COST AND TRAFFIC ANALYSIS OF DEMAND ACCESS SATELLITE NETWORKS

Thesis by
James Laurens Latimer

In Partial Fulfillment of the Requirements
For the Degree of
Doctor of Philosophy

California Institute of Technology
Pasadena, California

1978

(Submitted December 16, 1977)

Acknowledgement

I have received the valuable help of numerous individuals, corporations, and countries in obtaining traffic and cost data presented here. They are listed in the appendices. In particular, I must express my appreciation to Ed Posner for his valuable suggestions. Furthermore, I have drawn heavily from a number of published and unpublished works in writing this thesis. For example, substantial portions of section 3.2.2 are based on James Spilker's recent book with his permission. I have attempted to give adequate credit to the authors of the cited material and apologize to any author not adequately credited. Carrie Humphrey typed the equations.

The generous financial support of Hughes Aircraft Company through their fellowship program is gratefully acknowledged. I also appreciate the vocational experience at Bell Laboratories and Hughes Space and Communications Group. In addition, the grant by the Gilbert Fitzhugh Visiting Fellowship Committee enabled me to meet many interesting individuals who are doing research in the same field.

My parents gave me curiosity and their support, my friends gave me encouragement, and M.Y. gave me the dark side of reality. Finally, I must admit that without the patience and encouragement of my thesis adviser, John R. Pierce, I would never have finished.

"Nos hac novimus esse nihil"

Abstract

A method using a Markov process model is described and used to analyze the traffic capacity and call blockage probability of an SCPC (single channel per carrier) satellite network. The model is shown to be suitable for telephone traffic. The traffic density for certain networks whose states can be ordered in a linear fashion (birth/death process) is derived and shown to exhibit an Erlang B distribution. The application of the Erlang B distribution to PA (preassigned) and limited cases of FVDA (fully-variable demand access) and SVDA (semi-variable demand access) is demonstrated. The Erlang B equation does not apply when an SVDA network exhibits partial sharing of channels by different earth stations. An example of this is given and the capacity and blockage probability solved exactly by the use of steady state Markov statistics. The traffic capacity and blockage for a satellite with channel switching capability in the repeater is shown to be equivalent to an SVDA network.

SCPC should be used in place of preassigned frequency division multiplex (FDM) channels whenever the traffic load of the earth station is very light. In this case it is better to use a few expensive SCPC channels than many less expensive FDM channels. FDM is probably better if two stations have traffic between them requiring more than 15-25 channels.

A simulation program was written to determine the blockage probability for those networks too large to do any other way. The goodness of the random number generator was tested and the results of the simulation were compared to known solutions.

Traffic data obtained from Latin American and Caribbean nations was used to estimate the number of satellite channels and earth station modems required and the cost of such a network. We have also presented as an example the improvement of SVDA over PA in the case of Jamaica where 1/4th as many modems would be required.

A rough analysis of earth station costs was made based upon information obtained from several companies. It is shown that a large part of the network cost is in the modems. We estimate the cost of a regional network for Latin America and the Caribbean to be about $25M. The earth stations represent about 40% of this cost, and 65% of the earth station expense is in the modems.

Table of Contents

1.0 INTRODUCTION

1.1 Overview Of The Problem

There are many locations in the world where it is not economically practical to establish a large scale terrestrial communication network. Frequently this is because the originating traffic is of low density and emanates from widely separate geographical locations. Under such circumstances a demand access satellite network may be attractive.

Such a satellite communication network should be only as complex as is necessary to accomplish its task. Today, however, the performance and cost for different network configurations is only partially understood and system designs are often empirical.

In a satellite network for telephone traffic, with which this thesis is primarily concerned, one is constrained to a design in which the probability that a user is blocked from making a call is less than some specified value. Knowing the relationship between the complexity of the network and probability of blockage for a given traffic density pattern, one can make a reasonable estimate as to the expected cost and optimal configuration.

1.2 Goal And Objectives

Goal - To provide the design engineer with a method of designing an efficient satellite network.

Objectives -
1) To determine the probability of blockage for a given traffic density pattern and network configuration.
2) To determine what factors affect the cost for a given network configuration.
3) To use the above to present a network configuration suited to Latin America and the Caribbean using actual traffic statistics.

1.3 Summary Of Results

This thesis is comprised of three parts. There is the background information and description of the makeup of a satellite system and some cost analysis. Then there is a very detailed mathematical technique to compute the probability that a call is blocked for some symmetric networks. Unfortunately actual networks will be only approximately symmetrical. Finally, a not so rigorous method is used to estimate the number of channels and modems required by an actual network. Heuristic reasons are given for believing the estimation to be valuable for the first stage of a design process, preceding a detailed simulation. Such a method of simulation is given and verified by comparison with closed form solutions of simple problems. It could be used for problems of more capacity.

The telephony network described herein is characterized by a single satellite acting as a repeater for many earth stations, each of which desires to communicate with several others.

The network could be set up with preassigned (PA) or demand assigned (DA) channels. Demand assignment is the form of channel assignment in which voice channels are assigned on demand between a pair of earth stations. There are two methods of demand assignment access, semi-variable (SVDA) and fully-variable (FVDA). The efficiency and cost of the system depends on the channel utilization and the choice of time division multiple access (TDMA) or frequency division multiple access (FDMA). A fully-variable system has a lower probability of call blockage for the same number of channels, but the added cost of additional ground terminal equipment and complexity may dictate a compromise. In practice a hybrid of both PA and DA channels could be used.

The satellite onboard switching is assumed to be capable of connecting any uplink frequency to any downlink frequency, and can make the connection for all such distinct pairs simultaneously. Under such conditions the probability that a connection cannot be made on the uplink is independent of a similar probability on the downlink. In practice a call is dropped unless connection can be made on both the uplink and downlink. This couples the traffic density distributions of the two links. For instance, if blockage occurs on the downlink side but not on the uplink side, then the call is dropped and an uplink channel is also freed. If, however, one is willing to obtain a slightly pessimistic answer, he can compute the blockage probability and capacity for the uplink and downlink halves of the system separately and combine appropriately for the total throughput blockage probability.

If one assumes Poisson call arrival statistics and exponential holding time, the system can be modelled as a Markov process. Examining the steady state solution of the probability state space, one can in principle determine both the capacity and blockage probability. For some simple networks, the probability of going from n channels in use to n+1 channels in use (and vice versa) is independent of the exact topological configuration of the channels. The states of the system can then be expressed simply as the number of channels in use. Since the states can be numbered sequentially and transitions can occur only between adjacent states, this is a birth/death process and is easily amenable to solution. For others, however, the lack of symmetry precludes a tractable solution, even with the aid of computers, in any network of more than a few earth stations and several channels because of the immense number of states.

One can obtain numerical results for a particular network by means of simulation. A good random number generator should be used in this case. This is because the manner in which the system is loaded depends strongly upon the interarrival times of calls occurring at about the same time. Thus the autocorrelation properties of the random number generator are very important.

In practice, the non-ideal nature of actual traffic data makes modelling a network very difficult if the density of traffic is low, precisely the conditions which favor a demand assignment network. Therefore, approximations must be used in an actual design.

If the actual probability of blockage is low, then to first order approximation, blockage at each of the earth stations is independent of that of the other earth stations and of blockage on the satellite. In this case we can use the Erlang B equation to compute blockage of each part separately. This approximation is not very good if the number of channels on the satellite is very small. In this case, however, one can use the numerical simulation method. The systems designer should take care to evaluate the adequacy of these approximations for his particular application.

Telephone traffic data was obtained from the various Latin American and Caribbean nations and calculations performed to estimate the channel capacity of a satellite to serve this region and the approximate size and cost of the earth stations. The configuration analyzed was a frequency division multiplex, frequency division multiple access, single channel per carrier (FDM,FDMA,SCPC) network.

Because the traffic density for most of the countries is very low and therefore subject to a proportionally large fluctuation, any model is at best qualitative and it is necessary to provide adequate margin for error. Earth station costs have been derived based upon current market prices obtained from a large number of vendors of the electronic subunits. It is likely that they could be made for less. It appears that the modem cost is a very large part of the overall network cost and designs that reduce this cost would be worth the effort.

2.0 TECHNOLOGICAL BACKGROUND

2.1 Background Of Satellite Telecommunications

With satellite communications, as with any other discipline, knowing the historical development aids understanding the framework of logic about the makeup.

In recent years, the importance of satellite communications has increased rapidly. Satellite communication is now a major means for domestic as well as international communications over long and moderate distances. Various governments and non-military organizations provide operational satellite communication service, including the Intelsat Consortium, the U.S. Government, the Indonesian government, and the Canadian domestic satellite communications network. Existing communication satellite networks are being enlarged, and several other countries are implementing their own domestic satellite communication networks.

In the past satellites have been built simply and conservatively because the price of any failure was too high to permit a technology that had not been proven reliable, and extensive testing was often expensive. Now, however, the scientific understanding of various technologies has advanced, making elaborate and complex technology more reliable. Under such circumstances the greater demand for service and performance may in fact dictate a sophisticated design because the overall costs may be less. Several examples may illustrate this point.

Recent satellite designs have incorporated shaped beam antenna patterns which are produced by a two dimensional array of horns and a parabolic reflector.[1] Since the currents in the reflector dish do not lie in a plane, the resultant aperture field distribution cannot be modelled as a linear array of point sources. The design of these shaped

[1] We are restricting the discussion to 4/6 and 11/14 GHz. Other methods are used at lower frequencies. The downlink is always the lower of the two frequencies.

beams requires extensive nonlinear computer modelling and optimization. After fabrication, extensive far-field range measurements and "tweaking" are used to meet the design specifications. The use of dual polarization also requires extensive computer analysis and manufacturing techniques that were unavailable until recently.

The understanding of intermodulation products is a second area in which advanced analysis has been fruitful in allowing greater performance. These "sideband-like" frequency components appear as noise whenever several modulated carriers are passed through a TWT (traveling-wave tube) because of its inherent nonlinearity. The problem is particularly irksome when the satellite traffic consists of many SCPC (single channel per carrier) signals. Two techniques to decrease intermodulation products, using smaller bandwidths per TWT (hence more TWT's) and "backing off" the power output of the TWT, both sacrifice the capacity of the system if carried to extremes. As in the case of the antenna designs, complex computer analysis may be used to distribute the channels optimally among the various TWT's and throughout the bandwidth so as to maximize the number of voice channels allowed and still allow a high power output from the TWT.

As in other areas of electronic design, microprocessors are becoming increasingly popular for satellite systems, not so much as space hardware (low volume demand and radiation hardening make them relatively unattractive) but as an important part of the earth station segment. In a satellite system with many earth stations, such as a demand access system in which the traffic pattern is reconfigured each minute, a microprocessor is an inexpensive means for controlling modem usage, channel allocation, and monitoring the status of each earth station site.

As of today only two commercial demand access satellite systems are in use, the Intelsat SPADE system and the Indonesian "Palapa" network.[2]

---

[2] The Aloha system is more experimental in nature.

The rest of the world's traffic is carried by channels each dedicated to be used solely by a pair of earth stations. Such a network, allocating channels on a permanent or "preassigned" basis, does not need some of the expensive equipment required by a network in which channels are allocated on a temporary or "on demand" basis. Because of the statistical nature of telephone traffic, a preassigned channel, particularly one allocated for use between two low traffic earth stations, will frequently be inactive. In a demand access system another earth station would be free to use the channel. The remaining question is, "how many channels are required and what is the cost?"

2.2 Description Of A Telephone Network

In order to understand the function of the satellite in a communication network, it is helpful to be familiar with some of the fundamentals of a typical terrestrial telephone system.

86% of international satellite traffic is telephone, telegraph, and data transmission (Lusignan, 1975). This thesis deals exclusively with telephone traffic.

When placing a call, the telephone is connected to a local central office via a subscriber loop. Each central office handles switching between a few hundred to 50,000 subscribers in the same area. Heavy traffic between two central offices in the same metropolitan area is carried by interoffice trunks. Sometimes the next level of hierarchy is used so that traffic from several central offices is directed to a tandem office.

Other levels of switching hierarchy include primary, sectional, and regional toll offices. Between different offices (of the same or different levels of hierarchy) the traffic is carried on trunks. What these trunks really are physically and the form of the voice signals on the trunks varies widely. From the telephone set to the central office a pair of wires is used to carry the signal. The central office employs concentrators, electrical equipment that essentially connects all phones in use at one time to a small number of distributing trunks. (This is sometimes called space switching.) At this point, time or frequency division multiplex is used to send several signals over the same circuit. There are numerous signal forms employed, including single sideband, frequency modulation, pulse code modulation, and phase modulation to transmit over wire, microwave links, or coaxial cables. In the case of FDM-FM, industry has standardized the FM carriers to sizes of 24, 60, and 132 channels. At each level in the network hierarchy, incoming signals are separated, amplified, and passed on to the right place. In the simplest sense an earth station acts as does any other office and the earth station - satellite - earth station radio link is just another circuit. In the broader sense it substitutes for hundreds of circuits by sharing its channels among many earth stations.

2.3 Description Of A Satellite Communication System

The basic system consists of a satellite acting as a repeater for many earth stations, each of which desires to communicate with several others.

Major elements in a satellite communications system include the satellite, a network of earth stations, and multiple-access communications equipment by which many earth stations can operate through a single satellite. In this thesis, emphasis is on the demand access satellite system, its traffic, and cost; many of the satellite and earth station subsystems not specifically related are omitted.

All commercial communications satellites are in orbits that are synchronous with the earth rotation. This allows considerable system simplification since each earth station then points continuously at the same satellite position. For the purposes of the following discussion, we are assuming that only single hops (that is, with the signal following a single earth-satellite-earth path) are allowed. Double

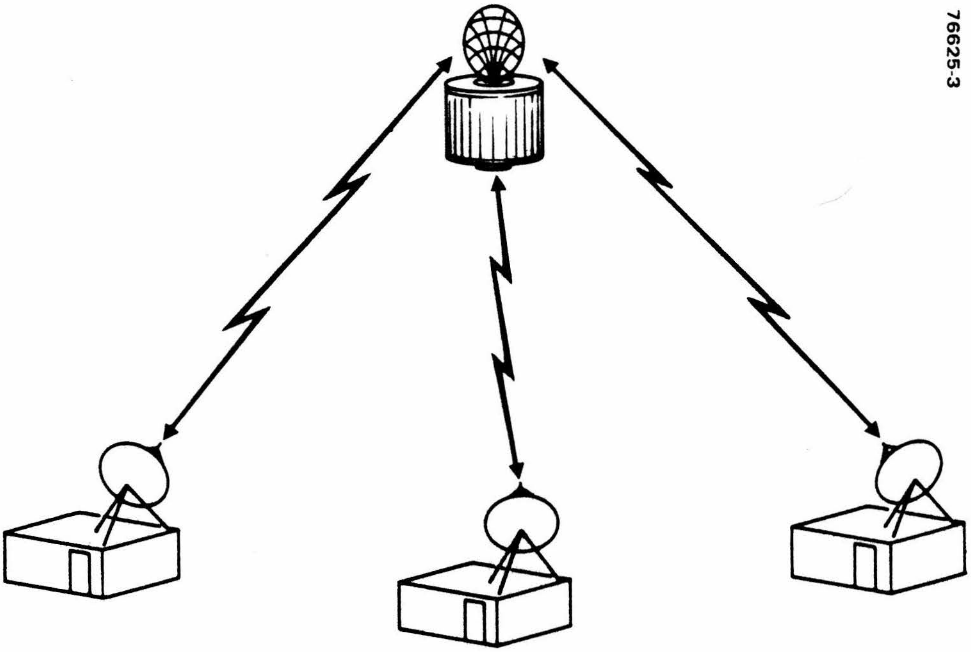


FIGURE 2-1. TYPICAL NETWORK

hops, in which the path of the signal makes two round trips to a satellite are not suitable for voice traffic because the delay is too great. Anyway, for a regional network one wouldn't need double hops.

Typically, at a given earth station a set of duplex analog (or digital) lines enters an earth station complex. These analog lines are filtered and multiplexed together.

Inputs can arrive at the earth station in a variety of analog and digital forms. These original information sources, analog and digital, commonly include: analog (voice, multiplexed voice, video, facsimile); digital (teletype, muliplexed teletype, computer input/output, digital television or facsimile, digitized voice).

The resulting information sources are then fed into the appropriate digital or analog modem that modulates a carrier at some convenient IF frequency (usually 70 MHz). Each modulated carrier is then up-converted to the appropriate RF frequency for transmission to the satellite. In some earth stations the modulation of the carrier takes place directly at RF.

It is usually convenient to multiplex signals that have the same ultimate destination into one single frequency block for transmission on a single RF carrier. This is called frequency division multiplex (FDM) and is not a form of demand access.

The modulated signals from several earth stations then arrive at the synchronous satellite transponder where they are relayed to the appropriate destination earth station by means of either an earth-coverage or narrow-beam satellite antenna.

Transponders used in satellite communications are usually channelized into separated frequency channels. Each has a peak-power-limited amplifier which operates in a quasi-linear or limiting mode and has a frequency translating repeater. Each channel operates with multiple carriers from separate earth stations. Bandpass filters are used to separate carrier channels and to provide isolation between high-level satellite power output and low-level inputs.

Multiple carriers arrive at, and must be relayed by, the satellite. Most communication satellites contain several parallel transponders, often with several narrow beam antennas to aid in the multiple access problem, particularly where the received signal levels differ widely for different classes of users. (The power levels may vary because of different size dishes, high power amplifiers of differing power outputs, or adjustments made to reduce intermodulation products.) Satellite transponders used for multiple access are often "multichannel" in configuration. That is, the transponder is channelized by frequency-selective filters to allow different frequency bands to be handled by separate amplifiers and antennas. This allows parallel amplification by smaller TWT's, switching to different downbeams, decrease in intermodulation effects, individual downlink frequency selection, and other nice things.

2.3.1 Network Components & Types Of Call Blockage -

In a single channel per carrier[3] (SCPC) network consisting of one satellite and a number of earth stations, the traffic capacity and call blockage of the system depend on the particular DA network configuration. This thesis first presents a generalized DA network configuration. Then assumptions and simplifications are made which lead to the various classes of commonly used DA configurations. Some of these can be modeled as linear processes (where the Erlang B equation applies) and some can't. Of the latter, a simplified case is solved using a Markov process.

The major components of a network are the incoming traffic, the earth station, the modem, the channel, and the satellite (see figure 2-2). The hardware description of these components will be left for later. Rather than presenting the network as a whole, it will be partitioned into subunits.

The beginning and end of a call's progress throughout the system is the earth station (see figure 2-3). The earth station connects calls from the telephone exchange to the modems. One SCPC modem is required for every call passing through the system. For the moment, one can think of a modem (short for modulator/demodulator) as upconverting and/or downconverting an incoming telephone call to the appropriate

---

[3] We are referring to a voice channel, 4kHz wide. "Channel" is an ambiguous term indicating some specified chunk of frequency space; sometimes it is a voice channel, sometimes a whole transponder, 40MHz wide of which only 36 MHz are available for use.

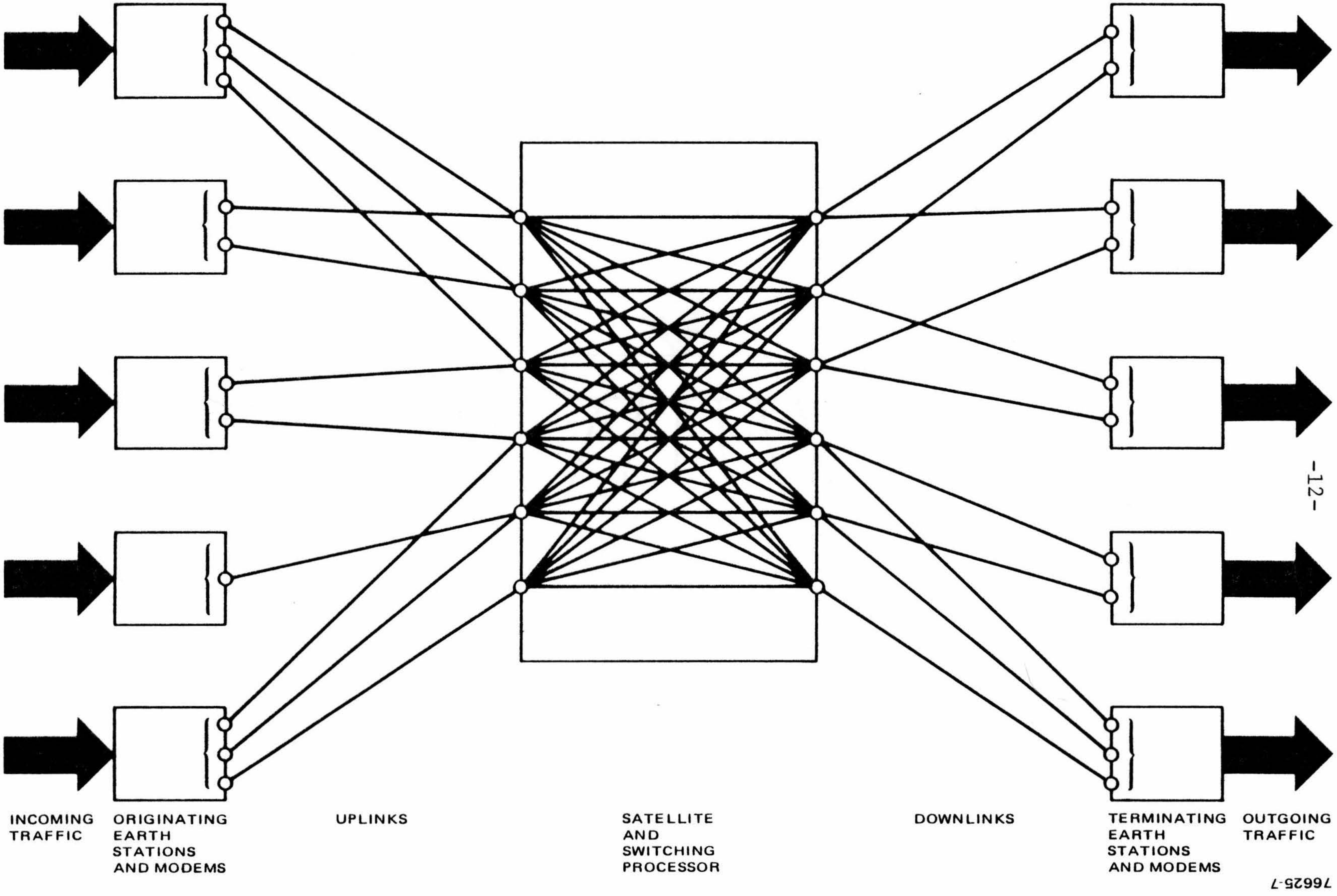


FIGURE 2-2. COMMUNICATION SATELLITE REPEATER

channel frequency. In a one-way telephone exchange, modems handle calls in only one direction; in a two-way telephone exchange full-duplex modems handle both incoming and outgoing calls.

Calls arriving from the exchange and destined to the satellite may be assumed to obey a Poisson traffic distribution.[4] As many calls as there are modems could be serviced and the rest will be blocked and lost. Those that are blocked are <u>modem blocked calls.</u>

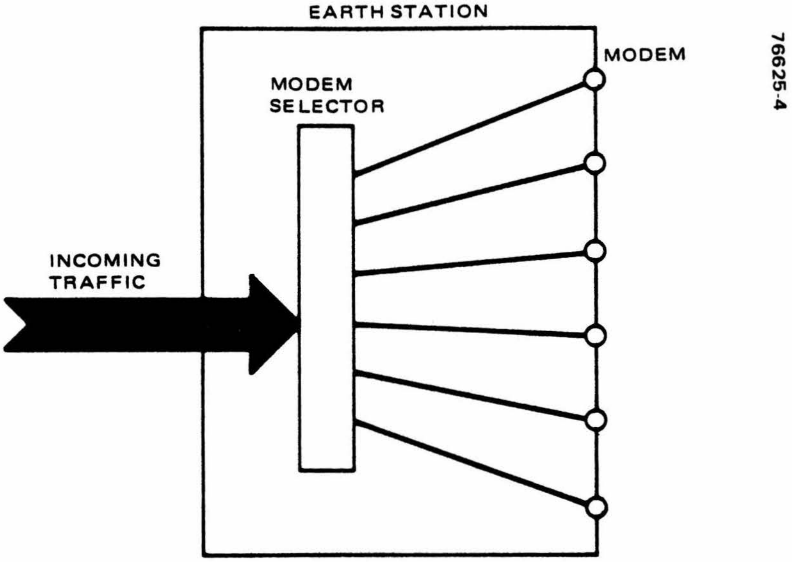


FIGURE 2-3.

An outgoing call which enters a modem can access the satellite on any of the channel frequencies available to that modem (see figure 2-4). If all channels available to that modem are in use, the call must try another available modem. If the channels from all available modems are in use, then the call is blocked and lost. Similarly, a call could try to reach an earth station from a given channel and, although there might be available modems, none of these could receive on that particular frequency. In either case this is a <u>channel blocked call.</u> (Note that in the first case the satellite itself may have a free channel but it

---

[4] For a reference on the Poisson distribution see Feller p. 156.

cannot be accessed from any of the available modems of the earth station.) Choice of a channel or modem could either be made randomly or in a preselected order.

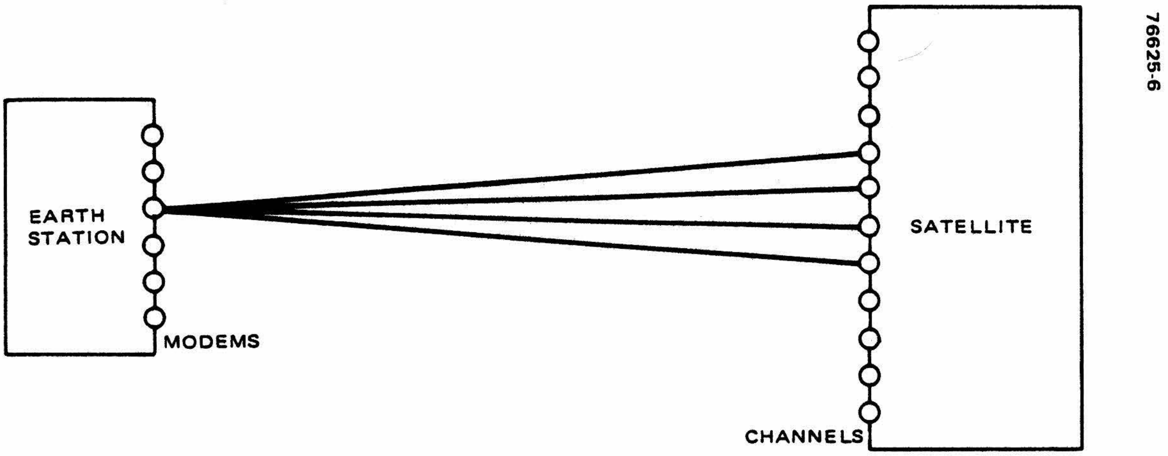


FIGURE 2-4.

Each channel can be accessed by various modems of different earth stations (see figure 2-5). They would compete for use of channel on a first come first served demand access basis. Once accessed, a channel is in use until the termination of a call.

On a conventional satellite there is a unique downlink channel assigned to each uplink channel as shown in figure 2-6 (solid lines). A satellite with onboard switching has the capability of connecting any uplink channel to any downlink channel (broken lines).

It could occur that there be both a free modem and a free channel but that for a call from a given earth station to another an available path does not exist. This is a network blocked call. If the cause is lack of connectivity on the satellite (in the case of a conventional satellite without onboard switching) then it is simultaneously a satellite blocked call.

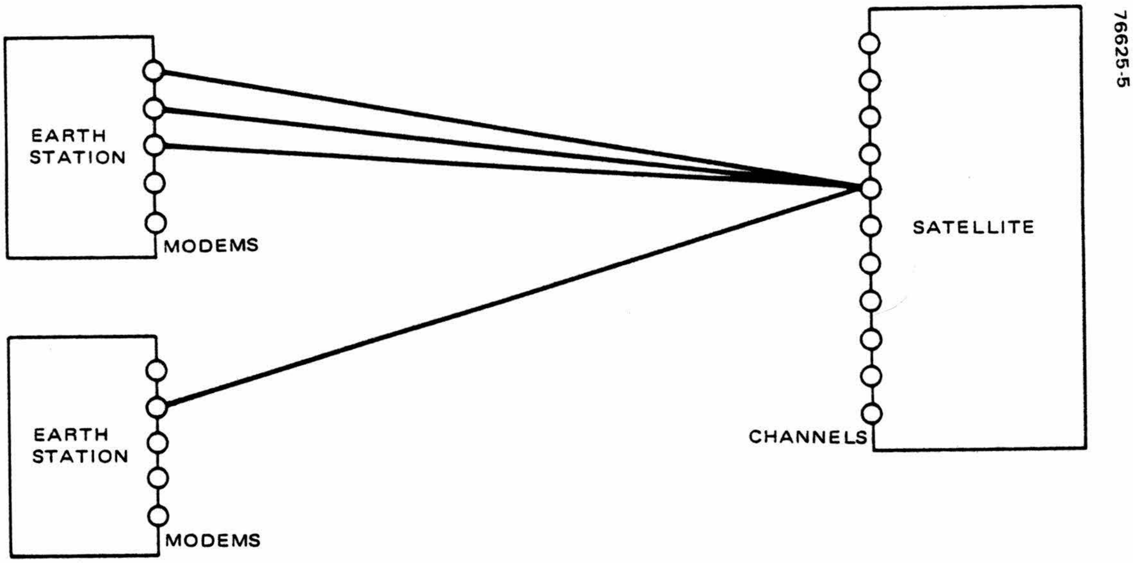


FIGURE 2-5.

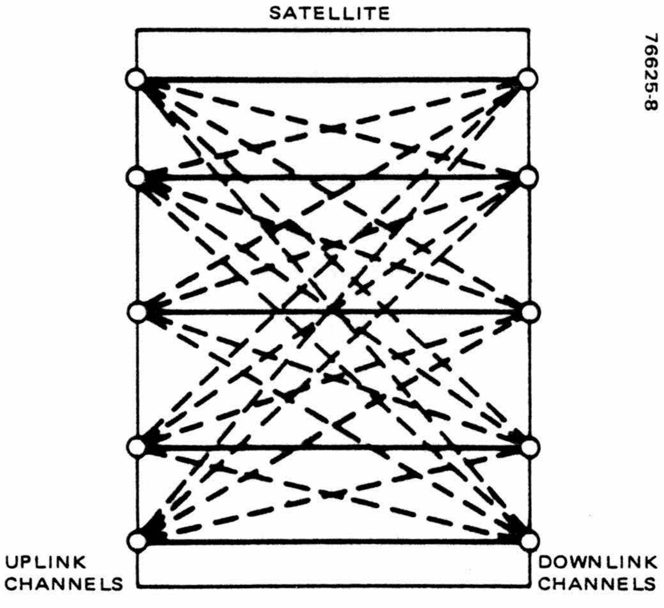


FIGURE 2-6. ONBOARD CONNECTIVITY

2.3.2 Types Of Channel Assignment -

At this point it is appropriate to describe frequency-division multiple access (FDMA) and time-division multiple access (TDMA). Although this thesis is concerned primarily with FDMA, a brief description of TDMA is given. Single channel per carrier (SCPC) is also discussed.

a) Preassigned vs. Demand Access

A channel between two earth stations that is preassigned is used exclusively for carrying traffic between them. If all the channels in a network are preassigned then there must be at least one channel for every pair of earth stations that wish to communicate. Furthermore a pair of earth stations must have as many channels as they have simultaneous conversations. For example, if ten earth stations each wished to communicate with the others, with a maximum of 5 calls in use at any time, the system would need 275 voice channels or a little over a megahertz in bandwidth. Very seldom would all 275 channels be in use at one time. In fact, it is likely that more than 200 are in use simultaneously less than 1% of the time. Demand access channel assignment permits any earth station to use any channel as long as the station needs it. After a particular call is finished, the channel is freed and may be grabbed by any other earth station. In this manner the network will require a substantially smaller number of channels.

b) TDMA

Time-division multiple access (TDMA) is a multiple-access technique that permits individual earth station transmissions to be received by the satellite in separate nonoverlapping time slots. Each earth station must determine satellite system time and range so that the transmitted signals are timed to arrive at the satellite in the proper time slots. Needed are: buffer storage and timing, A/D converters, frame format, and carrier recovery. A network time standard for a limited net of earth stations covered by one satellite would require either a precision clock on board or a master timing station which relays the time to the earth stations via the satellite. (Perhaps the earth stations set their clocks by locking on to a synch in the preamble of the time bursts.) Possibly the Global Positioning System (GPS) may serve in a useful capacity once it is fully operational and shown to be reliable over an extended period of time.[5]

Time-division multiple access can achieve efficiencies in satellite-power utilization of 90 percent or more compared to the 3- to 6-dB loss in power utilization that is typical of FDMA operation, due to the need in FDMA to minimize the intermodulation effects by backing off the power of the TWTs.

Use of TDMA allows the use of a single up- and down-converter, since all the information is on one IF carrier. Phase linearity of the converters is, however, more important than for FDMA.

c) FDMA

Frequency division multiple access (FDMA) is a multiple-access technique whereby several signals are transmitted on carriers at different RF center frequencies. These signals may come from the same or different earth stations.

There are many forms of FDMA that are commonly used in satellites. The particular format of the FDMA channel utilization depends on the total traffic of the originating earth station, the flexibility of the modulators, modems, and upconverters, and the signal distortion, adjacent channel interference, and intermodulation effects caused by the satellite transponder nonlinearities.

---

[5] (See Navstar Global Positioning System, Astronautics & Aeronautics, April 1976)

In one format we might transmit only one carrier per earth station. In this case all the incoming traffic to the earth station might be multiplexed together and upconverted to be transmitted on a single carrier. The total bandwidth of this multiplexed carrier would be called a channel, and no other earth station in the network would transmit in that frequency range.

An alternate approach would be that separate carriers might be transmitted by each earth station for each receiving earth station the signal was being transmitted to. In this case each transmitting earth station would be assigned a portion of the total bandwidth for each channel, so as not to interfere with other earth stations transmitting on the same frequency. This latter approach has the advantage that it requires the receiving earth station to demodulate only the data intended for it, but it may not have any power or efficiency advantage. This technique is sometimes called destination oriented.

In the variant of FDMA chosen for investigation here, one modulates each carrier by a single voice channel. This single-channel per carrier (SCPC) system has the advantage that if used in a demand assignment mode, it improves the efficiency of the system by sharing the frequency space between a number of earth stations according to their needs at a given time. These SCPC carriers can also be voice-activated such that carrier power is turned on only during time intervals when the voice envelope exceeds a certain threshold level. Typically, the incoming signal is modulated by an SCPC modem to an intermediate frequency (IF) which the master control center assigns uniquely to that call. (In some cases the earth station grabs any available frequency slot and then informs everyone else, with some low probability that two earth stations simultaneously grab the same frequency.) The signal is then upconverted and transmitted on a specific carrier. Although each earth station may transmit several voice signals on the same carrier and use a single transponder (TWT) on the satellite, each signal is assigned a separate nonoverlapping frequency channel on the carrier. As before, the carrier can either be destination oriented or carry data destined for different earth stations. An earth station may transmit either none, one, or several of these single-channel carriers, depending on its traffic demand at a given time and the capability of the existing equipment.

The network itself is composed of the earth stations, channels, and satellite. The system consists of a number of transmitting earth stations (each capable of transmitting to the satellite on a number of different frequencies), a satellite (with either a translating repeater or onboard switching that can connect any uplink frequency to any downlink frequency), and receiving earth stations (each capable of receiving on a number of different frequencies) as is depicted in figure 2-2. Each connection between an earth station and satellite channel is called a link.

Although more than one earth station could in general use the same frequency, only one could use it at a time. If an earth station has sole use of a channel frequency, that link is said to be preassigned.

For simplicity, let us assume that a call is only one-way, i.e. unidirectional, from an originator to a destinee. For two-way conversations we could establish the system with a receive frequency paired with each transmit and vice versa in an identical configuration. Thus one need not consider the return paths.

In order to complete a call from one earth station to another, the system must provide a free uplink and downlink pair of frequencies on the satellite that are associated with each other, a corresponding free modem for the uplink frequency at the originating earth station, and a corresponding free modem for the downlink frequency at the terminating earth station.

Let us consider first a satellite with a translating repeater (no onboard switching). In this case each uplink frequency is permanently connected on the satellite to a specific downlink frequency. The assignment of channels at the earth stations can, however, vary.

Networks can be established with preassigned (PA) or demand-assigned (DA) channel allocation. There are two types of demand-assignment systems, semi-variable (SVDA) and fully-variable (FVDA).

A preassigned (PA) network (see figure 2-7) dedicates to each earth station a number of fixed-tuned transmit and receive frequencies that are different from those of any other earth station, In order for a call to be established, the system must find an uplink frequency allocated to the originating earth station and a downlink frequency allocated to the receiving earth station which have a common connection on the satellite.

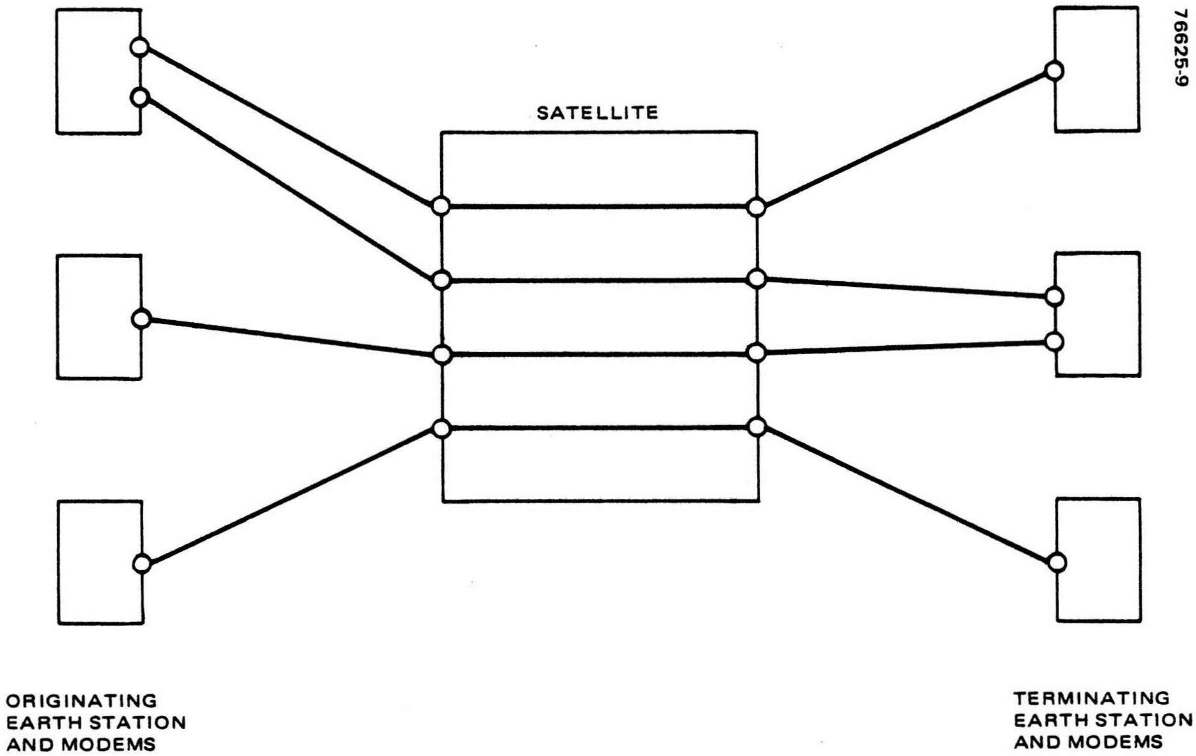


FIGURE 2-7. PA NETWORK

Thus each pair of earth stations that wishes to communicate must have an uplink and downlink frequency dedicated for such use.

The fully-variable demand access (FVDA) system (see figure 2-8) gives greater flexibility in that each earth station can transmit and receive on an arbitrary pair of frequencies.

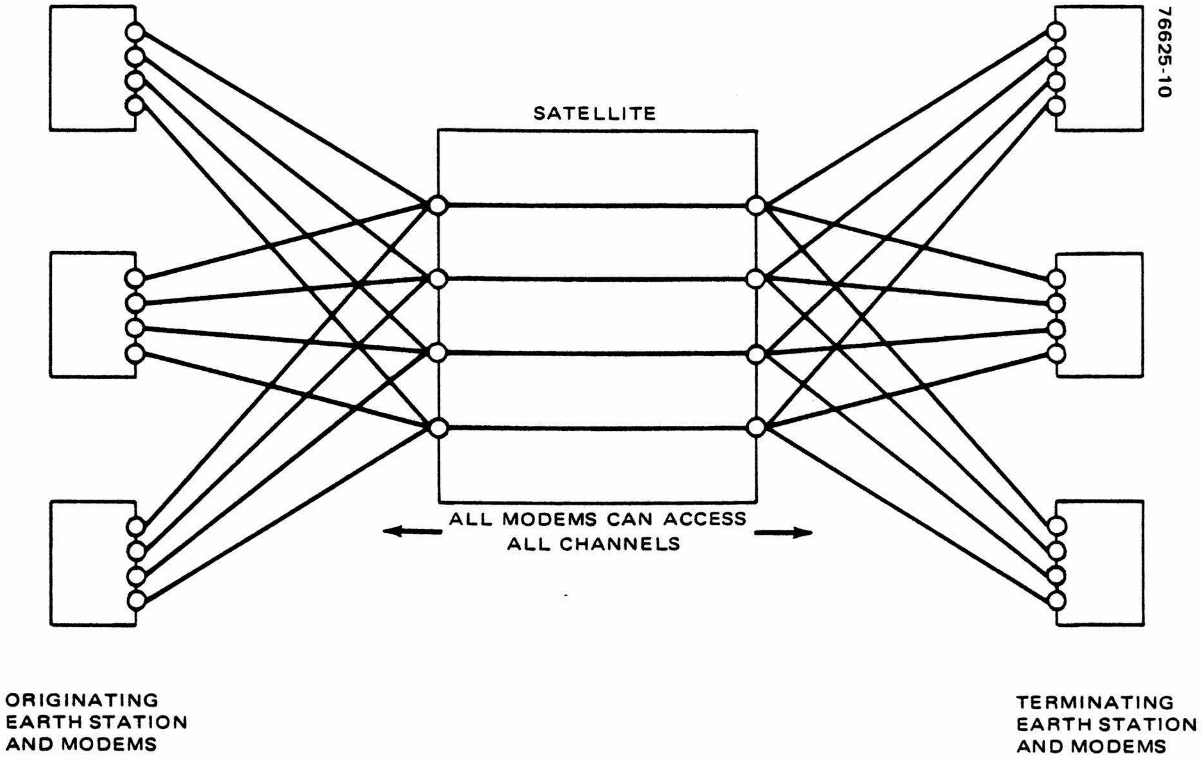


FIGURE 2-8. FVDA NETWORK

It suffices that the satellite have a pair of uplink and downlink frequencies that aren't in use provided that the originating and terminating earth stations each have a free modem. The disadvantage of this system is that each earth station must be capable of transmitting and receiving on all channels. This greatly increases the per unit expense of the ground equipment. If both the originating and terminating earth stations have free modems, then in the case of a fully-variable system an incoming call is blocked only if the total capacity of the satellite is exceeded (i.e., every channel is in use).

The semi-variable demand access (SVDA) system can be classed as either variable destination (VDDA) or variable origin (VODA). The VDDA system has a specified number of fixed-tuned preassigned uplink frequencies assigned to each earth station and downlink receivers at the earth stations which are tunable over the entire range of system receive frequencies. As before, uplink and downlink frequencies are connected on the satellite. In essence, such a system has preassigned uplink frequencies and fully-variable downlink frequencies. A call can be completed between a pair of stations whenever the terminal earth station has at least one receiver modem available. The VODA system is simply the reverse of the VDDA system. An SVDA system has flexibility in the choice of frequencies used while the PA system does not. The variable destination (VDDA) system is more attractive than the variable origin because a receiver with full capability is less expensive than a similar transmitter. The VODA, however, may be of use in some cases because an earth station can pick a certain channel on the satellite and the master control station does not need to be sent the destination code, thereby saving time in setting up the call. (This assumes that the destination code uniquely determines the earth station.)

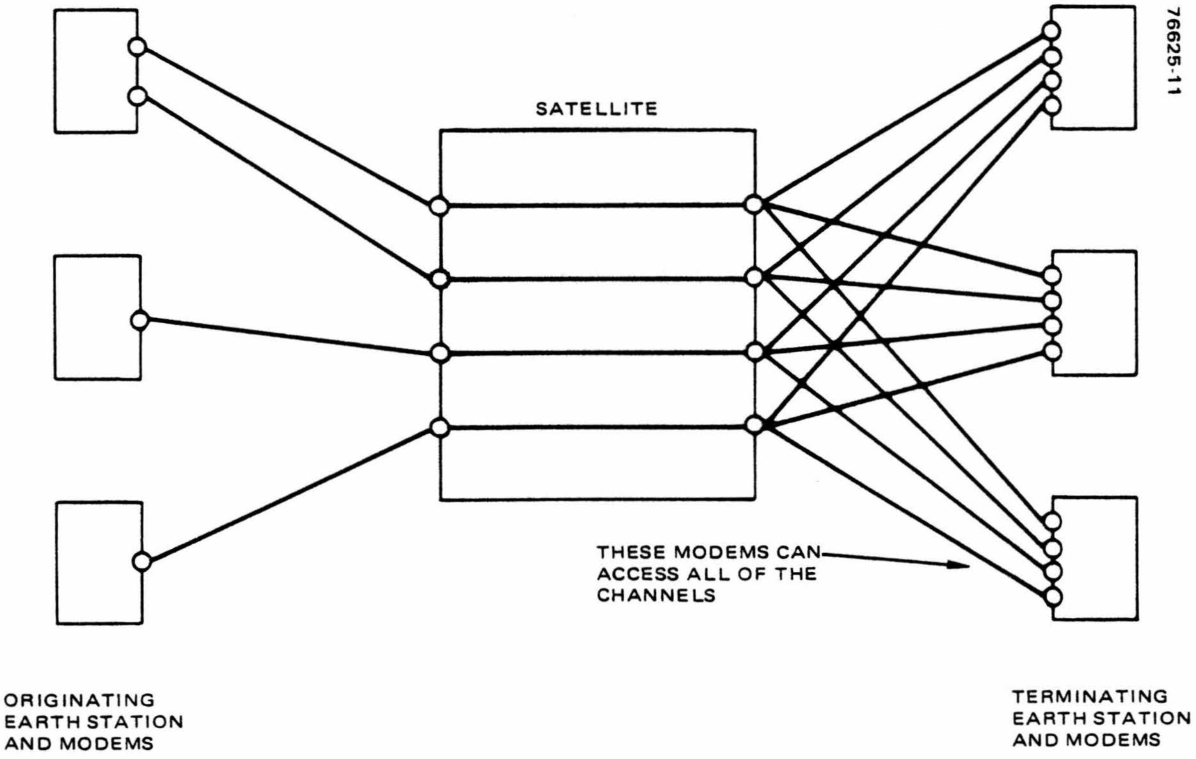


FIGURE 2-9. SVDA NETWORK (WITHOUT OVERLAP)

The simplest form of SVDA is that in which the preassigned uplink (VDDA) or downlink (VODA) frequencies are unique to a particular earth station, (see figure 2-9) so that there is no competition for channels. This system is particularly easy to analyze for capacity and blocking probability. Like the FVDA system the SVDA system is limited not only by the capacity of the satellite, but also by the number of preassigned links dedicated to each earth station.

The second form of SVDA has an overlap of the fixed-tuned frequencies assigned to each station (see figure 2-10).

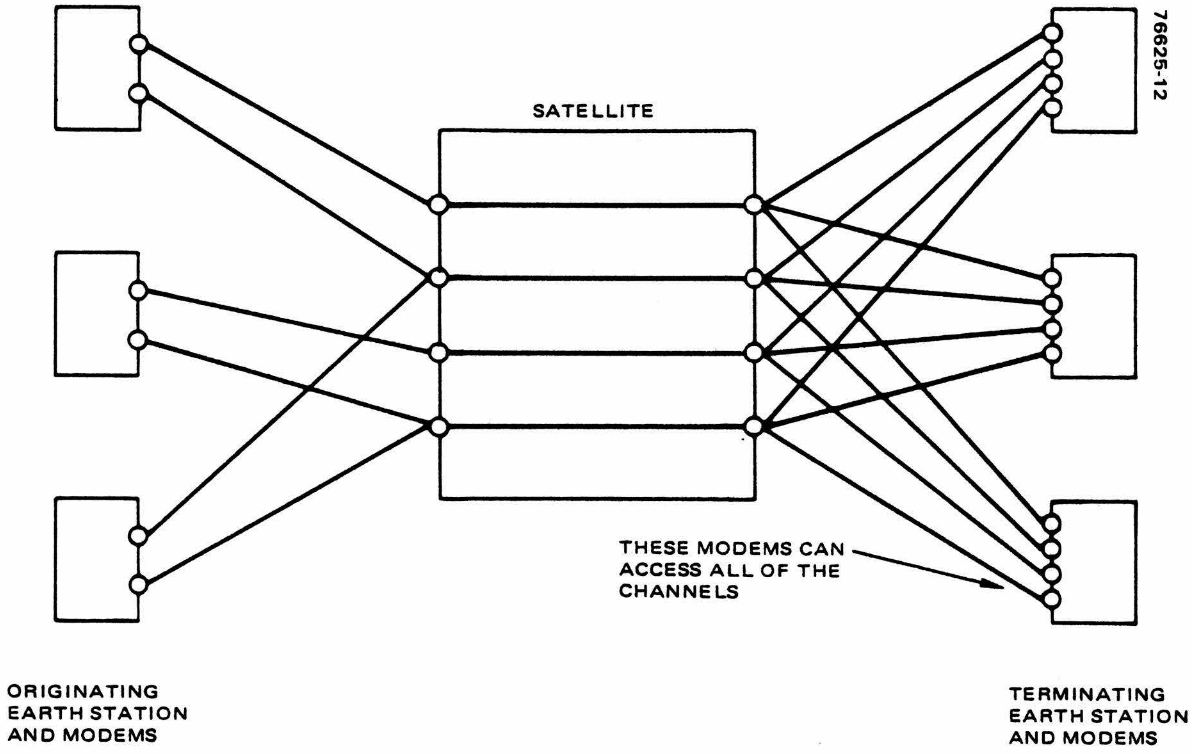


FIGURE 2-10. SVDA NETWORK (WITH OVERLAP)

This SVDA system limits the tunable receiver to some subset of the entire range of system receive frequencies, reducing the cost by decreasing the number of crystals, etc. required. This latter system is not only limited by the capacity of the satellite and number of preassigned links at each earth station, but also by the topological configuration of up/downlinks of the network at any given time because of tuning constraints.

The third form of SVDA is a hybrid of SVDA and PA giving each earth station some preassigned "dedicated line" channels and using the semi-variable option for "overflow" calls (see figure 2-11).

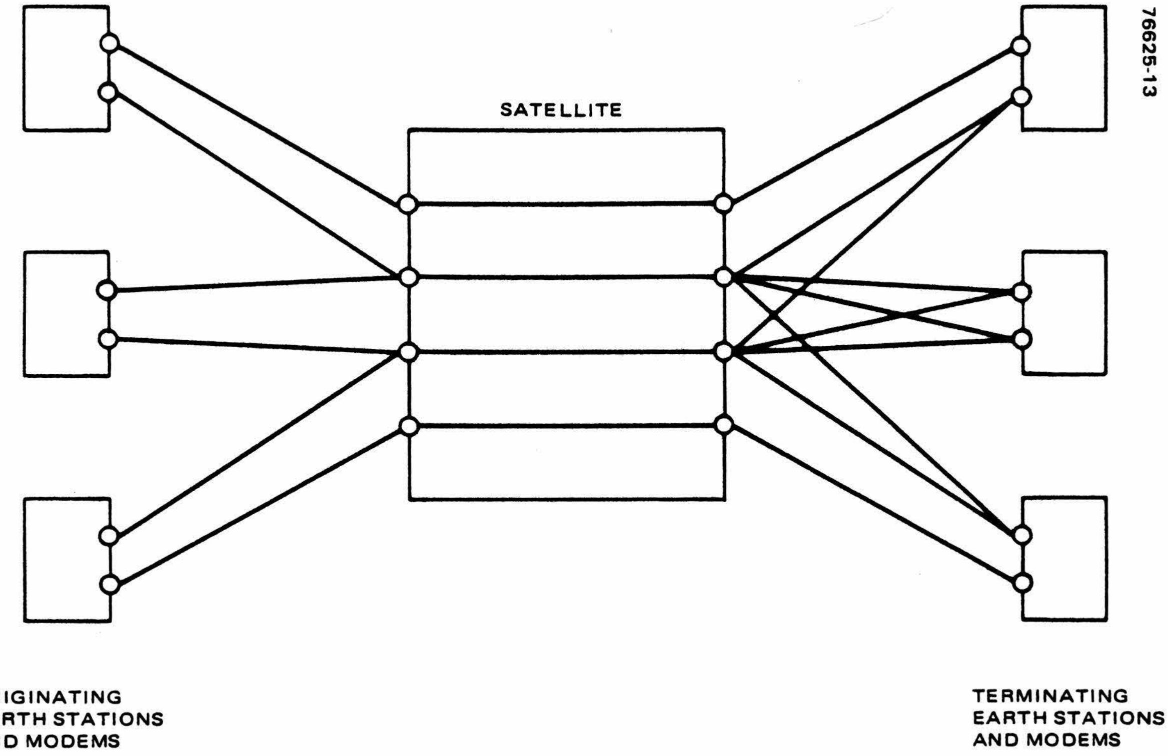


FIGURE 2-11. HYBRID NETWORK

2.3.3 Onboard Switching -

So far we have assumed each uplink frequency to be connected permanently on the satellite to a specific downlink frequency. If we include on the satellite the capability of switching any uplink to any downlink, then there is often less likelihood that a call will be blocked. Switching adds greater flexibility only in those channel assignment systems in which neither the uplink nor downlink has fully-variable modems. The PA system is an example. In contrast, there is no advantage to adding onboard switching to a fully-variable system.

One could also conceive of a system not discussed above which would benefit by switching. Assume both the uplinks and downlinks to be fixed as in the PA system, but the total number of modems in the network to exceed the number of channels on the satellite, so that there is an overlap of the fixed-tuned frequencies. Frequencies are not uniquely assigned and hence two earth stations may compete for usage of the same frequency. This might be reasonable for a number of earth stations each of which is presented with a light traffic load. An earth station with the option of several earth to satellite paths may select them either randomly or in some predetermined order. (One form of random search would be to pick randomly a permutation of possible channels and search in that order.)

If the switching allows any uplink frequency to be connected to any downlink frequency then it suffices to find any free uplink and downlink frequency, since the up and downlinks do not have a predetermined connection on the satellite.

Let us call a symmetric system one for which the topology of paths in the reverse direction is identical to that in the forward direction. Such a network has an attractive attribute for analysis, namely that the blockage of a call depends only on the forward blockage and one need not consider the return path. (We are assuming that the symmetry of the forward and reverse paths is retained when calls are established. Random searches, for instance, would only be made for the forward path, and the reverse path would be set up accordingly.) With full connection onboard switching available, the probability that a connection cannot be made on the uplink is independent of a similar probability on the downlink. The satellite onboard switching is assumed to be capable of connecting any uplink frequency to any downlink frequency, and can make the connection for all distinct such pairs simultaneously. (This of course assumes that calls can be switched while others are in progress.) In order to complete a call it suffices that both the originating and terminating earth station be able to connect to the satellite. Thus one can compute the blockage probability and capacity of the uplink and downlink separately and combine appropriately to determine that for the the total system.

It follows that a network with onboard switching has the same traffic characteristics and blockage probability and is thus equivalent to an SVDA network, provided both have the same number of receive and transmit modems and in addition both have the same overlap (or lack of it) of fixed uplink frequencies. Hence the same mathematical analysis applies for a network with switching as for an SVDA system. And under the assumption that uplink and downlink blockage probabilities are independent, each half of the system can be treated separately.

3.0 COST CONSIDERATIONS

3.1 System Tradeoffs

We are interested in a satellite system which provides regional coverage of a nation or group of nations. Usually such a satellite must have high effective-isotropic-radiated-power (EIRP) in order to permit the use of low-cost earth stations and to provide high-density traffic.

The microwave earth stations, on the other hand, must be carefully designed to make the most effective use of the satellite power without undue cost to the total communications system. Clearly, the larger the earth station G/T, the more efficient is the use made of a given satellite power. However, earth station cost increases as the G/T increases because of the added cost in the size of the structure and reflector and the cost of the low-noise receiver. Hence, in a total system, the costs of earth stations and satellites must be weighed against each other.

3.1.1 Comparing Preassigned Network With Demand Access Network -

A preassigned network is somewhat simpler in terms of the equipment needed since frequencies do not have to be allocated and the individual earth stations do not have to be informed of the status of other parts of the system. The demand assignment, however, provides more efficient utilization of the capacity of the satellite and more flexible routing as well as lower cost through the use of common units to serve several different links. In essence, the transponders are on the average filled with a larger number of active voice channels.

If there is a large amount of traffic between two earth stations, then it is cheaper to use multiplex equipment (FDM-FM) because it can handle several calls at the same time, whereas SCPC equipment requires a separate modem for each call. Overall, however, SCPC demand access will be cheaper if the traffic load is light, since one SCPC modem can share the load to two or more earth stations, whereas the FDM equipment is dedicated to a particular destination. Use of SCPC adds about $700 per modem to the cost of an earth station, but fewer modems are required (see Appendix C). Section 6 gives an example of how great a savings SCPC can be. Some studies have shown that FDM is cheaper than SCPC if the volume of traffic between two earth stations is over 10 to 25 erlangs.

The second consideration is the total capacity of a satellite transponder for FDM/FM and SCPC operations. It is well known that as the number of FDM/FM carriers accessing a single transponder increases the transponder capacity (number of voice channels) decreases for a fixed TWT power level. This satellite capacity must be carefully calculated when designing an actual SCPC system. In conclusion, if the overall originating traffic is sufficiently sparse, a single channel per carrier mode of operation coupled with the flexibility afforded by the demand assignment of voice channel leads to a more efficient use of both the earth station equipment and satellite.

3.1.2 Comparing TDMA With FDMA -

We will not dwell on this subject, since it is not central to this thesis, but feel we should mention a few points.

As was pointed out earlier, TDMA allows the TWTs to be operated at a higher power level without the side effects of intermodulation interference. For a fixed power level, therefore, TDMA allows more channel accesses of a transponder. One of the future values of TDMA is that it allows single or multiple beam steering that provides time isolation of several beams in addition to spatial and polarization isolation. This means greater G/T for each of the links and perhaps smaller and cheaper earth stations.

FDMA, on the other hand, is currently being used by a large number of countries. The FDMA equipment is generally less expensive unless the arriving telephone traffic is already in digital form, and many of the smaller countries are reluctant to invest large sums of capital to convert their systems. Thus it is cheaper for existing systems.

3.2 Subsystem Tradeoffs

3.2.1 Satellite -

a) Hardware

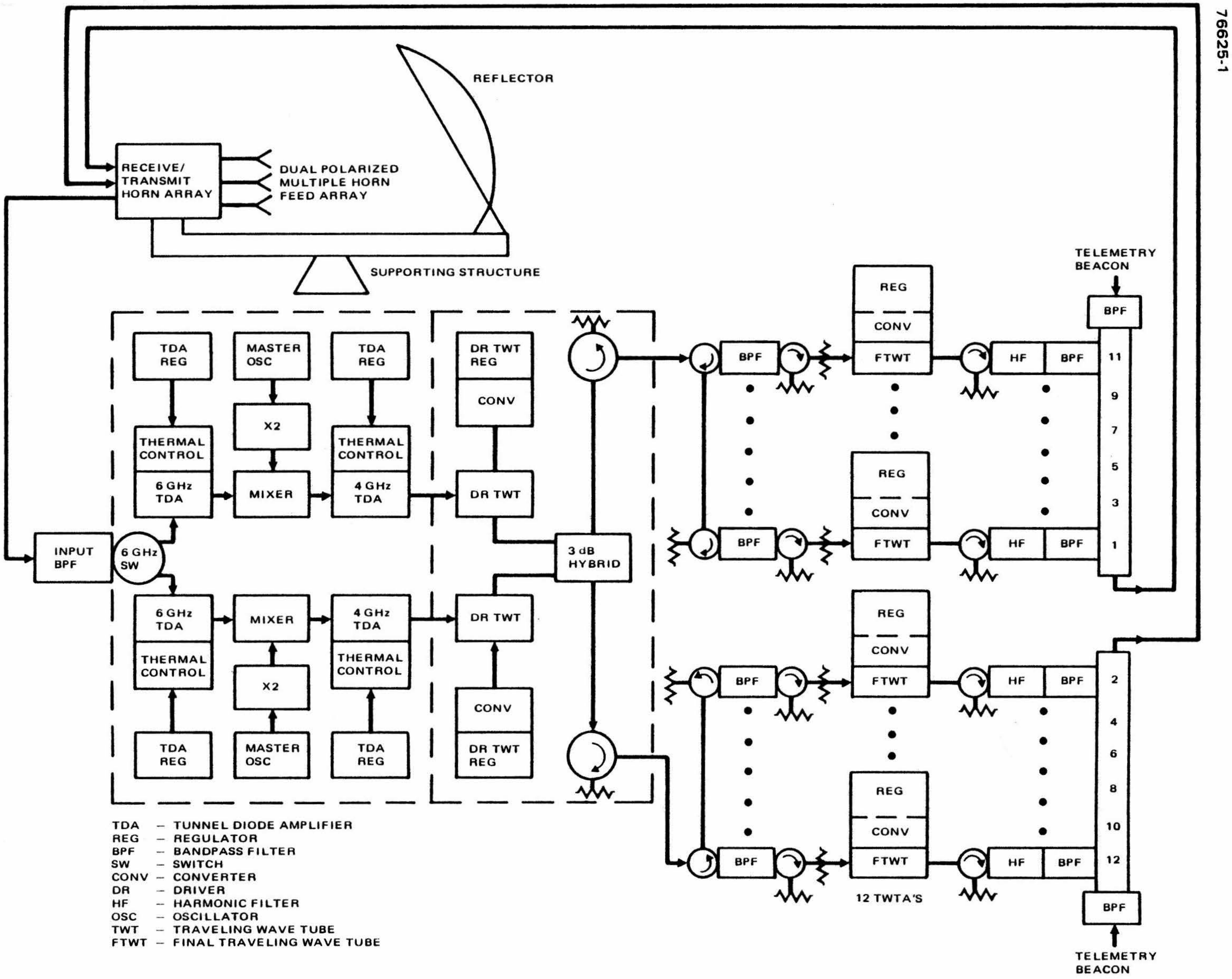
This section describes the equipment which would commonly be found in a demand assignment satellite network for telephone traffic. Appendix B and Appendix C provide detailed information on the corporations who furnished information and typical costs for such equipment.

The basic satellite is a microwave, fixed gain, multiplexed repeater. A simplified block diagram of the communication system is shown in figure 3-1. The twelve RF channels are spaced 40 MHz apart, including 4 MHz guard bands. The satellite receives a 6 GHz carrier upbeam and transmits a 4 GHz downbeam.

The bandwidth available for an individual satellite is limited to 500 MHz in the commercial C-band region (5.925-6.425 GHz uplink and 3.7-4.2 GHz downlink). Some commercial satellites are now being designed to operate in the K-band region as well.

The elements of the repeater are:

1. Antenna (reflector, feed system)
2. Receiver (input BPF, 6 GHz TDA, mixer, 4 GHZ TDA, driver TWT, output hybrid and isolators)
3. Input multiplexers (circulators, channel BPF's, isolators)
4. TWTA's (TWT and its power supply)



TDA – TUNNEL DIODE AMPLIFIER
REG – REGULATOR
BPF – BANDPASS FILTER
SW – SWITCH
CONV – CONVERTER
DR – DRIVER
HF – HARMONIC FILTER
OSC – OSCILLATOR
TWT – TRAVELING WAVE TUBE
FTWT – FINAL TRAVELING WAVE TUBE

FIGURE 3-1. SATELLITE REPEATER BLOCK DIAGRAM

5. Output multiplexer (isolators, harmonic filters, bandpass filters, combiners).

The antenna reflector focuses the upbeam energy into the feed horns. Frequently one reflector is used for both up and downbeams. A typical reflector is between 1. and 2.5 meters in diameter.

The receiver includes the input bandpass filter through the output isolators. The next stage of the receiver is redundant, and either half can be used. The receiver first provides amplification at the receive frequencies (6 GHz) in a tunnel diode amplifier (with a noise temperature of 290°K). The received signal is then translated down to the transmit band by the mixer. Further amplification is achieved by the 4 GHz tunnel diode amplifier and the driver TWT. Finally, the output signal is evenly split in order to feed both the even and odd channel input multipliers. This is done because the filters of the next stage are not ideal.

The input multiplexers serve to separate the signal into 12 channels. If all the received signals were transmitted by one TWT, the intermodulation noise (generated by the TWT) would cause a drastic reduction in overall satellite capacity. In order to avoid, as much as possible, the intermodulation distortion caused by putting multiple signals through one power amplifier, the repeater is divided into 12 separate channels.

The "final" TWT provides the amplification to high power levels for downbeam transmission. It is usually "backed off" to avoid excessive intermodulation. These TWTs are typically about 5 watts each. The amount of backoff depends strongly on the desired signal to noise ratio of the entire system, but is usually many dB.

The output multiplexers include the two combiners and the isolators, harmonic filters, and bandpass filters for each channel. Again, there are separate even and odd multiplexers. The outputs from the even and odd multiplexer outputs are then fed to the antenna.

(Actually, for "dual mode" systems there must be separate paths to the feed and separate feed horns for the downbeam. The contiguous multiplexer does not require this, but seems to be more complicated to implement.) This amplified signal is then beamed back to earth.

b) Launch

There are three launch vehicles commonly used for communications satellites, the Titan IIIC, Atlas, and Delta. The shuttle will naturally be added to the list shortly.

The payload can weigh up to .5 tons and have a diameter of 2.4 meters. Both the weight and size constrain the number of transponders, the size of the solar array, the size of the antenna dish, and the quantity of onboard fuel for station keeping, to name a few items.

3.2.2 Earth Station -

The earth station is the link between the terrestrial network and the satellite. It is comprised of the physical facilities, the control and monitoring equipment, the telephone interface equipment, the RF equipment, and the antenna. (Information on manufacturers may be found in Appendix B).

Figure 3-2 shows a typical earth station schematic.

a) Physical Facilities

An earth station must have a shelter to house the electronic equipment under adequate temperature and humidity conditions and a source of power. Power supplies must be used to convert the line voltage to a form usable by the various items of electronic equipment. For TDMA the peak power may be much greater than the average power, but there is not much difference for FDMA. There must also be tracking and antenna drive equipment. The construction cost for these facilities

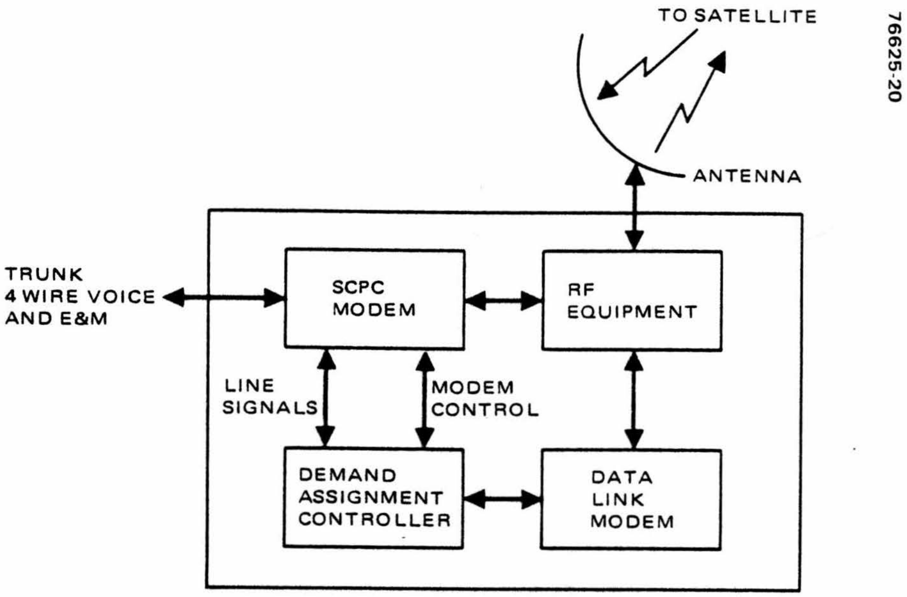


FIGURE 3-2. TYPICAL EARTH STATION

depends on the local climate, cost of labor, etc.

b) Control & Monitoring Equipment

Information concerning the type of call, routing, and status are transmitted to the earth station in order to establish the link.

In a decentralized system, as each call is requested or completed by the user earth station and channels are assigned or released by a demand assignment unit using the common signaling channel, each earth station updates a current log of available frequencies. This log is updated continually by each earth station, which monitors the common signalling channel to determine which channels are being utilized by other terminals. The earth station initiating the request must wait the roundtrip propagation time of .24 seconds before the channel request is received by the destination earth station. The initiating earth station has requested at random a frequency pair of those remaining unassigned in the frequency log. If during the propagation time of the channel request another terminal requests the same frequency, the channel is considered busy and the transmitting earth station must initiate another frequency request. The random selection of a frequency channel makes it unlikely that two terminals will simultaneously request the same channel.

Alternatively, one can use a master control station to direct all channel allocations. This method can simplify equipment at the remote earth station and control the assignment of each channel equally well or better than the separate assignment at each local earth station. Since the more complex control equipment is then located only at the master control station, the remote earth stations can be simplified.

The master control station acts like the switching equipment at a transit trunk center, but instead of controlling switching paths through an exchange, the master control station directs the assignment of radio frequencies to the SCPC equipment in order to connect telephone trunks at different earth stations with each other. The master control station does this by means of slave units at each of the earth stations. It should be mentioned that equipment to control both the earth stations and network is expensive and its cost difficult to estimate.

c) Modems & Telephone Interface

The demand assigned switching subsystem at each earth station responds to the requests for service, allocates an unused frequency to the user, and notifies other earth stations of its use through the common signaling channel.

FDMA generally requires a multiplicity of frequency converters with full tunability over the satellite bandwidth or whatever bandwidth is to be used by the earth station. This may not be necessary if the modem can tune the signal on the IF over a sufficiently wide range.

Basically, a call entering the system on an access trunk is connected by the DA equipment to a trunk at another location on the basis of the destination code. In order to do this, the user must provide the earth station with 3 different types of signals, line signals that indicate that a call is starting or finishing, register signals that indicate its destination, and of course the voice signal of the subscriber.

A "seize" line signal on the originating trunk indicates a circuit is being requested. A "clear" line signal indicates the call is terminated. Register signals are usually digits indicating the destination code (country code, area code, and exchange code) and local subscriber telephone number.

The DA equipment must act as an interface to interpret these control signals and send the information to the master control station so it can establish the necessary path.

Typically, line signals are single frequencies whose presence in a given direction indicates the trunk status. Sometimes different frequencies are used and sometimes line signals are distinguished by pulse duration or order of occurrence. Register signals are multifrequency (MF) pairs of frequencies,[6] each pair representing a number. In the United States both line and register signals are sent without regard for whether the next switching center is receiving them properly, it being assumed that the chance of an error is low and anyway the subscriber can try again if a connection is not made. In most other countries a handshaking method is used whereby a return MF signal declares that each digit has been received. The register signals are thus sent digit by digit.

The handshaking method is useful over a noisy path where the error rate is high, but it is a nuisance over a satellite link because of the round trip time delay of the digits. There are two ways to get around this. One is to fake out the system by returning a false echo to the originating trunk center and absorbing the return echo from the

[6] They may also be "decadic" signals, pulses like those of a regular dial telephone.

destination trunk center. The second method is to use a converter and transmit the digits in a burst.

Once the earth station receives a frequency assignment, the modem can be tuned to the proper frequency. The SCPC modem provides for the transmission of four wire voice and line signals from a telephone trunk over the satellite system. There is one SCPC modem for every trunk entering the DA system. The modem utilizes FM modulation[7] to encode the voice and line signal information onto an IF (intermediate frequency) carrier. Each voice channel in the transmission system uses its own RF carrier, and hence the name single channel per carrier (SCPC). For the full duplex operation required by a telephone circuit, a modem will transmit voice information on one satellite channel, and receive voice information on another adjacent channel. Two such channels used in this manner constitute a satellite circuit.

Usually the modems can be tuned to select any of the frequency slots within a 36 MHz satellite transponder. This is accomplished by a frequency synthesizer using phase locked loop techniques. To select a satellite frequency pair, the synthesizer is programmed either by a set of thumbwheel switches on the front panel or by a serial stream from the DA controller. If fixed crystals are used, then the modems can only access a limited subset of frequencies over the transponder range. In this case switches may be used to select the proper frequency.

It should be noted that the modems can transmit line signals over the satellite circuit just as well as the voice signal.

A preemphasis filter may be used prior to modulation to shape (whiten) the voice spectrum. A deemphasis filter is then used by the receiving earth station.

[7] The current Intelsat SPADE system uses pulse code modulation (PCM) and quadrature phase shift keying QPSK. This is more efficient than FDM/FM/FDMA (Edelson and Weith, 1972) but the modems are more complex.

A compander may be used to put loss in tte circuit when the voice is not present and increase the subjective quality of the link.

A voice detector can be added to "activate" the carrier used for the voice channel, depending on whether or not speech is present. Empirically about 40% of the carriers are active at any particular time. The use of such a device results in power savings on the satellite and improvement in the system C/N and transponder capacity. The voice detector operates by sensing the harmonic content in voiced sounds as opposed to the more random nature of some background noise. A short delay prevents clipping of the first syllables.

Echo suppressors or cancellers can be used to minimize the satellite link roundtrip echo.

Carriers from the modems are at the IF frequency, usually 70 MHz. (But the signal will be anywhere in the 52-88 MHz bandwidth.)

d) RF & Antenna

The final stage between the modems and the antenna is the RF equipment. A typical configuration is shown in figure 3-3.

From the modems an IF patch connects a single multiplexed channel to an upconverter. The patch enables a user carrier to access any one of a number of frequency converters.

Each frequency converter up-converts the IF signal in either one or two steps (single or double conversion) to the microwave band for further amplification and transmission. Each up-converter also has an independent power control to allow adjustment of the transmitted carrier level. The IF filters in the frequency converters are often equalized in amplitude and group delay over the full frequency range.

In complex earth stations, tunable frequency synthesizers are used in the converters. These are all phase locked to a frequency standard (usually cesium) and multiplexed to the desired frequency by a

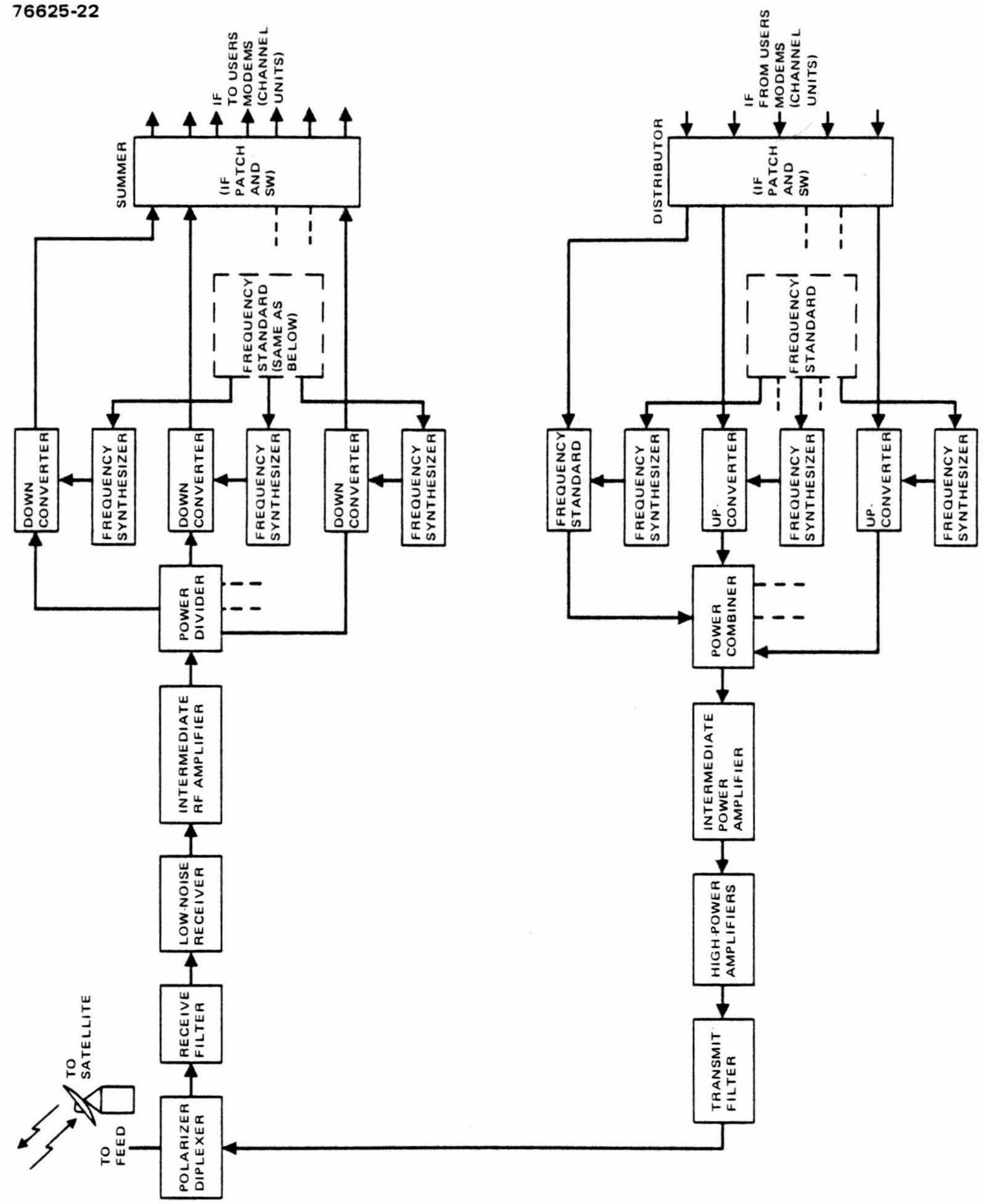


FIGURE 3-3. RF EQUIPMENT OF TYPICAL EARTH STATION

phase-locked multiplier. In simpler earth stations, frequency conversion to the RF transmission frequency is often performed by a fixed frequency local oscillator (LO) rather than by a more complex microwave frequency synthesizer. Tunability, if any, can then be provided in the IF frequency range.

The difference in these two approaches may make some difference in both the cost of the earth station and the total structure of the network. For instance, fixed crystals may be cheaper than synthesizers. If this is the case, the SVDA with partial overlap, as outlined in section 2.3.2c, may be the best to use.

Amplification at the microwave transmission frequency then takes place in the intermediate power amplifier (IPA) and high power amplifier (HPA), both of which are commonly wide bandwidth amplifiers (often TWTs).

The output signal power in the transmit band is then heavily filtered to reduce undesired intermodulation and spurious components in the receive band, which otherwise could saturate the receive amplifiers for multicarrier transmission. Signal power is then passed through waveguide to the feed diplexer and emitted from the feed with appropriate polarization to illuminate the reflector itself.

At the receive end of the earth station there must be an extremely high level of isolation between transmit and receive signals provided by the diplexer and transmit and receive filters.

The received signals pass through the diplexer and receive filter and are amplified by a wide-bandwidth, low-noise receiver (LNR), usually a tunnel-diode amplifier (TDA) or paramp, and an intermediate level amplifier (ILA). The microwave output of these (ILA) amplifiers is then passed to a power divider, which distributes the signals to the set of down-converters.

Down-conversion then usually translates the microwave output to the same IF frequency used in the up-converter. The frequency synthesizer is phased locked to the same frequency standard used for the transmit up-converter.

Rainfall attenuation can have severe effect on the signal strength at frequencies above 10 GHz, and is particularly bad for low earth station antenna slant angles where sidelobes cannot be made low enough. Many tropical countries receive between 200 and 400 cm of rain annually and tropical thunderstorms may extend 8 km in height. Any satellite system, therefore, must provide adequate rain margin in the overall G/T. Intelsat, for instance, specifies a .01% off-the-air allowance due to atmospheric propagation disturbances in designing their system. Rain degrades the system in two ways, both of which are worst for low antenna slant angles.

In a system utilizing both polarizations, rain causes cross-polarization interference which if unaccounted for will cause severe degradation in S/N. Adaptive depolarization filters can be used, however, to orthogonalize the two signals.

Of more interest to us is the degradation in noise temperature, which can be calculated from black body radiation. The approximate relation between absorption and noise temperature is as follows: Let T_m be the temperature of the rain; T_n be the noise temperature; and A be the absorption in dB for a given frequency. Then

$$T_n = T_m(1 - 10^{-A/10})$$

At 4 GHz, A= 2dB absorbtion, T_m=290°K. $\Rightarrow$ T_n=100°K. (This is the incremental temperature above the galactic background which is about 4°K.) Thus rain will add about 100°K. to the total noise temperature of the system. Typically the system is designed with excess dB so degradation of the system does not fall below the specifications. The existence of such a noise temperature also suggests that there is not much advantage in using a very low temperature LNA (Crane, 1971). In calculating the total G/T, the receiver temperature and the space temperature are added, so the degradation is worse for cooler receivers using a fixed ratio of G/T. (Appendix C shows a plot of db degradation vs. receiver noise temperature.)

Since earth stations are designed for multicarrier operation, there must be adequate filtering to isolate the low-noise receiver from the high-power transmitter, just as there is in satellite transponders. The frequency converters must be designed to minimize spurious mixer cross-products, and the power amplifier may have to be backed off from full power level to reduce transmitted intermodulation products.

There are several sizes of antennas available and price is roughly proportional to the area (and hence gain). This information may be found in Appendix C. An antenna that is 4 to 5 meters in diameter can be used for a small SCPC earth station if a good low noise transistor amplifier is used.

3.2.3 Master Control System -

A typical demand assignment network has one master control station and from 10 to 1000 earth stations. The master control station must keep track of the status of the overall system. It must also interpret the routing code and direct each of the earth stations. Having the master control station interpret the destination codes requires fewer register/interpreters than if it were to be done by the individual earth stations, since use of this common equipment can be shared among all the earth stations,

Figure 3-4 shows a typical master control station combined with one earth station. Such a facility capable of directing 120 earth stations costs between \$1.5M and \$2.0M.

3.3 Cost Conclusions And Summary

A detailed cost accounting for an operational network should include the recurring and nonrecurring expenses for both the space segment and earth stations. This thesis does not discuss the operating

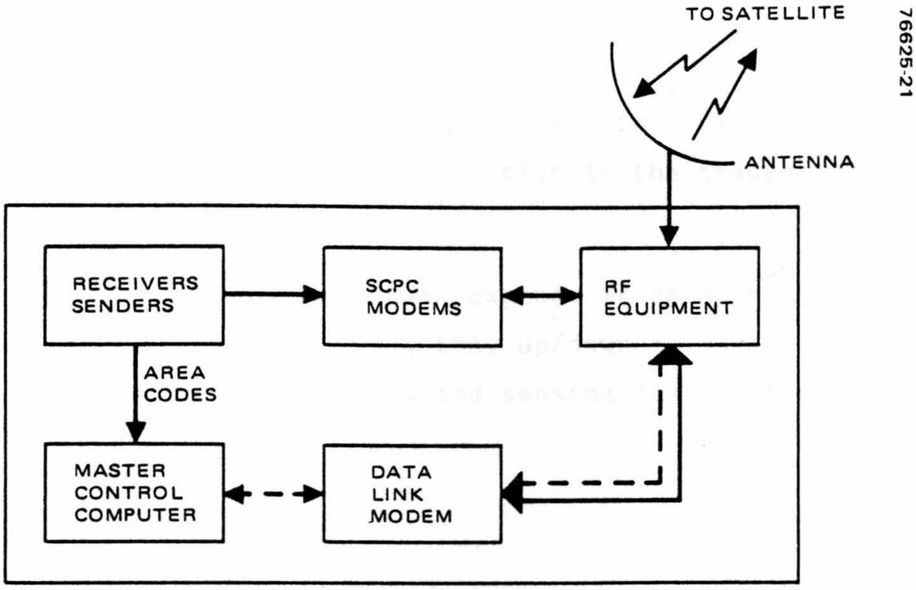


FIGURE 3-4. MASTER CONTROL STATION

expenses and gives only ballpark figures for the rest.

The satellite represents approximately 10-20% of the total cost of a typical communication system. (Satellite, earth stations, and launch costs). A small satellite costs about $10M to $15M. It is also possible to lease all or part of an existing satellite. The price is about $800K per year for a 36 MHz transponder (as long as one doesn't have to pay Intelsat prices). In addition there is the launch cost and insurance (about 7%). The Delta booster and apogee motor cost about $18M.

Complete cost breakdown for a typical earth station can be found in Appendix C. From this we find that the cost of an earth station is about $100K + $3.5K/modem. We note that earth stations such as would be used by the regional network described in section 6 would range in price from $107K to $1062K depending on the amount of traffic, and almost all would cost less than $300K. In truth, an actual system would probably use FDM for heavy traffic routes and DA for thin traffic routes. For large traffic loads, FDM equipment is cheaper, but if used exclusively for thin route systems, would imply a ridiculous number of modems and satellite channels.

Appendix C also notes the necessity of rain margins in designing a system. This is an important consideration in the tradeoff between the LNA temperature and antenna size.

The user may encounter a hidden expense if he opts for redundancy in the system. The redundant HPA, LNA, up/down converters, combiners, dividers, and associated switching and sensing devices are all available .... for a price.

4.0 STATISTICAL THEORY OF TRAFFIC IN A DEMAND ACCESS SATELLITE SYSTEM

This section reviews a well known method, using the Erlang B equation, to calculate the traffic capacity of a fixed number of channels. Then there is a very detailed mathematical technique to compute blockage probability which is not tractable for large networks. For these cases an estimation of the capacity (outlined in section 6.3) can be used, to be followed by the computer simulation method of section 4.5.

4.1 Introduction To General Assumptions

In the model which is of use in describing a satellite network, traffic is assumed to arrive at each earth station according to a Poisson distribution. Any call that cannot get through because there is not an available path is said to be blocked. Blocked calls are immediately lost from the system and the calls that are established terminate after a time period that obeys the exponential distribution. The two assumptions of Poisson arrival rates and exponential holding times are essential for the system to be "memoryless" and crucial for the development of a steady state solution derived later on.[8]

The fundamental measured quantity of a telephone traffic network is its call density measured in erlangs and defined as the mean number of calls per unit time (call rate) times the mean duration of each call. If the mean arrival rate is λ calls per second and the mean holding time is $1/\mu$ seconds, then the mean traffic is $\rho=\lambda/\mu$ erlangs. Typical holding times are about 3 minutes for international calls and 5 minutes for domestic calls. Throughout this discussion we will assume that the

[8] A constant call length distribution would not exhibit this property and in fact a steady state solution could not be obtained since knowing how long a channel had been in use would give information as to when it would be free, and the system would not be ergodic.

traffic demand ρ is a given constant. (That is, it is not a complex renewal problem.) Practically, the assumption of long transients or semi-equilibrium makes the problem tractable.

For any network the relationship between traffic and blockage probability is:

$$\text{Total Traffic} = \rho(1-P_B)$$

where "Total traffic" is that traffic which is not blocked, "ρ" is the traffic presented to the system, and "P_B" is the probability of blockage.

Normally the system would have several earth stations, several satellite channels, and a multiplicity of paths which a call could take to reach its destination. One would like to examine an arbitrary system with its associated topology, arrival rates, and holding times and determine the mean traffic density and the probability of a call being blocked at each of the earth stations. One method of deriving this is by means of Markov chains.

Some simple networks have the property that the probability of going from n channels in use to n+1 channels in use (and vice versa) is independent of the exact topological configuration of the channels. The states of the system can then be expressed simply as the probability that n channels are in use. Since the states can be numbered sequentially and transitions can only occur between adjacent states, this is a birth/death process and is easily amenable to solution (Feller, 1968).

By definition, in the general birth/death process states can be ordered linearly and only transitions between adjacent states can occur:

$$n-1 \longleftrightarrow n \longleftrightarrow n+1$$

Both the "telephone channel" (described below) and the "teller and waiting customers" are examples of birth/death processes. A "telephone channel" may be described as a system in which there is a semi-infinite number of queues, but each queue can only have a length of one or zero. (One or no calls being served.) In general each queue could have a different service time, but there is only one arrival statistic.

The state of the system is the number of calls being handled at a given time.

In a time dependent process the probability that a call arrives or that one departs is given by:[9]

$$P\{n \longrightarrow n+1 \text{ in time } dt\} = \lambda_n dt$$
$$P\{n \longrightarrow n-1 \text{ in time } dt\} = \mu_n dt$$

Then if P_n is the probability of the system being in state n:

$$P_n(t+dt) = P_n(t)\ (1-\lambda_n dt - \mu_n dt) + P_{n+1}(t)\mu_{n+1}\ dt + P_{n-1}(t)\ \lambda_{n-1} dt$$

$$P'_n = \lambda_{n-1}P_{n-1} - (\lambda_n+\mu_n)\ P_n + \mu_{n+1}P_{n+1}$$

Solution then depends on the particular boundary conditions, and a steady state condition permits the use of "telescoping equations".

4.2 Statistical Characteristics Of Telephone Traffic

For many applications a telephone channel can be modelled as a simple birth/death process and will give rise to Poisson and Erlang B statistics (Feller, 1968). Assume that all of the states have the same arrival and departure statistics. Then we may let $\lambda_n=\lambda$ and $\mu_n=\mu$ for all n. Suppose there are c channels. Then the probability of n calls at time t+dt is:

$$\begin{aligned} P_n(t+dt) = {} & P_{n-1}(t)P\ \{\text{1 new call, none of n-1 lines released in dt}\} \\ & + P_n(t)P\ \{\text{no new calls, none of n lines released in dt}\} \\ & + P_{n+1}(t)P\ \{\text{no new calls, one of n+1 line released in dt}\} \\ & + o(dt) \end{aligned}$$

---

[9] Probability of more than one arrival goes as o(dt) and such terms can be dropped in the subsequent differential equation.

$$\begin{aligned} P_n(t+dt) = {} & P_{n-1}(t)\ \lambda dt(1-\mu dt)^{n-1} \\ & + P_n(t)\ (1-\lambda dt)\ (1-\mu dt)^n \\ & + P_{n+1}(t)\ (1-\lambda dt) \binom{n+1}{1} (1-\mu dt)^n\ \mu dt \\ & + o(dt) \end{aligned}$$

or using the notation of a general birth/death process,

$$\lambda_n = \lambda,\ \mu_n = n\mu \quad \text{for } n = 1,\ldots,c-1$$

$$\lambda_0 = \lambda,\ \mu_o = 0; \quad \lambda_c = 0,\ \mu_c = c\mu$$

$$P'_n(t) = \lambda P_{n-1}(t) - (\lambda+n\mu)\ P_n(t) + (n+1)\ \mu P_{n+1}(t) \qquad n = 1,\ldots,c-1$$

$$P'_o(t) = -\lambda P_o(t) + \mu P_1(t) \qquad n = 0$$

$$P'_c(t) = \lambda P_{c-1}(t) - c\mu P_c(t) \qquad n = c$$

Letting $\rho = \lambda/\mu$, the steady state equations are:

$$(n+1)\ P_{n+1} - nP_n = \rho(P_n - P_{n-1})$$

$$P_1 = \rho P_o$$

$$cP_c = \rho P_{c-1}$$

$$\sum_{k=1}^{n-1} (k+1)\ P_{k+1} - kP_k = \rho \sum_{k=1}^{n-1} (P_k - P_{k-1})$$

$$nP_n - P_1 = \rho(P_{n-1} - P_o)$$

$$P_n = \frac{\rho}{n}\ P_{n-1} + \cancelto{0}{(P_1 - \rho P_o)}$$

Using the condition that $\sum_{n=0}^{c} P_n = 1$, we find that if $c = \infty$ then

$$P_n = \frac{\rho^n e^{-\rho}}{n!} \quad \text{or Poisson distribution}$$

If $c \neq \infty$

$$P_n = \frac{\frac{\rho^n e^{-\rho}}{n!}}{\sum_{k=0}^{c} \frac{\rho^k e^{-\rho}}{k!}} = \frac{\frac{\rho^n}{n!}}{\sum_{k=0}^{c} \frac{\rho^k}{k!}} \quad \text{or Erlang B distribution}$$

For such a system with c channels, the blocking probability is the probability that all channels are in use at the time a call arrives. This is

$$P_c = \frac{\frac{\rho^c e^{-\rho}}{c!}}{\sum_{k=0}^{c} \frac{\rho^k e^{-\rho}}{k!}} = \frac{\frac{\rho^c}{c!}}{\sum_{k=0}^{c} \frac{\rho^k}{k!}}$$

This is the Erlang B blocking probability. Although both are queues, the telephone system (blocked calls cleared operation) is different from a "teller and customers" queue in that the teller has one service time and a maximum of c people in line whereas a telephone system has c queues and a maximum of one person in each. Each queue may have a different service time.

4.3 Formulation Of The General Traffic Analysis Problem

4.3.1 Application Of Erlang B Equation -

We can now compute the traffic density for some types of channels and networks. The Erlang B blockage is not universally applicable. Here are specific cases where it can be applied. Assume circuits have one-way traffic.

On a preassigned network one must sum the traffic on each link between two earth stations.

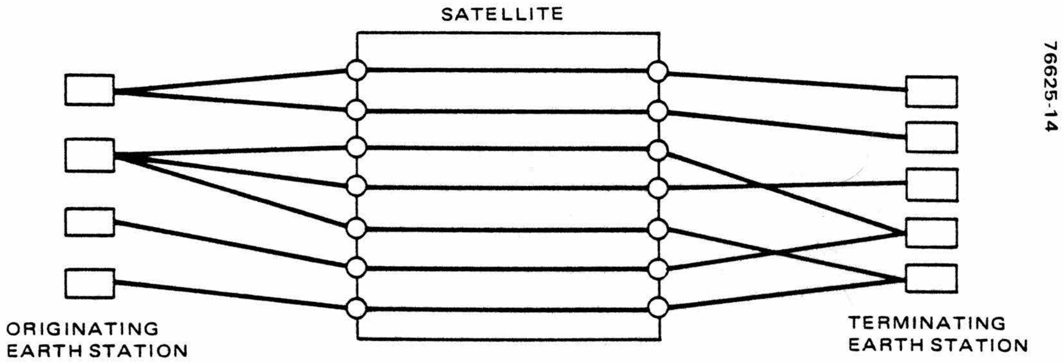


FIGURE 4-1. PA NETWORK

Since each is dedicated there will be no sharing of channels and hence we need not be concerned in addition with existence of a free modem. Note that in general each dedicated channel could handle several conversations. If there are N links each of which is presented with a traffic of ρ_i erlangs and the i^{th} has a capacity for up to c_i calls, then the total traffic will be the sum of traffic on each.

$$\text{Traffic}_{PA} = \sum_{i=1}^{N} \text{Traffic}(i^{th} \text{ channel}) = \sum_{i=1}^{N} \sum_{j=1}^{c_i} j\, P_j(\rho_i)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{c_i} j \left(\frac{\frac{\rho_i^{\,j}}{j!}}{\sum_{k=0}^{c_i} \frac{\rho_i^{\,k}}{k!}} \right)$$

The probability that a call will be blocked is:

$$P_B = 1 - \prod_{i=1}^{N} \left(1 - \frac{\frac{\rho_i^{\,c_i}}{c_i!}}{\sum_{k=0}^{c_i} \frac{\rho_i^{\,k}}{k!}} \right)$$

On a semi-variable network with no sharing of circuits, one must sum the traffic from each earth station. Assume enough receive modems exist to handle all calls presented to the satellite.

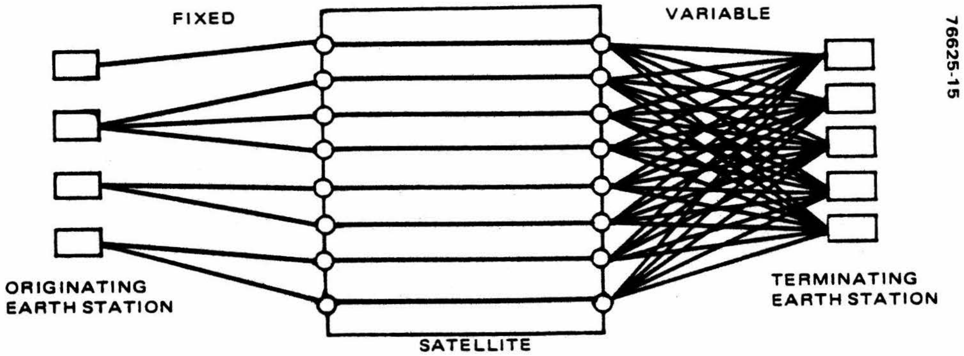


FIGURE 4-2. SVDA NETWORK

The i^{th} earth station has access to M_i channels. Of the M_i channels, the j^{th} has a capacity for up to C_{ij} calls. Thus the i^{th} earth station has a capacity call for up to $D_i = \sum_{j=1}^{M_i} C_{ij}$ calls. If there are N earth stations and each is presented with a traffic of ρ_i erlangs, then the total traffic will be:

$$\text{Traffic}_{sv} = \sum_{i=1}^{N} \text{Traffic } (i^{th} \text{ earth station})$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{D_i} j \left(\frac{\frac{\rho_i^{\,j}}{j!}}{\sum_{k=0}^{D_i} \frac{\rho_i^{\,k}}{k!}} \right)$$

The probability that a call will be blocked is:

$$P_B = 1 - \prod_{i=1}^{N} \left(1 - \frac{\frac{\rho_i^{\,D_i}}{D_i!}}{\sum_{k=0}^{D_i} \frac{\rho_i^{\,k}}{k!}} \right)$$

On a fully-variable network one must sum the traffic into the satellite. (Just look at the total equivalent erlangs on the up or down link.) Assume enough receive and transmit modems so that calls are not blocked at the earth station.

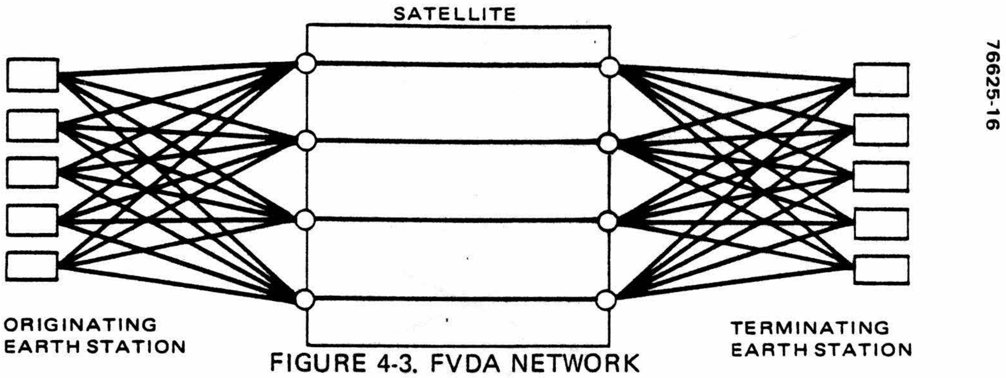


FIGURE 4-3. FVDA NETWORK

ρ is then the total traffic presented to the system.

$$E = \sum_{i=1}^{N} D_i = \sum_{i=1}^{N} \sum_{j=1}^{M} C_{ij} = \text{total capacity of the system.}$$

$$\text{Traffic}_{fv} = \sum_{j=1}^{E} j \left(\frac{\frac{\rho^j}{j!}}{\sum_{k=0}^{E} \frac{\rho^k}{k!}} \right)$$

The probability that a call will be blocked is:

$$P_B = \frac{\frac{\rho^E}{E!}}{\sum_{k=0}^{E} \frac{\rho^k}{k!}}$$

Although the number of erlangs on a trunk between two points is easy to compute, it usually only applies in cases in which no one else is competing for the use of that trunk (for instance a preassigned system) and calls are set up on a random basis. If in addition some lines are dedicated or a predetermined selection order is used, the Erlang B distribution, as it is called, is not the one to use.

On some networks symmetry allows one to combine certain call densities (ignoring modem blockage). On a semi-variable system one can combine all call densities from a given earth station. On a fully-variable system one can combine all incoming call densities. In order to combine these call densities, the system must be divisible into independent subsystems of earth stations and channels, each earth station of which can access each channel and vice versa.

The following is an illustrative example.

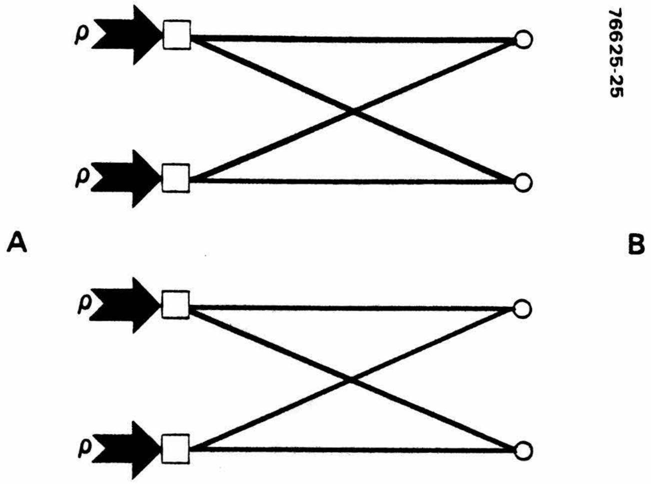


FIGURE 4-4. EXAMPLE OF ERLANG B NETWORK

The mean traffic from A to B is:

$$2\sum_{n=0}^{2} n\,P_n = 2\,P_1 + 4\,P_2$$

$$= 2\,\frac{(2\rho)}{1+(2\rho)+\frac{(2\rho)^2}{2}} + 4\,\frac{\frac{(2\rho)^2}{2}}{1+(2\rho)+\frac{(2\rho)^2}{2}}$$

$$= \frac{4\rho + 8\rho^2}{1+2\rho+2\rho^2}$$

4.3.2 Non-Erlang B Networks -

Before we assumed that the probabilities that each of two adjacent channels is in use are independent events. This is not always the case. If there is asymmetry or only partial symmetry (partial overlap of channel links) in the the way the links between the earth stations and the satellite channels are arranged, then the individual links are neither independent nor totally dependent and consequently have traffic density probabilities that are coupled. Were one to use the link traffic densities as the states of the system, the resulting transition matrix would have joint probabilities. The proper Markov chain model to use defines a state as a set of vectors, each indicating a channel and the earth station, if any, which is using it. These states are independent. The state space is the set of all combinations of connections that the network can have. In general this model is not a birth/death process since states cannot be ordered in a linear fashion; the state space is multidimensional.

In general a network will exhibit a very large number of states. Sometimes, however, symmetry allows one to use a much smaller subset of these states and solve for the state probabilities with a much smaller transition matrix since the probabilities and transition for some states are identical to those of others. This smaller matrix will be considerably easier to invert.

The topology is important. Two networks may have the same number of channels and earth stations; each earth station may be able to access the same number of channels on the satellite; and yet they have different traffic densities because of different configurations.

For the general network one should first examine the state diagram to determine what symmetry exists and reduce the number of equations necessary to solve the Markov state probabilities.

An example is given in the following section.

One further point. If one could switch call routing in the middle of a call, one could obtain slightly more efficient loading of channels and less probability of blockage. This is because mid-call switching would reduce slightly the dependence on the past set up. Presumably this could be done, but the complexities probably would offset the gains.

4.4 Analytic Solution Using Markov Analysis (Topology & Symmetry)

4.4.1 Explanation Of Method -

We are interested in those networks which do not follow Erlang B statistics and have partial overlap of frequency channels. The state of each channel is either no call or one call.

Let calls arrive at the i^{th} earth station according to the Poisson distribution with parameter λ_i, and let the service time (holding time) for a call that arrived at the i^{th} earth station and was connected to the j^{th} earth station be of exponential distribution with parameter μ_{ij}. Then the mean arrival rate is λ_i and the mean holding time is $1/\mu_{ij}$.

We will define a state to be the status of each of the satellite channels, i.e. whether it is in use, and if so by which earth station. (In fact, if all of the earth stations receive traffic with the same holding times, then it suffices to know whether or not a channel is in use.)

The forward probability transition equations provide that the probability of being in a state at time t+dt is the sum of the probabilities of being in each of the states at time t times the probability of a transition (call arriving or terminating) in that interval dt. In the steady state these probabilities are independent of time. We thus obtain a set of homogeneous equations and a constraint that the sum of the probabilities equals unity which we can solve exactly. This can perhaps best be illustrated by an example.

4.4.2 Limitations -

This method of finding the traffic and probability of blockage does quite well as long as there are not too many states of the system. Ultimately one has to invert a matrix whose dimensions are the number of states of the system (minus one). It is possible to describe the system in 2^{N_c} states, where N_c is the number of channels, and this can be reduced further by means of symmetry, but even so if $N_c = 10$ this matrix far exceeds the capacity of any computer (see also Appendix D).

If there are not too many states, but the matrix is too big to invert by hand, there are some non-numerical computer languages which can be used to obtain an analytical solution. In this fashion, we were able to invert a 6 by 6 matrix symbolically. One such language is Altran which was developed at Bell Laboratories. The other language is Reduce. Both allow one to define variables symbolically, and they do all the "garbage collection" for you.

4.4.3 Application Of Markov Analysis To A Particular Network - with Partial Sharing of Channels

Consider the following network comprised of three earth stations, each being presented with ρ erlangs of traffic and having access to two of the three satellite channels.

The network is:

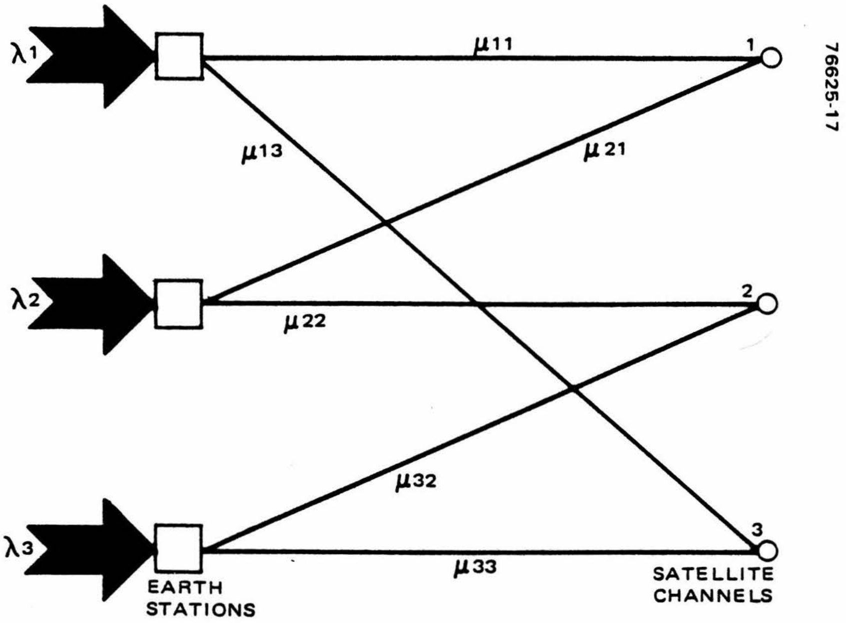


FIGURE 4-5. NETWORK CONFIGURATION

Since each channel can receive either no calls or a call from one of two different earth stations, it can be in one of three states. The state of a channel is independent of the state of another channel, so the network can be in one of $3^3=27$ states of the network.

As calls arrive, the earth station could search for a free channel by either random or preferential search. The derivation given is for a random search. The change from a random selection of channel frequencies to a predetermined order of choice alters the equations only by changing certain probabilities from fractions to ones or zeros, but the resulting equations will in general lose their symmetry and the resulting simplicity of solution. (Perhaps one could set up a similar scheme but let each link be several channels/frequencies capable of handling more than one call.)

Because of the topological symmetry of the network there are seven distinct probability states. They are:

(000)
(100) (001) (200) (020) (030) (003)
(101) (220) (033)
(130) (021) (203)
(120) (023) (103) (201) (031) (230)
(121) (223) (133) (131) (221) (233)
(123) (231)

(ijk) means that the 1st channel is accessed by the i^{th} earth station, etc.

The time dependent equations for the probability state space are:[10]

$$
\begin{aligned}
P_{000}(t+dt) &= \\
&= P_{000}(t)\ (1-\lambda_1 dt)\ (1-\lambda_2 dt)\ (1-\lambda_3 dt) \\
&\quad +P_{100}(t)\ (1-\lambda_1 dt)\ (1-\lambda_2 dt)\ (1-\lambda_3 dt)\ \mu_{11} dt \\
&\quad +P_{200}(t)\ (1-\lambda_1 dt)\ (1-\lambda_2 dt)\ (1-\lambda_3 dt)\ \mu_{21} dt \\
&\quad +P_{030}(t)\ (1-\lambda_1 dt)\ (1-\lambda_2 dt)\ (1-\lambda_3 dt)\ \mu_{32} dt \\
&\quad +P_{020}(t)\ (1-\lambda_1 dt)\ (1-\lambda_2 dt)\ (1-\lambda_3 dt)\ \mu_{22} dt \\
&\quad +P_{003}(t)\ (1-\lambda_1 dt)\ (1-\lambda_2 dt)\ (1-\lambda_3 dt)\ \mu_{33} dt \\
&\quad +P_{001}(t)\ (1-\lambda_1 dt)\ (1-\lambda_2 dt)\ (1-\lambda_3 dt)\ \mu_{13} dt
\end{aligned}
$$

$$
\begin{aligned}
P_{100}(t+dt) &= P_{000}(t)\ (1/2)\ \lambda_1 dt\ (1-\lambda_2 dt)\ (1-\lambda_3 dt) \\
&\quad +P_{100}(t)\ (1-\lambda_1 dt)\ (1-\lambda_2 dt)\ (1-\lambda_3 dt)\ (1-\mu_{11} dt) \\
&\quad +P_{103}(t)\ \mu_{33} dt \\
&\quad +P_{120}(t)\ \mu_{22} dt \\
&\quad +P_{130}(t)\ \mu_{32} dt \\
&\quad +P_{101}(t)\ \mu_{13} dt
\end{aligned}
$$

---

[10] Superfluous multiplicative terms will be omitted.

$$\begin{aligned} P_{101}(t+dt) = {} & P_{101}(t)\ (1-\lambda_2 dt)\ (1-\lambda_3 dt)\ (1-\mu_{11} dt)\ (1-\mu_{13} dt) \\ & +P_{100}(t)\ \lambda_1 dt \\ & +P_{131}(t)\ \mu_{32} dt \\ & +P_{121}(t)\ \mu_{22} dt \\ & +P_{001}(t)\ \lambda_1 dt \end{aligned}$$

$$\begin{aligned} P_{130}(t+dt) = {} & P_{130}(t)\ (1-\lambda_1 dt)\ (1-\lambda_3\ dt)\ (1-\mu_{11} dt)\ (1-\mu_{32} dt) \\ & +P_{030}(t)\ \lambda_1 dt(1/2) \\ & +P_{100}(t)\ \lambda_3 dt(1/2) \\ & +P_{133}(t)\ \mu_{33} dt \\ & +P_{131}(t)\ \mu_{13} dt \end{aligned}$$

$$\begin{aligned} P_{120}(t+dt) = {} & P_{120}(t)\ (1-\lambda_1 dt)\ (1-\lambda_3 dt)\ (1-\mu_{11} dt)\ (1-\mu_{22} dt) \\ & +P_{100}(t)\ \lambda_2 dt \\ & +P_{123}(t)\ \mu_{33} dt \\ & +P_{020}(t)\ \lambda_1 dt(1/2) \\ & +P_{121}(t)\ \mu_{13} dt \end{aligned}$$

$$\begin{aligned} P_{121}(t+dt) = {} & P_{121}(t)\ (1-\mu_{11} dt)\ (1-\mu_{22} dt)\ (1-\mu_{13} dt) \\ & +P_{101}(t)\ \lambda_2 dt \\ & +P_{021}(t)\ \lambda_1 dt \\ & +P_{120}(t)\ \lambda_1 dt \end{aligned}$$

$$\begin{aligned} P_{123}(t+dt) = {} & P_{103}(t)\ \lambda_2 dt \\ & P_{023}(t)\ \lambda_1 dt \\ & P_{120}(t)\ \lambda_3 dt \\ & P_{123}(t)\ (1-\mu_{11} dt)\ (1-\mu_{22} dt)\ (1-\mu_{33} dt) \end{aligned}$$

If we rewrite in matrix form, this becomes:

$$\begin{bmatrix} \\ \\ \tilde{M}' \\ \\ \\ \end{bmatrix} \underset{\text{all 27 states}}{\begin{pmatrix} P_{000} \\ P_{100} \\ P_{001} \\ \vdots \\ P_{233} \end{pmatrix}} = 0 \qquad \text{where} \sum_{\substack{\text{all} \\ 27 \\ \text{states}}} P_{ijk} = 1$$

Actually there are only 7 distinct (non-reducible by symmetry) states, so

$$\begin{bmatrix} \\ \\ \\ \tilde{M} \\ \\ \\ \\ \end{bmatrix} \begin{pmatrix} P_{000} \\ P_{100} \\ P_{120} \\ P_{130} \\ P_{101} \\ P_{123} \\ P_{121} \end{pmatrix} = 0$$

where

$$P_{000}+6P_{100}+6P_{120}+3P_{130}+3P_{101}+2P_{123}$$

$$+6P_{121} = 1$$

$$\text{i.e.} \sum P_{ijk} w_{ijk} = 1$$

This may be rewritten as

$$\begin{bmatrix} & & \\ & M & \\ & & \end{bmatrix} \begin{pmatrix} P'_{000} \\ P'_{100} \\ P'_{120} \\ P'_{130} \\ P'_{101} \\ P'_{123} \\ P'_{121} \end{pmatrix} = 0$$

where

$$P_{000}+P_{100}+P_{120}+P_{130}+P_{101}+P_{123}+P_{121} = 1$$

$$P'_{000}=P_{000},\ P'_{100} = 6P_{100},\ P'_{120} = 6P_{120},\ \text{etc.}$$

$$\text{i.e.} \sum P'_{ijk} = 1 \qquad P'_{ijk} = P_{ijk}w_{ijk}$$

Call this by the notation

$$\begin{bmatrix} M_{00} & M_{01} & M_{02} & M_{03} & M_{04} & M_{05} & M_{06} \\ M_{10} & M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} \\ M_{20} & M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} \\ M_{30} & M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & M_{36} \\ M_{40} & M_{41} & M_{42} & M_{43} & M_{44} & M_{45} & M_{46} \\ M_{50} & M_{51} & M_{52} & M_{53} & M_{54} & M_{55} & M_{56} \\ M_{60} & M_{61} & M_{62} & M_{63} & M_{64} & M_{65} & M_{66} \end{bmatrix} \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{pmatrix} = 0$$

By virtue of $\sum P_{ijk} = \sum P_j = 1$

$$\begin{bmatrix} M_{000} & M_{01} & \cdots \\ M_{10} & M_{11} & \cdots \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ & \cdot & \end{bmatrix} \begin{pmatrix} 1 - \sum\limits_{j=1}^{6} P_j \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{pmatrix} = 0$$

Writing the sum out explicitly

$$M \begin{pmatrix} 1 \\ P_1 \\ \vdots \end{pmatrix} - M \begin{pmatrix} P_1 \\ 0 \\ \vdots \end{pmatrix} - M \begin{pmatrix} P_2 \\ 0 \\ \vdots \end{pmatrix} - \cdots\cdots - M \begin{pmatrix} P_6 \\ 0 \\ \vdots \end{pmatrix} = 0$$

or

$$M \begin{pmatrix} 1 \\ P_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ P_6 \end{pmatrix} - \begin{bmatrix} 0 & M_{00} & 0 & 0 & \cdot & \cdot \\ 0 & M_{10} & 0 & & \cdot & \cdot \\ 0 & M_{20} & 0 & & & \\ 0 & M_{30} & 0 & & & \\ 0 & M_{40} & 0 & & & \\ 0 & M_{50} & 0 & & & \\ 0 & M_{60} & 0 & & & \end{bmatrix} \begin{pmatrix} 1 \\ P_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ P_6 \end{pmatrix} - \begin{bmatrix} 0 & 0 & M_{00} & 0 & \cdot & \cdot \\ 0 & 0 & M_{10} & & & \\ & & \cdot & & & \\ & & \cdot & & & \\ & & \cdot & & & \\ & & \cdot & & & \\ & & \cdot & & & \end{bmatrix} \begin{pmatrix} 1 \\ P_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ P_6 \end{pmatrix} - \cdots = 0$$

$$\therefore$$

$$\begin{bmatrix} M_{00} & M_{01}-M_{00} & M_{02}-M_{00} & M_{03}-M_{00} & \cdots \\ M_{10} & M_{11}-M_{10} & M_{12}-M_{10} & \cdot & \\ M_{20} & M_{21}-M_{20} & M_{22}-M_{20} & \cdot & \\ M_{30} & M_{31}-M_{30} & M_{32}-M_{30} & \cdot & \\ M_{40} & M_{41}-M_{40} & M_{42}-M_{40} & \cdot & \\ M_{50} & M_{51}-M_{50} & M_{52}-M_{50} & \cdot & \\ M_{60} & M_{61}-M_{60} & M_{62}-M_{60} & \cdot & \end{bmatrix} \begin{pmatrix} 1 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{pmatrix} = 0$$

$$\begin{bmatrix} M_{11}-M_{10} & M_{12}-M_{10} & \cdot\ \cdot\ \cdot \\ M_{21}-M_{20} & M_{22}-M_{20} & \cdot \\ M_{31}-M_{30} & M_{32}-M_{30} & \cdot \\ M_{41}-M_{40} & M_{42}-M_{40} & \cdot \\ M_{51}-M_{50} & M_{52}-M_{50} & \\ M_{61}-M_{60} & M_{62}-M_{60} & \end{bmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{pmatrix} = -\begin{pmatrix} M_{10} \\ M_{20} \\ M_{30} \\ M_{40} \\ M_{50} \\ M_{60} \end{pmatrix}$$

Returning from the generalization to the specific problem and letting $\lambda_i = \lambda \; \forall_i$; $\mu_{ij} = \mu \forall i,j$ one can write the linear equations relating the steady state probabilities as:

$$\begin{bmatrix} -3\lambda & 6\mu & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2}\lambda & -3\lambda-\mu & 2\mu & \mu & \mu & 0 & 0 \\ 0 & \frac{3}{2}\lambda & -2\lambda-3\mu & 0 & 0 & \mu & \mu \\ 0 & \lambda & 0 & -2\lambda-2\mu & 0 & 0 & 2\mu \\ 0 & 2\lambda & 0 & 0 & -2\lambda-3\mu & 0 & 2\mu \\ 0 & 0 & 3\lambda & 0 & 0 & -3\mu & 0 \\ 0 & 0 & \lambda & \lambda & \lambda & 0 & -3\mu \end{bmatrix} \begin{pmatrix} P_{000} \\ P_{100} \\ P_{120} \\ P_{130} \\ P_{101} \\ P_{123} \\ P_{121} \end{pmatrix} = 0$$

$$\begin{array}{cc} \begin{bmatrix} -3\lambda & \mu & & & & & \\ 3\lambda & -3\lambda-\mu & 2\mu & 2\mu & 2\mu & & \\ & \frac{3}{2}\lambda & -2\lambda-2\mu & & & 3\mu & \mu \\ & \frac{1}{2}\lambda & & -2\lambda-2\mu & & & \mu \\ & \lambda & & & -2\lambda-2\mu & & \mu \\ & & \lambda & & & -3\mu & \\ & & \lambda & 2\lambda & 2\lambda & & -3\mu \end{bmatrix} & \begin{pmatrix} 1 \cdot P_{000} \\ 6 \cdot P_{100} \\ 6 \cdot P_{120} \\ 3 \cdot P_{130} \\ 3 \cdot P_{101} \\ 2 \cdot P_{123} \\ 6 \cdot P_{121} \end{pmatrix} = 0 \\ \begin{array}{ccccccc} \overline{0} & \overline{0} & \overline{0} & \overline{0} & \overline{0} & \overline{0} & \overline{0} \end{array} & \overline{27} \end{array}$$

$$\begin{bmatrix} -3\rho & 1 & & & & & \\ 3\rho & -3\rho-1 & 2 & 2 & 2 & & \\ & \frac{3}{2}\rho & -2\rho-2 & & & 3 & 1 \\ & \frac{1}{2}\rho & & -2\rho-2 & & & 1 \\ & \rho & & & -2\rho-2 & & 1 \\ & & \rho & & & -3 & \\ & & \rho & 2\rho & 2\rho & & -3 \end{bmatrix} \begin{pmatrix} 1 \cdot P_{000} \\ 6 \cdot P_{100} \\ 6 \cdot P_{120} \\ 3 \cdot P_{130} \\ 3 \cdot P_{101} \\ 2 \cdot P_{123} \\ 6 \quad P_{121} \end{pmatrix} = 0$$

$$\begin{bmatrix} -6\rho-1 & -3\rho+2 & -3\rho+2 & -3\rho+2 & -3\rho & -3\rho \\ \frac{3}{2}\rho & -2\rho-2 & 0 & 0 & 3 & 1 \\ \frac{1}{2}\rho & 0 & -2\rho-2 & 0 & 0 & 1 \\ \rho & 0 & 0 & -2\rho-2 & 0 & 1 \\ 0 & \rho & 0 & 0 & -3 & 0 \\ 0 & \rho & 2\rho & 2\rho & 0 & -3 \end{bmatrix} \begin{pmatrix} 6 \cdot P_{100} \\ 6 \cdot P_{120} \\ 3 \cdot P_{130} \\ 3 \cdot P_{101} \\ 2 \cdot P_{123} \\ 6 \cdot P_{121} \end{pmatrix} = \begin{pmatrix} -3\rho \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$6 \cdot P_{100} = 6\rho/(6\rho^3 + 9\rho^2 + 6\rho + 2)$$

$$6 \cdot P_{120} = 9\rho^2/(12\rho^3 + 18\rho^2 + 12\rho + 4)$$

$$3 \cdot P_{130} = (9\rho^3 + 6\rho^2)/(24\rho^4 + 60\rho^3 + 60\rho^2 + 32\rho + 8)$$

$$3 \cdot P_{101} = (9\rho^3 + 12\rho^2)/(24\rho^4 + 60\rho^3 + 60\rho^2 + 32\rho + 8)$$

$$2 \cdot P_{123} = 3\rho^3/(12\rho^3 + 18\rho^2 + 12\rho + 4)$$

$$6 \cdot P_{121} = 9\rho^3/(12\rho^3 + 18\rho^2 + 12\rho + 4)$$

Total traffic out of the system

$$= 1\cdot 6\cdot P_{100} + 2\cdot 6\cdot P_{120} + 2\cdot 3\cdot P_{130} + 2\cdot 3\cdot P_{101} + 3\cdot 2\cdot P_{123} + 3\cdot 6\cdot P_{121} = \frac{18\rho^3 + 18\rho^2 + 6\rho}{6\rho^3 + 9\rho^2 + 6\rho + 2}$$

The probability that an incoming call is blocked[11] =

$$\begin{aligned} &P_{233} + P_{203} + P_{223} \\ &+P_{231} + P_{201} + P_{221} \\ &+P_{133} + P_{103} + P_{123} \\ &+P_{131} + P_{101} + P_{121} \end{aligned} = \begin{aligned} &P_{101} + P_{130} + 2P_{120} \\ &+6P_{121} + 2P_{123} \end{aligned} = \frac{6\rho^3 + 3\rho^2}{6\rho^3 + 9\rho^2 + 6\rho + 2}$$

Note that this agrees with the equation:

$$\text{Total Traffic} = (1-P_B)3\rho$$

---

[11] Since we have assumed that the satellite has onboard switching, then the only way that a call can be blocked is if it arrives at an earth station and all of the frequencies for that earth station are in use.

4.5 Computer Simulation Solution

4.5.1 Explanation Of Method -

As pointed out in section 4.4.2, for other than the simplest of networks, there are too many states to be able to solve or even set up the corresponding Markov state transition equations. In this case is is necessary to resort to a time domain computer simulation to obtain the total traffic and blockage probability of a given network as a function of traffic presented to the earth stations. This is permissible since the statistics are ergodic and memoryless. However, the random number algorithm used to generate the traffic statistics of the network must be quite good. This is because the manner in which the system is loaded depends strongly upon the interarrival times of calls arriving at about the same time. Thus the autocorrelation properties of the random number generator are very important. In addition, variations in the mean of the exponentially distributed random variables will change the time scale.

Our algorithm to simulate the traffic statistics of a satellite network addresses the problem in which calls arrive at the earth station each having Poisson arrival statistics and exponential holding times. The arrival statistics at each earth station may vary but all calls are assumed to obey the same exponential holding time distribution. (In fact bilingual conversations are usually longer, but this is a second order effect.) The program picks two random numbers from an exponential distribution for each incoming call. One is used to determine the interarrival time from the last call, and the other is used to determine the duration of the call. Calls are then placed on a channel or taken off again in the order of their respective times. Searching for an available channel may take place either randomly or in a preselected order.

A listing of the program, written in Fortran for the PDP DEC-10 computer is given in Appendix L. The particular example shown assumes all earth stations are presented with the same arrival traffic density.

Another numerical technique to solve for the probabilities could be used if the state equations of the network are known. This is the method of over-relaxation. M. Eisenberg has empirically derived parameters for such a method and suggests working inward from the boundary. This method will bog down if the number of states is too large.

We have also conceived of a technique which could be used both by a simulation program and an actual system to assign frequencies quickly without having to search for an available channel. The method of search is neither random nor predetermined and is as follows: Make a pushdown stack of available frequencies/channels. "Pop" when a call arrives and "push" when it terminates.

4.5.2 Arguments Of Validity (Tests & Goodness) -

Picking a random number from an ensemble which obeys an exponential distribution can be accomplished by first obtaining a number from a uniform distribution and applying the proper transformation. The random number generator "RAN" used on the DEC-10 is not very random, and decoding the time function (which is used extensively for the computer games) is likewise not uncorrelated enough for our purposes. Instead, an algorithm by R. C. Tausworthe called RANDOM written in assembler code for the JPL language, MBASIC™[12], has been modified for our purposes.

Two methods to transform to an exponential distribution were used. The first was to use the mapping $\frac{-\ln(x)}{\lambda}$ where x is a uniformly distributed random variable and λ is the exponent. (It may be derived using the cumulative distribution function $1-e^{-\lambda x'}$ of the exponential distribution.) The second method may be somewhat faster. It is a

---

[12] Trademark of the California Institute of Technology

computational algorithm that appears in Knuth (1969, Vol.2) which we have programmed in Fortran and have called, REXP.F4 (see Appendix L).

Because the traffic statistics of a network with sharing of frequency channels is a function of traffic from more than one presumably independent source, it is important that successively generated random numbers be uncorrelated. In addition, the cumulative distribution function observed should be close to the theoretical value. Samples taken from the exponential distribution were compared with expected results.

By the central limit theorem, the sum of a sequence of mutually independent random variables with a common distribution tends to follow a normal distribution with mean $n\mu$ and variance $n\sigma^2$. (Feller, 1968, chapter X). Table 4-6 shows the sample mean of the distribution as a function of n, the number of sample points, and the probability of observing such a statistic. We see that there is between 51% and 82% probability of observing a sample mean as small as or smaller than was obtained.

n	mean	$\lvert 1-\text{mean}\rvert \sqrt{n}$	probability
1000	.97780	.702	.51
10000	1.01265	1.265	.79
20000	1.00532	.752	.55
30000	1.00608	1.053	.71
40000	1.00637	1.274	.79
50000	1.00604	1.350	.82
60000	1.00297	.727	.53
70000	1.00385	1.019	.69
75000	1.00439	1.202	.77

Table 4-6. MEAN AND CONFIDENCE BOUND OF RANDOM VARIABLES

Suppose n samples are obtained that come from a known distribution of random variables. If a sample cumulative distribution function is obtained, then the maximum absolute value of the difference between the sample cumulative distribution function and the known distribution function obeys the Kolmogorov-Smirnov distribution (see Fisz, 1963). Let F(x) be the true CDF and let $F_n(x)$ be the fraction of observations less than or equal to x. Then:

$$P(\lambda) = \sum_{-\infty}^{\infty} (-1)^k \, e^{-2k^2\lambda^2}$$

where

$$\lambda = \sqrt{n} \sup_{-\infty<x<\infty} \left| F_n(x) - F(x) \right|$$

Figure 4-7 shows typical values of the Kolmogorov-Smirnov function as a function of λ. This is the probability that a value of λ could occur as large as or larger than that observed. Testing the random generator with 1000 samples, we obtained a Kolmogorov-Smirnov statistic of .9975. The above formula shows that there is a .7267 probability of observing a statistic this small or smaller.

As was pointed out earlier, it is important that sequential values taken from the random number generator be uncorrelated. Table 4-8 shows a typical autocorrelation for 75000 sample points. Although not shown in the table, autocorrelations of the sequence shifted by four through ten places were also computed and found to be well within the expected range. The observed values, mean, variance, and standard deviation are given for the sample mean, sample variance, and correlation of neighboring random variables taken from an exponential distribution with parameter μ. Appendix F derives the expectation value for these observed statistics.

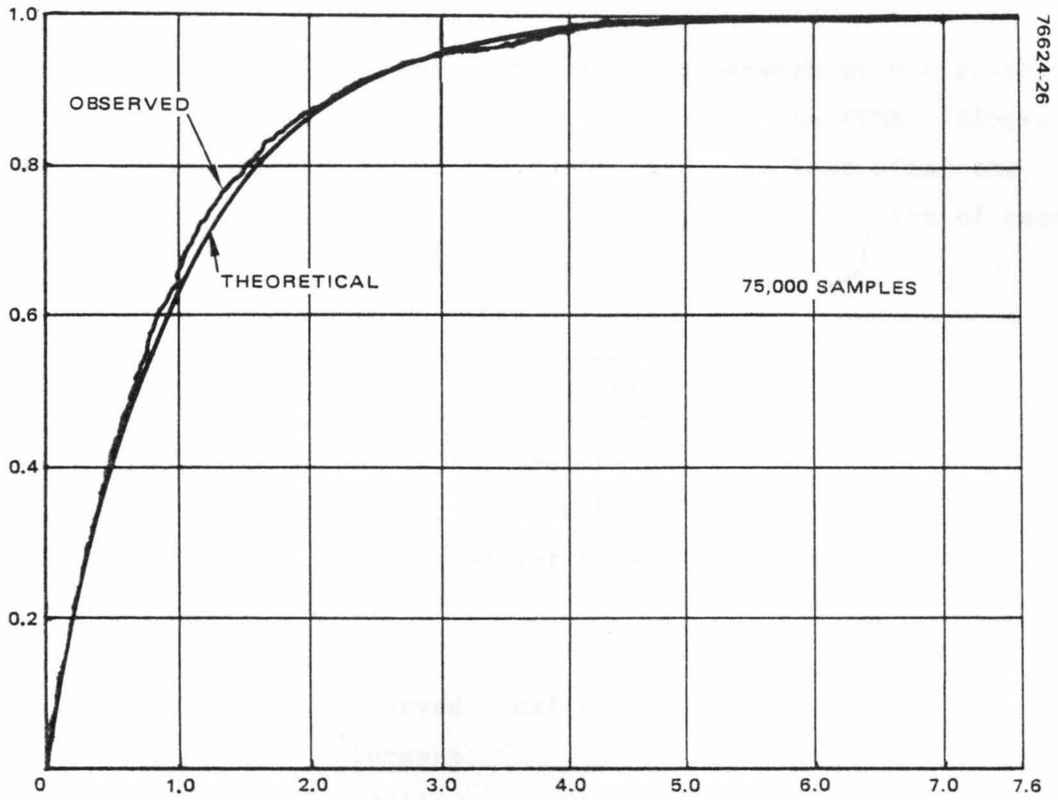


FIGURE 4-7. CUMULATIVE DISTRIBUTION FUNCTION

This table shows $S_{N,p}^2$, the sample autocorrelation function of the random variable sequence $\{x_i\}$ of N points shifted by p. Given are value observed and values calculated in Appendix F. N=75000 sample points were used.

		Calculated Expectation	
p	observed $S_{N,p}^2$	mean	standard deviation
0	1.01541	.99987	.01032
1	-.00082	-.00001	.00632
2	.00542	-.00001	.00632
3	.00038	-.00001	.00632
4	.00260	-.00001	.00632

Table 4-8. AUTOCORRELATION OF RANDOM VARIABLES

4.5.3 Results -

The computer simulation was obtained of a network with 4 earth stations and 3 channels. The network is assumed to be FVDA. Blockage probability was estimated to be the number of calls lost after the system had been presented with 5000 calls. Using the results of section 4.3.1 the exact formula for the blockage probability is:

$$P_B = \frac{\frac{(4\rho)^3}{6}}{1 + 4\rho + \frac{(4\rho)^2}{2} + \frac{(4\rho)^3}{6}}$$

A comparison of results for certain values of ρ is given in Table 4-9. The random number generator seems to be biased high, but not very much. This simulation method may be used for networks with more channels.

observed ρ	observed blockage	calculated ρ	calculated blockage
.05	.0011	.0507	.0012
.10	.0072	.1012	.0074
.15	.0198	.1497	.0212
.20	.0387	.2034	.0421
.25	.0625	.2529	.0712
.30	.0898	.3061	.0954
.35	.1192	.3443	.1210
.40	.1496	.4011	.1520
.45	.1803	.4468	.1790
.50	.2105	.5110	.2174
.55	.2400	.5555	.2364
.60	.2684	.6067	.2797

Table 4-9. RESULTS OF SIMULATION

As a last comment, it is possible that convergence of the simulator might be faster if the traffic load were increased gradually.

4.6 Simplified Analytic Solution

This section has been written as an afterthought, not because it is less significant than the rest of the thesis (to the contrary) but because it was thought of afterwards.

Appendix D shows the total number of states of a satellite network to be $(1+N_{cc})^{N_c}$. In fact many networks can be described by a much smaller number of states. If the traffic from all of the earth stations has the same mean holding time, then the transition probability of a channel is not a function of the origin of a call currently in progress, so each channel really has only two states, in use or not in use. In this case the total number of states of the network is 2^{N_c}. Now for many networks, symmetry of the configuration further implies that states exhibiting the same number of active channels all have the same probability. Then the total number of states of the network is only N_c+1.

For this latter case, one can write the relationship between state probabilities as the familiar birth/death process equation:

$$\lambda_{n-1} P_{n-1} - (\lambda_n+\mu_n)P_n + \mu_{n+1} P_{n+1} = 0$$

By the use of telescoping equations one obtains

$$\lambda_{n-1} P_{n-1} = \mu_n P_n$$

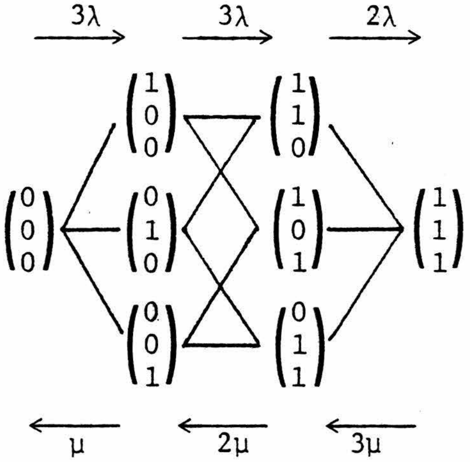
This implies that by examining the forward and reverse transition rates one can obtain the ratios of adjacent states. Since the sum of the probabilities must add to unity, all states can be found exactly without the use of a matrix inversion.

The reverse rate μ_n, is proportional to the number of calls in progress. The forward rate, λ_n, is proportional to the number of earth stations which can access the remaining channels for any particular arrangement of n channels in use. (If two arrangements, each with n channels in use, had different probabilities this method wouldn't work.) Graphically the general case is:

Forward rate		$\xrightarrow{N_e\lambda}$		$\xrightarrow{N_1\lambda}$		$\xrightarrow{N_2\lambda}$		$\xrightarrow{N_3\lambda}$		$\xrightarrow{N_4\lambda}$	
Channel usage	0		1		2		3		4		5
Reverse rate		$\xleftarrow[\mu]{}$		$\xleftarrow[2\mu]{}$		$\xleftarrow[3\mu]{}$		$\xleftarrow[4\mu]{}$		$\xleftarrow[5\mu]{}$	

where N_i = Number of earth stations that can access the remaining channels if i channels are in use.

We will clarify this using the example of section 4.4.3. There are 3 channels. The diagram is:



In the righthand transition $\lambda_2 = 2\lambda$ because only 2 of the 3 earth stations can access the remaining channel. We see then that

$$P_1/P_0 = 3\rho; \qquad P_2/P_1 = 3\rho/2; \qquad P_3/P_2 = 2\rho/3$$

$$\therefore \quad P_0 = \frac{1}{1+3\rho+\frac{9}{2}\rho^2+3\rho^3} \qquad P_1 = \frac{3\rho}{1+3\rho+\frac{9}{2}\rho^2+3\rho^3}$$

$$P_2 = \frac{9/2\rho^2}{1+3\rho+9/2\rho^2+3\rho^3} \qquad P_3 = \frac{3\rho^3}{1+3\rho+9/2\rho^2+3\rho^3}$$

$$TT = \frac{3\rho+9\rho^2+9\rho^3}{1+3\rho+9/2\rho^2+3\rho^3} \qquad P_B = 1-\frac{TT}{3\rho} = \frac{3/2\rho^2+3\rho^3}{1+3\rho+9/2\rho^2+3\rho^3}$$

Suppose we were to use the same network in which each earth station has two modems but those modems are able to access all of the channels.

Then the diagram is:

$$\overset{3\lambda}{\underset{\mu}{\rightleftarrows}} \qquad \overset{3\lambda}{\underset{2\mu}{\rightleftarrows}} \qquad \overset{\frac{8}{3}\lambda}{\underset{3\mu}{\rightleftarrows}}$$

$\lambda_2=\frac{8}{3}\lambda$ because the earth stations offer traffic at rate 3λ and there is 8/9th probability that not all three calls will come from the same earth station. In this case:

$$TT = \frac{3\rho + 9\rho^2 + 12\rho^3}{1 + 3\rho + 9/2\rho^2 + 4\rho^3}$$

$$P_B = 1 - \frac{TT}{3\rho} = \frac{1/2\rho^2 + 4\rho^3}{1 + 3\rho + 9/2\rho^2 + 4\rho^3}$$

We can use these calculations as a heuristic justification of a technique common to traffic theory calculations and used in section 6.3. It is stated there that the network blockage probability is equal to the sum of the modem blockage probability and the satellite blockage probability obtained using the Erlang B equation if the traffic load is sufficiently light. Using Erlang B and the above network,

$$P\binom{\text{satellite}}{\text{blockage}} = P_S = \frac{\frac{(3\rho)^3}{3!}}{1+3\rho+\frac{(3\rho)^2}{2}+\frac{(3\rho)^3}{3!}} = \frac{27\rho^3}{6+18\rho+27\rho^2+27\rho^3} = \frac{9}{2}\rho^3 + \ldots$$

$$P\binom{\text{modem}}{\text{blockage}} = P_m = \frac{\frac{\rho^2}{2!}}{1+\rho+\frac{\rho^2}{2!}} = \frac{\rho^2}{2+2\rho+\rho^2} = \frac{\rho^2}{2} - \frac{\rho^3}{2} + \ldots$$

$$P\binom{\text{network}}{\text{blockage}} = 1 - (1-P_s)(1-P_m) = \frac{\rho^2}{2} + 4\rho^3 + \ldots$$

The exact relationship is:

$$P_B = \frac{\frac{1}{2}\rho^2 + 4\rho^3}{1+3\rho+9/2\rho^2+4\rho^3} = \frac{\rho^2}{2} + \frac{5}{2}\rho^3 + \ldots$$

So the approximation is good if $\rho \ll 1$, but this should be verified in an actual design, perhaps by simulation.

5.0 DERIVED DESIGN PARAMETERS

5.1 Methodology For Designing A Network Configuration

5.1.1 Factors Affecting Blockage Probability -

In most networks, the acceptable probability of blockage is chosen to be 1%. The actual blockage depends on the following factors:

Traffic Density. Blockage increases with the increase in number of calls. The probability is zero if the amount of incoming traffic is zero, is unity if the traffic is infinite, and is usually linear with traffic for a light load.

Number of Satellite Channels. Blockage decreases with the number of channels, but the effect is more pronounced if there are just a few channels to start off with. Furthermore, increasing the number of channels is of little help if most of the blockage occurs at the earth station.

Types of Demand Access. Fully-variable demand access achieves less blockage than semi-variable, which in turn is better than preassigned. The more overlap of the semi-variable, the closer it approximates the fully-variable case. Figure 5-1 shows the number of satellite circuits required for a fixed blockage probability. (Courtesy M. Horstein)

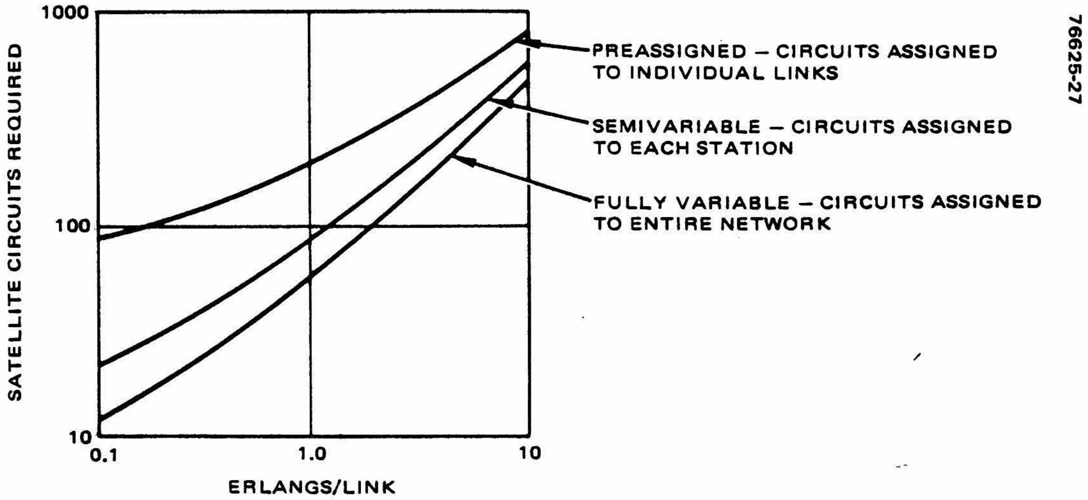


FIGURE 5-1. REDUCED SATELLITE CIRCUIT REQUIREMENT WITH DEMAND ASSIGNMENT

Number of Earth Stations. Varying the number of earth stations alone does not necessarily affect the blockage, but, in general, more earth stations means more traffic and this affects the channel and satellite blockage. A large number of earth stations also tends to even out the traffic and decreases the peakedness of the overall traffic.

Number of Modems per Earth Station. Current opinion is that a demand access network should be designed with most of the blockage occurring at the modem. This is for two reasons. In the first place, should the system expand, it is easier to add modems at the earth station than to add channels on the satellite. Secondly, the satellite has a large amount of traffic and its statistics are thus quite smooth. The earth station on the other hand exhibits traffic which is likely quite peaked and adding even a single modem usually makes a large difference in the blockage. Appendix J shows an example of this effect. Since SCPC operation is more economical than FDM only when the traffic is light, and otherwise not used, changes in the number of modems usually makes a large difference in the blockage.

5.1.2 Typical Values For Parameters -

An SCPC network could have up to 12000 voice channels if one is operating in the 4/6 GHz band and using FM modulation. This number would be somewhat smaller if TV or dedicated data traffic were to use the same satellite.

One would expect a regional network to have between 5 and 100 earth stations using FDM and today's technology. It is possible, and has been considered in the case of Iran, that networks could have over 1000 earth stations. The tradeoff between more terrestrial microwave links and more earth stations is an important issue beyond the scope of this thesis.

In the case of Latin America and the Caribbean, only two countries reported more than 60 erlangs of busy-hour traffic. This corresponds to about 80 modems. It should be pointed out that realistically some of the traffic would be more economically carried by terrestrial links or dedicated lines and that a maximum of 40 to 50 modems is more likely.

5.2 Probability Of Blockage Vs. Traffic Density Graphs

As was discussed previously, calls can be lost due to modem blockage, channel blockage, and satellite blockage.

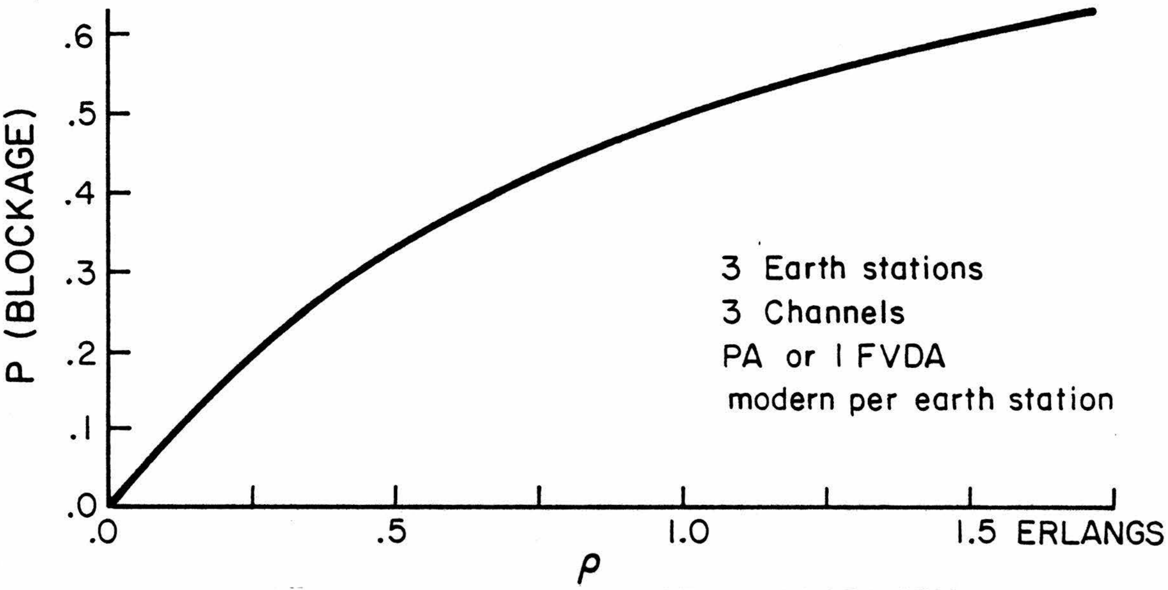


FIGURE 5-2. BLOCKAGE PROBABILITY

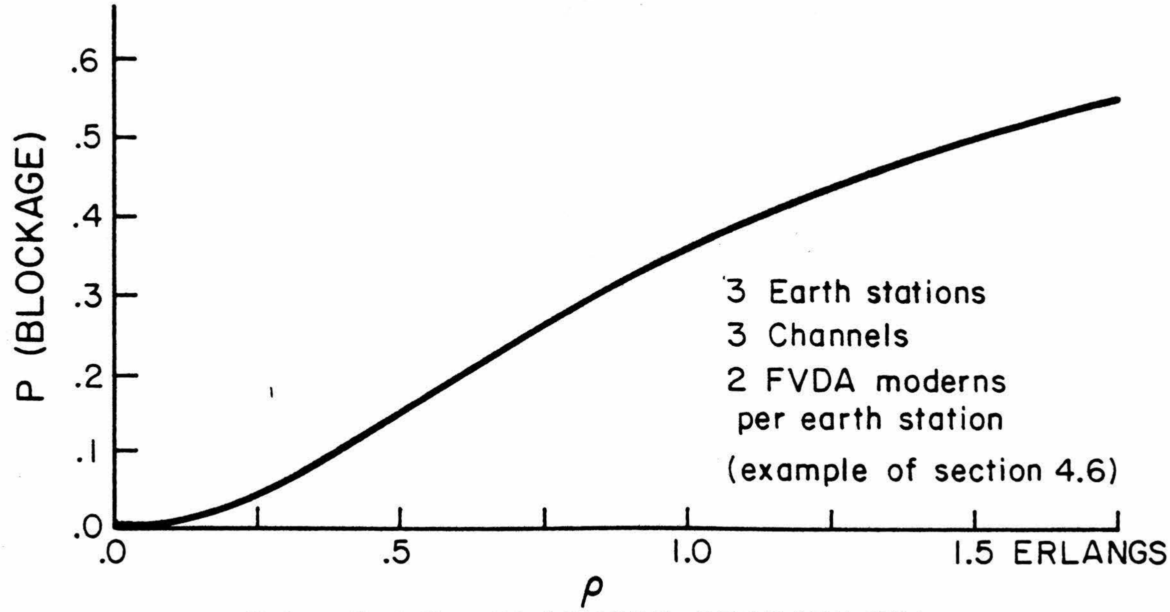


FIGURE 5-3. BLOCKAGE PROBABILITY

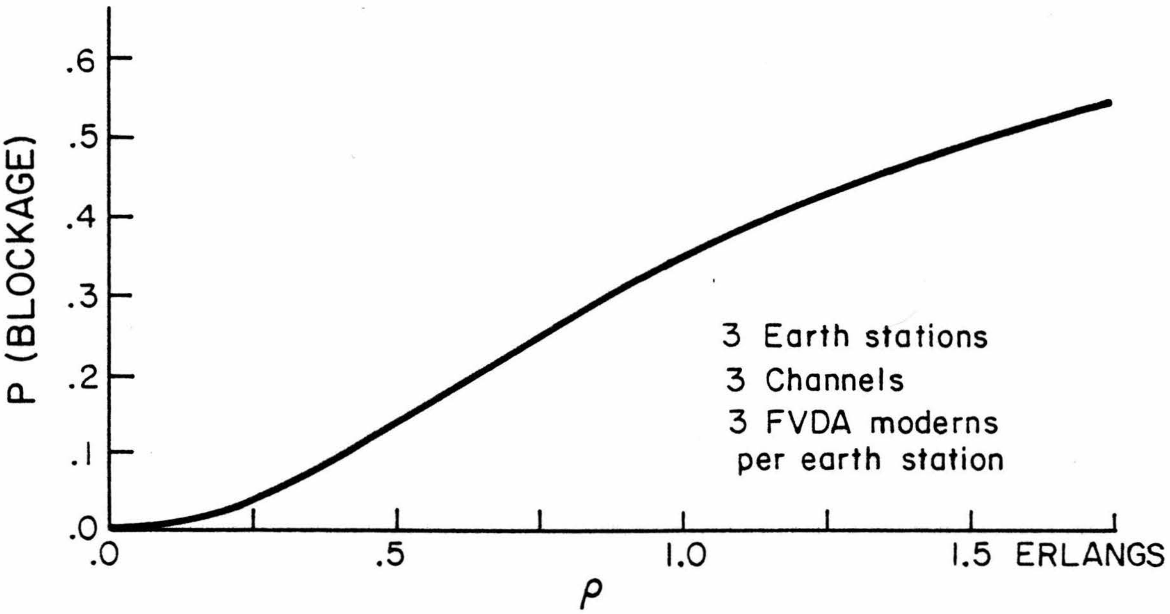


FIGURE 5-4. BLOCKAGE PROBABILITY

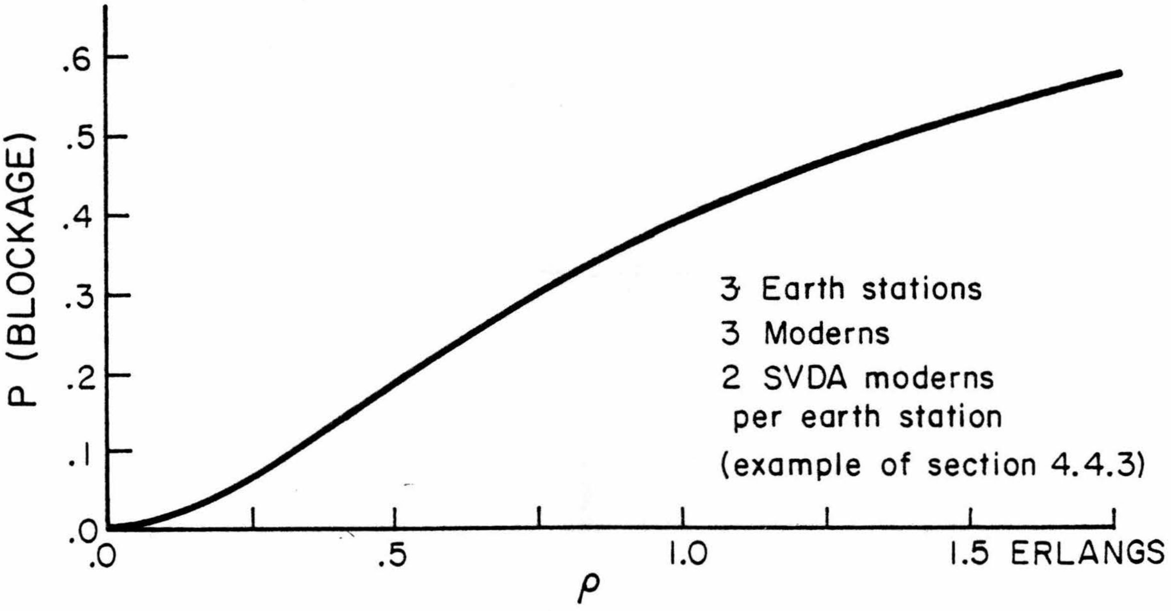


FIGURE 5-5. BLOCKAGE PROBABILITY

5.3 Asymptotic Approximations to Traffic Calculations

This section shows approximations to various traffic formulas valid for very light or very heavy traffic loads.

Erlang B:

$$\frac{\frac{\rho^c}{c!}}{1+\rho+\frac{\rho^2}{c!}+\ldots+\frac{\rho^c}{c!}} = \frac{\rho^c}{c!} - \frac{\rho^{c+1}}{c!} + \frac{\rho^{c+2}}{2c!} - \ldots \qquad \begin{cases} \rho \text{ small} \\ c \geq 2 \end{cases}$$

$$= 1 - c\left(\frac{1}{\rho}\right) + c\left(\frac{1}{\rho^2}\right) + \ldots \qquad \begin{cases} \rho \text{ large} \\ c \geq 2 \end{cases}$$

Example from Section 4.4.3:

$$\text{Blockage:} \quad \frac{6\rho^3+3\rho^2}{6\rho^3+9\rho^2+6\rho+2} = \frac{3}{2}\rho^2 - \frac{3}{2}\rho^3 - \frac{9}{4}\rho^4 + 9\rho^5 + \ldots \qquad \{\rho \text{ small}$$

$$= 1 - \frac{1}{\rho} + \frac{1}{2\rho^2} - \frac{1}{12}\,\frac{1}{\rho^3} - \frac{1}{24}\,\frac{1}{\rho^4} + \ldots \qquad \{\rho \text{ large}$$

$$\text{Traffic:} \quad \frac{18\rho^3+18\rho^2+6\rho}{6\rho^3+9\rho^2+6\rho+2} = 3\rho - \frac{9}{2}\rho^3 + \frac{9}{2}\rho^4 + \frac{27}{4}\rho^5 - 27\rho^6 + \ldots \qquad \{\rho \text{ small}$$

$$= 3 - \frac{3}{2}\,\frac{1}{\rho} + \frac{1}{4}\,\frac{1}{\rho^2} + \frac{1}{8}\,\frac{1}{\rho^3} + \ldots \qquad \{\rho \text{ large}$$

for ρ large

Light Traffic Limit:

Assume that the network exhibits either one or no calls. As in previous calculations we have N_e earth stations and N_c channels on the satellite. Each earth station can access N_{ec} channels and each channel can be accessed by N_{cc} earth stations. Let $P_{0000\ldots}$ be the probability of no calls on the system and $P_{1000\ldots}$ be the probability that the first channel is accessed by the first earth station.

Assuming there is only one or no calls on the network, then:

$$P_{1000\ldots} = P_{0000\ldots}\left(\frac{\rho}{N_{ec}}\right)$$

$$\sum P_{ijkl\ldots} = 1 \quad \text{and there are } N_e N_{ec} \text{ links.}$$

$$\therefore \quad P_{0000\ldots} = \frac{1}{N_e N_{ec}\left(\frac{\rho}{N_{ec}}\right) + 1} = \frac{1}{\rho N_e + 1}$$

$$P_{1000\ldots} = \frac{\rho}{N_{ec}(\rho N_e + 1)}$$

$$\text{Total Traffic} = \frac{\rho N_e}{\rho N_e + 1} = \rho N_e - (\rho N_e)^2 + \ldots$$

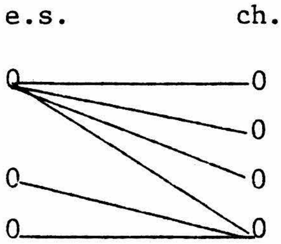
$$P_B = 1 - \frac{TT}{N_e\ \rho} = 1 - \frac{1}{(\rho N_e + 1)} = \frac{\rho N_e}{\rho N_e + 1}$$

$$= \rho N_e - (\rho N_e)^2 + \ldots$$

Note that this is not a good approximation for the example of Sec. 4.4.3.

Heavy Traffic Limit:

Assume that if a channel is free and a call arrives at an earth station which can access that channel, then all other channels accessible by that earth station are busy ⇒ The arriving call must go on the only free channel.



$$\begin{aligned} P(\text{ch. busy};t+dt) &= P(\text{ch. busy};t)\ P(\text{no call drop}) \\ &\quad + P(\text{ch. not busy};t)\ P\begin{pmatrix}\text{call arrives at} \\ \text{one of } N_{cc} \text{ stations} \\ \text{which access channel}\end{pmatrix} \end{aligned}$$

$$= P(\overset{\text{ch.}}{\text{busy}};\ t)\ (1-\mu dt) + [1 - P(\overset{\text{ch.}}{\text{busy}};t)]\quad N_{cc}\lambda dt$$

Steady state: $$P(\text{ch. busy}) = \frac{N_{cc}\rho}{1+N_{cc}\rho} = 1 - \frac{1}{N_{cc}\rho} + \left(\frac{1}{N_{cc}\rho}\right)^2 - \ldots$$

$$\text{Total Traffic} = (N\ \text{ch.})\ P(\text{ch. busy}) = \frac{N_{cc}N_c\rho}{1+N_{cc}\rho} = N_c - \frac{N_c}{N_{cc}\rho} + \ldots$$

$$P_B = 1 - \frac{TT}{N_e\rho} = 1 - \frac{N_{ce}}{1+N_{cc}\rho} = 1 - \frac{N_{ce}}{N_{cc}\rho} + \frac{N_{ce}}{(N_{cc}\rho)^2} - \frac{N_{ce}}{(N_{cc}\rho)^3} + \ldots$$

(First three terms of series agree with example section 4.4.3)

6.0 EXEMPLARY APPLICATION TO A REGIONAL NETWORK

6.1 Introduction

For the purposes of an example, a regional network for Latin America and the Caribbean has been analyzed. Traffic data was obtained from the various countries and calculations have been made to determine the approximate size and cost of the earth stations and satellite.

The Americas, excluding the United States and Canada, are composed of some 32 nations and semi-autonomous regions of a variety of historical, cultural, and economic backgrounds. The people speak 5 major languages (Dutch, English, French, Portuguese, and Spanish) and a multitude of lesser tongues. The countries have a total of 13,877,000 telephones in service which amounts to 4.2 telephones per 100 people and 3.6% of the world's telephones (The World's Telephones, Jan. 1976) Certainly these will increase in the forthcoming years, but probably not uniformly in all countries.

The reader wishing background on the individual nations is invited to read the Department of State background notes, the Latin America Political Report, and the Latin America Economic Report (see references).

The traffic pattern among the various countries is affected most by linguistic, economic, cultural, and political conditions. Often traffic is heavier between countries whose people speak the same language than between countries which are geographically close. For instance, Jamaica has much more traffic with the West Indies than with the nearby Central American nations. Most of Martinique's calls are with Guadeloupe. Some countries have reduced traffic loads because there is heavy tax on calls or because conversations are monitored. The volume of international trade is a major stimulus to international telephone calls. Several other factors also influence international calls. The farther apart countries are the smaller the number of calls because distant subscribers are less likely to be a part of the same community of interest. Time differences also restrict traffic. For instance, two persons in different time zones who wish to communicate via the telephone may be restricted to the overlap of their respective work hours.

Price is likely to be the main limiting factor for international calls (see figure 6-1). In comparison with letters, international telephone calls are an expensive means of communication and will therefore be rarely used if a letter will suffice. This gap in price is closing rapidly in those countries where labor costs are rising.

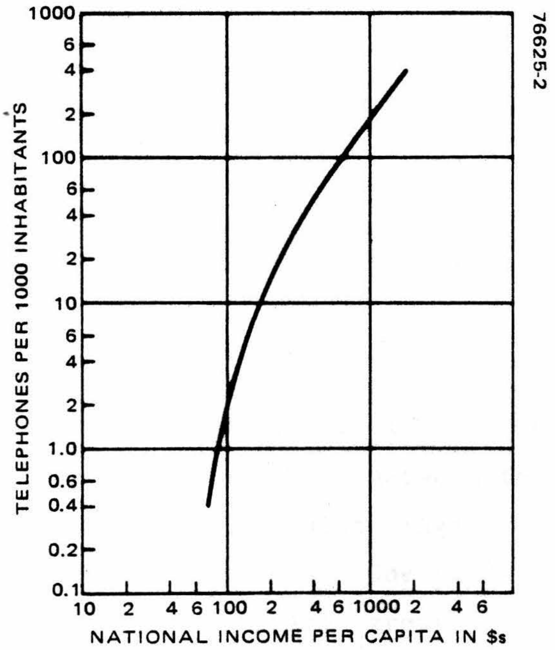


FIGURE 6-1. INCOME VS TELEPHONE DENSITY

By mutual agreement, any international telephone traffic must be answerable to CCITT regulations, although the rules are flexible if there is sufficient cause. In addition, since 18 of the countries are members of the Intelsat consortium (see Appendix H), an arrangement must be made with this organization to permit such traffic.

The signatory of each nation that joins Intelsat (Comsat in the case of the U.S.) agrees not to use a non-Intelsat satellite which would cause "substantial erosion" of Intelsat traffic (Smith, 1976). Thus Intelsat would have no objection to a satellite that is entirely for domestic use but would probably object to a regional satellite over which it was not the controlling organization. The policy of the U.S. State Department is not to interfere in any way which would undermine Intelsat. Accordingly, NASA will not launch a communications satellite for international use without the approval of Intelsat.

Let us use as an example the task of designing a demand access network to carry the international traffic of Latin America and the Caribbean. Although in practice one should consider the economic tradeoffs involved in locating the earth stations and determining the boundary of each geographical area whose aggregate traffic is served by a given earth station, we will divide the traffic according to political boundaries. We will use the 36 political areas listed in Appendix H.

6.2 Evaluation Of The Traffic Data

Discussions with various individuals in the field seem to indicate that the practical design of a telephone network that is profit making involves a mixture of experience and black magic. Traffic never originates as an ideal Poisson process. One finds diurnal peakedness, seasonal fluctuations, and unpredictable growth. Extremely low arrival rates typically exhibit greater peakedness, producing a larger number of busy hour erlangs.

The telephone traffic used in this model was obtained from a variety of sources. ATT kindly lent their only copy of the CCITT General Plan for the Development of the Regional Latin American Network (1973). This information was helpful, but five years outdated and somewhat incomplete. Accordingly, letters were sent to the various foreign governments requesting their assistance in obtaining the necessary information. All but three or four answered my letters, and almost a half sent the information I requested. These letters requested the annual minutes and erlangs of outgoing traffic between each of the countries for the year 1976 and as projected for the years 1978, 1980, and 1986. The data hides the fact that undoubtedly some traffic is already being blocked and the true "a priori" traffic is different. Also the relationship between annual minutes and erlangs reveals that either calculations were made differently or actual peak traffic figures were quoted. This is surmised because different countries showed widely varying ratios. Averaging over the data, the annual traffic (in thousands of minutes) divided by the busy-hour erlangs was about 102. This ratio was used for data where only the annual minutes were provided (or where the number of erlangs given was off by more than an order of magnitude).[13] Actually, one would like to use the peak coincident traffic data, but this is unavailable.

It was at first surprising to note that a couple of Central American countries reported more outgoing traffic than Brazil, until I realized that most of it was with their neighbors. Their international traffic was all local traffic!

Checking the information sent with the 1972 CCITT predictions, all of the countries reported more traffic in 1976 than had been predicted. I have been told that the reason for this is political, namely that it is safer to underestimate traffic and end up with too few circuits that are heavily in use, than to overestimate the traffic and have too many circuits which cost a lot of unnecessary money.

Appendix G shows confidence bounds for estimating the parameter of a Poisson process. From this we see that even for an ideal Poisson process, assuming three minute holding times, one could only estimate the Poisson parameter for an annual traffic of 210 minutes to within 10%

---

[13] In my dealings with many people for this thesis, I found that I did not appreciate two types of individuals, those who were not very helpful, and those who were helpful but wrong.

at the 90% confidence level. Data could not possibly be accurate to four significant figures, although such accuracy was reported by some countries. At the same time, if one is designing for a 1% blockage probability, one should also like to be able to claim 99% reliability, otherwise the numbers don't make sense.

In essence, traffic data with low data rates is unreliable and in designing a network with 1% acceptable call blockage one should be careful to provide plenty of margin for error.

The reader is also directed to Feller(1968, Vol. 1) which shows that if an unlimited trunk system starts in state $M(0)=i$, then after time t, $\mathrm{Var}= \rho(1-e^{-\mu t})+ie^{-\mu t}$. In our case $1/\mu = 3$ minutes and $t = 1$ year.

6.3 Formulation Of A Model

The ultimate task of a systems engineer is to outline the design requirements necessary to provide adequate service for a minimum cost over the lifetime of the system (or to point out the questions that must be answered in order to decide on the requirements). This thesis does not attempt to accomplish that task. Rather we will attempt to present a heuristic approach towards designing a frequency division multiple access single channel per carrier (FDMA-SCPC) communication network for a regional satellite network. We will assume that the system is semi-variable demand access (SVDA) with partial overlap of frequency channels that are allocated to the various earth stations.

Using the existing data for 1976, we will assume that all the traffic from each of the 36 political areas is directed to a single earth station. We will furthermore assume that each SCPC modem can access only one frequency channel. Thus in our previous terminology, a modem blocked call is simultaneously a channel blocked call. (But vice versa may not necessarily be true.) An incoming call can be blocked either because there is no available channel on the satellite or because none of the frequency channels allocated to that earth station are free. For small probabilities of blockage, the two types of blockage are approximately independent. Thus, the total blockage probability is the sum of the respective blockage probabilities.

$$P(\text{total blockage}) = P(\text{channel blockage}) + P(\text{satellite blockage})$$

If the design criterion is to achieve a 1% blockage probability, we may distribute this between the channel blockage and satellite blockage. How this distribution is to be made is largely an economic decision; one would like to find that ratio for which the overall cost is minimized. During the design phase, it usually is easier to decrease blockage on the satellite. Also, should the system have to expand later, it is easier to add capacity at each of the earth stations. The solution to this problem can be obtained by computer optimization, given the proper parameters, and will not be demonstrated in this thesis.

A heuristic method to design the network is to choose a blockage probability for the satellite and use the Erlang B formula to find the number of channels required to meet that blockage level, given the total traffic of the network. (Sum the traffic from all earth stations.) Secondly, given the channel blockage and traffic at each earth station, use the Erlang B to find the number of frequencies to be allocated to it. If the sum of the number of channels allocated to each earth station equals the total number of channels, we have SVDA with no overlap. If the sum is less, then some of the satellite channels will never be used (and probably we've made a mistake in the calculations). If the sum is more (and it should be in a practical system) then we have SVDA with overlap.

In order to make the network most efficient, one should spread the traffic equally among all the channels. How this is done is a critical part of the network design.

It is not sufficient merely to design a network which meets the network probability of blockage criterion; one must also meet certain minimum blockage requirements for each of the earth stations. Otherwise an earth station with a large amount of traffic may saturate a given channel, completely blocking a smaller earth station sharing that same channel. For example, if the smaller earth station has a very light traffic load and uses only that channel, blockage of a single call will be percentage-wise much greater than for the larger earth station. Likewise, adding more modems to a large earth station after the system is operational will likely degrade service of the smaller earth stations more than would be expected if one examines only the increase in network probability of blockage. So it is really the worst blockage over all earth stations that interests us.

In order to avoid these problems, we propose the following method of distributing the traffic among the various earth stations.

Divide the total traffic presented to the network by the number of channels (as calculated above). This is the average traffic per channel. Now for each earth station divide its total traffic by the number of modems (allocated frequency channels). This is the partial traffic per channel. Now allocate frequency channels to each earth station such that:

1) The earth stations with the largest partial traffics are allocated channels first.

2) The sum of the partial traffics allocated to each channel is very close to the average traffic per channel.

3) The selection distributes the channels among the earth stations so that the number of channels which they mutually share is minimized, for every n earth stations, n=2,3,4,...

This particular scheme is based upon the following assumption. When an earth station is in operation and is presented with an incoming call, the selection of an available channel is made randomly with equal probability of choosing one of the available channel frequencies.

Naturally one could also select channel frequencies in an ordered fashion. This is analogous to the overflow circuits in the standard terrestrial network. It is not clear which is more advantageous. One cannot alter the statistics of the holding times of calls on a given channel, but by restructuring the assignment of frequency channels to the individual earth stations and the selection of channels within the earth station, one can modify the interarrival time statistics of a channel.

Selecting channels randomly tends to average the traffic statistics over all channels. It may, however, be better for an earth station with heavy traffic always to attempt to use a certain channel for which it has exclusive right for by doing so that channel will have a large probability of a call being placed on it and it will show a larger traffic density than usual. One could also use a random selection technique in which not all channels are picked with equal probability.

6.4 Results

Using the existing traffic data for 1976, there are a total of 783.85 outgoing erlangs for the entire system. Following the logic of the previous section to assign a greater part of the total blockage to the earth stations, we will allocate .5% probability of blockage to each of the earth stations and .1% to the satellite. As shown in Appendix J, a blockage probability of .1% on the satellite dictates that it have 1590 channels.[14] Appendix J also shows the number of channel frequencies to be allocated by each earth station in order to achieve the required earth station blockage probability. If there is .5% blockage by both the originating and destination earth stations (or 1% blockage for their combined traffic), then the network requires a total of 2031 modems. This gives the system a total blockage probability of 1.1%.

[14] Note that 2 channels are required on the satellite for each outgoing call since the conversation is two-way.

The 1590 channels required would not fill the satellite and the exact number of transponders required would depend on the round trip signal to noise ratio of the system. As a comparison, however, the Intelsat IV satellites have a capacity of 4000 telephone circuits. Intelsat IV A and Intelsat V which incorporate frequency reuse have capacities of 6000 and 12000 telephone circuits respectively.

There is too much data to give a complete comparison of the savings of demand access over preassigned channels. As a particular example, in order to achieve the same blockage probability, Jamaica, which can be served by 30 demand access circuits, would require 133 preassignment circuits. In this case demand assignment is a 4 to 1 improvement and typical of how DA saves in circuitry.

In such a system, the cost of the satellite, launch, booster, and insurance is about $15M.[15] The earth stations, not including the modems, have a combined cost of $3.8M. The combined cost of the modems amounts to $7.1M. We see then that 65% of the earth station cost (or 27% of the network cost) is in the modems.

---

[15] Actually it is $30M but we are assuming that the satellite is half filled.

APPENDIX A

Definition of Terms

Amplifier. A device which amplifies an electrical signal.

Antenna. A device which radiates and/or receives electromagnetic waves. An antenna acts both as a matching device between two different propagation mediums and focuses energy from/to a particular region of space.

Backoff. The reduction in TWT power levels to reduce nonlinearities in the output.

Baseband. The information signal before modulation.

Blocked Call. A call is said to be blocked if it arrives at a switching center and there is no available path for it to take to get to its destination.

Carrier. A single frequency radio signal which is modulated by the modulating waveform in a communications system.

CCITT. International Consulative Committee for Telegraph and Telephone. An International organization which makes recommendations dealing with international radio communications.

Channel. A segment of frequency or time domain capable of carrying circuits.

Clear. A line signal which indicates that the subscriber has disconnected.

Communication link. A path between two points in a communications system over which an electrical signal may propagate.

Communications channel. A band of frequencies which are used by a communications system in transferring information from the source to the destination.

Compander. Compressor/Expander - Used to improve the subjective signal to noise level of a voice channel.

Concentrators. Switching equipment that allows many incoming lines to share a smaller number of outgoing lines.

Deemphasis. Alteration of the power spectrum of the demodulated information signal to simultaneously cancel the preemphasis of the signal and alter the noise spectrum thus increasing the signal-to-noise ratio.

Demand Access (DA). A method of allocating the use of a channel as it is required by an earth station.

Demand Assignment. A method whereby a satellite circuit is assigned to a trunk when a call begins. The trunk can thus be connected to any trunk at another earth station.

Demodulate. Extract the information signal from the modulated carrier.

Downconverter. Equipment that demodulates a signal to a lower frequency band.

Downlink. The link between the TWT output and the earth station receiver input.

Earth Station. A facility located at some location on the earth whose purpose is to transmit and receive traffic between a trunk center and the satellite.

Echo Suppressor. A voice operated device for connection to a telephone circuit to attenuate echo currents in one direction caused by signal currents in the other direction.

Frequency Division Multiple Access (FDMA). A form of frequency division multiplex in which the individual channels are shared among a group of users according to their needs at a particular time.

Frequency Division Multiplex (FDM). Several signals are put through a channel using a separate frequency band for each.

Frequency Pair. A circuit which consists of a pair of frequencies, the originating frequency for the A modem, and the emanating frequency for the B modem.

G/T. The ratio of the gain of a receiver to the noise temperature.

Geostationary satellite. A satellite in a circular equatorial orbit, whose orbital velocity equals the rotational velocity of the earth, and which, therefore, occupies a fixed position over a specific location on the earth.

Intermediate frequency (IF). A frequency between baseband and RF. Certain system functions are done at IF because it is easier to build hardware this way. IF frequencies for an FDM/FM signal are from 52 to 88 MHz, or a nominal value of 70 MHz.

Intermodulation. The interaction of signals in a nonlinear device resulting in signal components at the device output which are products of two or more of the input signals.

Line Signals. Signals sent between sets of switching equipment to indicate the status of a trunk.

Modem. Modulator/demodulator. Modem is the term for SCPC modems used in the DA system.

Preemphasis. Alteration of the power spectrum of an information signal before using it to modulate a carrier.

Probability of Blockage. The ratio of blocked calls to total number of calls. It is sometimes called the "grade of service".

Register Signals. Signals sent between sets of switching equipment to cause routing of a call through the telephone network.

RF. Radio frequency.

Satellite. In this thesis we are referring to a communications satellite in synchronous equatorial orbit capable of relaying telephone traffic from one earth station to another.

Seize. A line signal which indicates a circuit request.

Signalling. In a telephone network, the transmission of information required to set up and control a connection and charge the calling party.

Single Channel Per Carrier. A transmission system in which each voice channel modulates a separate RF carrier. SCPC transmission is commonly used in satellite communication systems when many ground stations have low traffic volumes between many destinations.

Time Division Multiple Access (TDMA). A form of time division multiplex in which the individual time slots are shared among a group of users according to their needs at a particular time.

Time Division Multiplex (TDM). A method of putting several signals through a channel by assigning a separate time slot to each signal.

Traffic. Information flowing in a communication network, usually generated on a random basis. Incoming and Outgoing traffic refers to that traffic arriving at or leaving a switching point, respectively. Traffic also refers to the average density of telephone calls in progress simultaneously.

Transponder. One of the 12 channels of the satellite repeater.

Traveling-wave tube (TWT). An amplifier whose operation depends on the interaction of an electron beam and a signal traveling along a conductor which spirals around the path of the electron beam.

Trunk Center. A long distance telephone switching facility.

Upconverter. Equipment that modulates a signal to a higher frequency band.

Voice Activation. A process by which an SCPC carrier is transmitted only when a modulating voice signal is present.

Voice Channel. In SCPC transmission, a voice channel is the frequency band used for the transmission of a carrier modulated by one voice waveform. In an FDM/FM system, a voice channel is typically 40 kHz wide.

APPENDIX B

List of Corporations Supplying Earth Station Components and Systems

Aeroneutronic Ford

Amplica

Aydin

Bell Telephone Manufacturing (Belgium)

California Microwave

Collins Radio Gp.; Rockwell International

Comsat General

General Electric

Harris

Hughes Aircraft

ITT Spacecom

Jet Propulsion Labs

Keltec

Lenkurt-GTE

LNR

Logos

NEC America

RCA American Communications

Scientific Atlanta

Scientific Communications

SPAR (Canada)

Westinghouse

Western Union

APPENDIX C

Cost Breakdown for a Typical Earth Station

The following figures and tables show the breakdown of costs of a typical earth station.

Table C-1 shows the range of costs for all of the earth station except the modem. The righthand column shows values picked to estimate the cost of the network described in section 6. Table C-2 shows typical costs of a modem/channel unit. The figures shows cost vs. performance curves for the HPA, LNA, and antenna.

Equipment	Price Range	Price used in Example
Shelter	13K-20K	15K
Power Supply and Cables	8K-16K	12K
Antenna (including feed and diplexer)	6K-50K	18K; 5 meters
Diplexer	1K-2K	-
Horn feed and diplexer		
4.5m	1K-1.5K	
10m	3K-4K	-
HPA TWT	10K-30K	19K; 150 Watts
Up converter (and ref. oscill.)	6K-9K	6.5K
LNA	1.5K-35K	6K; 90#K.
Down converter (price is a function of LO stability)	6.5K-9.5K	7.5K
Low Noise Switch	800-2K	-
Low power Switch	800-2K	-
High Power Switch	3K-6K	-
Subtotal:		84K
+20% for everything else		16K
Total:		100K

Table C-1. Earth Station Costs

Equipment	Price Range	Price used in Example
Channel Unit/Modem (2-way)	2.5K-5.0K	2.7K
Frequency Synthesizer	600-900	
Modem	2.2K-2.6K	
Codec/Echo Suppressor	400-500	
Tone disabler	50-100	
Compander	100	
Reference Oscillator (included in modem price)	2K	
Summer/Distributor Rack	400-500/channel unit	
Channel Rack (SCPC)	800-1200/channel unit	800
Channel adaptors (PA)	500-1500	
(DA)	1000-3000	
Total:		3.5K

In general demand access modems are about $700 more expensive than preassigned modems.

Miscellaneous:

TV receiver	17K-20K
DA controller	4K-6K
Data link modem	5K

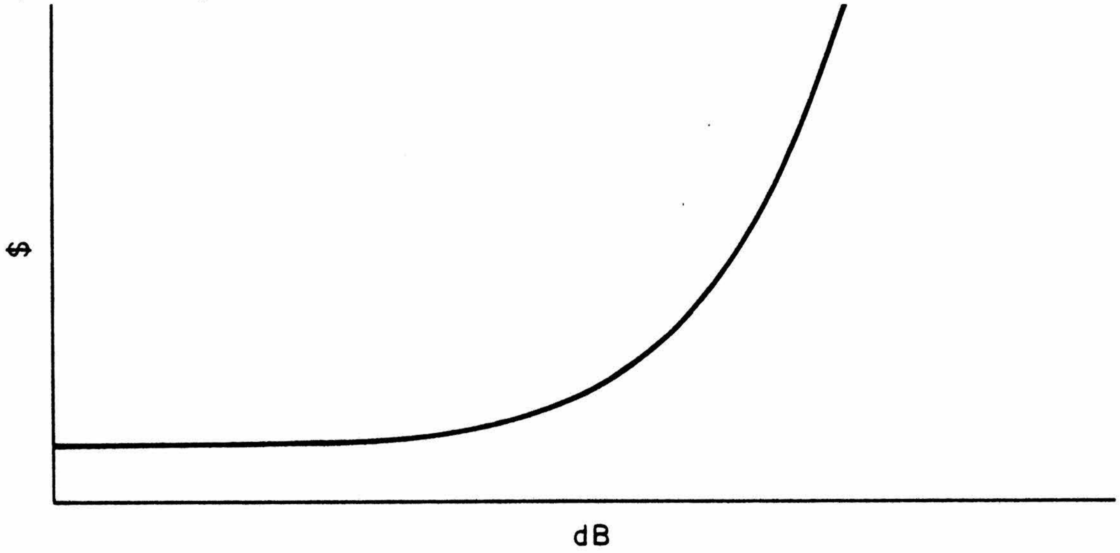
Table C-2. Modem/Channel Unit Costs

Several points can be made. First of all, many earth stations have redundant RF equipment. Such equipment plus automatic switching circuitry may add another $25K-$35K to the cost of the earth station. Secondly, some designs require a separate up/down converter for each modem. This adds appreciably to the cost of an earth station.

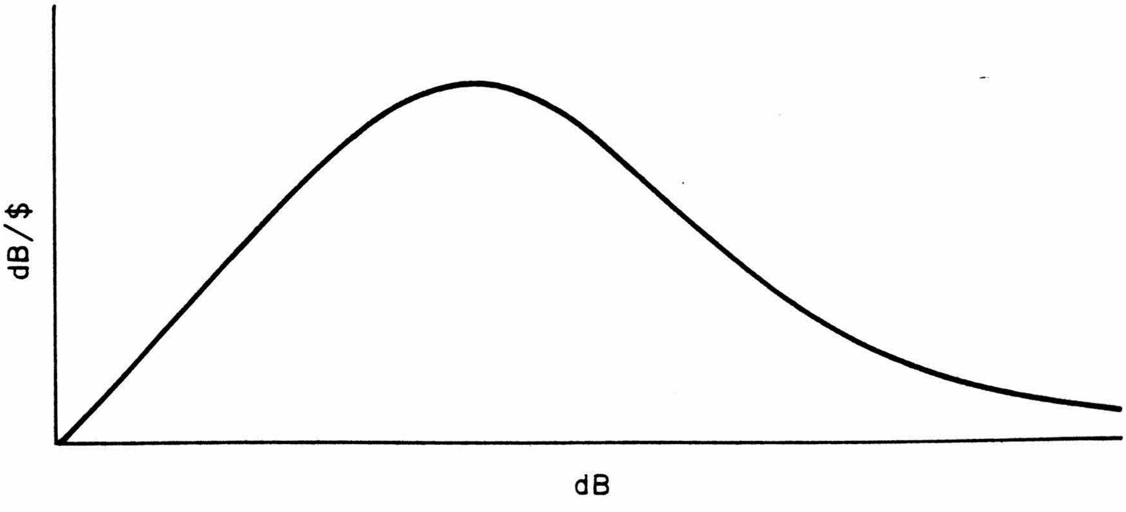
As was mentioned in section 3.2.2d, rain margin must be considered when designing a system. Figure C-6 shows degredation of the system (in dB's) due to the presence of rain for various LNA temperatures. This may be used in connection with statistical information on rain attenuation to determine the maximum rain-noise temperature which must be tolerated.

Looking at Figures C-4 and C-5 we find that the best performance to price ratio is 1.3 dB/$1K for the LNA and 1.0 dB/$1K for the antenna. It might therefore be advantageous to use this optimal LNA and use a suboptimal dish size if the rain does not degrade the system too much at 100°K receiver noise temperatures.

Finally, a comment will be made on prices in general. The general shape of cost performance curves is:



So the shape of performance/$ vs. performance is:



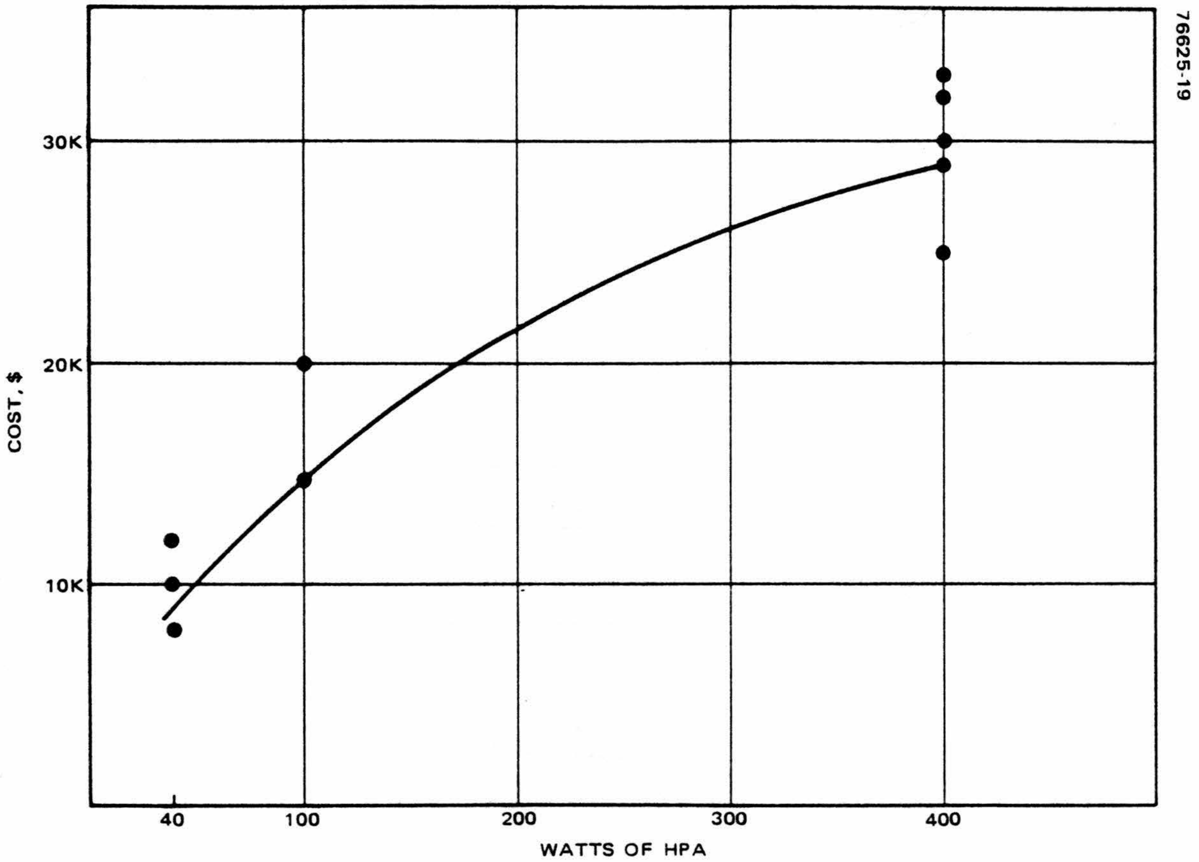


FIGURE C-3. COST VERSUS POWER RELATIONSHIP FOR HIGH POWER AMPLIFIERS (HPA)

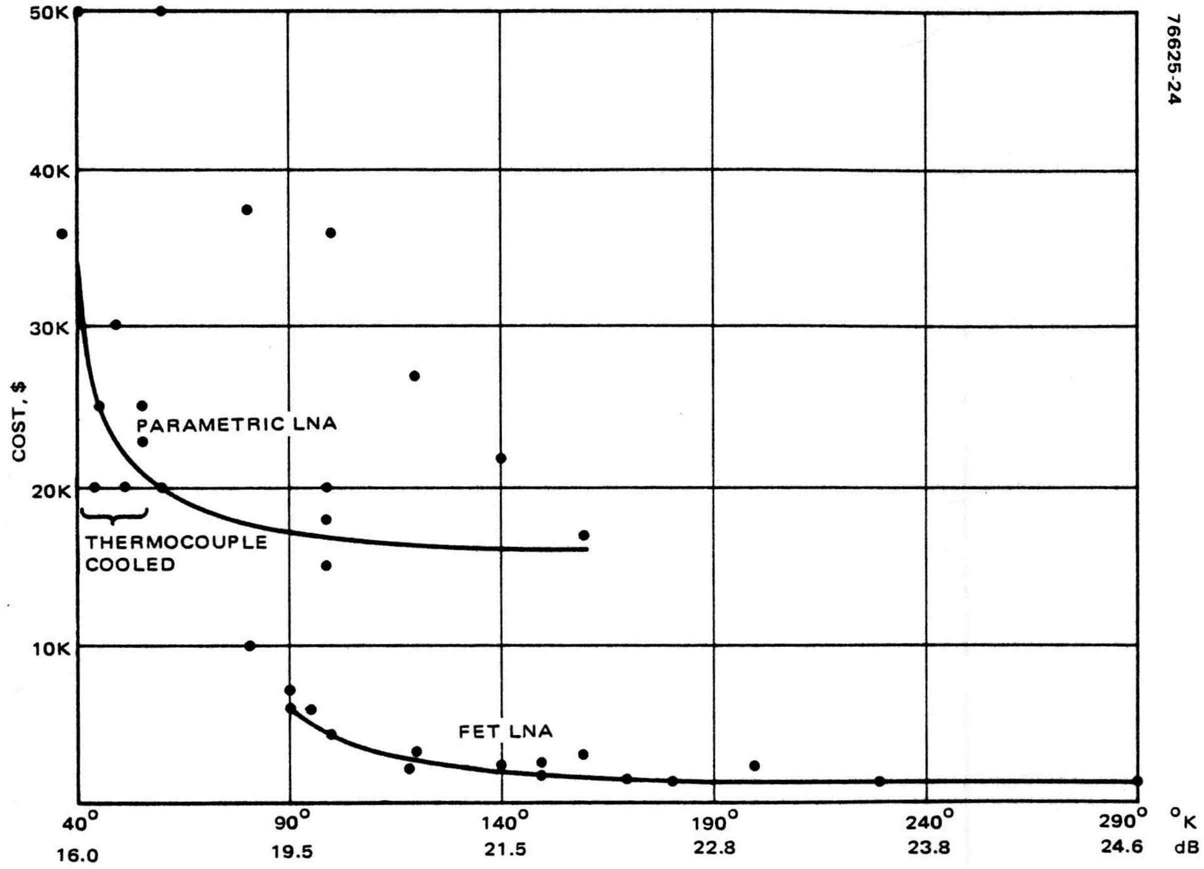


FIGURE C-4. LOW NOISE AMPLIFIER (LNA) COST VERSUS TEMPERATURE AND LOSS

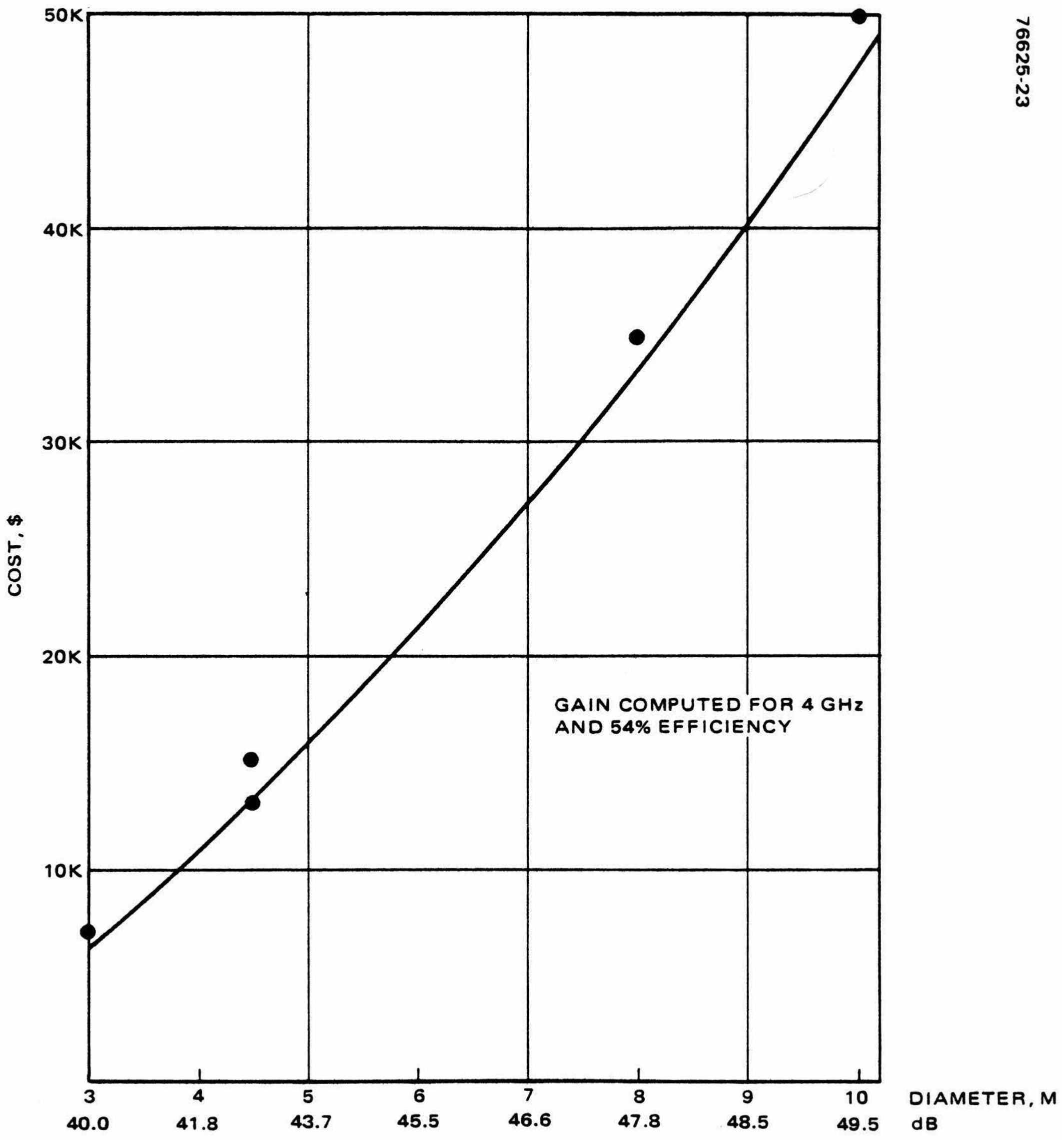


FIGURE C-5. ANTENNA COST VERSUS SIZE AND GAIN

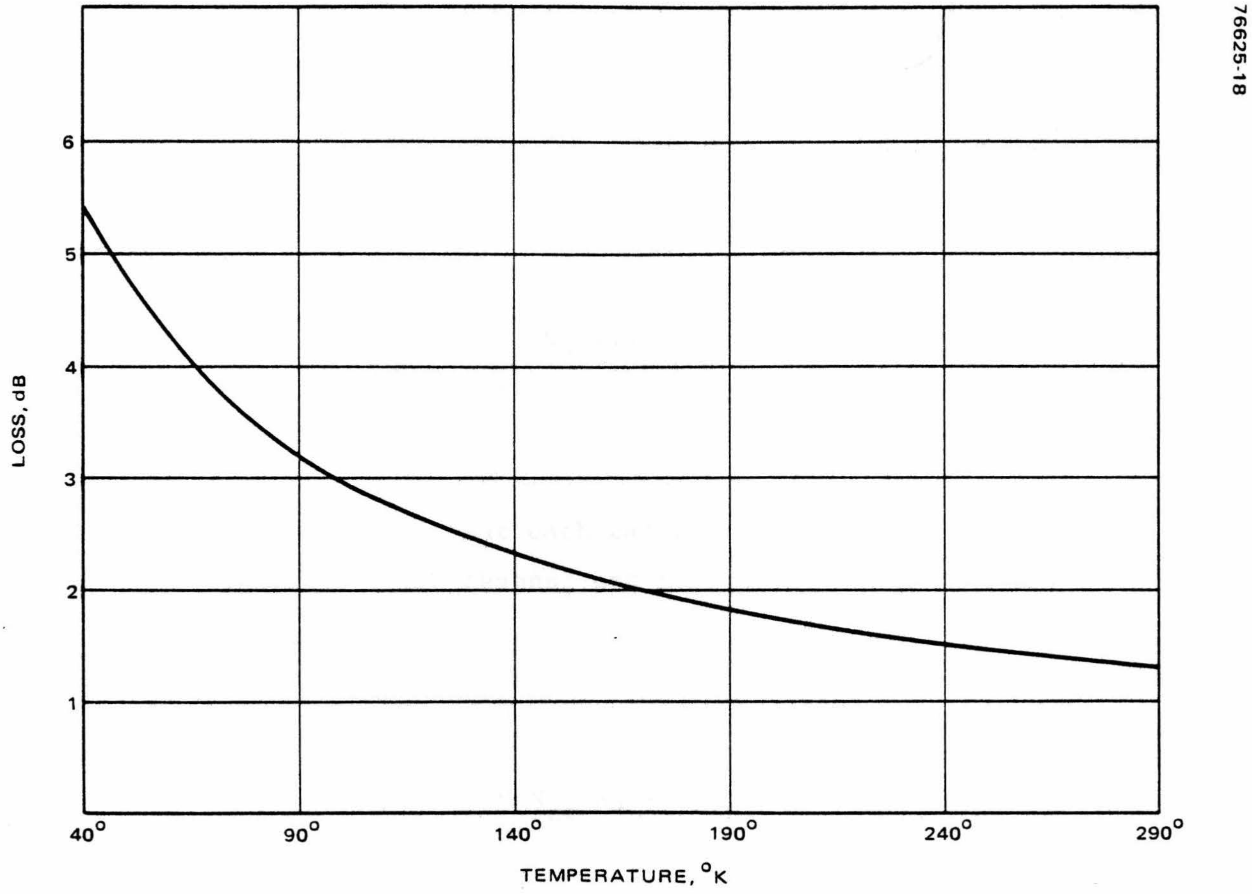


FIGURE C-6. DB DEGRADATION DUE TO 100°K RAIN-NOISE

APPENDIX D

Counting the States of a Satellite Network

In general the uplink paths might consist of N_e earth stations, $N_{ce}(i)$ links from the i^{th} earth station to the satellite, N_c satellite channels, and links to the j^{th} channel from $N_{cc}(j)$ different earth stations. The total number of paths (though only N_c can be used at any one time) is thus:

$$N_{path} = \sum_{i=1}^{N_e} N_{ce}(i) = \sum_{j=1}^{N_c} N_{cc}(j)$$

If the system is such that each earth station can access the same number of channels and each channel can be accessed by the same number of earth stations, then,

$$N_{path} = N_{ce} N_e = N_{cc} N_c$$

and since a given channel is either connected to one of the N_{cc} earth stations or is not in use, the total number of states of the system is:

$$N_{states} = (1 + N_{cc})^{N_c}$$

APPENDIX E

Double Source Traffic Statistics

This section outlines two mathematical results in the case where two sources of traffic, both Poisson, but with different mean holding times, are presented to a network with blocking. An example of this is a telephone network which is used both for voice conversations and computer data, since computer connections are usually much longer than ordinary calls. Blockage for such a case depends only on the total traffic in erlangs. This result may also be extended to an arbitrary number of sources.

The first result uses an example to argue that, in general, traffic which comes from two sources is not memoryless unless it is tagged. The second result shows the Markov state equations for double source traffic and derives the appropriate blockage probabilities.

a)

The following example serves to show that a double source traffic is not in general memoryless. We will compute the expectation value of the holding time of a call given that it has already been in progress for time t.

The statistics of each of the sources is as before and there is one channel. By Bayes's rule,

$$P\left(\begin{matrix}\text{source}\\ k\end{matrix}\;\middle|\;\begin{matrix}\text{in duration}\\ \text{time } t\\ \text{or greater}\end{matrix}\right) = \frac{P\left(\begin{matrix}\text{in duration}\\ \text{time } t\\ \text{or greater}\end{matrix}\;\middle|\;\begin{matrix}\text{source}\\ k\end{matrix}\right) \quad P\left(\begin{matrix}\text{source}\\ k\end{matrix}\right)}{\sum_i P\left(\begin{matrix}\text{in duration}\\ \text{time } t\\ \text{or greater}\end{matrix}\;\middle|\;\begin{matrix}\text{source}\\ i\end{matrix}\right) \quad P\left(\begin{matrix}\text{source}\\ i\end{matrix}\right)}$$

In this case,

$$P\left(\begin{matrix}\text{in duration}\\ \text{time } t\\ \text{or greater}\end{matrix}\;\middle|\;\begin{matrix}\text{source}\\ k\end{matrix}\right) = \int_t^\infty e^{-\mu t'} dt' = e^{-\mu t}$$

So, the expectation value of the holding time for a fixed value of t is:

$$E_t\left(\frac{1}{\mu}\right) = \frac{\frac{\lambda_1}{\lambda_1+\lambda_2} e^{-\mu_1 t}}{\frac{\lambda_1}{\lambda_1+\lambda_2} e^{-\mu_1 t} + \frac{\lambda_2}{\lambda_1+\lambda_2} e^{-\mu_2 t}} \frac{1}{\mu_1} + \frac{\frac{\lambda_2}{\lambda_1+\lambda_2} e^{-\mu_2 t}}{\frac{\lambda_1}{\lambda_1+\lambda_2} e^{-\mu_2 t} + \frac{\lambda_2}{\lambda_1+\lambda_2} e^{-\mu_2 t}} \frac{1}{\mu_2}$$

$$E_t\left(\frac{1}{\mu}\right) = \frac{\rho_1 e^{-\mu_1 t} + \rho_2 e^{-\mu_2 t}}{\lambda_1 e^{-\mu_1 t} + \lambda_2 e^{-\mu_2 t}}$$

$$E_o\left(\frac{1}{\mu}\right) = \frac{\rho_1+\rho_2}{\lambda_1+\lambda_2} \qquad (\text{if } \lambda_1 = \lambda_2;\ \frac{1}{2}\,(\frac{1}{\mu_1} + \frac{1}{\mu_2}))$$

To find

$$E_\infty\left(\frac{1}{\mu}\right) = \lim_{t\to\infty} E_t\left(\frac{1}{\mu}\right) \qquad \text{let } \mu^* = \min(\mu_1,\mu_2)$$

Then

$$E_\infty\left(\frac{1}{\mu}\right) = \lim_{t\to\infty} \frac{\rho_1 e^{(\mu^*-\mu_1)t} + \rho_2 e^{(\mu^*-\mu_2)t}}{\lambda_1 e^{(\mu^*-\mu_1)t} + \lambda_2 e^{(\mu^*-\mu_2)t}}$$

$$= \frac{1}{\min(\mu_1,\mu_2)} = \max\left(\frac{1}{\mu_1}, \frac{1}{\mu_2}\right)$$

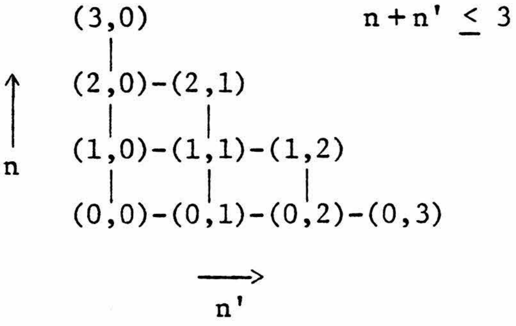
since the other terms →0.

If $\mu_1 = \mu_2$ then $E_t(1/\mu)$ is a constant and the system is memoryless. If each call is tagged so that one knows at all times which source statistics it obeys, then the system is likewise memoryless.

b)

In this section we will derive the general Markov chain solution to the case where a trunk of c channels is presented with two Poisson sources of different arrival rates and holding times. The following sets up the Markov state equations of a double Poisson source.

Suppose we have c channels and two sources with arrival rates and departure rates λ, μ and λ', μ' and traffic densities $\rho = \lambda/\mu$ and $\rho' = \lambda'/\mu'$. The state transitions are (for $c = 3$):



The general steady state equations are:

$$(\lambda_{n'-1}P_{n,n'-1} - \mu_{n'}P_{n,n'}) - (\lambda_{n'}P_{n,n'} - \mu_{n'+1}P_{n,n'+1})$$

$$+ (\lambda_{n-1}P_{n-1,n'} - \mu_n P_{n,n'}) - (\lambda_n P_{n,n'} - \mu_{n+1}P_{n+1,n'}) = 0$$

where

$$\begin{cases} \mu_n = n\mu; \quad \mu_{n'} = n'\mu' \\ \lambda_n = \lambda; \quad \lambda_n{}' = \lambda' \end{cases}$$

and appropriate boundary conditions:

$n + n' \leq c$
$n \geq 0; \; n' \geq 0$

It may be shown that the solution to these equations is:

$$\lambda_{n'-1}P_{n,n'-1} - \mu_{n'}P_{n,n'} = 0 \qquad 0 \leq n,\ 1 \leq n';\quad n+n' \leq c$$

$$\lambda_{n-1}P_{n-1,n'} - \mu_{n}P_{n,n'} = 0 \qquad 1 \leq n,\ 0 \leq n';\quad n+n' \leq c$$

Thus

$$P_{n,n'} = \left(\frac{\lambda^n}{n!\mu^n}\right)\left(\frac{\lambda'^{n'}}{n'!\mu'^{n'}}\right)P_{o,o} = \frac{\rho^n\rho'^{n'}}{n!n'!}P_{o,o}$$

and since the probability must sum to unity,

$$P_{n,n'} = \frac{\frac{\rho^n}{n!}\ \frac{\rho'^{n'}}{n'!}}{\sum\limits_{0 \leq i+j \leq c} \frac{\rho^i\rho'^j}{i!j!}}$$

If $c = \infty$

$$P_{n,n'} = \left(\frac{\rho^n}{n!}\,e^{-\rho}\right)\left(\frac{\rho'^{n'}}{n'!}\,e^{-\rho'}\right)$$

For $c \neq \infty$ the blockage probability is

$$P_B = \frac{\sum\limits_{i+j=c} \frac{\rho^i}{i!}\ \frac{\rho'^j}{j!}}{\sum\limits_{0 \leq i+j \leq c} \frac{\rho^i}{i!}\ \frac{\rho'^j}{j!}}$$

$$= \frac{\frac{1}{c!}(\rho+\rho')^{c}}{\sum_{n=0}^{c} \frac{1}{n!}(\rho+\rho')^{n}}$$

which is an Erlang B equation, showing that the probability of blockage depends only on the total traffic, $\rho+\rho'$.

APPENDIX F

Mean and Variance of the Sample Autocorrelation Function

The use of pseudorandom number generators frequently requires that sequential PN generated samples are nearly uncorrelated. Herein are the formulas to test the goodness from the observed sample autocorrelation.

Analysis

Let $\{x_i\}$ $i \varepsilon (-\infty,\infty)$ be statistical observations of a random variable.

Let $\mu_n = E[x_i^n]$ be the nth moment

$$(\mu = \mu_1, \quad \sigma^2 = \mu_2 - \mu_1^2)$$

Let $S_N^2 = \frac{1}{N}\sum_{i=1}^{N} x_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)\left(\frac{1}{N}\sum_{j=1}^{N} x_i\right)$ be the sample variance.

$$\mu(S_N^2) = E[S_N^2] = E\left[\frac{1}{N}\sum_{i=1}^{N} x_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)\left(\frac{1}{N}\sum_{j=1}^{N} x_j\right)\right]$$

$$= \frac{1}{N} N\, E[x_i^2] - \frac{1}{N^2}\left(N\, E[x_i^2] + (N^2-N)\, E\,[x_i]\;\; E[x_j]\right)$$

$$= (\mu_2 - \mu_1^2)(1 - \frac{1}{N})$$

$$= \sigma^2\ (1 - \frac{1}{N})$$

$$\sigma^2(S_N^2) = E\left[\frac{1}{N}\sum_{i=1}^{N} x_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)\left(\frac{1}{N}\sum_{j=1}^{N} x_j\right)\right]^2 - \left(E\left[\frac{1}{N}\sum_{i=1}^{N} x_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)\left(\frac{1}{N}\sum_{j=1}^{N} x_j\right)\right]\right)^2$$

$$= E\left[\frac{1}{N^2}\sum_{i=1}^{N}x_i^2\sum_{j=1}^{N}x_j^2 - \frac{2}{N^3}\sum_{k=1}^{N}x_k^2\sum_{i=1}^{N}x_i\sum_{j=1}^{N}x_j + \frac{1}{N^4}\sum_{i=1}^{N}x_i\sum_{j=1}^{N}x_j\sum_{k=1}^{N}x_k\sum_{\ell=1}^{N}x_\ell\right] - \left((\mu_2-\mu_1^2)(1-\tfrac{1}{N})\right)^2$$

$$= \frac{1}{N^2}\left[N\mu_4 + (N^2-N)\mu_2^2\right] - \frac{2}{N^3}\left[N\mu_4 + N(N-1)\mu_2^2 + 2N(N-1)\mu_3\mu_1 + N(N-1)(N-2)\mu_2\mu_1^2\right]$$

$$+ \frac{1}{N^4}\left[N\mu_4 + 4N(N-1)\mu_3\mu_1 + 3N(N-1)\mu_2^2 + 6N(N-1)(N-2)\mu_2\mu_1^2 + N(N-1)(N-2)(N-3)\mu_1^4\right]$$

$$- \left((1-\tfrac{1}{N})^2\ \mu_2^2 - 2(1-\tfrac{1}{N})^2\ \mu_2\mu_1^2 + (1-\tfrac{1}{N})^2\ \mu_1^4\right)$$

$$= \left(\frac{1}{N}-\frac{2}{N^2}+\frac{1}{N^3}\right)\mu_4 + \left(-\frac{4}{N}+\frac{8}{N^2}-\frac{4}{N^3}\right)\mu_3\mu_1 + \left(-\frac{1}{N}+\frac{4}{N^2}-\frac{3}{N^3}\right)\mu_2^2$$

$$+ \left(\frac{8}{N}-\frac{20}{N^2}+\frac{12}{N^3}\right)\mu_2\mu_1^2 + \left(-\frac{4}{N}+\frac{10}{N^2}-\frac{6}{N^3}\right)\mu_1^4$$

Let $S_{N,p}^2 = \frac{1}{N}\sum_{i=1}^{N}x_i(x_{i-p}) - \left(\frac{1}{N}\sum_{i=1}^{N}x_i\right)\left(\frac{1}{N}\sum_{j=1}^{N}x_{j-p}\right)$ be the sample autocorrelation function of the random variable sequence $\{x_i\}$ shifted by p. $(p \neq 0)$

The mean

$$\mu(S_{N,p}^2) = E\left[S_{N,p}^2\right] = E\left[\frac{1}{N}\sum_{i=1}^{N}x_i(x_{i-p}) - \left(\frac{1}{N}\sum_{i=1}^{N}x_i\right)\left(\frac{1}{N}\sum_{j=1}^{N}x_{j-p}\right)\right]$$

$$= \begin{cases} \mu_1^2 - \dfrac{(N-p)}{N^2}\mu_2 - \dfrac{N^2-(N-p)}{N^2}\mu_1^2 & N \geq p \\ 0 & N \leq p \end{cases}$$

$$= \begin{cases} \left(-\frac{1}{N} + \frac{p}{N^2}\right)\mu_2 + \left(\frac{1}{N} - \frac{p}{N^2}\right)\mu_1^2 & N \geq p \\ 0 & N \leq p \end{cases}$$

The variance

$$\sigma^2(S_{N,p}^2) = E\left[(S_{N,p}^2)^2\right] - \left[E(S_{N,p}^2)\right]^2$$

$$= E\left[\left(\frac{1}{N}\sum_{i=1}^{N} x_i x_{i-p} - \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)\left(\frac{1}{N}\sum_{j=1}^{N} x_{j-p}\right)\right)^2\right] - \left(E\left[\frac{1}{N}\sum_{i=1}^{N} x_i x_{i-p} - \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)\left(\frac{1}{N}\sum_{j=1}^{N} x_{j-p}\right)\right]\right)^2$$

$$= E\left[\frac{1}{N^2}\sum_{i=1}^{N} x_i(x_{i-p}) \sum_{j=1}^{N} x_j(x_{j-p}) - \frac{2}{N^3}\sum_{i=1}^{N} x_i(x_{i-p}) \sum_{j=1}^{N} x_j \sum_{k=1}^{N} x_{k-p}\right.$$

$$\left. + \frac{1}{N^4}\sum_{i=1}^{N} x_i \sum_{j=1}^{N} x_{j-p} \sum_{k=1}^{N} x_k \sum_{\ell=1}^{N} x_{\ell-p}\right] - \left(\mu(S_{N,p}^2)\right)^2$$

$$= \frac{1}{N^2}\left[N\mu_2^2 + 2\{N-p\}\mu_2\mu_1^2 + (N^2 - N - 2\{N-p\})\mu_1^4\right]^{\dagger}$$

$$- \frac{2}{N^3}\left[2\{N-p\}\mu_3\mu_1 + N\mu_2^2 + \{N-2p\}\mu_2^2\right.$$

$$+ 2(N^2 - N - \{N-p\})\mu_2\mu_1^2 + 2(\{N-p\}N - \{N-p\} - \{N-2p\})\mu_2\mu_1^2$$

$$\left. + (N^3-2\{N-p\}-N-\{N-2p\}-2(N^2-N-\{N-p\})-2(\{N-p\}N-\{N-p\}-\{N-2p\}))\mu_1^4\right]$$

$$+ \frac{1}{N^4}\left[\{N-p\}\mu_4 + 4(\{N-p\}N-\{N-p\})\mu_3\mu_1 + (N^2-\{N-p\})\mu_2^2 + 2(\{N-p\}^2-\{N-p\})\mu_2^2\right.$$

$$\left. + 4(\{N-p\}N^2 - (\{N-p\}^2 - \{N-p\}) - 2(\{N-p\}N-\{N-p\}))\mu_2\mu_1^2\right]$$

[†] The following notation is used:

$$\{N-p\} = \begin{cases} N-p & \text{if } N \geq p \\ 0 & \text{if } N \leq p \end{cases} \qquad \{N-2p\} = \begin{cases} N-2p & \text{if } N \geq 2p \\ 0 & \text{if } N \leq 2p \end{cases}$$

$$+\ 2(N^3 - (N^2-\{N-p\}) - 2(\{N-p\}N-\{N-p\}))\mu_2\mu_1^2$$

$$+\ (N^4-\{N-p\} - 4(\{N-p\}N-\{N-p\}) - (N^2-\{N-p\}) - 2(\{N-p\}^2 -\{N-p\})$$

$$- 4(\{N-p\}N^2 - (\{N-p\}^2 -\{N-p\}) - 2(\{N-p\}N-\{N-p\}))$$

$$- 2(N^3 - (N^2-\{N-p\}) -2(\{N-p\}N-\{N-p\})))\mu_1^4\Big]$$

$$- \frac{1}{N^4}\{N-p\}^2 (\mu_2^2 - 2\mu_2\mu_1^2 + \mu_1^4)$$

$$= \frac{\{N-p\}}{N^4}\mu_4 + \left(\frac{-4\{N-p\}}{N^4}\right)\mu_3\mu_1 + \left(\frac{1}{N} - \frac{1}{N^2} - \frac{2\{N-2p\}}{N^3} + \frac{\{N-p\}^2-3\{N-p\}}{N^4}\right)\mu_2^2$$

$$+\left(-\frac{2}{N} + \frac{2+2\{N-p\}}{N^2} + \frac{4\{N-2p\}-4\{N-p\}}{N^3} + \frac{18\{N-p\}-2\{N-p\}^2}{N^4}\right)\mu_2\mu_1^2$$

$$+\left(\frac{1}{N} + \frac{-1-2\{N-p\}}{N^2} + \frac{4\{N-p\}-2\{N-2p\}}{N^3} + \frac{-12\{N-p\}+\{N-p\}^2}{N^4}\right)\mu_1^4$$

$$= \begin{cases} \left(\frac{1}{N^3} - \frac{p}{N^4}\right)\mu_4 + \left(\frac{-4}{N^3} + \frac{4p}{N^4}\right)\mu_3\mu_1 + \left(\frac{1}{N} - \frac{2}{N^2} + \frac{-3+2p}{N^3} + \frac{3p+p^2}{N^4}\right)\mu_2^2 \\ \quad + \left(\frac{-2p}{N^2} + \frac{18}{N^3} + \frac{-2p^2-18p}{N^4}\right)\mu_2\mu_1^2 \\ \quad + \left(-\frac{1}{N} + \frac{2+2p}{N^2} + \frac{-12-2p}{N^3} + \frac{12p+p^2}{N^4}\right)\mu_1^4 & \text{if } N \geq 2p,\ N > p \\ \\ \left(\frac{1}{N^3} - \frac{p}{N^4}\right)\mu_4 + \left(\frac{-4}{N^3} + \frac{4p}{N^4}\right)\mu_3\mu_1 + \left(\frac{1}{N} + \frac{-3-2p}{N^3} + \frac{3p+p^2}{N^4}\right)\mu_2^2 \\ \quad + \left(\frac{-4-2p}{N^2} + \frac{18+8p}{N^3} + \frac{-18p-2p^2}{N^4}\right)\mu_2\mu_1^2 \\ \quad + \left(-\frac{1}{N} + \frac{4+2p}{N^2} + \frac{-12-6p}{N^3} + \frac{12p+p^2}{N^4}\right)\mu_1^4 & \text{if } N < 2p,\ N \geq p \\ \\ \left(\frac{1}{N} - \frac{1}{N^2}\right)\mu_2^2 + \left(-\frac{2}{N} + \frac{2}{N^2}\right)\mu_2\mu_1^2 + \left(\frac{1}{N} - \frac{1}{N^2}\right)\mu_1^4 & \text{if } N < 2p,\ N < p \end{cases}$$

Example

Consider the case of an exponential distribution. Then the n^{th} moment, $\mu_n = \int_0^\infty t^n \rho e^{-\rho t} dt$.

$$\mu_1 = \frac{1}{\rho} \qquad \mu_2 = \frac{2}{\rho^2} \qquad \mu_3 = \frac{6}{\rho^3} \qquad \mu_4 = \frac{24}{\rho^4}$$

If $\rho = 1$ then

$$\mu(S_N^2) = 1 - \frac{1}{N}$$

$$\sigma^2(S_N^2) = \frac{8}{N} - \frac{14}{N^2} + \frac{6}{N^3}$$

$$\mu(S_{N,p}^2) = \begin{cases} -\frac{1}{N} + \frac{p}{N^2} & N \geq p \\ 0 & N \leq p \end{cases}$$

$$\sigma^2(S_{N,p}^2) = \begin{cases} \frac{3}{N} + \frac{-6-2p}{N^2} + \frac{12+6p}{N^3} + \frac{-12p+p^2}{N^4} & \begin{array}{l} N \geq 2p \\ N > p \end{array} \\ \frac{3}{N} + \frac{-4-2p}{N^2} + \frac{12+2p}{N^3} + \frac{-12p+p^2}{N^4} & \begin{array}{l} N < 2p \\ N \geq p \end{array} \\ \frac{1}{N} - \frac{1}{N^2} & \begin{array}{l} N < 2p \\ N < p \end{array} \end{cases}$$

For large N, the standard deviation

$$s_d(S_{N,p}^2) = \sqrt{\frac{3}{N}}$$

APPENDIX G

Confidence Bounds for Estimating a Poisson Parameter

A confidence bound for estimating the parameter of a Poisson process has been found. The same technique may be used with a risk function to obtain the utility.

Background

A Poisson process $\{N(t)\}$ is a stationary stochastic process in which $N(t)$ represents the total number of events that have occurred up to time t. It furthermore must satisfy[16]

$$\text{i)}\quad P\,\{N(t+\Delta t) - N(t) \geq 2\} \;=\; o(\Delta t)$$

$$\text{ii)}\quad P\,\{N(t+\Delta t) - N(t) = 1\} \;=\; \lambda\Delta t + o(\Delta t)$$

where λ is the rate of the process.

If we define the probability of n arrivals in time t to be

$$P_n(t) = P\;\{N(t+t_o) - N(t_o) = n\}$$

Then it follows that

$$P_n(t) = \frac{(\lambda t)^n\, e^{-\lambda t}}{n!}$$

Now suppose we wish to estimate λ. If we observe m arrivals in time t, then we can estimate λ by the average rate of observed arrivals.

$$\overline{\lambda} = \frac{m}{t}$$

[16] A function f is said to be $o(\Delta t)$ if $\lim\limits_{\Delta t \to 0} \frac{f(\Delta t)}{\Delta t} = 0$

The mean and variance are given by

$$\mu(\bar{\lambda}) = E(\frac{m}{t}) = \frac{1}{t}\ (\lambda t) = \lambda$$

$$\sigma^2(\bar{\lambda}) = E\left((\frac{m}{t})^2\right) - \left[E(\frac{m}{t})\right]^2 = \frac{1}{t^2}(\lambda t) = \frac{\lambda}{t}$$

and the standard deviation

$$\sigma(\bar{\lambda}) = \sqrt{\frac{\lambda}{t}} = \frac{\sqrt{m}}{t}$$

Analysis

We wish to obtain confidence bounds on $\bar{\lambda}$, the average rate of observed arrivals, as an estimator of λ. We will calculate the probability that given $\bar{\lambda}$ observed, $\lambda \leq \tilde{\lambda}$ for some $\tilde{\lambda}$.

Bayes's theorem is invoked with a uniform prior distribution assumed for the distribution of λ. The reader is cautioned to decide for himself whether this assumption is justified for his particular application.

The average rate of observed arrivals,

$$\bar{\lambda} = \frac{m}{t}$$

By Bayes's theorem, for the discrete case, the posterior probability of the k^{th} choice of λ,

$$P\ (\lambda_k|\bar{\lambda}) = \frac{P(\bar{\lambda}|\lambda_k)\ P\ (\lambda_k)}{\sum_j P(\bar{\lambda}|\lambda_j)\ P\ (\lambda_j)}$$

Extending to the continuum, the conditional cumulative distribution function,

$$P\ (\lambda \leq \tilde{\lambda}|\bar{\lambda}) = \frac{\int_0^{\tilde{\lambda}} d\lambda'\ P(\bar{\lambda}|\lambda')\ P(\lambda')}{\int_0^{\infty} d\lambda'\ P(\bar{\lambda}|\lambda')\ P(\lambda')}$$

where $P(\overline{\lambda}|\lambda')$ is the conditional probability density function. For fixed t, $P(\overline{\lambda}|\lambda') = P(m|\lambda')$, so letting $P(\lambda') = 1 \; \forall \lambda'$

$$P\,(\lambda \leq \tilde{\lambda}|\overline{\lambda}) = \frac{\int_0^{\tilde{\lambda}} d\lambda' \left(\frac{(\lambda' t)^m \, e^{-\lambda' t}}{m!}\right)}{\int_0^{\infty} d\lambda' \left(\frac{(\lambda' t)^m \, e^{-\lambda' t}}{m!}\right)}$$

$$= 1 - \frac{\sum_{r=0}^{m} \frac{(\tilde{\lambda} t)^r}{r!}}{e^{\tilde{\lambda} t}}$$

This is an incomplete gamma function and may also be expressed as

$$1 - \frac{(\tilde{\lambda} t)^m \, e^{-\tilde{\lambda} t}}{m!} \left[1 + \frac{m}{\tilde{\lambda} t} + \frac{m(m-1)}{(\tilde{\lambda} t)^2} + \ldots\right]$$

or as

$$1 - \frac{1}{\frac{a}{(\tilde{\lambda} t)^n} + \frac{b}{(\tilde{\lambda} t)^{n+1}} + \frac{c}{(\tilde{\lambda} t)^{n+2}} + \ldots}$$

where

$$a = m!$$

$$b = m! \left(1 - \frac{a}{(m+1)!}\right)$$

$$c = m! \left(1 - \frac{b}{(m+1)!} - \frac{a}{(m+2)!}\right)$$

Figure G-1 shows $P(\lambda \leq \tilde{\lambda} | \bar{\lambda})$ for various values of t.

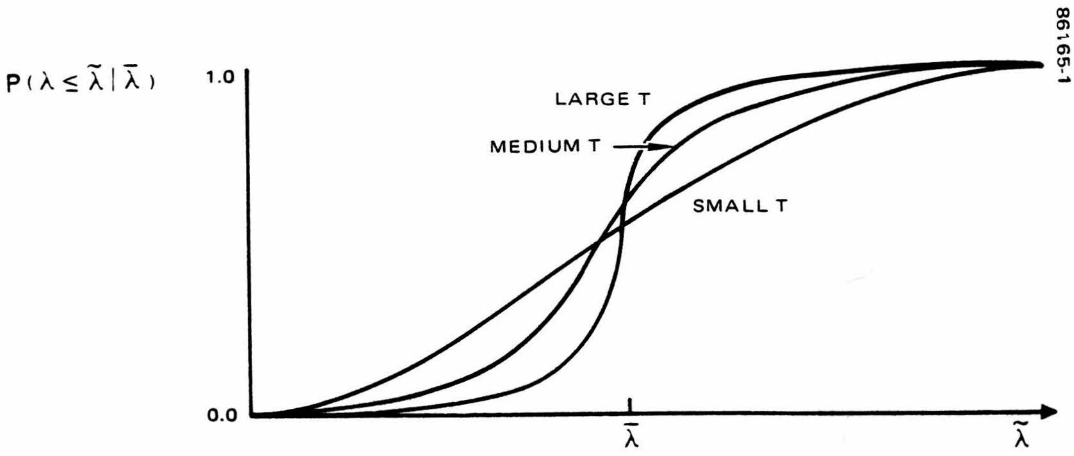


FIGURE G-1. CUMULATIVE DISTRIBUTION FUNCTION

The probability distribution function is

$$\frac{d}{d\tilde{\lambda}} P(\lambda \leq \tilde{\lambda} | \bar{\lambda}) = \frac{\tilde{\lambda}^m \, t^{m+1}}{m! \, e^{\tilde{\lambda} t}}$$

It may be used with the risk function to obtain the utility. Table G-2 gives the solution to this equation showing $\tilde{\lambda}t$ as a function of m for the 90% and 99% confidence levels.

90% CONFIDENCE LEVEL

m	$\tilde{\lambda}t$
0	2.303
1	3.890
2	5.323
3	6.681
4	7.994
5	9.275
6	10.533
7	11.771
8	12.995
9	14.206
10	15.407
11	16.598
12	17.782
13	18.958
14	20.129
15	21.293
16	22.452
17	23.607
18	24.757
19	25.903
20	27.045
21	28.184
22	29.320
23	30.454
24	31.584
25	32.712
26	33.837
27	34.960
28	36.081
29	37.199
30	38.316
31	39.431
32	40.544
33	41.654
34	42.764
35	43.872
36	44.978
37	46.083
38	47.187
39	48.290
40	49.391
41	50.491
42	51.589
43	52.687
44	53.783
45	54.878
46	55.972
47	57.066
48	58.158
49	59.249
50	60.340
51	61.429
52	62.518
53	63.606
54	64.693
55	65.780
56	66.865
57	67.949
58	69.034
59	70.116
60	71.200
61	72.281
62	73.363
63	74.443
64	75.523
65	76.603
66	77.681
67	78.760
68	79.837
69	80.914
70	81.990

99% CONFIDENCE LEVEL

m	$\tilde{\lambda}t$
0	4.606
1	6.639
2	8.407
3	10.046
4	11.605
5	13.109
6	14.571
7	16.000
8	17.403
9	18.784
10	20.145
11	21.491
12	22.822
13	24.140
14	25.447
15	26.744
16	28.031
17	29.310
18	30.581
19	31.846
20	33.104
21	34.355
22	35.601
23	36.842
24	38.078
25	39.309
26	40.535
27	41.757
28	42.975
29	44.190
30	45.401
31	46.609
32	47.813
33	49.015
34	50.213
35	51.409
36	52.602
37	53.792
38	54.979
39	56.165
40	57.348
41	58.529
42	59.707
43	60.884
44	62.059
45	63.231
46	64.402
47	65.571
48	66.738
49	67.903
50	69.068
51	70.230
52	71.391
53	72.550
54	73.708
55	74.864
56	76.019
57	77.172
58	78.325
59	79.476
60	80.626
61	81.774
62	82.921
63	84.067
64	85.212
65	86.356
66	87.498

TABLE G-2.

APPENDIX H

Names of Latin American and Caribbean Nations

Argentina	Spanish
Bahamas (UK)	English
Barbados	English
Belize	English
Bolivia	Spanish
Brazil	Portuguese
British West Indies (UK)	English
Leeward Islands	English
(Antigua, Dominica)	English
Grenada	English
Cayman Islands (UK)	English
Chile	Spanish
Colombia	Spanish
Costa Rica	Spanish
Cuba	Spanish
Dominican Republic	Spanish
Ecuador	Spanish
El Salvador	Spanish
Falkland Islands (UK)	English
French Guiana	French
Guadeloupe (F)	French
Guatemala	Spanish
Guyana	English
Haiti	French
Honduras	Spanish
Jamaica	English
Martinique (F)	French
Mexico	Spanish
Netherlands Antilles (N)	Dutch
(Aruba, Bonaire, Curacao)	Dutch
Nicaragua	Spanish
Panama	Spanish
Paraguay	Spanish
Peru	Spanish
Puerto Rico (US)	Spanish
Surinam	Dutch
Trinidad and Tobago	English
Uruguay	Spanish
Venezuela	Spanish

INTELSAT: The International Telecommunications Satellite Organization as of 31 March 1976 (Latin American and Caribbean members)

Country/Signatory

ARGENTINA
Empresa Nacional de Telecomunicaciones de la Republica Argentina (ENTEL)

BARBADOS
Cable & Wireless (West Indies) Ltd.

BOLIVIA
Empresa Nacional de Telecomunicaciones (ENTEL)

BRAZIL
Empresa Brasileira de Telecomunicaciones S.A. (EMBRATEL)

CHILE
Empresa Nacional de Telecomunicaciones S.A. (ENTEL)

COLOMBIA
Empresa Nacional de Telecomunicaciones de Colombia (TELECOM)

COSTA RICA
Instituto Costarricense de Electricidad

DOMINICAN REPUBLIC
Compania Dominicana de Telefonos, C. por A.

ECUADOR
Instituto Ecuatoriano de Telecomunicaciones (IETEL)

GUATEMALA
Government of Guatemala

HAITI
Telecommunication d'Haiti, S.A.

JAMAICA
Jamaica International Telecommunications, Ltd. (JAMINTEL)

MEXICO
Government of Mexico

NICARAGUA
Compania Nicaraguense de Telecomunicaciones por Satelite (NICATELSAT)

PANAMA
Intercontinental de Comunicaciones por Satelite (INTERCOMSA)

PERU
Empresa Nacional de Telecomunicaciones del Peru (ENTEL PERU)

TRINIDAD & TOBAGO
Trinidad & Tobago External Telecommunications Company, LTD. (TEXTEL)

VENEZUELA
Venezuelan Telephone Company

APPENDIX I

Names and Addresses of Foreign Dignitaries (Spring 1977)

Jorge Videla
Presidente
Buenos Aires, Argentina

Gen. Albano E. Harquindeguy
Minister of Interior
Buenos Aires, Argentina

Arnaldo T. Musich
Ambassador from Argentina
Washington, D.C.

Gen. Hugo Banzer Suárez
Presidente
La Paz, Bolivia

Col. Julio Trigo
Minister of Transportation and Communications
La Paz, Bolivia

Alberto Crespo Gutiérrez
Ambassador from Bolivia
Washington, D.C.

Gen. Augusto Pinochet Ugarte
Presidente
Santiago, Chile

Sergio Fernandez
Minister of Public Works
Santiago, Chile

Manuel Trucco
Ambassador from Chile
Washington, D.C.

Dr. Alfonso López Michelsen
Presidente
Bogotá, Colombia

Dr. Sara Ordoñez de Loñdono
Minister of Communications
Bogotá, Colombia

Daniel Odubar Quirós
Presidente
San José, Costa Rica

Alvaro Jenkins Morales
Minister of Public Works and Transportation
San José, Costa Rica

Rodolfo Silva Vargas
Ambassador from Costa Rica
Washington, D.C.

Maj. Fidel Castro R.
First Secretary; Premier and Presidente
Havana,Cuba

Maj. Jesus Montane Oropesa
Minister of Communications
Havana,Cuba

Dr. Joaquín Balaguer
Presidente
Santo Domingo, Dominican Republic

Michael Douglas
Minister of Public Works and Communicatons
Santo Domingo, Dominican Republic

Dr. Horatio Vicioso Soto
Ambassador from Dominican Republic
Washington, D.C.

V.Adm. Alfredo Poreda
Supreme Council of Governors
Quito, Ecuador

Gen. Angel Polivio Vega
Minister of Public Works
Quito, Ecuador

Col. Arturo Armando Molina
Presidente
San Salvador, El Salvador

Antonio Jorge Seaman
Minister of Development and Public Works
San Salvador, El Salvador

Dr. Francisco Bertrand Galindo
Ambassador from El Salvador
Washington, D.C.

Gen. Kjell Eugenio Laugerud
Presidente
Guatemala City, Guatemala

Ricardo Arguedas Martínez
Minister of Communications and Public Works
Guatemala City, Guatemala

Col. Abundio Maldonado
Ambassador from Guatemala
Washington, D.C.

Gen. Juan Alberto Melgar
Chief of State
Tegucigalpa, Honduras

Lt. Col. Mario Flores Theresín
Minister of Communications and Public Works
Tegucigalpa, Honduras

Dr. Roberto Lazarus
Ambassador from Honduras
Washington, D.C.

José López Portillo
Presidente
Mexico City, Mexico

Emilio M. Montoya
Minister of Communicatons and Transportation
Mexico City, Mexico

Hugo B. Margain
Ambassador from Mexico
Washington, D.C.

Gen. Anastasio Somoza-Debayle
Presidente
Managua, Nicaragua

Armel Gonzales E.
Minister of Development and Public Works
Managua, Nicaragua

Dr. Guillermo Sevilla-Sacasa
Ambasador from Nicaragua
Washington, D.C.

Gen. Omar Torrijos Herrera
Chief of Government
Panama City, Panamá

Tomás Guerra
Minister of Public Works
Panama City, Panamá

Nicolás Gonzales-Revilla
Ambassador from Panama
Washington, D.C.

Gen. of the Army Alfredo Stroessner
Presidente
Asunción, Paraguay

Gen. Juan Antonio Cáceres
Minister of Public Works and Communications
Asunción, Paraguay

Div. Gen. Francisco Moráles Bermúdez Cerratti
Presidente
Lima, Perú

Brig. Gen. Elivio Vanini Chumpitazi
Minister of Trasportaion and Communications
Lima, Perú

Dr. Carlos García-Bedoya
Ambassador from Peru
Washington, D.C.

Dr. Aparicio Mendez
Presidente
Montevideo, Uraguay

Arturo Sampson
Minister of Public Works
Montevideo, Uraguay

Brig. Gen. José Perez Caldez
Ambassador from Uraguay
Washington, D.C.

Carlos Andres Perez
Presidente
Caracas, Venezuela

Dr. Jesus Vivas Casanova
Minister of Transportations and Communications
Caracas, Venezuela

Dr. Ignacio Iribarren Borgas
Ambassador from Venezuela
Washington, D.C.

Sir Milo B. Butler Sr.
Governor General
Nassau, Bahamas

Sineon L. Bowe
Minister of Works and Utilities
Nassau, Bahamas

Livingston B. Johnson
Ambassador from the Bahamas
Washington, D.C.

Sir Deighton Lisle Ward
Governor General
Bridgetown, Barbados

L.B. Brathwaite
Minister of Comunications and Works
Bridgetown, Barbados

Peter Donovan McEntee
Governor
Belmopan, Belize

Louis S. Sylvestre
Minister of Energy and Communications
Belmopan, Belize

Leo DeGale
Governor General
St. George's, Grenada

Eric Gairy
Prime Minister
St. George's, Grenada

Herbert Preudhomme
Dept. Prime Minister of Communications and Works
St. George's, Grenada

Arthur Chung
President
Georgetown, Guyana

Steve Narine
Minister of Communications
Georgetown, Guyana

Laurence Everil Mann
Ambassador from Guyana
Washington, D.C.

Florizel A. Glaspole
Governor General
Kingston, Jamaica

Horace Clarke
Minister of Public Utilities
Kingston, Jamaica

Alfred A. Rattray
Ambassador from Jamaica
Washington, D.C.

Sir Ellis Emmanuel Innocent Clarke
President
Port of Spain, Trinidad and Tobago

Hector McLean
Minister of Works and Communications
Port of Spain, Trinidad and Tobago

Victor C. McIntyre
Ambassador from Trinidad and Tobago
Washington, D.C.

Anthony Crosland
Secretary of State for Commonwealth Affairs
London, United Kingdom

Judith Hart
Minister for Overseas Development
London, United Kingdom

Sir Peter Ramsbotham
London, Ambassador from the United Kingdom
Washington, D.C.

Herve Bourseiller
Prefect
Cayenne, French Guiana

Jacque Kosciusko Morizet
Ambassador from France
Washington, D.C.

Olivier Stirn
French State Sec. for Overseas Departments
Paris, France

Jean-Claude Arousseau
Prefect
Basse-Terre, Guadeloupe

Jean-Claude Duvalier
President for Life
Port-au-Prince, Haiti

Pierre Biamby
Minister of Interior and National Defense
Port-au-Prince, Haiti

George Salomon
Ambassador from Haiti
Washington, D.C.

Paul Noirot Cosson
Prefect
Fort-de-France, Martinique

D.C. Martina
Lt. Governor
Willemstad, Curacao, Netherlands Antilles

Age Robert Tammenems Bakker
Ambassador from the Netherlands
Washington, D.C.

Johan Henri Eliza Ferrier
President
Paramaribo, Surinam

Dr. A. G. Karamet Ali
Minister of Public Works and Communications
Paramaibo, Surinam

Roel F. Karamat
Ambassador from Surinam
Washington, D.C.

Gen. Ernesto Geisel
Presidente
Brasilia, Brazil

Capt. Enclides Quandt de Oliveira
Ministro de Telecomunicacoes
Brasilia, Brazil

Joâo Baptista Pinheiro
Ambassador from Brazil
Washington, DC

APPENDIX J

Tables of Traffic Data and Blockage Probabilities

Tables J-1, J-2, and J-3 show the traffic data obtained from the various nations for the year 1976, as well as the calculated number of modems required by each nation. Traffic is measured in erlangs. Blockage probabilities have been calculated using the Erlang B equation.

PEAK HOUR TRAFFIC IN ERLANGS 1976 TO► FROM ▼	Argentina	Bahamas	Barbados	Belize	Bolivia	Brazil	British West Indies (UK)	Leeward Islands (Antigua, Dominica)	Grenada	Cayman Islands (UK)	Chile	Colombia	Costa Rica	Cuba	Dominican Republic	Ecuador	El Salvador	Falkland Islands (UK)	French Guiana	Guadeloupe (F)	Guatemala	Guyana	Haiti	Honduras	Jamaica	Martinique (F)	Mexico	Netherlands Antilles (N) (Aruba, Bonaire, Curacao)	Nicaragua	Panama	Paraguay	Peru	Puerto Rico (US)	Surinam	Trinidad and Tobago	Uruguay	Venezuela
Argentina	-	.005	.005	.0006	2.26	26.64	.06	.02	0	.001	11.57	1.43	.32	.005	.002	.95	.084	.02	.002	.003	.05	.01	.002	.04	1.05	.15	2.72	.003	.17	1.16	7.76	5.36	.42	.01	.008	70.5	5.39
Bahamas	.005	-	.05	.0310	.005	.06	5.58	1.65	0	.04	.008	.05	.018	.005	.005	.029	.029	0	.001	.09	.01	.47	.001	.01	.400	.29	.01	.05	.01	.0449	.005	.009	.18	.005	.046	.005	0
Barbados	.005	.05	-	.0167	.005	.01	5.58	1.65	.58	.04	.001	0	.004	.005	.005	.001	.003	0	.001	.09	.01	.47	.001	.01	.59	.29	.01	.05	.01	.005	.005	.01	.18	.005	2.07	.005	.16
Belize	.0006	.0310	.0167	-	0	0	.0043	.0015	.0006	.0219	0	.0165	.0198	0	.0022	.0006	.0717	0	0	0	.1291	.0034	.0002	.0525	.1663	.0023	.0357	.0004	.0044	.0139	0	.0066	.0023	0	.0059	0	.0039
Bolivia	2.26	.005	.005	0	-	.80	0	.0001	0	.0005	.64	.03	.003	.04	.01	.039	.030	0	.0005	.001	.03	.001	.002	.01	.133	.0005	.02	.009	.01	.048	.08	.433	.04	0	.0001	.08	.189
Brazil	13.12	.06	.01	.001	.80	-	.002	.001	.001	.01	2.35	1.22	.15	.01	.11	.45	.10	.0002	.02	.003	.15	.03	.02	.08	.03	.02	2.52	.03	.11	.60	3.20	1.50	.17	.05	.05	5.30	2.74
British West Indies (UK)	.06	6.24	6.24	.0043	0	.002	-	1.05	.451	.3	0	.03	0	.06	.06	0	0	0	.01	.05	0	.39	.01	0	.57	.3	.1	.6	0	.0019	.06	0	.1	.06	4.74	0	.12
Leeward Islands (Antigua, Dominica)	.02	2.18	2.18	.0015	.0001	.001	1.29	-	.176	.12	0	0	.001	.02	.02	0	0	0	.004	.20	0	.30	.004	0	.45	.17	.04	.24	0	.0027	.02	.0001	1.02	.02	1.56	.0001	.017
Grenada	0	0	.72	0	0	0	.451	.176	-	0	0	0	0	0	0	0	0	0	0	0	0	.069	0	0	.078	0	0	.010	0	0	0	0	0	0	.980	0	.02
Cayman Islands (UK)	.0007	.04	.04	.0219	.0005	.01	.3	.12	0	-	0	0	.044	0	.002	.001	.002	0	0	.0002	.002	.003	.0002	.001	2.37	.002	.03	.0004	.01	.0078	0	.001	.002	0	.016	.001	.007
Chile	8.32	.008	.001	0	.64	4.2	0	0	0	0	-	1.4	.12	.002	.02	.71	.039	0	0	.001	.059	.001	0	.02	.005	0	.48	.008	.02	.33	.18	.94	.049	.001	.005	.36	4.4
Colombia	.7	.05	0	0	.03	.6	.03	0	0	0	.4	-	.4	.03	.1	2.1	.1	0	0	0	.1	0	.01	.03	.03	0	1.3	.4	.1	1.9	.01	.86	.3	0	.02	.05	5.2
Costa Rica	.158	.018	.004	.016	.003	.135	0	.001	.001	.044	.152	.581	-	.052	.068	.161	4.391	0	0	.0002	5.071	.002	.008	2.255	.040	.001	3.058	.057	5.223	4.644	.004	.233	.259	0	.005	.009	.570
Cuba	.005	.005	.005	0	.04	.01	.06	.02	0	0	.37	.05	.13	-	.005	.007	.014	0	.0002	.0002	.014	.0002	.0007	.007	.267	.0002	.42	.0002	.003	.02	.02	.07	.02	.0002	.004	.07	.09
Dominican Republic	.002	.005	.005	.0022	.01	.11	.06	.02	0	.002	.02	.1	.068	.005	-	.039	.086	0	.0002	.0002	.086	.0002	.01	.04	.133	.0002	.67	.0002	.08	.1449	.007	.031	17.14	.0002	.013	.031	1.70
Ecuador	1.14	.029	.001	0	.039	.72	0	0	0	.001	1.66	5.41	.25	.007	.039	-	.039	0	0	0	.069	0	.003	.029	.006	0	.91	.039	.029	1.34	.020	2.90	.10	0	.008	.059	1.57
El Salvador	.084	.029	.003	.074	.030	.131	0	0	.001	.002	.079	.591	9.296	.014	.086	.081	-	0	0	.0002	24.255	.0004	.004	2.180	.021	0	3.409	.007	8.173	1.658	.001	.100	.104	.001	.004	.003	.044
Falkland Islands (UK)	.02	0	0	0	0	.0002	0	0	0	0	0	0	0	0	0	0	0	-	0	0	0	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0
French Guiana	.002	.001	.001	0	.0005	.02	.01	.004	0	0	0	0	0	.0002	.0002	0	0	0	-	.0002	0	.02	0	0	0	.002	.001	.002	0	.000	.0002	.001	.005	.01	.002	.001	.03
Guadeloupe (F)	.003	.29	.29	0	.001	.003	.05	.35	0	.0002	.001	0	.0002	.0002	.0002	0	.0002	0	.0002	-	.0002	.01	.0002	.0002	.133	.4	.0002	.05	0	.001	.0002	.0003	.05	.0002	.028	.0002	.005
Guatemala	.084	.01	.01	.1291	.03	.15	0	0	0	.002	.059	.1	5.071	.014	.086	.069	24.255	0	0	.0002	-	.0004	.004	5.42	.133	0	6.96	.007	6.7	.4005	.001	.067	.104	.001	.006	.003	.16
Guyana	.01	.47	.47	.0034	.001	.03	.39	.21	.069	.003	.001	0	.002	.0002	.0002	0	.0004	0	.02	.01	.0004	-	.005	.0002	.533	.01	.01	.002	0	.001	.0005	.002	.07	.05	2.16	.002	.06
Haiti	.002	.001	.001	.0002	.002	.02	.01	.004	0	.0002	0	.01	.008	.0007	.01	.003	.004	0	0	.0002	.004	.005	-	.002	.400	.07	.0006	.01	.01	.0152	.001	.004	.653	.01	.005	.004	.01
Honduras	.04	.01	.01	.0525	.01	.08	0	0	0	.001	.02	.03	3.81	.007	.04	.029	1.599	0	0	.0002	5.42	.0002	.002	-	.133	0	1.6	.003	5.04	.326	.0005	.023	.05	.0005	.002	.023	.02
Jamaica	.133	.400	.533	.267	.133	.133	.400	0	0	.800	.133	.133	.133	.267	.133	.133	.133	0	0	.133	.133	.533	.400	.133	-	.133	.267	.133	.133	.267	.133	.133	.667	.133	.933	0	.400
Martinique (F)	.15	.29	.29	.0023	.0005	.02	.3	.36	0	.002	0	0	.001	.0002	.0002	0	0	0	.002	.4	0	.01	.07	0	.133	-	.05	.002	0	.0001	.0002	.001	.07	.002	.069	.001	.08
Mexico	1.7	.01	.01	.64	.02	4.3	.1	.04	0	.6	2.21	1.57	3.43	.71	.67	.05	2.48	0	.001	.0002	6.96	.01	.0006	.16	.28	.05	-	.002	2.62	2.47	.032	2.5	.05	.001	.039	.18	2.61
Netherlands Antilles (N) (Aruba, Bonaire, Curacao)	.003	.07	.07	.0004	.009	.03	.6	.24	.010	.0004	.008	.62	.057	.0002	.0002	.039	.007	0	.002	.05	.007	.002	.01	.003	.133	.002	.002	-	.02	.054	.004	.018	.63	.16	.108	.018	4.92
Nicaragua	.17	.01	.01	.03	.01	.12	0	0	.001	.01	.07	.55	11.5	.003	.08	.09	8.3	0	0	0	6.7	0	.01	3.9	.01	0	7.6	.02	-	2.6	.004	.2	.2	.001	.003	.01	.4
Panama	.2756	.0449	.005	.006	.048	.308	.0019	.0027	.0009	.0078	.326	2.499	3.115	.049	.1449	.7089	.273	.000	.000	.001	.4005	.001	.015	.326	.073	.0001	1.5848	.054	.684	-	.026	.468	.2347	.0058	.016	.0369	.8719
Paraguay	7.12	.005	.005	0	.08	10.9	.06	.02	0	0	.19	.01	.004	.02	.031	.020	.001	0	.001	.0002	.001	.0005	.001	.0005	.133	.0005	.05	.004	.004	.04	-	.04	.02	0	.0003	.21	.4
Peru	2.394	.009	.004	0	.443	1.466	0	.0001	.0002	.001	1.92	1.266	.200	.082	.031	1.91	.049	0	.001	.0003	.067	.002	.004	.023	.011	.001	1.12	.018	.036	.714	.043	-	.083	0	.007	.008	1.85
Puerto Rico (US)	.34	.18	.18	.0023	.04	.17	.1	2.635	1.028	.002	.049	.3	.259	.446	17.14	.10	.104	0	.005	.05	.104	.07	.653	.05	.667	.07	.05	.982	.2	.2347	.02	.083	-	.01	2.548	.083	2.466
Surinam	.01	.005	.005	0	0	.02	.06	.02	0	0	.001	0	0	.0002	.0002	0	.001	0	.01	.0002	.001	.012	.01	0	.133	.002	.001	.07	.001	.0058	0	0	.01	-	.02	0	.02
Trinidad and Tobago	.008	.046	1.429	.003	.0001	.042	0	.479	.907	.016	.005	.051	.018	.004	.013	.019	.002	0	.002	.028	.006	.889	.005	.002	.807	.069	.039	.108	.003	.025	.0003	.007	.206	.029	-	.0006	1.240
Uruguay	70.5	.005	.005	0	.08	5.19	0	.001	0	.001	.8	.31	.009	.08	.031	.059	.003	0	.001	.0002	.003	.002	.004	.023	0	.001	.42	.018	.01	.11	.31	.26	.083	0	.0006	-	.43
Venezuela	7.40	0	.293	.004	.189	3.635	1.730	.017	.027	.007	3.39	15.740	.885	.453	1.70	.989	.175	0	.03	.005	.16	.06	.01	.5	.24	.16	2.96	5.29	.028	1.33	.4	1.85	3.31	.02	2.63	.72	-

TABLE J-I. TRAFFIC TABLE

Country	Outgoing Traffic	Incoming Traffic	Total Traffic	No. of Modems @ 1% Blockage	No. of Modems @ .5% Blockage
Argentina	138.18	116.24	254.42	273	275
Bahamas	9.20	10.61	19.81	30	31
Barbados	11.90	12.91	24.81	36	37
Belize	.61	1.30	1.91	7	7
Bolivia	4.96	4.96	9.92	18	18
Brazil	35.02	60.07	95.09	112	115
British West Indies (UK)	21.61	17.23	38.84	52	54
Leeward Islands (Antigua, Dominica)	10.06	9.09	19.15	30	31
Grenada	2.49	3.25	5.74	12	13
Cayman Islands (UK)	3.04	2.04	5.08	11	12
Chile	22.32	26.43	48.75	62	65
Colombia	14.85	34.10	48.95	63	65
Costa Rica	27.22	39.33	66.55	82	85
Cuba	1.76	2.39	4.15	10	11
Dominican Republic	20.62	20.65	41.27	54	57
Ecuador	16.42	8.79	25.21	36	38
El Salvador	50.47	42.38	92.85	110	112
Falkland Islands (UK)	.02	.02	.04	2	2
French Guiana	.11	.12	.23	3	3
Guadeloupe (F)	1.67	1.02	2.69	8	8
Guatemala	50.04	50.00	100.04	117	120
Guyana	4.60	3.38	7.98	15	16
Haiti	1.28	1.29	2.57	8	8
Honduras	18.38	15.31	33.69	46	48
Jamaica	8.53	10.29	18.82	29	30
Martinique (F)	2.31	2.20	4.51	11	11
Mexico	36.51	38.45	74.96	91	94
Netherlands Antilles (N) (Aruba, Bonaire, Curacao)	7.91	8.28	16.19	26	27
Nicaragua	42.61	29.44	72.05	88	91
Panama	12.62	20.52	33.14	45	47
Paraguay	19.37	12.35	31.72	44	46
Peru	13.76	18.12	31.88	44	46
Puerto Rico (US)	31.42	26.58	58.00	73	75
Surinam	.42	.59	1.01	5	5
Trinidad and Tobago	6.51	18.11	24.62	35	37
Uraguay	78.75	77.77	156.52	175	177
Venezuela	56.34	38.19	94.53	112	114
TOTAL	783.85	783.85	1567.70	1975	2031

Note: Values for no. of modems which are greater than 120 are approximate and may be low as much as 5%.

TABLE J-2. MODEM REQUIREMENT

Country	Outgoing Traffic	Incoming Traffic	Total Traffic	No. of Modems @ .5% Blockage
Argentina	.133	1.05	1.183	6
Bahamas	.400	.4	.800	5
Barbados	.533	.59	1.123	5
Belize	.267	.166	.433	4
Bolivia	.133	.133	.266	3
Brazil	.133	.03	.163	3
British West Indies (UK)	.400	.57	.970	5
Leeward Islands (Antigua, Dominica)	0	.45	.45	4
Grenada	0	.078	.078	2
Cayman Islands (UK)	.800	2.37	3.170	9
Chile	.133	.005	.138	3
Colombia	.133	.03	.163	3
Costa Rica	.133	.04	.173	3
Cuba	.267	.267	.534	4
Dominican Republic	.133	.133	.266	3
Ecuador	.133	.006	.139	3
El Salvador	.133	.021	.154	3
Falkland Islands (UK)	0	0	0	1
French Guiana	0	0	0	1
Guadeloupe (F)	.133	.133	.266	3
Guatemala	.133	.133	.266	3
Guyana	.533	.533	1.066	5
Haiti	.400	.400	.800	5
Honduras	.133	.133	.266	3
Martinique (F)	.133	.133	.266	3
Mexico	.267	.28	.547	4
Netherlands Antilles (N) (Aruba, Bonaire, Curacao)	.133	.133	.266	3
Nicaragua	.133	.01	.143	3
Panama	.267	.073	.340	3
Paraguay	.133	.133	.266	3
Peru	.133	.011	.144	3
Puerto Rico (US)	.667	.667	1.334	6
Surinam	.133	.133	.266	3
Trinidad and Tobago	.933	.807	1.740	8
Uraguay	0	0	0	1
Venezuela	.400	.24	.640	4
TOTAL	8.53	10.29	18.82	133

TABLE J-3. PA MODEM REQUIREMENT FOR JAMAICA

APPENDIX K

Copies of Letters Requesting Information

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

ELECTRICAL ENGINEERING 116-81

1 June 1977

Dear Sir:

Telecommunications is of vital importance to the progress and solidarity of any nation. The use of radio and television is well-established and unlikely to change much in the next ten or twenty years, but the demand for telephone service is growing rapidly. The number of calls between neighboring nations in particular will increase along with increased economic activity, and can indeed aid economic growth.

A regional communication satellite network offers a possible solution to this demand for service and is the subject of my current research. I am therefore asking for your assistance in obtaining some necessary information related to the number of calls made between your country and its neighbors.

For partial fulfillment of my Ph.D. thesis in Electrical Engineering at California Institute of Technology, I am investigating the cost and utility of a demand access satellite network for the Americas. To accomplish this, I must first know the amount of telephone traffic between nations. From this I will be able to compute the number of channels, earth station transmitters, etc. that are required.

I would be greatly indebted to you if you would have the enclosed table completed to show current and predicted telecommunications traffic. I believe that this information could be of great value to the future of satellite communication in the Americas.

Sincerely,

James Laurens Latimer

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

ELECTRICAL ENGINEERING 116-81

1 June 1977

Excelentísimo Señor,

La telecomunicación es de vital importancia al progreso y solidaridad de cualquier nación. El uso de radio y televisión está bien establecido y con toda seguridad esto no va a cambiar en los próximos diez o veinte años, pero la demanda para servicios telefónicos crece rapidamente. El número de llamadas entre naciones vecinas aumentará con el incremento de actividad económica, y puede, por si mismo, ayudar al crecimiento económico.

Una red regional de telecomunicaciónes vía satélite ofrece una posible solución a esta demanda de servicio y es el objecto de mi presente investigación. Por lo tanto, le pido a Ud. su asistencia en obtener cierta información necesaria relacionada al número de llamadas hechas entre su país y los países vecinos.

Como requerimiento parcial de mi tesis para título de doctor en Ingeniería Eléctrica en el Instituto Tecnológico de California, estoy investigando el costo y la utilidad de una red vía satélite de acceso en demanda (demand access) para el continente americano. Para poder hacerlo, debo enterarme primero del volumen de tráfico telefónico entre naciones. Con esa información podré computar el número de canales, transmisores de estación terrena, etc., que se necesita.

Le agradecería de sobremanera si Ud. pudiera hacer completar el formulario o tabla adjunta indicando tanto el tráfico actual como el que se prevee para el futuro. Estoy seguro de que esta información será de un gran valor para el futuro de la comunicación vía satélite en el continente americano.

Agradeciendole sinceramente su atto. y s.s.,

James Laurens Latimer

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

ELECTRICAL ENGINEERING 116-81

1 June 1977

Cher Monsieur:

Les télécommunications ont une importance capitale pour le progrès et l'unité des nations. L'usage de la radio et de la télévision a été bien établi, et un changement dans les dix ou vingt prochaines années n'est pas probable. Toutefois la demande pour le service du téléphone augmente rapidement. La quantité des appels entre les nations voisines suivra l'augmentation de l'activité économique.

Un réseau de satellites de communication regional offre une solution possible à cette demande. C'est le sujet de mes recherches en ce moment. Pour cette raison, je voudrais vous demander votre assistance afin d'obtenir quelques informations nécessaires au sujet de la quantité des appels qui sont faits entre votre pays et les pays voisins.

Afin d'achever une partie de ma thèse de doctorate d'ingènieur électricien au California Institute of Technology, je cherche le coût et l'utilité d'un "demand access" système de satellites pour l'Amérique du Nord et du Sud. Pour accomplir cela, il faut que je sache d'abord le nombre de communications téléphoniques entre les nations. Muni de ces informations, je peux calculer la quantité de voies, émetteurs terrestres, etc., qui seront nécessaires.

Je vous serais très reconnaissant si vous pouviez faire compléter la table ci-jointe avec les informations au sujet du volume des télécommunications en ce moment et dans le futur. Je crois que ces informations auront une grande valeur pour le futur des communications par satellites sur le continent américain.

En attendant votre réponse, je vous prie d'agréer, Monsieur, mes salutations distinguées.

James Laurens Latimer

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

ELECTRICAL ENGINEERING 116-81

1 June 1977

Hoog Geachte Heer:

Telecommunicatie is van levensbelang voor de vooruitgang en solidariteit van elk land. Het gebruik van radio en televisie heeft een vaste plaats verworven en zal waarschijnlijk in de komende tien tot twintig jaar slechts in geringe mate veranderen, maar de behoefte aan telefoonverbindingen neemt snel toe. Meer bepaald, het aantal gesprekken tussen buurlanden zal stijgen met de toenemende ekonomische aktiviteit en kan bijdragen tot de ekonomische groei.

Een regionaal communicatiesatellietennet biedt een mogelijke oplossing voor die behoefte en is het onderwerp van mijn lopend onderzoek. Om die reden ben ik zo vrij U om enige hulpte vragen bij net verkrijgen van de noodzakelijke gegevens betreffende het aantal telefoongesprekken tussen Uw land en de buurlanden.

In het kader van mijn dissertatie in de Afdeling Elektrotechniek van het California Institute of Technology ben ik bezig met een onderzoek naar de kosten en het nut van een `demand access` satelietennet, voor Noord- en Zuid-Amerika. Informatie betreffende de intensiteit van het telefoonverkeer tussen verschillende landen is hiervoor noodzakelijk. Op grond hiervan kan ik vervolgens het aantal benodigde kanalen, grond station zenders, enz. berekenen.

Ik zou het bijzonder op prijs stellen indien U zou zorgen voor het invullen van de ingesloten tabel, die het huidige en verwachte telecommunicatieverkeer aangeeft. Naar mijn mening kan die informatie van grote waard zijn voor de toekomst van de communicatie per sateliet in Noord- en Zuid-Amerika.

Hoogachtend,

James Laurens Latimer

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

ELECTRICAL ENGINEERING 116-81

1 June 1977

Prezado Senhor,

Telecomunicações é vital importância para o progresso e solidariedade de qualquer nação. O uso do rádio e da televisão está bem estabelecido e positavamente não mudará muito nos próximos dez ou vinte anos, mas a demanda para o serviço telefônico está crescendo rápidamente. Particularmente, o número de telefonemas entre nações vizinhas crescerá junto com o crescimento de atividades econômicas e pode portanto auxiliar o crescimento econômico.

Uma rêde satélite de comunicação regional oferece uma solução possível para a demanda desse serviço, assim sendo, êsse é o têma da minha pesquiza. Portanto, estou pedindo a assistência de V.Sa. para obter informações necessárias, relacionando o número de telefonemas feito entre seu país e seus vizinhos.

Para o preenchimento parcial da minha tese de Ph.D. no Instituto de Tecnologia da Califórnia em Engenharia Elétrica, estou investigando o custo e a utilidade de uma rêde satélite com acesso à demanda (demand access) para as Américas. No sentido de conseguir êsse objetivo, primeiramente preciso saber qual a duração média e o número de chamadas telefônicas entre as nações, o que me permitirá computar o número de canais, estações transmissoras da terra, etc, necessárias.

Agradeceria se V.Sa. pudesse completar o formulário incluso indicando o tráfico atual e o previsto para o futuro. Acredito que tais informações seriam de grande valor para uma futura comunicação por satélites entre as Américas.

Aproveito a oportunidade para apresentar à V.Sa. meus protestos de elevada estima e consideração.

James Laurens Latimer

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

ELECTRICAL ENGINEERING 116-81

Dear Sirs:

I am investigating the cost of a demand access satellite network. I am particularly interested in the cost and availability of earth station equipment such as antennas, tracking subsystems, RF synthesizers, LN amplifiers, RF mixers, time and frequency standards, modems, concentrators, and so forth.

Kindly send me information showing what you have to offer, particularly that which represents the state of the art either in terms of performance or in terms of low cost.

Sincerely,

James L. Latimer

APPENDIX L

List of Computer Programs

```
C       THIS PROGRAM COMPUTES THE CERTAINTY LEVEL IN ESTIMATING THE
C       PARAMETER OF A POISSON PROCESS FROM M OBSERVATIONS.
      REAL L
      TYPE 10
   10 FORMAT(1X,'90% CERTAINTY LEVEL')
C       CHANGE THE FORMAT IF COMPUTING THE 99% CERTAINTY LEVEL.
      L=.001
      DO 50 M=0,70
   20 L=L+.001
      IF(FN(M,L).LT..9) GO TO 20
C       CHANGE THE ABOVE STATEMENT IF COMPUTING THE 99% CERTAINTY LEVEL.
      PRINT 30, M,L
      TYPE 30, M,L
   30 FORMAT(1X,I3,F10.3)
   50 CONTINUE
      END
      FUNCTION FN(M,L)
      REAL L
      SUM=1
      IF(M.EQ.0) GO TO 20
      DO 10 I=1,M
   10 SUM=1.+(L/(M+1-I))*SUM
   20 CONTINUE
      FN=1.-SUM/EXP(L)
      RETURN
      END
```

```
C       THIS PROGRAM COMPUTES THE NUMBER OF CHANNELS REQUIRED TO ACHIEVE
C       A .5% PROBABILITY OF BLOCKAGE FOR A GIVEN NUMBER OF ERLANGS OF
C       TRAFFIC.
C       IC IS THE NUMBER OF CHANNELS
C       RHO IS THE NUMBER OF ERLANGS
      TYPE 2
    2 FORMAT(1X,'  RHO   CH PR(BL)')
      PB=.005
      DRHO=.001
      MAXCH=1000
      NRINTV=MAXCH/DRHO
      LASTIC=1
      DO 100 I=1,NRINTV
      RHO=DRHO*I
      DO 50 IC=LASTIC,MAXCH
      SUM=1.
      SUMFAC=1.
      DO 20 J=1,IC
      RHOIC=RHO/(IC+1.-J)
      SUM=SUM*RHOIC
      SUMFAC=SUMFAC*RHOIC
      SUM=SUM+1.
   20 CONTINUE
      PRBLK=SUMFAC/SUM
      IF(PRBLK.LT.PB) GO TO 60
   50 CONTINUE
   60 CONTINUE
      PRBLK=PRBLK*100.
      IF(RHO.GT..001.AND.IC.EQ.LASTIC) GO TO 90
```

```
      TYPE 99, RHO,IC,PRBLK
   99 FORMAT(1X,F7.3,I3,F7.3)
   90 LASTIC=IC
  100 CONTINUE
      END

          ------------------------------
          ------------------------------

C        THIS PROGRAM COMPUTES THE CORRELATION FUNCTION OF RANDOM
C        VARIABLES.
C
      DIMENSION RING(10),SUMXX(10),SUMX(10),CORREL(10)
C        THE FOLLOWING INITIALIZES THE RANDOM NUMBER GENERATOR.
      CALL RNDSET(0)
      DO 10 I=1,10
      SUMXX(I)=0.
      SUMX(I)=0.
   10 RING(I)=-ALOG(RANDOM(IDUM))
      DO 100 I=1,75000
      N=I
      IM10=MOD(I-1,10)+1
      DO 40 J=1,10
      SUMXX(J)=SUMXX(J)+RING(IM10)*RING(MOD(IM10-J+10,10)+1)
   40 SUMX(J)=SUMX(J)+RING(MOD(IM10-J+10,10)+1)
      IF(MOD(N,1000).NE.0) GO TO 100
      DO 80 K=1,10
   80 CORREL(K)=SUMXX(K)/N-SUMX(1)*SUMX(K)/N/N
      ZZZ=SUMX(1)/N
      PRINT 90,N,ZZZ,(CORREL(K),K=1,10)
      TYPE 90,N,ZZZ,(CORREL(K),K=1,10)
   90 FORMAT(1X,I5,11(1X,F8.5))
  100 RING(IM10)=-ALOG(RANDOM(IDUM))
      END

          ------------------------------
          ------------------------------

C        THIS PROGRAM COMPUTES THE CUMULATIVE DISTRIBUTION FUNCTION
C        OF RANDOM VARIABLES.
C
      COMMON /HPPLOT/HPPLOT
      DIMENSION VAR(5000),IDX(5000),XI(5000),TXI(5000)
      DIMENSION DD(3)
      DATA DD/'F=P(','Y<X)',0.0/
      N=1000
      CALL RNDSET(0)
C        0 FOR SCOPE AND 1 FOR PLOTTER
      HPPLOT=0
      CALL ERASE
      DO 10 I=1,N
      XI(I)=FLOAT(I)/N
   10 VAR(I)=-ALOG(RANDOM(IDUM))
      CALL SORTI2(VAR,N,IDX)
      XN=FLOAT(N)
      DO 15 I=1,N
   15 TXI(I)=1.-EXP(-VAR(I))
      RMAX=0.
      DO 30 I=1,N
   30 RMAX=AMAX1(RMAX,ABS(TXI(I)-XI(I)))
      RMAX=RMAX*SQRT(XN)
      TYPE 40, RMAX
   40 FORMAT(1X,F10.6)
      CALL LABEL(0.0,0.0,0.0,VAR(N),15.0,12,'PROB',4,0)
      CALL LABEL(0.0,0.0,0.0,XN,10.0,10,'OCCURENCE',9,1)
      CALL XYPLOT(N,VAR,XI,0.0,VAR(N),0.0,1.0,DD,0)
      CALL XYPLOT(N,VAR,TXI,0.0,VAR(N),0.0,1.0,DD,1)
```

```
      END
```

```
C      THIS PROGRAM COMPUTES THE PROBABILITY OF BLOCKAGE FOR IC CHANNELS
C      AS A FUNCTION OF THE INCOMING TRAFFIC USING THE ERLANG B
C      EQUATION.
C
      DIMENSION X(100),Y(100)
      RHODEL=.05
      IC=3
      DO 50 I=1,100
      RHO=I*RHODEL
      X(I)=RHO
      SUM=1.
      SUMFAC=1.
      DO 20 J=1,IC
      RHOIC=RHO/(IC+1.-J)
      SUM=SUM*RHOIC
      SUMFAC=SUMFAC*RHOIC
      SUM=SUM+1.
   20 CONTINUE
      Y(I)=SUMFAC/SUM
   50 CONTINUE
      TYPE 99,(X(I),Y(I),I=1,100)
   99 FORMAT(1X,2F10.6)
      END
```

```
        TITLE UNIFORM RANDOM NUMBER GENERATOR
        ENTRY RNDSET,RANDOM
        WTAU: BLOCK D19
        R0=0
        R1=1
        T0=2
        T1=3
        RNDIZ=[D41475557]
        XN=[0]
        I=[0]
RNDSET:
        PUSH    17,R1
        PUSH    17,T1
        PUSH    17,T0
        MOVE    R1,@(16)
        CAIN    R1,0
        MOVE    R1,RNDIZ
        MOVEM   R1,XN
        MOVE    T1,XN
        MOVEM   T1,WTAU
        MOVNI   T0,D18
RND.02: IMUL    T1,RNDIZ
        MOVEM   T1,WTAU+D19(T0)
        AOJL    T0,RND.02
        SETZM   I
        POP     17,T0
        POP     17,T1
        POP     17,R1
        POPJ    17,0
RANDOM:
        PUSH    17,T0
        MOVE    T0,I
        CAIN    T0,D19
        PUSHJ   17,RAND1
        MOVE    R0,WTAU(T0)
```

```
        TLZ     R0,776000
        LSH     R0,-1
        FSC     R0,200
        AOS     I
        POP     17,T0
        POPJ    17,0
RAND1:
        PUSH    17,T1
        PUSH    17,R1
        SETZB   T0,I
        MOVNI   T1,D16
RAN.02: MOVE    R0,WTAU+D17(T1)
        MOVE    R1,WTAU+D18(T1)
        LSH     R1,D8
        LSHC    R0,D9
        XORM    R0,WTAU+D16(T1)
        AOJLE   T1,RAN.02
        MOVE    R0,WTAU+D18
        MOVE    R1,WTAU
        LSH     R1,D8
        LSHC    R0,D9
        XORM    R0,WTAU+D17
        MOVE    R0,WTAU
        MOVE    R1,WTAU+1
        LSH     R1,D8
        LSHC    R0,D9
        XORM    R0,WTAU+D18
        POP     17,R1
        POP     17,T1
        POPJ    17,0
        END
```

```
C        REXP.F4
C        THIS FUNCTION MAY BE USED TO GENERATE RANDOM VARIABLES THAT ARE
C        EXPONENTIALLY DISTRIBUTED.  USE OF THE FUNCTION SHOULD BE
C        PROCEEDED BY THE STATEMENT:  CALL RNDSET(0)
      FUNCTION REXP(IDUM)
      DIMENSION P(25),Q(25),QT(25),R(25)
      R(1)=1.
      QT(1)=1.
      Q(1)=QT(1)/(EXP(1.)-1.)
      P(1)=1.-1./EXP(1.)
      DO 10 J=2,25
      R(J)=R(J-1)/FLOAT(J)
      QT(J)=QT(J-1)+R(J)
      Q(J)=QT(J)/(EXP(1.)-1.)
      P(J)=1.-1./EXP(FLOAT(J))
   10 CONTINUE
      J=1
      U0=RANDOM(IDUM)
      U1=RANDOM(IDUM)
      X=U1
   20 IF(U0.LT.Q(J))GO TO 40
      J=J+1
      UJ=RANDOM(IDUM)
      IF(X.GT.UJ) X=UJ
    8 FORMAT(I5)
      GO TO 20
   40 U=RANDOM(IDUM)
      J=1
   50 IF(U.LT.P(J)) GO TO 70
      J=J+1
      X=X+1.
      GO TO 50
```

```
   70 REXP=X
      RETURN
      END

         ------------------------------
         ------------------------------

C        THIS PROGRAM SIMULATES THE TRAFFIC CHARACTERISTICS OF A
C        SATELLITE COMMUNICATION NETWORK.  THE NETWORK CONSISTS OF A
C        DEMAND ACCESS SATELLITE WITH NCH CHANNELS AND NES EARTH
C        STATIONS.  EACH EARTH STATION IS ASSIGNED CERTAIN LINKS BY
C        WHICH IT MAY ATTEMPT TO ACQUIRE A CHANNEL.  THE METHOD OF
C        SEARCHING FOR AN AVAILABLE CHANNEL MAY EITHER BE SEQUENTIAL OR
C        RANDOM.  ACCUMULATED CALL BLOCKAGE PROBABILITIES ARE COMPUTED
C        AFTER EACH CHANGE IN THE USAGE OF THE SATELLITE CHANNELS.
C
C        STRT(I)     START TIME OF THE CALL CURRENTLY AT THE I TH EARTH
C                    STATION.
C        IR          =0 IF ORDERED SEARCH OF AVAILABLE CHANNELS.
C                    =1 IF RANDOM SEARCH OF AVAILABLE CHANNELS.
C        NES         THE NUMBER OF EARTH STATIONS.
C        NCH         THE NUMBER OF SATELLITE CHANNELS.
C        NLES(I)     THE NUMBER OF LINKS FROM THE I TH EARTH STATION.
C        IESCH(I,J)  THE NUMBER OF THE CHANNEL WHICH IS CONNECTED TO THE
C                    J TH LINK OF THE I TH EARTH STATION.
C        ICOUNT      CURRENT TOTAL NUMBER OF INCOMING CALLS.
C        IBLOCK      CURRENT TOTAL NUMBER OF BLOCKED CALLS.
C        STRTM       IS THE MINIMUM START TIME OF CALLS CURRENTLY
C                    ARRIVING AT THE EARTH STATIONS.
C        STPM        IS THE MINIMUM STOP TIME OF CALLS CURRENTLY BEING
C                    SERVICED BY THE CHANNELS.
C        MES         IS THE NUMBER OF THE EARTH STATION WITH THE MINIMUM
C                    START TIME.
C        STP(I)      THE STOP TIME OF THE CALL CURRENTLY ON THE I TH
C                    CHANNEL.
C                    =0. IF THERE IS NO CALL CURRENTLY ON THE I TH
C                    CHANNEL.
C        MCH         IS THE NUMBER OF THE CHANNEL WITH THE MINIMUM STOP
C                    TIME.
C        IPERM       IS AN ARRAY WHICH PRAND USES TO OUTPUT A RANDOM
C                    ORDER OF INTEGERS.
      DIMENSION STRT(10),NLES(10),IESCH(10,50),IES(10),STP(50),IPERM(12)
      CALL RNDSET(0)
      CALL ERRSET(0)
      TIME=0.
      RHO=.05
      DO 2 I=1,4
    2 STRT(I)=-ALOG(RANDOM(IDUM))/RHO
      IR=1
      DO 5 I=1,12
    5 IPERM(I)=I
C      THE FOLLOWING LINES ARE FOR AN FVDA NETWORK WITH 4 EARTH STATIONS
C      AND 3 CHANNELS.  THEY SHOULD BE CHANGED FOR OTHER NETWORK
C      CONFIGURATIONS.
      NES=4
      NCH=3
      DO 10 I=1,NES
      NLES(I)=NCH
      DO 10 J=1,NCH
   10 IESCH(I,J)=J
C           ---
      DO 40 I=1,NCH
   40 STP(I)=0.
      ICOUNT=0
      IBLOCK=0
   60 STRTM=10000000.
      STPM=10000000.
```

```
      ITYPE=0
      IF(MOD(ICOUNT,500).EQ.0) ITYPE =1
      DO 80 I=1,NES
      IF(STRT(I).GT.STRTM) GO TO 80
      MES=I
      STRTM=STRT(I)
   80 CONTINUE
      DO 120 I=1,NCH
      IF(STP(I).EQ.0.) GO TO 120
      IF(STP(I).GT.STPM) GO TO 120
      MCH=I
      STPM=STP(I)
  120 CONTINUE
      IF(STRTM.GE.STPM) GO TO 200
      IF(IR.EQ.1) CALL PRAND(IPERM,NLES(MES))
      ICOUNT=ICOUNT+1
      DO 140 I=1,NLES(MES)
      IE=IESCH(MES,IPERM(I))
      IF(STP(IE).NE.0) GO TO 140
      STP(IE)=STRT(MES)-ALOG(RANDOM(IDUM))
      GO TO 180
  140 CONTINUE
      IBLOCK=IBLOCK+1
      IF(ITYPE.EQ.1) TYPE 160
  160 FORMAT(1X,'BLOCKED CALL')
  180 STRT(MES)=STRT(MES)-ALOG(RANDOM(IDUM))/RHO
  200 IF(STPM.LE.STRTM) STP(MCH)=0
      IF(STPM.EQ.STRTM) TYPE 220,STPM,STRTM
  220 FORMAT(1X,'ERROR',2F15.9)
      PRBLK=FLOAT(IBLOCK)/FLOAT(ICOUNT)
      RHOEXP=ICOUNT*ALOG(EXP(1.))/(NES*AMIN1(STRTM-TIME,STPM-TIME))
      IF(ITYPE.EQ.1) TYPE 240,ICOUNT,MCH,MES
  240 FORMAT(1X,10I5)
      IF(ITYPE.EQ.1) TYPE 260,STRTM,STPM,RHOEXP,PRBLK
  260 FORMAT(1H+,30X,4F10.5)
      IF(ICOUNT.LE.5000) GO TO 60
      ICOUNT=0
      IBLOCK=0
      TIME=AMIN1(STRTM,STPM)
      RHO=RHO+.05
      GO TO 60
      END
C
C
C       THIS SUBROUTINE TAKES THE INTEGERS 1 THROUGH N
C       (N<13) AND PUTS THEM INTO THE ARRAY M IN RANDOM ORDER.
      SUBROUTINE PRAND(M,N)
      DIMENSION IFAC(12),LST(12)
      INTEGER M(50)
      DO 10 I=1,12
   10 LST(I)=I
      IFAC(1)=1
      IFAC(2)=2
      IFAC(3)=6
      IFAC(4)=24
      IFAC(5)=120
      IFAC(6)=720
      IFAC(7)=5040
      IFAC(8)=40320
      IFAC(9)=362880
      IFAC(10)=3628800
      IFAC(11)=39916800
      IFAC(12)=479001600
      NF=IFAC(N)
C       THE FOLLOWING GIVES US A RANDOM INTEGER BETWEEN 1 AND
C       N FACTORIAL.
      IX=INT(RANDOM(IDUM)*NF)+1
```

```
      IR=MOD(IX,NF)+1
C      NOW WE WILL FIND THIS PARTICULAR PERMUTATION OF INTEGERS.
      DO 40 I=1,N-1
      IFC=IFAC(N-I)
      IP=(IR-1)/IFC+1
      IR=MOD(IR-1,IFC)+1
      M(I)=LST(IP)
      IF(IP.GT.N-I) GO TO 40
      DO 20 J=IP,N-I
   20 LST(J)=LST(J+1)
   40 CONTINUE
      M(N)=LST(1)
      RETURN
      END
```

```
C      THIS PROGRAM PLOTS PROBABILITY OF BLOCKAGE AS A FUNCTION OF
C      THE PRESENTED TRAFFIC IN ERLANGS.
      REAL DEN(10),NUM(10)
      REAL DN,NM
      DIMENSION X(1000),PR(1000)
      COMMON /HPPLOT/HPPLOT
      DIMENSION DD(3)
      DATA DD/'    ','    ',0.0/
C      0 FOR SCOPE AND 1 FOR PLOTTER
      HPPLOT=1
      CALL ERASE
      TYPE 5
    5 FORMAT(1X,'ORDER OF POLYNOMIAL?')
      ACCEPT 10,N
   10 FORMAT(I)
      N=N+1
      TYPE 15
   15 FORMAT(1X,'NUM  DEN?')
      ACCEPT 20,(NUM(I),I=1,N)
   20 FORMAT(F)
      ACCEPT 20,(DEN(I),I=1,N)
      TYPE 25,(NUM(I),I=1,N),(DEN(I),I=1,N)
   25 FORMAT(1X,24F5.2)
      CALL ERASE
      NMAX=100
      DO 40 J=1,NMAX
      X(J)=(J-1)*.05
      NM=0.
      DN=0.
      DO 30 I=1,N
      NM=NM*X(J)+NUM(N-I+1)
   30 DN=DN*X(J)+DEN(N-I+1)
      PR(J)=NM/DN
   40 CONTINUE
      CALL LABEL(0.0,0.0,0.0,5.,15.0,10,'RHO',3,0)
      CALL LABEL(0.0,0.0,0.0,1.,10.0,10,'P(BLOCKAGE)',11,1)
      CALL XYPLOT(NMAX,X,PR,0.0,5.,0.0,1.0,DD,1)
      END
```

REFERENCES

Abromowitz, M., and I. A. Stegun. Handbook of Mathematical Functions, 1964, pp. 260-263.

"Alternate Routing of Telephony Traffic and Regional Network Planning," Australian Post Office Engineering Instruction, Planning Traffic J1200, 1962.

Annual Cumulative Bulletin, Public Affairs Information Service, 1975/1976.

Basic Telephone Traffic Theory, Australian Post Office Engineering Instruction, Planning Traffic B1100, 1960.

Bent Pipe Feasibility Study, Interim Report 890-68, Jet Propulsion Laboratory.

Brandes, R. D. et al. "Impact of Expanded Communication Services on Satellite System Design," 6th AIAA Satellite Communications Systems Conference, April 1976.

Burtt, J. E., et al. Technology Requirements for Communication Satellites in the 1980's, NASA-CR-114680, by Lockheed Missiles and Space Company for Ames Research Center, Oct. 1973.

Calculation of Best Arrangement for Alternative Routing, CCITT Recommendation, International Telecommunication Union, p. 93.

Carnahan, Luther and Wilkes. Applied Numerical Methods, Ch. 5, Wiley.

Collins, Arthur A. and Robert D. Pederson. A Time for Innovation, Merle Collins Foundation, p. 19.

"Communications Satellite and Earth Station Hardware Review," Communications Satellite Planning Center, Vol. II, T. R. #2, Stanford Electronics Laboratories, Aug. 1975.

Crane, R. K. "Propagation Phenomena Affecting Satellite Communication Systems," Proc. IEEE, Feb. 1971, pp. 178-188.

Current World Leaders Almanac, Spring 1977.

Curves Showing the Relation Between the Traffic Offered and the Number of Circuits Required, IVth Plenary Assembly, Geneva: CCITT, International Telecommunication Union, 1969, Supplement 8, pp. 1-2.

Demand Assignment System Controller's Manual, Vol. I, Hughes Aircraft Co., March 1977.

Department of State Background Notes Publication.

Dill, G. D. "Comparison of Circuit Call Capacity of Demand-Assignment and Preassignment Operation," Comsat Technical Review, Vol. 2, No. 1, Spring 1972, p. 243.

Dutton, E. J. Earth-Space Attenuation Prediction Procedures at 4 to 16 GHz, Office of Telecommunications, U. S. Department of Commerce, O. T. Report 77-123.

Economic Studies at the National Level in the Field of Telecommunications (1964-1968), CCITT, International Telecommunication Union, July 1968.

Edelson, Burton I. "Global Satellite Communications," Scientific American, Feb. 1977, pp. 58-73.

Edelson, Burton I. and A. M. Werth. "SPADE System Progress and Applications," Comsat Technical Review, Vol. 2, No. 1, Spring 1972, pp. 221-241.

Eisenberg, M. "Double Switching in Multimachine Systems," Bell Telephone Labs Technical Memorandum (MM 68-3123-8), 26 Aug. 1968.

Feller, William. An Introduction to Probability Theory and Its Applications, 3rd Ed., Vol. I, 1968.

Fisz, Marek. Probability Theory and Mathematical Statistics, J. Wiley, 3rd Ed., 1963.

Fraser, W. Telecommunications, MacDonald: London, 1967, pp. 782-784.

General Plan for Development of the Regional Latin American Network, Vol. I and II, CCITT, International Telecommunication Union, 1973.

Harris, J. and Steinhorn, J. P. "Effects of Rain on Communications Satellite Systems Tradeoffs," International Conference on Communications, July 1976.

Heathcote, W. R. Erlang Function Table, Hughes Aircraft Co. IDC, 31 Oct. 1972.

Horstein, M. Thin-Route Satellite Telephony Networks, Proceedings of 1st Iranian Congress of Electrical Engineering, Vol. 2, May 1974, pp. 1442-1455.

"The Intelsat Organization Annual Report to the President and Congress."

Joyce, Heathcote and Nilsen. Hughes IDC BRZ-035.

Kemeny and Snell. Finite Markov Chains, Van Nostrand.

Kleinrock, L. Queuing Theory, Vol. 1 and 2.

Knuth. The Art of Computer Programming, Vol. 2, Seminumerical Algorithms, Addison-Wesley, 1969, p. 114.

Kraus, J. D. Radio Astronomy, McGraw Hill, 1966, QB475 K7.

Latin America, Latin America Political Report, Latin America Economic Report, Latin America Commodities Report, Newsletters (Weekly), 6/7 New Bridge St., London EC4V 6HR.

Lee, A. M. Applied Queuing Theory, Mac Millian, 1966, QA273.1 L4.

Lusignan, Bruce. "Demand Assignment Multi Access System for Thin Route Satellite Telephone System," Dec. 1976.

Lusignan, Bruce. "Investment of Traffic Assignment in a Hybrid Ground and Special Communications Network,".

Lusignan, Bruce. "Planning Optimization and Utilization of Preassigned and Demand Assigned Services in Domestic Satellite Telecommunications Systems," Dec. 1976.

Lusignan, Bruce. "Rural Communications Planning Methodology for Integrated Satellite and Terrestrial Facility," Dec. 1976.

Lusignan, Bruce. "Satellite and Ground Station Intermodulation in SCPC Systems."

Lusignan, Bruce. "Use of Satellite in Meeting Telecommunications Needs in Developing Countries," June 1975.

"Measurement and Recording of Traffic," CCITT Recommendation E.500, International Telecommunication Union, Vol. IIA, Q.80, Vol. VI.

Meyer, S. L. Data Analysis for Scientists and Engineers, Ch. 24, Wiley, 1975.

"A Model Digital Satellite Network," Memorandum for Record, Bell Telephone Laboratories, 12 Sept. 1975.

Morse, P. M. Queues, Inventories, and Maintenance, Wiley, 1958.

National Telephone Networks for the Automatic Service, III Plenary Assembly, CCITT, International Telecommunication Union, 1964.

"Navstar Global Positioning System," Astronautics and Aeronautics, April 1976, pp. 26-32.

Palapa. System Growth Study, Phase 2, Hughes Aircraft Company.

Parzen. Stochastic Processes, Holden-Day, 1962, QA273.

Pierce, John R. Waves and Messages. Doubleday Anchor Books, 1967, pp. 109-118.

Proceedings of Seminar on Small Earth Terminals, Office of Telecommunication Policy (Ex. Office of President), 31 Oct. 1975.

Puente, J. G., W. G. Schmidt, and A. M. Werth. "Multiple Access Technique for Commercial Satellites," Proc. IEEE, Feb. 1971, pp. 218-229.

Puente, J. G. and A. M. Werth. "Demand-Assigned Service for the Intelsat Global Network," IEEE Spectrum, Vol. 8, No. 1, Jan. 1971, pp. 59-69.

Ralston and Wilf. Mathematical Methods for Digital Computers, Wiley, Vol. 2, QA 76.5 R13.

Reinhart, E. E. R-1463-NASA, May 1974.

Ross, S. Applied Probability Models with Optimization Applications, Holden-Day, 1970, QA273 R67.

Saaty, T. L. Elements of Queuing Theory, MacGraw Hill, 1961, QA273 S218.

Sinha, A. "A Model for TDMA Burst Assignment and Scheduling," Comsat Technical Review, Vol. 6, No. 2, Fall 1976, pp. 219-251.

Smith, Delbert. Communications Via Satellite, A Vision in Retrospect, A. W. Sijhoff Publisher, 1976.

The South American Handbook, Trade and Travel Publication, Mendy Press, Parsonage Lane, Bath BA1 1EN, England; Distributor: Rand McNalley, San Francisco.

Spilker, James J. Jr. Digital Communication by Satellite, Prentice Hall, 1977.

Table of the Erlang Lass Formula, IVth Plenary Assembly, International Telegraph and Telephone Consulative Committee (CCITT), Mar Del Plata, 1968, White Book, Vol. VI, Supplement 7, p. 1.

Talley, David. Basic Telephone Switching Systems, Hayden Book Co., 1969.

"Traffic Problems in Telephone Networks via Satellite with Preassigned and Demand Assigned Circuits," Intelsat/IEE International Conference on Digital Satellite Communications, 1969, p. 494, TK5101 I6.

Wilkinson, R. J. Theories for Toll Traffic Engineering in the U.S.A. BSTJ, March 1956, pp. 421-514.

"The World's Telephones," Overseas Operations, AT&T Long Lines.

THESIS INDEX