ESSAYS IN FORWARD MARKETS AND THE URANIUM INDUSTRY

Thesis by

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In Partial Fulfillment of the Requirements
for the degree of
Doctor of Philosophy

California Institute of Technology
Pasadena, California
1979

(Submitted October 12, 1978)

ACKNOWLEDGMENTS

I wish to thank the members of my committee, Forest Nelson, Roger Noll, Louis Wilde, and expecially the chairman, James Quirk, for advice and guidance in the writing of this thesis. I have also had helpful discussions with Robert Forsythe and David Montgomery. Elizabeth Hoffman has contributed substantively and with her unending support. Dana Leimbach provided excellent typing.

ABSTRACT

This thesis brings together two papers, one of which is primarily empirical and one of which is theoretical. The first estimates long run costs of uranium production. The second analyzes theoretically the impact of fixed price contracting on the decisions of a firm facing price uncertainty.

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INTRODUCTION

This thesis is comprised of two papers, one which is primarily empirical in nature and one which is exclusively theoretical. While the two papers are essentially distinct, they are related in the sense that the theoretical issues explored in the second paper arose in the course of interpreting the data in the first paper. The first paper focuses on the recent and extremely rapid increase in the price of uranium oxide. First it reviews the major developments in the uranium market. Then it uses published data on the costs associated with various stages of the production process to develop estimates of the long-run cost of uranium production. The production cost estimates were undertaken in order to provide benchmarks against which the recently observed prices may be compared. The purpose of such comparisons is to obtain an idea of whether current prices differ significantly from long-run production costs. If they do not, prices can be expected to remain close to current levels. If they are above or below long-run production costs, however, prices may be expected to decrease or increase, respectively, as the market tends toward long-run equilibrium.

The second paper is a theoretical study of the role of forward contract markets in influencing the investment and production decisions of firms which are uncertain about future prices. The issue of what those influences are arose in the course of studying the uranium market. In the early stages of the development of the commercial uranium market, there failed to develop a significant degree of forward contracting, even though there was considerable uncertainty, at least on the supply side, over what future prices would be. This fact led at least one observer[1] to conclude that uranium producers, because of their inability to secure contracts for future output, did not undertake investment sufficient to meet the future needs of an expanding nuclear generating industry. While such an argument has considerable intuitive appeal, it was decided that a more general theoretical investigation of how participation in contracting affects firm decisions was appropriate. The models in the second paper are of general firms and are not intended to be specifically representative of the uranium industry. However, with some qualifications, which relate primarily to technical assumptions, the conclusions reached are generally consistent with and supportive of the argument that the existence of more forward contracting would have led to more investment by the uranium producers.

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[1]Paul L. Joskow, "Commercial Impossibility, the Uranium Market and the Westinghouse Case," Journal of Legal Studies, VI, (January, 1977): 131-138.

CHAPTER 1

LONG-RUN COSTS IN URANIUM PRODUCTION

The purpose of this paper is to estimate long-run production costs and equilibrium prices for uranium oxide (U_3O_8). Concentrated uranium oxide, or yellowcake, as it is known, is the raw material which, after a number of intermediate processes, is manufactured into the fuel for nuclear reactors. The cost of nuclear fuel is of interest because it is not, as may have been thought in the past, insignificant in the economics of electric power generation using nuclear reactors. Joskow and Baughmon, for instance, have concluded that doubling the price of uranium, including enrichment charges, would have the effect of reducing nuclear capacity by 24 percent by 1995.[1]

A large increase in the spot price for a pound of uranium oxide has occurred in the last five years. In October 1972 the price per pound was $5.95.[2] By June 1977 it had reached $42.25.[3] This increase has led to concern and speculation over the course of prices in the future. While predicting prices is necessarily subject to a good deal of uncertainty, there are data available which allow production cost estimates to be made. These calculations will be presented in the final section of this paper. First, however, as background for these estimates, the steps in uranium production and the nuclear fuel cycle will be outlined and the organization and development of the market for uranium will be reviewed.

URANIUM PRODUCTION AND THE NUCLEAR FUEL CYCLE

The nuclear fuel cycle begins with exploration for and mining of uranium ore. Extensive drilling is required to locate and define ore deposits. Open pit or underground mining techniques are used to extract the ore, depending on the depth of the deposit. Currently productive deposits of ore generally contain on the order of .2 percent uranium oxide. Due to the low uranium content at this stage, the ore is milled near the mining site to avoid the cost of transporting the ore significant distances. In the milling operation, the uranium is extracted from the ore and concentrated as the semi-refined product referred to as yellowcake. At this stage the uranium is purchased by utilities and reactor manufacturers. The general practice has been for the manufacturers to supply a newly constructed reactor with the initial fuel core and one or more reloads.[4] There are about 20 mills currently in operation, with others in various stages of planning and construction.[5]

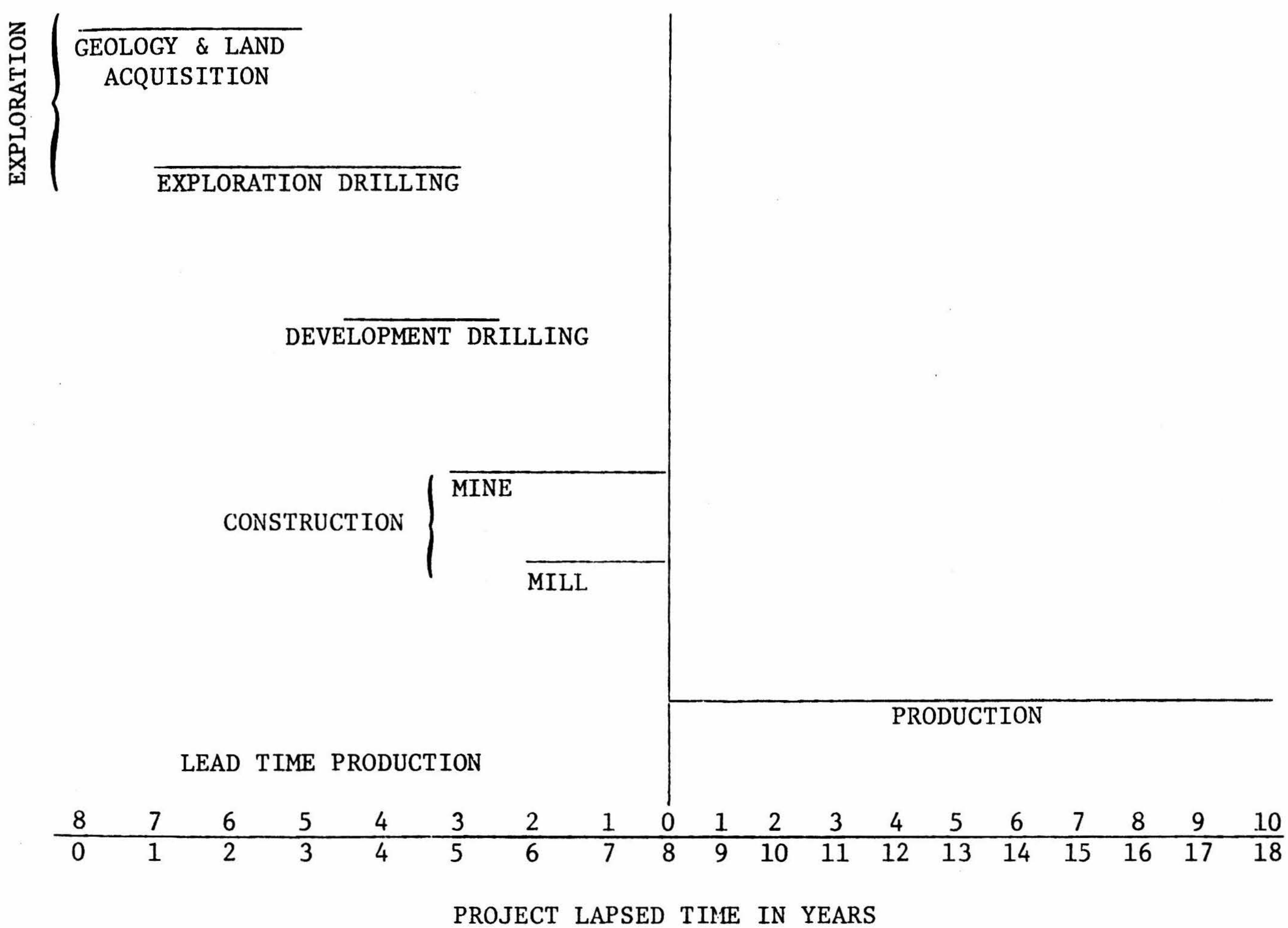
Additional stages in the front half of the fuel cycle include the conversion of the ore concentrate to uranium hexafluoride (UF_6), enrichment of the uranium in the isotope U_{235} from .7 percent to from 2 to 4 percent, the conversion of the enriched uranium into UO_2, and the processing of the UO_2 into pellets. The pellets are placed in rods which form the fuel assembly, or reactor core. There are two companies with facilities to convert yellowcake to UF_6.[6] Enrichment is performed in United States Government plants originally built to enrich uranium for defense purposes. The UO_2 conversion and manufacture of the fuel assemblies are done primarily by the reactor manufacturers and a small number of other companies.[7]

In the irradiation stage, the fissioning of the U^{235} in the fuel rods is the source of heat in a nuclear power generator. After the fuel irradiation, the spent fuel can be processed to obtain any uranium remaining or plutonium produced (although these are not currently being done), and the remainder of the spent fuel must be stored.

One of the characteristics of the uranium industry which is crucial to an understanding of the market is the length of time necessary for operations required to provide additional production capabilities. The time required from initial geological studies and exploratory drilling until the beginning of production has been estimated to be eight years.[8] Typical lead times for other production activities are shown in Figure 1. Depending on the amount of preproduction work which has already taken place in developing potential production sites, one of the implications of the lead times is that, if production capacity is being fully utilized, it may be extremely difficult, if not impossible, to satisfy unexpected short-term increases in demand. In economic terms, this means that the short-run supply function is inelastic for quantities greater than current production capacity. On the demand side, since fuel costs represent a relatively small part of the total cost of electricity production from a nuclear power plant, the short-run demand function is also likely to be inelastic. The combination of the inelastic short-run supply and demand functions implies that there is potential for spot prices to deviate dramatically

FIGURE 1

IDEALIZED LEAD TIME SCALE FOR PRODUCTION



SOURCE: John Klemenic, "Analysis and Trends in Uranium Supply," Uranium Industry Seminar, U.S. ERDA, Grand Junction Colorado, October 1976, p. 232.

from long-run equilibrium prices.

Theoretical considerations imply that those long-run equilibrium prices should include two components, the long-run marginal cost of production and a charge (shadow price) which reflects the scarcity of an exhaustible resource and serves to allocate that resource in an optimal fashion among time periods. Changes in the first component will occur over time as ore is mined from deposits which are deeper, more difficult to work or are of a lower grade, as well as when prices of factors of production change or as cost-saving technologies are developed. If the marginal costs of production are constant over time, then the second component of equilibrium prices should increase at the rate of interest.[9] Briefly, the reasons for the existence of the second component and its rate of increase can be illustrated by the following example. Assume that a technology which will replace the need for using the exhaustible resource will become available at a future date. As the transition to that technology is made and the exhaustible resource is used up, the price of the last units of the exhaustible resource will equal the cost of using the new technology. This will be true so long as the cost of the new technology is at least as high as the cost of producing the last units of the resource; otherwise the transition will be completed before the resource is exhausted. In an intertemporal competitive equilibrium, the difference between the price and production costs at any given time will be the difference at the transition time discounted back at the rate of interest. This is the equilibrium condition because producers will then be indifferent between current production in any period and withholding output in anticipation of higher prices in the future.

William D. Nordhaus has estimated the two components of optimal future prices for exhaustible energy supplies (petroleum, coal, shale oil, natural gas, and nuclear fuel), assuming that conversion to a backstop technology (the breeder reactor) is completed in the period 2120-2170.[10] Based on these calculations, which assume a 10 percent rate of interest, the shadow price for uranium is found to be relatively unimportant in the near term, representing less than 1 percent of the price in 1980. By 2000, however, it is estimated that it will represent over 6 percent of the price and will continue to increase from then on.[11]

Both the example given above and Nordhaus's analysis are based on the assumption that the total supply of the exhaustible resource is known in advance. Should significant unanticipated discoveries occur, the shadow price component of long-run prices would decrease, indicating that the resource has become less scarce. If this should be the case, shadow prices would represent an even smaller portion of long-run equilibrium prices than indicated above. In any event, a close approximation to long-run equilibrium prices in the next decade or so will be long-run marginal costs of production.

Deviations in spot prices from those prices will occur when production capacity differs from that required to meet current demands at those prices. During periods when demand is lower than production capacity, the spot price will be determined by the variable costs of production and the cost to the producer of storing the product until demand increases[12], and when demand exceeds production capacity, the spot price will rise to clear the market.

Relatively few of the transactions appear to occur on a strictly spot basis, however. The Nuclear Exchange Corporation (NUEXCO), a company specializing in coordinating sales and purchasing efforts in the uranium market, listed unfilled domestic demands for 1978 delivery as of June 30, 1977 at 850 tons.[13] Total 1978 requirements, based on the ERDA "mid"-case estimate assuming no breeder, are 23,100 tons.[14] Table 1 gives similar comparisons for the years 1977 through 1986. Clearly, most of the requirements for up to five years in the future have already been covered by contracts for future delivery and about one-half to one-third of the requirements have been contracted for for the period six to nine years hence. These represent higher proportions of forward contracting than had been true in previous years, however. These changes in forward contracting, along with changes in the terms of the contracts, will be discussed below, in connection with the history of the uranium market.

DEVELOPMENT OF THE URANIUM MARKET

Uranium oxide has been produced in the United States since 1948.[15] Until 1964, however, only the government could legally purchase uranium. Then the Private Ownership of Special Nuclear Materials Act of 1964 allowed commercial sales to take place.[16] Commercial deliveries did not begin until 1967, however, with all previous sales having been made to the AEC, at AEC-set prices.[17]

TABLE 1

UNFILLED REQUIREMENTS AS OF 6/30/77 AND
ESTIMATED TOTAL REQUIREMENTS, ANNUALLY

1,000 TONS OF URANIUM OXIDE

YEAR OF DELIVERY	ESTIMATED REQUIREMENTS	UNFILLED REQUIREMENTS	PERCENT UNFILLED
'77	14.9	.4	2.7
'78	23.1	.9	3.7
'79	28.3	3.6	13.2
'80	31.3	4.6	14.8
'81	33.4	7.1	21.2
'82	36.7	12.0	32.8
'83	36.5	19.0	52.0
'84	42.4	19.4	45.7
'85	45.9	24.4	53.1
'86	45.4	30.5	67.1

SOURCES: Requirements, "mid"-case ERDA estimate, cited in Klemenic; unfilled requirements, NUEXCO report, 6/30/77.

The trend in spot prices since the beginning of the commercial market is shown in Figure 2. A detailed analysis of the history of uranium production has been provided by Joskow.[18] He has divided the development of the industry into five periods:

1. 1948-1958. The AEC purchased uranium at prices ($9.00-$12.50) which encouraged exploration and production. Bonuses and direct subsidies were also provided.

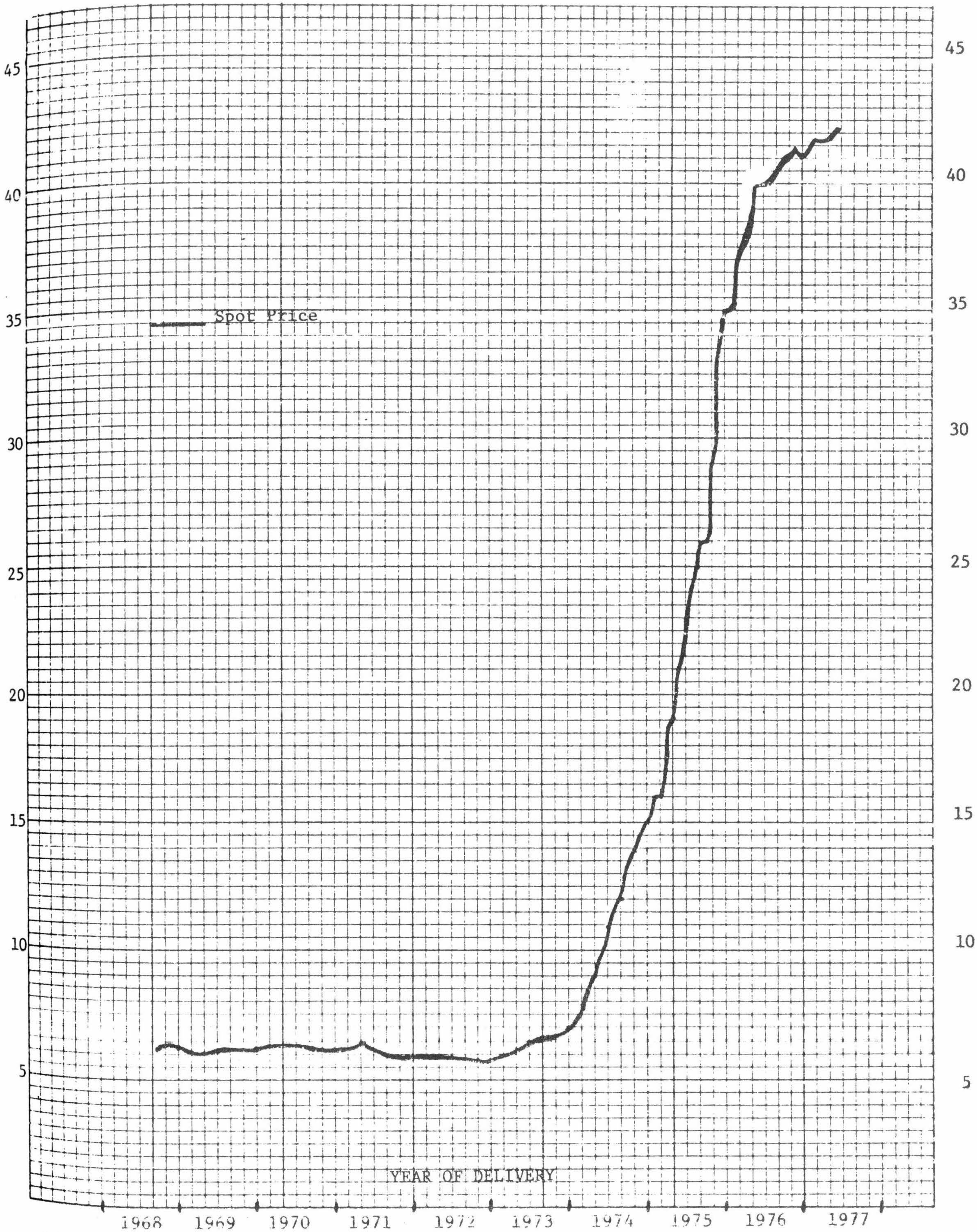
2. 1958-1962. The AEC purchased large quantities of uranium from existing facilities, but did not encourage further exploration or expansion. Prices paid during this period were lower than they had been previously and declined to below $9.00.

3. 1962-1969. The AEC paid $8.00 per pound for production from existing reserves in an effort to cover short-run costs and to keep the industry alive in anticipation of a commercial market.

4. 1969-1973. The commercial market begins and there is near-term excess capacity. However, there is significant exploration and expansion in 1969 and 1970 in anticipation of future increases in demand. The market does not develop quickly, though, as utilities are reluctant to make forward purchases at prices high enough to encourage further expansion. Exploration drops off and prices fall to under $6.00 in 1972.

FIGURE 2

SPOT PRICES FOR YELLOWCAKE IN DOLLARS PER POUND



SOURCE: NUEXCO Report, 6/30/77, pp. 3.2, 3.3.

5. 1973-1975. Two events which contributed to the price increase took place during this period: 1) The AEC changed its enrichment contracts to provide enrichment services only on a long-term, firm contract basis. The contracts run for ten years and had to be signed eight years in advance of initial deliveries. This is in sharp contrast with the previous method of contracting, which required less than one year's advance notice. This apparently encouraged utilities to enter the market to cover future requirements to a greater extent than they had previously and perhaps caused some buyers to contract for more uranium than would otherwise have been necessary in attempts to secure the limited enrichment contracts.[19]

2) Westinghouse announced that it would not deliver around 70 million pounds of U_3O_8, contracted for by its reactor customers and others at base prices of \$8-\$10 for delivery in 1975-1988, because it would be "commercially impracticable" to do so, due to its short position of about 40,000 tons for that period.[20] This further added to the price increases as utilities which believed their requirements were covered had to reenter the market.

Since 1975 the price has continued to increase, although from mid-1976 to mid-1977 the rate of increase has been considerably slower than it had been in 1974, 1975 and the first half of 1976.

Joskow's interpretation of the rapidity of the price increase during the period 1974-1976 is founded in large part on the observation that utilities failed to contract in the early 1970s for significant quantities of their future uranium requirements.[21] The argument runs as follows. From the suppliers point of view, there was uncertainty in the early 1970s surrounding the future price of uranium. This uncertainty was due to at least three sources: (1) delays in reaction construction made previous projections of nuclear capacity and, therefore, uranium demand less reliable; (2) the AEC had not yet determined what would be the disposition of its uranium stockpile, which it had accumulated during the 1960s and amounted to 50,000 tons; and (3) the possibility that, for the first time in the era of the commercial market, the embargo on foreign uranium would be lifted. In any event, the costs of exploration, development, and production from new facilities, which would be required in order to expand output, were projected to be above the $8 to $10 range, which meant that any long-term contracts the suppliers would have been willing to agree to would have to have been higher, as well.

On the other hand, the utilities apparently expected the price to remain at around $8, due both to current availability, both in the spot market and from Westinghouse, at or below that price, and to the publication by the AEC of statistics on reserves in the

$8 and $10 forward cost categories.[22] The utilities, therefore, were unwilling to sign contracts at prices as high as the producers considered necessary and little forward contracting took place. As a consequence, the uranium producers, apparently reluctant to bear all of the risk associated with the price uncertainty, refrained from exploratory and development drilling and from investment in new processing facilities. By the mid-1970s, however, it was apparent that the increase in the number of nuclear reactors meant that, by around 1980, there would be insufficient production capacity to fulfill the expected demand, and, as the utilities attempted to sign contracts to fulfill their future requirements, the price rose to clear the market.

The increase in the degree of forward contracting which has occurred can be seen in Table 2, which presents estimates of the percent of future requirements not yet contracted for as of the beginning of 1972 and as of mid-1977 for deliveries 4 to 8 years after 1972 and 3 1/2 to 8 1/2 years after 1977.

TABLE 2

PERCENT OF FUTURE REQUIREMENTS NOT YET CONTRACTED FOR
AS OF THE BEGINNING OF 1972 AND MID-1977

YEARS BEFORE DELIVERY	PERCENT OF ESTIMATED REQUIREMENTS NOT CONTRACTED FOR	
	1972	Mid-1977
3 1/2		21.1
4	39.4	
4 1/2		32.8
5	34.5	
5 1/2		52.0
6	78.8	
6 1/2		45.7
7	85.5	
7 1/2		53.1
8	90.1	
8 1/2		67.1

SOURCES: Contracts as of 1972, Statistical Data of the Uranium Industry, various years; collected in Joskow, "Commercial Impossibility, the Uranium Market and the Westinghouse Case," Journal of Legal Studies VI, (January 1977); p. 128.

Estimated requirements and unfilled requirements as of mid-1977, Table 1.

While the degree of forward contracting has increased to the point where a very small percentage of transactions takes place on a strictly spot basis, the spot market prices apparently play an important role in determining the actual prices paid under the contracts, since many of the contracts specify that delivery prices will be linked to spot prices at the time of delivery.

In a context of certainty, the theoretical equilibrium relationship between current spot prices and forward contract prices, specified to be paid at the time of future delivery, is that the contract prices will not exceed current spot prices, increased at the rate of interest, plus storage costs. Otherwise, customers for a nonperishable product would find it less costly to make purchases for future requirements on a current spot basis and bear the storage costs themselves. When there is less than equilibrium production capacity, however, as appears to have been the case recently, producers will be able to obtain contract terms more favorable to themselves, since it will be difficult for customers to make spot purchases to fill future requirements. Terms specified in recent formal contracts have been more favorable to producers than would be expected in long-run equilibrium. According to a recent NUEXCO report:

> . . . sellers continue to insist on the two-tier price formulas, which typically require the buyer to pay the higher of a floor or a price somehow related to market. Floor price variations are numerous: cost plus, single base adjusted by indices, multiple bases adjusted by indices and year-by-year fixed prices. Market price variations are equally numerous, but most continue to involve periodic negotiations under a defined procedure. The stabilization of prices during the past year has apparently, however, enabled some buyers to negotiate ceiling prices and other mechanisms designed to limit their market exposure.[23]

Thus, at the current stage in the development of the market, forward contracting appears to be basically a mechanism for linking suppliers and customers, with the customer still facing the possibility of having to pay prices determined, at least in part, by future spot prices. This indicates that there still exists a "seller's market," although it is perhaps becoming less so, as evidenced by the reduction in the rate of increase of spot prices and the successful introduction of ceiling prices on the part of some buyers. As the market more closely approaches equilibrium, forward prices should become more narrowly specified, serving as a hedge for both sides of the market.

In the next section, estimates of long-run equilibrium prices will be developed. Given Nordhaus's conclusion that shadow prices will be a very small percentage of total prices, the estimates will be based on estimates of long-run marginal costs of production.

LONG-RUN MARGINAL COST OF PRODUCTION

The estimates derived here are based on exploration costs (capitalized to the beginning of the production period), the capital costs of the mine and milling facilities, the variable costs of mining, hauling and milling, and provisions for taxes, profits and royalties. The procedure to be followed in the calculations will be to use estimates which provide upper and lower bounds on the various components of cost for what is likely to be a marginal production facility. Then, combining these estimates, the output prices which will yield zero excess profits will be solved for. The result will be a range of estimates which will, it is hoped, bracket true long-run equilibrium prices.

In the calculations, rates of return of 10 percent and 15 percent are assumed. The corporate income tax rate of 48 percent is incorporated, along with the depletion allowance. The depletion allowance provides that 22 percent of gross mining revenues can be deducted from taxable income, up to a deduction limit of 50 percent of the taxable income, before the tax is computed.[24] Since the figures used here imply that 22 percent of gross revenues exceeds 50 percent of taxable income, it will be assumed that the effective tax rate is 24 percent. If firms can manage to deduct the entire 22 percent, however, this will lower the price needed to obtain the required rate of return. Royalties are assumed to amount to 5 percent of gross revenues, which is apparently the typical amount paid to states and landowners.[25] The productive lifetime of the mine-mill complex is assumed to be 10 years, with 300 days of operations per year. See, however, the discussion below on the optimal productive lifetime of the plant from the firm's point of view. Although many firms use a form of accelerated depreciation, for computational simplicity, straight-line depreciation of capital assets is assumed for the productive lifetime of the plant. Use of the straight-line method biases the estimates upward somewhat. Since exploration costs are typically deducted on a current basis, the effective exploration costs will be assumed to be 76 percent of current costs, capitalized to the beginning of the production period and not included in deductions during the period of operation.[26] Combining these assumptions allows the price which will provide the necessary rate of return to be found by solving for p in the following equation.

$$\sum_{t=1}^{10} (1+r)^{-t}((pq - .05pq - vq) - .24(pq - .05pq - vq - .1C(q))) = C(q) + E(10q)$$

where:

- p is the price of output,
- q is the quantity of output per year,
- v is the variable cost per unit of output,
- C(q) is the capital cost of a facility capable of processing q units per year,
- E(10q) is the capitalized exploration cost of exploration sufficient for ten years of production, and
- r is the required rate of return.

The capital and exploration cost estimates are based in part on the testimony of J. Clayton Stephenson before the JCAE in

1974 during the hearings on the Future Structure of the Uranium Enrichment Industry.[27] Mr. Stephenson is President of the Mining and Metals Division of the Union Carbide Corporation and Chairman of the Uranium Advisory Council of the American Mining Congress. Additional figures are obtained from estimates supplied in a 1975 draft report on uranium supply prepared for the Edison Electric Institute's Nuclear Fuels Supply Study Program.[28] Ranges for the variable costs are also taken from the EEI report; other estimates are derived from a study by John Klemenic, Director of the Supply Analysis Division of ERDA's Grand Junction Office.[29]

In Mr. Stephenson's statement before the JCAE, he estimated that acquisition, exploration and exploration development costs would rise from a current average of $1.00 per pound to $2.30 by 1985, calculated in 1974 dollars.[30] Assuming that the exploration expenditures are deducted on a current basis, these figures can be reduced by 24 percent to obtain the effective cost, since this is assumed to be the effective marginal tax rate. This gives a range of costs for exploration of from $.76 to 1.75 per pound. These figures will be used in the range of cost estimates after being capitalized at a rate of 10 percent per year for five years, which is about the mean of the delay between drilling and initial production. This procedure gives capitalized exploration costs at the beginning of production of from $1.22 to $2.82 per pound.

Mr. Stephenson has also provided estimates of capital costs for plants and mines of various capacities. These estimates are reproduced in Table 3.

TABLE 3

CAPITAL COSTS FOR NEW URANIUM PRODUCTION (1974 DOLLARS)

(1) CAPACITY IN TONS OF ORE PER DAY	(2) AVERAGE PLANT COST	(3) AVERAGE MINE COST	(4) COST PER TON PER DAY
1,500	$18,600,000	$1,860,000	$13,640
2,000	$22,000,000	$2,200,000	$12,100
3,000	$28,200,000	$2,820,000	$10,340
5,000	$38,300,000	$3,830,000	$ 8,426

SOURCES: Columns (1) - (3), from Table 3, p. 304, Hearings on Future Structure of the Uranium Enrichment Industry. Column (4), ((2) + (3))/(1).

Inspection of Column (4) from Table 3 indicates the implicit assumption of economies of scale in these estimates. Thus, the question arises of the effect production from ore deposits of varying sizes has on the cost of production. It can be shown that, under economies of scale, a profit-maximizing firm will deplete larger deposits of ore in a shorter elapsed time, using more than proportionately larger mills to take advantage of the scale economies.

It is also the case that the output price which is required to cover variable costs and provide a given rate of return on capital investments is lower, the larger is the ore body (see appendix). Thus, to determine the length of the production horizon and the amount of capital investment required per unit of output, it is necessary to know what the size of the marginal ore deposit will be.

The representative "marginal plant" adopted here is one with a capacity of 2,000 tons of ore per day. This is slightly smaller than the average size of production centers, cited in Klemenic's work, which range in stage of development from having construction commitments to those which have had sufficient exploration to warrant the assumption of future production.[31] From Table 3, this gives a capital cost of $24,200,000 for a plant and mill. The EEI study estimates capital costs to be about $40,000,000 for a 2,000 ton per day facility.[32] To provide a range in capital costs, both figures will be used here. Since the construction of the mill takes place during the two years prior to production (see Figure 1), the capital costs will be increased by 10 percent to account for interest costs during construction.

It will be assumed that the mine/mill complex is productive for what appears to be the industry rule of thumb, ten years. Currently productive production centers average slightly larger than 2,000 tons per day capacity, and since it can probably be safely assumed that the ten year horizon has been based on recent experience, a ten year horizon is probably not too drastic a distortion.[33]

The range of variable costs used will be taken from the figures cited for mining, hauling and milling costs in the EEI study.[34] These are presented in Table 4.

These figures can be compared with data Klemenic provides from summarizing industry financial reports.[35] He has estimated 1976 operating costs to be (in millions) $181 for mining, $14 for hauling and $69 for milling, for 16,000 tons of U_3O_8 produced from ore with an average grade of .163 percent. This yields average variable costs per pound of U_3O_8 of $8.25. Scaled to an ore grade of .10 percent, this implies variable costs of $13.45 per pound. However, about 2/5 of the 1976 production was from open pit mines, which have lower operating costs than do underground mines. Therefore, since most new production in the future is expected to come from underground mines, the high end of the EEI range of variable costs should probably be relied on most heavily.

The estimates used in the calculations are summarized in Table 5. The costs stated in this table are expressed on the basis of an assumed .10 percent ore grade. This will not necessarily be the grade in a marginal facility, however. The average grade of ore

TABLE 4

VARIABLE COSTS OF PRODUCTION

DOLLARS PER POUND OF U_3O_8

ASSUMING .10 PERCENT ORE GRADE

	LOW	HIGH
Mining	$ 6.70	$10.72
Hauling	.50	.68
Milling	2.80	4.35
	$10.00	$15.75

SOURCE: S. M. Stoller Corporation, EEI report, p. 92.

TABLE 5

RANGE OF AVERAGE COSTS PER POUND

.10 PERCENT ORE GRADE

1974 DOLLARS*

Capitalized effective cost of exploration	$ 1.20	$ 2.82
Capitalized capital costs (per pound of production per year; ten year horizon	22.18	36.66
Variable Cost	$10.00	$17.75

*More accurately, these are probably partly 1974 dollars and partly 1975 dollars, since EEI cost data were collected in 1975. These were based on companies' "recent cost experience." It is not clear, however, how recent the figures were nor if they were adjusted to a current basis in any way.

delivered in 1975 was .17 percent, and the grade has been declining fairly steadily since the beginning of production in 1948.[36] Klemenic has estimated that the average grade of ore which could be produced at a forward cost of $15 or less will have an ore grade of .15 percent for at least the next two decades.[37] He has further calculated that sufficient U_3O_8 could be produced in this category to more than meet ERDA's "mid" case forecast of domestic requirements, at least through 1990.[38] Since the .15 percent grade represents an average of future grades, a marginal facility might well process ore which is of a lower grade. Some observers have suggested that the ore grade for marginal procedures may be as low as .10 percent.[39] The calculated prices are quite sensitive to the ore grade which is assumed; a price based on a .10 percent grade is 50 percent higher than one based on a .15 percent grade. The approach taken here will be to consider the .15 percent figure as a somewhat optimistic (from the customer's point of view) assumption and the .10 percent figure as somewhat pessimistic.

Table 6 presents the calculated values for long-run marginal costs, based on the various cost figures and assuming .10 percent and .15 percent ore grades. Middle-range calculations are also presented; they are based on an assumed ore grade of .125 percent, which is midway between the other two (steps in the calculations are presented in the appendix).

The calculations using the low exploration costs have been included here primarily to illustrate the effect on equilibrium output prices of a change in exploration costs, rather than to

TABLE 6

CALCULATED VALUES FOR LONG-RUN MARGINAL COSTS
1974 DOLLARS

"Pessimistic"

.10% ORE GRADE, HIGH EXPLORATION COSTS

	LOW CAPITAL VARIABLE			HIGH CAPITAL VARIABLE	
	Low	High		Low	High
10%	21.14	27.20	10%	23.93	29.98
15%	23.69	29.74	15%	27.21	33.26

.10% ORE GRADE, LOW EXPLORATION COSTS

	LOW CAPITAL VARIABLE			HIGH CAPITAL VARIABLE	
	Low	High		Low	High
10%	17.49	23.55	10%	20.28	26.33
15%	19.22	25.27	15%	22.74	28.79

"Midrange"

.125% ORE GRADE, HIGH EXPLORATION COSTS

	LOW CAPITAL VARIABLE			HIGH CAPITAL VARIABLE	
	Low	High		Low	High
10%	16.91	21.76	10%	19.14	23.98
15%	18.95	23.79	15%	21.77	26.61

.125% ORE GRADE, LOW EXPLORATION COSTS

	LOW CAPITAL VARIABLE			HIGH CAPITAL VARIABLE	
	Low	High		Low	High
10%	13.99	18.84	10%	16.22	21.06
15%	15.38	20.22	15%	18.19	23.03

TABLE 6

(CONTINUED)

"Optimistic"

.15% ORE GRADE, HIGH EXPLORATION COSTS

	LOW CAPITAL VARIABLE Low	LOW CAPITAL VARIABLE High		HIGH CAPITAL VARIABLE Low	HIGH CAPITAL VARIABLE High
10%	14.09	18.13	10%	15.95	19.99
15%	15.79	19.83	15%	18.14	22.17

.15% ORE GRADE, LOW EXPLORATION COSTS

	LOW CAPITAL VARIABLE Low	LOW CAPITAL VARIABLE High		HIGH CAPITAL VARIABLE Low	HIGH CAPITAL VARIABLE High
10%	11.66	15.70	10%	13.52	17.55
15%	12.81	16.85	15%	15.16	19.19

estimate future costs. Similarly, for reasons discussed above, the calculations incorporating the low variable costs are not likely to be realistic. This leaves ranges of estimates of $27.20 to $33.26, $21.76 to $26.61 or $18.13 to $22.17, depending on whether a .10 percent, .125 percent or .15 percent ore grade is assumed.

The low and midrange figures are in basic agreement with those cited by representatives from two of the largest uranium producers in testimony before the JCAE in 1974. Douglas M. Johnson, President of United Nuclear Corporation, testified in a prepared statement:

> If the ore produced drops from a grade of 4 pounds per ton (about the U.S. average today) to 2 pounds per ton, the cost of production almost doubles and the selling price will also need to nearly double. To retain the same uranium production capacity, the investment in facilities will be much greater, although probably not double because there are some economies of scale available.
>
> With all of this, a price of $20.00 to $25.00 per pound, in today's dollars is a level we should expect to see by late in this decade if our nation is to be assured of adequate future reserves and production capacity.[40]

In response to Chairman Price's question about what price would be necessary to stimulate capital investment for new mining and milling facilities, he said:

My guess is that by the latter part of the decade to bring in the lower grade reserves that will be necessary for marginal producers we will need something in the neighborhood of $20 to $25, based on today's dollars.[41]

D. A. McGee, Chairman of the Board and Chief Executive Officer of Kerr-McGee Corporation, responded to Chairman Price's question in essentially the same way:

I think it is around $20 a pound in today's dollars. At this price, there will be economic incentives for both exploration and production.[42]

Other estimates and predictions include EEI's range of $18 to $21 by 1980, in 1975 dollars;[43] Joskow's estimate of $30 to $40, based partly on the EEI data;[44] and Vince Taylor's conclusion that future prices will be under $20 per pound, which is based largely on Klemenic's data.[45] These estimates basically bracket those which were derived in this paper.

In order to understand where current prices stand in relation to the various estimates, it is necessary to adjust for the inflation which has occurred since 1974. According to NUEXCO, the indices most frequently used in uranium sales contracts are the Bureau of Labor Statistics' Wholesale Price Index for Industrial Commodities and the Gross Average Hourly Earning of Production or Non-Supervisory Workers for Metal Mining, Non-Ferrous Metals, and Chemicals and Allied Products.[46]

By April 1977, the various indices had increased on the order of 30 percent over the 1974 averages and 13 percent to 18 percent over the 1975 averages.[47] Based on the increase over the 1974 indices, this implies that the estimates derived in this paper translate into ranges of $35.36 to $43.18, $28.29 to $34.59 and $23.57 to $28.82 for the .10 percent, .125 percent and .15 percent ore grades, expressed in 1977 dollars. These may be somewhat conservative increases, however, given that costs per foot for drilling were estimated to have increased by two-thirds since 1975 and are expected to continue to increase at a rapid rate, as is indicated in Table 7.

A price of $42.25 in mid-1977 is thus higher than all but the "pessimistic" range of prices derived in this paper. Thus, a tentative conclusion which can be drawn and one which is consistent with the slowing of the rate of increase of prices and the beginning of changes in contract terms in favor of the buyers, which were discussed above, is that recent prices are probably still somewhat above long-run levels, but that a large decline in prices should not be expected.

TABLE 7

COSTS PER FOOT
EXPLORATION DRILLING

YEAR	COST
1975	$2.90
1976	$3.13
1977	$4.80*
1978	$5.24*

SOURCE: NUEXCO report, 6/30/77.

*ERDA estimate.

APPENDIX ON PRODUCTION HORIZONS

1. To show that, under economies of scale, the production horizon which maximizes profits decreases as the size of the ore body increases:

Let X = size of the ore body, in units of output,

T = length of the production horizon,

q = rate of production,

C(q) = cost of mill with capacity to produce at rate q,

p = price of output,

v = variable cost of extraction and processing,

r = interest rate.

The firm's objective is to maximize the present value of its net cash flow:

$$\pi = \int_0^T (p\cdot q - v\cdot q)e^{-rt}dt - C(q). \qquad (1)$$

It is assumed that the firm produces at rate q throughout the entire production horizon and that at the end of the horizon the ore body is depleted:

$$\int_0^T q dt = X. \qquad (2)$$

The rate of output can thus be expressed as a function of the size of the ore body and the length of the production horizon:

$$q = \frac{X}{T}. \tag{3}$$

Substituting for q in (1) gives:

$$\pi = \int_0^T (p - v)\frac{X}{T} e^{-rt}dt - C(\frac{X}{T}). \tag{4}$$

The firm's problem is to choose the length of the production horizon (and, therefore, the mill capacity and rate of output) which maximizes π. The first-order condition is:

$$\frac{\partial\pi}{\partial T} = - \frac{X}{T^2}(p - v) \int_0^T e^{-rt}dt + \frac{X}{T}(p - v)e^{-rT} + \frac{X}{T^2}C' = 0. \tag{5}$$

Differentiating (5) with respect to T and X gives:

$$\frac{\partial^2\pi}{\partial T^2}dT = \{ - \frac{1}{X}\frac{\partial\pi}{\partial T} - \frac{X}{T^3}C''\} dX. \tag{6}$$

Assuming the second-order condition holds, $\frac{\partial^2\pi}{\partial T^2}$ is negative; and from (5), $\frac{\partial\pi}{\partial T}$ is zero. Therefore, $\frac{\partial T}{\partial X}$ is negative for $C'' < 0$ and zero for $C'' = 0$.

2. To show that, under economies of scale, the price which is just sufficient to cover long-run costs decreases as the size of the ore body increases, assuming that the firm acts to maximize profits.

Covering long-run costs implies:

$$\pi = \int_0^T (p - v)\frac{X}{T} e^{-rt} dt - C(\frac{X}{T}) = 0. \tag{7}$$

Solving for p from (7) gives:

$$p = \frac{\frac{T}{X} C \left(\frac{X}{T}\right)}{\int_0^T e^{-rt} dt} + v. \tag{8}$$

The derivative of price with respect to the size of the ore body, given that profits remain at zero and the production horizon is adjusted in a profit-maximizing manner, is:

$$\frac{dp}{dX} = \left.\frac{\partial p}{\partial T}\right|_{\pi = 0} \cdot \frac{\partial T}{\partial X} + \left.\frac{\partial p}{\partial X}\right|_{\pi = 0}. \tag{9}$$

From the previous problem, $\frac{\partial T}{\partial X} < 0$ for $C'' < 0$.

The partial derivative of p with respect to T in (8) is:

$$\left.\frac{\partial p}{\partial T}\right|_{\pi = 0} = \frac{\left(\frac{C}{X} - \frac{C'}{T}\right)\int_0^T e^{-rt} dt - \frac{T}{X} Ce^{-rT}}{\left(\int_0^T e^{-rt} dt\right)^2} \tag{10}$$

From the first-order conditions for profit mazimization:

$$C' = (p - v)[\int_0^T e^{-rt}dt - Te^{-rT}]. \qquad (11)$$

From (7):

$$C = \frac{X}{T}(p - v)\int_0^T e^{-rt}dt. \qquad (12)$$

Substituting in (10) for C and C',

$$\Big|_{\pi = 0} = \frac{\frac{1}{T}(p - v)\int_0^T e^{-rt}dt(\int_0^T e^{-rt}dt - Te^{-rT}) - \frac{1}{T}(p - v)(\int_0^T e^{-rt}dt - Te^{-rT})\int_0^T e^{-rt}dt}{(\int_0^T e^{rt}dt)^2} \qquad (13)$$

which equals zero.

Thus,

$$\frac{dp}{dX} = 0 \cdot \frac{\partial T}{\partial X} + \frac{\partial p}{\partial X}\Big|_{\pi = 0} = \frac{\frac{-T}{X^2}C + \frac{C'}{X}}{\int_0^T e^{-rt}dt}. \qquad (14)$$

The numerator in (14) can be written as $\frac{1}{X}(C' - \frac{T}{X}C)$. Assuming $C(0) = 0$, the sign of $(C' - \frac{T}{X}C)$ is negative, since for C strictly concave ($C'' < 0$), average cost $(C(\frac{X}{T}) \div \frac{X}{T})$ is greater than marginal cost $(C'(\frac{X}{T}))$ for $\frac{X}{T} > 0$. Thus, $\frac{dp}{dX} < 0$ for $C'' < 0$.

APPENDIX ON CALCULATIONS

EXPLORATION COSTS

The exploration costs cited in the text were first reduced by 24 percent, which is the assumed effective marginal tax rate, and then multiplied by $(1.1)^5$ to obtain effective exploration costs, capitalized to the beginning of the production period.

CAPITAL COSTS

The capital cost figures cited in the text are in terms of tons of ore per day. Production capacity was converted to pounds of U_3O_8 per year, assuming 300 days of operations per year and ore grade of .10 percent. The costs were then divided by the output per year, to determine the capital cost per pound of output per year. This is to be interpreted as the cost of building a facility capable of processing ore sufficient to produce one pound of U_3O_8 per year for ten years. This figure is then increased by 10 percent to account for interest payments during construction.

VARIABLE COSTS

The variable costs are already stated in terms of dollars per pound, the appropriate unit.

DETERMINATION OF PRICE

To obtain the estimate of long-run cost per pound, in the equation in the text, p was solved for, after setting q equal to 1.

Thus,

$$p = \left(\frac{1}{.95}\right)\left(\frac{[C(1) + E(10)]/\sum_{t=1}^{10} (1+r)^{-t} \quad - \quad .024C(1)}{.76} \quad + \quad v\right).$$

FOOTNOTES

1. Paul L. Joskow and Martin L. Baughman, "The Future of the U.S. Nuclear Energy Industry," The Bell Journal of Economics 7 (Spring 1976): 19.

2. Nuclear Exchange Corporation, "Significant Events in the Uranium Market: 1969-1976," report, October 15, 1976; reprinted in Nuclear News, December 1976, p. 46.

3. Nuclear Exchange Corporation, "NUEXCO Monthly Report to the Nuclear Industry," 107, Menlo Park, California, June 30, 1977.

4. U.S. Atomic Energy Commission, The Nuclear Industry, 1974, WASH 1174-74: 53.

5. John Klemenic, "Analysis and Trends in Uranium Supply," Uranium Industry Seminar, U.S. ERDA, Grand Junction, Colorado, October 1976, pp. 264-265.

6. Nuclear Industry, 1974, pp. 44-45.

7. Nuclear Industry, 1974, p. 53.

8. Klemenic, "Analysis and Trends in Uranium Supply," p. 232.

9. If the marginal cost is not constant over time, then the change in the shadow price is a function of the change in the marginal cost as well as the rate of interest.

10. William D. Nordhaus, "The Allocation of Energy Resources," Brookings Papers on Economic Activity 3, 1973: 529-576.

11. Nordhaus, "The Allocation of Energy Resources," pp. 544-554.

12. It is likely that, should there be periods of slack demand in the future, storage would take place after production, rather than in the mine, since there are problems with damage from flooding in inactive mines.

13. NUEXCO, Monthly Report 107, p. 8.

14, Klemenic, "Analysis and Trends in Uranium Supply," p. 254.

15. United States Atomic Energy Commission, Statistical Data of The Uranium Industry 1974, GJO-100 (74), Grand Junction, Colorado.

16. United States, Congress, Senate, Amending the Atomic Energy Act of 1954 to Provide for Private Ownership of Special Nuclear Materials,

S. Rept. 1325 to Accompany S.3075, 2d Session, 1964. An additional provision of the act was a prohibition of enrichment of foreign uranium intended for use in domestic reactors. This provision was apparently designed primarily to protect the domestic uranium industry from foreign competition until a domestic commercial market developed. The prohibition remained in effect until the following schedule, allowing a gradual introduction of foreign uranium, was adopted in 1974. Feed material for enrichment by a customer for use in a domestic reactor may not exceed 10 percent in 1977, 15 percent in 1978, 20 percent in 1979, 30 percent in 1980, 40 percent in 1981, 60 percent in 1982, 80 percent in 1983, with no restrictions thereafter.

17. U.S. Energy Research and Development Administrator, *Statistical Data of the Uranium Industry* 1976, GJO-100 (76), Grand Junction, Colorado.

18. Paul L. Joskow, "Commercial Impossibility, the Uranium Market and the Westinghouse Case," *Journal of Legal Studies* VI, (January 1977): 125-143.

19. For a discussion of this point of view, see Vince Taylor, *How the U.S. Government Created the Uranium Crisis (and The Coming Uranium Bust)*, Pan Heuristics, Los Angeles, June 1977 (preliminary).

20. Joskow, "Commercial Impossibility, the Uranium Market and the Westinghouse Case." Westinghouse's market position is analyzed in detail by Joskow.

21. Joskow, p. 131-138.

22. Forward costs, as defined by the AEC, are simply any costs of producing which have not yet been borne by the producer. Therefore, such items as exploration and development costs are excluded in cases where exploration and development drilling has already occurred. Taxes are not included in forward cost, either.

23. NUEXCO, Monthly Report 107, p. 14.

24. S.M. Stoller Corporation, "Report on Uranium Supply, Task III of EEI Nuclear Fuels Supply Study Program," Appendices to the Report of the Edison Electric Institute on Nuclear Fuels Supply, (New York: Edison Electric Institute, [1976]), p. 101.

25. Battelle Pacific Northwest Laboratories, Assessment of Uranium and Thorium Resources in the United States and the Effect of Policy Alternatives, (Richland, Washington, [1974]), p. 7.26.

26. S.M. Stoller Corporation, "Report on Uranium Supply, Task III of EEI Nuclear Fuels Supply Study Program," p. 87.

27. United States, Congress, Joint Committee on Atomic Energy, Future Structure of the Uranium Enrichment Industry, Hearings before the Joint Committee on Atomic Energy, 93d Cong., 2nd Sess., 1974, Part 3: Vol. I, Phase III: AEC, Industry, and Finance Witnesses, pp. 295-310.

28. S.M. Stoller Corporation, "Report on Uranium Supply, Task III of EEI Nuclear Fuels Supply Study Program," p. 92.

29. Klemenic, "Analysis and Trends in Uranium Supply," pp. 249, 267.

30. Hearings, p. 299.

31. Klemenic, "Analysis and Trends in Uranium Supply," p. 263.

32. S.M. Stoller Corporation, "Report on Uranium Supply, Task III of EEI Nuclear Fuels Supply Study Program," p. 92.

33. Klemenic, "Analysis and Trends in Uranium Supply," p. 265.

34. S.M. Stoller Corporation, "Report on Uranium Supply, Task III of EEI Nuclear Fuels Supply Study Program," p. 92.

35. Klemenic, "Analysis and Trends in Uranium Supply," pp. 249, 267.

36. Joskow, "Commercial Impossibility, the Uranium Market and the Westinghouse Case," p. 122.

37. Klemenic, "Analysis and Trends in Uranium Supply," p. 265.

38. Klemenic, "Analysis and Trends in Uranium Supply," pp. 236, 249, 254. Conclusions about market prices should not be drawn at this point, however, since forward costs exclude not only sunk costs, but also interest, taxes, and profits.

39. Hearings, pp. 317, 330 and Joskow, "Commercial Impossibility, the Uranium Market and the Westinghouse Case," p. 148.

40. Hearings, p. 317.

41. Hearings, p. 330.

42. Hearings, p. 337.

43. S.M. Stoller Corporation, "Report on Uranium Supply, Task III of EEI Nuclear Fuels Supply Study Program," pp. 100-102.

44. Joskow, "Commercial Impossibility, the Uranium Market and the Westinghouse Case," p. 149.

45. Vince Taylor, The Myth of Uranium Scarcity, Pan Heuristics, Los Angeles, April 1977, p. 28.

46. NUEXCO, Monthly Report 107, p. 1.6.

47. NUEXCO, Monthly Report 107, p. 4.

CHAPTER 2

FIXED-PRICE CONTRACTS AND THE THEORY OF THE FIRM UNDER UNCERTAINTY

Introduction

Firms typically are uncertain about future product or input prices. Nevertheless, to produce output for sale in future periods, a firm may have to decide in advance on the quantities of, for example, equipment or structures to employ when significant installation or construction times are required. Firms which must make such advance commitments may seek to secure contracts which guarantee the purchase or sale of specific quantities of an input or output at a given price. This is especially true of firms which are risk averse. The primary reason a risk averse firm would desire forward contracts is that the variation in net revenues is thereby reduced and, over some range of contract prices, the expected utility of profits is increased. This is true for both buyers and sellers of the product. Thus, when there exist contract prices which enable expected utilities to be increased on both sides of the market simultaneously, there is reason to believe that a forward market will exist.

Participation in a forward market does more than simply alter the distribution of the firm's profits for given production choices, however. Since the distribution of profits is changed, the incentives are also changed, and firms may respond by making different input or total production choices. It is the objective of this paper to model the behavior of the firm faced with uncertainty and able to take part in a forward market.

Both the theory of the firm under uncertainty and the theory of optimal forward positions have been developed in a number of recent articles. The analyses are similar in that in both, the assumptions are made that agents are risk averse and make decisions in the current period in order to maximize the expected utility of profits in a future period. However, the theory of optimal forward positions has been developed primarily to explain transactions in currency markets. Therefore, the effects of forward markets on production decisions have not yet been considered. By incorporating the main features of the two theories, this paper extends the theory of the firm under uncertainty to consider those effects. Before turning to the discussion of forward contracting by firms, the major developments in the theories of the firm under uncertainty and optimal forward positions will be reviewed.

Theory of the Firm Under Uncertainty

Models of the firm under uncertainty share the assumption that some or all of the decisions of the firms must be made before the uncertainty is resolved. The firm is also assumed to possess a subjective probability distribution over prices, or whatever parameters are subject to uncertainty; in one of the models, the firm is assumed to possess a degree of monopoly power so that it faces an uncertain demand function, rather than simply an uncertain price. A somewhat more controversial assumption is that the firm behaves as though it has a von Neumann-Morgenstern utility function over profits. This is a problem if decisions are made by committees rather than individuals because of the difficulty of insuring the existence of a group utility function. The approach might be justified, however, by the fact that in many firms decisions are made by individuals, or by claiming that such a utility function is simply an approximation of reality.

One of the original papers on the theory of the firm under uncertainty is by Sandmo (1971). The model developed in that paper assumes that the firm is a price-taker, knows with certainty its cost function, and must commit itself to the quantities of all inputs it will employ before observing the output price. Given these assumptions, one of the results is that, under price uncertainty, the quantity of output which maximizes the expected utility of profits will be smaller than the quantity which would be produced under certainty if the certain price were equal to the mean of the distribution of prices.

Sandmo also suggests looking at the effect of stretching the probability distribution about the mean, i.e., making the distribution of prices "riskier". This is accomplished by introducing two shift parameters in the probability distribution, one multiplicative and one additive. Increasing risk about a constant mean is a matter of multiplying prices by a constant greater than one and simultaneously subtracting a positive constant from each price. Ishii (1977) has been able to show that such an increase in uncertainty reduces the expected utility-maximizing output of the firm.

By further restricting the firm's utility function so that Arrow's (1965) measure of absolute risk aversion (-U"/U') is a decreasing function of income, Sandmo finds that an increase in fixed costs (or what is the same thing, a lump sum tax) also decreases the firm's output.

Leland (1972) has extended Sandmo's basic model to consider the more general case of the firm which faces a generalized uncertain demand function. Sandmo's results on reduced output under uncertainty and the effect of increased fixed costs are found to hold in the general case. Additionally, firms with the flexibility to determine price or output after observing the actual demand curve will prefer to do so to setting both price and output *ex ante*.

Batra and Ullah (1974) examine the case of a competitive, risk averse firm which employs two inputs in the production of its output and which must choose the quantities of both inputs before observing the price it will receive for its output. Results claimed with this model include: 1) a risk averse firm will choose smaller quantities of inputs and output compared to the firm facing a certain output price equal to the mean of the price distribution, and 2) for a firm with a utility function which has absolute risk aversion decreasing in income, an increase in uncertainty reduces the firm's output. However, Hartman (1975) has shown that the first of these conclusions is incorrect with respect to the inputs.

Coes (1977), in the context of the models proposed by Sandmo and Leland, has shown that a sufficient condition for an increase in demand uncertainty to decrease output is that absolute risk aversion be non-increasing.

The theory of the two-input, competitive firm facing output price uncertainty has been extended by Hartman (1976) and Turnovsky (1973) to allow for adjustment of one of the inputs after observing the output price. The two approaches differ in that Hartman assumes that the level of the variable input is chosen after observing the selling price, while Turnovsky, using a cost-function rather than a production function, assumes that the firm must make its basic production plan before observing the selling price, but that after observing it, modifications can be made, subject to additional adjustment costs. It turns out that little can be said about the firm's decisions using the latter approach, other than that the production plans and output will, in general, be different if the firm possesses *ex post* flexibility than if it does not.

No unambiguous results are obtained in Hartman's analysis, either. However, the model is one which explicitly takes a dynamic programming approach and is essentially the one used in this paper to analyze the effects of forward contracting.

Finally, in his survey on probabilistic microeconomics, McCall (1972) includes futures contracts under the section on insurance, noting only that

> The existence of futures contracts permits the farmer or food processor to specialize in production, while the speculator specializes in risk bearing.

Optimal Forward Positions

Feldstein (1968) is the first author to explicitly analyze forward exchange positions using the assumption that traders act to maximize expected utility. The analysis was specifically designed to investigate behavior under uncertainty in foreign currency exchange markets. The rate of exchange in such markets is the price, in foreign currency, of one unit of domestic currency. The spot rate is the current rate of exchange and the forward rate is the rate specified in contracts to deliver currency at a given future date.

Traders are assumed to have a subjective probability distribution over future spot rates and a risk-averse utility function over net assets at the end of any trading period. The increase (decrease) in net assets resulting from participation in a given forward market is the excess of the contract rate for delivery at a future date over the spot rate prevailing at that date times the number of units of currency the trader has contracted to deliver (receive). Given the probability distribution over future spot rates and the utility function for the trader, there is associated with each number of contracts to be delivered or received an expected value of the utility function. The trader is assumed to contract to deliver or receive the number of units which maximizes that expected value.

Feldstein has restricted his analysis to consider only those combinations of utility functions and subjective probability distributions which allow expected utility to be expressed as a function of the mean and variance of the distribution alone. Additionally, it is assumed that expectations may depend, in part, on the forward rate, and that they are, in the terms of the literature, inelastic with respect to the forward rate. As used in the exchange literature, this means that when the forward rate increases, any increase in the distribution which results leaves the mean increased by less than the change in the forward rate.

The conclusions of this model for the case of a single foreign currency are: (1) An increase in expected gain per unit of speculation with constant variance is likely to increase speculation but may decrease it. (2) An increase in expected gain per unit of speculation with constant relative variance decreases speculation. (3) An increase in the variance of the future spot rate with constant mean is likely to decrease speculation but may increase it.

For the case of multiple exchange opportunities, the conclusions are: (1) For any set of expected gains and variances per unit of exchange, the relative sizes of the optimum forward position in each currency are independent of the overall forward position. (2) Even if each forward rate exceeds its expected spot rate, some currencies may be bought forward. (3) An increase in expected gain for one currency is likely to increase speculation in that currency, but may decrease it. (4) An increase in one expected gain with constant relative risk decreases both the optimum total speculation and the relative share of that currency. (5) An increase in one variance with constant expected gain is likely to decrease speculation.

Leland (1975) has generalized Feldstein's analysis by relaxing the restriction that expected utility be expressed as a function of the mean and variance alone and by considering alternative investment opportunities. In his analysis, traders are assumed to have utility functions over terminal wealth and subjective probability distributions over spot prices, with the only restriction being the concavity of the utility functions. Feldstein's results for the case of one forward market are found to generalize to the unrestricted case. (1) Increases in the expected gains of a unit of speculation, with all other moments constant, probably increases speculation, but may decrease it. Leland has shown that the effect of such an increase can be divided into a positive substitution effect and a positive, zero, or negative income effect, depending on whether absolute risk aversion increases, is constant, or decreases as a function of terminal wealth. (2) A proportional change in all possible per unit profits or losses, corresponding to Feldstein's constant relative variance case, changes speculation in inverse proportion to the magnitude of the change. (3) "Stretching" the distribution about a constant mean is likely to decrease speculation, but in some cases may increase it. Feldstein's results about multiple markets are similarly extended.

In a somewhat different context, Baron (1976) has extended the forward exchange literature to a general equilibrium model of international trade under exchange rate uncertainty. In his model, both firms and investors participate in spot and forward exchange markets in the various currencies and the investors also participate in capital asset markets for firm shares. He has shown that, at an investment equilibrium, all investors are indifferent to changes in the forward positions of firms, since, for any changes in those forward positions, the investors may rearrange their own forward positions to obtain distributions of returns identical to those in effect before the changes by the firms. Consequently, there do not exist unique equilibrium values for the amounts of foreign exchange "covering" firms will choose.

An analogue to such a general equilibrium model of international trade exists in the analysis of forward contracting for the output of firms. That analogue involves the explicit introduction of investors who, along with the firms, participate in the spot and forward markets for the goods which firms produce, as well as in the capital asset markets for firm shares. A result in this model which is analogous to the indifference by investors to the covering decisions of firms in the foreign exchange markets is that investors are indifferent to the covering decisions by firms in the forward contract markets. While there are investors who do participate in forward markets for certain goods, such as those represented on commodity exchanges, there are also many other investors who do not participate in such markets and many markets which have as their primary or only participants the firms which produce or utilize the products traded. Little or no participation by individual investors in forward markets may result from such factors as information and transactions costs and imperfections in capital markets. Whatever the reasons, however, the investors in firms participating in such incomplete markets are presumably not, in general, indifferent to the forward positions of those firms. Therefore, while there are the usual difficulties associated with aggregating the preferences of more than one investor, if we model firm choices as resulting from the maximization of a single risk averse utility function, as is typically done in analyzing the behavior of individual firms under uncertainty, the forward positions of those firms will be uniquely determined.

Forward Contracting and Production Decisions

Participation by the firm in a forward contracts market requires it to make simultaneously the two kinds of decisions which were analyzed in the theory of the firm under uncertainty and the theory of optimal forward positions. Consider the firm which is uncertain about the spot price which it will receive for its product in a future period, yet must make some of its input decisions before observing that spot price. If there is a forward market for its product, it must decide, before observing the spot price, how much of those inputs to employ and how much output to commit to the contract market. This paper models the decisions under uncertainty of firms which engage in contracting and make some or all of their input decisions before the uncertainty is resolved.

In the models of contracting developed here, as in the models of the firm under uncertainty and the models of forward positions, it will be assumed that the firm possesses a subjective probability distribution over the spot price and that it has a risk-averse utility function.

These assumptions imply that it is about to associate with each possible combination of decisions about production and contract positions an expected utility. It is then assumed that the firm chooses to implement that combination which gives the highest expected utility. In the case of the firm which possesses some degree of flexibility to make adjustments after observing the actual spot price, the dynamic programming assumption is made that a firm will make decisions after observing the spot price to maximize short-run profits, given the values of the decisions it made before observing the spot price and cannot adjust. Further, in making the decisions it cannot adjust after observing the spot price, it will take into account the effects on the short-run decisions in such a way that expected utility is maximized.

Among the effects to be considered are those a contract price increase would have on the decisions of the firm. At this stage in the analysis, it will be assumed that changes in the contract price will not alter the firm's subjective distribution over spot prices. Clearly, this cannot literally be true; firms probably form price expectations based on information from many sources, certainly not the least of which are forward prices. However, there are justifications for assuming independence at this stage. One is that it makes the analysis much less complex. Another and perhaps more compelling reason is that we are interested here in the pure economic effects of the existence of and changes in forward markets, as distinct from the informational effects. Future research efforts may, however, be appropriately directed toward relaxation of the independence assumption and consideration of what may be the full impact of forward price changes on firm decisions.

A couple of additional caveats merit mention here. Both have to do with the limitation of the analysis to what is essentially one period; investment and contracting decisions are made at the beginning of the period and production and transactions are carried out at the end of the period, after the random variables have been observed. The first difficulty with this is that investments of a fixed nature are rarely intended to last for only one production period. While the analysis could be made using a multi-period dynamic programming approach, which would perhaps more closely approximate the true context of investment decisions, it is doubtful that this would provide significant improvements in insight, while the complexity would be greatly increased. It is perhaps better to view the production portion of the single period as a somewhat less good approximation to the productive lifetime of the investment. The second difficulty is the lack of a provision for carrying output as inventory into future periods. Again, to do so would require a large increase in complexity, which may be appropriate in future research. It should be pointed out, however, that the inventory problem is not unique to the analysis of contracting by the firm. It is shared by the current state of the theory of the firm under uncertainty without contracting. Since consideration of contracting does not necessarily make the issue of inventory carry-over more pressing, it may be that the extension of the theory in the direction of inventory decisions should first be made without the added complication of contract decisions.

We now turn to the models of firms engaged in forward contracting. The first section models a firm which makes all of its production decisions before the spot price is observed and may contract to sell some or all of its output at a fixed price. This model provides an introduction to the analysis of contracting decisions without the modeling complexity necessitated by the assumption that input adjustments may be made after the spot price is observed. The second section allows for such ex post flexibility and develops the analysis utilizing a dynamic programming model. In the third section we turn to an analysis of a firm which may contract for a variable input before observing the spot price. The model developed in that section also assumes ex post flexibility and is a demand side analogue of the supplying firm in the second section. Finally, the fourth section assumes that the output of the firm modeled in the second section is an intermediate good employed as a variable input by the firm modeled in the third section. That section considers the equilibrium conditions in the contract market for the intermediate good and compares the decisions made by the contracting firms with those which would be made by a vertically integrated firm.

I. CONTRACTING UNDER PRODUCT PRICE UNCERTAINTY WITHOUT EX POST FLEXIBILITY

In this model it is assumed both that the firm is uncertain about the price which will prevail for its product in the spot market and that it does not have the ability to adjust its production decisions once the spot price has been observed. The firm does, however, have the option of contracting to sell a quantity of its output at a fixed price before observing the spot price. The spot and contract markets in which the firm participates are assumed to be competitive, meaning that the firm perceives the prices in those markets as parameters which it is not able to influence by its decisions.

The notation employed and the further specific assumptions made are summarized as follows:

IA) The spot and contract prices are p and P, respectively. Prices are non-negative and the firm has a subjective probability density function over p.

IB) Spot and contract sales are denoted by q and Q respectively. The firm is free to set q and Q at will before observing p.

IC) Output and the cost of output are denoted by X and $C(X)$, respectively. The cost function is convex, with C' and C'' both positive.

ID) The utility function over profits, π, is $\phi(\pi)$ and is strictly concave, with ϕ' positive and ϕ'' negative.

IE) The expected value of the spot price is greater than the marginal cost of production at zero output: $E\{p\}>C'(0)$.

For portions of the analysis which follows, it will also be assumed that

IF) The contract price is greater than or equal to the expected value of the spot price: $P \geq E\{p\}$.

The firm's profit for a given spot price is the sum of spot and contract revenues, minus the cost of production:

$$\pi = p\,q + P\,Q - C(X) \tag{1}$$

Production is the sum of spot and contract sales:

$$X = q + Q \tag{2}$$

Substituting for q from (2) into (1), it can be seen that the firm's problem is reduced to choosing X and Q to maximize the expected utility of profits:

$$E\{\phi(\pi)\} = E\{\phi(p(X-Q) + PQ - C(X))\} \tag{3}$$

The first order conditions for a maximum are

$$\frac{\partial\{E\phi(\pi)\}}{\partial Q} = E\{\phi'(P - p)\} = 0 \tag{4}$$

$$\frac{\partial\{E\phi(\pi)\}}{\partial X} = E\{\phi'(p - C')\} = 0. \tag{5}$$

Assumption IE) insures that the firm will, in the absence of contracting, choose to produce a positive amount of output. To show this, we note that the derivative of the expected utility of profits with respect to output, for Q equal to zero, and evaluated at X equal to zero, is $E\{\phi'(-C(0))\cdot(p-C'(0))\}$. Here, marginal utility is a function of a constant, $(-C(0))$, and is, therefore, itself, a constant. Since, by ID), ϕ' is positive, the derivative of the expected utility of profits with respect to X, at Q and X equal to zero, is positive, zero, or negative as $E\{p\}$ is greater than, equal to, or less than $C'(0)$. Thus, assumption IE), that $E\{p\} > C'(0)$, implies that the expected utility of profits for the firm reaches a maximum at a value of X greater than zero. Having insured that the firm we are considering will exist, that is, choose to produce a positive amount of output, we now turn to the investigation of the comparative statics of the decisions of that firm.

Totally differentiating (4) and (5) gives

$$\begin{bmatrix} E\{\phi''(P-p)^2\} & E\{\phi''(P-p)(p-C')\} \\ E\{\phi''(P-p)(p-C')\} & E\{\phi''(p-C')^2-\phi'C''\} \end{bmatrix} \begin{bmatrix} dQ \\ dX \end{bmatrix} = \begin{bmatrix} -E\{\phi''(P-p)Q+\phi'\} \\ -E\{\phi''(p-C')Q\} \end{bmatrix} \begin{bmatrix} dP \end{bmatrix} \quad (6)$$

The second order conditions are that $\Delta =$
$E\{\phi''(P-p)^2\}\cdot E\{\phi''(p-C')^2-\phi'C''\} - E\{\phi''(P-p)(p-C')\}\cdot E\{\phi''(P-p)(p-C')\}$
be positive and that $E\{\phi''(P-p)^2\}$ and $E\{\phi''(p-C')^2-\phi'C''\}$ be negative. The latter two conditions may be verified directly by recalling IC) and ID). Given IC) and ID), it can be shown as follows that the former condition is also satisfied. First, note from (4) and (5) that $E\{\phi'p\} = P\cdot E\{\phi'\} = C'\cdot E\{\phi'\}$. This implies that $P = C'$. Substituting P for C' in (6) allows the second order conditions to be written as $-E\{\phi'(P-p)^2\}\cdot E\{\phi'C''\} > 0$, which is satisfied, since ϕ'' is negative and $(P-p)^2$, ϕ' and C'' are all positive.

Given the model outlined above, we can now examine the effects on the firm's decisions of both a contract price change and the introduction of the contracting market itself. The effect of a contract price increase on the quantity of output produced is found by solving for $\frac{\partial X}{\partial P}$ from (6):

$$\frac{\partial X}{\partial P} = \frac{1}{\Delta}\,[E\{\phi''(P - p)^2\} \cdot E\{\phi''(P - p)Q\} - E\{\phi''(P - p)^2\}E\{\phi''(P - p)Q+\phi'\}]$$

$$= \frac{1}{\Delta}\,[-E\{\phi''(P - p)^2\cdot E\{\phi'\}] \qquad (7)$$

Given the assumptions in ID), and using the fact that Δ is positive, $\frac{\partial X}{\partial P} > 0$, we can state the following result.

<u>Proposition 1</u>: Given a strictly convex cost function and a strictly concave utility function over profits, a competitive, expected utility-maximizing firm facing uncertainty over output spot prices, unable to adjust production decisions after observing the spot

price and participating in a competitive forward contract market, will increase its total output as the result of an increase in the contract price.

We will now consider the effect of an increase in the contract price on the quantity of contracts the firm chooses to sell. From (6):

$$\frac{\partial Q}{\partial P} = \frac{1}{\Delta}[-E\{\phi''(P-p)Q+\phi'\}\cdot E\{\phi''(P-p)^2-\phi'C''\}+ E\{\phi''(P-p)Q\}\cdot E\{\phi''(P-p)^2\}]$$

$$= \frac{1}{\Delta}[Q\cdot E\{\phi''(P-p)\}\cdot E\{\phi'C''\}+E\{\phi'\}E\{\phi'C''\}-E\{\phi'\}E\{\phi''(P-p)^2\}] \qquad (8)$$

Note that Q can be placed outside the expectation because it does not vary with p. Two of the three terms in the numerator can be signed without further assumptions. Since ϕ' and C" are both positive, $E\{\phi'\}E\{\phi'C''\}$ is positive, and since ϕ'' is negative and $(P-p)^2$ is positive, $E\{\phi'\}\cdot E\{\phi''(P-p)^2\}$ is negative.

The sign of the term $Q\cdot E\{\phi''(P-p)\}\cdot E\{\phi'C''\}$ depends on the signs of Q and $E\{\phi''(P-p)\}$, since $E\{\phi'C''\} > 0$. Q is not restricted to be non-negative and it may be that a firm would choose to purchase output for resale on the spot market if the contract price were low enough. Therefore, the cases where Q is positive, negative and zero must be considered individually.

The sign of $E\{\phi''(P-p)\}$ can be shown to depend on how the firm's attitude toward risk-taking changes as its wealth changes. To formalize how the firm's attitude toward risk changes, we introduce the concept of absolute risk aversion, which was developed by Arrow (1965). The measure of absolute risk aversion is defined as $R_A = \frac{-\phi''(\pi)}{\phi'(\pi)}$. Operationally, R_A is positively related to the amount a firm is willing to pay to avoid a given "fair" bet. If R_A is an increasing (constant) (decreasing) function of π, that amount increases (is constant) (decreases) as π increases.

If we restrict the analysis to utility functions for which $\frac{\partial R_A}{\partial \pi}$ remains positive, zero or negative over their entire ranges, then with the addition of one assumption, $E\{\phi'(P-p)\}$ can be signed for each of these types of utility function. The additional assumption is that the firm does not assume a short position in the market, i.e., that it does not contract to sell more than it intends to produce, thereby being required to purchase the difference on the spot market. If, however, a firm were to assume a short position, it is apparent that the results of the following derivations would simply have the opposite signs.

To sign $E\{\phi''(P-p)\}$, first consider $E\{\phi'(P-p)\}$. From (4), this expectation is known to be zero. We define Z to be a positive constant such that at $p = P$, $-\phi''Z = \phi'$. When absolute risk aversion does not vary with π, $\frac{-\phi''}{\phi'} = \frac{1}{Z}$ for all spot prices and

$$E\{\phi''(P-p)\} = E\{\frac{-\phi'}{Z}(P-p)\} = \frac{-1}{Z} E\{\phi'(P-p)\} = 0.$$

Thus, from (8)

$$\left.\frac{\partial Q}{\partial P}\right|_{\substack{\text{constant}\\ \text{absolute}\\ \text{risk aversion}}} = \frac{E\{\phi'\}\cdot E\{\phi' C''\} - E\{\phi'\}\cdot E\{\phi''(P-p)^2\}}{\Delta} > 0. \quad (9)$$

We can now state the following proposition.

Proposition 2: Given the assumptions stated in Proposition 1 and assuming that the firm's utility function has the property that absolute risk aversion does not vary with income, an increase in the contract price will result in the firm choosing to sell a greater quantity of contracts.

Before considering the cases where absolute risk aversion is increasing or decreasing, we note that the assumption that Q is not greater than X implies profit is a nondecreasing function of the spot price, since, from (1) and (2):

$$\frac{\partial \pi}{\partial p} = X - Q \geq 0 \text{ for } X \geq Q. \quad (10)$$

Under decreasing absolute risk aversion, if $\frac{\partial \pi}{\partial p} > 0$, $\frac{-\phi''}{\phi'} < \frac{1}{Z}$ for p < p and $\frac{-\phi''}{\phi'} > \frac{1}{Z}$ for p < P. Since (P-p) < 0 for p > P and (P-p) > 0 for p < P, $-\phi''(P-p)Z > \phi'(P-p)$ for all p not equal to P; if p=P, they are both zero. Therefore $E\{-\phi''(P-p)Z\} > E\{\phi'(P-p)\} = 0$ and $E\{\phi''(P-p)\} \leq 0$, with strict inequality for $p \neq P$. If $\frac{\partial \pi}{\partial P} = 0$, i.e., when X = Q, $\frac{-\phi''}{\phi'}$ does not vary with p and $E\{\phi''(P-p)\} = 0$. Under increasing absolute risk aversion with $\frac{\partial \pi}{\partial p} > 0$, $\frac{-\phi''}{\phi'} > \frac{1}{Z}$ for p > P and $\frac{-\phi''}{\phi'} < \frac{1}{Z}$ for p < P. Therefore, $-\phi''(P-p)Z < \phi'(P-p)$ for all p not equal to P, and $E\{\phi''(P-p)\} > 0$.

We can now summarize the various cases where the term $Q \cdot E\{\phi''(P-p)\} \cdot E\{\phi' C''\}$ from (8) may be signed. It is negative when: Q is positive and there is decreasing absolute risk aversion with $Q < X$; Q is negative and there is increasing absolute risk aversion with $Q < X$. It is zero when: Q is zero; $Q = X$; there is constant absolute risk aversion. It is positive when: Q is positive and there is increasing absolute risk aversion; Q is negative and there is decreasing absolute risk aversion. Clearly, only in the cases where the term is zero or positive can $\frac{\partial Q}{\partial P}$ be signed unambiguously. This result can be summarized in the following proposition:

<u>Proposition 3</u>: Given the assumptions stated in Proposition 1, an increase in the contract price will result in the firm choosing to sell a greater (purchase a smaller) quantity of contracts when any of the following hold: (1) Q is zero; (2) $Q = X$; (3) Q is positive and there is increasing absolute risk aversion; (4) Q is negative and there is decreasing absolute risk aversion; (5) there is constant absolute risk aversion.

While it is not possible to sign $\frac{\partial Q}{\partial P}$ in every case, we can identify two distinct types of effects which a change in the contract price has on the quantity of contracts sold. The first of these reflects the change in the relative expected profitability of contract sales versus spot sales when the contract price changes. The second reflects the effect on the firm's willingness to take risks when its wealth position is changed.

To illustrate the second type of effect, we introduce a shift parameter, α, which represents additional certain income to the firm. Let α have an initial value of zero so that increases in it represent lump-sum additions to the certain income of the firm. The firm's utility function can be written as $\phi(\pi+\alpha)$. Differentiating (4) and (5) with respect to Q, X, and α gives, after substituting P for C',

$$\begin{bmatrix} E\{\phi''(P-p)^2\} & -E\{\phi''(P-p)^2\} \\ -E\{\phi''(P-p)^2\} & E\{\phi''(P-p)^2-\phi'C''\} \end{bmatrix} \begin{bmatrix} dQ \\ dX \end{bmatrix} = \begin{bmatrix} -E\{\phi''(P-p)\} \\ E\{\phi''(P-p)\} \end{bmatrix} d\alpha \qquad (11)$$

Thus, the effect of an increase in certain income on the supply of contracts is

$$\frac{\partial Q}{\partial \alpha} = \frac{E\{\phi''(P-p)\} \cdot E\{\phi'C''\}}{\Delta} \qquad (12)$$

Now, substituting from (12) and (9) into (8),

$$\frac{\partial Q}{\partial P} = \frac{\partial Q}{\partial \alpha} \cdot Q + \left.\frac{\partial Q}{\partial P}\right|_{\substack{\text{constant} \\ \text{absolute} \\ \text{risk aversion}}} \qquad (13)$$

For a given number of contracts outstanding, the partial derivative of the certain income of the firm with respect to P is simply Q, since the certain income is P·Q. Thus, $\left(\frac{\partial Q}{\partial \alpha} \cdot Q\right)$ can be interpreted as the effect of an increase in certain income on the number of contracts sold times the effect of a price increase on the amount of the firm's certain income, and might well be termed the "income" effect. $\left.\frac{\partial Q}{\partial P}\right|_{\substack{\text{constant}\\ \text{absolute}\\ \text{risk aversion}}}$ is the effect that a price increase has on the number of contracts sold in the absence of any change in the firm's willingness to take risks. It might be termed the "substitution" effect, since it represents substitution by the firm of contract sales for spot sales as the result of a price change.

These interpretations of the ways a constant price increase influences the quantity of contracts sold are summarized in the following proposition:

Proposition 4: Given the assumptions of Proposition 1, the change in the number of contracts sold as the result of a change in the contract price is the sum of an income effect, which results from changes in the attitude of the firm toward risk taking as a result of changes in the certain income of the firm, and a substitution effect, which results from the change in the expected profitability of contract sales relative to spot sales as the contract price changes. The income effect is of indeterminate sign and the substitution effect is always positive.

Proposition 4 is a result which is analogous to the demonstration by Leland (1975), in his analysis of optimal positions in forward currency markets, that the effects of parameter changes on optimal forward positions can be divided into "income" and "substitution" effects. Further, he also showed that the signs of the substitution effects are determinate, while the signs of the income effects depend on whether absolute risk aversion is an increasing or decreasing function of income. These results correspond to Proposition 3 and the discussion preceding it.

We turn now to an investigation of the effect the existence of a contract market has on the production decision of the firm. Up to this point, it has been assumed that the firm could either purchase or sell contracts for output, and, indeed, it is conceivable that situations might arise in which a firm would desire to purchase contracts in anticipation of reselling the product at a (presumably) higher spot price. However, the typical case must certainly be that the producing firm would desire to sell contracts, and it is that case which is of most interest when speaking of the effects of introducing a contract market where none existed previously. We therefore introduce an assumption which is a sufficient (but not necessary) condition for the firm to choose to sell a positive number of contracts. That assumption is IF), which is that the contract price is at least as great as the expected value of the spot price. This is certainly in accord with intuition, since a definition of risk aversion is that receiving the mean of a risky distribution is preferred to being subjected to the distribution itself.

That IF), in conjunction with the other assumption, insures that the firm will choose to sell a positive number of contracts can be demonstrated as follows. First note that the derivative of the expected utility of profits with respect to Q is $E\{\phi'\cdot(P-p)\}$, which can also be written as $E\{\phi'\}\cdot P-E\{\phi'\cdot p\}$. If the contract price, P, is equal to the expected value of the spot price, the derivative is $E\{\phi'\} \cdot E\{p\} - E\{\phi' p\}$, which is equal to the to the negative of the covariance of ϕ' and p, since $COV(a,b) = E\{a \cdot b\} - E\{a\} \cdot E\{b\}$. When Q is zero, the derivative of ϕ' with respect to p is $\phi'' \cdot X$, which is negative, since by ID), ϕ'' is negative and, as we have shown, IE) implies that X is positive. Therefore, at Q equal to zero, the covariance of ϕ' and p is negative and $E\{\phi'\} \cdot E\ p - E\{\phi' \cdot p\}$ is positive. Since ϕ' is positive, this expression will also be positive for values of P greater than $E\{p\}$. Since $E\{\phi'\} \cdot P - E\{\phi' \cdot p\}$ is the derivative of expected utility of profits with respect to Q and is positive at Q equal to zero for all values of P greater than or equal to $E\{p\}$, the expected utility of profits will attain a maximum at a value of Q greater than zero when the contract price is at least as great as the mean of the spot price. It should be emphasized here that $P \geq E\{p\}$ is a sufficient condition for Q to be positive, not a necessary one. Since Q is strictly positive at $P = E\{p\}$, there will generally be a range of values of P somewhat less than $E\{p\}$ for which Q will also be positive. Values of P in this range correspond to the notion of the willingness of the risk-averse firm to forego a "risk premium" to obtain a guaranteed price for its output.

In order to determine the effect that the introduction of a contract market would have, we introduce a constraint on the number of contracts the firm may sell and determine the effect of relaxing that constraint. While the analysis which follows would also apply for values of P less than E{p}, but still sufficiently high that the firm would choose to sell a positive number of contracts if it were allowed to do so, we will explicitly assume $P \geq E\{p\}$, so that W is assured to be strictly positive. We now introduce the parameter $\bar{Q}$, which is the maximum member of contracts the firm is allowed to sell. For a given value of P, $\bar{Q}$ is to be thought of as taking on values from zero up to the number of contracts the firm would choose to sell in order to maximize the expected utility of profits, if it were free to do so. The effect of introducing a contract market where none existed previously is thus the cumulative effect of allowing $\bar{Q}$ to vary from zero to its maximum value. To avoid additional complexity, this analysis will be carried out under the assumption of constant absolute risk aversion.

The constraint on the number of contracts the firm may sell is

$$\bar{Q} - Q \geq 0, \tag{14}$$

which will always hold with equality for $P \geq E\{p\}$ and $\bar{Q}$ taking on the values proscribed above.

The firm maximizes the expected utility of its profits subject to the constraint:

$$\mathcal{L} = E\{\phi(\pi)\} + \lambda(\bar{Q} - Q) \tag{15}$$

The first order conditions, which, give our assumptions, will hold with equality, are

$$\frac{\partial \mathcal{L}}{\partial Q} = E\{\phi'(P-p)\} - \lambda = 0 \qquad (16)$$

$$\frac{\partial \mathcal{L}}{\partial X} = E\{\phi'(p-c')\} = 0 \qquad (17)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \overline{Q} - Q = 0 \qquad (18)$$

Differentiating the first-order conditions with respect to Q, X, λ and $\overline{Q}$ gives:

$$\begin{bmatrix} E\{\phi''(P - p)^2\} & E\{\phi''(P - p)(p - C')\} & -1 \\ E\{\phi''(P - p)(p - C')\} & E\{\phi''(p - C')^2 - \phi'C''\} & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} dQ \\ dX \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} d\overline{Q} \qquad (19)$$

The effect on total production of relaxing the constraint on contracts is

$$\frac{\partial X}{\partial \overline{Q}} = \frac{E\{\phi''(P - p)(p - C')\}}{\Delta} \qquad (20)$$

where $\Delta = -E\{\phi''(p - C')^2 - \phi'C''\} > 0$.

From the first-order conditions, $P = \frac{\lambda}{E\{\phi'\}} + C'$. Substituting for P in (20) gives:

$$\frac{\partial X}{\partial \overline{Q}} = \frac{E\left\{\phi''\left(\frac{\lambda}{E\{\phi'\}} + C' - p\right)\left(p - C'\right)\right\}}{\Delta}$$

$$= \frac{-E\{\phi''(p - C')^2\} + E\left\{\phi'' \frac{\lambda}{E\{\phi'\}} \left(p - C'\right)\right\}}{\Delta} \qquad (21)$$

Since, from (17), $E\{\phi'(p - C')\} = 0$, constant absolute risk aversion implies that $E\{\phi''(p - C')\} = 0$. Further, $\frac{\lambda}{E\{\phi'\}}$ is a constant independent of p, so that constant absolute risk aversion allows (21) to be reduced to

$$\frac{\partial X}{\partial \overline{Q}} = \frac{-E\{\phi''(p - C')^2\}}{\Delta} > 0 \qquad (22)$$

The following proposition can now be stated.

Proposition 5: Given a strictly convex cost function and a strictly concave utility function over profits characterized by constant absolute risk aversion, a competitive, expected utility-maximizing firm facing uncertainty over output spot prices and unable to adjust production decisions after observing the spot price will increase total production as the result of introducing a market for the sale of contracts to deliver output at a fixed price which is at least a great as the mean of the spot price.

We again emphasize that IF) is not a necessary condition for Proposition 5, only a sufficient one. A necessary condition for the introduction of the market to increase production is that the contract price be sufficiently high that the firm would choose to sell contracts if it could do so.

CONTRACTING UNDER PRODUCT PRICE UNCERTAINTY WITH EX POST FLEXIBILITY

The model developed in this section relaxes the assumption made in the previous section that firms are unable to make adjustments in output after the spot price has been observed. In this model, the firm is assumed to employ two inputs in its production process and, while it must determine the level of one of those inputs before observing the spot price, it is free to choose the level of the other input after the spot price has been observed. These two inputs may be thought of as representing the firm's fixed and variable inputs, respectively.

Due to the added flexibility in this model, the final level of costs and the quantity of spot sales are not known at the time that investment decisions and contract sales must be made. Consequently, the firm's task of maximizing the expected utility of profits becomes more complex. To handle this increased complexity it is assumed that the firm can solve the following dynamic programming problem: 1) For whatever amount of the fixed input chosen and the number of contracts sold, the firm chooses the amount of the variable input to employ and the quantity of spot sales or purchases which maximize short-run profits. 2) Taking into account the decisions which would be made for each spot price in the short run for each combination of the fixed input and the quantity of contracts which might be chosen, the firm chooses that combination which maximizes the expected utility of profits.

All of the markets in which the firm participates, i.e., those for the inputs as well as for output, are assumed to be competitive, with the firm viewing all prices parametrically.

The notation employed and the further specific assumptions made are summarized as follows.

IIA) The spot and contract prices are p and P, respectively, Prices are non-negative and the firm has a subjective probability density function over p.

IIB) Spot and contract sales are denoted by q and Q, respectively. The firm chooses Q before and q after observing the spot price, and q and Q may take on negative values.

IIC) The production function for total output is f(K,L), where K is the fixed input and L is the variable input. The per unit prices for K and L are r and w, which are known with certainty. The production function is strictly concave, with f_K and f_L positive, f_{KK} and f_{LL} negative, and $(f_{KK} f_{LL} - f_{KL}^2)$ positive.

Further, it is assumed that the marginal physical products of the inputs, f_K and f_L, approach infinity as the respective quantities of the inputs approach zero. This condition insures that positive quantities of the inputs will always be hired, and, therefore, that production will be positive.

In addition to IIC, it will occasionally be convenient to assume the following regularity condition applies:

IIC') The production function f(K,L) is sufficiently regular that for all ranges of output $(f_K - f_L \frac{f_{KL}}{f_{LL}})$ is everywhere either positive or negative. This holds, for example, if K and L are normal inputs, given concavity of f with positive marginal products.

IID) The utility function over profits, π, is $\phi(\pi)$ and is strictly concave, with ϕ' positive and ϕ'' negative.

For portions of the analysis which follows, it will also be assumed that

IIE) The contract price is greater than or equal to the expected value of the spot price: $P \geq E\{p\}$.

For a specific spot price, profits of the firm are the sum of spot and contract revenues, minus the variable and fixed costs:

$$\pi = pq + PQ - wL - rK \tag{23}$$

The sum of spot and contract transactions equals the output of the firm:

$$q + Q = f(K,L) \tag{24}$$

Since setting L in the short run determines output, and therefore spot sales, q can be eliminated as a short-run decision variable, and, after substitution from (24) into (23), profits become a function of L and the variables K and Q, which are fixed before the observation of p,

$$\pi = p(f(K,L) - Q) + PQ - wL - rK \tag{25}$$

For given values of K and Q and any observed value of p, short run profits are maximized by choosing L to satisfy the first-order condition:

$$\frac{\partial \pi}{\partial L} = pf_L - w = 0 \tag{26}$$

We will denote the optimal short run value of L by L^*, which can be written as a function of K, p and w:

$$L^* = L^* (K,p,w) \tag{27}$$

The partial derivatives of L^* are found by totally differentiating (26):

$$\frac{\partial L^*}{\partial K} = \frac{-f_{LK}}{f_{LL}} \tag{28}$$

$$\frac{\partial L^*}{\partial p} = \frac{-fL}{pf_{LL}} \tag{29}$$

$$\frac{\partial L^*}{\partial w} = \frac{1}{pf_{LL}} \tag{30}$$

By concavity of the production function, (29) is positive and (30) is negative; without further assumptions (28) is of indeterminate sign.

Assuming that L will be chosen optimally in the short run, profits can be written as:

$$\pi = p(f(K,L^*(K,p,w)) - Q) + PQ - wL^*(K,p,w) - rK \qquad (31)$$

Choosing K and Q to maximize the expected utility of profits requires satisfying the first order conditions:

$$\frac{\partial E\phi(\pi)}{\partial K} =\{E\ \phi'[p(f_K + f_L \frac{\partial L^*}{\partial K}) - w \frac{\partial L^*}{\partial K} - r]\} = 0$$

$$= E\{\phi'[pf_K + \frac{\partial L^*}{\partial K}(pf_L - w) - r]\} = 0$$

$$= E\{\phi'[pf_K - r]\} = 0 \qquad (32)$$

$$\frac{\partial E\phi(\pi)}{\partial Q} = E\{\phi'(P-p)\} = 0 \qquad (33)$$

Totally differentiating the first order conditions gives:

$$\begin{bmatrix} E\left\{\phi''(pf_K - r)^2 + \phi' p\left(f_{KK} - \frac{f_{KL}^2}{f_{LL}}\right)\right\} & E\{\phi''(pf_K - r)(P-p)\} \\ E\{\phi''(pf_K - r)(P-p)\} & E\{\phi''(P-p)^2\} \end{bmatrix} \begin{bmatrix} dK \\ dQ \end{bmatrix} =$$

$$\begin{bmatrix} -E\{\phi''Q(pf_K - r)\} & E\{\phi''K(pf_K - r) + \phi'\} & E\left\{L^*\phi''(pf_K - r) - \phi' \frac{f_{KL}}{pf_{LL}}\right\} \\ -E\{\phi''Q(P-p) + \phi'\} & E\{\phi''K(P-p)\} & E\{L^*\phi''(P-p)\} \end{bmatrix} \begin{bmatrix} dP \\ dr \\ dw \end{bmatrix} \quad (34)$$

The second-order conditions for a maximum are that $E\{\phi''(p_{f_K} - r)^2 + \phi' p(f_{KK} - \frac{f_{KL}^2}{f_{LL}})\}$ and $E\{\phi''(P-p)^2\}$ are negative and that

$$\Delta = E\{\phi''(pf_K - r)^2 + \phi'_p(f_{KK} - \frac{f_{KL}^2}{f_{LL}})\} \cdot E\{\phi''(P-p)^2\}$$

$$- E\{\phi''(pf_K - r)(P-p)\}^2 \quad (35)$$

is positive. IIC) and IID) insure the negativity of the first two expectations. It is assumed that Δ is positive.

CONSTANT ABSOLUTE RISK AVERSION

For purposes of the present analysis, we will make the following additional assumption:

IIE) The firm's utility function is characterized by constant absolute risk aversion. That is, $\frac{\partial R_A}{\partial \pi} = 0$, where $R_A = \frac{-\phi''(\pi)}{\Phi'(\pi)}$

The implications of relaxing IIE) will be discussed later in this section. Under IIE), the terms $-E\{\phi''Q(pf_K-r)\}$, $E\{\phi''K(pf_K-r) + \phi'\}$, $-E\{\phi''Q(P-p)+\phi'\}$ and $E\{\phi''K(P-p)\}$ in (34) are found to be zero, positive, negative and zero, respectively. These can be shown by noting that constancy of R_A implies $\frac{-\phi''(\pi)}{\phi'(\pi)} = c$, a constant.

Therefore, substituting $(-c\phi')$ for ϕ'' in $E\{\phi''(pf_K-r)\}$ and $E\{\phi''(P-p)\}$ gives, by (32) and (33)

$$E\{\phi''(pf_K-r)\} = -c\cdot E\{\phi'(pf_K-r)\} = 0 \qquad (36)$$

and

$$E\{\phi''(P-p)\} = -c\cdot E\{\phi'(P-p)\} = 0 \qquad (37)$$

Since K and Q are fixed values, we thus have

$$-E\{\phi''Q(pf_K-r)\} = -Q\ E\{\phi''(pf_K-r)\} = 0 \qquad (38)$$

$$E\{\phi'K(pf_K-r) + \phi'\} = E\{\phi'\} > 0 \qquad (39)$$

$$-E\{\phi''Q(P-p) + \phi'\} = -E\{\phi'\} < 0 \tag{40}$$

$$E\{\phi''K(P-p)\} = KE\{\phi''(P-p)\} = 0 \tag{41}$$

We can now use (38) - (41) to evaluate the following comparative statics terms from (34) under the assumption of IIE):

$$\frac{\partial K}{\partial P} = \frac{E\{\phi''Q(P-p)+\phi'\}\cdot E\{\phi''(pf_K-r)(P-p)\}-E\{\phi''Q(pf_K-r)\}E\{\phi''(P-p)^2\}}{\Delta}$$

$$= \frac{E\{\phi'\}\cdot E\{\phi''(pf_K-r)(P-p)\}}{\Delta} \tag{42}$$

$$\frac{\partial K}{\partial r} = \frac{E\{\phi''K(pf_K-r)+\phi'\}\cdot E\{\phi''(P-p)^2\}-E\{\phi''K(P-p)\}\cdot E\{\phi''(pf_K-r)(P-p)\}}{\Delta}$$

$$= \frac{E\{\phi'\}\cdot E\{\phi''(P-p)^2\}}{\Delta} < 0 \tag{43}$$

$$\frac{\partial K}{\partial w} = \frac{E\{L^*\phi''(pf_K-r)-\phi'\frac{f_{KL}}{pf_{LL}}\}E\{\phi''(P-p)^2\}-E\{L^*\phi''(P-p)\}E\{\phi''(pf_K-r)(P-p)\}}{\Delta} \tag{44}$$

$$\frac{\partial Q}{\partial P} = \frac{1}{\Delta}\Big[E\{\phi''(pf_K-r)(P-p)\}\cdot E\{\phi''Q(pf_K-r)\} - E\{\phi''(pf_K-r)^2 + \phi'p(f_{KK}-\frac{f_{KL}}{f_{LL}})^2\}\cdot E\{\phi''Q(P-p)+\phi'\}\Big]$$

$$= \frac{-E\{\phi''(pf_K-r)^2+\phi'p(f_{KK}-\frac{f_{KL}^2}{f_{LL}})\}\cdot E\{\phi'\}}{\Delta} > 0 \tag{45}$$

$$\frac{\partial Q}{\partial r} = \frac{1}{\Delta}\Big[E\{\phi''(pf_K-r)^2+\phi' p(f_{KK} - \frac{f_{KL}{}^2}{f_{LL}})\}\cdot E\{\phi'' K(P-p)\}$$

$$-E\{\phi''(pf_K-r)(P-_p)\}E\{\phi'' K(pf_K-r)+\phi'\}\Big]$$

$$= \frac{-E\{\phi''(pf_K-r)(P-p)\}\cdot E\{\phi'\}}{\Delta} \qquad (46)$$

$$\frac{\partial Q}{\partial w} = \frac{1}{\Delta}\Big[E\{\phi''(pf_K-r)^2+\phi' p(f_{KK} - \frac{f_{KL}{}^2}{f_{LL}})\}\cdot E\{L^*\phi''(P-p)\}$$

$$-E\{\phi''(pf_K-r)(P-p)\}\cdot E\{L^*\phi''(pf_K-r) - \phi\frac{f_{KL}}{pf_{LL}}\}\Big] \qquad (47)$$

Signing (43) and (45) allows the following two propositions to be stated:

Proposition 6: Assume an expected utility-maximizing firm with a two-input production function facing output price uncertainty and participating in a competitive forward contract market for its output. Assume that the firm must determine the level of one input before observing the spot price for output and that it may determine the level of the second input after observing the spot price and assume IIA)-IIE) which includes, in particular, the consumption of constant absolute risk aversion. Then an increase in the per-unit cost of the fixed input will decrease the amount of that input hired.

Proposition 7: Given the assumptions of Proposition 6, an increase in the contract price will result in the firm choosing to sell a greater quantity of contracts.

The restriction imposed on the production function by IIC'), that is, that $(f_K - f_L \frac{f_{KL}}{f_{LL}})$ is everywhere either negative or positive, implies that, under constant absolute risk aversion, the off-diagonal terms in the coefficient matrix in (34) have the same sign as $(f_K - f_L \frac{f_{KL}}{f_{LL}})$. To prove this statement, first note that

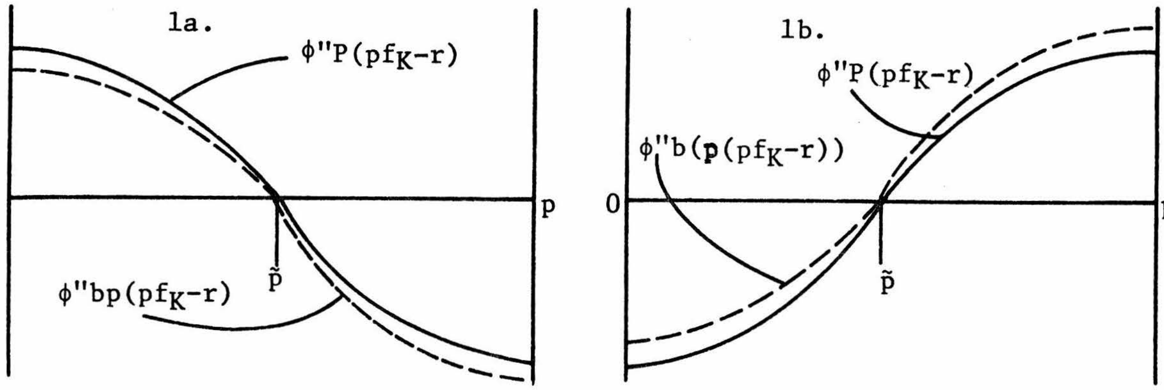
$$E\{\phi''(pf_K-r)(P-p)\} = E\{\phi''P(pf_K-r)\} - E\{\phi''p(pf_K-r)\}$$

$$= -E\{\phi''p(pf_K-r)\}, \qquad (48)$$

since P is a constant and, from (37), $E\{\phi''(pf_K-r)\} = 0$. We now want to determine how (pf_K-r) changes as p increases. The derivative of (pf_K-r) with respect to p is

$$\frac{\partial(pf_K-r)}{\partial p} = f_K + pf_{KL}\frac{\partial L}{\partial p} = f_K - f_L \frac{f_{KL}}{f_{LL}} \qquad (49)$$

Notice that if inputs are normal, so that f_{KL} is positive, (49) will always be positive. Further, IIC') implies that (pf_K-r) is monotonic in p. For the case where (pf_K-r) is increasing in p, the graph of $\phi''P(pf_K-r)$ resembles Figure 1a, and for the case where it is decreasing in p, the graph of $\phi''P(pf_K-r)$ resembles Figure 1b.



Note that $\phi''P(pf_K-r)$ is not necessarily monotonic in p, since ϕ'' is not monotonic in p. However, since ϕ'' is always negative, $E\{\phi''P(pf_K-r)\}$ is zero and (pf_K-r) is monotonic in p, there is one <u>and only</u> one p such that $\phi''P(pf_K-r) = 0$. We denote this spot price by $\tilde{p}$ and define b to be a positive constant such that $b\tilde{p} = P$. Then, for $p>\tilde{p}$, $bp>P$ and for $p<\tilde{p}$, $bp<P$. This means that in 1a, for all values of p, $\phi''bp(pf_K-r) \leq \phi''P(pf_K-r)$, and since $E\{\phi''P(pf_K-r)\} = 0$, $E\{\phi''bp(pf_K-r)\} < 0$. Since b is a positive constant, $E\{\phi''p(pf_K-r)\}<0$, as well. Conversely, in 1b, for all p, $\phi''bp(pf_K-r) \geq \phi''P(pf_K-r)$, and $E\{\phi''p(pf_K-r)\} > 0$. Thus, when $(f_K-f_L \frac{f_{KL}}{f_{LL}})$ is positive, $\frac{\partial(pf_K-r)}{\partial p}$ and $E\{\phi''(pf_K-r)(P-p)\}$ are also positive, and when $(f_K-f_L \frac{f_{KL}}{f_{LL}})$ is negative, $\frac{\partial(pf_K-r)}{\partial p}$ and $E\{\phi''(pf_K-r)(P-p)\}$ are negative.

With these additional results, the comparative statics terms in (42) and (46) can be signed if an assumption is made about the sign of $(f_K-f_L \frac{f_{KL}}{f_{LL}})$: $\frac{\partial K}{\partial P}$ is positive (negative) when

$(f_K - f_L \frac{f_{LK}}{f_{LL}})$ is positive (negative): $\frac{\partial Q}{\partial r}$ is negative (positive) when $(f_K - f_L \frac{f_{LK}}{f_{LL}})$ is positive (negative).

The following propositions can now be stated:

<u>Proposition 8:</u> Given the assumptions of Proposition 6 and IIC'), an increase in the contract price will increase the amount of the fixed input hired when $(f_K - f_L \frac{f_{KL}}{f_{LL}})$ is positive and decrease the amount hired when $(f_K - f_L \frac{f_{LK}}{f_{LL}})$ is negative.

<u>Proposition 9:</u> Given the assumptions of Proposition 8, an increase in the per-unit cost of the fixed input will decrease the number of contracts sold when $(f_K - f_L \frac{f_{LK}}{f_{LL}})$ is positive and increase the number sold when $(f_K - f_L \frac{f_{LK}}{f_{LL}})$ is negative.

Since normal inputs imply that $(f_K - f_L \frac{f_{LK}}{f_{LL}})$ is positive, we also have the following two corollaries:

<u>Corollary 1:</u> Given the assumptions of Proposition 8, and assuming normal inputs, an increase in the contract price will increase the amount of the fixed input hired.

<u>Corollary 2:</u> Given the assumptions of Proposition 8, and assuming normal inputs, an increase in the per unit cost of the fixed input will decrease the number of contracts sold.

Notice, however, that assuming that inputs are competitive, that is, that f_{LK} is negative, does not imply that the results in Corollaries 1 and 2 are reversed, since $\left(f_K - \frac{f_L f_{LK}}{f_{LL}}\right)$ will not be negative unless $f_{LK} < \frac{f_{LL} \cdot f_K}{f_L}$.

We are now in a position to determine the effect of an increase in the contract price on the total output of the firm. Since the output of the firm varies with the spot price, this effect must be stated in terms of the output for a given spot price. For a given p, the derivative of production with respect to P is

$$\frac{\partial f(K,L^*(p,K,w))}{\partial P} = f_K\Big|_p \frac{\partial K}{\partial P} + f_L\Big|_p \frac{\partial L}{\partial P}\Big|_p \tag{50}$$

However, $\frac{\partial L}{\partial P}\Big|_p$ can also be written as $\frac{\partial L}{\partial K}\Big|_p \frac{\partial K}{\partial P}$.

Substituting for $\frac{\partial L}{\partial P}\Big|_p$ in (50) gives

$$\frac{\partial f}{\partial P}\Big|_p = \frac{\partial K}{\partial P}\left(f_K\Big|_p + f_L\Big|_p \frac{\partial L}{\partial K}\Big|_p\right) \tag{51}$$

From (28), $\frac{\partial L}{\partial K}\Big|_p = -\frac{f_{LK}}{f_{LL}}\Big|_p$.

Therefore, (51), can be written as

$$\left.\frac{\partial f}{\partial P}\right|_p = \frac{\partial K}{\partial P}\left(\left.f_K\right|_p - \left.f_L\right|_p \left.\frac{f_{LK}}{f_{LL}}\right|_p\right) \tag{52}$$

From Proposition 8, constant absolute risk aversion and IIC' imply that $\frac{\partial K}{\partial P}$ has the same sign as $f_K - f\,\frac{f_{LK}}{f_{LL}}$. Therefore the production of the firm will be increased by an increase in P for every spot price. This result is summarized in the following proposition:

<u>Proposition 10</u>: Given the assumptions of Proposition 8, the total amount of output the firm produces for each spot price will be increased by an increase in the contract price.

Notice that output is increased by an increase in P even if the increase in P decreases the amount of the fixed input which is hired. Clearly, the amount of fixed input hired would only decrease as the result of an increase in P when the inputs are competitive in the extreme. In that case the decrease in the fixed input would be more than offset by the increase in the variable input hired so that output would still be increased.

It is also possible to determine from this model the effects of the existence of the contract market itself on the firm's investment and production decisions. As in the case of the firm without <u>ex post</u> flexibility, we will do this by introducing a constraint on the amount of contracting the firm may do and determine the effects of relaxing that constraint.

Also as in the case of the firm without <u>ex post</u> flexibility, we will concentrate the analysis on the firm which would sell a positive number of contracts if it were allowed to do so. Assumption IIE), that the contract price is at least as great as the mean of the spot price, is a sufficient condition for the firm to choose to sell contracts. The demonstration of this is virtually identical to that in the case without <u>ex post</u> flexibility, and an abbreviated version will be reproduced here: the derivative of the expected utility of profits with respect to Q is $E\{\phi'\cdot(P-p)\} = E\{\phi'\}P-E\{\phi'\cdot p\}$, which is equal to the negative of the covariance of ϕ' and p when $P = E\{p\}$. When Q is zero, the covariance between ϕ' and p is negative, since $\partial\phi'/\partial p = \phi''\cdot f(K,L(p))$, which is negative, since, by IID), ϕ' is negative, and, by IIC), production is positive. Therefore, $E\{\phi'\}\cdot P - E\{\phi'\cdot p\}$ is positive at Q equal to zero for $P = E\{p\}$. This expression is also positive when P is greater than $E\{p\}$, since ϕ' is always positive. Therefore, the derivative of expected utility of profits with respect to Q at Q equal to zero is positive for $P \geq E\{p\}$, which implies that the value of Q which maximizes the expected utility of profits is greater than zero. It is again the case that $P \geq E\{p\}$ is a sufficient condition for Q to be positive, not a necessary one, and that there will generally be values of $P < E\{p\}$ for which the firm would choose to sell contracts.

We again introduce the parameter $\overline{Q}$, which is the maximum number of contracts the firm is allowed to sell. For a given value of P, $\overline{Q}$ is to be thought of as ranging in value from zero up to the number of contracts the firm would choose to sell in order to maximize the expected utility of profits, if it were free to do so. The effect of introducing a contract market where none existed previously is thus the cumulative effect of allowing $\overline{Q}$ to vary from zero up to its maximum value.

The constraint on the number of contracts the firm may sell is thus written

$$\overline{Q} - Q \geq 0, \qquad (53)$$

which will always hold with equality for $P \geq E\{p\}$ and $\overline{Q}$ taking on the values prescribed above. The firm's problem is to solve the constrained dynamic programming problem,

$$\max \mathcal{L} = E\{\phi(\pi)\} + \lambda(\overline{Q} - Q)$$

where λ is the Legrange multiplier associated with the constraint. The first order conditions, which, given our assumptions, will hold with equality, are

$$\frac{\partial \mathcal{L}}{\partial K} = E\{\phi'(pf_{K-r})\} = 0 \qquad (55)$$

$$\frac{\partial \mathcal{L}}{\partial Q} = E\{\phi'(P-p)\} - \lambda = 0 \qquad (56)$$

$$\frac{\partial}{\partial\lambda} = \bar{Q} - Q = 0 \tag{57}$$

Totally differentiating these first-order conditions gives:

$$\begin{bmatrix} E\left\{\phi''(pf_K-r)^2 + \phi' p\left(f_{KK} - \frac{f_{KL}{}^2}{f_{LL}}\right)\right\} & E\{\phi''(pf_K-r)(P-p)\} & 0 \\ E\{\phi''(pf_K-r)(P-p)\} & E\{\phi''(P-p)^2\} & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} dK \\ dQ \\ d\lambda \end{bmatrix} \tag{58}$$

$$= \begin{bmatrix} 0 & -E\{\phi''Q(pf_K-r)\} & E\{\phi''K(pf_K-r) + \phi'\} & E\left\{L^*\phi''(pf_K-r) - \phi'\frac{f_{KL}}{pf_{LL}}\right\} \\ 0 & -E\{\phi''Q(P-p)+\phi'\} & E\{\phi''K(P-p)\} & E\{L^*\phi''(P-p)\} \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} d\bar{Q} \\ dP \\ dr \\ dw \end{bmatrix}$$

The effect on the amount of the fixed input hired of allowing the firm to sell more contracts is

$$\frac{\partial K}{\partial \bar{Q}} = \frac{E\{\phi''(pf_K-r)(P-p)\}}{\Delta} \tag{59}$$

where $\Delta = -E\{\phi''(pf_K-r)^2 + \phi' p(f_{KK} - \frac{f_{KL}{}^2}{f_{LL}})\} > 0.$

As was demonstrated earlier, $E\{\phi''(pf_K-r)(P-p)\}$ has the same sign as $(f_K - f_L \frac{f_{LK}}{f_{LL}})$, under constant absolute risk aversion and IIC'). Therefore, $\frac{\partial K}{\partial \bar{Q}}$ also has the same sign as $(f_K - f_L \frac{f_{LK}}{f_{LL}})$. The effect of introducing the contracting market on the amount of the fixed input hired is stated in the following proposition and corollary:

Proposition 11: Assume the conditions of Proposition 8, except that there does not exist a contract market. The introduction of a contract market with a contract price at least as great as the mean of the spot price, will increase the amount of the fixed price, will increase the amount of the fixed input hired if $(f_K - f_L \frac{f_{LK}}{F_{LL}})$ is positive and will decrease the amount hired if $(f_K - f_L \frac{f_{LK}}{f_{LL}})$ is negative.

Corollary 3: Given the assumption of Proposition 11 and normal input, the introduction of a contract market with the contract price at least as great as the mean of the spot price, the introduction of a contract market will increase the amount of the fixed input hired.

The effect of introducing the contracting market on the total output of the firm can be found by looking at the effect on firm production of relaxing the constraint on $\overline{Q}$. Since output depends on the particular spot price, the effect of relaxing the constraint must be examined for a given spot price. The derivative of total production with respect to $\overline{Q}$ for any given spot price is the sum of the effects on production of the changes in K and L resulting from the change in $\overline{Q}$:

$$\left.\frac{\partial f}{\partial \overline{Q}}\right|_p = \left.f_K\right|_p \frac{\partial K}{\partial \overline{Q}} + \left.f_L\right|_p \left.\frac{\partial L}{\partial K}\right|_p \frac{\partial K}{\partial \overline{Q}} \tag{60}$$

Substituting $\left.\frac{-f_{LK}}{f_{LL}}\right|_p$ for $\left.\frac{\partial L}{\partial K}\right|_p$ in (60), we have

$$\left.\frac{\partial f}{\partial \bar{Q}}\right|_p = \left(\left.f_K\right|_p - \left.f_L\right|_p \left.\frac{f_{LK}}{f_{LL}}\right|_p \frac{\partial K}{\partial \bar{Q}}\right) \tag{61}$$

Since $\frac{\partial K}{\partial \bar{Q}}$ has the same sign as $(f_K - f_L \frac{f_{LK}}{f_{LL}})$, $\left.\frac{\partial f}{\partial \bar{Q}}\right|_p > 0$ for all p, under constant absolute risk aversion and IIC'). The effect of the introduction of the contracting market on output is stated in the following proposition:

<u>Proposition 12:</u> Given the assumptions of Proposition 11, the introduction of a contract market with a contract price at least as great as the mean of the spot price will increase the total production of the firm for all spot prices.

As was true for the model without <u>ex post</u> flexibility, the assumption that the contract price is at least as great as the mean of the spot price is only a sufficient condition to state the effects of the introduction of a contract market. The conclusions of Propositions 11 and 12 and Corollary 3 would also hold for any contract price sufficiently large that the firm would choose to sell contracts at that price, even if the contract price were somewhat less than the mean of the spot price.

VARIABLE ABSOLUTE RISK AVERSION

In this section we relax the assumption that the firm's utility function has the property of constant absolute risk aversion and derive expressions for the effects of changes in P and r on the *ex ante* decision variables, K and Q.

As in the model without *ex post* flexibility, we introduce the shift parameter, α, to represent additions to the certain income of the firm. The parameter has an initial value of zero and increases in it represent lump-sum additions to the firm's wealth position. The firm's utility function may now be written as $\phi(\pi+\alpha)$. Differentiating (32) and (33) with respect to α and the *ex ante* decision variables gives:

$$\begin{bmatrix} E\{\phi''(pf_K-r)^2 + \phi'p(f_{KK}-\frac{f_{KL}{}^2}{f_{LL}})\} & E\{\phi''(pf_K-r)(P-p)\} \\ E\{\phi''(pf_K-r)\ (P-p)\} & E\{\phi'(P-p)^2\} \end{bmatrix} \begin{bmatrix} dK \\ dQ \end{bmatrix}$$

$$= \begin{bmatrix} -E\{\phi''(pf_K-r)\} \\ -E\{\phi''(P-p)\} \end{bmatrix} \begin{bmatrix} d\alpha \end{bmatrix} \qquad (62)$$

Solving from (62), the effects of a change in the firm's wealth position on the amount of the fixed input hired and the number of contracts sold can be expressed as:

$$\frac{\partial K}{\partial \alpha} = \frac{E\{\phi''(P-p)\}\cdot E\{\phi''(pf_K-r)(P-p)\}-E\{\phi''(pf_K-r)\}\cdot E\{\phi''(P-p)^2\}}{\Delta} \qquad (63)$$

$$\frac{\partial Q}{\partial \alpha} = \frac{1}{\Delta}\Big[E\{\phi''(pf_K - r)\}\cdot E\{\phi''(pf_K - r)(P-p)\} - E\{\phi''(P-p)\}\cdot E\{\phi''(pf_K - r)^2$$

$$+ \phi' p(f_{KK} - \frac{f_{KL}^2}{f_{LL}})\}\Big] \quad (64)$$

Using (63) and (64), we can now express equations (42), (43), (45), and (46) as follows:

$$\frac{\partial K}{\partial P} = \left.\frac{\partial K}{\partial P}\right|_{\substack{\text{constant}\\ \text{absolute}\\ \text{risk aversion}}} + Q \cdot \frac{\partial K}{\partial \alpha} \quad (65)$$

$$\frac{\partial K}{\partial r} = \left.\frac{\partial K}{\partial r}\right|_{\substack{\text{constant}\\ \text{absolute}\\ \text{risk aversion}}} + K\ \frac{\partial K}{\partial \alpha} \quad (66)$$

$$\frac{\partial Q}{\partial P} = \left.\frac{\partial Q}{\partial P}\right|_{\substack{\text{constant}\\ \text{absolute}\\ \text{risk aversion}}} + Q \cdot \frac{\partial Q}{\partial \alpha} \quad (67)$$

$$\frac{\partial Q}{\partial r} = \left.\frac{\partial Q}{\partial r}\right|_{\substack{\text{constant}\\ \text{absolute}\\ \text{risk aversion}}} + K \cdot \frac{\partial Q}{\partial \alpha} \quad (68)$$

Thus, as was true for the model without *ex post* flexibility, when absolute risk aversion is not constant, the comparative statics expressions may be written as sums of two types of effects. The first of these results from alterations in incentives and corresponds to the effect when absolute risk aversion is constant. This may be termed the substitution effect, since it involves the firm substituting the fixed input for the variable input or contract sales for spot sales. The second effect results from changes in the firm's attitude toward risks as its wealth position changes due to changes in the prices. Unlike the model without *ex post* flexibility, however, we have not been successful in signing this second type of effect from knowledge of whether absolute risk aversion is an increasing or decreasing function of profits.

The difficulty in signing the second type of effect under the assumption of *ex post* flexibility is present because of that flexibility itself. Without flexibility the firm is either a seller in the spot market for all spot prices or a buyer in the spot market for all spot prices, and the quantity bought or sold does not vary with the spot price. With flexibility, however, the firm buys in the spot market if the spot price is low and sells in the spot market if the spot price is high, and the quantity bought or sold varies with the spot price. In the case without flexibility, the sign of the effect for the firm which makes spot purchases is the opposite of that for the firm which makes spot sales. In the case with flexibility, however, since the firm makes spot purchases over some portion of the distribution of spot prices and spot sales over the remainder, both effects are present and which dominates, in general, has not been determined.

The appendix provides some numerical examples which illustrate the ranges of the spot price distribution over which the firm makes spot purchases and spot sales.

III. CONTRACTING FOR A VARIABLE INPUT.

In this section we model a firm which is uncertain about the future spot price of a variable input, but which may purchase at a fixed price contracts for future delivery of quantities of that input. As in the previous section, it is assumed that the firm has the flexibility to adjust the total quantity of its variable input via participation in the spot market after the spot price is observed and that it is able to solve the dynamic programming problem of choosing the ex ante decision variable to maximize the expected utility of profit. Also, as in the previous sections, the input and output markets are assumed to be competitive, with the firm viewing all prices parametrically. In this model only the variable input spot price is subject to uncertainty. However, uncertainty over the firm's output price could be incorporated without adding a great deal of complexity.

The notations and further assumptions of the model of contracting for a variable input are:

IIIA) The price of the firm's output is b, which is known with certainty.

IIIB) Spot and contract prices for the variable input are p and P, respectively. Prices are non-negative and the firm has a subjective probability density function over p.

IIIC) Spot and contract purchases of the variable input are q and Q, respectively. The firm chooses Q before and q after observing the spot price. Q and q may individually be negative. However, their sum is non-negative.

IIID) The fixed-input quantity and per unit cost are J and r, respectively.

IIIE) The production function is strictly concave, with g_J and $g_Q = g_q$ positive, where J is the fixed input and Q + q is the total amount of the variable input employed. The production function is strictly concave, with g_J and g_Q positive, g_{JJ} and g_{QQ} negative, and $(g_{JJ}\, g_{QQ} - g_{JQ}^2)$ positive. Further, for any given production function, g_{JQ} is assumed positive everywhere or negative everywhere.

It is also assumed that the marginal physical products of the inputs, g_J and g_Q, approach infinity as the respective quantities of the inputs approach zero. This condition insures that the total quantity of each of the two inputs hired will always be positive, and, therefore, that production will be positive.

IIIF) The utility function over profits, π, is $\psi(\pi)$ and is strictly concave, with ψ' positive and ψ'' negative.

For a specific spot price for the variable input, profits of the firm are sales revenues, minus the spot and contract expenditures for the variable input and minus the cost of the fixed input:

$$\pi = b \cdot g(J, Q + q) - pq - PQ - rJ \qquad (69)$$

For portions of the analysis which follows, it will also be assumed that

IIIG) The contract price is less than or equal to the expected value of the spot price: $P \leq \{E\}p$.

After the variable input spot price has been observed, the quantity of spot purchases is chosen to satisfy the short-run profit-maximizing condition

$$\frac{\partial \pi}{\partial q} = bg_Q - p = 0 \tag{70}$$

The optimal short-run value of q is denoted by q*, which can be written as a function of J, Q, b and p:

$$q^* = q^*(J, Q, b, p) \tag{71}$$

The partial derivatives of q* are found by totally differentiating (70):

$$\frac{\partial q^*}{\partial J} = \frac{-g_{QJ}}{bg_{QQ}} \tag{72}$$

$$\frac{\partial q^*}{\partial Q} = -1 \tag{73}$$

$$\frac{\partial q^*}{\partial b} = \frac{-g_Q}{bg_{QQ}} \tag{74}$$

$$\frac{\partial q^*}{\partial p} = \frac{1}{bg_{QQ}} \tag{75}$$

By concavity of the production function, (74) is positive and (75) is negative. (73) indicates that, for a given level of the fixed input, the total amount of the variable input employed will be independent of the amount contracted for. Without further assumptions on the production function, (72) cannot be signed.

Assuming that q will be chosen optimally in the short run, profits can be written as:

$$\pi = b\cdot g(J,Q+q^*(J,Q,b,p))-pq^*(J,Q,b,p)-PQ-rJ \qquad (76)$$

The first-order conditions for maximizing the expected utility of profits with respect to the <u>ex ante</u> decision variables, J and Q, are:

$$\frac{\partial E\psi(\pi)}{\partial J} = E\{\psi'(bg_J-r)\} = 0 \qquad (77)$$

$$\frac{\partial E\psi(\pi)}{\partial Q} = E\{\psi'(bg_Q-P)\} = 0 \qquad (78)$$

Totally differentiating the first-order conditions gives:

$$\begin{bmatrix} E\{\psi''(bg_J-r)^2+\psi' bg_{JJ}\} & E\{\psi''(bg_J-r)(bg_Q-P)+\psi' bg_{JQ}\} \\ E\{\psi''(bg_J-r)(bg_Q-P)+\psi' bg_{JQ}\} & E\{\psi''(bg_Q-P)^2 + \psi' bg_{QQ}\} \end{bmatrix} \begin{bmatrix} dJ \\ dQ \end{bmatrix}$$

$$\begin{bmatrix} E\{\psi''Q(bg_J-r)\} & E\{\psi''J(bg_J-r)+\psi'\} & -E\{\psi''g\ (bg_J-r)+\psi' g_J\} \\ E\{\psi''Q(bg_Q-P)+\psi'\} & E\{\psi''J(bg_Q-P)\} & -E\{4''g\cdot(bg_Q-P)+\psi' g_Q\} \end{bmatrix} \begin{bmatrix} dP \\ dr \\ db \end{bmatrix} \qquad (79)$$

The second-order conditions for a maximum are that $E\{\psi''(bg_J-r)^2+\psi'bg_{JJ}\}$ and $E\{\psi''(bg_Q-P)^2 + \psi'bg_{QQ}\}$ are negative and that

$$\Delta = E\{\psi''(bg_J-r)^2+\psi bg_{JJ}\}\cdot E\{\psi''(bg_Q-P)^2+\psi'bg_{QQ}\}$$

$$- E\{\psi''(bg_J-r)(bg_Q-P) + \psi'bg_{JQ}\}^2 \qquad 80)$$

is positive. IIIE) and IIIF) insure that the first two expectations are negative and Δ is assumed positive.

CONSTANT ABSOLUTE RISK AVERSION

As in the analysis of contracting for the sale of output, we will consider as a special case the firm with a utility function characterized by constant absolute risk aversion. This assumption will be relaxed later in the section and variable absolute risk aversion will be considered. Formally, the assumption is stated as:

IIIG) the firm's utility function has the property that the measure of constant absolute risk aversion, $\frac{-\psi''(\pi)}{\psi'(\pi)}$, does not vary with π.

Constancy of absolute risk aversion and (77) and (78) imply that the terms $E\{\psi''(bg_J-r)\}$ and $E\{\psi''(bg_Q-P)\}$ are zero. Therefore, since Q and J do not vary with p, the coefficients of dP and dr from (79) are evaluated as follows:

$$E\{\psi''Q(bg_J-r)\} = 0 \tag{81}$$

$$E\{\psi''J(bg_J-r) + \psi'\} = E\{\psi'\} > 0 \tag{82}$$

$$E\{\psi''Q(bg_Q-P) + \psi'\} = E\{\psi'\} > 0 \tag{83}$$

$$E\{\psi''J(bg_Q-P)\} = 0 \tag{84}$$

Without further restrictions on the production function, the signs of the coefficients of db in (79) are indeterminate.

Using (81) - (84), the comparative statics terms from (79), under constant absolute risk aversion, reduce to:

$$\frac{\partial J}{\partial P} = \frac{1}{\Delta}\Big[E\{\psi''Q(bg_J-r) \quad E\ \psi''(bg_Q-P)^2+\psi' bg_{QQ}\}$$

$$-E\{\psi''Q(bg_Q-P)+\psi'\}\cdot E\{\psi''(bg_J-r)(bg_Q-P)+bg_{JQ}\}\Big]$$

$$= \frac{-\ E\{\psi'\}\cdot E\{\psi''(bg_J-r)(bg_Q-P)+bg_{JQ}\}}{\Delta} \tag{85}$$

$$\frac{\partial Q}{\partial P} = \frac{1}{\Delta}\Big[E\{\psi''Q(bg_Q-P)+\psi'\}\cdot E\{\psi''(bg_J-r)^2+\psi\ bg_{JJ}\}$$

$$-E\{\psi''Q(bg_J-r)\}\cdot E\{\psi''(bg_J-r)(bg_Q-P)+\psi\ bg_{JQ}\}\Big]$$

$$= \frac{E\{\psi'\}\cdot E\{\psi''(bg_J-r)^2+\psi' bg_{JJ}\}}{\Delta} < 0 \tag{86}$$

$$\frac{\partial J}{\partial r} = \frac{1}{\Delta}\Big[E\{\psi''J(bg_J-r)+\psi'\}\cdot E\{\psi''(bg_Q-P)^2+\psi' bg_{QQ}\}$$

$$- E\{\psi''J(bg_Q-P)\}\cdot E\{\psi''(bg_J-r)(bg_Q-P)+\psi' bg_{JQ}\}\Big]$$

$$= \frac{E\{\psi'\}\cdot E\{\psi''(bg_Q-P)^2+\psi' bg_{QQ}\}}{\Delta} < 0 \tag{87}$$

$$\frac{\partial Q}{\partial r} = \frac{1}{\Delta}\Big[E\{\psi''J(bg_Q-P)\cdot E\{\psi''(bg_J-r)^2+\psi' bg_{JJ}\}$$

$$- E\{\psi''J(bg_J-r)+\psi'\}\cdot E\{\psi''(bg_J-r)(bg_Q-P)+\psi' bg_{JQ}\}\Big]$$

$$= - \frac{E\{\psi'\}\cdot E\{\psi''(bg_J-r)(bg_Q-P)+\psi' bg_{JQ}\}}{\Delta} \tag{88}$$

$$\frac{\partial J}{\partial b} = \frac{1}{\Delta}\Big[E\{\psi''g\cdot(bg_Q-P)+\psi' g_Q\}\cdot E\{\psi''(bg_J-r)(bg_Q-P)+\psi' bg_{JQ}\}$$

$$- E\{\psi''g\cdot(bg_J-r)+\psi' g_J\}\cdot E\{\psi''(bg_Q-P)^2+\psi' bg_{QQ}\}\Big] \tag{89}$$

$$\frac{\partial Q}{\partial b} = \frac{1}{\Delta}\Big[E\{\psi''g\cdot(bg_J-r)+\psi'g_J\}E\{\psi''(bg_J-r)(bg_Q-P)+\psi'bg_{JQ}\}$$

$$- E\{\psi''g\cdot(bg_Q-P)+\psi'g_Q\}\cdot E\{\psi''(bg_J-r)^2+\psi'bg_{JJ}\}\Big] \quad (90)$$

The two comparative statics terms which can immediately be signed, (86) and (87), imply the following two propositions.

Proposition 13: Assume an expected utility-maximizing firm with a two-input production function facing uncertainty over the price of the variable input and participating in a competitive forward market for that input. Then IIIA) - IIIF), including the assumption of constant absolute risk aversion, imply that an increase in the price of a forward contract for the variable input will decrease the number of those contracts purchased.

Propostion 14: Given the assumptions of Proposition 13, an increase in the cost of the fixed input will decrease the quantity of that input employed.

Additionally, (85) and (88) will both be negative when inputs are normal and positive when inputs are competitive. To show this, we note that (85) and (88) have the opposite sign of $E\{\psi''(bg_J-r)(bg_Q-P)+\psi'bg_{JQ}\}$, which can also be written as $[E\{\psi''bg_Q(bg_J-r)\} - E\{\psi''P(bg_J-r)\} + E\{\psi'bg_{JQ}\}]$. Under constant absolute risk aversion, the second expectation is zero. The first expectation can be signed as follows. The derivative of $(bg_J - r)$ with respect to p is

$$\frac{\partial(bg_J - r)}{\partial p} = bg_{JQ}\frac{\partial q^*}{\partial p} = \frac{g_{JQ}}{g_{QQ}} \tag{91}$$

Thus, when inputs are normal, that is, when g_{JQ} is positive, (91) is negative and when inputs are competitive and g_{JQ} is negative, (91) is positive. We define $\tilde{p}$ to be the spot price at which $bg_J - r = 0$. Therefore, since ψ'' is negative, under normal inputs, for $p > \tilde{p}$, $\psi''(bg_J - r) > 0$ and for $p < \tilde{p}$, $\psi''(bg_J - r) < 0$; under competitive inputs, for $p > \tilde{p}$, $\psi''(bg_J - r) < 0$ and for $p < \tilde{p}$, $\psi''(bg_J - r) > 0$. Now define d to be a positive constant such that at $\tilde{p}$, $d = bg_Q$. From (70), $bg_Q = p$. Therefore, for $p > \tilde{p}$, $bg_Q > d$ and for $p < \tilde{p}$, $bg_Q < d$. For the case of normal inputs, we thus have $\psi'' bg_Q(bg_J - r) > \psi'' d(bg_J - r)$ for all $p \neq \tilde{p}$; if $p = \tilde{p}$, then they are equal and when inputs are competitive, $\psi'' bg_Q(bg_J - r) < \psi'' d(bg_J - r)$ for all $p \neq \tilde{p}$. Since d is a constant, $E\{\psi'' d(bg_J - r)\}$ is zero under constant absolute risk aversion. Therefore, $E\{\psi'' bg_Q(bg_J - r)\}$ is positive when inputs are normal and negative when inputs are competitive. The third expectation, namely $E\{\psi'' bg_{JQ}\}$, is also positive when inputs are normal and negative when inputs are competitive, from the fact that ψ' is positive and the definition of normal and competitive inputs. Thus, the entire term $E\{\psi''(bg_J-r)(bg_Q-P) + \psi' bg_{JQ}\}$ is positive for normal inputs and negative for competitive inputs. Therefore, (85) and (88) are negative for normal inputs and positive for competitive inputs and the following propositions can be stated.

Proposition 15: Given the assumptions of Proposition 13, an increase in the price of contracts for the variable input will decrease the quantity of the fixed input when inputs are normal and decrease the quantity of the fixed input when inputs are competitive.

Proposition 16: Given the assumptions of Proposition 13, an increase in the price of the fixed input will decrease the number of contracts purchased for the variable input when inputs are normal and increase the number of contracts purchased when inputs are competitive.

We now turn to an investigation of the effects of the existence of the contract market itself on the firm's input and output decisions. As in the models of the firm selling contracts when there is output price uncertainty, we do this by introducing a constraint on the amount of contracting the firm may do and determine the effects of relaxing that constraint.

In the models of the firms which produce output subject to price uncertainty it was argued that it was appropriate to restrict the analysis of the effects of introducing a contract market to situations in which those firms would choose to sell rather than purchase contracts for future delivery. For firms which purchase inputs subject to price uncertainty, we make the analoguous argument that attention is appropriately restricted to situations in which those firms would purchase contracts for future delivery of those inputs. A sufficient condition for the firm which employs the variable input subject to price uncertainty to choose to purchase a positive number of contracts is assumption IIIG), which is that the contract price is less than or equal to the expected value of the spot price. This can be demonstrated as follows. The derivative of the expected utility of profits with respect to Q is $E\{\psi'\cdot(bg_Q-p)\}$. However, from equation (70) $bg_Q=p$, so that the derivative can be written as $E\{\psi'\cdot(p-P)\} = E\{\psi'\cdot p\} - E\{\psi'\}\cdot P$, which is equal to the covariance between ψ' and p when $P = E\{p\}$. When Q is zero, the covariance between ψ' and p is positive, since $\partial\psi'/\partial p = -\psi''\cdot q$, which is positive, since, by IIIF), ψ'' is negative, and, by IIIE), q is positive when Q is zero. When P is less than $E\{p\}$, $E\{\psi'\cdot p\} - E\{\psi'\}\cdot P$ will also be positive, since $E\{\psi'\}$ is positive. Therefore, the derivative of the expected utility of profits with respect to Q, when evaluated at Q equal to zero, is positive for $P \leq E\{p\}$, and the expected utility of profits is at a maximum for a value of Q greater than zero. Since Q is strictly positive at $P = E\{p\}$, it will generally also be the case that there will be values of $P > E\{p\}$ for which the firm will choose to purchase a positive number of contracts, so it can be seen that $P \leq E\{p\}$ is only a sufficient condition for the firm to purchase contracts. Values of P in excess of $E\{p\}$, but sufficiently low that the firm still chooses to purchase contracts, corresponds to the notion that the risk averse firm is willing to advance a "risk premium" to avoid the uncertainty of the spot market input price.

We again introduce the parameter $\overline{Q}$, which is the maximum number of contracts the firm is allowed to purchase. For a given value of P, $\overline{Q}$ is to be thought of as ranging in value from zero to the number of contracts the firm would choose to purchase in order to maximize the expected utility of profits, if it were free to do so. The effect of introducing a contract market where none existed previously is thus the cumulative effect of allowing $\overline{Q}$ to vary from zero up to its maximum value.

The constraint on the number of contracts the firm may purchase is thus written

$$\overline{Q} - Q \geq 0, \qquad (92)$$

which will always hold with equality for $P \leq E\{p\}$ and $\overline{Q}$ taking on the values prescribed above. The firm's problem is to solve the constrained dynamic programming problem,

$$\max \mathcal{L} = E\{\psi(\pi)\} + \mu\ (\overline{Q}-Q), \qquad (93)$$

where μ is the Lagrange multiplier associated with the constraint. The first order conditions, which, given our assumptions, will hold with equality, are

$$\frac{\partial \mathcal{L}}{\partial J} = E\{\psi' \cdot (bg_{J-r})\} = 0 \qquad (94)$$

$$\frac{\partial \mathcal{L}}{\partial Q} = E\{\psi' \cdot (bg_Q - P)\} - \mu = 0 \qquad (95)$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = \overline{Q} - Q \geq 0 \qquad (96)$$

Totally differentiating the first-order conditions gives:

$$\begin{bmatrix} E\{\psi''(bg_J-r)^2 + \psi'bg_{JJ}\} & E\{\psi''(bg_J-r)(bg_Q-P) + \psi'bg_{JQ}\} & 0 \\ E\{\psi''(bg_J-r)(bg_Q-P) + \psi'bg_{JQ}\} & E\{\psi''(bg_Q-P)^2 + \psi'bg_{QQ}\} & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} dJ \\ dQ \\ d\mu \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -d\overline{Q} \end{bmatrix} \qquad (97)$$

The effect of allowing the firm to purchase more contracts for the variable input on the amount of the fixed input hired is

$$\frac{\partial J}{\partial \overline{Q}} = \frac{E\{\psi''(bg_J-r)(bg_Q-P)+\psi'bg_{JQ}\}}{\Delta} \qquad (98)$$

where

$$\Delta = -E\{\psi''(bg_J-r)^2 + \psi'bg_{JJ}\} > 0 \qquad (99)$$

As was shown previously, under the assumption of constant absolute risk aversion, $E\{\psi''(bg_J-r)(bg_Q-P) + \psi' bg_{JQ}\}$ is positive when inputs are normal and negative when inputs are competitive. Therefore, the following proposition can be stated:

Proposition 17: Assume the conditions of Proposition 13, except that there does not exist a constant market. The introduction of a market in forward contracts for the variable input with the contract price less than or equal to the mean of the spot price, will increase the quantity of the fixed input hired when inputs are normal and decrease the quantity of the fixed input hired when inputs are competitive.

The effect of allowing the firm to purchase more contracts for the variable input on the total amount of the variable input hired for a given spot price is:

$$\left.\frac{\partial(Q+q)}{\partial\bar{Q}}\right|_P = \frac{\partial Q}{\partial\bar{Q}} + \left.\frac{\partial q}{\partial Q}\right|_P \frac{\partial Q}{\partial\bar{Q}} + \left.\frac{\partial q}{\partial J}\right|_P \frac{\partial J}{\partial\bar{Q}} \tag{100}$$

From (73), $\frac{\partial q}{\partial Q} = -1$. Therefore (100) reduces to

$$\frac{\partial(Q+q)}{\partial\bar{Q}} = \left.\frac{\partial q}{\partial J}\right|_P \frac{\partial J}{\partial\bar{Q}} \tag{101}$$

From (72), $\left.\frac{\partial q}{\partial J}\right|_p = \left.\frac{-g_{QJ}}{g_{QQ}}\right|_p$. Therefore $\left.\frac{\partial q}{\partial J}\right|_p$ is positive for normal inputs and negative for competitive inputs. From (98), $\frac{\partial J}{\partial \bar{Q}} = \frac{E\{\psi''(bg_J-r)(bg_Q-P) + \psi' bg_{JQ}\}}{\Delta}$, which was also found to be positive for normal inputs and negative for competitive inputs. Therefore both derivatives in (101) are positive for normal inputs and negative for competitive inputs and their product is always positive. The following proposition can thus be stated:

<u>Proposition 18:</u> Given the conditions of Proposition 17, the introduction of a market in forward contracts for the variable input with the contract price less than or equal to the expected value of the spot price will increase the total quantity of the variable input employed at each spot price.

The effect on output of allowing the firm to purchase more contracts for the variable input, for a specific spot price, is

$$\left.\frac{\partial g}{\partial \bar{Q}}\right|_p = \left.g_Q\right|_p \left(\frac{\partial Q}{\partial \bar{Q}} + \left.\frac{q^*}{Q}\right|_p \frac{\partial Q}{\partial \bar{Q}}\right) + \left.g_J\right|_p \frac{\partial J}{\partial \bar{Q}} \tag{102}$$

Since $\frac{\partial q^*}{\partial Q} = -1$, the coefficient of g_Q is zero, reflecting the fact that, for a given level of the fixed input and a specific spot price, the total quantity of the variable input hired (Q+q*) will be independent of the quantity purchased by contract. The entire effect on output of relaxing $\overline{Q}$ is thus transmitted through the effect on the amount of the fixed input hired. Since g_J is positive, Proposition 18 implies the following:

Proposition 19: Given the conditions of Proposition 17, the introduction of a market in forward contracts for the variable input with the contract price less than or equal to the expected value of the spot price will, for any spot price, increase the output of the firm when inputs are normal and decrease the output of the firm when inputs are competitive.

As was true for the models of the firm selling contracts, the restriction on the contract price relative to the mean of the spot price is only a sufficient condition to state the effect of the introduction of the contract market. The conclusions of propositions 17, 18, and 19 would also hold for any contract price sufficiently low that the firm would choose to purchase contracts at that price, even if the contract price were somewhat greater than the mean of the spot price.

VARIABLE ABSOLUTE RISK AVERSION

In this section we relax the assumption that the firm's utility function has the property of constant absolute risk aversion and derive expressions for the effects of changes in P and r on the *ex ante* decision variables, J and Q. The shift parameter, α, is again introduced to represent lump-sum additions to the firm's income. The firm's utility function is written as $\psi(\pi+\alpha)$. Differentiating (77) and (78) with respect to α and the *ex ante* decision variables gives:

$$\begin{bmatrix} E\{\psi''(bg_J-r)^2 + \psi' bg_{JJ}\} & E\{\psi''(bg_J-r)(bg_Q-P) + \psi' bg_{JQ}\} \\ E\{\psi''(bg_J-r)(bg_Q-P) + \psi' bg_{JQ}\} & E\{\psi''(bg_Q-P)^2 + \psi' bg_{QQ}\} \end{bmatrix} \begin{bmatrix} dJ \\ dQ \end{bmatrix}$$

$$= \begin{bmatrix} -E\{\psi''(bg_J-r)\} \\ -E\{\psi''(bg_Q-P)\} \end{bmatrix} \begin{bmatrix} d\alpha \end{bmatrix} \qquad (103)$$

Solving from (103), the effects of a change in the firm's wealth position on the amount of the fixed input hired and the number of contracts for the variable input purchased are:

$$\frac{\partial J}{\partial \alpha} = \frac{1}{\Delta}\,[E\{\psi''(bg_Q-P)\}\cdot E\{\psi''(bg_J-r)(bg_Q-P) + \psi' bg_{JQ}\}$$

$$-E\{\psi''(bg_J-r)\}\cdot E\{\psi''(bg_Q-P)^2+\psi' bg_{QQ}\}] \qquad (104)$$

$$\frac{\partial Q}{\partial \alpha} = \frac{1}{\Delta}\,[E\{\psi''(bg_J-r)\}\cdot \{\psi''(bg_J-r)(bg_Q-P) + \psi' bg_{JQ}\}$$

$$-E\{\psi''(bg_Q-P\}\cdot E\{\psi''(bg_J-r)^2+ ' bg_{JJ}\}] \qquad (105)$$

Using (104) and (105), equations (85) - (88) may be expressed as

$$\frac{\partial J}{\partial P} = \left.\frac{\partial J}{\partial P}\right|_{\substack{\text{constant}\\ \text{absolute}\\ \text{risk aversion}}} - \frac{\partial J}{\partial \alpha}\cdot Q \qquad (106)$$

$$\frac{\partial Q}{\partial P} = \left.\frac{\partial Q}{\partial P}\right|_{\substack{\text{constant}\\ \text{absolute}\\ \text{risk aversion}}} - \frac{\partial Q}{\partial \alpha}\cdot Q \qquad (107)$$

$$\frac{\partial J}{\partial r} = \left.\frac{\partial J}{\partial r}\right|_{\substack{\text{constant}\\ \text{absolute}\\ \text{risk aversion}}} - \frac{\partial J}{\partial \alpha}\cdot J \qquad (108)$$

$$\frac{\partial Q}{\partial r} = \left.\frac{\partial Q}{\partial r}\right|_{\substack{\text{constant}\\ \text{absolute}\\ \text{risk aversion}}} - \frac{\partial Q}{\partial \alpha} \cdot J \qquad (109)$$

As in the models of contracting under output price uncertainty, when absolute risk aversion varies with income, the effects of changes in the parameters on the firm's ex ante decision variables may be expressed as sums of two types of effects. The first of these is the result of changes in the incentives the firm faces and are the same as those present when absolute risk aversion does not change. This may be termed the substitution effect, since it involves the firm's substituting the fixed input for the variable input or contract purchases for spot purchases. The second type of effect results from a change in the degree of the firm's risk aversion as the certain income of the firm changes due to a change in the cost of an input. Because this effect is due to changes in the profit of the firm, it might be termed the income effect. Again, as in the case of the firm contracting under uncertainty with ex post flexibility, the sign of the income effect has not been determined, even when absolute risk aversion is assumed an increasing or decreasing function of income.

IV. EQUILIBRIUM IN A FIXED-PRICE CONTRACT MARKET

We have considered in the two previous sections the behavior, under the assumption of *ex post* flexibility, both of firms which sell contracts for their output and of firms which purchase contracts for their variable inputs. In this section we link those two types of firms by assuming that the output of the first type is an intermediate good which is the variable input of the second type. Ignoring the possibility of participation by outside speculators, the agents in the contract market for the intermediate good are thus these two types of firms.

We define an equilibrium contract price in such a competitive contract market to be a price such that the total quantity of contracts which the producing firms are willing to supply is equal to the total number of contracts which the purchasing firms demand, each firm having chosen the quantity of the fixed input to employ and the number of contracts to buy or sell in such a way that the expected utility of its profits is maximized. For simplicity, we will not consider here questions of exit or entry, and take the number of firms on each side of the market to be determined independently of the existence of and changes in the contract market. Additional simplifying assumptions which we make are that all the firms on each side of the market are identical and that there are equal numbers of supply and demand firms so that we may consider one representative firm on each side of the market and the transactions between them.

The notation employed in this section is the same as that introduced in the sections on the individual firms under _ex post_ flexibility. Distinctions between supply and demand variables will be denoted by the subscripts S and D, respectively.

Recall from equations (23), (26), (32) and (33) that the profits and the _ex ante_ and _ex post_ first-order conditions of the supply firm are

$$\pi_S = pq_S + PQ_S - wL - rK \tag{110}$$

$$\frac{\partial E\phi(\pi_S)}{\partial K} = E\{\phi'(pf_K - r)\} = 0 \tag{111}$$

$$\frac{\partial E\phi(\pi_S)}{\partial Q} = E\{\phi'(P-p)\} = 0 \tag{112}$$

$$\frac{\partial \pi_S}{\partial L} = pf_L - w = 0 \tag{113}$$

and from equations (69), (70), (77) and (78) that the profits and the _ex ante_ and _ex post_ first-order conditions of the demand firm are

$$\pi_D = b \cdot g(J, Q_D + q_D) - pq_D - PQ_D - rJ \tag{114}$$

$$\frac{\partial E\psi(\pi_D)}{\partial J} = E\{\psi'(bg_J - r)\} = 0 \tag{115}$$

$$\frac{\partial E\psi(\pi_D)}{\partial Q} = E\{\psi'(bg_Q - P)\} = 0 \tag{116}$$

$$\frac{\partial \ddot{\pi}_D}{\partial q} = bg_Q - P = 0 \qquad (117)$$

For the firms participating in the markets for the intermediate good, there are two distinct equilibria. The first is an ex ante equilibrium in the contract market, which occurs before the spot price is observed or any spot transactions take place. The usual case for an equilibrium in the contract market is for Q* to take on a positive value. However, such an equilibrium will not exist when the supply firm's subjective probability distribution over spot prices is so much higher than the demand firm's subjective probabilty that no intersection of $Q_S(P)$ and $Q_D(P)$ occurs at a positive value of Q.

$$Q_S(P^*) = Q_D(P^*) = Q^* \qquad (118)$$

where $Q_S(P)$ and $Q_D(P)$ are the supply and demand functions for contracts. These supply and demand equations for contracts result from the maximization of the expected utilities of (110) and (114), respectively, and satisfy equations (111), (112), (115) and (116) ex ante, given that (113) and (117) will be satisfied ex post. Such an equilibrium will not exist when the supply firm's subjective probability distribution over spot prices is so much higher than the demand firm's subjective probability distribution that no intersection of $Q_S(P)$ and $Q_D(P)$ occurs. If the equilibrium does exist, however, and if the firms have constant absolute risk aversion utility functions, it will be a unique and stable equilibrium, since, from Propositions 7 and 13, the supply function for contracts is strictly upward-sloping and the demand function is strictly downward-sloping. Furthermore, a condition which comes very close to being a sufficient condition for an equilibrium at a positive value of Q when firms have constant absolute risk aversion utility function is that the supply firm's mean of spot prices be not greater than the demand firm's mean. This is the case because, as we have seen, the supply firm will want to supply a positive quantity of contracts at its evaluation of the mean and the demand firm will demand a positive quantity at its evaluation of the mean. Further, the supply function is positively sloped and the demand functions is negatively sloped. Therefore, if the supplier's evaluation of the mean is not greater than the demander's, the intersection of the supply and demand functions will occur at a value of Q not lower than the minimum of the quantities the suppliers would desire to supply and the demanders would desire to demand at their respective evaluations of the mean of the spot price. The only case which is not ruled out by these conditions is that the intersection would not occur at a finite value of P. This unusual case would occur only if the supply and demand functions approached values of Q asymptotically such that no intersection occurs.

Excluding this case, however, a sufficient condition for an equilibrium to exist at a positive value of Q is that the supplier's evaluation of the mean be less than or equal to the demander's evaluations of the mean. Obviously, however, it is not a necessary condition, since there exist the possibility of equilibrium values of Q which are positive even if the mean of the supplier's is greater than the mean of the demander's.

Another interesting question regarding the contract market equilibrium concerns the value of the equilibrium contract price in relation to the mean or means of the spot price distributions. Without more specific restrictions, however, no qualitative predictions can be made, for in all the cases mentioned, the equilibrium contract price could occur at virtually any of the possibilities: greater than, less than, or equal to the means of the distributions of either side. Thus, an empirical finding that forward prices do or do not include a "risk premium," one way or the other, relative to the means of the spot prices, has very little meaning.

The other equilibrium is the ex post spot market equilibrium. This equilibrium is characterized by a spot price, p*, and a spot quantity, q*, such that

$$q_S(p^*) = q_D(p^*) = q^* \tag{119}$$

where $q_S(P)$ and $q_D(P)$ are the supply and demand functions for spot quantities, respectively. Since the quantities of the fixed inputs for both the supply and demand firms and the number of contracts bought and sold are determined before the observation of the spot price, the spot market supply and demand functions result from the maximization of equations (110) and (114), with the ex ante variables treated parametrically. These functions satisfy equations (113) and (117), respectively.

If the contract market is not operative, then the only ex ante decision variables are the fixed inputs. The profits of the supply firm without contracting are

$$\pi_S = pq_S - wL - rK \tag{120}$$

and the first-order conditions are (111) and (113). The profits of the demand firm are

$$\pi_D = b \cdot g(J, q_D) - pq_D - rJ \tag{121}$$

and the first-order conditions are (115) and (117).

Equations (111) and (115) indicate that, for both supply and demand firms, with and without contracting, the fixed input will be hired up to the point where the expected change in utility associated with the change in revenue due to an increase in the fixed input equals the expected change in utility associated with the increase in costs. However, the quantities of the fixed inputs which satisfy equations (115) and (117) will not, in general, be the same with and without contracting, since, from equations (110), (114), (120), and (121), the profit functions, and, therefore, the marginal utilities of profits, are different. Since the quantities of the fixed input hired will not, in general, be the same, neither will the quantities of the variable input hired and the production levels of the firms remain the same with the introduction of contracting. This will be the case because the levels of the ex post variable depend on the levels of the ex ante inputs chosen. In particular, constant absolute risk aversion implies that the introduction of a contract market would result in an equilibrium such that a positive number of contracts would be bought and sold by the demand and supply firms, respectively. Then the total amount of transactions for the intermediate good at any spot price would be increased by the introduction of the contract market. This statement is clearly true when the equilibrium contract price is equal to the mean of spot distributions of both firms From Proposition 12, when the contract price is greater than or equal to the mean of the spot prices, the total amount of the intermediate good produced at any spot price will be increased by the introduction of a contract market. Similarly, from Proposition 18, when the contract price is less than or equal to the mean of the spot prices, the total amount of the intermediate good purchase will be increased by the introduction of a contract market. However, the conclusions of Proposition 12 and 18 also hold for all values of the contract price such that positive contract transactions would result from the introduction of a contract market. Therefore, under constant absolute risk aversion, when the introduction of a contract market would result in positive contract transactions, the total amount of transactions for the intermediate good would be increased for any spot price by the introduction of a contract market.

The Contract Market and the Vertically Integrated Firm

The fundamental way in which participation in the contract market alters the incentives for the firms is that, for the supply firm the revenues from, and for the demand firm the costs of, the inframarginal units of the intermediate good are made certain. If only the spot market exists, an increase in the spot price results in what is essentially a lump-sum transfer from the demand firm to the supply firm for the inframarginal units, and vice-versa for a decrease in the spot price. No such transfers take place when the inframarginal units are contracted for. If the firms were merged, rather than separate, so that a single firm both produced the intermediate good and employed it in the production of the final good (or another intermediate good), then such lump-sum transfers would not take place, either. Having observed this similarity between the role of the contract market and that of the vertically integrated firm in eliminating some of the uncertainty associated with uncertain spot prices, a question which may be asked is: Are the investment and production decisions made by the integrated firm with a given utility function the same as those made by two individual firms, each with that same utility function and engaged in contracting, if all have the same probability density function over spot prices? The answer to this question is that there exist configurations of spot price distributions, production functions and utility functions for which the decisions are all identical. However, for other configurations, the decisions may be different. The following two examples illustrate one configuration for which the decisions are the same and one for which they are slightly different. For ease of computation, the supply firm in the following examples is assumed to have a production function which exhibits constant returns to scale. Constant returns to scale production functions were not considered in the text, since firm size is not necessarily determinant in such cases. However in these examples, the decreasing returns to scale production function of the demand firm insures that the equilibrium sizes of the two firms will be finite.

EXAMPLE 1

In this example, the utility of profits for all firms is $\pi^{\frac{1}{2}}$ which has the property of decreasing absolute risk aversion. The other assumptions are listed below.

$p = 1$ with probability .5

$p = 3$ with probability .5

$w = 1$

$r = 1$

$b = 8$

$$f(K,L) = K^{1/2} L^{1/2}$$

$$q(J,Q + q) = J^{1/4} (Q + q)^{1/2}$$

When the firms engage in contracting, the profit functions are

$$\pi_S = p(K^{1/2}L^{1/2} - Q) + PQ - L - K \qquad (122)$$

$$\pi_D = 8\ J^{1/2}(Q + q)^{1/2} - PQ - pq - rJ \qquad (123)$$

The <u>ex ante</u> equilibrium values which result from maximizing the expected utilities of π_S and π_D are

P = 1.75

Q = 51.2

J = 36

K = 51.2

If the firms are integrated, the joint profit function is

$$\pi_J = 8\ J^{1/4}(K^{1/2}\ L^{1/2} + q)^{1/2} - L - pq - r(K + J)$$

The ex ante values are

J = 36

K = 51.2

Thus, the quantities of the fixed inputs hired are the same for the integrated firm as they are for the separate firms engaged in contracting.

EXAMPLE 2

In this example, the utility function for all firms is $-e^{-\pi}$, which has the property of constant absolute risk aversion. All the other assumptions are retained from the previous example.

With this utility function, the ex ante equilibrium values under contracting are

$P = 1.75$

$Q = 32.255413$

$J = 36$

$K = 32.510826$

For the integrated firm, the ex ante fixed inputs are

$J = 36$

$K = 32.255413$

In this example the fixed input hired by the demand firm is the same as that hired by the joint firm, but that hired by the supply firm is slightly higher. Thus, while the difference is small in this case, this example does show there exist configurations such that contracting firms make different ex ante decisions than would integrated firms.

V. CONCLUDING REMARKS ON FORWARD CONTRACTING AND THE URANIUM INDUSTRY

The motivation for developing the theoretical models of the effects of forward contracting on firm decisions arose out of the suggestion, outlined in the first paper, that peculiarities in the development of the contracting market in the early years of the commercial uranium industry led to underinvestment by uranium producing firms. Since forward contracting is a widespread business practice, however, the models developed in the second paper were designed to apply to a wider class of firms than simply those producing uranium. In developing those models, various assumptions were made about the characteristics of firms engaged in contracting. One assumption simply guaranteed that firms would be of finite size, while others were found to be sufficient to allow the derivation of specific results. Given the different assumptions employed, it is important to ask at this point whether those assumptions describe the actual features of the uranium industry and, therefore, whether the models help explain decisions in that industry.

Before examining the assumptions themselves, let us outline for the uranium industry the economic variables which correspond to those in the models. First, the firms in the industry are in large measure integrated mining and milling firms. This description of what constitutes a typical firm is supported by the fact that, in 1970, 95.5 percent of the ore milled was mined captively (U.S. Federal Trade Commission, 1974). That is, each milling firm mined nearly all of the ore that it milled.

Since the firms are integrated, they must decide both how much exploration and development and how much mining and milling capacity to invest in. These variables are seen as the variables in the model with *ex post* flexibility as the variables which are chosen before spot prices are observed and production begins. When production begins the firms can vary output by varying the quantities and grades of ore mined and milled. Once the mining and milling capacities are built, it requires more ore and associated variable inputs to obtain a given amount of yellowcake, as lower grades of ore are processed. In other words, the possibility exists for substitution between the variables considered fixed and those considered variable. For example, in the long run, increased production may be brought about either by developing a new ore body and processing it in a new mill or by using more variable inputs to process lower grades of ore with the same fixed inputs.

Turning now to a comparison between the assumptions of the model and the characteristics of uranium production, an assumption was made in the model that production processes were characterized by decreasing returns to scale in order to insure that firm size was finite. However, the actual uranium production technology is not characterized by decreasing returns to scale over all ranges of output. The data on capital costs indicate that larger mills can be built for lower capital costs per unit of ore processed. This implies that for a given grade of ore the production process is subject to increasing returns to scale. This does not mean that ever lower average costs can be obtained by expanding indefinitely, however. In producing from a given ore body, average costs may decline over some range of plant size, but as more and more output is obtained from a given ore body, the average grade of ore used declines and the ore body itself becomes exhausted. As that happens, the unit costs must eventually rise as plant size rises.

The preceding remarks indicate that the average cost curve for production from a given ore body is U-shaped. A U-shaped cost curve for production from a given ore body does not insure, however, that the firm itself will be of finite size since firms might simply replicate least cost-sized plants indefinitely. In order to replicate identical plants, all ore bodies would have to be of equal ore grades and sizes and available in unlimited numbers. In reality, however, ore bodies differ in grades, sizes, and production costs, and those with the lowest unit costs tend to be exploited first. Thus, while a firm may have a U-shaped cost curve for each plant, its average costs will tend to increase as it increases the number of ore bodies it exploits. Thus, a firm can reasonably be expected to face decreasing returns to scale, viewed in long-run terms.

A second important assumption of the models in the second paper is that firms participating in the contract market are risk averse. Risk aversion insures that firms that wish to participate in the contract market will choose to supply a finite quantity of contracts at the contract price. Direct evidence on the risk preferences of firms is difficult to come by. It might be argued that firms were reluctant to undertake new investment because of losses sustained in the 1960s, when the anticipated commercial market failed to develop. Such behavior is not necessarily attributable to risk aversion, however, since firms might simply, because of this previous experience, have attached somewhat more likelihood to the possibility of the market again failing to materialize than they otherwise would have. Thus, while risk aversion may not be an unreasonable assumption, it remains an empirical question whether uranium producing firms actually were risk averse.

Risk aversion alone was not found to be sufficient to derive comparative statics results in the theoretical models, however. A necessary condition for some of the results was that the fixed and variable inputs not be competitive in the extreme. While evidence is not available to test that assumption, it is probably not unreasonable for most production processes.

In addition, the assumption of constant absolute risk aversion was found to be a sufficient but not a necessary condition in deriving comparative statics results. In fact, it does not appear that a well-defined set of necessary conditions on utility functions can be enumerated. As a practical matter, we do know, however, that when absolute risk aversion is not constant, the comparative statics results are reversed only when the changes in the risk preferences of the firms as income changes outweigh the pure substitution effects due to changes in incentives. While such income effects cannot be ruled out theoretically, it is probably not unreasonable to assume, in the absence of evidence to the contrary, that they are small relative to the substitution effects. Thus, if we are willing to assume simply that uranium producers are risk averse, the predictions of the models seem reasonable hypotheses about their behavior.

Finally, the most difficult aspect to judge is what the expectations of the potential sellers were with respect to future spot prices, since empirical evidence on expectations is unavailable. Subject to the qualifications discussed above, and holding expectations constant, an equilibrium contract price sufficiently high to have induced uranium producers to supply contracts would have resulted in more investment in mining and milling capacity. However, if Westinghouse's willingness to sell contracts for forward delivery at $8-$10 per pound, a price considerably below long-run costs, affected the expectations of uranium producers over spot prices as well as their ability to sell expected utility-increasing contracts, then an increase in the contract price would have induced even more new investment by raising their spot price expectations.

In conclusion, the suggestion raised in the first paper that if Westinghouse had not kept the equilibrium contract price significantly below the long-run costs of uranium producers, then uranium producers would have sold contracts and undertaken additional new investment as a result seems to be supported by the predictions of the models developed in the second paper. However, we cannot rule out the possibility that underinvestment occurred simply because Westinghouse's behavior influenced the expectations of the other producers by leading them to believe that there was a significant chance that spot prices for uranium would be below long-run costs.

APPENDIX

This appendix provides specific examples which illustrate the choices made by the supply firm under ex post flexibility. In the first example, the distribution over spot prices is symmetric about the contract price, which is assumed equal to the mean of the distributions. In the second example the contract price is assumed equal to the mean of an asymmetric distribution.

EXAMPLE 1

Assume that $\phi(\pi) = \pi^{\beta}$, with $0 < \beta < 1$ and that $f(K, L) = K^{\alpha} L^{\frac{1}{2}}$, with $0 < \alpha < \frac{1}{2}$.

The utility of profits is thus

$$\phi(\pi) = [p(K^{\alpha} L^{\frac{1}{2}} - Q) + PQ - wL - rK]^{\beta} \tag{A1}$$

Maximizing short run profits implies

$$\frac{\partial \pi_{SR}}{\partial L} = \frac{1}{2} pK^{\alpha}L^{-\frac{1}{2}} - w = 0 \tag{A2}$$

Solving A2 for $L^{\frac{1}{2}}$ and L gives

$$L^{\frac{1}{2}} = \frac{1}{2} \frac{p}{w} K^{\alpha} \tag{A3}$$

and

$$L = \frac{1}{4} \frac{p^2}{w^2} K^{2\alpha} \tag{A4}$$

Substituting for L in the profit function gives

$$\phi(\pi) = \left[p\left(\frac{1}{2}\frac{p}{w} K^{2\alpha} - Q\right) + PQ - \frac{1}{4}\frac{p^2}{w} K^{2\alpha} - rK\right]^{\beta}$$

$$= \left[\frac{1}{4}\frac{p^2}{w} K^{2\alpha} + Q(P - p) - rK\right]^{\beta} \quad \text{(A5)}$$

The first order conditions are

$$\frac{\partial E\phi(\pi)}{\partial K} = E\left\{\beta\left[\frac{1}{4}\frac{p^2}{w} K^{2\alpha} + Q(P-p) - rK\right]^{\beta-1} \left[\frac{\alpha}{2}\frac{p^2}{w} K^{2\alpha-1} - r\right]\right\} = 0 \quad \text{(A6)}$$

$$\frac{\partial E\phi(\pi)}{\partial Q} = E\left\{\beta\left[\frac{1}{4}\frac{p^2}{w} K^{2\alpha} + Q(P-p) - rK\right]^{\beta-1} \left[P - p\right]\right\} = 0 \quad \text{(A7)}$$

Now assume there are three possible spot prices, each of which might occur with probability 1/3. The contract price is assumed equal to one of the spot prices, as well as the mean of the distribution of the spot prices. Denoting the difference between the contract price and the extreme spot prices by a, the three spot prices can be written as

$$p_1 = P - a$$

$$p_2 = P \quad \text{(A8)}$$

$$p_3 = P + a$$

Substituting the specific values of the spot prices from A8 into A7 gives

$$\frac{1}{3}\beta\left[\frac{1}{4}\,\frac{(P-a)^2}{w}\,K^{2\alpha}+Qa-rK\right]^{\beta-1}\left[a\right] = \frac{1}{3}\beta\left[\frac{1}{4}\,\frac{(P+a)^2}{w}\,K^{2\alpha}-Qa-rK\right]^{\beta-1}\left[a\right] \quad \text{(A9)}$$

Solving for Q from A9 gives

$$Q = \frac{P}{2w}\,K^{2\alpha} \quad \text{(A10)}$$

For any spot price, the output which the firm chooses to produce is

$$f(K,\,L(p)) = \frac{1}{2}\,\frac{p}{w}\,K^{2\alpha} \quad \text{(A11)}$$

Thus, in this example, the firm makes spot purchases when the spot price is below the contract price, makes no spot transactions when the spot price is equal to the contract price, and produces for spot sales when the spot price is higher than the contract price.

EXAMPLE 2

The only difference between this example and the previous one is the distribution over spot prices. The distribution is assumed to be

p_1 = P-2a, with probability 1/6

p_2 = P, with probability 1/2

p_3 = P+a, with probability 1/3

The equation which corresponds to (A9) in this example is

$$\frac{1}{6}\beta \; \frac{1}{4}\left[\frac{(P-2a)^2}{w} \; K^{2\alpha} + 2\;Qa - rK\right]^{\beta-1} \left[2a\right]$$

$$= \frac{1}{3}\beta\left[\frac{1}{4}\;\frac{(P+a)^2}{w}\;K^{2\alpha} - Qa - rK\right]^{\beta-1}\left[a\right] \qquad \text{(A12)}$$

Solving for Q from (A12) gives

$$Q = \frac{(P-\frac{a}{2})}{2w}\;K^{2\alpha} \qquad \text{(A13)}$$

Comparing (A13) with (A11), when the spot price is equal to the contract price, production for spot sales are found to be $\frac{a}{4w}\;K^{2\alpha}$. Thus, for this example, an asymmetric distribution implies that the firm will make spot transactions at the contract price, even though it is equal to the mean of the distribution.

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