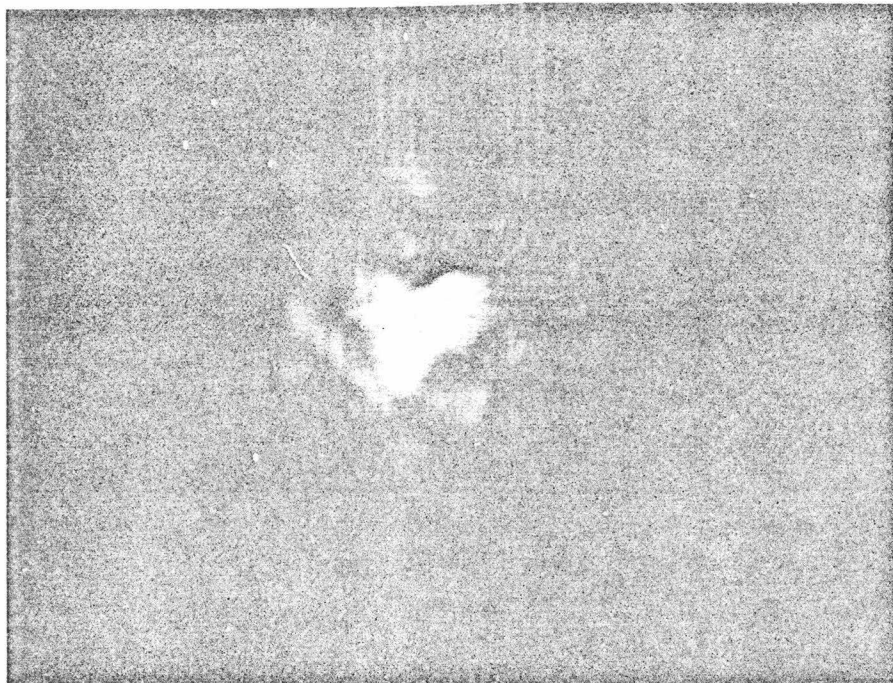
Conjugate beam with the same aberrator
in the path

Fig. 3.4



(a)

Reflection from a mirror with an aberrator
in the path of the beam



(b)

Conjugate beam with the same aberrator
in the path

Fig. 3.5

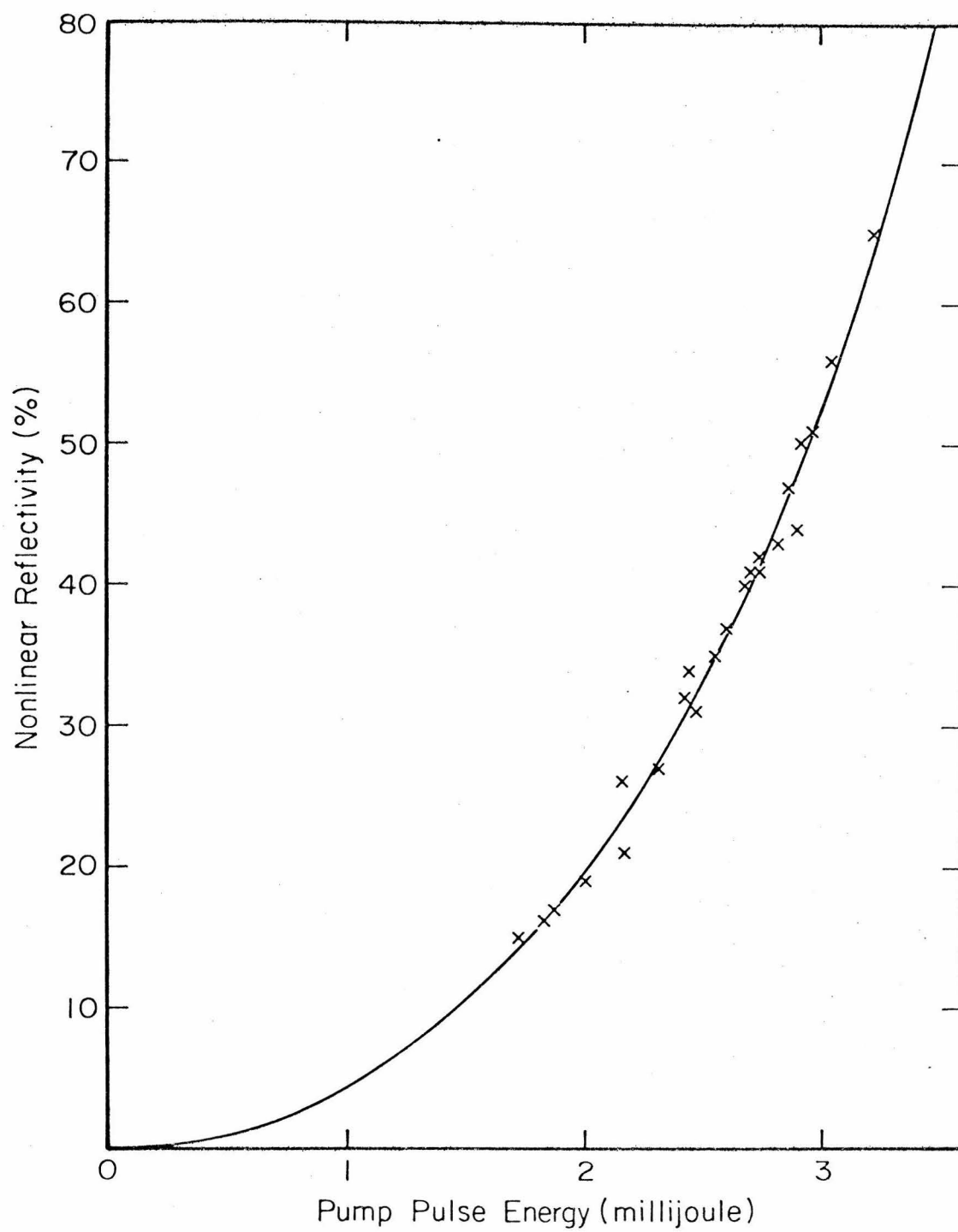


Fig. 3.6 The nonlinear reflectivity versus the pump pulse energy

3.4 Conclusion

In Section 3.2, the theoretical background of the experiment was presented. The radiated field due to the mixing of three pulsed Gaussian beams in the MBBA was derived. The derivation assumed that the two pump waves and the signal wave remained unchanged, and therefore any coupling between the signal and its conjugate wave was neglected. It was shown that the radiated field would also be a Gaussian beam with a smaller beam waist than that of the input beams, but it would have a pulse width identical to that of the input pulses. It was a small signal analysis corresponding to the linear region of the coupled wave formalism of Chapter 2. It predicted a nonlinear reflectivity that would be proportional to the square of the pump energies.

The experimental results described in Section 3.3 verified the qualitative features of the analysis. The observed conjugate wave was Gaussian and had the correct temporal behavior. Its ability to compensate for phase distortion was also demonstrated. There were, however, two discrepancies between the analysis and the experiment. The nonlinear reflectivity varied as $\tan^2(0.209 \epsilon_p)$ instead of ϵ_p^2 . This suggested that the pump pulse energies were high enough such that the small signal analysis was not adequate to describe the interaction taking place inside the medium. It showed a functional dependence of the nonlinear reflectivity on the pump pulse energies similar to that predicted by the coupled wave formalism for the CW plane wave case.

The second discrepancy was that the nonlinear coefficient β , deduced from the nonlinear reflectivity, was about 10 times smaller than

the reported value [6]. This was attributed to the misalignment of the beams and the presence of self-focusing inside the medium. Both factors combine to reduce the interaction region and hence the energy of the conjugate wave.

Chapter 3 - REFERENCES

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Chapter 4

PHASE CONJUGATION BY PHOTON ECHOES

All the optical mixing schemes for phase conjugation require the simultaneous illumination of a medium with several intense optical beams. There is, however, another phase reversal technique which uses the finite "memory" time of the atoms in the "mixing" medium. This technique does not require the optical fields to be present simultaneously.

Heer and McManamon [1] first pointed out that wave front correction and phase conjugation can take place in a photon-echo geometry involving forward traveling noncoincident four-wave mixing. Shiren [2] later showed that a backward conjugate echo can result when two of the noncoincident input pulses are opposite to each other.

In this chapter, these ideas will be explored and a perturbation theory will be used that can be applied to more general situations. The treatment results in a formulation of the problem that is similar to the perturbational description of CW nonlinear optical mixing [3]. It also includes the effects of collisional relaxation (T_1, T_2) on the process of conjugate echo formation. In Chapters 2 and 3, the nonlinear susceptibility was taken to be a phenomenological constant. The analysis presented in this chapter will illustrate how the nonlinear susceptibility of an atomic system can be derived quantum mechanically.

4.1 Photon Echo in Two-Level Atomic Systems

Consider an ensemble of two-level atoms with energy levels E_s and E_m ($E_m > E_s$). It will be assumed for the moment that no collisions take place so that the relaxation times are taken as infinite. The atoms are

subjected to three pulses of duration δ of optical radiation of the same frequency ω but traveling along arbitrary directions

$$\vec{E}_i(\vec{r}, t) = \frac{1}{2} \vec{E}_i(t) \exp[i(\omega t - \vec{k}_i \cdot \vec{r})] + \text{c.c.}, \quad i=1,2,3 \quad (4.1)$$

The pulse amplitude envelopes $E_i(t)$ are shown in Figure 4.1. The pulses may or may not coincide and the pulse areas are taken to be much smaller than π . To establish a bridge between the problem described above and the conventional nonlinear optics (where the three fields exist simultaneously) the problem will be solved using a perturbation expansion of the time evolution operator [4,5]. The objective here is to find the polarization induced in the atomic medium which is third order in the applied fields.

The wave function $\psi(t)$ at time t is related to its value $\psi(t_0)$ at an earlier time t_0 , according to

$$\begin{aligned} \psi(t) &= \exp[-i \frac{\hat{H}_0}{\hbar} (t-t_0)] \psi(t_0) \\ &\quad - \frac{i}{\hbar} \left\{ \int_{t_0}^t \sum_g |g\rangle \langle g| \exp[-i\omega_g(t-t')] \hat{V}(t') \exp[-i \frac{\hat{H}_0}{\hbar} (t'-t_0)] dt' \right\} \psi(t_0) \\ &= [\hat{U}^{(0)}(t-t_0) + \hat{U}^{(1)}(t-t_0)] \psi(t_0) \end{aligned} \quad (4.2)$$

where $\omega_g \equiv E_g/\hbar$. The total Hamiltonian is the sum of the unperturbed value $\hat{H}_0$ and the perturbation $\hat{V}(t)$, and

$$\hat{V}(t) = - \hat{\vec{\mu}} \cdot \vec{E}(\vec{r}, t) \quad (4.3)$$

with $\hat{\vec{\mu}}$ being the atomic dipole moment operator and $\vec{E}(\vec{r}, t)$ treated as a classical variable. Only the zero and first order terms in the expansion of the time evolution operator are included.

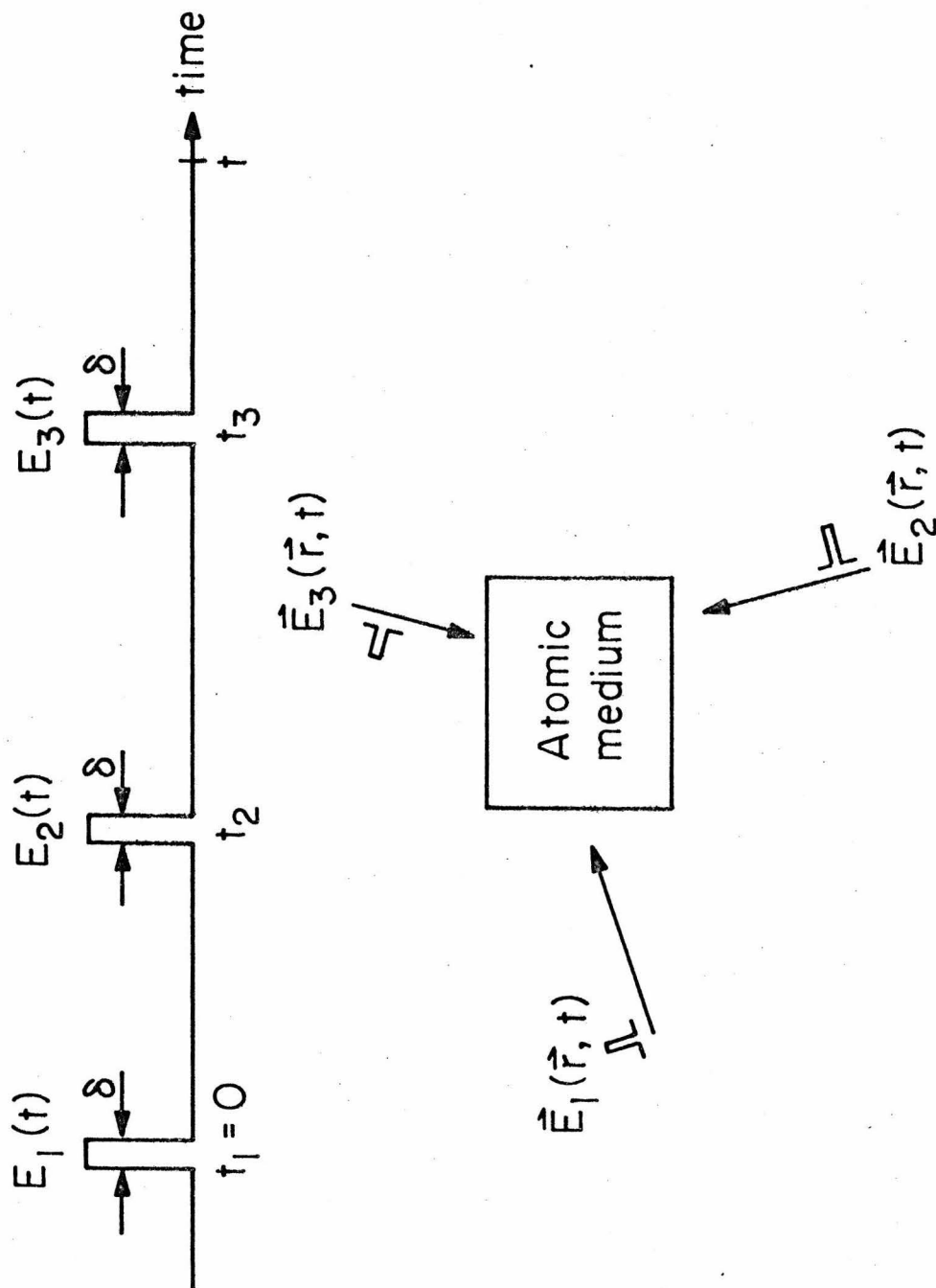


Fig. 4.1 The geometry for the interaction of the three-pulse sequence with the atomic medium

In the limit of small "area" pulses ($\mu E \delta / \hbar$) $\ll 1$ (δ = pulse duration) and negligible dephasing during the pulse $(\omega_m - \omega_s - \omega) \delta \ll 1$, the operator $\hat{U}^{(1)}(t-t_0)$ which represents the effect (to first order in E) of a pulse at t_i on the wave function becomes

$$\hat{U}^{(1)}(t_i) = \frac{i\delta}{2\hbar} \exp[\pm i(\omega t_i - \vec{k}_i \cdot \vec{r})] \sum_g |g\rangle \langle g| E_j^\pm \mu_j \quad (4.4)$$

$g = m$ or s , $E_j^- \equiv E_j^*$, $E_j^+ \equiv E_j$, $j = x, y$, or z . We take $(\hat{\mu}_j)_{ms} = \mu_{jms}$ for $m \neq s$ and zero otherwise. It follows that if at t_i the atom is in the ground state $|s\rangle$ then $\hat{U}^{(1)}(t_i)$ causes it to make a transition to the excited state $|m\rangle$ and vice versa. In the first case we use the lower (-) superscript, while in the second case we use the upper (+) superscript. The choice of $E_j^+(\equiv E_j)$ or $E_j^-(\equiv E_j^*)$ is due to the fact that only terms with near vanishing resonance denominators are retained in the derivation leading to equation (4.4).

We define the eigenfunction $\psi^{(1,0,3)}$, as an example, as that component of $\psi(t)$ that is due to "scattering" from $|s\rangle$ to $|m\rangle$ at t_1 (by E_1) and from $|m\rangle$ back to $|s\rangle$ at t_3 (by E_3), i.e.,

$$\psi^{(1,0,3)}(t) = \hat{U}^{(0)}(t-t_3) \hat{U}^{(1)}(t_3) \hat{U}^{(0)}(t_3-t_1) \hat{U}^{(1)}(t_1)|s\rangle \quad (4.5)$$

We can now use the formalism to calculate the j component of the induced atomic dipole moment $\langle \mu_j(t) \rangle$ following the sequence of three pulses. Specifically, we will look first for a dipole moment which is proportional to $E_1^* E_2 E_3$. Such a dipole moment results from $\langle \psi^{(0,0,3)} | \mu_j | \psi^{(1,2,0)} \rangle$ and from $\langle \psi^{(0,2,0)} | \mu_j | \psi^{(1,0,3)} \rangle$. To calculate the first term we apply equation (4.4) to obtain

$$\psi^{(1,2,0)} = \left(\frac{i\delta}{2\hbar}\right)^2 \mu_{1ms} \mu_{2sm} E_1^* E_2 \exp\{-i[(\omega t_1 - \vec{k}_1 \cdot \vec{r}) + \omega_m(t_2 - t_1) - (\omega t_2 - \vec{k}_2 \cdot \vec{r}) + \omega_s(t - t_2)]\} |s\rangle \quad (4.6)$$

$\psi^{(1,2,0)}$ thus represents a component of $\psi(t)$ which underwent a scattering $|s\rangle \rightarrow |m\rangle$ absorbing a photon from E_1 followed by $|m\rangle \rightarrow |s\rangle$ scattering due to the emission of a photon to E_2 . Similarly,

$$\psi^{(0,0,3)} = \frac{i\delta}{2\hbar} \mu_{3ms} E_3^* \exp\{-i[(\omega t_3 - \vec{k}_3 \cdot \vec{r}) + \omega_m(t - t_3) + \omega_s(t_3 - t_1)]\} |m\rangle \quad (4.7)$$

When the time t_1 is taken without loss of generality, as zero, the induced dipole moment at $t > t_3$ is

$$\begin{aligned} \langle \mu_j \rangle &= \langle \psi^{(0,0,3)}(t) | \mu_j | \psi^{(1,2,0)}(t) \rangle \\ &= \frac{i\delta^3}{8\hbar^3} \mu_{1ms} \mu_{2sm} \mu_{3sm} \mu_{jms} E_1^* E_2 E_3 \exp\{i[\omega t - (-\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \cdot \vec{r} + (\omega_{ms} - \omega)(t - t_2 - t_3)]\} \end{aligned} \quad (4.8)$$

where $\omega_{ms} \equiv \omega_m - \omega_s$.

A calculation of the contribution due to $\langle \psi^{(0,2,0)} | \mu_j | \psi^{(1,0,3)} \rangle$ gives a result identical to (4.8). The total value of $\langle \mu_j \rangle$ is thus twice that given by (4.8). Let the resonance offset parameter be $\Delta \equiv (\omega_{ms} - \omega)$. An inhomogeneous distribution of resonant frequencies is described by a normalized lineshape function $g(\Delta)$. The induced polarization P_j is obtained by summing $\langle \mu_j \rangle$ over all Δ . The result is

$$\begin{aligned}
 P_j &= N \int_{-\infty}^{\infty} \langle \mu_j \rangle g(\Delta) d\Delta \\
 &= \frac{i\delta^3}{4\hbar^3} N \mu_{1ms} \mu_{2sm} \mu_{3sm} \mu_{jms} E_1^* E_2 E_3 \exp\{i[\omega t - (-\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \cdot \vec{r}]\} S(t-t_2-t_3)
 \end{aligned} \tag{4.9}$$

where $S(t) = \int_{-\infty}^{\infty} g(\Delta) e^{i\Delta t} d\Delta$ is the Fourier transform of the normalized lineshape function $g(\Delta)$, and N is the dipole moment density.

For an even $g(\Delta)$ the function $S(t-t_2-t_3)$, hence P_j , is essentially zero except for a duration $\sim 2\pi/\Delta_{1/2}$ centered on

$$t = t_2 + t_3 \tag{4.10}$$

where $\Delta_{1/2}$ is the width of $g(\Delta)$. During this period the sample will radiate a photon echo provided the phase matching conditions are satisfied. If the pulses E_2 and E_3 are exactly opposite, $\vec{k}_2 + \vec{k}_3 = 0$, then the radiated echo at $t = t_2 + t_3$ will propagate according to (4.9) in the direction $-\vec{k}_1$, i.e., the reverse of the direction of E_1 . These results have been noted by Shiren [2], who applied the formalism of photon echoes. Our perturbation approach brings out specifically the field dependence $E_1^* E_2 E_3$ which is in a form identical to that used to describe four-wave conjugation in conventional nonlinear optics terminology [3]. It should be recalled, however, that here $E_1(t)$, $E_2(t)$, and $E_3(t)$ do not necessarily coincide in time.

By comparing equation (4.9) with equation (2.12) or (2.14), it is obvious that the induced polarization given in (4.9) can be expressed as a product of a third order nonlinear susceptibility $\chi^{(3)}$ and the applied fields. The analysis presented in this section shows how $\chi^{(3)}$ can be

derived. The formalism is general and is applicable to describe the interaction of other systems with pulsed or continuous optical waves.

The output wave radiated by the polarization (4.9) is the complex conjugate of the first wave $E_1(t)$. In holographic terms we can describe the process as one whereby a "hologram" is written into the atomic medium by pulses $E_1(t)$ and $E_2(t)$ which do not coincide in time. An interrogation pulse at t_3 opposite in direction to $E_2(t)$ gives rise to the backward read-out echo at $t = t_3 + t_2$ which is the conjugate replica of $E_1(t)$. If pulses E_2 and E_3 are parallel to each other as well as to pulse E_1 , i.e., $\vec{k}_1 = \vec{k}_2 = \vec{k}_3$, then the induced echo has, according to (4.9), a spatial dependence of $\exp[i(\omega t - \vec{k}_1 \cdot \vec{r})]$. It is thus radiated in the forward direction.

The special case of $\vec{E}_2(\vec{r}, t) = \vec{E}_3(\vec{r}, t)$ is of interest. The pulse formed is proportional to $E_1^* E_2^2 \exp\{i[\omega t - (2\vec{k}_2 - \vec{k}_1) \cdot \vec{r}]\}$ and occurs, according to (4.10), at $t = 2t_2$. If $\vec{k}_1 = \vec{k}_2$ then the pulse is radiated in the forward direction and can be recognized as the usual forward photon echo [6]. It is interesting to note that although only two pulses are applied, the phenomenon involves a third order mixing, except that two of the fields (E_2 and E_3) are degenerate. The demonstrations of phase conjugation by photon echo have been reported recently [7,8].

It is also of interest to inquire about the possibility of an echo proportional to $E_1 E_2^* E_3$ in which the second pulse is conjugated. An analysis similar to the above reveals that such an echo occurs at $t = t_3 - t_2 < t_3$ and is thus unrealizable except for the special case $t_2 = 0$, i.e., pulses E_1 and E_2 coincide.

4.2 The Effect of Relaxation Time

To determine the effect of the population relaxation time (T_1) and dephasing collisions (T_2) on the intensity of the radiated pulse, the induced dipole moment can be alternatively obtained as

$$\begin{aligned} \langle \mu_j \rangle &= \text{trace}(\hat{\rho} \hat{\mu}_j) \\ &= \rho_{12}(\mu_j)_{21} + \rho_{21}(\mu_j)_{12} \end{aligned} \quad (4.11)$$

where $\hat{\rho}$, the density matrix operator, can be expressed as a perturbation series, i.e., $\rho_{ij} = \rho_{ij}^{(0)} + \rho_{ij}^{(1)} + \rho_{ij}^{(2)} + \dots$. Specifically, $\rho_{ij}^{(1)}$ obeys the following equation of motion,

$$\begin{aligned} \frac{d\rho_{ij}^{(1)}}{dt} - i\omega_{ji} \rho_{ij}^{(1)} + \frac{\rho_{ij}^{(1)}}{T_{ij}} &= \frac{-i}{\hbar} [\hat{V}(t), \hat{\rho}^{(0)}]_{ij} \\ i, j &= 1, 2 \end{aligned} \quad (4.12)$$

For $i=j$, T_{ij} equals T_1 , and $\rho_{ii}^{(0)}$ is determined by the i^{th} energy level thermal equilibrium population. For $i \neq j$, T_{ij} equals T_2 , and $\rho_{ij}^{(0)} = 0$. Equation (4.12) is obtained by using a perturbation expansion of the density matrix operator [9] in terms of the applied fields, and we shall look for a term in ρ_{12} that is proportional to the product of the three applied fields.

Upon the incidence of the first pulse, only the off diagonal density matrix elements, ρ_{12} and ρ_{21} , are perturbed to first order in E_1 . The driving term on the right hand side of (4.12) is proportional to $(\rho_{11}^{(0)} - \rho_{22}^{(0)})E_1(t)$. After the perturbation due to the first pulse, ρ_{12}

evolves as $\exp[t(i\omega_{ms} - \frac{1}{T_2})]$ and in the limit of small δ ,

$$\begin{aligned} \rho_{12}^{(1)}(t_3 > t > t_1) = & -i \frac{\mu_{1sm}}{2\hbar} \delta(\rho_{11}^{(0)} - \rho_{22}^{(0)}) E_1 \\ & \times \exp[i(\omega t_1 - \vec{k}_1 \cdot \vec{r})] \exp[(i\omega_{ms} - \frac{1}{T_2})(t-t_1)] \end{aligned} \quad (4.13)$$

The second pulse affects the diagonal elements, ρ_{11} and ρ_{22} , giving rise to perturbation terms proportional to $E_1^* E_2$. In this case, the driving term in (4.12) is the product of $E_2(t)$ and the $\rho_{12}^{(1)}(t)$ given in (4.13). ρ_{11} and ρ_{22} then evolve as $\exp(-t/T_1)$ after the second pulse.

$$\begin{aligned} \rho_{11}^{(1)}(t > t_2) = & \frac{-\mu_{1ms} \mu_{2sm}}{4\hbar^2} \delta^2(\rho_{11}^{(0)} - \rho_{22}^{(0)}) E_1^* E_2 \exp[i\omega(t_2-t_1)] \\ & - i(\vec{k}_2 - \vec{k}_1) \cdot \vec{r} \exp[-(i\omega_{ms} + \frac{1}{T_2})(t_2-t_1)] \exp[-\frac{1}{T_1}(t-t_2)] \end{aligned} \quad (4.14)$$

When the third pulse arrives, ρ_{12} and ρ_{21} are affected by driving terms proportional to $E_3(t)(\rho_{11}^{(1)}(t) - \rho_{22}^{(1)}(t))$ when the on-diagonal elements are given in (4.14). Thus ρ_{12} and ρ_{21} are proportional to $E_1^* E_2 E_3$ and evolve as $\exp[t(i\omega_{ms} - \frac{1}{T_2})]$ for $t > t_3$. Setting $t_1 = 0$, the final expression for the induced polarization is

$$\begin{aligned} P_j = & \frac{i\delta^3}{4\hbar^3} N \mu_{1ms} \mu_{2sm} \mu_{3sm} \mu_{jms} E_1^* E_2 E_3 \exp\{i[\omega t - (-\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \cdot \vec{r}]\} \\ & \times S(t-t_2-t_3) \exp[-\frac{1}{T_2}(t+t_2-t_3) - \frac{1}{T_1}(t_3-t_2)] \end{aligned} \quad (4.15)$$

At the height of the echo at time $t = t_2 + t_3$, the effect of relaxation is thus to reduce the radiated field by the factor $\exp[-\frac{1}{T_2}(2t_2) - \frac{1}{T_1}(t_3-t_2)]$. It is thus necessary that the time delay t_2 between the first and second pulses be short compared with the homogeneous relaxation time T_2 .

4.3 Phase-Conjugation in a Three-Level Atomic System

The above formalism can be easily applied to describe the interaction between a multilevel atomic system and a multipulse sequence (provided all the pulses have areas very much smaller than π). We shall next consider an interesting case of a three-level system with energy levels E_a , E_b and E_c ($E_c > E_b > E_a$). The three atomic states are connected by the nonzero dipole matrix elements μ_{ab} and μ_{bc} , while μ_{ac} is taken to be zero. The atoms are subjected to three pulses, each of width δ at times t_1 , t_2 , and t_3 , respectively. The first two pulses are of the same frequency $\omega_1 = \omega_2 \approx \omega_{ba}$, but can have different polarizations and wave vectors. They modulate the atomic state populations, which, in turn, scatter the third applied pulse which has frequency $\omega_3 \approx \omega_{cb}$. The induced dipole moment that involves all the three applied fields is caused by $\langle \psi^{(0,2,0)} | \mu_j | \psi^{(1,0,3)} \rangle$ and $\langle \psi^{(1,0,0)} | \mu_j | \psi^{(0,2,3)} \rangle$ resulting in

$$\begin{aligned} \langle \mu_j \rangle = & \left(\frac{i\delta}{2\hbar} \right)^3 \mu_{1ab} \mu_{2ba} \mu_{3bc} \mu_{jcb} E_1 E_2^* E_3 \\ & \times \exp\{i[\omega_3 t - (\vec{k}_1 - \vec{k}_2 - \vec{k}_3) \cdot \vec{r} + (\omega_{ba} - \omega_1)(t_2 - t_1) + (\omega_{cb} - \omega_3)(t - t_3)]\} \\ & + \left(\frac{i\delta}{2\hbar} \right)^3 \mu_{1ab} \mu_{2ba} \mu_{3bc} \mu_{jcb} E_1^* E_2 E_3 \\ & \times \exp\{i[\omega_3 t - (-\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \cdot \vec{r} - (\omega_{ba} - \omega_1)(t_2 - t_1) + (\omega_{cb} - \omega_3)(t - t_3)]\} \quad (4.16) \end{aligned}$$

When summed over the inhomogeneous distribution of the resonant frequencies of the atoms, it is observed that a maximum superradiant superposition of all the individual atomic polarizations is formed at t_3 , provided that

$t_2 = t_1$. To include the relaxation effect, the density matrix operators are solved and the induced polarization at $t > t_3$ and for the case $t_2 = t_1 = 0$ is

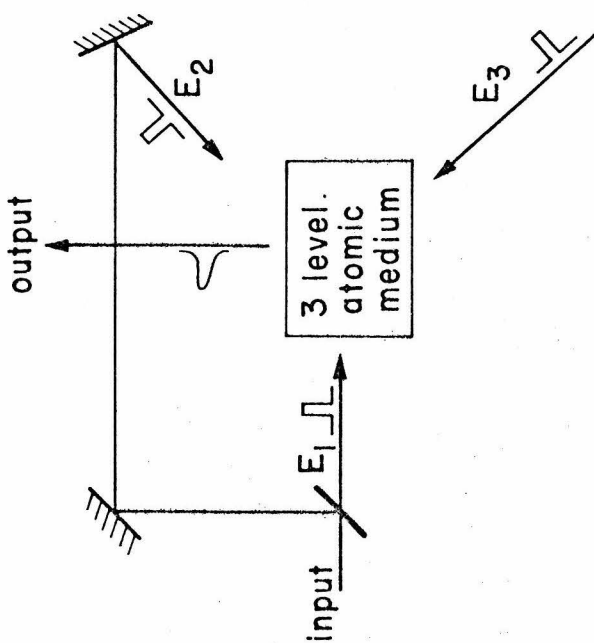
$$\begin{aligned} \langle \mu_j \rangle = & \left(\frac{i\delta}{2\hbar} \right)^3 \mu_{lab} \mu_{2ba} \mu_{3bc} \mu_{jcb} \exp \left\{ -\frac{1}{T_2} (t-t_3) - t_3 \frac{1}{T_1} + i(\omega_{cb} - \omega_3)(t-t_3) \right\} \\ & \times \left\{ E_1 E_2^* E_3 \exp \{ i[\omega_3 t - (\vec{k}_1 - \vec{k}_2 + \vec{k}_3) \cdot \vec{r}] \} \right. \\ & \left. + E_1^* E_2 E_3 \exp \{ i[\omega_3 t - (-\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \cdot \vec{r}] \} \right\} \end{aligned} \quad (4.17)$$

The first term of this polarization gives rise to a radiated field of frequency ω_3 and wave vector $\vec{k}_r$ with $k_r = \frac{\omega_3}{c} n(\omega_3)$, provided

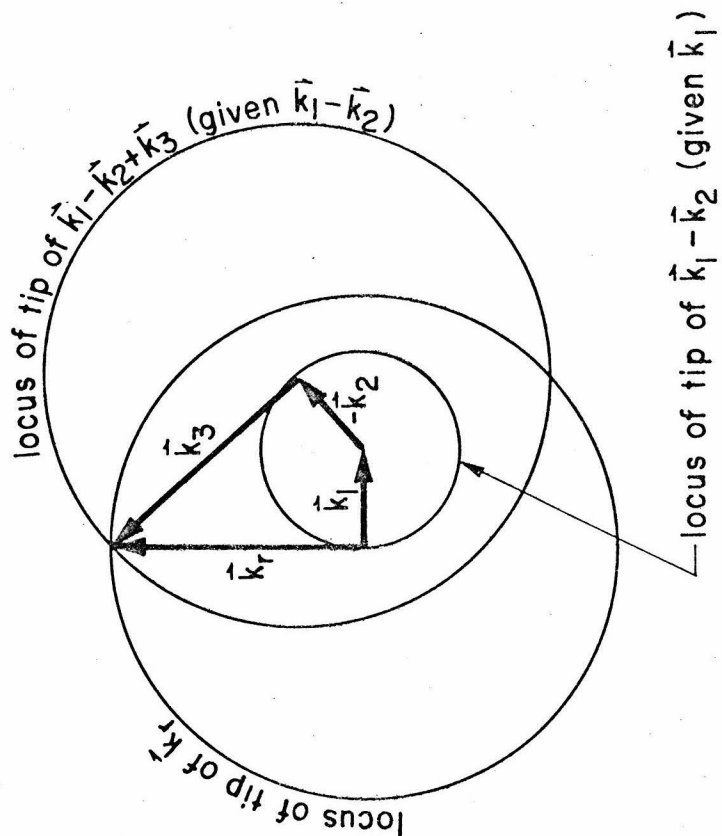
$$\vec{k}_1(\omega_1) - \vec{k}_2(\omega_2) + \vec{k}_3(\omega_3) = \vec{k}_r(\omega_3) \quad (4.18)$$

where $n(\omega_3)$ is the material index of refraction at ω_3 . The roles of E_1 and E_2 are reversed in the second term of the induced polarization. In an atomic medium in which $\omega_{ba} = \omega_{cb}$, ω_3 can be made equal to ω_1 , and (4.18) is easily satisfied. In particular, E_1 and E_3 can be chosen to be the two counterpropagating pump fields and a backward conjugate replica of E_2 is produced independent of the direction of $\vec{k}_2$.

The phase matching condition of (4.18) can also be met in a birefringent or an isotropic medium. Figure 4.2 shows a graphic construction for satisfying the phase matching condition in an isotropic medium for $\omega_3 > \omega_2 = \omega_1$. Given some choice of $\vec{k}_1(\omega_1)$ and $\vec{k}_2(\omega_1)$ the direction of $\vec{k}_3(\omega_3)$ and the radiated field $\vec{k}_r(\omega_3)$ is determined as shown.



(b)



(a)

Fig. 4.2 (a) A graphic construction for satisfying the phase-matching condition in an isotropic medium for $\omega_3 > \omega_2 = \omega_1$; (b) the geometry of the interaction of the three-pulse sequence with the 3-level atomic medium.

Chapter 4 - REFERENCES

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Chapter 5

APPLICATIONS OF PHASE CONJUGATE OPTICS

5.1 Corrections of Phase Aberrations

The most frequently quoted example for the application of phase conjugation is the correction of aberrations that arise when an optical wave passes through a phase distorting medium. Consider the example of sending spatial information which is encoded on an optical carrier wave. After propagating through a communication channel, such as a multimode optical fiber, the information is scrambled due to undesired phase distortion. In the case of a multimode fiber, this arises from the fact that each mode propagates with a different phase velocity. If a conjugate wave is generated at this point to reverse the phases and then allowed to propagate through another similar section of the fiber, the phase acquired by each fiber mode is cancelled, and the information thus recovered. Another important potential application of phase conjugation involves the corrections of aberrations in laser amplifiers and in other dynamic optical systems in real time. A possible method is illustrated in Figure 5.1. The figure shows a configuration designed to focus the output of a high power pulsed laser amplifier on a very small target, shown in the figure.

If the output of the amplifier were to be focused directly on the target, the distortion of the amplifier medium during the pump pulse would make it impossible to obtain a diffraction limited spot size thus reducing the available intensity.

In the compensation scheme the target 2 is illuminated by the output of a low power laser oscillator 1 (CW or pulsed). The reflected light 3 is collimated by lens 4 and traverses the amplifier 5 in reverse (away from the target). The distorted wave front 6 is incident on the conjugator 7 and is reflected as a conjugate wave 8. The wave retraverses the amplifier 5, emerging as a plane wave 9 which is focused on the target as a diffraction limited spot.

A most important attribute of this scheme is its ability to track a moving target provided the distance traversed during the round trip time from the target to the conjugator is small compared to the spot size. As a matter of fact, if the gain of the amplifying medium is sufficiently high and/or the reflection of the conjugator is strong, then an oscillation mode can exist in which the target serves as one reflector while the conjugate reflector completes the resonator. In this mode of oscillation [1] the beam autotracks the moving target. The low power illuminating laser (1 in Fig. 5.1) is not needed in this mode of oscillation.

5.2 Real-Time Spatial Information Processing

The analogy between real-time holography and four-wave mixing was pointed out in Chapter 2. It is natural to expect that four-wave mixing will have applications that are normally associated with conventional holography. Conventional holographic techniques are used extensively in coherent image processing such as pattern recognition and spatial filtering. Various schemes to use nonlinear optical effects for performing image processing have been proposed [2-4]. In this

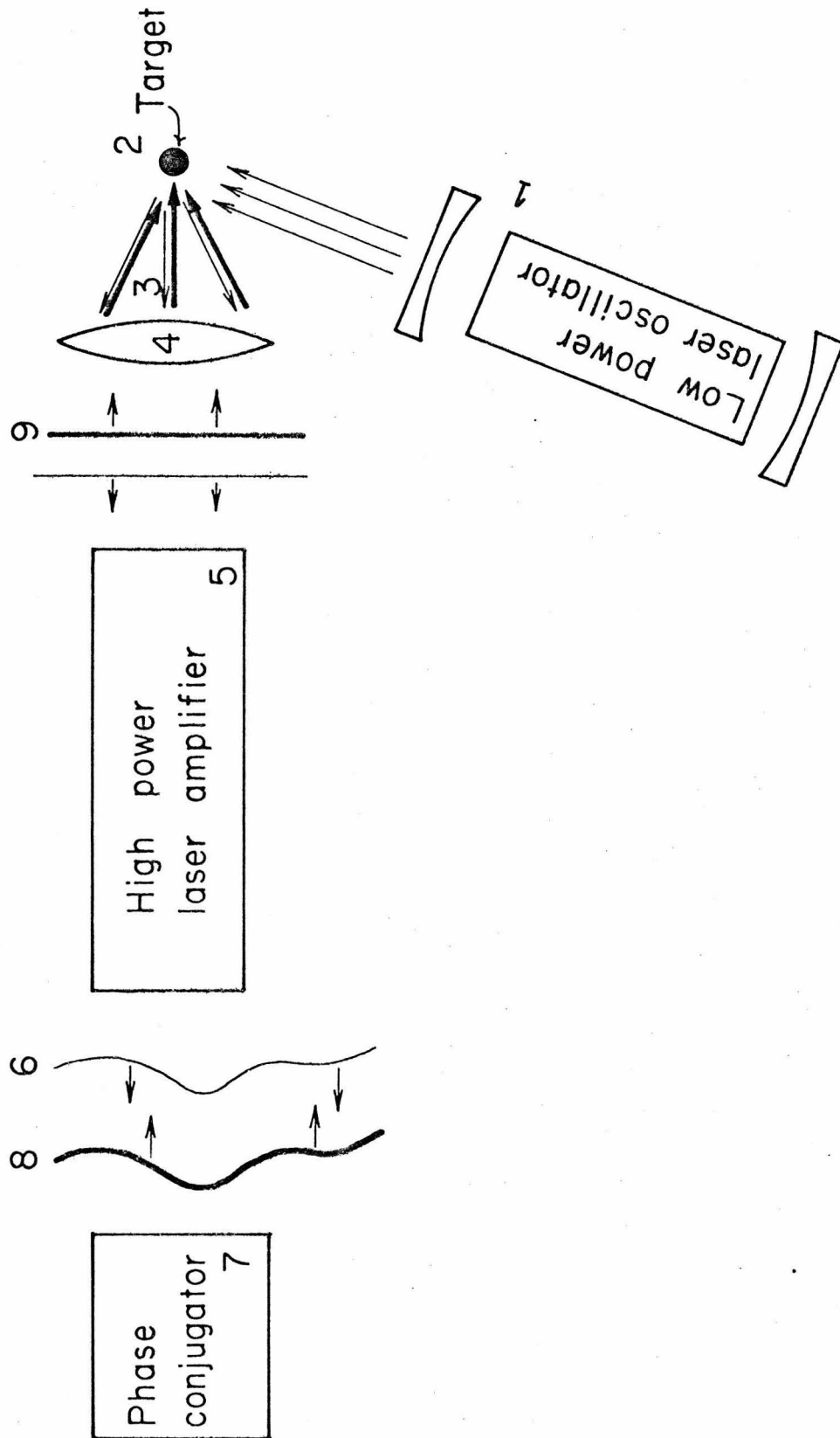


Fig. 5.1 A configuration designed to focus the output of a high power pulsed laser on a target

section, a scheme will be presented to show how four-wave mixing can be utilized to perform, in real time, the spatial convolution and correlation of images.

Consider the nature of the resultant field produced by simultaneously mixing three optical fields, all of radian frequency ω , incident upon a thin medium possessing a third order nonlinear optical susceptibility, $\chi^{(3)}$, centered at the common focal plane of two identical lenses of focal length f . The geometry is illustrated in Figure 5.2. Each field is specified spatially at the front focal plane of its respective lens with the following amplitudes:

$$\begin{aligned} E_1 &= \frac{1}{2} A_1(x,y,z) e^{i(kz-\omega t)} + \text{c.c.} \\ E_2 &= \frac{1}{2} A_2(x,y,z) e^{-i(kz+\omega t)} + \text{c.c.} \\ E_4 &= \frac{1}{2} A_4(x,y,z) e^{i(kz-\omega t)} + \text{c.c.} \end{aligned} \quad (5.1)$$

where $A_{1,4}(x,y,0) \equiv u_{1,4}(x,y)$, and $A_2(x,y,4f) \equiv u_2(x,y)$. Fields E_1 and E_2 are essentially counterpropagating, with E_4 being parallel to these fields and separated either spatially [e.g., shifted by (x_s, y_s)] or via orthogonal polarizations. The u_i contain the input information to be convolved or correlated, which can, for example, be in the form of phase and/or amplitude transparencies.

For a given thin lens, with an index of refraction n , a maximum thickness Δ and a focal length f , its effect on an incident optical wave is represented by a transmission function [5]

$\exp[ikn\Delta - \frac{ik}{2f}(x^2+y^2)]$. Furthermore, for a given electromagnetic wave

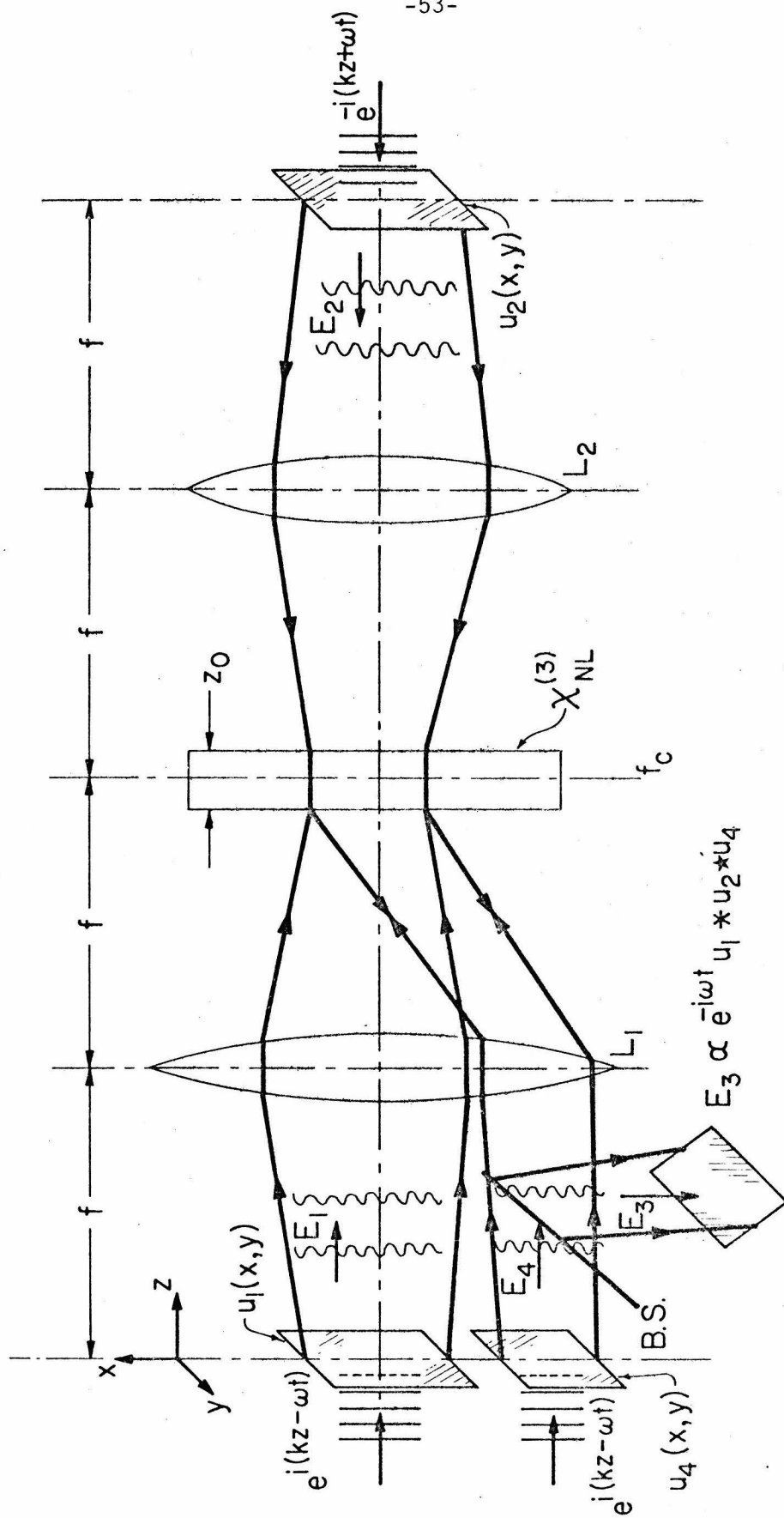


Fig. 5.2 The geometry for the spatial convolution and correlation of three optical fields

distribution $u(x,y)$ at the plane $z=0$, the scalar diffraction theory states that the wave distribution for $z > 0$, in the Fresnel approximation, will be [5]

$$u(x,y,z) = \frac{1}{i\lambda z} \exp(ikz) \exp\left[\frac{ik}{2z} (x^2+y^2)\right] \times \iint_{-\infty}^{\infty} dx' dy' u(x',y') \exp\left[\frac{ik}{2z} (x'^2+y'^2)\right] \exp\left[-\frac{ik}{z} (xx'+yy')\right] \quad (5.2)$$

Taking $z=0$ to be the front focal plane of the lens L_1 in Fig. 5.2, it can be shown that a field $A_1(x,y)$ defined in equation (5.1), after traveling from the front focal plane to the lens L_1 and passing through the lens, is of the form

$$A_1(x,y, f \leq z \leq 3f) = \frac{-i}{\lambda f} \exp[ik(n\Delta+z)] \iint_{-\infty}^{\infty} dx' dy' u_1(x',y') \times \exp\left[\frac{ik}{2f} \left(2 - \frac{z}{f}\right) (x'^2+y'^2)\right] \exp\left[-\frac{i2\pi}{\lambda f} (xx'+yy')\right] \quad (5.3)$$

Similarly, A_2 after propagating through lens L_2 has the form

$$A_2(x,y, f \leq z \leq 3f) = \frac{-i}{\lambda f} \exp[ik(n\Delta+4f-z)] \iint_{-\infty}^{\infty} dx' dy' u_2(x',y') \exp\left[\frac{ik}{2f} \left(\frac{z}{f} - 2\right) (x'^2+y'^2)\right] \exp\left[-\frac{i2\pi}{\lambda f} (xx'+yy')\right] \quad (5.4)$$

Anticipating the mixing process of interest, the complex conjugate of A_4 (after propagating through lens L_1) is

$$A_4^*(x,y, f \leq z \leq 3f) = \frac{+i}{\lambda f} \exp[-ik(n\Delta+z)] \iint_{-\infty}^{\infty} dx' dy' u_4^*(x',y') \exp\left[-\frac{ik}{2f} \left(2 - \frac{z}{f}\right) (x'^2+y'^2)\right] \exp\left[\frac{i2\pi}{\lambda f} (xx'+yy')\right] \quad (5.5)$$

The fields A_1 , A_2 , and A_4^* become directly proportional to the Fourier transforms of their respective u_i at the back focal plane of the lenses, i.e., $z=2f$, and are approximately the Fourier transforms of the u_i if the first exponents in the integrands of the three above equations can be neglected. This happens when

$$|z-2f| \ll \frac{2f^2\lambda}{r_i^2} \quad (5.6)$$

where r_i is the spatial extent of u_i .

We now place a medium centered at the focal plane f_c of thickness z_0 [satisfying the constraint (5.6)], which possesses a third order non-linear optical susceptibility, $\chi^{(3)}$. Without loss of generality, we assume a transparent, lossless medium, and neglect linear refractive effects.

The complex amplitude of nonlinear polarization at $\omega = \omega + \omega - \omega$ generated by the mixing of the three waves is, according to equation (2.12)

$$P_{NL_i} = \chi_{ijkl}^{(3)} A_{1j} A_{2k} A_{4l}^* \quad (5.7)$$

where repeated indices are summed over the field polarizations. The resultant field, A_3 , is given by

$$A_3(\vec{x}) = - \frac{4\pi\omega^2}{c^2} \int P_{NL}(\vec{x}') G(\vec{x}, \vec{x}') d^3\vec{x}' \quad (5.8)$$

where

$$G(\vec{x}, \vec{x}') = - \frac{1}{4\pi} \frac{e^{ikr}}{r} \quad (5.9)$$

is the Green function that satisfies the wave equation. Using (5.3),

(5.4), and (5.5) in (5.7) and (5.8), as well as condition (5.6), and integrating over the volume of the nonlinear medium, it is found that A_3 , after having passed through the lens L_1 , is

$$\begin{aligned}
 A_3(x, y, z = 0) &= - \frac{i2\pi\omega}{c} \frac{z_0 \chi^{(3)}}{\lambda^2 f^2} \exp[i2k(n\Delta + 2f)] \\
 &\times \iint_{-\infty}^{\infty} dx_1 dy_1 \iint_{-\infty}^{\infty} dx_4 dy_4 u_1(x_1, y_1) u_2(x_4 - x - x_1, y_4 - y - y_1) u_4^*(x_4, y_4) \\
 &= - \frac{i2\pi\omega}{c} \frac{z_0 \chi^{(3)}}{\lambda^2 f^2} \exp[i2k(n\Delta + 2f)] u_1(-x, -y) * u_2(-x, -y) \star u_4(-x, -y)
 \end{aligned} \tag{5.10}$$

The symbols $*$ and $\star$ denote the standard operations of convolution and correlation, respectively [5]. It has been assumed that $\chi^{(3)}$ is uniform throughout the nonlinear medium. A degenerate four-wave mixing interaction in the common focal volume of a two-lens system can therefore lead to the operations of spatial convolution and correlation of optically encoded fields. The extension of these concepts to real time matched filters, pattern recognition, and other forms of image processing follows directly.

Equation (5.10) is the primary result. The spatial convolution of $u_1(-x, -y)$ and $u_2(-x, -y)$ is obtained by making $u_4(-x, -y)$ a point source. Similarly, the correlation operation is performed by placing information on fields u_1 and u_4 , with u_2 being a point source. The point source will actually be a diffraction limited small Gaussian spot; and the finite spot size, in the focal plane f_c in Fig. 5.2, of the "point source" input will ultimately limit the spatial frequency bandwidth of the above operations.

The approximations used in the above analysis place upper limits on both the resolution (or spatial frequency bandwidth), and the efficiency (or nonlinear gain) of the interaction. The Fresnel approximation is related to $f_{\max}$, the greatest spatial frequency present, by

$$f_{\max} < \left(\frac{4}{\pi \lambda^3 f} \right)^{1/4} \quad (5.11)$$

This same approximation also places an upper limit on the input field spot size, which is given by

$$d < \left(\frac{4 \lambda f^3}{\pi} \right)^{1/4} \quad (5.12)$$

Hence, the maximum number of resolution elements possible is derivable through (5.11) and (5.12) yielding

$$N_{\max} = \frac{16f}{\pi \lambda} \quad (5.13)$$

Using values of 10 cm and 0.5μ for f and λ , respectively, leads to a value of 10^3 cm^{-1} for $f_{\max}$, which corresponds to a grid of 1000×1000 resolution elements per square centimeter.

The second constraint, (5.6), yields an upper limit to the nonlinear gain of the interaction. From (5.6) we obtain

$$z_0 \ll b_0 = \frac{2\pi^2 \omega_0^2}{\lambda} \quad (5.14)$$

where ω_0 is the spot size in the focal plane. For Gaussian beams, the value of b_0 corresponds to 2π times the confocal parameter [6].

The maximum number of resolution elements given in (5.13) has been derived with the assumption that the waves can be of infinite extent. In

actuality, the finite spot size of the Gaussian "point source" in the focal plane determines the maximum spatial frequency of the convolution or correlation operation. It can be shown that the number of resolution elements is

$$N \approx (d_1/d_2)^2 \quad (5.15)$$

where d_1 is the spatial extent of the input field, and d_2 is the input aperture size of Gaussian "point source," assuming both are satisfying the Fresnel approximation (5.12). It follows that for a given input power and choice of lens focal length, the output power with information present in the two input fields is related to that for no information present by

$$P_{\text{out_info}} \approx P_{\text{out_no info}} \cdot N \quad (5.16)$$

Thus, we see that there is a tradeoff between the spatial bandwidth product, N , and the output power (or nonlinear gain) of the interaction.

5.3 A Phase Conjugate Resonator

It has been mentioned that a nonlinear material, when used in the four-wave mixing geometry, can be treated as a conjugate mirror. If such a mirror is used to replace one of the two conventional mirrors that form a laser cavity, it has been observed that there is no stability condition necessary for oscillation to occur [7]. It also compensates phase distortion contributed by any poor intracavity optical components which otherwise would degrade the optical quality of the laser output beam.

In this section, an analysis is presented to derive the stability criterion and mode spectra of a laser resonator of which one of the mirrors is replaced by a phase conjugator. The phase-conjugate reflection is taken to be caused by a four-wave nonlinear interaction. The geometry of such a phase conjugate resonator is shown in Figure 5.3.

Consider an incident Gaussian field

$$E_i = E_i(\vec{r}) \exp[i(\omega t - kz - \frac{kr^2}{2\rho}) - \frac{r^2}{w^2}] + \text{c.c.} \quad (5.17)$$

with a complex amplitude $E_i(\vec{r})$, a radius of curvature ρ , and a spot size w , which is also frequently written as

$$E_i = E_i(\vec{r}) \exp[i(\omega t - kz - \frac{kr^2}{2q_i})] + \text{c.c.} \quad (5.18)$$

The complex radius of curvature q_i in (5.18) is defined as

$$\frac{1}{q_i} = \frac{1}{\rho} - \frac{i\lambda}{\pi w^2} \quad (5.19)$$

The effect of a phase-conjugate mirror is to reflect such an incident field to give its conjugate replica, leaving the wave front and the spot size unchanged. The reflected field is

$$E_r \propto E_i^*(\vec{r}) \exp[i(\omega t + kz + \frac{kr^2}{2\rho}) - \frac{r^2}{w^2}] + \text{c.c.} \quad (5.20)$$

which can also be expressed as

$$E_r \propto E_i^*(\vec{r}) \exp[i(\omega t + kz - \frac{kr^2}{2q_r})] + \text{c.c.} \quad (5.21)$$

The reflected field complex radius of curvature is

$$\frac{1}{q_r} = -\frac{1}{\rho} - \frac{i\lambda}{\pi w^2} = -\frac{1}{q_i^*} \quad (5.22)$$

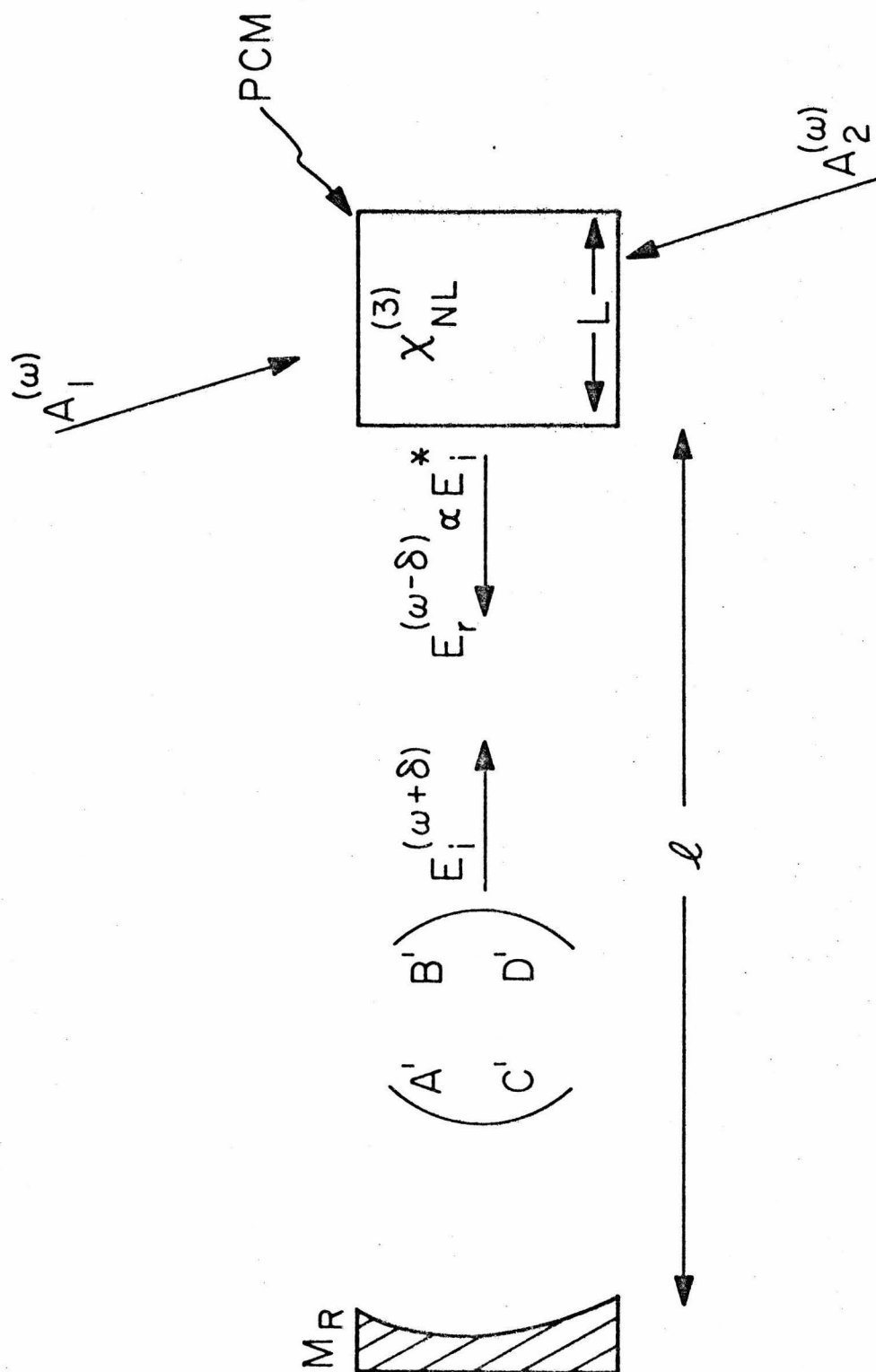


Fig. 5.3 A phase conjugate resonator

Using the conventional ray matrix formalism [6] which describes the effects of optical components on Gaussian beam propagation, the phase-conjugate mirror can be represented by a "ray" matrix

$$M \approx \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (5.23)$$

with the output and input complex radii of curvature related by

$$q_r = \frac{Aq_i^* + B}{Cq_i^* + D} \quad (5.24)$$

The conjugation operation on the q_i in this case is a unique feature of the conjugate mirror and is absent in the conventional usage of a ray matrix. It follows directly that the ordinary ray matrix formalism [6] for treating the propagation of Gaussian beams through a sequence of optical components can be applied also in the case when one of the elements in the sequence is a phase conjugate mirror. Using the matrix in (5.23) to represent the conjugate mirror, the complex output radius of curvature q_{out} at any plane following the conjugate mirror is related to the input complex radius of curvature, q_i , by

$$q_{out} = \frac{A_T q_i^* + B_T}{C_T q_i^* + D_T} \quad (5.25)$$

where $\begin{pmatrix} A_T & B_T \\ C_T & D_T \end{pmatrix}$ is the product of all the ray matrices representing the optical components, including the conjugate mirror, in the sequence. All the matrices in the sequence are assumed to be real and the conjugation operation on q_i can be performed at any plane.

Consider the situation sketched in Figure 5.3. The resonator is bounded on one end by a mirror having a radius of curvature, R , containing arbitrary intracavity optical components described collectively by an $A'B'C'D'$ matrix, $\underline{M}'$, for optical propagation from left to right, and again by an $A''B''C''D''$ matrix, $\underline{M}''$, for propagation from right to left. The resonator is bounded on the other end by a phase-conjugate mirror. In order to investigate the stability criterion for such a cavity, the standard self-consistent formalism is applied and the complex radius of curvature of the beam is required to reproduce itself after one round trip. Choosing a plane to the immediate right of the real mirror, we trace a beam that propagates to the right and get, after one round trip, the following matrix product:

$$\begin{aligned} \underline{M}_{\approx 1} &= \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \begin{pmatrix} A'' & B'' \\ C'' & D'' \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned} \quad (5.26)$$

where we have used the relation

$$\underline{M}''\underline{M}' = \underline{M}(\underline{M}')^{-1} \quad (5.27)$$

which can be shown straightforwardly, using the reciprocity property of the group of optical elements represented by $\underline{M}'$ (or $\underline{M}''$). The result is quite obvious if it is realized that the combination of any lossless and passive optical components and a phase conjugate mirror can be represented as the conjugate mirror itself, due to its time reversing nature. Self-

consistency demands the field to reproduce itself at the aforementioned plane after one round trip, and hence,

$$q = \frac{A_1 q^* + B_1}{C_1 q^* + D_1} \quad (5.28)$$

Upon solving for the beam parameters it is found that

$$\rho = -R \quad (5.29)$$

that is, the radius of curvature of the Gaussian beam matches that of the real mirror at the plane of the mirror, which can either be a concave or a convex mirror. There is no constraint on the spot size of the beam and also no stability condition for such a cavity. The result obtained is independent of the cavity length and the intracavity optical components, in marked contrast to the properties of a conventional Fabry-Perot resonator. A typical Gaussian mode which is self-consistent for one round trip is sketched in Figure 5.4a.

In conventional resonators, all allowed eigenmodes are obtained by demanding a single round trip self-consistent solution. However, in the case of a phase-conjugate resonator, it is possible to have allowed modes which will reproduce themselves after two round trips. Furthermore, by simple ray tracing, it can be shown that, due to the time-reversing nature of the conjugate mirror, the rays repeat themselves after two round trips. Using the same procedure as before, the ray matrix describing the beam propagation for two round trips is

$$(M_1)^2 = I \quad (5.30)$$

where $\tilde{I}$ is the identity matrix. Since the conjugate mirror is encountered twice in this case, the conjugation operation on q is cancelled. Hence, in contrast to the single round trip case, any complex radius of curvature (i.e., both ρ and w) at the initial plane will yield a self-consistent solution. The resonator is stable for any real convex or concave mirror. In Figure 5.4b, a Gaussian beam satisfying the two-round-trip self-consistent requirement is sketched.

It has been shown above that there exists a large range of Gaussian beam parameters that satisfy the self-consistency criterion for both the one- and two-round-trip case. This multitude of acceptable solutions follows from the very nature of the phase-conjugate mirror. Since only one constraint is specified within the resonator, in this case the curvature of the real mirror, and together with the fact that the conjugate mirror replicates any field incident upon its surface, no unique set of ρ and w exists within the phase conjugate resonator.

In the above analysis, the frequency ω of the Gaussian beam is the same as that of the pump waves illuminating the nonlinear medium where the four-wave mixing takes place. If the incident wave is at a frequency $\omega + \delta$ while the pump waves are at the frequency ω , the induced nonlinear polarization and hence the reflected conjugate wave, is at a frequency $\omega + \omega - (\omega + \delta) = \omega - \delta$. The frequency deviation δ can be greater or smaller than zero. Specifically, if the incident Gaussian beam is

$$E_i = E_i(\vec{r}) \exp\{i[(1 + \frac{\delta}{\omega})\omega t - (1 + \frac{\delta}{\omega})kz - (1 + \frac{\delta}{\omega})\frac{kr^2}{2q_i}]\} + \text{c.c.} \quad (5.31)$$

where $k = \omega n/c$, the reflected field from a conjugate mirror located at $z = 0$ is

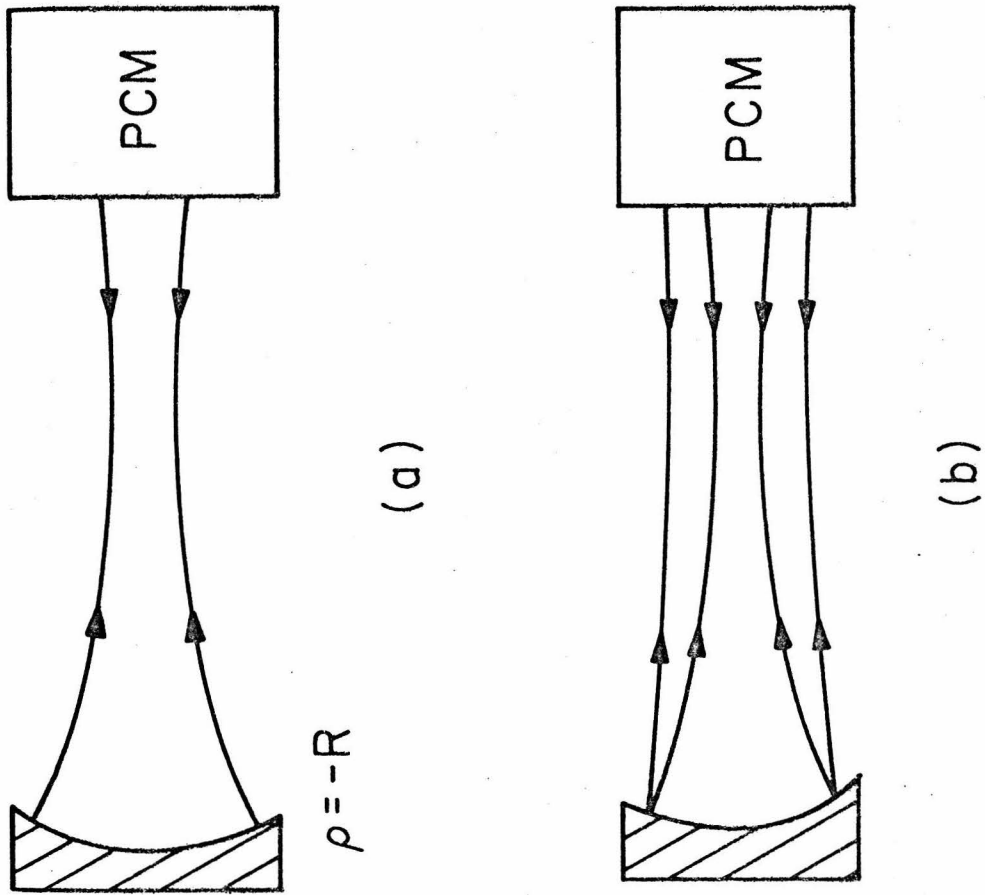


Fig. 5.4 A Gaussian mode which is self-consistent for (a) one, and (b) two round trips in a phase conjugate resonator

$$E_r \propto E_i^*(\vec{r}) \exp\{i[(1 - \frac{\delta}{\omega})\omega t + (1 - \frac{\delta}{\omega})kz - (1 + \frac{\delta}{\omega}) \frac{kr^2}{2(-q_i^*)}]\} + \text{c.c.} \quad (5.32)$$

which can be rewritten as

$$E_r \propto E_i^*(\vec{r}) \exp\{i[(1 - \frac{\delta}{\omega})\omega t + (1 - \frac{\delta}{\omega})kz - (1 - \frac{\delta}{\omega}) \frac{kr^2}{2q_r}]\} + \text{c.c.} \quad (5.33)$$

where

$$q_r = -q_i^* \frac{(1 - \frac{\delta}{\omega})}{(1 + \frac{\delta}{\omega})} \quad (5.34)$$

The reflected field, at the frequency $\omega - \delta$, propagates with a slightly different radius of curvature. The ray matrix, relating q_i to q_r in the same way as given in (5.24), is represented by

$$M \approx \begin{pmatrix} 1 - \frac{\delta}{\omega} & 0 \\ 0 & -(1 + \frac{\delta}{\omega}) \end{pmatrix} \quad (5.35)$$

which reduces to the matrix in (5.23) when δ is zero. Using the same self consistent formalism, this ray matrix can be used to find the constraints on ρ and w for the one- and two-round-trip case. It can be shown in a straightforward way that no solution exists for the one round-trip case, and a unique solution exists for the two-round-trip case, leading to interesting longitudinal and transverse mode spectra. However, the reflectivity of the conjugate mirror decreases rapidly with increasing $|\delta|$ due to phase mismatching [8] and hence these spectra are not easily observable and will probably be of limited practical importance.

The phase conjugate resonator is useful mainly when used in the degenerate case when all oscillating modes are locked to the same frequency ω as that of the pump beam. It will correct for any intracavity

phase aberrations and has been experimentally found to allow oscillation even in a conventional unstable cavity configuration [9].

Other applications of phase conjugate optics include optical filtering [8,10], compensation for group velocity dispersion in communication channels [11], optical gating, and others. Active research is still going on in this relatively new area. Many promising and exciting applications are yet to be explored.

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PART II

NONLINEAR OPTICAL PHENOMENA IN OPTICAL FIBERS

Chapter I
INTRODUCTION

1.1 Motivations for Studying Nonlinear Optical Effects Inside Fibers

The use of low-loss optical fibers for communication purposes has already become a reality. Large research efforts [1] have been devoted to the study of fiber properties such as the materials, scattering, and absorption losses, dispersion, mode characteristics, and fabrication techniques, etc. The objective is to understand and thus to control the transmission of light inside fibers under different conditions. The problems are primarily concerned with the linear response of the fibers to optical radiation. There exist, however, reasons for investigating nonlinear effects inside the fiber [2]. One reason is that of pure academic interest. An optical fiber is capable of guiding electromagnetic waves over long distances, with little attenuation, and simultaneously confining them to a small cross sectional area. This results in a long interaction region of high optical intensity even with small incident optical power. This situation is favorable for observing nonlinear optical phenomena, making the experiments and the measurements of nonlinear material properties a great deal easier. There are also practical motivations. In optical communication, it is desirable to increase the distances between repeater stations, which necessitates the use of large optical power for signal transmission. It is thus crucial to know the limitations of the fibers in handling large optical powers [3]. For example, it will be important to know the threshold power level for

various nonlinear effects and their effect on signal transmission. There are also devices built or proposed, such as the fiber oscillators [4,5] and some fiber gyro, based on the principle of nonlinear optical effects. The study of nonlinear optical phenomena in fibers will help to improve the performance of such devices and the invention of new devices.

1.2 Properties of Optical Fibers

The properties of optical fibers and their use for information transmission are reviewed briefly in this section. The structure of a typical optical fiber consists of a cylindrical core surrounded by a cladding layer which in turn is protected by an outer jacket region. When an electromagnetic wave travels along such a fiber, its energy propagates mainly inside the core medium, the wave being guided by total internal reflection at the core and cladding interface because the refractive index of the core is made slightly higher than that of the cladding. A component of the electric field propagating inside the fiber can be written, for example, as

$$E_x = \psi \exp(i\omega t) + \text{c.c.} \quad (1.1)$$

where ψ , the complex field amplitude, obeys the wave equation,

$$\nabla^2 \psi + \frac{\omega^2}{c^2} n^2(x,y) \psi = 0 \quad (1.2)$$

ω is the oscillation frequency of the wave and c is the velocity of light in vacuum. $n(x,y)$ is the index of refraction profile of the fiber and is uniform along the guiding direction z . The general

solution of equation (1.2) is

$$\psi = \sum_{mn} A_{mn} \mathcal{E}_{mn}(x,y) \exp(-\alpha_{mn}z - i\beta_{mn}z) \quad (1.3)$$

A_{mn} is an arbitrary constant and $\mathcal{E}_{mn}(x,y)$ is the (m,n) eigenmode of the fiber. α_{mn} and β_{mn} are the attenuation constant and the propagation constant of the (m,n) mode, respectively. The functional form of $\mathcal{E}_{mn}(x,y)$ depends on the particular index of refraction profile of the fiber [6]. In general, there are two kinds of modes, the guided modes and the radiation modes, analogous to the bound and free states of a particle in a potential well in quantum mechanics. The energy of the electromagnetic wave in a guided mode is confined in the fiber core region, while the energy radiates out of the fiber for a radiation mode. These eigenmodes form a complete set so that any electromagnetic wave distribution can be expanded in terms of a linear combination of them. The modes can also be normalized such that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dy \mathcal{E}_{ij}^*(x,y) \mathcal{E}_{mn}(x,y) = \delta_{im} \delta_{jn} \quad (1.4)$$

The propagation constant of a guided mode lies in the range $kn_c \leq \beta_{mn} \leq kn_0$ where n_c is the refractive index of the cladding layer and n_0 is the refractive index at the core center. If β_{mn} is smaller than kn_c , then the mode is a radiation mode. The number of guided modes depends on the geometry of the fiber, the wavelength of the propagating light, and the difference in refractive indices between the core and the cladding.

The frequently used method for information transmission through optical fibers consists of encoding the data into a train of light pulses,

sending them through a fiber, and then decoding the pulse sequence to recover the data at the receiving end. Because of group velocity dispersion, the pulses spread out in width as they travel along the fiber and eventually will overlap each other, resulting in a loss of information. This effect thus imposes a limit on the amount of information that can be transmitted per second. When direct pictorial transmission is desired, a bundle of fibers is used to transmit the spatial distribution of the light intensity of the illuminated object or picture. The resolution of the image that can be transmitted is limited by the diameter of the individual fiber in the bundle; and only two-dimensional pictorial information can be transmitted. In principle, three-dimensional pictorial information, i.e., both amplitude and phase, can be transmitted through a single multimode fiber [7]. The resolution of the picture is determined, in this case, by the number of guided modes of the fiber. However, the incident picture field excites many fiber modes, each of which propagates with a slightly different phase velocity. At the receiving end of the fiber, each mode has acquired a different phase, and the pictorial information is lost if the relative phase differences exceed 2π . This condition imposes a limit on the length of the fiber that can be used and the number of excited fiber modes, hence the resolution of the picture, allowed.

The technology of fiber manufacturing has made significant progress lately. Fiber loss is now approaching the limit of Rayleigh scattering, with attenuation less than one dB/km. Also, by controlling the refractive index profile of the fiber, it is possible to reduce the modal phase and

group velocity dispersions. It has been discovered that both the absorption and material dispersion go through a minimum at a wavelength of 1.3 μm . This has led to the development of new coherent sources operating at this wavelength.

1.3 The Main Features of Nonlinear Optical Effects Inside Fibers

Nonlinear optics was actively pursued only after the development of high power lasers because these nonlinear effects are small and require large optical intensities in order to be observed. Typically, the signal is proportional to the product of the light intensity I and the length L of the interaction region [2]. Large optical intensity can be achieved by focusing a laser beam with a lens into a small spot within the nonlinear medium. However, this beam diffracts rapidly, and the length over which the high optical intensity can be maintained is limited. For a given laser power P , the product

$$I \cdot L = \frac{P}{\pi w_0^2} \cdot \frac{\pi w_0^2}{\lambda} = \frac{P}{\lambda} \quad (1.5)$$

w_0 is the radius of the spot size and the interaction length is limited by diffraction to be the confocal parameter, $\frac{\pi w_0^2}{\lambda}$ [8]. Thus $I \cdot L$ is a constant and is independent of how strongly the beam is focused.

The nonlinear effects which occur inside optical fibers are not different from those observed in the bulk material. The only difference is in the geometry of the interaction region. If the same laser power P is coupled by a lens into a fiber, the product

$$I \cdot L = \frac{P}{\pi r_0^2} \cdot L \quad (1.6)$$

where r_0 is the core radius and L is the fiber length. For a single mode fiber, r_0 is on the order of several wavelengths; and for a several meter long fiber, the term in equation (1.6) is six orders of magnitude larger than the term in equation (1.5). This implies that nonlinear effects which require pulsed, Q-switched megawatt power lasers in order to be observed in the bulk can be observed with CW powers of several watts in a fiber. The power requirement can be further reduced if a longer fiber is used. There are, however, a couple of limitations on the length of the fiber to be used. The first limitation is the loss in the fiber itself. Each fiber has a loss coefficient α expressed in cm^{-1} or dB/km. If the fiber length is longer than $1/\alpha$, the signal and the input powers are severely attenuated and the output will be small. Another limitation is the coherence length of the incoming light. Some nonlinear effects are proportional to the product of the amplitudes of the pump waves instead of their intensities. This implies that in order for the signal generated in different sections of a fiber to add up in phase, the pump waves have to maintain a constant phase throughout the whole interaction region. Therefore, if the length of the fiber is longer than the coherence length of the pump waves, destructive interference will occur to reduce the amplitude of the output signal. Nonlinear processes such as the stimulated Raman scattering depend on the intensity of the pump wave and are therefore not subject to this constraint.

Fiber materials such as glass or liquid for liquid-filled hollow fiber are amorphous. These substances have a macroscopic inversion symmetry. This implies that nonlinear processes such as second harmonic

generation and three-wave mixing, where the induced nonlinear polarization is even order in the electric field, cannot occur inside these fibers [8]. The processes that will be discussed in this part of the thesis are third order nonlinear processes which can occur in any material.

1.4 Outline of Thesis Part II

In Chapter 2, a theory and an experiment for phase-conjugate wave front generation by degenerate four-wave mixing in a liquid filled fiber are described. In Chapter 3, a model for describing the stimulated Raman scattering will be given. Emphasis will be on fused silica fiber. A theory for CW Raman oscillation in optical fibers will be presented in Chapter 4. The analyses of Raman scattering and Raman oscillation discussed in Chapters 3 and 4 are easily applicable to describe the stimulated Brillouin scattering process. These two stimulated processes are the most intensively studied nonlinear effects inside fibers in the last ten years. Other nonlinear effects will be mentioned in Chapter 5, and their applications will be discussed.

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Chapter 2

PHASE CONJUGATION BY DEGENERATE FOUR-WAVE MIXING IN OPTICAL FIBERS

2.1 Analysis of Degenerate Four-Wave Mixing inside an Optical Fiber

The phenomenon of four-wave mixing is a third order nonlinear optical process; generally it needs high-laser-beam intensities and long interaction lengths in order to be observable. These conditions become less constraining when the interaction takes advantage of near-resonant atomic transitions. However, due to the ability of an optical fiber to guide light within a small cross-sectional area and over a long distance, even a small incident light power results in a sufficient intensity inside the fiber for nonresonant nonlinear phenomena and devices to be studied. The application of four-wave mixing to the generation of phase-conjugate wave fronts was discussed in detail in Part I. In this section the possibility of four-wave mixing in multimode optical fibers will be discussed. It will be found that complex image fields can be phase-conjugated and amplified without loss of spatial information, i.e., without mode mixing. A calculation shows that this can be done on a CW basis with moderate (~ 1 watt) pump powers in a few meters of fiber. This suggests that four-wave mixing in fibers is a serious candidate for real-time holographic applications including image transmission and compensation for distorting media.

The basic geometry involved in the experiment is illustrated in Figure 2.1. An input field E_i is incident on a fiber whose core medium possesses an appreciable third-order nonlinear coefficient $\chi^{(3)}$. The

input field E_i may correspond to an image field transmitted by another fiber or to some other field whose phase conjugate is sought. The fiber is multimode with a number of propagating modes essentially equal to the number of resolution elements contained in the incident field E_i . The fiber is pumped simultaneously by two strong, oppositely traveling fields, E_{p1} and E_{p2} whose frequency is the same as that of E_i . A reflected field E_r , generated by the four-wave mixing interaction, will be shown to be a complex conjugate of the input field. For sufficiently intense pump waves, E_r will be an amplified phase conjugate version of the input field E_i . The pump waves are taken as

$$\begin{aligned} E_{p1} &= \frac{1}{2} E_{p1}(x,y) e^{i(\omega t - \beta_1 z)} + \text{c.c.} \\ E_{p2} &= \frac{1}{2} E_{p2}(x,y) e^{i(\omega t + \beta_1 z)} + \text{c.c.} \end{aligned} \quad (2.1)$$

These pump fields will be coupled ideally into the lowest order propagating mode of the fiber so that their confinement over the full interaction length is assured. The input signal wave E_i and the reflected output wave E_r are expanded in terms of the propagating eigenmodes $\mathcal{E}_\ell(x,y)$ of the fiber as

$$\begin{aligned} E_i &= \frac{1}{2} \sum_m B_m(z) \mathcal{E}_m(x,y) e^{i(\omega t - \beta_m z)} + \text{c.c.} \\ E_r &= \frac{1}{2} \sum_\ell A_\ell(z) \mathcal{E}_\ell(x,y) e^{i(\omega t + \beta_\ell z)} + \text{c.c.} \end{aligned} \quad (2.2)$$

The complex amplitudes $B_m(0)$ are determined by the input conditions. We are seeking a solution for $B_m(z)$ and $A_m(z)$. The wave equation is

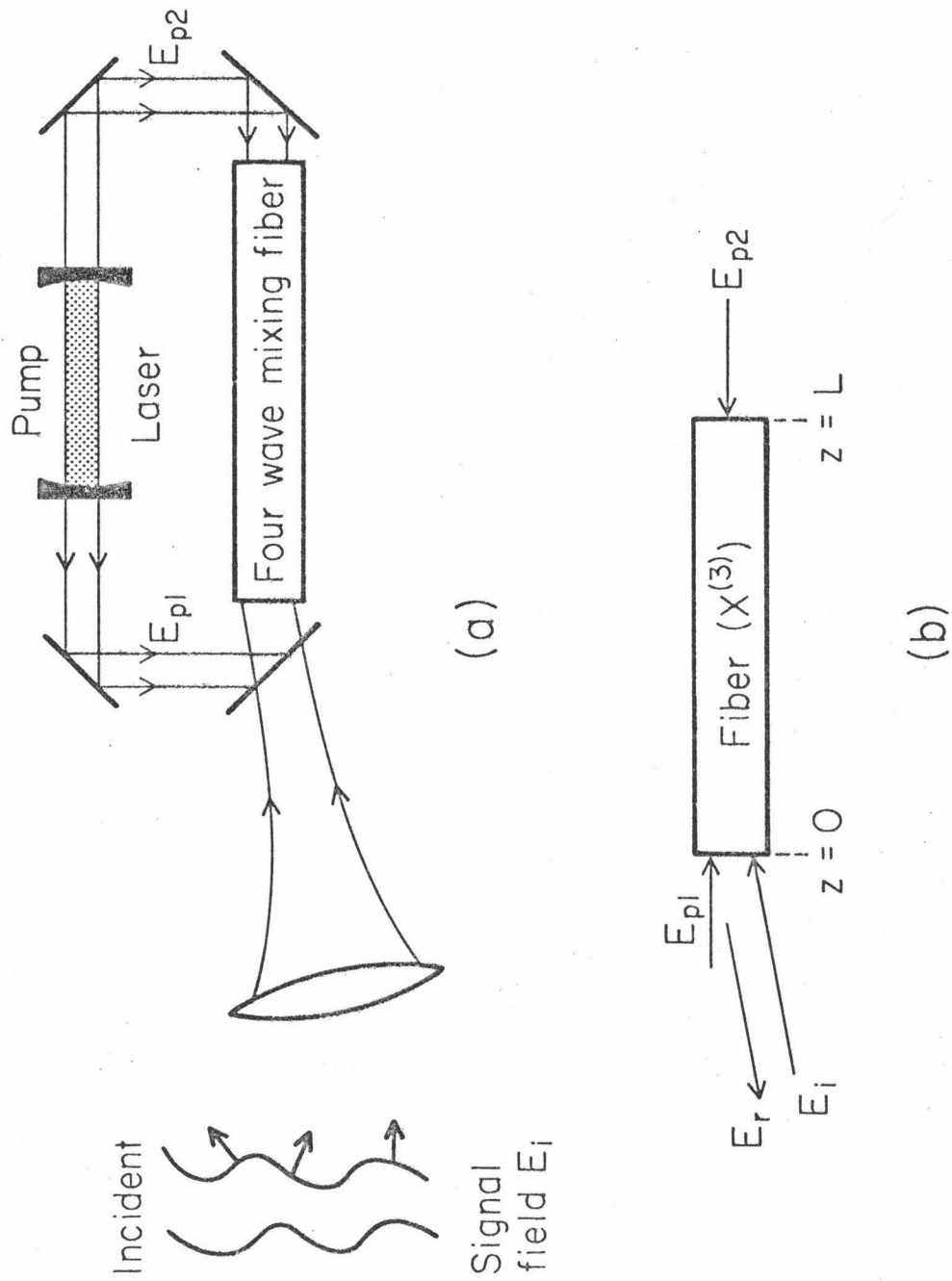


Fig. 2.1 The geometry for four-wave mixing in an optical fiber

$$\nabla^2 E - \mu \epsilon(\vec{r}) \frac{\partial^2 E}{\partial t^2} = \mu \frac{\partial^2}{\partial t^2} P_{NL}(r, t) \quad (2.3)$$

The functions $\mathcal{E}_\ell(x, y)$ satisfy (2.3) with $P_{NL} = 0$.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \beta_\ell^2 \right) \mathcal{E}_\ell(x, y) + \omega^2 \mu \epsilon(\vec{r}) \mathcal{E}_\ell(x, y) = 0 \quad (2.4)$$

and are orthonormal such that

$$\iint_{-\infty}^{\infty} dx dy \mathcal{E}_\ell(x, y) \mathcal{E}_m(x, y) = \delta_{\ell m} \quad (2.5)$$

Substituting equation (2.2) into equation (2.3),

$$\begin{aligned} \sum_{\ell} \left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \beta_\ell^2 + \omega^2 \mu \epsilon(\vec{r}) \right) \frac{A_\ell}{2} + i \beta_\ell \frac{dA_\ell}{dz} + \frac{1}{2} \frac{d^2 A_\ell}{dz^2} \right] \mathcal{E}_\ell \exp[i(\omega t + \beta_\ell z)] + c.c. \\ = \frac{\mu \partial^2}{\partial t^2} P_{NL}(\vec{r}, t) \end{aligned} \quad (2.6)$$

Using (2.4) to eliminate the first factor inside the square bracket, and assuming that $\left| \frac{d^2 A_\ell}{dz^2} \right| \ll |\beta_\ell A_\ell|$ leads to

$$\sum_{\ell} i \beta_\ell \frac{dA_\ell}{dz} \mathcal{E}_\ell \exp[i(\omega t + \beta_\ell z)] + c.c. = \frac{\mu \partial^2}{\partial t^2} P_{NL}(\vec{r}, t) \quad (2.7)$$

In a similar fashion we obtain for the input field

$$E_i = \sum_m -i \beta_m \frac{dB_m}{dz} \mathcal{E}_m \exp[i(\omega t - \beta_m z)] + c.c. = \frac{\mu \partial^2}{\partial t^2} P_{NL}(\vec{r}, t) \quad (2.8)$$

The induced nonlinear polarization is third order in the field amplitudes and is taken as

$$P_{NL}^{\omega=\omega+\omega-\omega} = \chi^{(3)} E_{p1} E_{p2} E_i^* \quad (2.9)$$

Substituting (2.9) into (2.7), we obtain

$$\sum_{\ell} \beta_{\ell} \frac{dA_{\ell}}{dz} \mathcal{E}_{\ell}(x,y) e^{i\beta_{\ell}z} = \frac{i\omega^2 \mu \chi^{(3)}}{2} E_{p1} E_{p2} \sum_m B_m^*(z) \mathcal{E}_m(x,y) e^{i\beta_m z} \quad (2.10)$$

In a similar fashion from (2.8),

$$\sum_m \beta_m \frac{dB_m}{dz} \mathcal{E}_m(x,y) e^{-i\beta_m z} = - \frac{i\omega^2 \mu \chi^{(3)}}{2} E_{p1} E_{p2} \sum_{\ell} A_{\ell}^*(z) \mathcal{E}_{\ell}(x,y) e^{-i\beta_{\ell} z} \quad (2.11)$$

or by conjugation (and taking E_{p1} , E_{p2} , and $\chi^{(3)}$ as real)

$$\sum_m \beta_m \frac{dB_m^*}{dz} \mathcal{E}_m(x,y) e^{i\beta_m z} = \frac{i\omega^2 \mu \chi^{(3)}}{2} E_{p1} E_{p2} \sum_{\ell} A_{\ell}(z) \mathcal{E}_{\ell}(x,y) e^{i\beta_{\ell} z} \quad (2.12)$$

Multiply (2.10) by $\mathcal{E}_s$ and integrate over the cross section using (2.5)

$$\frac{dA_s}{dz} = \frac{i\omega^2 \mu \chi^{(3)}}{2\beta_s} \sum_m B_m^*(z) e^{i(\beta_m - \beta_s)z} \int \int_{c.s.} E_{p1} E_{p2} \mathcal{E}_m \mathcal{E}_s dx dy \quad (2.13a)$$

and from (2.12)

$$\frac{dB_s^*}{dz} = \frac{i\omega^2 \mu \chi^{(3)}}{2\beta_s} \sum_m A_m(z) e^{i(\beta_m - \beta_s)z} \int \int_{c.s.} E_{p1} E_{p2} \mathcal{E}_m \mathcal{E}_s dx dy \quad (2.13b)$$

Equations (2.13) are the basic coupling equations. We note that a given mode A_s of the reflected field is coupled to all modes B_m of the incident field. This is undesirable, since it causes a loss of spatial information.

Two physical facts combine to remedy this mode scrambling: (a) We note that for a length of fiber $L \gg 2\pi(\beta_m - \beta_s)^{-1}$, modes A_s and B_m are grossly mismatched so that no cumulative power exchange between them can take place unless $\beta_m = \beta_s$. (b) In addition, if the pump fields $E_{p1}(x,y)$ and $E_{p2}(x,y)$ are more or less uniform over the cross section, then the overlap

integrals appearing in (2.13) are zero unless $m = s$. The combined effect of (a) and (b) is that a given reflected mode A_s of the reflected field couples almost exclusively to the incident field mode of the same index, i.e., to B_s . We thus have:

$$\begin{aligned}\frac{dA_s}{dz} &= \frac{i\omega^2 \mu \chi^{(3)}}{2\beta_s} E_{p1} E_{p2} B_s^* \\ \frac{dB_s^*}{dz} &= \frac{i\omega^2 \mu \chi^{(3)}}{2\beta_s} E_{p1} E_{p2} A_s\end{aligned}\quad (2.14)$$

Since the input field E_i is specified, the mode amplitudes B_s ($z=0$) are given. Another boundary condition is that the reflected field E_r is zero at the output end $z=L$ of the fiber, so that $A_s(L) = 0$. With these boundary conditions we can solve (2.14), obtaining

$$\begin{aligned}A_s(z) &= iB_s^*(0)(\sin \kappa_s z - \tan \kappa_s L \cos \kappa_s z) \\ B_s(z) &= B_s(0)(\tan \kappa_s L \sin \kappa_s z + \cos \kappa_s z)\end{aligned}\quad (2.15)$$

where $\kappa_s = \frac{\omega^2 \mu \chi^{(3)} E_{p1} E_{p2}}{2\beta_s}$. Equations (2.15) display the same amplification and oscillation features as the plane wave case [1]. We are particularly interested in what happens to the image information. We find that at $z=0$,

$$A_s(0) = -iB_s^*(0) \tan \kappa_s L \quad (2.16)$$

The total reflected image field can be reconstructed, using equation (2.2), as

$$E_r(x, y, z = 0) = -i \tan \kappa L \sum_{\ell} \frac{1}{2} B_{\ell}^*(0) \mathcal{E}_{\ell}(x, y) \exp(i\omega t) + \text{c.c.} \quad (2.17)$$

where, neglecting the weak dependence of κ_s on s , we took $\kappa_s \equiv \kappa$. The original input field is, according to (2.2)

$$E_i(x, y, z = 0) = \sum_{\ell} \frac{B_{\ell}(0)}{2} \mathcal{E}_{\ell}(x, y) e^{i\omega t} + \text{c.c.} \quad (2.18)$$

so that as far as the total complex field amplitudes are concerned, we have

$$E_r(x, y, z = 0) = -i \tan(\kappa L) E_i^*(x, y, z = 0) \quad (2.19)$$

The reflected field at the fiber input is thus an amplified (for $\kappa L > \pi/4$) complex conjugate replica of the input field.

The tremendous advantage of performing the phase conjugation inside a fiber is due to the fact that pump waves can be launched as fiber modes so that large pump intensities over the whole interaction path can result from moderate pump powers. If we consider, as an example, a multimode fiber with a core diameter of 20 μm which is filled with CS_2 and take the two pump wave powers as 1 watt, we obtain (using $\chi^{(3)} \approx 1.2 \times 10^{-12}$ esu) $\kappa \approx 5 \times 10^{-4} \text{ cm}^{-1}$, so that $\kappa L \approx 1$ is achieved with a fiber length of $L = 20\text{m}$. It should be noted that if the pump propagates as a low order mode, the overlap integrals of (2.13) may be appreciable for near-lying modes, i.e., incident and reflected modes of different indices can interact. This interaction, however, is negligible, since the phase mismatch $(\beta_s - \beta_m)L$ can be shown to be $\gg 2\pi$ in the above example.

The theory and example presented above clearly demonstrate the advantage of performing four-wave mixing in long fibers. Under such

conditions, it is important to consider the effect of optical absorption. If the intensity absorption coefficient of the four waves involved in the interaction is taken as α , we obtain instead of equation (2.19),

$$E_r(x,y,0) = \frac{-2i\kappa \tan(\kappa_e L) \exp(-\frac{1}{2}\alpha L)}{\alpha \tan(\kappa_e L) + 2\kappa_e} E_i^*(x,y,0) \quad (2.20)$$

where $\kappa_e = [\kappa^2 \exp(-\alpha L) - (\frac{1}{2}\alpha)^2]^{1/2}$. It follows from (2.20) that for substantial reflections we need fulfill $\alpha L \ll 1$, $\kappa > \frac{1}{2}\alpha$. If $\alpha L \gg 1$, then

$$\frac{E_r(x,y,0)}{E_i^*(x,y,0)} = -i \exp(-\frac{1}{2}\alpha L) \frac{\kappa}{\alpha} \quad (2.21)$$

The pump intensity in the above example is below that for the onset of stimulated Brillouin scattering in CS_2 and also is such that amplified spontaneous Stokes radiation is insignificant [2].

2.2 Demonstration of Degenerate Four-Wave Mixing in a Liquid-Filled Fiber

In this section, the observation of cw backward-wave generation using degenerate four-wave mixing in a nonresonant medium will be reported. The interaction took place inside a 3m long CS_2 -filled 4 μm i.d. optical fiber. Defining the backward-wave conversion efficiency as the ratio of the reflected wave intensity to the input wave intensity, a 0.45% conversion efficiency was measured with a pump power of 6 mW inside the fiber.

In the experiment sketched in Fig. 2.2, the pump and the probe waves originated from a cw single transverse and longitudinal mode argon ion

laser, operating at a wavelength of $5145\overset{0}{\text{\AA}}$. The laser output had a 10 m long coherence length, which ensured the coherence of the nonlinear interaction over the entire fiber length. The two fiber ends and the objective lenses were carefully aligned to allow maximum coupling of the two linearly polarized pump beams into the lower order fiber modes. Typically, fifty percent of the incident light was coupled into each end. Each fiber end was securely positioned inside a stainless steel CS_2 -filled cell (not shown in the figure). The CS_2 inside the cell was under a nitrogen pressure of 80 psi, which prevented the liquid from boiling when illuminated by the focused pump beams. The probe beam, whose power was roughly equal to that of the pump beams, was introduced nearly parallel to the forward-going pump beam, E_{p1} , and was coupled into the fiber by the same objective lens. Since the probe beam was slightly separated from the on-axis pump beam, the coupling efficiency was typically a factor of five lower than that of E_{p1} . All the beams were chopped (at 57 Hz) to reduce the duty cycle (and consequently to inhibit the degradation of CS_2) of the optical fields as well as to allow the usage of temporal discrimination techniques for the detection of the backward signal. The probe beam also passed through an optical isolator consisting of a linear polarizer and quarter-wave plate prior to coupling into the fiber. This isolator served two functions. First, it minimized any spurious Fresnel reflected probe signals from interfering with the backward-going wave. Second, being a polarization dependent phase "aberrator," the isolator would pass only the conjugate probe field, which could correct for this phase "distortion." Hence, the conjugate reflection of an RHCP field, which is also an RHCP wave, will be passed by the isolator in contrast

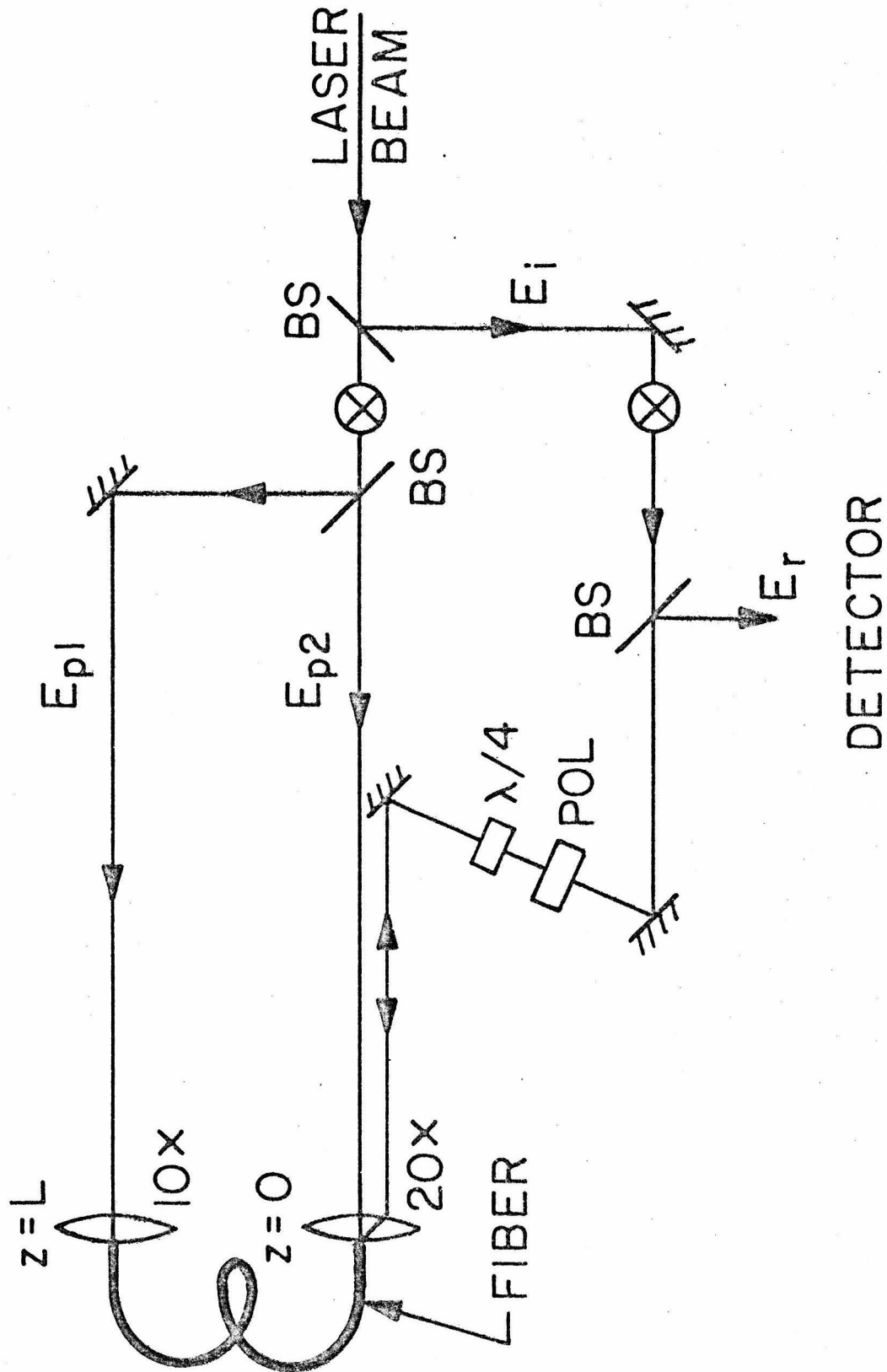


Fig. 2.2 The experimental set-up for four-wave mixing in a liquid-filled fiber

to an LHCP wave which is the result of a Fresnel reflection.

Due to the proximity of the probe and pump fields (constrained by the aperture of the objective lens), additional background noise arose from both the forward-going pump, E_{p1} (via Fresnel reflections), and the counter-propagating pump, E_{p2} (due to the residual throughput from the fiber). Attempts to reduce these background noise sources via polarization discrimination were not utilized due to polarization scrambling of the fields within the 3m long fiber. In order to extract the conjugate signal from the background by using temporal discrimination techniques, the pump beams and the probe beam were chopped with different "on" durations as shown in Fig. 2.3. The conjugate wave was generated only in the time interval when both the pumps and the probe passed through the chopper. During the interval when the beams did not overlap in time, the light detected was due to the Fresnel reflection of E_{p1} and the residual throughput of E_{p2} , plus residual Fresnel reflections of the probe wave that leaked through the isolator. The conjugate signal was obtained by subtracting the signal detected during the non-overlapping time intervals from that of the overlapping time interval. The conjugate signal disappeared when either pump beam was blocked. For an input power of 6 mW in each pump beam within each fiber end, a conversion efficiency of 0.45% was observed. For $n=1.628$, $\alpha = 2.5 \times 10^{-3} \text{ cm}^{-1}$ [3] and $(2\pi\chi^{(3)})/n = 1.1 \times 10^{-11} \text{ esu}$ [4], equation (2.20) predicts a conversion efficiency of 0.23%. The discrepancy between theory and experiment can be attributed to the rough estimate of the conjugate signal as well as the values of α and $\chi^{(3)}$ used in (2.20).

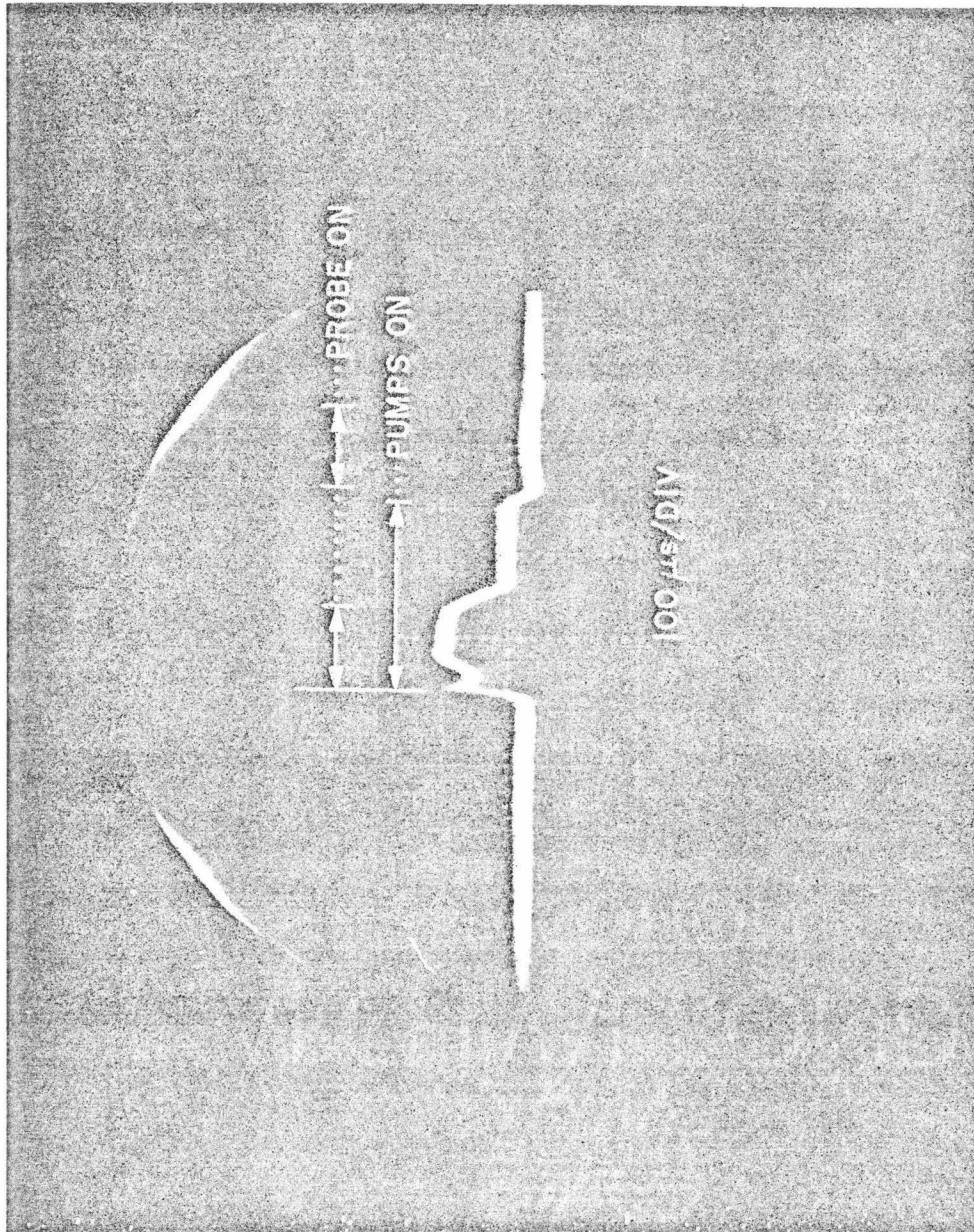


Fig. 2.3 Oscilloscope trace of the detector output

The pump powers we used were well below the threshold for both stimulated Brillouin scattering and Raman scattering [2]. The backward wave was observed only when both pump waves and the probe wave overlapped in time, even at a larger pumping power; thus other nonlinear effects such as optically induced birefringence or thermally induced index of refraction changes could not have generated the signal that was observed. Although there is polarization scrambling inside the fiber, a simple consideration of the magnitudes of the various third order susceptibility tensor elements [5] shows that the dominant backward-wave signal is indeed proportional to the complex conjugate of the probe signal.

In a separate experiment, we used only two waves, E_{p1} and E_{p2} , which were chopped with different duty cycles. In this case, a 1.6 m long 7 μ m i.d. CS₂-filled fiber was used. Prior to entering the objective lens, E_{p1} passed through a crosshair made of fine wires, so that its diffracted wave had additional structure surrounding the central spot. The purpose of the crosshair was to divert some energy of E_{p1} to higher order fiber modes. The light in the higher order modes essentially served as the probe wave as mentioned in the previous experiment. A backward-going wave was also detected as in the previous case. With E_{p1} blocked, a bright spot was observed that was due to the residual light from E_{p2} emanating from the fiber. When both E_{p1} and E_{p2} illuminated the fiber, we observed additional spatial structure in the form of a peripheral ring, which was not due to the Fresnel reflection of E_{p1} . At higher pump powers, distinct dots were seen that surrounded the central bright spot. This indicated that a backward-going wave with its energy mainly located in the higher order fiber modes was generated.

In conclusion, a cw backward-wave generation by four-wave mixing in optical fibers was demonstrated. In addition, the phase conjugate interaction was shown to be able to correct for polarization dependent aberrations and distortions such as optical component birefringence. The lifetime of the CS_2 -filled fiber was rather short (on the order of a few hours), especially when subjected to continuous laser beam illumination. Other Kerr media such as benzene or possibly a mixture of CS_2 and benzene may alleviate this problem and thus yield potential practical device applications. Finally, I would like to point out that high fidelity phase conjugation has been demonstrated with multimode pump waves [6,7].

Chapter 2 - REFERENCES

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Chapter 3

STIMULATED RAMAN SCATTERING IN OPTICAL FIBERS

3.1 Introduction

Stimulated Raman scattering is probably the most intensively studied nonlinear optical phenomenon in fibers. Since the first demonstration of a fiber Raman oscillator in 1970 [1], significant progress has been made in this area [2-10]. Raman scattering in waveguides made of CS_2 [1], benzene [3], hydrogen [11], and fused silica [12] have been investigated. Special attention has been devoted to silica fiber because of the broad band width of its Raman cross section, which offers a large tuning range for a silica fiber Raman laser. The Raman effect in a molecular medium is an inelastic photon scattering process where a pump laser photon is absorbed and a Stokes photon is emitted with the simultaneous excitation of a vibrational mode of the molecule. The frequency of the Stokes photon is down-shifted from that of the pump photon by the vibrational frequency. At sufficiently intense pump power, this process becomes a stimulated effect. The molecular medium in the presence of the pump wave can provide gain to an electromagnetic wave at the Stokes frequency and is capable of amplifying any spontaneously scattered light. The theory of stimulated Raman effect has been extensively discussed by many authors [13-16]. The usual analysis treats the pump wave as a constant throughout the medium so that only the wave equations for the vibrational wave and the Stokes wave need be considered. However, when the length of the medium is 100 m, or even 1 km which is the case for Raman scattering in an optical fiber, a new treatment including the

depletion of the pump wave is necessary to adequately describe the interaction.

This chapter considers the problem of forward Raman scattering process in a single transverse mode fiber. Both pump wave depletion and spontaneous scattering are considered in the analysis. Analytic solutions of the governing differential equations are obtained. The conditions under which a nondepleted pump approximation is valid are examined. Expressions are derived for the maximum fiber loss and the minimum fiber length, which allow significant pump to Stokes wave conversion. It is shown that for a given fiber length, there is an optimal pumping power, or at a given pump power, an optimal fiber length that yields maximum first order Stokes power output. Good agreement with published experimental results in threshold power prediction is obtained. The treatment is general enough to describe Raman scattering in any fiber, either silica or liquid-filled fiber.

3.2 The Small Signal Theory

Before discussing in detail a complete treatment of the stimulated Raman process in fiber, the small signal theory [17] will be summarized. The depletion of the pump wave due to the nonlinear process is neglected in this theory by limiting the discussion to cases of low conversion efficiency such that the generated Stokes-wave power is smaller than the pump-wave power at every point along the fiber. The pump power P_p is assumed to propagate along the fiber as

$$P_p(z) = P_{p0} \exp(-\alpha_p z) \quad (3.1)$$

P_{p0} is the input pump power at the fiber input end, $z = 0$. α_p is the fiber attenuation constant at the pump frequency. The governing equation for a forward traveling Stokes wave is

$$\frac{dP_s}{dz} = -\alpha_s P_s + \frac{g}{A} P_p P_s \quad (3.2)$$

P_s is the Stokes power, α_s is the attenuation coefficient of the fiber at the Stokes frequency. g is the frequency-dependent Raman gain coefficient in units of cm/Watt. A is the cross sectional area of the fiber. The solution to equation (3.2) is

$$P_s(z) = P_{s0} \exp\left\{-\alpha_s z + \frac{g P_{p0}}{\alpha_p A} [1 - \exp(-\alpha_p z)]\right\} \quad (3.3)$$

In an actual experiment, the Stokes power input P_{s0} is zero and any Stokes power output is due to amplified spontaneous scattering. It will be shown in the next section that this spontaneous scattering amplification can be modelled by assuming a fictitious input of one photon per mode (both transverse and longitudinal) of the fiber. Therefore, for $z > 0$,

$$P_s(z) = \sum_{\text{transverse mode}} \int_{-\infty}^{\infty} d\nu h\nu \exp\left\{-\alpha_s z + \frac{g P_{p0}}{\alpha_p A} [1 - \exp(-\alpha_p z)]\right\} \quad (3.4)$$

The frequency dependence of g can be taken as Lorentzian, i.e.,

$$g(\nu) = g_s \frac{(\Delta\nu_R/2)^2}{(\nu - \nu_s)^2 + (\Delta\nu_R/2)^2} \quad (3.5)$$

g_s is the peak gain centering at the central frequency ν_s and $\Delta\nu_R$ is the Raman linewidth. Since the major contribution to the integral in equation

(3.4) comes from a small frequency region $(\nu - \nu_s)$ centered on ν_s , the gain coefficient can be approximated as

$$g(\nu) \approx g_s \left[1 - \frac{(\nu - \nu_s)^2}{(\Delta\nu_R/2)^2} \right] \quad (3.6)$$

and thus

$$P_s(z) \approx \left[\sum_{\text{transverse mode}} (h\nu_s) \exp\left\{-\alpha_s z + \frac{g_s P_{po}}{\alpha_p A} [1 - \exp(-\alpha_p z)]\right\} \right] B_{\text{eff}} \quad (3.7)$$

where the effective bandwidth is

$$B_{\text{eff}} = \frac{\Delta\nu_R}{2} \left\{ \frac{\pi \alpha_p A}{g_s P_{po} [1 - \exp(-\alpha_p z)]} \right\}^{1/2} \quad (3.8)$$

The result in equation (3.7) implies that even though the Raman gain is frequency dependent as given in (3.5), the Stokes wave output behaves as if the gain is a constant, g_s , over a finite band width, B_{eff} , and zero elsewhere.

3.3 Spontaneous and Stimulated Raman Scattering in Long Low Loss Fibers

In this section, a model is presented for describing the stimulated Raman scattering process, in the forward direction, through a single transverse mode fiber. Pump wave depletion that is due to the nonlinear interaction, plus spontaneous scattering are both incorporated in our model. The previous analysis due to Smith [17], which neglects pump depletion by the stimulated process but allows for linear absorption of the pump, is found to be valid in the case of high loss fibers, which were the only ones available when Smith considered this problem.

The neglect of pump depletion is not valid in treating today's low loss fibers (< 4 dB/km); and the present analysis constitutes an extension to this practically important case.

Consider the total pump power to be concentrated in only one pump mode and to be injected into the end of a single transverse mode fiber at $z = 0$. As this pump wave propagates along the fiber, it is depleted by linear absorption in the fiber as well as by down conversion to Stokes photons. The differential equation governing n_p , the pump wave photon number, is

$$\frac{dn_p}{dz} + \alpha_p n_p = -\gamma_0 n_p (n_{s1} + 1 + n_{s2} + 1 + \dots + n_{sq} + 1) \quad (3.9)$$

Subscripts p and s refer to the pump and Stokes photons; α_p is the absorption constant of the fiber at the pump wave frequency; γ_0 is the nonlinear gain constant associated with the Raman process; n_{sj} is the photon number of the j^{th} longitudinal Stokes modes; q is the number of Stokes modes participating in the interaction. The integer 1 that goes with each Stokes mode accounts for the spontaneous scattering into that mode [16]. Each n_{sj} obeys the equation

$$\frac{dn_{sj}}{dz} + \alpha_s n_{sj} = \gamma_0 n_p (n_{sj} + 1) \quad (3.10)$$

The total Stokes photon number is

$$n_s = \sum_{j=1}^q n_{sj} \quad (3.11)$$

and it satisfies the equation

$$\frac{dn_s}{dz} + \alpha_s n_s = \gamma_o n_p (n_s + q) \quad (3.12)$$

α_s is the absorption constant of the fiber at the Stokes frequency. The initial conditions are

$$\begin{aligned} n_p(z=0) &= n_{po} \\ n_{sj}(z=0) &= n_{sjo} \\ n_s(z=0) &= \sum_{j=1}^g n_{sjo} = n_{so} \end{aligned} \quad (3.13a)$$

and we define

$$n_o = n_{po} + n_{so} \quad (3.13b)$$

to be the total input photon number.

In equations (3.9)-(3.13) we made the approximation that the gain constant γ is the same for all the q modes partaking in the interaction. The exact procedure would be to allow each Stokes mode j to have a gain $\gamma_j = \gamma(\nu_j)$ which is proportional to the Raman line shape. The nondepleted pump analysis shows that the correct result is obtained if we take γ_j to be a constant over an effective Stokes input bandwidth B_{eff} and take it as zero elsewhere. This is due to the combined frequency dependence of $\gamma(\nu_j)$ and the exponential nature of the amplification process, which, together discriminate strongly against Stokes modes removed from line center. In the presence of a finite Stokes wave input of bandwidth $\Delta\nu_s$, the effective bandwidth following the derivation in the last section is

$$B_{\text{eff}} = \frac{\Delta\nu_R}{2} \left\{ \frac{\pi}{\frac{\gamma_0 n_{p0}}{\alpha_p} [1 - \exp(-\alpha_p z)] + (\Delta\nu_R/\Delta\nu_s)^2} \right\}^{1/2} \quad (3.14)$$

$\Delta\nu_R$ is the Raman linewidth. In the absence of a Stokes wave input, any significant Stokes wave buildup at the fiber output end is due to single-pass superradiant emission. In this case,

$$B_{\text{eff}} = \frac{\Delta\nu_R}{2} \left\{ \frac{\pi}{\frac{\gamma_0 n_{p0}}{\alpha_p} [1 - \exp(-\alpha_p z)]} \right\}^{1/2} \quad (3.15)$$

for large z . The effective number of Stokes modes q is taken as the number of longitudinal fiber modes falling within B_{eff} and is

$$q = n_{\text{eff}} \frac{L}{c} B_{\text{eff}} \quad (3.16)$$

where n_{eff} is the effective mode index of refraction (in practice essentially the fiber core index), and L is the length of the fiber.

We have compared the results of the analysis based on the effective bandwidth concept to a numerical analysis using a frequency dependent gain. The results, described further in this chapter, justify the use of B_{eff} .

In silica fibers, Stokes shifts are confined to a frequency less than several hundred wave numbers [12], and hence α_p is nearly equal to α_s . This is especially true in the 1 μm wavelength region where α is near its minimum and $d\alpha/d\lambda$ is small. We shall assume that this is indeed the case, and let $\alpha_p = \alpha_s = \alpha$. Adding equation (3.9) and equation (3.12), we get

$$\frac{d}{dz} (n_p + n_s) + \alpha(n_p + n_s) = 0 \quad (3.17)$$

Hence, the total photon number varies as

$$n_p + n_s = n_0 \exp(-\alpha z) \quad (3.18)$$

This is merely a photon conservation statement indicating that the non-linear interaction conserves photons; and a decrease in the total photon number is due solely to absorption in the fiber medium. Substituting equation (3.18) into equations (3.9) and (3.12), we have

$$\frac{dn_p}{dz} + n_p[\alpha + q\gamma_0 + \gamma_0 n_0 \exp(-\alpha z)] = \gamma_0 n_p^2 \quad (3.19)$$

and

$$\frac{dn_s}{dz} + n_s[\alpha + q\gamma_0 - \gamma_0 n_0 \exp(-\alpha z)] = -\gamma_0 n_s^2 + q\gamma_0 n_0 \exp(-\alpha z) \quad (3.20)$$

Equations (3.19), (3.20), and (3.10) with the initial conditions (3.13) can be solved exactly. The solutions (whose derivations are given in the Appendix) are

$$n_p(z) = \frac{n_0 \exp(-\alpha z)}{1 + \xi + \eta} \quad (3.21)$$

$$n_s(z) = n_0 \exp(-\alpha z) \frac{\xi + \eta}{1 + \xi + \eta} \quad (3.22)$$

$$n_{sj}(z) = \frac{n_0 \exp(-\alpha z)}{1 + \xi + \eta} \left(\frac{n_{sj0}}{n_{so}} \xi + \frac{1}{q} \eta \right) \quad (3.23)$$

where

$$\xi = \frac{n_{so}}{n_{po}} \exp\{q\gamma_0 z + \frac{\gamma_0 n_0}{\alpha} [1 - \exp(-\alpha z)]\} \quad (3.24)$$

and

$$\eta = q\gamma_0 \exp[q\gamma_0 z - \frac{\gamma_0 n_0}{\alpha} \exp(-\alpha z)] \sum_{\ell=0}^{\infty} \frac{(\frac{\gamma_0 n_0}{\alpha})^{\ell} \{1 - \exp[-(\ell\alpha + q\gamma_0)z]\}}{(\ell\alpha + q\gamma_0)^{\ell}} \quad (3.25)$$

$\xi(z)$ describes the conversion from pump to Stokes photons due to a finite Stokes photon input n_{s0} . $\eta(z)$, on the other hand, describes amplified spontaneous scattering.

For typical pumping powers of a few watts, $n_{po} \gg q$ and $n_{po} \gg n_{s0}$, so that $n_0 \approx n_{po}$. The series in Eq. (3.25) can very well be approximated for large z by $\frac{1}{\gamma_0 n_{po}} \exp(\frac{\gamma_0 n_0}{\alpha})$, and

$$\eta = \frac{q}{n_{po}} \exp\{q\gamma_0 z + \frac{\gamma_0 n_0}{\alpha} [1 - \exp(-\alpha z)]\} \quad (3.26)$$

We note that in this case η is identical to ξ as given by equation (3.24) provided the input Stokes photon number n_{s0} is taken to be q . That is, the spontaneous emission output can be viewed as due to an input of one Stokes photon per mode. It is apparent that the amplified spontaneous scattering can be modelled alternatively by ignoring the spontaneous effect altogether in the differential equations, provided an equivalent input of 1 photon per mode is assumed.

In order to relate our model to the experimental results, equations (3.21)-(3.26) are rewritten in terms of power. P_p and P_s are used to denote the power in the pump and Stokes waves, respectively. The changes in the following notation is self explanatory. Equations (3.21)-(3.26) become

$$P_p(z) = \frac{P_o \exp(-\alpha z)}{1 + \xi + \eta} \quad (3.27)$$

$$P_s(z) = \frac{v_s}{v_p} P_o \exp(-\alpha z) \frac{\xi + \eta}{1 + \xi + \eta} \quad (3.28)$$

$$P_{sj}(z) = \frac{v_s}{v_p} \frac{P_o \exp(-\alpha z)}{1 + \xi + \eta} \left(\frac{P_{sjo}}{P_{so}} \xi + \frac{1}{q} \eta \right) \quad (3.29)$$

$$\xi = \frac{v_p}{v_s} \frac{P_{so}}{P_{po}} \exp\left\{ \frac{g_s P_o}{\alpha A} [1 - \exp(-\alpha z)] \right\} \quad (3.30)$$

$$\eta = \frac{v_p}{v_s} \frac{q h v_s v_g}{P_o L} \exp\left\{ \frac{g_s P_o}{\alpha A} [1 - \exp(-\alpha z)] \right\} \quad (3.31)$$

In equations (3.30) and (3.31) we have omitted a factor $\exp(q\gamma_o z)$ which is close to 1 even for very large z . P_o is defined to be $P_{po} + \frac{v_p}{v_s} P_{so}$. v_p and v_s are the pump and Stokes wave frequencies, respectively. v_g is the group velocity in the fiber medium. A is the effective cross sectional area of the fiber. g_s is the Raman power gain coefficient normally measured in experiments. h is Planck's constant.

Let us consider some of the consequences of equations (3.27)-(3.31). When $\xi + \eta \ll 1$,

$$P_p(z) \approx P_o \exp(-\alpha z) \quad (3.32)$$

In this limit, the conversion to Stokes photons is negligible and $P_p(z)$ according to equation (3.32) is attenuated only by the fiber loss α .

When $\xi + \eta \ll 1$,

$$P_s(z) \approx \frac{v_s}{v_p} P_o \exp(-\alpha z) \quad (3.33)$$

The conversion is completed and all the pump power has been transferred to Stokes power. When $\frac{\nu_s}{\nu_p} (\xi + \eta) = 1$, P_p equals P_s . We define this condition as the threshold condition. The input pump power which causes $\frac{\nu_s}{\nu_p} (\xi + \eta) = 1$ at $z = L$ is referred to as the threshold power. When $P_o \approx P_{po} \gg P_{so}$,

$$\frac{\nu_s}{\nu_p} (\xi + \eta) = \frac{1}{P_o} (P_{so} + \frac{qh\nu_s \nu_p g}{L}) \exp\left\{\frac{g_s P_o}{\alpha A} [1 - \exp(-\alpha z)]\right\} \quad (3.34)$$

Figure 3.1 shows $\frac{\nu_s}{\nu_p} [\xi(z) + \eta(z)]$ for different values of α . The parameters used in plotting the curves are $P_o = 1$ watt, $P_{so} = 0$, $\lambda_p = 1.06 \mu\text{m}$, $\lambda_s = 1.12 \mu\text{m}$, $\Delta\nu_R = 400 \text{ cm}^{-1}$, $g_s = 5 \times 10^{-10} \text{ cm/W}$, and $A = 10^{-7} \text{ cm}^2$ [17].

For $\alpha z \gg 1$,

$$\frac{\nu_s}{\nu_p} (\xi + \eta) = \frac{P'}{P_o} \alpha^{1/2} \exp\left(\frac{g_s P_o}{\alpha A}\right) \quad (3.35)$$

where

$$P' = h\nu_s \frac{\Delta\nu_R}{2} \left(\frac{\pi A}{\gamma_o P_o}\right)^{1/2} \quad (3.36)$$

and is independent of z . If it is smaller than 1, then as shown in Fig. 3.3, no significant stimulated conversion from pump to Stokes photons will take place. The expression in Eq. (3.35) equals 1 when $\alpha = \alpha_{\text{max}}$, which is defined as

$$\alpha_{\text{max}} = \frac{g_s P_o}{A \left[\ln\left(\frac{P_o}{P'}\right) - \frac{1}{2} \ln(\alpha_{\text{max}}) \right]} \quad (3.37)$$

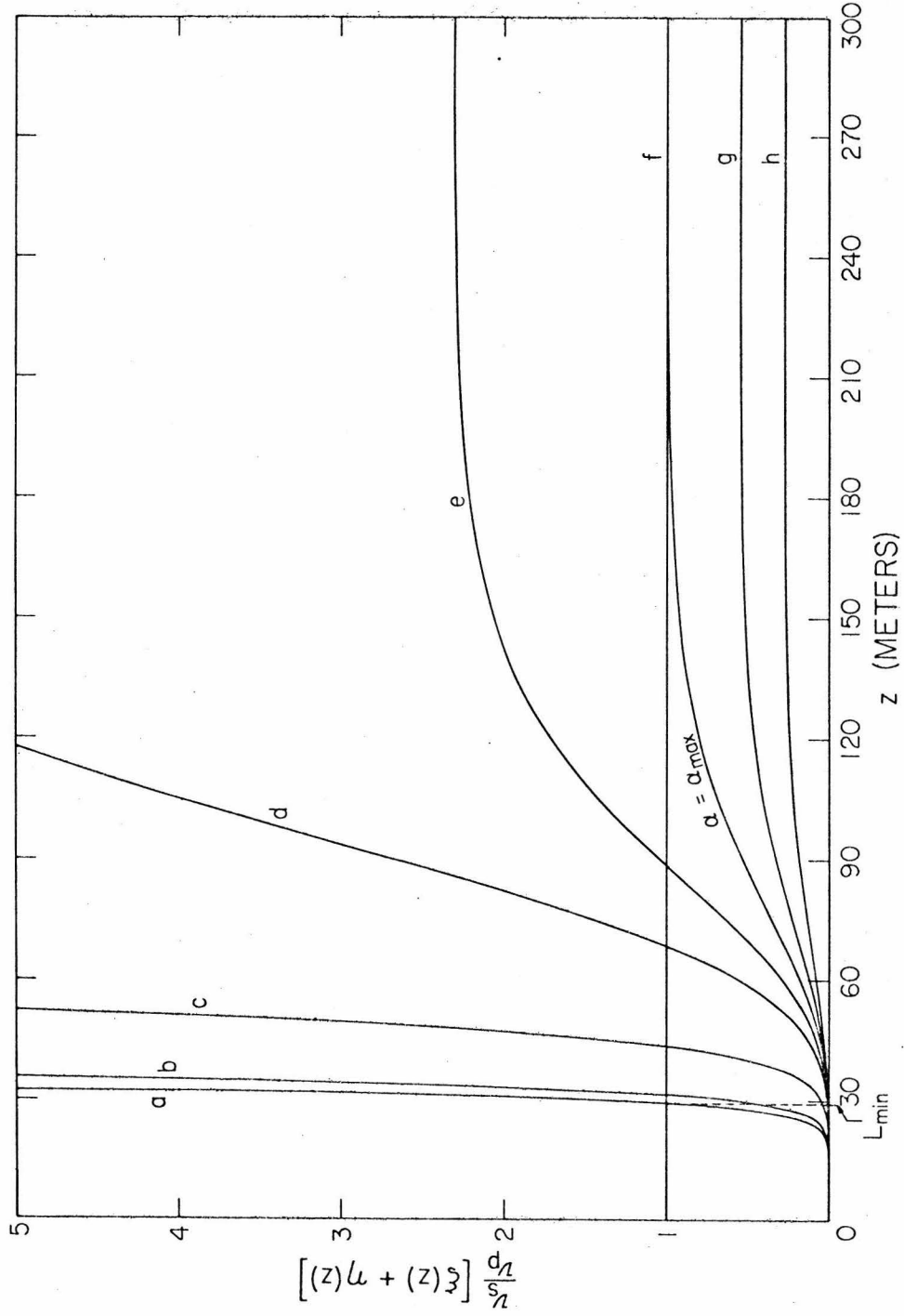


Fig. 3.1 The attenuation constant of the fiber, α , is: (a) $5 \times 10^{-6} \text{ cm}^{-1}$, (b) $5 \times 10^{-5} \text{ cm}^{-1}$, (c) $2 \times 10^{-4} \text{ cm}^{-1}$, (d) $3 \times 10^{-4} \text{ cm}^{-1}$, (e) $3.25 \times 10^{-4} \text{ cm}^{-1}$, (f) $\alpha = \alpha_{\max} = 3.445 \times 10^{-4} \text{ cm}^{-1}$, (g) $3.6 \times 10^{-4} \text{ cm}^{-1}$, (h) $3.8 \times 10^{-4} \text{ cm}^{-1}$

$\alpha_{\max}$ is therefore the upper limit of fiber loss for efficient photon conversion. Curve (f) in Figure 3.1 shows $\frac{v_s}{v_p} (\xi + \eta)$ at $\alpha_{\max}$. For those given input and fiber parameters, $\alpha_{\max}$ is 150 dB/km. In the limit $\alpha z \ll 1$,

$$\frac{v_s}{v_p} (\xi + \eta) = \frac{P'}{P_0} z^{-1/2} \exp\left(\frac{g_s P_0 z}{A}\right) \quad (3.38)$$

and is independent of α . If it is smaller than 1, no significant stimulated conversion will take place, no matter how small α is. The expression in equation (3.38) equals 1, i.e., threshold obtains at a fiber length $z = L_{\min}$, given by

$$L_{\min} = \frac{A}{g_s P_0} \left[\ln\left(\frac{P_0}{P'}\right) + \frac{1}{2} \ln(L_{\min}) \right] \quad (3.39)$$

$L_{\min}$ is interpreted as the minimum fiber length required for strong conversion even at zero loss. For the case shown in Figure 3.1, $L_{\min}$ is 29 meters. $\alpha_{\max}$ and $L_{\min}$ are related by the equation

$$\alpha_{\max} L_{\min} = 1 \quad (3.40)$$

Figure 3.2 shows $P_p(z)$ and $P_s(z)$ for $\alpha = 5 \times 10^{-5} \text{ cm}^{-1}$ (~ 20 dB/km), a high conversion efficiency case. We note the severe pump depletion occurring at $z > 30$ m. Figure 3.3 shows $P_p(z)$ and $P_s(z)$ for $\alpha_{\max}$. In this small conversion efficiency case, $P_p(z)$ varies roughly as $\exp(-\alpha z)$ throughout the whole fiber. In Figures 3.2 and 3.3, we also plotted $P_0 \exp(-\alpha z)$, which, when compared with $P_p(z)$, indicates the region in which a nondepleted pump approximation is valid. The same input and

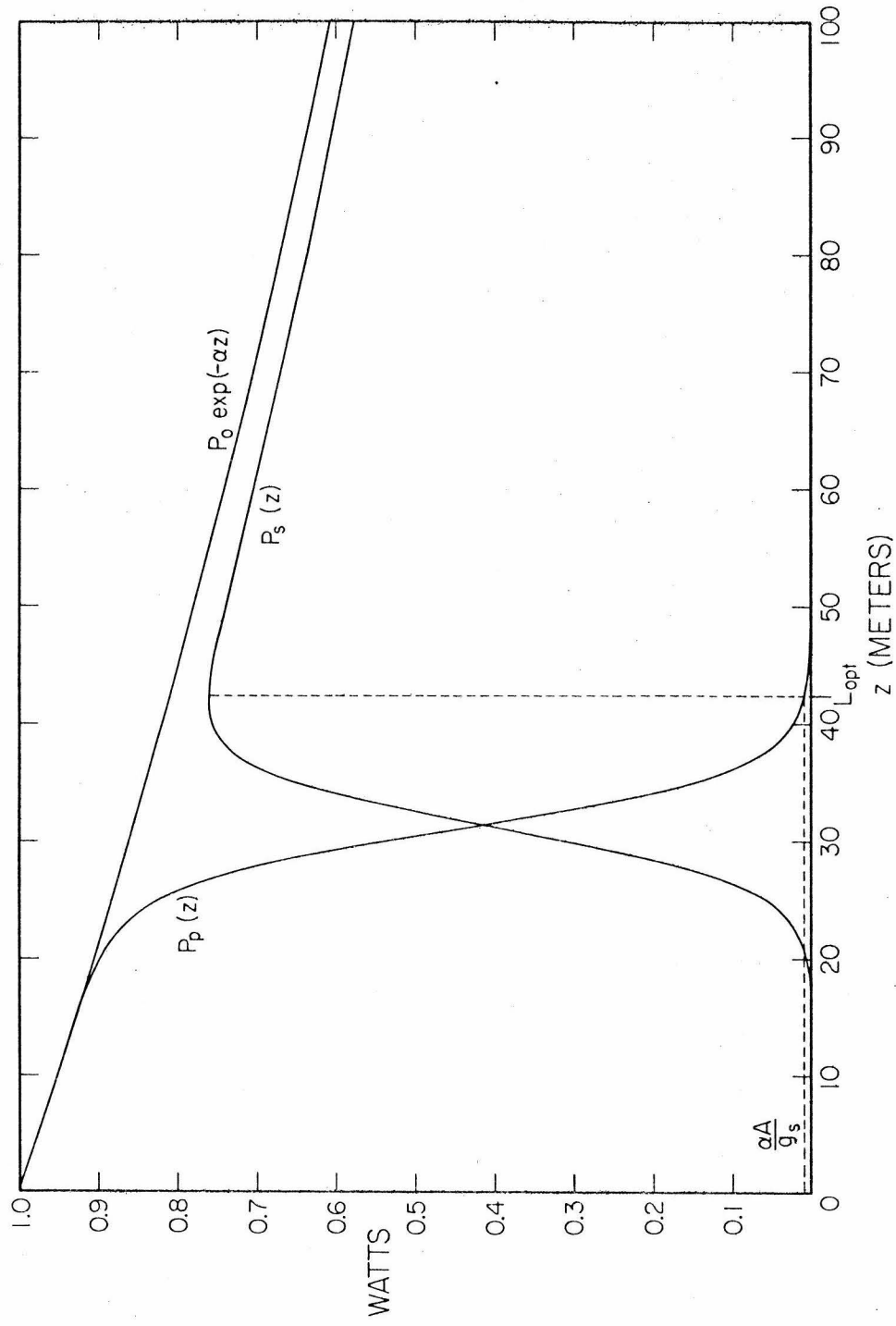


Fig. 3.2 $P_p(z)$ and $P_s(z)$ for $\alpha = 5 \times 10^{-5} \text{ cm}^{-1}$

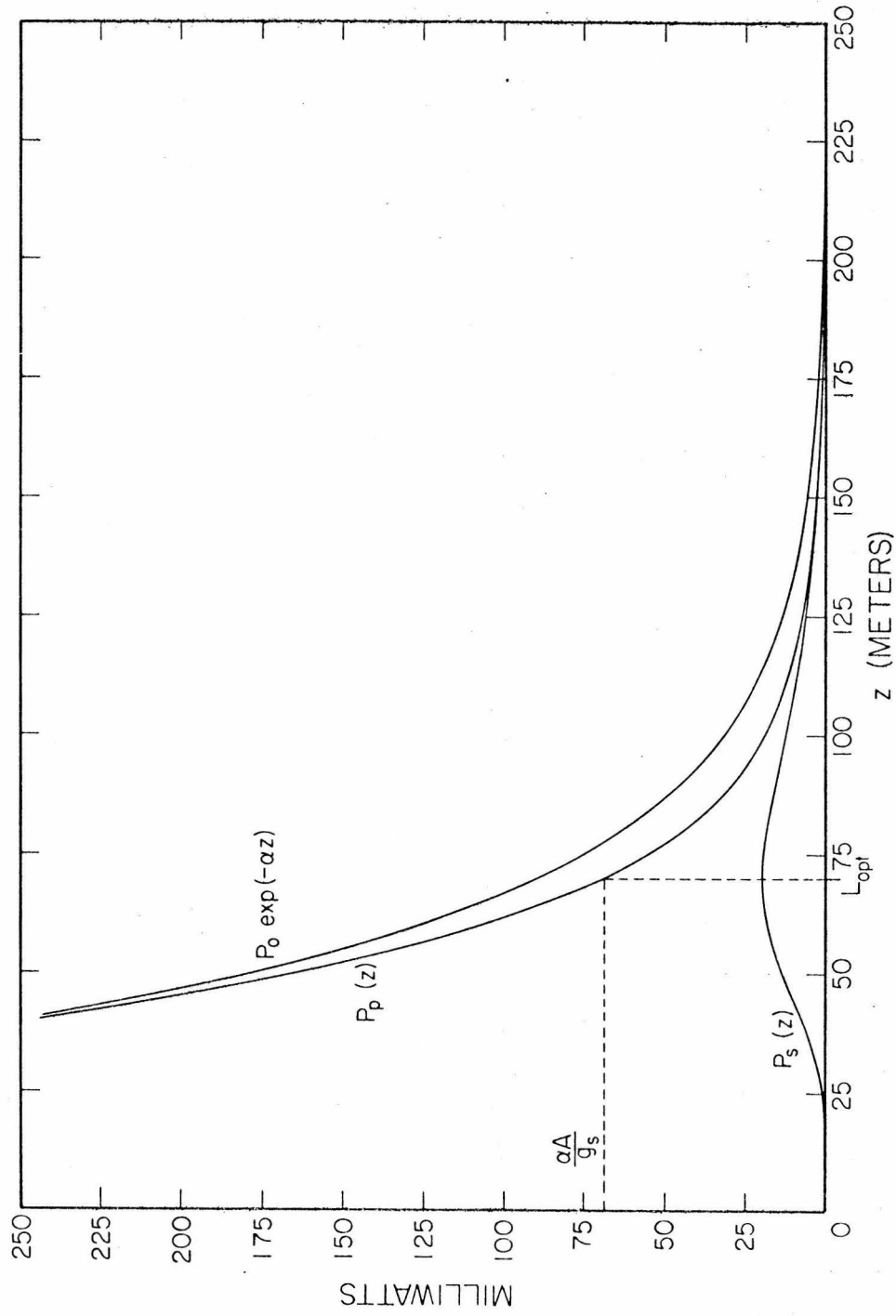


Fig. 3.3 $P_p(z)$ and $P_s(z)$ for $\alpha = \alpha_{\max} = 3.445 \times 10^{-4} \text{ cm}^{-1}$

fiber parameters are used for Figures 3.1, 3.2, and 3.3. It is apparent from Figures 3.2 and 3.3 that for a given input pump power there is an optimal fiber length L_{opt} , which results in maximum Stokes power output. Conversely, for a given fiber length L , there is an optimal pump power that will give maximum first order Stokes power output. Maximum Stokes power occurs when $\frac{dn_s}{dz} = 0$. Equation (3.12) tells us that $n_p \approx \alpha/\gamma_0$ at that point. Thus.

$$P_p(L_{opt}) = \frac{\alpha A}{g_s} \quad (3.41)$$

and from equation (3.28),

$$P_s(L_{opt}) = \frac{\nu_s}{\nu_p} [P_0 \exp(-\alpha z) - \frac{\alpha A}{g_s}] \quad (3.42)$$

From these relations, we can readily calculate the optimal pump power or optimal fiber length.

To test the validity of our model we solved numerically for $n_p(z)$ and $n_{sj}(z)$, for a particular case, assuming a Lorentzian Raman gain profile $\gamma(\nu_j)$ centered on the Stokes frequency ν_s . A Stokes photon input with a Gaussian spectral profile is assumed. It has a bandwidth $\Delta\nu_s$ equal to 800 cm^{-1} centered on ν_s . We divide the frequency space into equal segments 40 cm^{-1} wide. γ is then taken to be constant in each segment but different from one segment to another. The behavior of n_p and the injected Stokes photons is described by the equations

$$\frac{dn_p}{dz} + \alpha_p n_p = - \sum_{j=-10}^{10} \gamma_j (N_{sj} + p) n_p \quad (3.43)$$

and

$$\frac{dN_{sj}}{dz} + \alpha_s N_{sj} = \gamma_j n_p (N_{sj} + p) \quad (3.44)$$

where $\gamma_j = \gamma(\nu_j)$.

It is assumed that the input pump photon number is $n_p(z=0) = 10^{14}$. N_{sj} is the number of Stokes photons in the j^{th} frequency segment which is $40j \text{ cm}^{-1}$ away from ν_s . The initial excitation of the Stokes modes is taken as

$$N_{sj}(z=0) = 10^{10} \exp \left[- \left(\frac{40j}{\Delta\nu_s/2} \right)^2 \right] \quad (3.45)$$

The number of Stokes modes, p , in one 40 cm^{-1} segment is 6×10^6 for a 1 km long fiber. The Raman gain constant of the j^{th} segment is

$$\gamma_j = \frac{\gamma_0 \left(\frac{\Delta\nu_R}{2} \right)^2}{(40j)^2 + \left(\frac{\Delta\nu_R}{2} \right)^2} \quad (3.46)$$

where γ_0 is 10^{-17} cm^{-1} and the Raman line width $\Delta\nu_R$ is 400 cm^{-1} . The absorption constant of the fiber is assumed to be $\alpha_p = \alpha_s = 10^{-5} \text{ cm}^{-1}$.

Figure 3.4 shows the numerical solutions of $N_{sj}(z)$ for j from 0 to 10.

The plot for $j = -10$ to 0 is a mirror image and is omitted. In the figure,

$10^{-10} N_{sj}(z)$ is shown in steps of 5 meters from $z = 0$ to $z = 150$ meters.

At $z = 150$ meters, N_{s0} is 2.5×10^{13} and it drops, 80 cm^{-1} away, to N_{s2} ,

equal to 8×10^{12} , indicating a full spectral width at half maximum at

$z = 150 \text{ m}$ smaller than 160 cm^{-1} . The numerical solution shows that the

Stokes wave bandwidth is a function of distance, decreasing with z until

maximum conversion is reached. At this point the linear absorption begins

to dominate and the bandwidth stays constant for the remainder of the fiber

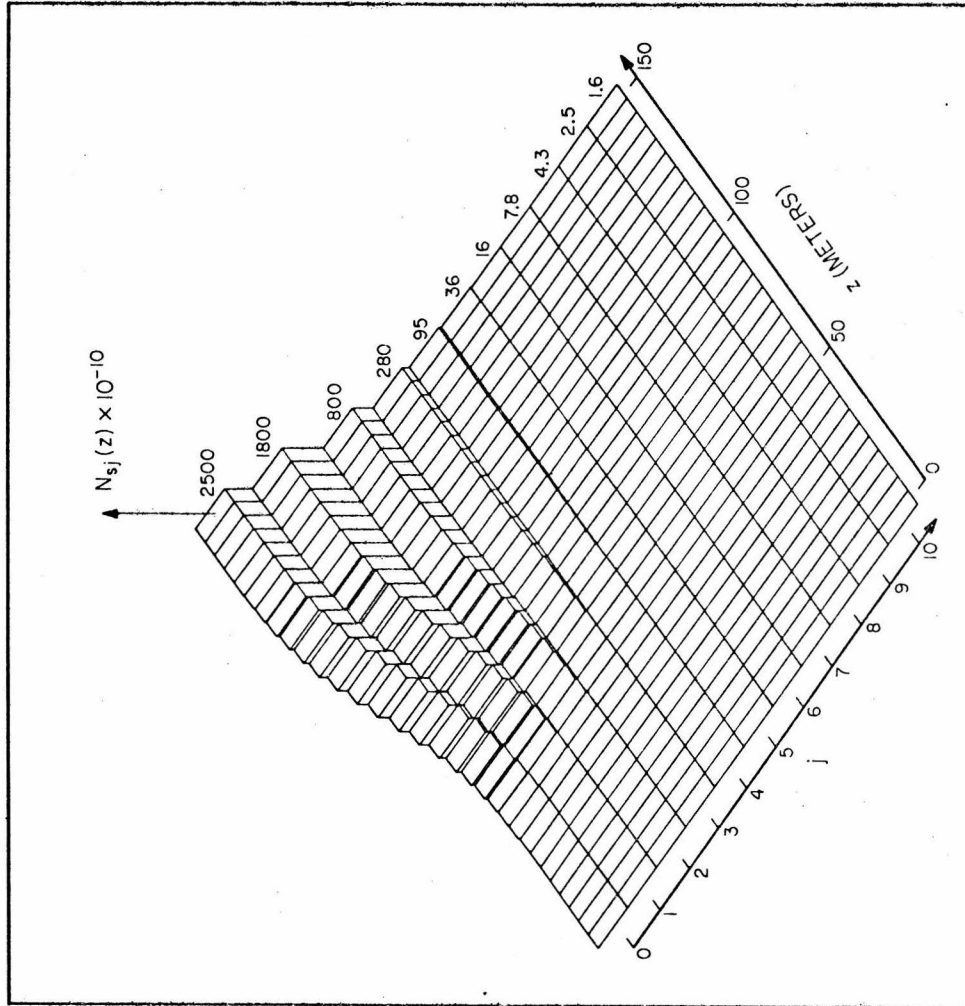


Fig. 3.4 $N_{sj}(z) \times 10^{-10}$ for j from 0 to 10, and for z from 0 to 150 m

lengths. Figure 3.5 shows $n_p(z)$ and $n_s(z)$ obtained both numerically and analytically. The close agreement, especially for large z , between the two results is apparent. This justifies the use of the effective bandwidth (3.14); however, the analytical solution does not yield detailed information about the spectral evolution of the Stokes radiation.

As a further test of the validity of the model, our solutions are compared with published experimental results. At small Stokes power, $P_s(L)$ is proportional to $\exp\{\frac{g_s P_0}{\alpha A} [1 - \exp(-\alpha L)]\}$. The rapid nonlinear rise in P_s as P_0 is increased was indeed observed and reported [3]. Another test is the prediction of threshold pumping power of the single-pass superradiant emission. Threshold occurs when $\frac{\nu_s}{\nu_p} \eta = 1$ or

$$\frac{q h \nu_s \nu_g}{P_{\text{threshold}} L} \exp\left\{\frac{g_s P_{\text{threshold}}}{\alpha A} [1 - \exp(-\alpha L)]\right\} = 1 \quad (3.47)$$

$q \frac{h \nu_s \nu_g}{L}$ is the power of the fictitious input of one photon per mode for the superradiant emission. q is $\frac{n_{\text{eff}} L}{C} B_{\text{eff}}$, and B_{eff} is given by equation (3.15). Thus the threshold occurs when

$$\frac{h \nu_s \Delta \nu_R}{2 P_{\text{threshold}}} \left[\frac{\pi \alpha A}{g_s P_{\text{threshold}} [1 - \exp(-\alpha L)]} \right]^{1/2} \times \exp\left\{\frac{g_s P_{\text{threshold}}}{\alpha A} [1 - \exp(-\alpha L)]\right\} = 1 \quad (3.48)$$

Consider the case of a recent experiment [18]. A silica fiber, 175 m long with an effective core area of $4.58 \times 10^{-7} \text{ cm}^2$ was pumped at $1.06 \mu\text{m}$. Stokes wave output was observed at $1.12 \mu\text{m}$. The loss coefficient of

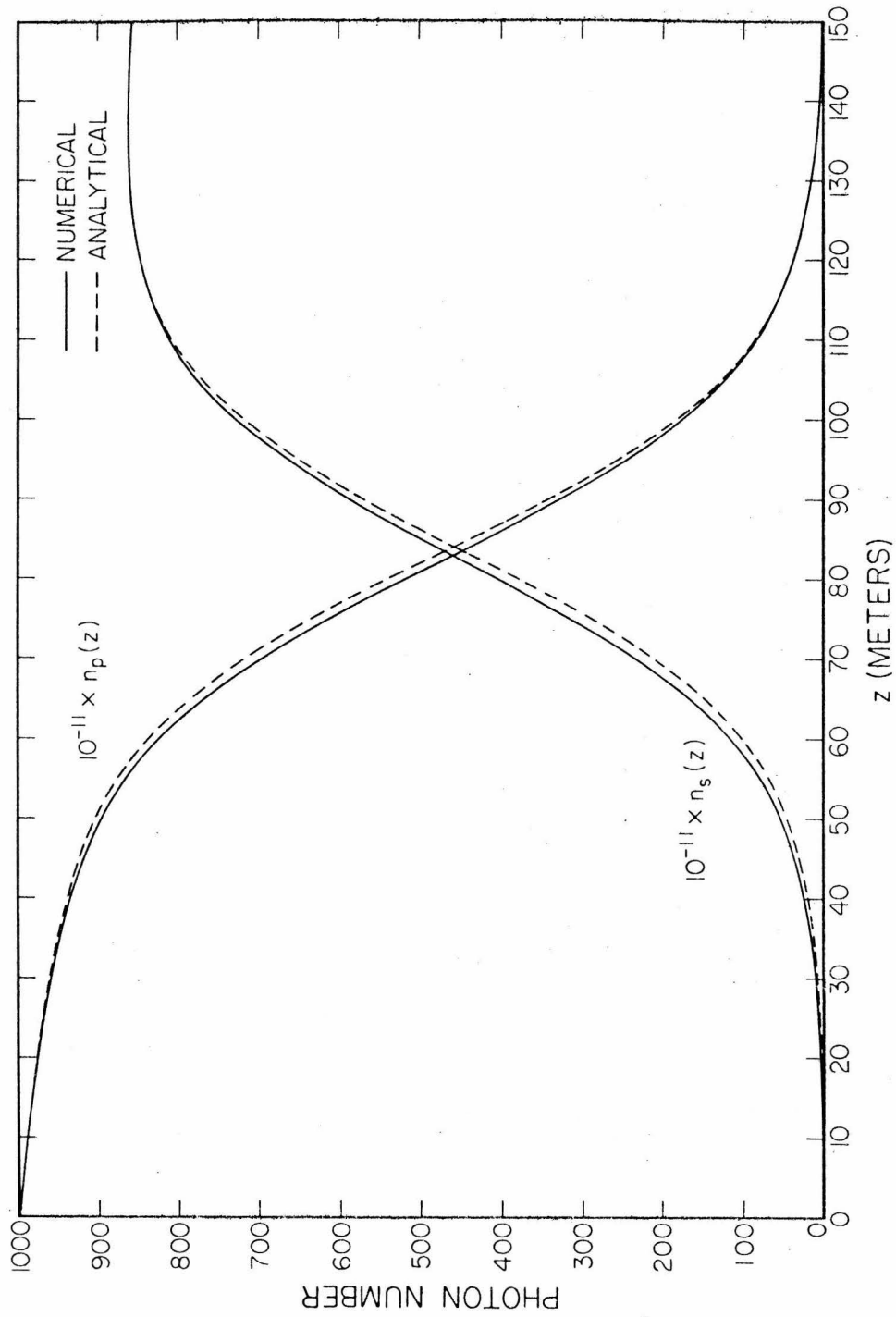


Fig. 3.5 Numerical and analytical solutions of $10^{-11} \times n_p(z)$ and $10^{-11} \times n_s(z)$

of the fiber was 4 dB/km ($\alpha = 0.92 \times 10^{-5} \text{ cm}^{-1}$), g_s was $0.92 \times 10^{-11} \text{ cm/W}$, and the Raman line width $\Delta\nu_R$ was taken to be 400 cm^{-1} . Threshold was reported to occur at $P_0 = 70 \text{ W}$. Our calculation, using equation (3.48) and the above data, gives $P_{\text{threshold}} = 58 \text{ W}$. A similar experiment was performed using a pure-germania (GeO_2) core fiber of length 1.5 m and area $7 \times 10^{-8} \text{ cm}^2$. Stokes output was observed at $1.116 \mu\text{m}$ when, again, pumped at $1.06 \mu\text{m}$. The exact value of g_s is not available. It is, however, known to be ~ 10 times that of silica [19]. $\Delta\nu_R$ is roughly 100 cm^{-1} . The fiber is lossy (300 dB/km) with $\alpha = 6.9 \times 10^{-4} \text{ cm}^{-1}$. The measured $P_{\text{threshold}}$ was 135 W and our calculation gives 112 W. The agreement with theory is reasonable. The 20% discrepancy may be due to possible polarization scrambling in the fiber and to the presence of backward scattering.

To summarize, we have presented a model for describing the forward Raman scattering in single mode fibers. We accounted for pump wave depletion and spontaneous scattering in the model. Analytic solutions of the governing differential equations were obtained. In the limit of negligible pump depletion, our results reduce to those of the small signal theory. The dependence of the pump and Stokes waves' evolution on the fiber losses, α , was examined. We showed that a nondepleted pump approximation is valid when $\xi + \eta \ll 1$, which is true for large α or short fiber length. In particular, if $\alpha > \alpha_{\text{max}}$, pump power does vary as $\exp(-\alpha z)$ throughout the whole length of even a very long fiber. This may require α to exceed 100 dB/km in some cases, and this attenuation factor is larger than that of today's low loss fibers. However, if

$L < L_{\min}$, the nondepleted pump approximation is valid even for very low loss fibers. We calculated the optimal pumping power or the optimal fiber length resulting in a maximum first order Stokes power output. Finally, the theory was compared with experimental results.

Appendix

To solve equation (3.19)

$$\frac{dn_p}{dz} + n_p[\alpha + q\gamma_0 + \gamma_0 n_0 \exp(-\alpha z)] = \gamma_0 n_p^2 \quad (A.1)$$

we change variable

$$n_p = 1/x \quad (A.2)$$

and obtain

$$\frac{dx}{dz} - x[\alpha + q\gamma_0 + \gamma_0 n_0 \exp(-\alpha z)] = -\gamma_0 \quad (A.3)$$

Introducing an integrating factor $\exp[-(\alpha + q\gamma_0)z + \frac{\gamma_0 n_0}{\alpha} \exp(-\alpha z)]$, we get

$$\begin{aligned} x = \exp[(\alpha + q\gamma_0)z - \frac{\gamma_0 n_0}{\alpha} \exp(-\alpha z)] \{ \int -\gamma_0 \exp[-(\alpha + q\gamma_0)z' \\ + \frac{\gamma_0 n_0}{\alpha} \exp(-\alpha z')] dz' + C \} \end{aligned} \quad (A.4)$$

with C as the integration constant to be determined. Using integration by parts,

$$\begin{aligned} x = \frac{1}{n_0} \exp(\alpha z) + \exp[(\alpha + q\gamma_0)z - \frac{\gamma_0 n_0}{\alpha} \exp(-\alpha z)] \\ \{ \int \frac{q\gamma_0}{n_0} \exp[-q\gamma_0 z' + \frac{\gamma_0 n_0}{\alpha} \exp(-\alpha z')] dz' + C \} \end{aligned} \quad (A.5)$$

Letting

$$u = \frac{\gamma_0 n_0}{\alpha} \exp(-\alpha z) \quad (A.6)$$

the integral in Eq. (A.5) becomes $\frac{-q\gamma_0}{\alpha n_0} \left(\frac{\alpha}{\gamma_0 n_0}\right)^{\frac{q\gamma_0}{\alpha}} \int u^{\frac{q\gamma_0}{\alpha} - 1} \exp(u) du$.

This can be integrated easily when the integrand is rewritten in terms of a series. After substituting back equation (A.6), we obtain

$$x = \frac{1}{n_0} \exp(\alpha z) \left\{ 1 - q\gamma_0 \exp[q\gamma_0 z - \frac{\gamma_0 n_0}{\alpha} \exp(-\alpha z)] \right. \\ \left. \times \sum_{\ell=0}^{\infty} \frac{\left(\frac{\gamma_0 n_0}{\alpha}\right)^{\ell} \exp[-(\ell\alpha + q\gamma_0)z]}{(\ell\alpha + q\gamma_0)\ell!} \right\} + C \exp[(\alpha + q\gamma_0)z - \frac{\gamma_0 n_0}{\alpha} \exp(-\alpha z)] \quad (A.7)$$

C is now determined by the initial condition

$$x(z=0) = \frac{1}{n_{p0}} \quad (A.8)$$

Finally, we have

$$n_p = \frac{n_0 \exp(-\alpha z)}{1 + \xi + \eta} \quad (A.9)$$

where

$$\xi = \frac{n_{s0}}{n_{p0}} \exp\{q\gamma_0 z + \frac{\gamma_0 n_0}{\alpha} [1 - \exp(-\alpha z)]\} \quad (A.10)$$

and

$$\eta = q\gamma_0 \exp[q\gamma_0 z - \frac{\gamma_0 n_0}{\alpha} \exp(-\alpha z)] \sum_{\ell=0}^{\infty} \frac{\left(\frac{\gamma_0 n_0}{\alpha}\right)^{\ell} \{1 - \exp[-(\ell\alpha + q\gamma_0)z]\}}{(\ell\alpha + q\gamma_0)\ell!} \quad (A.11)$$

Now

$$n_s = n_0 \exp(-\alpha z) - n_p \quad (A.12)$$

from equation (3.18); therefore

$$n_s = n_o \exp(-\alpha z) \frac{\xi + \eta}{1 + \xi + \eta} \quad (\text{A.13})$$

n_{sj} can now be written by inspection. It is verified that

$$n_{sj} = \frac{n_o \exp(-\alpha z)}{1 + \xi + \eta} \left(\frac{n_{sjo}}{n_{so}} \xi + \frac{1}{q} \eta \right) \quad (\text{A.14})$$

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Chapter 4

THEORY OF CW RAMAN OSCILLATION IN OPTICAL FIBERS

In the last chapter, both the pump depletion and the spontaneous scattering effects were incorporated into the analysis of the forward stimulated Raman scattering in a single-mode fiber. It was shown that the nondepleted pump approximation is invalid for low loss fibers. The consideration of pump depletion is even more critical in understanding the oscillations of the Stokes and the Brillouin shifted waves. A simplified oscillator theory has been presented that does account for pump depletion in the case of CW Brillouin ring oscillators [1]. We present here an analysis of the CW stimulated Raman oscillation in optical fibers. A method is derived to calculate the steady state output power of the SRS wave when the input pump power and the relevant parameters of the fiber resonant cavity are known. Good agreement with published experimental results is obtained. The evolution of the pump and the Stokes waves along the fiber are calculated. An expression for the threshold pump power is obtained. The oscillation condition and higher order Stokes waves generation are discussed.

4.1 Analysis

Consider a situation as depicted in Figure 4.1. A laser beam is coupled by a lens L_1 into a single mode fiber inside a Fabry-Perot resonator. The presence of this pump beam inside the fiber medium gives rise to a gain for an electromagnetic wave at a different (Stokes) frequency because of the stimulated Raman effect. Mirrors M_1 and M_2 are highly

reflective at this Stokes frequency. As the pump power is increased, the round trip gain for the Stokes wave will eventually exceed the loss and result in oscillation. The prism couples out a small fraction of the Stokes wave and also prevents the residual pump beam from being reflected back into the pumping laser. A large pump to Stokes wave conversion is usually attained, and hence the loss of the unused pump power in this geometry is unimportant.

There are three waves propagating inside the fiber. A pump wave of frequency ν_p travels in the forward direction. It generates a forward and a backward traveling Stokes wave at a frequency ν_s , which is down-shifted from ν_p by the vibrational frequency of the molecular medium. A classical treatment [2] of the stimulated Raman scattering process shows that the optical powers of the three waves obey the following equations:

$$\frac{dP_p}{dz} = -\alpha P_p - \frac{\nu_p}{\nu_s} \frac{g}{A} P_p (P_f + P_b) \quad (4.1)$$

$$\frac{dP_f}{dz} = -\alpha P_f + \frac{g}{A} P_p P_f \quad (4.2)$$

and

$$\frac{dP_b}{dz} = \alpha P_b - \frac{g}{A} P_p P_b \quad (4.3)$$

P_p , P_f , and P_b refer to the pump wave power, the forward Stokes wave power, and the backward Stokes wave power, respectively. The first term on the right side of each equation is due to the linear absorption in the fiber medium, where we have assumed the absorption coefficient, α , to be identical for the three waves. The second terms on the right

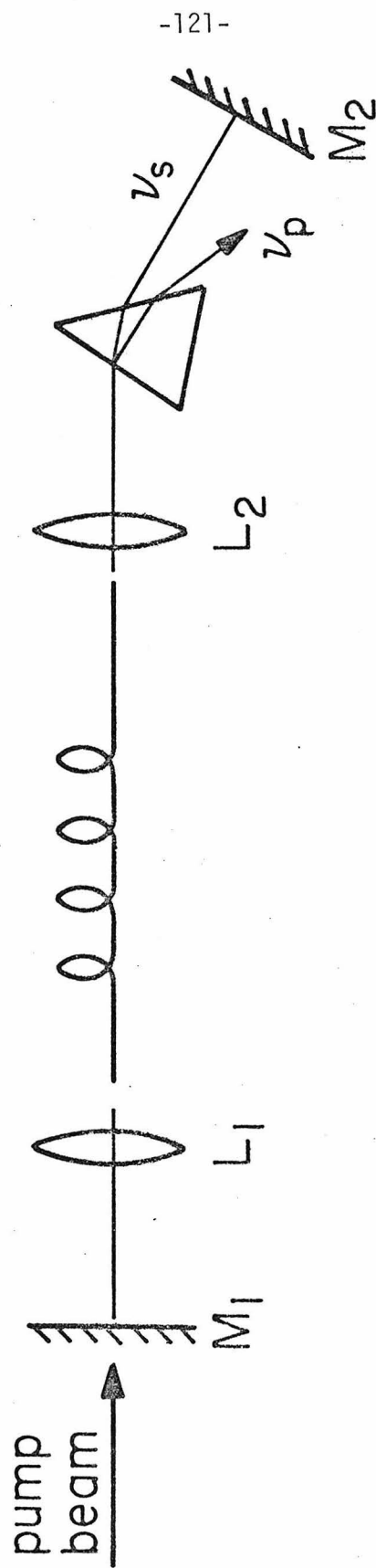


Fig. 4.1 The geometry of a typical fiber Raman oscillator

sides of the equations describe the nonlinear stimulated process and couple the three waves together. A is the effective core area of the single mode fiber, and g is the Raman gain constant in units of cm/W. z is the distance measured along the fiber with $z=0$ being the end at which the pump wave is injected into the fiber. We assume a CW steady state situation, with the Stokes wave oscillation already established. The problem of how the oscillation builds up from the spontaneous Raman scattering is not considered here. During oscillation, the intracavity Stokes-photon flux is so high that stimulated scattering is the dominant photon-conversion process. We are therefore justified in neglecting the spontaneous emission terms in our equations. We note that because of the depolarization effects inside a fiber, a slightly smaller Raman gain constant will have to be used in the above equations.

The boundary conditions of the waves are:

$$P_p(z=0) \equiv P_{p0} \quad (4.4)$$

$$P_f(z=0) \equiv P_{f0} = R_1 P_b(z=0) \equiv R_1 P_{b0} \quad (4.5)$$

and

$$P_b(z=L) \equiv P_{bL} = R_2 P_f(z=L) \equiv R_2 P_{fL} \quad (4.6)$$

L is the total length of the fiber, and P_{p0} is the amount of pump power actually coupled into the input end of the fiber. R_1 and R_2 are the effective power reflectivities relating the counter-propagating Stokes waves at the two fiber ends; the values of the reflectivities have taken into account the lens coupling loss, the fiber end loss, the prism loss, and the mirror transmission loss on each side of the fiber.

As the Stokes wave propagates along the fiber, it is attenuated exponentially as $\exp(-\alpha z)$ due to scattering and linear absorption in the medium. At the same time, it is amplified exponentially as $\exp[\frac{g}{A} \int_0^z P_p(z') dz']$ due to the stimulated Raman effect. The evolution of each Stokes wave is governed by the same physical mechanisms; and equations (4.2) and (4.3) differ only because of the counterpropagating nature of the two waves. It follows directly from (4.2) and (4.3) that

$$P_f(z) P_b(z) = P_{fo} P_{bo} = P_{fL} P_{bL} = K \quad (4.7)$$

where K is a z -independent constant. Using the boundary conditions in (4.5) and (4.6), it follows directly from (4.7) that

$$\frac{P_{fo}}{P_{fL}} = \frac{P_{bL}}{P_{bo}} = (R_1 R_2)^{1/2} \quad (4.8)$$

This equation specifies a relation between the end values of each Stokes wave. An alternative relation can be obtained by integrating (4.2) and (4.3) with the result

$$\frac{P_{fo}}{P_{fL}} = \frac{P_{bL}}{P_{bo}} = \exp[\alpha L - \frac{g}{A} \int_0^L P_p(z) dz] \quad (4.9)$$

Equating the above two relations immediately leads to an oscillation condition for the Stokes waves, namely,

$$(R_1 R_2)^{1/2} \exp[-\alpha L + \frac{g}{A} \int_0^L P_p(z) dz] = 1 \quad (4.10)$$

Equation (4.10) is a statement of the fact that for oscillation the round

trip gain of the Stokes wave is equal to its loss which consists of absorption inside the fiber medium and other cavity losses accounted for by R_1 and R_2 . The pump "profile" $P_p(z)$ will adjust itself so that (4.10) is satisfied, even when the pump power exceeds the threshold value for oscillation. This is the basis of gain saturation in a Raman oscillator.

The Stokes output power is proportional to P_{fL} and no distinction shall be made between them hereafter. We now proceed to solve for the Stokes output power, P_{fL} , and the residual pump power, $P_p(z=L) \equiv P_{pL}$, for a given input pump power, P_{po} . Substitute $P_f = K/P_b$ into (4.1), divide (4.3) by (4.1), and then integrate the resulting equation. We obtain

$$\begin{aligned} P_p(z) P_b(z) \exp \left\{ - \frac{g v_p}{\alpha A v_s} \left[\frac{v_s}{v_p} P_p(z) + P_f(z) - P_b(z) \right] \right\} \\ = P_{po} P_{bo} \exp \left\{ \frac{-g v_p}{\alpha A v_s} \left[\frac{v_s}{v_p} P_{po} + P_{fo} - P_{bo} \right] \right\} = C \end{aligned} \quad (4.11)$$

C is a constant and (4.11) describes a parametric trajectory in the space of P_p , P_f , and P_b . It relates the three waves at an arbitrary z within the fiber. In particular, at $z=L$, (4.11) can be reduced to

$$P_{fL} = \frac{\ln[(R_1 R_2)^{1/2} P_{pL}/P_{po}] + \frac{g}{\alpha A} (P_{po} - P_{pL})}{\frac{g}{\alpha A} \frac{v_p}{v_s} [1 - R_2 - (R_1 R_2)^{1/2} (1 - \frac{1}{R_1})]} \quad (4.12)$$

Equations (4.5), (4.6), and (4.8) are used in obtaining (4.12). We have thus expressed the Stokes output power P_{fL} as a function of the input and residual pump powers, P_{po} and P_{pL} , respectively. Equation

(4.12) provides one of the two equations needed for solving P_{fL} and P_{pL} .

It can be shown from (4.1), (4.2), and (4.3) that

$$\frac{dv}{dz} = -\alpha u \quad (4.13)$$

where $v = \frac{v_s}{v_p} P_p + P_f - P_b$ and $u = \frac{v_s}{v_p} P_p + P_f + P_b$. This is a statement of the fact that the total photon number is conserved during the Raman process and the only loss mechanism is due to the absorption by the fiber medium. We note that solving for u , v , and K [defined in (4.7)] is equivalent to solving P_p , P_f , and P_b , each of which can be expressed in terms of the former three parameters. In particular, when (4.11) is written in terms of u , v , and K , and using (4.13) to eliminate u in the resulting equation, leads to the differential equation

$$\frac{dv}{dz} = -\left\{ \alpha^2 v^2 + 4\alpha^2 K + 4\alpha^2 \frac{v_s}{v_p} P_{po} P_{bo} \exp\left[\frac{-gv_p}{\alpha A v_s} (v_0 - v)\right] \right\}^{1/2} \quad (4.14)$$

where v_0 is $v(z=0)$. Integrating (4.14) results in

$$\int_{v(z)}^{v_0} dv' \left\{ \alpha^2 v'^2 + 4\alpha^2 K + 4\alpha^2 \frac{v_s}{v_p} P_{po} P_{bo} \exp\left[\frac{-gv_p}{\alpha A v_s} (v_0 - v')\right] \right\}^{-1/2} = z \quad (4.15)$$

This equation will enable us later on to calculate P_p , P_f , and P_b within the fiber. At $z=L$ we have

$$\int_{v_L}^{v_0} dv' \left\{ \alpha^2 v'^2 + 4\alpha^2 K + 4\alpha^2 \frac{v_s}{v_p} P_{po} P_{bo} \exp\left[\frac{-gv_p}{\alpha A v_s} (v_0 - v')\right] \right\}^{-1/2} = L \quad (4.16)$$

where $v_L \equiv v(z=L)$. v_0 , v_L , and K can all be expressed in terms of P_{fL} and P_{pL} , and (4.16) is the second equation, which together with (4.12) will allow us to solve for the two unknowns, P_{fL} and P_{pL} . The integral in (4.16) can be performed by straightforward numerical techniques. It is also discovered that a linear approximation of the exponential term in the integrand makes (4.16) integrable and the P_{fL} and P_{pL} values thus calculated agree closely with the exact values obtained by the numerical technique. A method has thus been derived that allows the prediction of the Stokes output and the residual pump powers, when the input pump power and the fiber-resonator parameters are known. We will illustrate the usefulness of this method by applying it to a typical example. It will also be shown that in addition to finding P_{fL} and P_{pL} , our treatment enables us to calculate $P_p(z)$, $P_f(z)$, and $P_b(z)$ at all z 's along the fiber.

To illustrate how to make use of the results derived in the preceding section, we shall consider the case of a recently reported high efficiency CW Raman-fiber oscillator [3]. The experimental setup was the same as that depicted in Figure 4.1. From the provided data, $g = 0.9 \times 10^{-11} \text{ cm/W}$, $\alpha = 3.9 \times 10^{-5} \text{ cm}^{-1}$, $A = 9.9 \times 10^{-8} \text{ cm}^2$, $c/v_p = 0.5145 \text{ } \mu\text{m}$, $c/v_s = 0.5280 \text{ } \mu\text{m}$, $R_1 = 0.7$, and $R_2 = 0.4$. The pumping power P_{po} was 4 watts. A P_{fL} is calculated according to (4.12) for an assumed value of P_{pL} . This pair of P_{pL} and P_{fL} in conjunction with equations (4.5), (4.6), and (4.8) is then used to find the corresponding v_0 and v_L which, in turn, when substituted into (4.16), determine L . This L will be the length of a fiber having these particular P_{pL} and P_{fL} values at its output surface for the given pump power and resonator parameters. Hence, by varying

P_{pL} from zero to P_{po} , we are able for each value of P_{pL} to calculate the corresponding Stokes output power and also determine the fiber length required to observe such an output. According to (4.12), some P_{pL} values, which are close to either P_{po} or zero, result in negative values for P_{fL} . This occurs either when L is too short to provide a sufficient interaction length, or when L is too long so that attenuation per pass is too large. In both cases, there is no oscillation and they will not concern us here. We have plotted in Figure 4.2, using the above mentioned data, P_{pL} and P_{fL} for different pump power values with L along the horizontal axis. In the figure, the P_{pL} and P_{fL} at a particular L value are the predicted residual pump and Stokes output powers if an experiment is done using a fiber length of that L value. In the reported experiment the length of the fiber used was 100 m. At a pump power of 4 watts the estimated Stokes power after lens L_2 (see Fig. 4.1) was reported to be 2W, corresponding to a P_{fL} of 2.3 W. P_{pL} was estimated to be 0.5 W. Our calculation shown in Figure 4.2 indicates that for $L = 100$ m and $P_{po} = 4$ W, P_{fL} is 2.5 W and P_{pL} is 0.1 W. The agreement between the calculated and the reported values is quite reasonable.

In addition to predicting the Stokes output and the residual pump powers, it is also possible to calculate for a given fiber length L , P_p , P_f , and P_b at each point along the fiber. For example, in the above mentioned case for which P_{pL} and P_{fL} and hence K , v_o , and v_L have been calculated, a $v(z)$ value is substituted into (4.15) to find z . This z is then the distance measured along the fiber where $v(z)$ has that particular value. Moreover, $u(z)$ at this point is also known as provided

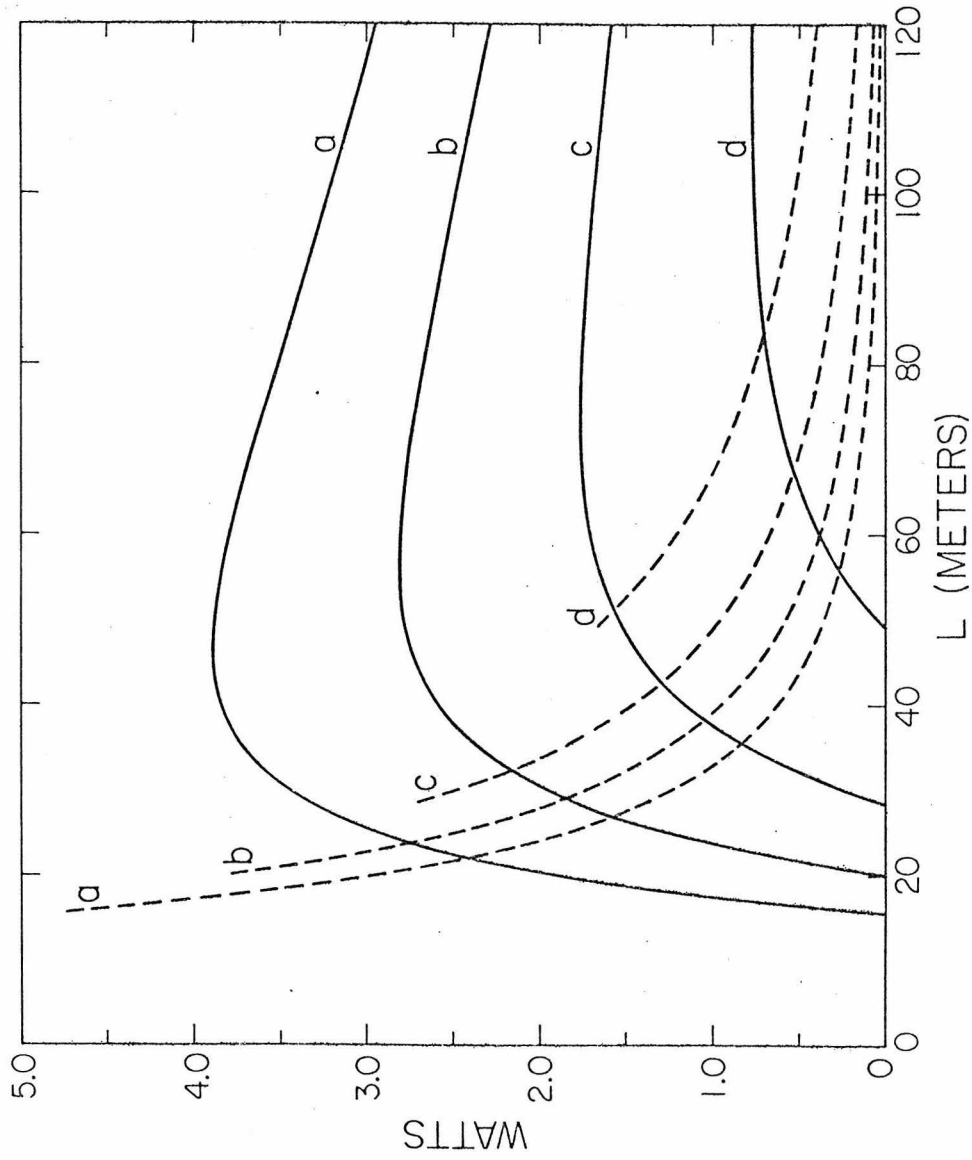


Fig. 4.2 Solid lines are the Stokes output powers and the broken lines are the residual pump powers. The input powers are (a) 5W, (b) 4W, (c) 3W, and (d) 2W

by (4.13) and (4.14). Consequently, P_p , P_f , and P_b are obtained in terms of the $v(z)$, $u(z)$, and K values. Therefore, by varying $v(z)$ from v_0 to v_L , we are able to determine $P_p(z)$, $P_f(z)$, and $P_b(z)$ for each z calculated. Figure 4.3 shows the evolution of the three powers along the 100 m fiber of our example for a 4W input pump power. The rapid depletion of the pump wave due to the conversion to Stokes photons is evident.

4.2 Discussion

It is apparent, as shown in Figure 4.2, that for a given input pump power there is an upper limit to the amount of Stokes output power obtainable from a fiber Raman oscillator. According to (4.12), when $P_{pL} = \frac{\alpha A}{g}$, the Stokes output is maximum and is equal to

$$P_{fL}(\max) = \frac{\ln[(R_1 R_2)^{1/2} \frac{\alpha A}{g P_{po}}] + \frac{g}{\alpha A} P_{po} - 1}{\frac{g}{\alpha A} \frac{v_p}{v_s} [1 - R_2 - (R_1 R_2)^{1/2} (1 - \frac{1}{R_1})]} \quad (4.17)$$

The correct fiber length, L_{opt} , to obtain this maximum Stokes output, can be determined from (4.16), since P_{pL} and P_{fL} are known. A fiber length that is shorter than L_{opt} yields smaller Stokes output power because the interaction length is shorter, resulting in smaller gain per pass. A fiber length longer than L_{opt} also results in smaller Stokes output power because the loss per pass is larger. L_{opt} , at 4 watt pumping power of our example, is about 57 meters. Using a fiber of this length should produce, at the specified pump power, higher Stokes output than what was reported. When designing a fiber Raman resonator which is intended to be pumped at a constant input power, L_{opt} is obviously an important parameter, although as shown in Figure 4.2, small deviations

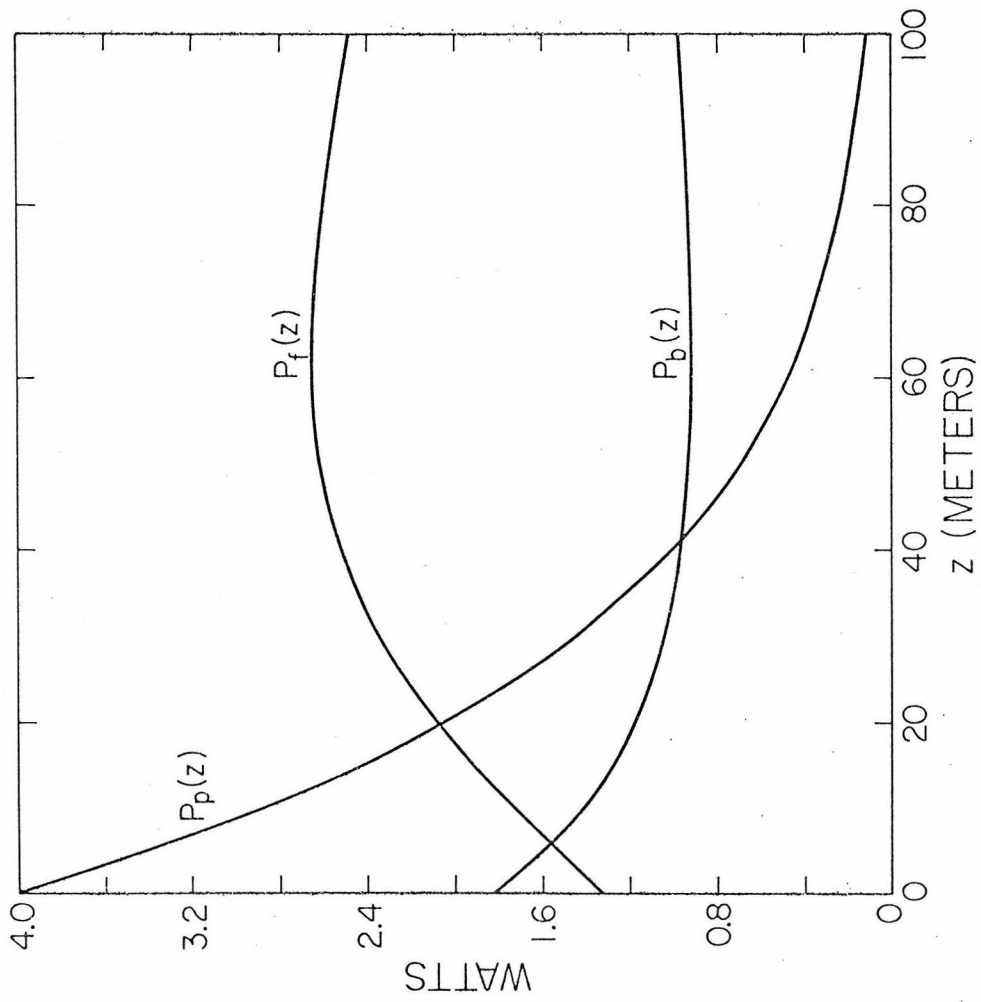


Fig. 4.3 The pump and the Stokes power inside the 100 m long fiber at a 4W pump power input

from this optimum value are easily tolerated.

We derived in (4.10) the oscillation condition for the fiber Raman laser. When the pump power is increased from zero to a value such that this condition is barely satisfied, the oscillating intracavity Stokes power is very small. At this threshold situation, the pump power will behave as $P_{pt} \exp(-\alpha z)$, where P_{pt} is the threshold input pump power. The exponential behavior of the pump follows directly from (4.1), the pump beam being attenuated only by the absorption of the medium in the absence of any Stokes wave. Upon substituting this threshold pump power behavior into (4.10), the required threshold pump input for oscillation is obtained as

$$P_{pt} = \frac{\alpha A}{g} \left[\frac{\alpha L - \frac{1}{2} \ln(R_1 R_2)}{1 - \exp(-\alpha L)} \right] \quad (4.18)$$

This is calculated to be 1.35 W when the data provided in the reported experiment of our example are used. The actual threshold value measured in the experiment was reported to be 1 W.

An interesting feature which results from (4.10) is that above threshold ($P_{po} > P_{pt}$), $\int_0^L P_p(z) dz$ is independent of the input pump power P_{po} . The total absorption of the pump power by the fiber is then

$$\alpha \int_0^L P_p(z) dz = P_{pt} [1 - \exp(-\alpha L)] \quad (4.19)$$

When the input pump power exceeds the threshold value, saturation effect keeps the single pass gain constant so as to maintain the oscillation condition described by (4.10). This is achieved because, as the Stokes intensity increases, the nonlinear coupling also increases,

resulting in an extremely fast draining of the photons from the pump wave. The pump wave becomes very much depleted after a short distance, so that the Stokes wave gain in the rest of the fiber is reduced, and the overall gain per pass remains clamped. Thus pump depletion is the dominant saturation effect in the fiber Raman laser.

In our analysis, we have neglected the higher order Stokes waves generation, which arises because the forward and backward propagating Stokes waves inside the fiber can act as pump waves to generate their own down-shifted Stokes waves. For simplicity, we shall consider only the second order forward Stokes wave and demonstrate that it is insignificantly small. The second order backward Stokes wave is of the same order of magnitude as its forward counterpart, while the third and higher order Stokes waves are even smaller. The power of the second order forward Stokes wave, denoted by P'_f obeys the equation

$$\frac{dP'_f}{dz} = -\alpha P'_f + \frac{g'}{A} P'_f (P_f + P_b) \quad (4.20)$$

the solution of which at $z = L$ is

$$P'_{fL} = P'_{fo} \exp\left\{-\alpha L + \frac{g'}{A} \int_0^L [P_f(z) + P_b(z)] dz\right\} \quad (4.21)$$

Equations (4.2) and (4.3) that describe P_f and P_b will have additional loss terms to account for the second order nonlinear effect. However, the original solutions of (4.2) and (4.3) will be the upper limits to the solutions when the second order effect is incorporated. Therefore, for our purpose, which is to estimate the maximum second order Stokes

wave that can possibly be generated, it is adequate to simply use the previous calculated values of P_f and P_b in (4.21). Remembering our definition of u ,

$$\int_0^L [P_f(z) + P_b(z)] dz = \int_0^L u(z) dz - \frac{v_s}{v_p} \int_0^L P_p(z) dz \quad (4.22)$$

Upon using (4.13), (4.18), and (4.19), equation (4.22) becomes

$$\int_0^L [P_f(z) + P_b(z)] dz = \frac{1}{\alpha} [v_o - v_L] - \frac{v_s}{v_p} \frac{A}{g} [\alpha L - \frac{1}{2} \ln(R_1 R_2)] \quad (4.23)$$

which is expressed in terms of previously calculated values. Thus P'_{fL} can be obtained from (4.21) and (4.23). Since there is actually no finite P'_{fo} , and any second-order Stokes power build-up is due to amplified spontaneous scattering, we shall arbitrarily assume one photon per mode as a fictitious input for P'_f [4]. Using a Lorentzian gain constant with a 400 cm^{-1} line width [5] and the data provided in our example, P'_{fL} is estimated to be only about $30 \text{ } \mu\text{W}$. This justifies our neglect of the higher order Stokes waves in the analysis.

In conclusion, we have presented a detailed treatment of a stimulated Raman fiber laser. We have demonstrated how the Stokes output power and the residual pump power can be calculated from a known input power and the resonator parameters. In addition, the evolution of the pump and the Stokes waves along the fiber are calculated. Our result is compared with published experimental data and reasonable agreement is obtained. The threshold pump power and the oscillation condition at and above threshold are discussed. Higher order Raman effect is shown

to be negligible, confirming that pump depletion is the dominant saturation effect.

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5.1 Other Nonlinear Optical Phenomena in Fibers

Stimulated Brillouin Scattering

Next to Raman scattering, Brillouin scattering has been most intensively studied inside fibers [1-6]. It is similar to Raman scattering except for the fact that the light scatters off an acoustic wave in the material. The shift in frequency of the scattered light is typically small, on the order of gigahertz, and it has a highest gain in the backward direction. Thus in an optical fiber, stimulated Brillouin scattering is observed in the backward direction, opposite to that of the pump wave. The equations governing the pump power P_p and the scattered power P_B are

$$\frac{dP_p}{dz} = -\alpha P_p - \frac{g_B}{A} \frac{v_p}{v_B} P_p P_B \quad (5.1)$$

and

$$\frac{dP_B}{dz} = \alpha P_B - \frac{g_B}{A} P_p P_B \quad (5.2)$$

The frequency shift $\nu_p - \nu_B$ is $2n\nu \frac{v_p}{c}$. n is the index of refraction of the material, and v is the acoustic wave velocity. Equations (5.1) and (5.2) are identical to (4.1) and (4.3) except that the quantities associated with the Stokes waves are replaced by those associated with Brillouin waves. The small signal theory given in Section 3.2 can be directly applied to describe the situation at hand. A complete treatment including the nonlinear pump depletion can only be carried out numerically. No analytic solution has been found except for the trivial case

of zero fiber loss ($\alpha = 0$). However, the theory of CW Raman oscillation presented in the last chapter can be easily used to describe the oscillation of the Brillouin shifted waves inside fiber. Experimentally, considerable research efforts have been devoted to this area. CW operation of Brillouin fiber ring oscillators [3] have been demonstrated with input pump powers less than 1 W. Recently, mode locking of the radiation in the first order Brillouin-shift wave of a fiber Brillouin oscillator was reported [6].

There are several other nonlinear optical phenomena observed inside fibers. These include nondegenerate four-wave mixing [7-9], self-phase modulation [10,11], optical Kerr effect [12], and coherent anti-Stokes Raman scattering [13]. They are all interesting third order nonlinear effects observed in either glass or liquid fibers, and have important implications for fiber optic communication.

5.2 Applications to Fiber Optic Communication

It was pointed out in Chapter 1 that one reason for looking into nonlinear optical phenomena inside fibers is to understand the limitation of fiber capability in handling large optical power. It has been shown in the last two chapters that it is possible for most of the energy carried by a pump wave to be transferred to a Stokes wave at a different frequency. This may be undesirable for communication purposes because a detector located at the receiving end of a fiber may have a frequency dependent response. Furthermore, the cross section for stimulated Brillouin scattering is very large in glass, implying that this effect will impose a limit on the amount of power that can be transmitted through a

fiber. Other effects such as spectral broadening and pulse shaping by means of self-phase modulation in fibers are also important factors to be considered in fiber optic communications.

The Raman oscillator provides laser oscillation at the Stokes frequency, and because of the large bandwidth of the Raman cross section in silica, the laser output of such a fiber Raman oscillator can be tuned over a large frequency range. This has already led to the use of this device as a source for probing fiber properties [14,15] in the near infrared region ($1.3\ \mu\text{m}$) where silica fibers have minimum absorption and dispersion.

Chapter 5 - REFERENCES

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