

Agile Slew Maneuvers of Ultralight Space Structures with Momentum Actuators

Thesis by
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ABSTRACT

Ultralight deployable spacecraft structures are at least an order of magnitude lighter and more volume efficient for launch than state-of-the art spacecraft structures of the same size, thanks to advancements in thin lightweight composites and new packaging schemes. As a result, they are suitable for a wide range of applications requiring extremely large apertures. For example, one of the application that we are currently interested in is in-space computer servers. These ultralight spacecraft could not only support photovoltaics that generate large amounts of power, but at the same time act as extremely large radiators for dissipating heat. Similarly, for space-based solar power missions, they can hold functional layers of photovoltaics and phased array antennas to provide uninterrupted energy transmission to any location on Earth.

However, this improvement in mass/volume efficiency comes at a price. Ultralight spacecraft exhibit high structural compliance posing new challenges in spacecraft attitude control, which includes residual vibrations in the flexible structure and decreased pointing accuracy, leading to new requirements in structure/control design and testing of these architectures. The present work demonstrates new maneuver techniques and testing methodologies for Caltech's Space Solar Power Project (SSPP) architecture.

Real-world structures are imperfect and require ground tests to characterize their in-space behavior. We present an experimental methodology to perform slew maneuvers to help study the simultaneous behavior of the actuator, control scheme, and the flexible structure response. We present the design of a momentum device prototype and a small-scale, lightweight, flexible structure built for performing slew maneuvers. The flexible structure is designed to be highly compliant along the maneuver direction, allowing the study of its elastic deformation. Smooth polynomial maneuvers are performed with the actuator-structure system mounted on an air bearing; the central hub orientation and flexible structure response is measured. Assuming deformations to be comprised of a single dominant mode, input shaping is applied to the smooth polynomial maneuver, using Zero Velocity (ZV) shaper to suppress vibrations post maneuver. Input-shaping experiments show a remarkable reduction in structural vibrations for high-agility rotations, which directly contributes to an increased power transmission capability in full-scale spacecraft.

Although small-scale prototypes are useful for demonstrating control techniques and system-level performance, it is quite complex to predict results for full-sized space structures given the lack of detailed similitude models for a combination of actuator-structure behavior and real-world defects. Moreover, full-scale structures are difficult to test on the ground in their deployed configuration, due to limitations in the current gravity compensation techniques. With increases in computational capability, data-centric system identification is the solution to robust modeling of real-world structures. These techniques are powerful not only in decomposing spatial-temporal variations but also in finding complex nonlinear behaviors. In the present work, reduced-order models are created from actuator-flexible structure experiments with equation-based nonlinear extensions to existing data-driven techniques. These models are then used to explore vibration-optimal agile maneuvers without making any assumptions about structural properties. These maneuvers, when tested in the experiments, result in fast rotations with a significant reduction of residual vibrations. This work directly contributes to improving the robustness of existing control schemes, enabling scalability to larger spacecraft and decreasing sensitivity to variations in the deployed structure over time.

Together, these results demonstrate that large ultralight spacecraft can be designed and robustly controlled while addressing unwanted effects due to their low stiffness. This development enables their application to next-generation space missions such as space-based solar power, in-space servers, solar sail demonstrations, and much more.

PUBLISHED CONTENT AND CONTRIBUTIONS

- [1] Soni, D., Gdoutos, E., and Pellegrino, S., “Experiments With a Momentum Exchange Actuator for Ultralight Flexible Spacecraft,” *AIAA SCITECH 2026 Forum*, 2026, p. 0411. <https://doi.org/https://doi.org/10.2514/6.2026-0411>, d.S. developed theoretical framework for the prototype, performed closed-loop quaternion feedback, designed the flexible structure and wrote the manuscript.
- [2] Soni, D., Gdoutos, E., and Pellegrino, S., “Deployment and Attitude Control Device for Large Flexible Spacecraft,” Presented at the ASME Aerospace Structures, Structural Dynamics, and Materials Conference, Houston, TX, 2025.
- [3] Soni, D., Gdoutos, E., and Pellegrino, S., “Deployment and Attitude Control Device for Large Lightweight Spacecraft,” (In preparation). D.S. developed theoretical framework for the prototype, performed actuator sizing, simulated rigid body closed loop maneuver tracking, and wrote the manuscript.
- [4] Soni, D., and Pellegrino, S., “Model Identification and Control of Large Flexible Ultralight Spacecraft with Momentum Exchange Actuator,” (In preparation). D.S. developed theoretical framework for nonlinear DMDc, developed flexible-body maneuver optimization, and wrote the manuscript.
- [5] Soni, D., and Pellegrino, S., “Data-driven Agile Slew Maneuver Experiments for Ultralight Flexible Spacecraft with Momentum Exchange Actuator,” (In preparation). D.S. developed theoretical framework for nonlinear DMDc, developed flexible-body maneuver optimization, and wrote the manuscript.

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Chapter 1

INTRODUCTION

1.1 Large deployable structures and space solar power

Space structures are no longer constrained to the size of launch vehicle fairings, thanks to the era of modern deployable structures. Large space reflectors and solar arrays enable extreme science objectives such as sample collection and return missions from adjacent planets or surrounding asteroids, all of which were beyond imagination a few decades ago. From communication satellites to deep space landers, deployable structures have proven to be quite effective.

Another area of interest where these deployable structures are extremely beneficial is space-based solar power. The objective is to provide energy to terrestrial power grids which is not only renewable but also economically competitive compared to conventional sources. The idea of harnessing Solar Power was first proposed[1] in 1968 by Dr. Peter Glaser. By that time, deployable structures were not in fashion. As a result, the first engineering proposal[2] by NASA in 1979 (Figure 1.1, left) was not only large ($\approx 6\text{ km}$), it required huge amount of resources in terms of materials and astronauts for assembly. Three decades later, about thirty new architectures[3] had emerged at the NASA Advanced Project's office (e.g. NASA Sun tower concept in Figure 1.1, right).

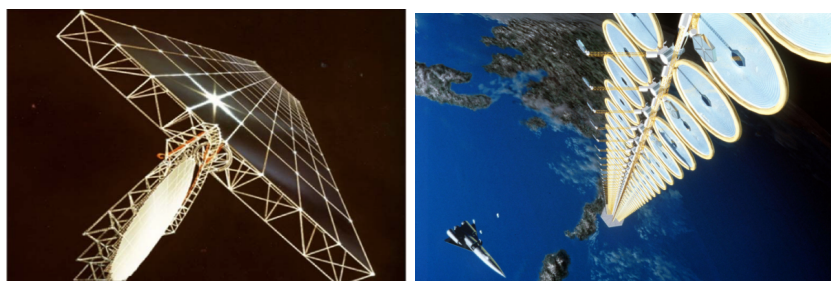


Figure 1.1: 1979 Reference SSPS(left) and NASA Sun tower concept (right).

Several more proposals were submitted in the next four decades with unique structural architectures. More recently, the California Institute of Technology proposed Space Solar Power Project[4] (SSPP) with a radically different approach. Rather than assembling an extremely large space structure in space, it calls for several small spacecraft flying together to coherently collect and transmit power wirelessly to

earth (Figure 2.1). Each spacecraft is proposed to be 60 m by 60 m in size. This makes the mission to be feasible economically[5].

Caltech's Space Solar Power Satellite (SSPS) consists of an *ultralight* structure with a multilayered planar architecture, which can efficiently collect solar power and beam the energy wirelessly in the form of electromagnetic radiation to a ground station on earth. Major advantages include power collection unaffected by the remote location or even during disaster events. Several versions of SSPS are anticipated in the near future including $10\text{ m} \times 10\text{ m}$, $30\text{ m} \times 30\text{ m}$, $60\text{ m} \times 60\text{ m}$ and so on. Each version is significantly challenging[6] in terms structural design and requires a thorough analysis regardless of the similarities in the architecture.

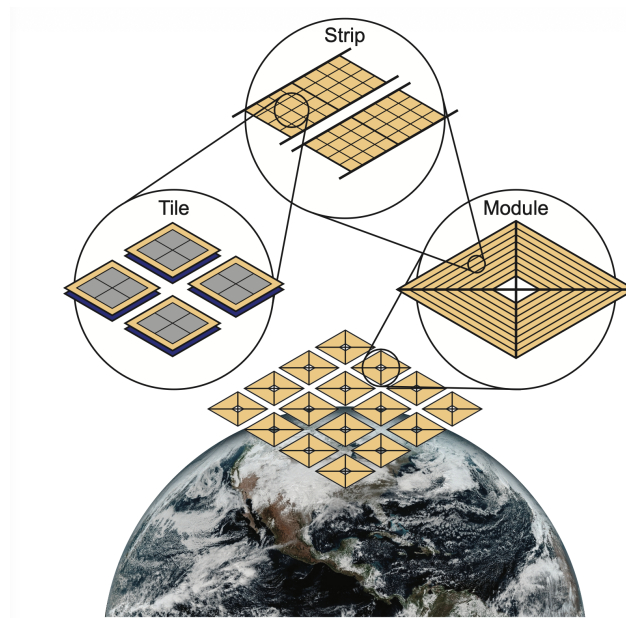


Figure 1.2: Overview of Caltech Space Solar Power Project concept originally introduced in[4].

These *ultralight spacecraft* are made possible by recent advances within materials and structures. They fall into a new category of spacecraft which exhibit higher performance and lighter weight by the virtue of unfurling into extremely large area apertures. These space structures are more flexible than the traditional stiff bus spacecraft due to tradeoff between packaging efficiency and structural stiffness.

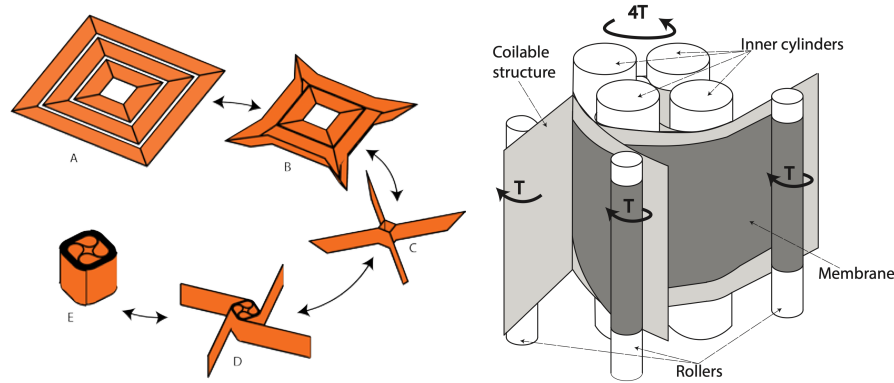


Figure 1.3: SSPS structure two-dimensional packaging scheme[7] (left) and deployment mechanism[8] (right).

A significant amount of work has been done towards the deployment of SSPS structure. A two-dimensional packaging scheme [7] allows folding each quadrant of the structure in Z-folds to develop an intermediate star shape with four arms. These arms are then be coiled around using pressure wrapping between polyimide membrane to achieve an efficiently packaged structure within near cylindrical volume. Coiling [8] is achieved by pressure wrapping the folded structure between four inner cylinders and corresponding outer rollers as shown in Fig. 1.3 (right). A smaller prototype 1.7 m x 1.7 m called Deployable on-Orbit ultraLight Composite Experiment (DOLCE) of SSPS structure was succesfully launched and demonstrated in early 2024. As mentioned earlier, because these *ultralight* structures are built for packaging efficiency, they have relatively low stiffness. As a result their first natural modes are several orders of magnitude lower than the current state-of-the-art [9] in space structures and this introduces new challenges in the light of spacecraft dynamics and control.

1.2 Spacecraft attitude control

The attitude of the spacecraft tends to change as the spacecraft separates from the payload adapter of the launch vehicle. Further into the mission life, it is often perturbed due to internal motion of the payload, sloshing propellant, which requires careful control methodologies for successful completion of the mission. Thus, attitude changes are necessary, be it orbit raising or changing orientation. Orientation change maneuvers also called *slew maneuvers* are quite commonly executed to achieve specific mission objectives such as sun acquisition, maintain line of sight on a ground station etc. In contrast to rigid spacecraft, these rotations excite structural dynamics in the flexible spacecraft and need to be studied in depth.

Apart from the external perturbations, sometimes mission requirements are the driving force for on-orbit station keeping and pointing. In the case of Caltech's SSPS architecture, power optimal guidance[10] can provide efficient power collection and transmission depending on the functional layers in the architecture. For e.g., a single layer PhotoVoltaics (PV) and a single layer Radio Frequency phased array (RF) requires the spacecraft to be slewed periodically during each orbit to maximize collection of solar power along with maintaining line-of-sight towards the ground station shown in Figure 1.4. This optimal solution necessitates a requirement for rapid slew maneuvers.

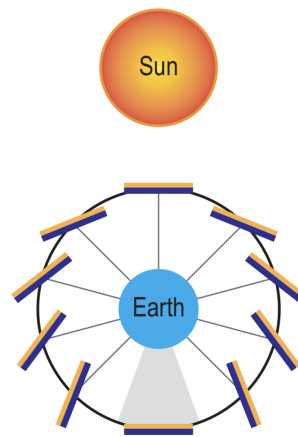


Figure 1.4: SSPS optimal attitude in GEO orbit[10] for a single PV, single RF architecture.

Earliest developments on slew maneuvers focused on utilizing a control strategy which could reorient the spacecraft towards a target direction. Often, maneuvers along the major or minor eigen-axes of the spacecraft were practiced and were termed as *eigenaxis* maneuvers. This was done utilizing a proportional-derivative control[11] in attitude angles. This technique even allowed to establish gain optimization based on flexible appendage characteristics using linear-quadratic-gaussian regulation on these spacecraft. However, this methodology was sensitive to modeling uncertainties and required further studies. Later, various attitude control strategies emerged from time-optimal (or fuel-optimal) control [12] to near-minimum time optimal control for flexible appendage[13] with root strain as feedback, slightly improving robustness to uncertainty. However, all these were performed assuming constant eigenaxis rotation. In 1990, Bilimoria and Wie [14] showed that an eigenaxis rotation about one of the principal axes is not time optimal. This led to several studies [15] on inertia symmetry within the satellite and optimal axis of rotation

for time optimal maneuvers. Later, more practical aspects were studied based on slew rate constraints and actuator saturation limits [16] or uncertainty in the inertia tensor[17] with linearly perturbed state update matrices. Another strategy explored by McInnes [18] was the inverse control which involves a combination of feedforward and feedback terms in the control law. This allowed control law design relative to a trajectory rather than a single final state.

These closed loop correction strategies were prone to instabilities due to uncertainties in the flexible body and actuator finite rise time. Hence, often open loop maneuvers were preferred in practice. Commonly, bang-bang control is practiced to minimize the time to reach desired line of sight. However, certain mathematical functions like smooth polynomials are proposed [19],[20] to allow for accommodation of actuator finite rise time by the means of gradually increasing the actuator effort. These smooth maneuvers are also shown to reduce residual vibrations for a rigid/flexible body system with low damping[9, Section 3]. Thus, they are far more robust towards reducing excitations in the flexible structure regardless of their known model details. This thesis will combine the robustness of these smooth polynomial maneuvers with the trajectory tracking methodology discussed by McInnes[18].

In general, it is necessary to know the spacecraft attitude at all times for feedback correction. The large angle orientation can be represented either by a direction cosine matrix, Euler angles, Rodrigues Parameters or quaternions. For the first time, Quaternion-based feedback control laws were developed by Wie and Barba[21] in 1984. Quaternions [22],[23], allow the description of the spacecraft orientation in all possible attitudes. They have no inherent geometrical singularity; there are no singularities in the kinematical differential equations; and successive rotations follow the quaternion multiplication rules. Moreover, quaternions are well suited for onboard real-time computation since only products and no trigonometric relations exist in the quaternion equations.

1.3 Momentum control system - a suitable actuator

Momentum devices are characterized by rotating or translating masses. They have a wide range of applications. For example, mechanical gyroscopes allow storage of energy[24] in case of flywheels by attaching a motor/generator at the end of high inertia rotor. They are also used for designing passive dampers called inerters[25], which work with opposing forces proportional to acceleration unlike conventional viscous dampers with velocity based forces. MEMS gyros use linearly vibrating

mass elements [26] to measure the acceleration on the system.

In case of spacecraft systems, momentum devices are used to perform attitude maneuvers using rotating discs called *rotors*. These devices make a closed system with the spacecraft allowing for a momentum exchange, hence they are called Momentum Control Devices (MCDs). The SSPP spacecraft portrays periodicity of attitude[10] with net zero rotation at the end of orbit (Figure 1.4) based on its power optimal guidance. The equal and opposite sense of the maneuvers allow for momentum recovery in these devices, reducing the extra de-saturation operation via attitude thrusters. Moreover, the projected mission life of 10-20 years [5] makes momentum devices much more feasible compared to conventional attitude thrusters.

Commonly, there are two kinds of these devices, Reaction Wheels Assemblies (RWAs) and Control Moment Gyros (CMGs). The former category works with increasing or decreasing the speed of the spinning rotor to exchange momentum in the spin direction. CMGs on the other hand, may or may not have the same capability plus they can gimbal the spinning rotors about another set of axes to exchange momentum. Compared to RWAs, CMGs have very efficient torque production based on power metrics, exhibit torque amplification allowing torques $> 10 \text{ Nm}$, and provide high pointing accuracy [27, Ch 3]. Especially Single Gimbal CMGs (SGCMGs) or *gyrodynes* are a popular choice due to their high agility and slightly lower complexity than Dual Gimbal CMGs (DGCMGs). Commercially available CMGs target usage from medium to large satellites. A set of available, large CMGs is shown in Table 1.1.

CMG	Torque [Nm]	Angular Momentum [Nms]	Mass [kg]	Size [m × m × m]
Airbus CMG 75	75	75	69	$0.5 \times 0.5 \times 0.5$
Honeywell M50 CMG	75	75	28	$0.6 \times 0.4 \times 0.2$
Blue Canyon CMG-12	12	12	18	$0.3 \times 0.4 \times 0.3$

Table 1.1: CMG market survey.

Airbus CMG 75 has two high inertia spinning rotors side by side attached to a central bracket which can be independently gimballed. This allows for great torque authority in any direction on a plane with perpendicular defined by the gimbal axis (perpendicular to base). Honeywell M50 CMG has a single gimbal axis perpendicular to spin axis of the rotor at the center. Again, this allows for torque authority in any direction in a plane defined by a perpendicular line along the gimbal axis. Blue Canyon CMG-12 is quite different than these two. A high inertia spinning



Figure 1.5: Left to right, Airbus CMG 75, Honeywell M50 CMG, Blue Canyon CMG-12.

rotor is placed on a Stewart-platform base enabling angular momentum vector to be changed in all three directions. The restricted range of motion in the linear actuators results in limited movement of the spin axis on a portion of surface of an imaginary unit sphere. This creates higher torque authority in-plane compared to the direction perpendicular to base. Isolating the spinning rotor from the base mounting bracket allows for a possibility of passive/active damping through the linear actuators useful to reduce jitter effects from mass imbalance in the rotor.

1.4 Ground tests

In a study of 156 spacecraft[28], 32% turned out to be failures due to Attitude and Orbit Control System (AOCS) failures. Out of those, around 50% were attributed to the actuators used for control and the rest remain unresolved due to lack of information or improper validation tests. In 2005 NASA launched the Demonstration of Autonomous Rendezvous Technology (DART) mission inspired from previous, successful demonstrations of the technology from JAXA and Roscosmos during that time. The DART mission failed due to accelerated end of mission life. Investigations[29] pointed towards incorrect estimation of spacecraft position after a expected flight software reset, leading to control authority saturation in the very early phase of mission life. As a result, Mishap Investigation Board (MIB) recommended that a validation of all simulation and math models should be carried out in all upcoming missions.

Thus, it is sufficient to say that, ground based Hardware In Loop (HIL) experimental testing is vital for demonstrating closed loop stability. It requires accurate models of the actuator, spacecraft and the sensors to attain acceptable predictions. This is achieved by real-world experiments, investigating actuator working, validating control algorithms in the presence of sensor/environment noise, etc.

However, it poses a unique set of challenges. Computer simulations must include

a variety of nonlinear phenomena found in momentum devices, which fundamentally limits the way these simulations can be architected. And, because momentum systems rely on the conservation of angular momentum for proper operation, hardware-in-the-loop test facilities must reproduce the dynamics of a free body with great precision. There are several real world imperfections which should be accounted by the simulation based on the current phase of project development. The University of Michigan developed one of the early testbeds[30] (Figure 1.6, left) in 2005 to perform attitude hardware-in-loop simulations. These facilities can also provide a method to emulate the effect of robotic appendages on spacecraft dynamics, for e.g. the Cepheus facility[31] at National Technical University, Athens (Figure 1.6, mid). Another purpose could be to test docking dynamics of two spacecrafts, as studied at the M-STAR simulation facility[32] at Caltech (Figure 1.6, right).

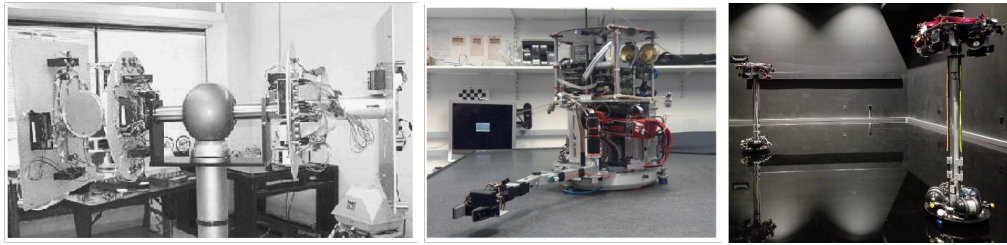


Figure 1.6: Left to right, University of Michigan's triaxial air bearing testbed, Robotics space emulator "Cepheus" operated at the National Technical university of Athens, M-STAR spacecraft simulator at Caltech.

1.5 Research objectives

The main objective of this thesis is to develop and demonstrate slew maneuvers which achieve faster pointing than state-of-the-art without exciting structural response in the next generation ultralight flexible spacecraft by the means of a momentum control system actuator specifically a Control Moment Gyroscope (CMG) array.

A flexible structure exhibits elastic deformations based on maneuver duration. A very slow maneuver results in very little or no deformations in a flexible body creating an almost rigid body rotation. The ratio of maneuver duration to the system's natural period ($\alpha = T/T_n$) is often used to design and classify slew maneuvers [33]. A heuristic practice for design of spacecraft slew maneuvers is $\alpha > 10$ or sometimes even $\alpha \approx 100$. However, for very large spacecraft like SSPP with very high natural period[6] $T_n \in [80 \text{ s}, 200 \text{ s}]$, this would result in very slow maneuvers with duration $T \in [30 \text{ minutes}, 5 \text{ hours}]$, reducing the geometric efficiency [10] of the SSPP

system. As a result, it becomes crucial to develop and test aggressive maneuvers with $\alpha < 10$ but also keep the residual vibrations in the structure to a minimum for coherent power transmission.

The computational tools developed in this thesis span from analysis of the flexible spacecraft response with a CMG actuator based on assumed physics-based models to an assumption-free real-world applicable identification framework allowing for robustness to the dynamic property variations of the system.

Then, the experimental procedures help identify challenges in 3-DOF testing of flexible body slew maneuvers with demonstration of attitude tracking using the developed closed-loop control law. As a motivating case study, the thesis focuses on next-in-line 10 m $\times$ 10 m version of Caltech Space Solar Power Project (SSPP) structural architecture[6] and a small-scale representative structure for experimental purposes.

The thesis addresses the following research objectives:

1. Theoretical formulation for flexible structure response to agile maneuvers accounting for actuator dynamics

Momentum control devices provide torque to the spacecraft by virtue of momentum exchange. As the momentum vector of the actuator changes in time, its time derivative results in the applied torque. Due to the local elastic deformation in the flexible structure near the actuator, the torque gets modified and results in a coupling between the two.

This thesis estimates the structural response of a flexible spacecraft structure accounting for this coupling with high fidelity modeling of thin lightweight composites.

2. Vibration-optimal agile slew maneuvers with assumed dynamic properties

It is possible to reduce structural oscillations based on assumptions about dynamic properties of flexible structures. *Flycast*[34] maneuvers were done in NASA's Space Radar Topography Mission (SRTM) shuttle with the X-band Synthetic Aperture Radar (SAR) antenna on a long boom. The lowest frequency of the boom was utilized to perform the maneuvers with a combination of pulses and delays at precise times allowing for net swift motion but no oscillations at the end of the maneuver.

However, by construction CMGs are unable to provide instantaneous changes in torque [27, Ch 3] because of their inability to provide instantaneous gimbal velocities and are thus more suitable for smooth variation of the setpoint torque function.

For these reasons, an input shaping framework adapted to momentum devices is proposed which not only results in smoother variations of maneuver angles but also retains the vibration suppression properties of the original impulse-based input shaping.

3. Data-driven vibration-optimal agile slew maneuvers

Real-world structures are imperfect. Their dynamic properties like stiffness and damping exhibit uncertainties. Thus, it is essential that they should be characterized before sending to space. However, very large spacecraft structures are difficult to test on ground due to limitations in the available gravity compensation procedures.

This is addressed by a novel system identification methodology which enables developing Reduced Order Models (ROM) of large flexible spacecraft and actuator on-the-go in space. A new set of observables is proposed accounting for various nonlinearities and the proposed methodology is used to optimize maneuvers for reducing elastic deformation in the flexible structure in the agile regime.

4. Experimental design of Hardware-In-Loop testbed to perform flexible body slew maneuvers

While extensive work[35][36] has been done for slew maneuver experiments with control moment gyroscopes, most of them allow only one Degree Of Freedom (DOF) rotational motion. There are others[37],[30],[38] with 3 DOF capability with a variety of gravity compensation techniques, but only for rigid body experiments. There has been no investigation of 3 DOF experiments with CMGs that addresses flexible body deformation during slew maneuvers.

There are several difficulties in designing such experiments. Firstly, real world flexible structure assemblies are not perfect and they show uncertain dynamic behaviors like change of mode frequencies, mode shapes, or damping etc. Secondly, the presence of gravity, in these experiments, makes flexible structures prone to buckling or structural instabilities. It is crucial to understand how these uncertainties combine with other sources of error like sensor noise, modeling error and control algorithm etc.

These challenges motivate the current work towards designing experimental procedures to simulate space-like conditions for demonstrating slew maneuvers with minimum interaction from the gravitational body loads and development of a compliant flexible structure robust to buckling.

1.6 Outline of the thesis

The remaining contents of this thesis are organized in six chapters. A brief summary of each chapter is provided below:

Chapter 2 provides an overview of the slew maneuvers considered and their properties in the context of flexible spacecraft. With assumed inertia, mode shapes, and linear superposition to describe the flexible spacecraft, a theoretical formulation is discussed to estimate response due to coupling effects of the actuator. Results for a $10\text{ m} \times 10\text{ m}$ Caltech's SSPP spacecraft are discussed.

Chapter 3 proposes equation-free data-centric methodology to approximate slew maneuver dynamics with measured data without any assumptions, thus addressing the real-world uncertainties in these systems. This framework is applied to the motivating $10\text{ m} \times 10\text{ m}$ Caltech's SSPP spacecraft example to estimate a Reduced-Order-Model (ROM). Then, this model is used in maneuver optimization to reduce elastic displacements and results are presented.

Chapter 4 presents design of a small-scale flexible structure, CMG array and relevant procedures for slew maneuver experiments. Proposed maneuvers from Chapter 2 are demonstrated and results compared.

Chapter 5 presents system identification results based on ROM proposed in Chapter 3 by disturbing the experimental prototype in closed-loop at different locations. Then, these ROMs are used to predict optimal maneuvers to decrease residual excitation in the flexible structure. These candidate maneuvers are then excited in the experiments and results are then compared to a baseline maneuver experiment.

Chapter 6 proposes a novel idea of integrating a momentum control device together with the existing deployment mechanism DOLCE[8] for volume efficient multifunctional actuator design for performing rest-to-rest maneuvers. Discussion on sizing of actuator is followed by efficient maneuver execution strategies and sensitivity studies with respect to initial conditions in the actuator.

Lastly, Chapter 7 summarizes the most important conclusions from the thesis and outlines possible directions for future research.

PHYSICS-BASED DYNAMICS MODELS

2.1 Flexible spacecraft model

Caltech's next-in-line SSPP spacecraft is proposed to be a $10\text{ m} \times 10\text{ m}$ that is similar to its previous counterpart DOLCE[8], although much larger. The structural architecture consists of composite ladder-like strips supporting pre-tensioned membranes with functional elements. These trapezoidal strips are arranged into four symmetric quadrants, to form concentric squares with a void at the center or hub, where the deployment mechanism is located.

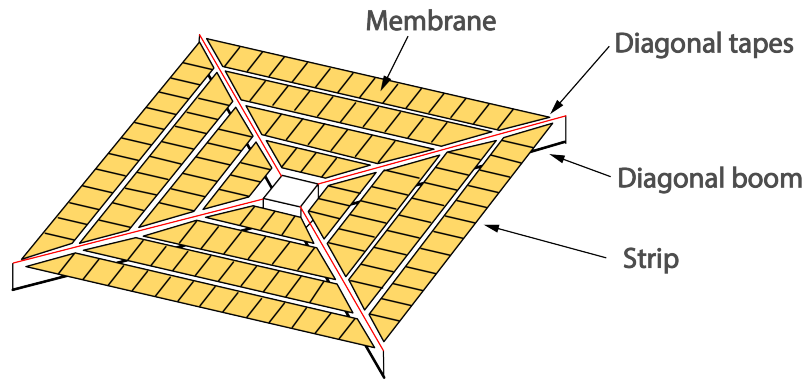


Figure 2.1: Next-in-line SSPP architecture.

Each strip is connected to two tapes running parallel to the diagonals on either side, forming a single quadrant. The array is supported by four diagonal booms deployed from a central mechanism. Recent work [6] on detailed finite element modeling provides us estimation of the natural frequencies and mode shapes of the system. For the purpose of slew maneuver dynamics, model M10-S4 is utilized with fully connected interface as described in [6]. This Abaqus FE model is then used to perform frequency analysis with free-free boundary conditions. First four elastic free-free mode shapes are shown in Figure 2.2.

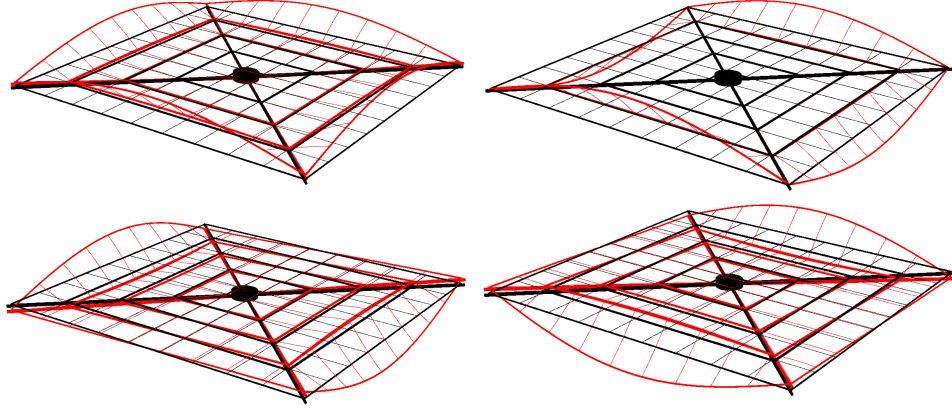


Figure 2.2: First four elastic mode shapes from top left - top right - bottom left - bottom right at 0.31 Hz, 0.33 Hz, 0.37 Hz and 0.37 Hz respectively.

2.2 Slew maneuvers

2.2.1 Baseline polynomial maneuver

Past research[20] shows that it is possible to achieve agility despite structural flexibility for ultralight flexible spacecraft by the means of tailoring the “shape” of the slew profile to decrease the excitation of the flexible modes. A smooth polynomial slew in angle ψ of degree seven $\mathbb{P}^7$ is known to perform well for reducing residual vibrations in the flexible structure. It is defined as follows:

$$\theta(t) = \sum_{k=0}^7 a_k t^k \quad (2.1a)$$

$$a_7 = -20\Delta\theta/T^7 \ ; \ a_6 = 70\Delta\theta/T^6 \ ; \ a_5 = -84\Delta\theta/T^5 \ ; \ a_4 = 35\Delta\theta/T^4 \quad (2.1b)$$

This is the lowest-order polynomial that can simultaneously satisfy boundary conditions on ψ and its velocity, acceleration, and jerk (the time derivative of acceleration) shown in Equation 2.2.

$$\psi(0) = 0 \ ; \ \dot{\psi}(0) = 0 \ ; \ \ddot{\psi}(0) = 0 \ ; \ \dddot{\psi}(0) = 0 \quad (2.2a)$$

$$\psi(T) = \Delta\psi \ ; \ \dot{\psi}(T) = 0 \ ; \ \ddot{\psi}(T) = 0 \ ; \ \dddot{\psi}(T) = 0 \quad (2.2b)$$

This maneuver will be used as baseline for comparing results with other techniques.

2.2.2 Ninth degree polynomial maneuver

Instead of fully defining the polynomial with the initial and the terminal conditions, one can also add more terms allowing for further flexibility in the shape change.

For example a ninth degree polynomial $\mathbb{P}^9$ can be used in combination with the conditions in Equation 2.2 to enable a simplest two d.o.f. basis for maneuver optimization purposes:

$$\theta(t) = \sum_{k=0}^9 a_k t^k \quad (2.3)$$

2.2.3 Input shaped polynomial maneuver

It is possible to reduce the oscillations, based on a system's dynamic characteristics. One can either perform a closed-loop feedback from the structure's strain measurements to develop the control law, or change the setpoint itself knowing the dynamic properties in advanced. This is called *input shaping*. An early form of modifying inputs was used in posicast control by Smith[39]. It involves breaking a step into two smaller steps, delayed in time, allowing for reduced settling time. An advanced version of this technique, called *Flycasting*[34] was done in NASA's Space Radar Topography Mission (SRTM) maneuvers with the X-band Synthetic Aperture Radar (SAR) antenna on a long boom. Maneuvers were performed by three pulses and two delays in between, using cold gas thrusters. However, by construction CMGs are unable to provide instantaneous change in torque [27, Ch 3] because of their inability to provide instantaneous gimbal velocities and are thus more suitable for smooth variation of the setpoint torque function.

Frequency shaping techniques were proposed by Gupta[40], and Junkins and Turner[41], involving derivative of the control input in a penalty function to produce smooth functions. Experiments were performed on near-minimum-time slew maneuver using combination of optimal control and root strain feedback[42] assuming a single dominant mode in the structure. It was shown to work well with the closed-loop feedback on the root strain. This technique exhibits robustness in presence of external disturbances but cannot accommodate system identification errors. A more advanced version of posicast control was developed by Singer and Seering[43], which allowed a combination of impulses (two or more) to reduce tracking errors and post-maneuver oscillations at the same time. They also showed that one can shape an already existing baseline maneuver using combination of these impulses with the help of signal convolution.

Thus, this technique is suitable on the present CMG because it allows modification of the already existing smooth polynomial maneuvers. Moreover, the properties of the original maneuver remain intact i.e. zero jerk, zero acceleration, zero velocity

at the terminal point of the maneuver. Hence, the resulting torque function remains smooth throughout the maneuver, making it possible to be executed using CMG. The simplest form of input shaping, called Zero-Velocity(ZV) shaper, which uses sum of two impulses, was adopted, assuming a single dominant mode of the system at ω_n and damping ratio of ζ . With this known information, one can superimpose the solutions from two impulses to achieve a net positive motion with no vibration after the input has ended

$$A_1 = \frac{1}{1+K} \quad ; \quad t_1 = 0 \quad (2.4)$$

$$A_2 = \frac{K}{1+K} \quad ; \quad t_2 = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \quad (2.5)$$

Where ,

$$K = e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}} \quad (2.6)$$

Applying convolution of these impulses with the smooth polynomial acceleration $\alpha(t) = \ddot{\psi}(t)$

$$\alpha_{is}(t) = \int_{s=-\infty}^{s=\infty} \alpha(s)(\delta_1(t-s) + \delta_2(t-s))ds \quad (2.7)$$

$$= \int_{s=-\infty}^{s=\infty} \alpha(s)(A_1\delta(t-t_1-s) + A_2\delta(t-t_2-s))ds \quad (2.8)$$

$$= A_1 H(t-t_1)\alpha(t-t_1) + A_2 H(t-t_2)\alpha(t-t_2) \quad (2.9)$$

2.3 Flexible structure dynamics

This section will develop a numerical framework to establish dynamics of a flexible structure using *mode displacement method*[44]. Instead of the absolute coordinates to describe the flexible body, it is possible to decouple the dynamics equations of motion by utilizing appropriate transformation to *principal coordinates*. Consider a momentum control actuator placed at the center of the spacecraft. Assuming the flexible body to be a multi-degree of freedom linear time invariant system with mass matrix $[\mathbf{M}]$ including flexible structure, actuator mass, and actuator inertia. Damping is assumed to be viscous with matrix $[\mathbf{C}]$ and stiffness matrix $[\mathbf{K}]$. These matrices describe motion of the system in *physical coordinates* $\mathbf{u} = [\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_N]$ at N number of nodes in Equation 2.10 relative to inertial frame $\mathcal{O}$. Body fixed frame $\mathcal{B}$ resides on the center of the spacecraft. External torque and loads are represented by vector f . Each coordinate has 6 DOF $\mathbf{q}_i = [x_i, y_i, z_i, \theta_i^x, \theta_i^y, \theta_i^z]$ (position

vector $\mathbf{x}$, finite rotation vector $\boldsymbol{\theta}$ [45]):

$$[M]\ddot{u} + [C]\dot{u} + [K]u = f \quad (2.10a)$$

$$u(0) = u_o \quad (2.10b)$$

$$\dot{u}(0) = v_o \quad (2.10c)$$

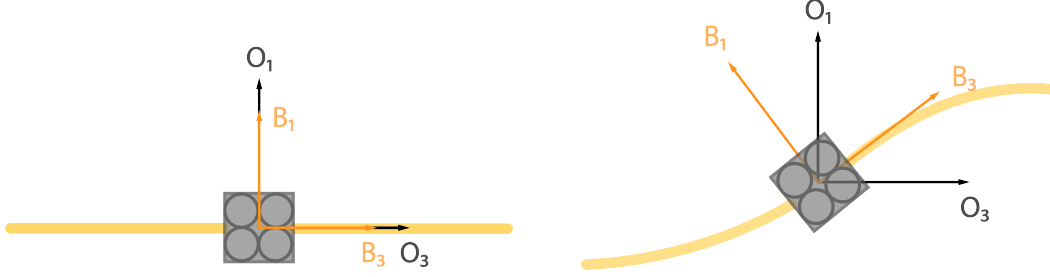


Figure 2.3: Flexible spacecraft with actuator at the center. Configuration at time $t = 0$ (left) and $t > 0$ (right).

2.3.1 Kinematics

Given the finite rotation angles of a node with position vector A relative to frame O , its location a relative to frame B can be calculated using the exponential map[46] $[\Lambda]$.

$$a = [\Lambda]A \quad (2.11)$$

$$[\Lambda] = \exp([\theta]_{\times}) = \cos\|\theta\|\mathbb{1} + \frac{\sin\|\theta\|}{\|\theta\|}[\theta]_{\times} + \frac{1 - \cos\|\theta\|}{\|\theta\|^2}\theta\theta^T \quad (2.12)$$

where $[\Lambda]$ is the rotation matrix and $[\theta]_{\times}$ is the skew symmetric matrix given by :

$$[\theta]_{\times} = \begin{bmatrix} 0 & -\theta_3 & \theta_2 \\ \theta_3 & 0 & -\theta_1 \\ -\theta_2 & \theta_1 & 0 \end{bmatrix} \quad (2.13)$$

Instantaneous angular velocity at any node can be acquired from finite rotation vector rates ($\dot{\theta}$) using the (symmetric) Hessian[47] [45]

$$\Omega = [H(\theta)]\dot{\theta} \quad (2.14a)$$

$$[H(\theta)] = \frac{1}{\|\theta\|^2}\theta\theta^T + \frac{\sin\|\theta\|}{\|\theta\|} \left[\mathbb{1} - \frac{1}{\|\theta\|^2}\theta\theta^T \right] + \frac{1 - \cos\|\theta\|}{\|\theta\|^2}[\theta]_{\times} \quad (2.14b)$$

2.3.2 Mode-displacement method

Assuming the spacecraft is free floating in the space with free-free boundary condition, it will exhibit rigid body modes. One can transform the system into principal coordinates using modal superposition with these modes as basis to describe the dynamics. This is achieved by *assuming that mode shapes remain constant* for the required range of deformations:

$$u = u_r + u_e = \begin{bmatrix} \Phi_r & \hat{\Phi}_e \end{bmatrix} \begin{bmatrix} \eta_r(t) \\ \eta_e(t) \end{bmatrix} = \Phi \eta \quad (2.15)$$

It is crucial to take all the rigid body modes and truncate to $\hat{N}_e$ elastic modes significant for response calculation stored in $\hat{\Phi}_e$. One can substitute the transformation map in Equation 2.15 in the equation of motion in Equation 2.10 and pre-multiply by $[\Phi]^T$ to get

$$\Phi^T [M] \Phi \ddot{\eta} + \Phi^T [C] \Phi \dot{\eta} + \Phi^T [K] \Phi \eta = \Phi^T f \quad (2.16)$$

Assuming *mass normalized mode shapes*, pre-multiplying by $[\Phi^T [M] \Phi]^{-1}$ in Equation 2.16 and by $\Phi^T [M]$ in Equation 2.15 yields two decoupled systems of equations [44, Ch 11] for rigid and elastic displacements each and appropriate initial conditions in principal coordinates, as follows:

$$\ddot{\eta}_r = \Phi_r^T f \quad (2.17a)$$

$$\ddot{\eta}_e + [\bar{C}] \dot{\eta}_e + [\bar{K}] \eta_e = \Phi_e^T f \quad (2.17b)$$

$$\eta(0) = \Phi^T [M] \mathbf{u}_o \quad (2.17c)$$

$$\dot{\eta}(0) = \Phi^T [M] \mathbf{v}_o \quad (2.17d)$$

where ,

- $[\bar{C}] = (\Phi^T [M] \Phi)^{-1} \Phi^T [C] \Phi = \text{diag}(2\zeta_7\omega_7, 2\zeta_8\omega_8, \dots)$ (only in case of modal damping)
- $[\bar{K}] = (\Phi^T [M] \Phi)^{-1} \Phi^T [K] \Phi = \text{diag}(\omega_7^2, \omega_8^2, \dots)$

Modal superposition for rigid body modes can be applied to a single node and then extrapolated later across the complete geometry. Center of the spacecraft is selected as this reference node (r). The extrapolation is as follows

$$\mathbf{x}(t) = \mathbf{x}_0 + \Lambda(\theta^{(r)}(t))[\mathbf{x}(0) - \mathbf{x}^{(r)}(0)] \quad (2.18a)$$

$$\theta(t) = \theta^{(r)}(t) \quad (2.18b)$$

2.3.3 State space formulation

The eigenvalue problem for decoupled system of equations is quadratic. One can develop a state space Linear Time Invariant (LTI) system from Equation 2.17. We follow an approach similar to [48] with state vector consisting of mode amplitudes η and velocities $\dot{\eta}$. We can write two LTI systems, one for rigid body motion and the other for elastic motion of the structure.

$$\dot{z}^e = [P]^e z^e + [Q]^e f_z \quad (2.19a)$$

$$\dot{z}^r = [P]^r z^r + [Q]^r f_z \quad (2.19b)$$

where,

$$[P]^e = \begin{bmatrix} \mathbb{0} & \mathbb{1} \\ -[\bar{K}] & -[\bar{C}] \end{bmatrix}, \quad [Q]^e = \begin{bmatrix} \mathbb{0} & \mathbb{0} \\ \mathbb{0} & (\Phi^e)^T \end{bmatrix}, \quad z^e = \begin{bmatrix} \eta^e \\ \dot{\eta}^e \end{bmatrix}, \quad f_z = \begin{bmatrix} \mathbb{0} \\ f \end{bmatrix} \quad (2.20a)$$

$$[P]^r = \begin{bmatrix} \mathbb{0} & \mathbb{1} \\ \mathbb{0} & \mathbb{0} \end{bmatrix}, \quad [Q]^r = \begin{bmatrix} \mathbb{0} & \mathbb{0} \\ \mathbb{0} & (\Phi^r)^T \end{bmatrix}, \quad z^r = \begin{bmatrix} \eta^r \\ \dot{\eta}^r \end{bmatrix}, \quad f_z = \begin{bmatrix} \mathbb{0} \\ f \end{bmatrix} \quad (2.20b)$$

After solving the two LTI systems in η^r and η^e , displacement in absolute coordinates is:

$$u = (\Phi^r)\eta^r(t) + (\Phi^e)\eta^e(t) \quad (2.21)$$

A momentum actuator is mounted at the center of the spacecraft and its goal is to provide the rigid body maneuver torque N_r ,

$$N_r = [0, I_{22}\ddot{\psi}, 0] \quad (2.22)$$

where ψ is the seventh degree polynomial maneuver defined in Equation 2.1 and inertia of the spacecraft in relaxed/reference state I_2 . Zero modal damping is assumed in the model for all modes:

$$\zeta_i = 0.001 \quad \forall i \in \{7, 8, 9, \dots\}$$

2.3.4 Coupling terms

One can use Newton's method to determine the coupling term. Consider the actuator used to perform the maneuvers to be System-I and the rigid hub altogether with the flexible structure as System-II. Actuator angular momentum " H_a " is written in frame $\mathcal{B}$ fixed on the hub rotating with " Ω_c " with respect to an inertial frame $\mathcal{O}$.

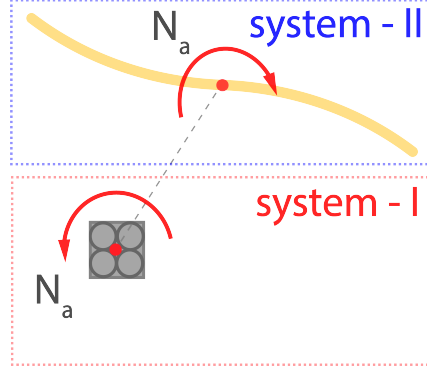


Figure 2.4: Torque generated by the actuator is applied to the rigid hub of the flexible structure.

The net torque exchange on the central hub is the derivative of the total angular momentum H_a of actuator (System-I)

$$N_a = \dot{H}_a + \Omega_c \times H_a \quad (2.23a)$$

where Ω_c can be calculated using Hessian matrix H from Equation 2.14

$$\Omega_c = H(\theta_c) \dot{\theta}_c \quad (2.24)$$

Hence, actuator torque becomes a function of local angles θ_c and angle derivatives $\dot{\theta}_c$

$$N_a(H_a, \dot{H}_a, \theta_c, \dot{\theta}_c) = \dot{H}_a + [H(\theta_c)] \dot{\theta}_c \times H_a \quad (2.25)$$

Rotation degrees of freedom at point C are function of local rotational mode shape Φ_c^θ , mode amplitudes $\eta(t)$, velocities $\dot{\eta}(t)$.

$$\theta_c = \Phi_c^\theta \eta ; \quad \dot{\theta}_c = \Phi_c^\theta \dot{\eta} \quad (2.26)$$

Thus the instantaneous actuator torque is as follows:

$$N_a(H_a, \dot{H}_a, \eta, \dot{\eta}, \Phi_c^\theta) = \dot{H}_a + [H(\Phi_c^\theta \eta)] \Phi_c^\theta \dot{\eta} \times H_a \quad (2.27)$$

To establish angular momentum and its derivative, we need state of angular momentum device δ . This is achieved by assuming rigid body maneuver ψ from Equation 2.22 and using the inverse dynamics discussed in Subsections 2.5.1 and 2.5.2.

This state δ and its derivative $\dot{\delta}$ when applied in the momentum device at the central hub excites the mode amplitudes η and mode velocities $\dot{\eta}$ as per the mass-normalized

mode shape Φ of the flexible body:

$$\ddot{\eta} + [\bar{C}]\dot{\eta} + [\bar{K}]\eta = -\Phi^T \begin{bmatrix} \mathbb{0} \\ N_a(\delta, \dot{\delta}, \eta, \dot{\eta}, \Phi_c^\theta) \end{bmatrix} \quad (2.28)$$

As mentioned earlier, free-free mass normal mode shapes of a 10 m x 10 m SSPP are utilized. Previous studies[6] provide a finite element model of the SSPP spacecraft with highly detailed flexible composite structure elements with their layup stiffness matrices and various preloads studies in the diagonal boom elements. Mode shapes from 10 m, fully connected structure are chosen for this analysis. Modal truncation is performed at 50 mode shapes by the means of convergence of simulation results. The first few flexible mode shapes are shown in the Figure 2.2.

2.3.5 Results for smooth maneuver

A smooth polynomial maneuver $\mathbb{P}^7$ of duration $T = 16$ s and $\Delta\psi = 90^\circ$ is simulated. The natural time period of the structure is $T_n = 3.2$ s. The time of simulation is twice the maneuver duration to observe dynamics post maneuver as well. Peak modal amplitudes η_p and velocities $\dot{\eta}_p$ are estimated over the entire simulation. These are shown in Figure 2.5. Two points at different locations (Figure 2.6, left) are observed on the spacecraft for elastic displacements. A time-history of these are shown in Figure 2.6, right. This maneuver excites specific mode shapes more than others called *dominant modes*. They are shown in Figure 2.8. This occurs because the chosen maneuver has more projection towards certain directions in the chosen basis of mass-normalized mode shapes.

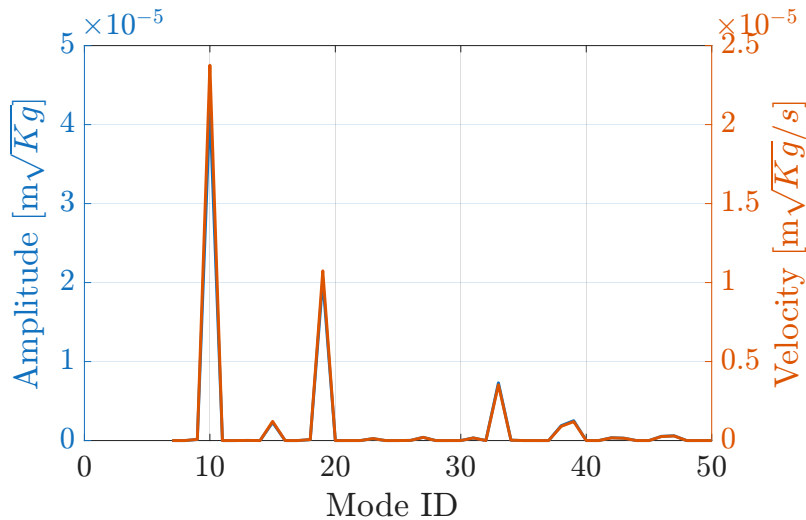


Figure 2.5: Peak mode amplitudes and velocities.

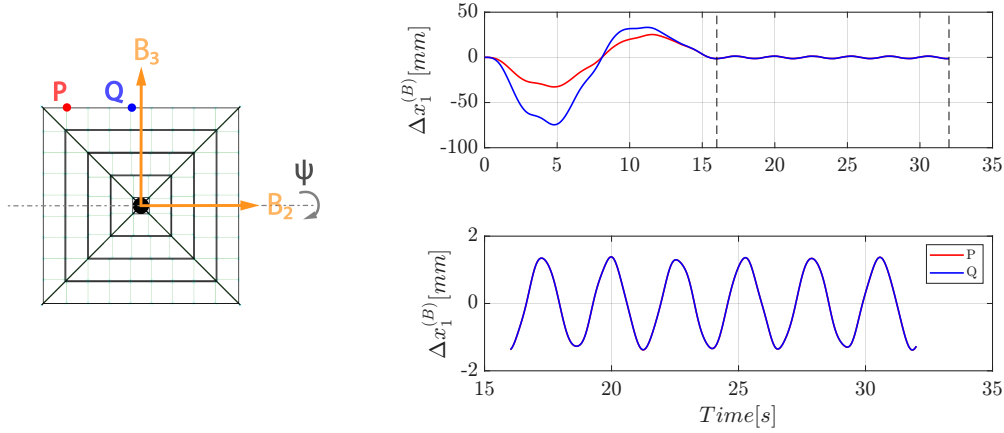


Figure 2.6: Out-of-plane elastic displacements at points P and Q.

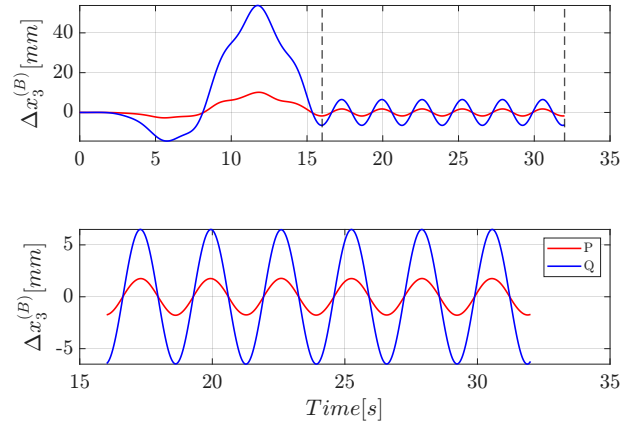


Figure 2.7: In-plane elastic displacements at points P and Q.

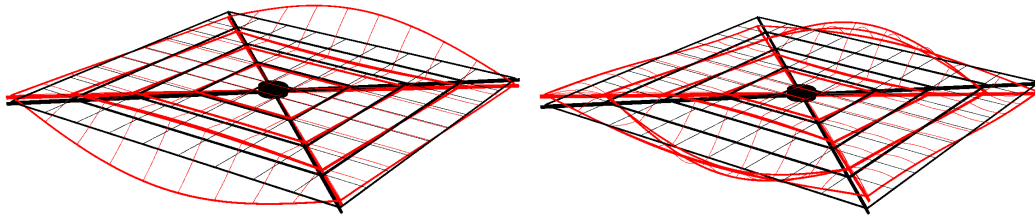


Figure 2.8: Dominant mode shapes, flexible mode 4 (left) and mode 13 (right) at 0.37 Hz and 0.65 Hz respectively.

2.4 Control scheme

A 3 DOF trajectory tracking problem in attitude is formulated for the actuator-spacecraft system. Usually, this is achieved with feedback from current state of

spacecraft allowing for the momentum device to act as per the error in state as shown in Figure 2.9. We use inverse dynamics[49] and quaternion/angular rates feedback together to ensure that the spacecraft follows the desired trajectory $\mathbf{p}_d$, $\mathbf{\Omega}_d$ i.e. smooth polynomial maneuver about body axis B_2

$$\mathbf{p}_d = \begin{bmatrix} 0 \\ \sin\left(\frac{\theta(t)}{2}\right) \\ 0 \\ \cos\left(\frac{\theta(t)}{2}\right) \end{bmatrix}; \quad \mathbf{\Omega}_d = \begin{bmatrix} 0 \\ \dot{\psi}(t) \\ 0 \end{bmatrix} \quad (2.29)$$

Using the nonlinear inverse control scheme allows perturbation in the state to converge to the reference trajectory rather than a reference state. The subscripts $()_d$ and $()_e$ represent desired and error signal in consideration. Spacecraft state $\mathbf{y}$ consists of angular velocity vector and the quaternion $[\mathbf{\Omega}, \mathbf{p}]$ For the purpose of our discussion, we will consider closed loop control of spacecraft attitude and assume that the momentum exchange actuator state δ is perfectly measurable and controllable i.e. $\delta = \delta_d$. Depending on over/under-actuation in the actuator to control attitude degrees of freedom, a steering law is utilized to develop relationship between error torque τ_e and actuator state δ . More discussion about steering law in Section 2.5. The closed loop quaternion control schematic is shown in Figure 2.9.

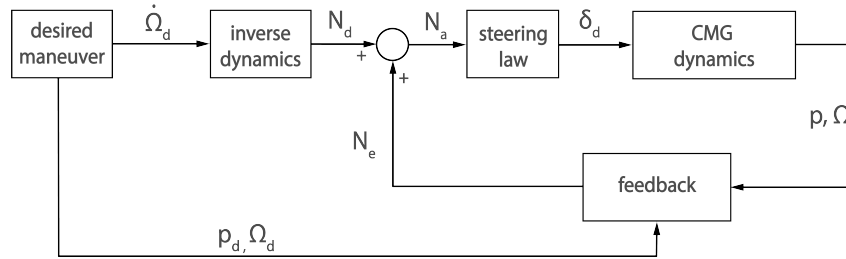


Figure 2.9: Closed loop quaternion control.

We use Hamiltonian right quaternions with unit magnitude $\mathbf{p} \in \mathbb{H}_1 = [\mathbf{p}_v \ p_s]$ with conjugate $\mathbf{p}^* = [-\mathbf{p}_v \ p_s]$. The quaternion product, denoted by $\otimes$, is used to implement rotations. Given quaternions $\mathbf{p}_1, \mathbf{p}_2 \in \mathbb{H}_1$, the quaternion product is defined as

$$\mathbf{p}_2 \otimes \mathbf{p}_1 = \mathbf{L}(\mathbf{p}_2)\mathbf{p}_1 \quad (2.30)$$

$$\mathbf{L}(\mathbf{p}_2) = \begin{bmatrix} p_s \mathbb{1}_{3 \times 3} + [\mathbf{p}_v]_{\times} & \mathbf{p}_v \\ -\mathbf{p}_v^T & p_s \end{bmatrix} \quad (2.31)$$

Where, $[\mathbf{v}]_{\times}$ is the skew symmetric matrix of a vector $\mathbf{v}$

$$[\mathbf{v}]_{\times} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \quad (2.32)$$

Procedure in [49] develops the control scheme by explicitly differentiating the constraint in terms of finite rotation angles relative to the desired trajectory of rotation angles. We develop a similar formulation with the constraint $\mathbf{s}(t) \in \mathbb{H}_1$ in terms of the desired quaternion $\mathbf{p}_d$ and the measured quaternion $\mathbf{p}$ instead. We chose our constraint

$$\mathbf{s}(t) = \mathbf{p}_d^*(t) \otimes \mathbf{p}(t) - \mathbb{1}_p \quad ; \quad \mathbf{s} \in \mathbb{H}_1 \quad (2.33)$$

Quaternions $\mathbf{p}_d$ and $\mathbf{p}$ can be propagated by respective angular rates $\mathbf{\Omega}_d$ and $\mathbf{\Omega}$ in the body frame as shown in [23] ,[50]. For example, the rate of change of the current quaternion

$$\dot{\mathbf{p}} = \frac{1}{2} \hat{\mathbf{\Omega}} \otimes \mathbf{p} \quad (2.34)$$

where, $\hat{\mathbf{\Omega}} \in \mathbb{H}_p$ is the pure quaternion constructed from the body fixed frame angular rate vector $\mathbf{\Omega}$ as $\hat{\mathbf{\Omega}} = \begin{bmatrix} \mathbf{\Omega} \\ 0 \end{bmatrix}$. Utilizing the product rule for derivatives and the aforementioned propagation of each quaternion as shown in Equation 2.34, the time derivatives of the constraint in Eqn-2.33 are calculated:

$$\dot{\mathbf{s}}(t) = \frac{1}{2} \mathbf{p}_d^* \otimes (\hat{\mathbf{\Omega}} - \hat{\mathbf{\Omega}}_d) \otimes \mathbf{p} \quad (2.35)$$

$$\ddot{\mathbf{s}}(t) = \frac{1}{2} \mathbf{p}_d^* \otimes \left[(\dot{\hat{\mathbf{\Omega}}} - \dot{\hat{\mathbf{\Omega}}}_d) + \frac{1}{2} (\hat{\mathbf{\Omega}} - \hat{\mathbf{\Omega}}_d) \otimes \hat{\mathbf{\Omega}} - \frac{1}{2} \hat{\mathbf{\Omega}}_d \otimes (\hat{\mathbf{\Omega}} - \hat{\mathbf{\Omega}}_d) \right] \otimes \mathbf{p} \quad (2.36)$$

Again , following the technique in [49] , instead of enforcing $\ddot{\mathbf{s}}(t)$ to be null, feedback terms are added using the lower derivatives of the constraint times the constant gains matrices λ_1, λ_2 as follows

$$\ddot{\mathbf{s}} + \lambda_1 \dot{\mathbf{s}} + \lambda_2 \mathbf{s} = \mathbb{0} \quad ; \quad \lambda_1, \lambda_2 \in \mathbb{M}^{4 \times 4}, \mathbf{s} \in \mathbb{H}_1 \quad (2.37)$$

Using equations-6.5,2.35,2.36 and substituting in 2.37 we can get a relationship between actuator torque and lower derivatives of the constraint. The vector part of

the final equation gives the actuator control law for our attitude trajectory tracking as follows

$$\mathbf{N}_a = \underbrace{\mathbf{I}\dot{\boldsymbol{\Omega}}_d + \mathbf{I}[\boldsymbol{\Omega}_d]_{\times}\boldsymbol{\Omega} + [\boldsymbol{\Omega}]_{\times}\mathbf{I}\boldsymbol{\Omega}}_{\text{feedforward}} - \underbrace{\mathbf{I}[\lambda_1(\boldsymbol{\Omega} - \boldsymbol{\Omega}_d) + 2\lambda_2(\mathbf{p}_d \otimes \mathbf{p}^*)]}_{\text{feedback}} \quad (2.38)$$

This control law consists of two kinds of terms. The first few are grouped together in the feedforward part of the control or $N_a^{(ff)}$, which come from the inverse dynamics. The second set of terms are grouped as the feedback or $N_a^{(fb)}$, which come from the constant gain times the lower derivatives of the constraints. Diagonal gain matrices are used. Diagonal values are selected based on assuming the second order error dynamics system to a damped second order system:

$$(\lambda_1)_{kk} = 2\zeta_{cl}\omega_{cl} \quad (2.39a)$$

$$(\lambda_2)_{kk} = \omega_{cl}^2 \quad (2.39b)$$

2.5 Control moment gyroscope equations of motion

A frequently used configuration is four CMGs arranged in a pyramid shape [51]. This configuration is selected because its torque envelope is close to a sphere allowing nearly uniform actuation in any direction [52]. The shape of this torque envelope can be adjusted by the means of skew angle β shown in Figure 2.10, increasing β results in an increase momentum exchange capability about B_3 axis and vice versa. The design parameter stored angular momentum H_a is designed to perform a zero-jerk smooth polynomial maneuver 2.1 based on maneuver momentum and torque, more discussion in Section 4.4.1.

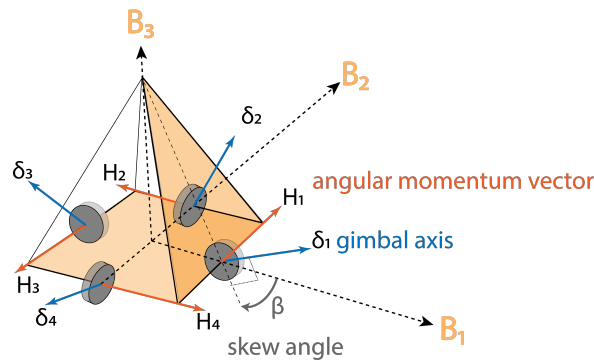


Figure 2.10: Pyramid CMG nomenclature.

2.5.1 Theoretical model of actuator

To develop a theoretical model, assume a body fixed cartesian coordinate frame B. The pyramid CMG consists of four rotors spinning at constant angular velocity

ω . The angular momentum magnitude is $h_o = J\omega$ for each rotor. Each angular momentum vector H_i can rotate on the surface of an imaginary pyramid, as shown in Fig. 2.10, along its respective gimbal axis δ_i . To perform a maneuver, one can change each of the four $\delta_i(t)$.

To develop a solution, one starts by writing the total angular momentum of the system H_T as the sum of the angular momentum of the supporting frame of inertia $\mathbf{I}$ and that of the CMG:

$$H_T = \mathbf{I}\Omega + H_a \quad (2.40)$$

where $\mathbf{I}$ is the inertia tensor for the assembly rotating with angular velocity Ω in frame B and h is the angular momentum of the CMG. For a closed system with no external loads, the angular momentum is conserved:

$$\frac{dH_T}{dt} = \dot{H}_T + \Omega \times H_T = 0 \quad (2.41)$$

and hence,

$$\mathbf{I}\dot{\Omega} + \dot{H}_a + \Omega \times (\mathbf{I}\Omega + H_a) = 0 \quad (2.42)$$

The angular momentum of the CMG actuator H_a can be written as a function of the gimbal angles δ_i and the pyramid angle β as follows

$$H_a = \sum_k^4 H^{(k)} = \underbrace{\begin{bmatrix} C_\beta S_{\delta_1} & C_{\delta_2} & -C_\beta S_{\delta_3} & -C_{\delta_4} \\ -C_{\delta_1} & C_\beta S_{\delta_2} & C_{\delta_3} & -C_\beta S_{\delta_4} \\ S_\beta S_{\delta_1} & S_\beta S_{\delta_2} & S_\beta S_{\delta_3} & S_\beta S_{\delta_4} \end{bmatrix}}_{\mathbf{B}^{(p)}} \underbrace{\begin{bmatrix} h_o \\ h_o \\ h_o \\ h_o \end{bmatrix}}_{\mathbf{H}^o} \quad (2.43)$$

and the time derivative can be calculated using the Jacobian of equation 2.43

$$\dot{H}_a = \underbrace{\begin{bmatrix} C_\beta C_{\delta_1} & -S_{\delta_2} & -C_\beta C_{\delta_3} & S_{\delta_4} \\ S_{\delta_1} & C_\beta C_{\delta_2} & -S_{\delta_3} & -C_\beta C_{\delta_4} \\ S_\beta C_{\delta_1} & S_\beta C_{\delta_2} & S_\beta C_{\delta_3} & S_\beta C_{\delta_4} \end{bmatrix}}_{\mathbf{A}^{(p)}} \begin{bmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \\ \dot{\delta}_4 \end{bmatrix} h_o \quad (2.44)$$

The matrices $\mathbf{B}^{(p)}$ and $\mathbf{A}^{(p)}$ are the *Pyramid configuration* and the *Pyramid Jacobian* matrices which depend on the orientation of the each CMG in the cluster. Using equations 2.40 through 2.44, one can develop a nonlinear system of ordinary differential equations in the state $\delta = [\delta_1 \ \delta_2 \ \delta_3 \ \delta_4]^T$. Given the maneuver angular

velocities Ω and angular accelerations $\dot{\Omega}$ with chosen initial values of gimbals $\delta(0)$, gimbal angle variation can be estimated. This requires solving for four gimbal angles using three equations from Equation 2.44. As the *Jacobian* loses rank, the gimbal rates necessary to produce a finite torque approach infinity. This describes the onset of a singularity in the CMG. Singularities are primarily classified in two different types. Saturation singularities at the limit of the array capability are common to any array. Such singularities, also known as *external singularities*, are accommodated in the sizing of the control system and cannot be avoided by numerical regularization techniques. Another class of singularities called *internal singularities* are of much greater importance. They arise by unique combination of gimbal orientations due to cancellation of contribution from each CMG in the array leading to net zero angular momentum. No incremental motion in any of the gimbal can recover from this state. Hence the notion of avoiding these states is important.

2.5.2 Steering algorithms

CMG steering algorithms[27][Ch 7] enable to pick a unique choice from these set of solutions based on required torque. Two different steering algorithms are implemented in the current work depending on theoretical modeling or hardware implementation.

1. Moore–Penrose inverse steering:

The most commonly used steering law in the literature is based on pseudoinverse of the Jacobian called the Moore–Penrose inverse. This is also known as the "perfect torque tracking" law where the sole objective is to reduce the norm of $\dot{\delta}$ given the solution exists. The biggest downside is that it is incapable of avoiding or escaping singularities and does not accommodate constraints on gimbal angles, rates, or accelerations. This makes it a bad choice for real world implementations but ideal for theoretical controlled simulations

$$\dot{\delta} = (A^{(p)})^T (A^{(p)} (A^{(p)})^T)^{-1} \dot{h} \quad (2.45)$$

2. Singularity escape steering:

Singularity-escape algorithms allow passing through the singularities and recovering with minimal (but not necessarily zero) deviation from the commanded torque. They work on the addition of a scalar or matrix enabling numerical conditioning of the matrix inverse. They are inexact mappings from commanded torque to gimbal rates. Several algorithm exist for the escaping singularities in single gimbal

CMGs. Singularity Robust inverse steering[53] regularizes the singular values of the matrix $A^{(p)}(A^{(p)})^T$ at the singularity so that the matrix to be inverted remains positive definite which makes it a good choice for experimental implementation. The steering law is described as follows:

$$\dot{\delta} = (A^{(p)})^T [A^{(p)}(A^{(p)})^T + \alpha_o e^{-\mu\sigma}]^{-1} \dot{h} \quad (2.46)$$

where, $\sigma = \sqrt{|A^{(p)}(A^{(p)})^T|}$ is called the singularity index. It provides a measure of the distance of the CMG array from singularity.

2.6 Discussion

This chapter has formulated a numerical model to estimate flexible spacecraft response accounting for coupling emanating from the momentum device by Newton's third law. Free-free mass normalized mode shapes are used to decoupled flexible structure deformations. Main findings include estimation of spatial and temporal variation of elastic displacements throughout the structure and identification of dominant modes for jerk-smooth polynomial maneuver of seventh degree. These results help in design improvement of next-generation 10 m $\times$ 10 m SSPP spacecraft structure and are relevant to target vibration-suppression techniques if required. The main takeaways are that the structural response depends on free-free mode shapes of the architecture and the deformation might not be dominated by lowest eigenvalue elastic mode of the system. Moreover, actuator-flexible structure coupling does not significantly change the dynamics in case of constant eigenaxis rest-to-rest maneuvers.

While the results focus on a specific class of architectures called *Ultralight flexible spacecraft*[9], the developed framework is agnostic to the type of flexible body, or actuator, or kind of control algorithm and it is robust with respect to finite element modeling in software of choice.

Different kind of maneuvers discussed in this chapter will be utilized in the upcoming Chapters 3, 4 and 5 to study structural response during slews. Pyramid CMG equations of motion, respective steering laws and closed-loop quaternion feedback trajectory tracking will all be used in real-world slew maneuver experiments of a small-scale flexible structure in Chapters 4 and 5.

Chapter 3

DATA-BASED DYNAMICS MODELS

Lightweight composites provide a great opportunity to develop larger and larger spacecraft. The Hubble Space Telescope, the Starshade, and the Advanced Composite Solar Sail Spacecraft are all examples of highly volume efficient deployable spacecraft with large surface area enabling challenging goals in contrast to conventional satellites. However, when it comes to testing of these spacecraft, only subsystem level testing or a small part of the big assembly is performed with space-like conditions. More specifically, attitude dynamics of such spacecraft is often analyzed from finite element models of flexible bodies which are governed by assumed deterministic design choices and measurements with uncertainties. These models are often corrected based on trial and error from the on orbit data collection and rely on heritage based design evolution. This demands several spacecraft of similar nature to be flown before converging to a robust design.

However, for a very large spacecraft like SSPP, multiple trials are not feasible economically. Moreover, the dynamic properties of the structure are prone to uncertainties either due to slight changes in the final shape in space through deployment mechanics or variation of material properties due to viscoelasticity or material degradation of composites. All this uncertainty leads to change in stiffness and damping of the structure. This demands a new methodology which is robust and updates the model parameters with measurements done online.

Unlike the conventional physics-based approach, data driven methods are projected to perform better because of their insensitivity to any underlying assumptions or equations. Many critical problems, such as climate prediction[54], predicting epidemics[55], or controlling turbulence for energy efficient power production[56], are all primed to take advantage of progress in the data-driven discovery of dynamics.

The method called Dynamic Mode Decomposition(DMD) originated in the fluid dynamics community to simplify complex flows into a decomposition based on spatiotemporal coherent structures. Schmid and Sesterhenn[57] first defined the DMD algorithm and demonstrated its ability to provide insights from high-dimensional fluids data. It became quite popular because it is an equation-free method which can provide an accurate decomposition to be used for short-time future-state prediction

and control. An extension of DMD called DMDc (DMD with control) provides the capability to disambiguate between measurement dynamics and input-output relationship simultaneously for closed loop system.

The purpose of this chapter is to identify and develop a reliable model identification framework based on DMDc which can create new maneuvers that not only are agile ($\alpha < 10$) but also decrease the residual vibrations in system without exact knowledge of the dynamic properties. Meaningful extensions to the existing algorithm are provided. This is followed by an optimization formulation to explore a subspace of maneuvers using the proposed Reduced-Order-Model (ROM).

3.1 Dynamic Mode Decomposition with Control (DMDC) algorithm

Assume a closed loop system with vector of observables $\mathbf{y}_i$ and input vector $\mathbf{u}_i$ at time t_i called measurement snapshot and input snapshot respectively. The temporal iteration of this discrete dynamics system can be defined as with contribution from previous measurement via matrix $\mathbf{A}$ and from the previous input through matrix $\mathbf{B}$:

$$\mathbf{y}_{k+1} = \mathbf{A}\mathbf{y}_k + \mathbf{B}\mathbf{u}_k \quad (3.1)$$

One can arrange time snapshots of measurements in the increasing order of time to form measurement snapshot matrix $\mathbf{Y}$ and shifted measurement snapshot matrix $\mathbf{Y}'$ as shown in equations 3.2 . Similarly, the input matrix $\mathbf{\Gamma}$ can be constructed with input snapshots, as shown in Figure 3.3,

$$\mathbf{Y} = \begin{bmatrix} | & | & & | \\ \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_{m-1} \\ | & | & & | \end{bmatrix}; \mathbf{Y}' = \begin{bmatrix} | & | & & | \\ \mathbf{y}_2 & \mathbf{y}_3 & \cdots & \mathbf{y}_m \\ | & | & & | \end{bmatrix} \quad (3.2)$$

$$\mathbf{\Gamma} = \begin{bmatrix} | & | & & | \\ \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_{m-1} \\ | & | & & | \end{bmatrix} \quad (3.3)$$

A compact way of describing Equation 3.1 for all times is possible through these snapshot matrices:

$$\mathbf{Y}' = \underbrace{\begin{bmatrix} \mathbf{A} & \mathbf{B} \end{bmatrix}}_{\mathbf{G}} \underbrace{\begin{bmatrix} \mathbf{Y} \\ \mathbf{\Gamma} \end{bmatrix}}_{\mathbf{S}} \quad (3.4)$$

$$\Rightarrow \mathbf{G} = \mathbf{Y}'\mathbf{S}^{-1}; \mathbf{G} \in \mathbb{R}^{n \times (n+q)} \quad (3.5)$$

Using SVD of $\mathbf{S} = \tilde{\mathbf{U}}\tilde{\Sigma}\tilde{\mathbf{V}}^*$ truncated to first $\tilde{r}$ singular values

$$\mathbf{G} = \mathbf{Y}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\tilde{\mathbf{U}}^* \quad (3.6)$$

Approximations of the operators $\mathbf{A}$ and $\mathbf{B}$ can be found by splitting the left singular vectors $\tilde{\mathbf{U}}$ into two separate components:

$$[\mathbf{A}, \mathbf{B}] \approx [\mathbf{Y}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\tilde{\mathbf{U}}_1^*, \mathbf{Y}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\tilde{\mathbf{U}}_2^*] \quad ; \quad \tilde{\mathbf{U}}_1 \in \mathbb{R}^{n \times \tilde{r}}, \quad \tilde{\mathbf{U}}_2 \in \mathbb{R}^{q \times \tilde{r}} \quad (3.7)$$

For high dimensional systems ($n \gg q$), DMDC cannot use truncated $\tilde{\mathbf{U}}$ to find the low-rank model of the dynamics and input matrix since $\tilde{\mathbf{U}}$ is defined for the *measurement-input augmented space*. Hence, a reduced order subspace is constructed from the *output measurements* $\mathbf{Y}'$ with truncation till r values ($r < \tilde{r}$)

$$\mathbf{Y}' = \hat{\mathbf{U}}\hat{\Sigma}\hat{\mathbf{V}}^* \quad ; \quad \hat{\mathbf{U}} \in \mathbb{R}^{n \times r}, \quad \hat{\Sigma} \in \mathbb{R}^{r \times r}, \quad \hat{\mathbf{V}} \in \mathbb{R}^{r \times m-1}, \quad (3.8)$$

Using the transformation $\mathbf{Y} = \hat{\mathbf{U}}\tilde{\mathbf{Y}}$, the following reduced-order approximations of $\mathbf{A}$ and $\mathbf{B}$ can be computed

$$\tilde{\mathbf{A}} = \hat{\mathbf{U}}^*\bar{\mathbf{A}}\hat{\mathbf{U}} = \hat{\mathbf{U}}^*\mathbf{Y}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\tilde{\mathbf{U}}_1^*\hat{\mathbf{U}} \quad ; \quad \tilde{\mathbf{A}} \in \mathbb{R}^{r \times r} \quad (3.9)$$

$$\tilde{\mathbf{B}} = \hat{\mathbf{U}}^*\bar{\mathbf{B}} = \hat{\mathbf{U}}^*\mathbf{Y}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\tilde{\mathbf{U}}_2^*, \quad ; \quad \tilde{\mathbf{B}} \in \mathbb{R}^{r \times q} \quad (3.10)$$

The Reduced-Order Model (ROM) can be defined as follows

$$\tilde{\mathbf{y}}_{k+1} = \tilde{\mathbf{A}}\tilde{\mathbf{x}}_k + \tilde{\mathbf{B}}\tilde{\mathbf{x}}_k \quad (3.11)$$

$$\tilde{\mathbf{y}}_1 = \hat{\mathbf{U}}^*\mathbf{x}_1 \quad (3.12)$$

Retrieve the measurements back in original coordinates

$$\mathbf{y}_k = \hat{\mathbf{U}}\tilde{\mathbf{y}}_k \quad (3.13)$$

Because of a central issue with DMD that was first observed in Tu et al. [58], a delay-augmentation[59][Ch-7] on the measurement is required. This is done due to the fact that DMDC fails to capture a single sine wave and cannot return the conjugate pair of complex eigenvalues and instead returns a single real eigenvalue. *Delay coordinates* refer to an augmented vector obtained by stacking the measurement $\mathbf{y}$ at the current time along with copies of $\mathbf{y}$ at future times. A delayed stack for both

input and state sequences with the augmented snapshot matrices is shown below:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_{m-1} \\ \mathbf{y}_2 & \mathbf{y}_3 & \cdots & \mathbf{y}_m \\ | & | & & | \\ \mathbf{y}_{ns} & \mathbf{y}_{ns+1} & \cdots & \mathbf{y}_{ns+m-2} \end{bmatrix}; \mathbf{Y}' = \begin{bmatrix} \mathbf{y}_2 & \mathbf{y}_3 & \cdots & \mathbf{y}_m \\ \mathbf{y}_3 & \mathbf{y}_4 & \cdots & \mathbf{y}_{m+1} \\ | & | & & | \\ \mathbf{y}_{ns+1} & \mathbf{y}_{ns+2} & \cdots & \mathbf{y}_{ns+m-1} \end{bmatrix}; \quad (3.14)$$

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_{m-1} \\ \mathbf{u}_2 & \mathbf{u}_3 & \cdots & \mathbf{u}_m \\ | & | & & | \\ \mathbf{u}_{ns} & \mathbf{u}_{ns+1} & \cdots & \mathbf{u}_{ns+m-2} \end{bmatrix} \text{ where , } ns = \# \text{ of stacks} \quad (3.15)$$

3.2 Proposed nonlinear DMDC algorithm

For DMDC to be able to resolve nonlinearities in the dynamics, one needs to find a group of nonlinear observables. It enables not just better *reconstruction* but also correct *predictions*. Here, reconstruction is defined as ROM capability for reproducing the training data. Whereas, prediction is referred to either extrapolating the training data for a longer time or estimating response to a completely different set of inputs than the training data.

Consider a flexible structure actuated by a CMG. For the inspiration to develop nonlinear observables, one can look at the rigid body equations of motion with torques applied due to the CMG actuator N_a , gravity N_g and from the flexible structure N_f :

$$[\mathbf{I}]\dot{\boldsymbol{\Omega}} + \underbrace{\boldsymbol{\Omega} \times [\mathbf{I}]\boldsymbol{\Omega}}_{\text{nonlinear}} = \mathbf{N}_a + \underbrace{\mathbf{N}_g}_{\text{nonlinear}} + \mathbf{N}_f \quad (3.16a)$$

$$-\mathbf{N}_a = \dot{\mathbf{H}}_a + \underbrace{\boldsymbol{\Omega} \times \mathbf{H}_a}_{\text{nonlinear}} \quad (3.16b)$$

Depending on the scenario whether it is a ground test or an actual spacecraft, additional terms like the gravitational torque N_g can influence the dynamics. It depends on the present orientation defined by passive rotation quaternion p with

center of mass eccentricity r_{com} relative to the center of rotation as follows:

$$N_g = \underbrace{r_{com} \times Cay^{-1}\{p \otimes [0, 0, -1, 0]^T \otimes p^*\}}_{\text{nonlinear}} Mg \quad (3.17a)$$

$$= \begin{bmatrix} p_1^2 r_2 + p_2^2 r_2 - (p_3^2 + p_4^2) r_2 + 2p_2 p_3 r_3 + 2p_1 p_4 r_3 \\ -p_1^2 r_1 - p_2^2 r_1 + (p_3^2 + p_4^2) r_1 - 2p_1 p_3 r_3 + 2p_2 p_4 r_3 \\ -2(p_2 p_3 r_1 + p_1 p_4 r_1 - p_1 p_3 r_2 + p_2 p_4 r_2) \end{bmatrix} Mg \quad (3.17b)$$

where the function $cay^{-1}()$ is known as the inverse Cayley map[60][Pg 62]. More discussion about gravitational torque in Section 5.2. Expanding the first nonlinear term $\Omega \times I \Omega$ in Equation 3.16, we see that the basis $\bar{\Omega}$ fully defines all the components:

$$\Omega \times I \Omega = \begin{bmatrix} I_{31}\Omega_1\Omega_2 + I_{32}\Omega_2^2 - I_{21}\Omega_1\Omega_3 - I_{22}\Omega_2\Omega_3 + I_{33}\Omega_2\Omega_3 - I_{23}\Omega_3^2 \\ -I_{31}\Omega_1^2 - I_{32}\Omega_1\Omega_2 + I_{11}\Omega_1\Omega_3 - I_{33}\Omega_1\Omega_3 + I_{12}\Omega_2\Omega_3 + I_{13}\Omega_3^2 \\ I_{21}\Omega_1^2 - I_{11}\Omega_1\Omega_2 + I_{22}\Omega_1\Omega_2 - I_{12}\Omega_2^2 + I_{23}\Omega_1\Omega_3 - I_{13}\Omega_2\Omega_3 \end{bmatrix} \quad (3.18a)$$

$$\bar{\Omega} = \begin{bmatrix} \Omega_1^2 & \Omega_2^2 & \Omega_3^2 & \Omega_1\Omega_2 & \Omega_2\Omega_3 & \Omega_3\Omega_1 \end{bmatrix}^T \quad (3.18b)$$

Similarly, a good basis for gravitational torque N_g is $\bar{p}$ (look at Equation 3.17b)

$$\bar{p} = \begin{bmatrix} p_1 p_2 & p_2 p_3 & p_3 p_4 & p_4 p_1 & p_1 p_3 & p_2 p_4 & p_1^2 & p_2^2 & p_3^2 & p_4^2 \end{bmatrix}^T \quad (3.19)$$

With all these ideas, following nonlinear ROM model is proposed with cross terms as state - state and state - input nonlinear adjustments in the derivative.

$$\mathbf{Y}' = \begin{bmatrix} \mathbf{A}_{n \times n} & \mathbf{B}_{n \times q} & \mathbf{P}_{n \times 3} & \mathbf{Q}_{n \times 6} & \mathbf{R}_{n \times 10} \end{bmatrix} \begin{bmatrix} \mathbf{Y} \\ \mathbf{\Gamma} \\ \Omega_h \times H_a \\ \bar{\Omega}_h \\ \bar{p}_h \end{bmatrix} \stackrel{\Delta}{=} \mathbf{GS} \quad (3.20a)$$

$$\mathbf{Y} = \begin{bmatrix} \Delta x \\ H_a \\ \Omega_h \\ p_h \end{bmatrix} ; \quad \mathbf{\Gamma} = \dot{H}_a \quad (3.20b)$$

where, the each column of measurement snapshot matrix $\mathbf{Y}$ consists of elastic displacements Δx at different spatial locations on the flexible structure relative to hub fixed frame, CMG actuator angular momentum H_a , hub angular velocities Ω_h and hub orientation quaternion p_h . Input snapshot matrix $\mathbf{\Gamma}$ has columns of partial derivative of CMG angular momentum $\dot{H}_a$. Nonlinear observables $\bar{\Omega}$ and $\bar{p}$ are measured at the hub. We assume the structure is deforming in the linear regime ($N_f \propto \Delta x$) and hence the choice of linear observables for elastic displacements.

The DMDc algorithm needs to be updated for estimates of the new matrices $\mathbf{P}$ and $\mathbf{Q}$. Using SVD of $\mathbf{S} = \tilde{\mathbf{U}}\tilde{\Sigma}\tilde{\mathbf{V}}^*$ truncated to first $\tilde{r}$ singular values

$$\mathbf{G} = \mathbf{Y}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\tilde{\mathbf{U}}^* ; \quad \tilde{\mathbf{U}} \in \mathbb{M}^{(n+q+3+6+10) \times \tilde{r}} \quad (3.21)$$

Approximations of the operators $\mathbf{A}$, $\mathbf{B}$, $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ can be found by splitting the left singular vectors $\tilde{\mathbf{U}}$ into five separate components :

$$[\mathbf{A}, \mathbf{B}, \mathbf{P}, \mathbf{Q}, \mathbf{R}] \approx [\mathbf{Y}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\tilde{\mathbf{U}}_1^*, \mathbf{Y}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\tilde{\mathbf{U}}_2^*, \mathbf{Y}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\tilde{\mathbf{U}}_3^*, \mathbf{Y}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\tilde{\mathbf{U}}_4^*, \mathbf{Y}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\tilde{\mathbf{U}}_5^*] \quad (3.22)$$

$$\tilde{\mathbf{U}}_1 \in \mathbb{M}^{n \times \tilde{r}}, \quad \tilde{\mathbf{U}}_2 \in \mathbb{M}^{q \times \tilde{r}}, \quad \tilde{\mathbf{U}}_3 \in \mathbb{M}^{3 \times \tilde{r}}, \quad \tilde{\mathbf{U}}_4 \in \mathbb{M}^{6 \times \tilde{r}}, \quad \tilde{\mathbf{U}}_5 \in \mathbb{M}^{10 \times \tilde{r}} \quad (3.23)$$

Similar to regular DMDc with high measurements compared to the inputs ($n \gg q$), a reduced order subspace is constructed from the *shifted output measurements* $\mathbf{Y}'$ with truncation till r values ($r < \tilde{r}$)

$$\mathbf{Y}' = \hat{\mathbf{U}}\hat{\Sigma}\hat{\mathbf{V}}^* ; \quad \hat{\mathbf{U}} \in \mathbb{R}^{n \times r}, \quad \hat{\Sigma} \in \mathbb{R}^{r \times r}, \quad \hat{\mathbf{V}} \in \mathbb{R}^{r \times m-1}, \quad (3.24)$$

Using the transformation $\mathbf{Y} = \hat{\mathbf{U}}\tilde{\mathbf{Y}}$, the following reduced-order approximations of $\mathbf{A}$ and $\mathbf{B}$ can be computed

$$\tilde{\mathbf{A}} = \hat{\mathbf{U}}^* \mathbf{A} \hat{\mathbf{U}} = \hat{\mathbf{U}}^* \mathbf{Y}' \tilde{\mathbf{V}} \tilde{\Sigma}^{-1} \tilde{\mathbf{U}}_1^* \hat{\mathbf{U}} ; \quad \tilde{\mathbf{A}} \in \mathbb{R}^{r \times r} \quad (3.25)$$

$$\tilde{\mathbf{B}} = \hat{\mathbf{U}}^* \mathbf{B} = \hat{\mathbf{U}}^* \mathbf{Y}' \tilde{\mathbf{V}} \tilde{\Sigma}^{-1} \tilde{\mathbf{U}}_2^* ; \quad \tilde{\mathbf{B}} \in \mathbb{R}^{r \times q} \quad (3.26)$$

$$\tilde{\mathbf{P}} = \hat{\mathbf{U}}^* \mathbf{P} = \hat{\mathbf{U}}^* \mathbf{Y}' \tilde{\mathbf{V}} \tilde{\Sigma}^{-1} \tilde{\mathbf{U}}_3^* ; \quad \tilde{\mathbf{P}} \in \mathbb{R}^{r \times 3} \quad (3.27)$$

$$\tilde{\mathbf{Q}} = \hat{\mathbf{U}}^* \mathbf{Q} = \hat{\mathbf{U}}^* \mathbf{Y}' \tilde{\mathbf{V}} \tilde{\Sigma}^{-1} \tilde{\mathbf{U}}_4^* ; \quad \tilde{\mathbf{Q}} \in \mathbb{R}^{r \times 6} \quad (3.28)$$

$$\tilde{\mathbf{R}} = \hat{\mathbf{U}}^* \mathbf{R} = \hat{\mathbf{U}}^* \mathbf{Y}' \tilde{\mathbf{V}} \tilde{\Sigma}^{-1} \tilde{\mathbf{U}}_5^* ; \quad \tilde{\mathbf{R}} \in \mathbb{R}^{r \times 10} \quad (3.29)$$

Discrete time nonlinear update in reduced coordinates is defined as follows

$$\tilde{\mathbf{y}}_{k+1} = \tilde{\mathbf{A}}\tilde{\mathbf{y}}_k + \tilde{\mathbf{B}}\mathbf{u}_k + \tilde{\mathbf{P}}([\Omega_h]_{\times} H_a)_k + \tilde{\mathbf{Q}}(\bar{\Omega}_h)_k + \tilde{\mathbf{R}}(\bar{p}_h)_k \quad (3.30)$$

$$\tilde{\mathbf{y}}_1 = \hat{\mathbf{U}}^* \mathbf{y}_1 \quad (3.31)$$

Retrieve the measurements back in original coordinates

$$y_k = \hat{\mathbf{U}} \tilde{y}_k \quad (3.32)$$

3.3 Maneuver optimization formulation

The baseline for slew maneuvers is a seventh degree polynomial $\mathbb{P}^7$ fully defined by the means of initial and terminal conditions (Equation 2.2) in time till the third derivative or jerk:

$$\psi(0) = 0 ; \dot{\psi}(0) = 0 ; \ddot{\psi}(0) = 0 ; \dddot{\psi}(0) = 0 \quad (3.33a)$$

$$\psi(T) = \Delta\psi ; \dot{\psi}(T) = 0 ; \ddot{\psi}(T) = 0 ; \dddot{\psi}(T) = 0 \quad (3.33b)$$

An extended polynomial basis is proposed which not only follows the smoothness properties of the baseline but also allows freedom to explore for an optimal behavior. A ninth degree polynomial is proposed $\mathbb{P}^9$ which gives two additional d.o.f. for optimizer if imposed with same initial and final value constraints. Unlike the input shaping technique discussed in Section 2.2.3, this method can address several requirements apart from residual deformation reduction in the flexible structure. Behaviors like terminal angle error in the rigid hub, elastic deformation of the structure during the maneuver, residual elastic deformation of the structure post maneuver, and residual vibration of the rigid hub post maneuver can all be tuned with appropriate penalty weights. Given the measurements of hub angles θ_h , maneuver angle change $\Delta\psi$, duration of maneuver T and elastic displacements in the flexible structure Δx , one can define a general objective function with parts of measurement snapshot matrix Y , input snapshot matrix Γ during discrete times $\{t_1, t_2, \dots, t_m, t_{m+1}, \dots, t_N\}$ ($t_m = T$) as follows:

$$J(\mathbf{Y}, \mathbf{\Gamma}) = \underbrace{\lambda ||(\theta_h)_m - \Delta\psi||^2}_{\text{terminal}} + \underbrace{\beta \left(\frac{\sum_{k=m}^N (\theta_h)_k^2 - \sum_{k=1}^m (\theta_h)_k}{m} \right)}_{\text{during}} + \underbrace{(1 - \lambda - \beta) \cdot \left(\frac{\sum_{k=m}^N (\theta_h)_k^2 - \sum_{k=m}^N (\theta_h)_k}{N - m} \right) + \left(\frac{\sum_{k=m+1}^N (\Delta x)_k^2 - \sum_{k=m+1}^N (\Delta x)_k}{N - m} \right)}_{\text{post}} \quad (3.34)$$

The summation terms together represent the signal variance. Because of the conditions in Equation 2.2, the terminal term in the objective function is automatically satisfied. Consider a special case when $[\lambda, \beta] = [0, 0]$ with only the second term in

the *post* terms. Then, this would minimize the residual deformation in the flexible structure. It can be used in combination with the reduced order model defined in Equation 3.20.

We define an optimization problem with the intention of finding a better trajectory $y^*(t) = [\Delta x^* \ H_a^* \ \Omega_h^* \ \theta_h^*] \in \mathbb{R}^n$ and control input $u^*(t) = \dot{H}_a^*$ as compared to our baseline resulted from seventh degree polynomial maneuver $u_o(t) = (\dot{H}_a)_o$. ROM is used to estimate states y_k during and post maneuver $t_k \in [0, 2T] \ \forall k \in \{1, N\}$ where, T is the maneuver duration. Following is a mathematical representation of the maneuver optimization problem. Given measurements $\mathbf{Y} = \{y_1, y_2, \dots y_N\}$ and inputs $\mathbf{\Gamma} = \{u_1, u_2, \dots u_N\}$

$$\begin{aligned}
 & \min_{a_i, b_i, c_i} \quad J(\mathbf{Y}, \mathbf{\Gamma}) \\
 & \text{s.t.} \quad \tilde{y}_{k+1} = \tilde{\mathbf{A}}\tilde{y}_k + \tilde{\mathbf{B}}u_k + \tilde{\mathbf{P}}([\Omega_h] \times H_a)_k + \tilde{\mathbf{Q}}(\bar{\Omega}_h)_k, \\
 & \quad \tilde{y}_1 = \hat{\mathbf{U}}^* y_1, \\
 & \quad y_k = \hat{\mathbf{U}}\tilde{y}_k, \\
 & \quad \psi_k = \left[\sum_{i=0}^9 a_i t_k^i, \sum_{i=0}^9 b_i t_k^i, \sum_{i=0}^9 c_i t_k^i \right]^T, \\
 & \quad u_k = (\dot{H}_a)_k, \\
 & \quad (H_a)_k = B^{(p)}(\delta_k)H^\circ \quad (\text{see Equation 2.43}), \\
 & \quad (\dot{H}_a)_k = -u_k - (\dot{\psi})_k \times (H_a)_k \quad (\text{see Equation 2.44}), \\
 & \quad \delta_{k+1} = \Delta t [A^{(p)}(\delta_k)]^\dagger (\dot{H}_a)_k / h_o, \\
 & \quad (.)^\dagger \text{denotes Moore-Penrose pseudo inverse,} \\
 & \quad 0 < t_k < 2T
 \end{aligned}$$

3.4 Numerical experiments

Theoretical model from Chapter 2 is used to perform identification experiments numerically. Elastic displacements Δx are measured at several points on the spacecraft whereas other measurements are performed at the hub as shown in Figure 3.1.

Traditionally, DMDC method is applied with snapshots during closed loop inputs as the observation period. However, in our system, for better disambiguation between the dynamics and the control, we perform our identification with special conditions by combining both open-loop phase and a closed-loop phase. A short disturbance is provided at the beginning and behavior of the system observed after. Open loop at the beginning helps estimate the matrix $\mathbf{A}$ without any control input from the

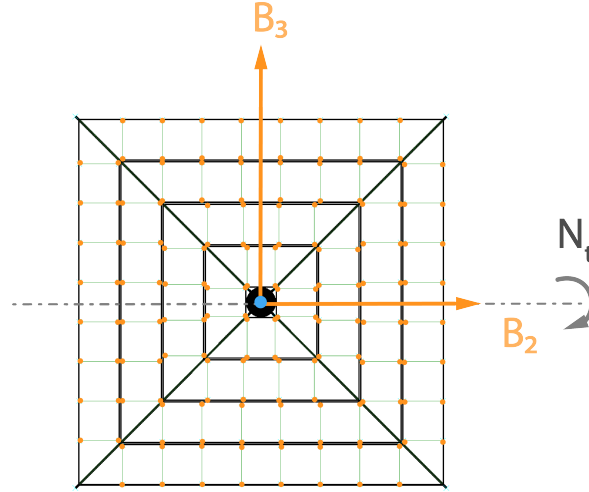


Figure 3.1: Elastic displacement measurement locations (in orange) and central hub measurements (in blue).

CMG. A time of 20 s after the impulse excitation is chosen for open-loop period and then the closed-loop begins. To develop sufficiency of excitation, we utilize a short impulse by thrusters to excite structural modes. We perform CMG-flexible structure stabilization in our numerical model about a reference orientation using Equation 2.38 with gains corresponding to $\zeta_{cl} = 0.5$, $\omega_{cl} = 2\pi 0.1$.

Excitation is applied as a torque at the central hub in the form of a finite time gaussian impulse which is equivalent to an attitude thruster firing for a real world scenario. Only the data corresponding to post excitation is utilized for developing a ROM. The thruster torque disturbance is assumed as follows:

$$N_t(t) = r_t \frac{A_o}{\sigma \sqrt{2\pi}} e^{-\frac{(t)^2}{2\sigma^2}} ; \quad 3\sigma = 0.1 \text{ s}, \quad A_o = 3 \text{ Ns}; \quad r_t = 0.3 \text{ m} \quad (3.35)$$

Where, A_o is the area of the impulse, σ is impulse shape parameter with 3σ equal to finite time width of the excitation applied at a moment arm r_t .

3.4.1 ROM results - reconstruction

The truncated singular values are shown in Figure 3.2. The measurement-input augmented matrix is set to a higher threshold than the measurement snapshot matrix. The algorithm is executed for data from the open-loop ($t < 20$) phase and a following closed-loop ($t > 20$) phase of the simulation. The number of stacks ns used as augmented measurement vector is 20.

Numerical model hub quaternion (solid) and the corresponding reconstructed components (dashed) are shown in Figure 3.3 with exceptional performance. Elastic

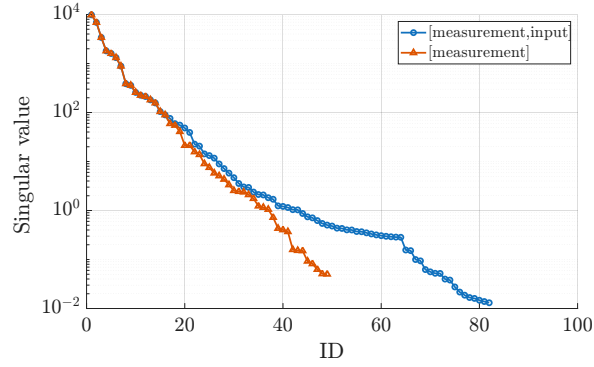


Figure 3.2: Singular values for measurement-input augmented matrix $\mathbf{S}$ and measurement snapshot matrix $\mathbf{Y}$.

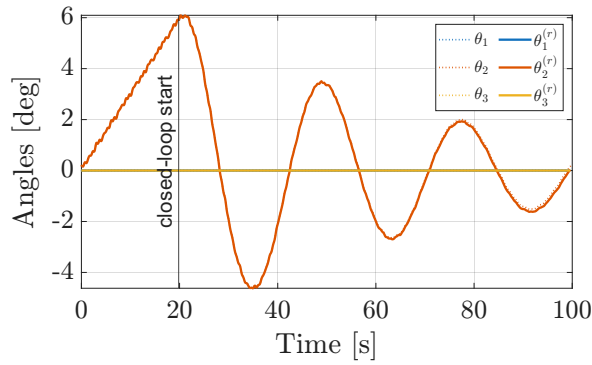


Figure 3.3: Reconstruction of hub root angles.

displacements from numerical simulation (solid) are also reconstructed (dashed) at the points P and Q in the structure as shown in Figure 3.4. The out-of-plane component Δx_1 is very well reconstructed whereas in-plane component Δx_3 suffers from slight underestimation in magnitude. Yet all components perform well in terms of capturing oscillations at the right time.

The estimated matrices for the ROM are shown in Figure 3.5. Matrix $\mathbf{A}$ is observed to have three high magnitude broad diagonals consisting of several elements along the width. This indicates that elastic displacements update on a point is predominantly dependent on the displacements of nearby points on the structure and corresponding points on the opposite side of maneuver axis. A low magnitude in matrix $\mathbf{P}$ suggests small state-input coupling $\Omega_h \times H_a$ between hub angular velocity Ω_h and actuator angular momentum H_a . Similarly, magnitude of components in matrix $\mathbf{Q}$ points to a small nonlinearity in the dynamics due to Coriolis torque $I \times I\Omega$. Within matrix $\mathbf{Q}$, higher values in columns one (Ω_1^2) and four ($\Omega_1\Omega_2$) representing the basis $\bar{\Omega}$ (see Equation 3.18b) is explained by small cross inertia terms I_{31} , I_{21} and inertia along

maneuver axis I_{22} . (See Equation 3.18a).

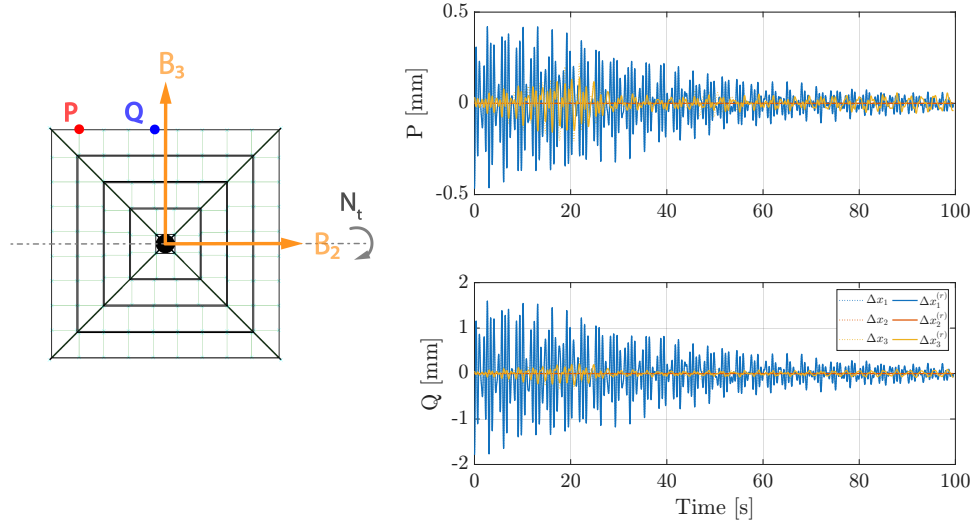


Figure 3.4: Reconstructed elastic displacements (superscript $(.)^{(r)}$) at points P (top) and Q (bottom).

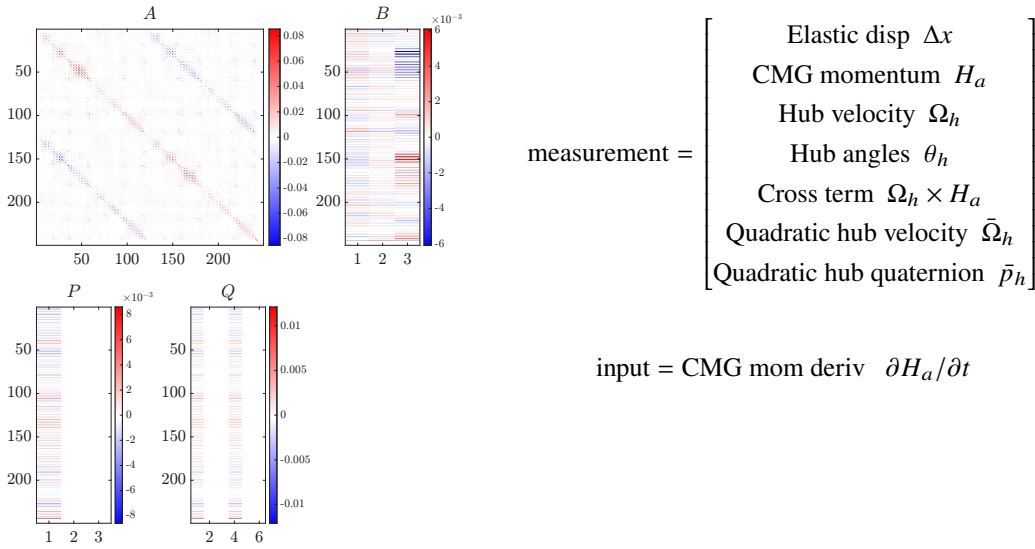


Figure 3.5: Estimated matrices.

3.4.2 ROM results - prediction

This ROM defined by matrices A, B, P and Q is then used for *prediction* with inputs corresponding to the baseline seventh degree polynomial slew maneuver about axis

B_2 . Then, the results are compared with the behavior from the full-fidelity numerical model obtained from Chapter 2.

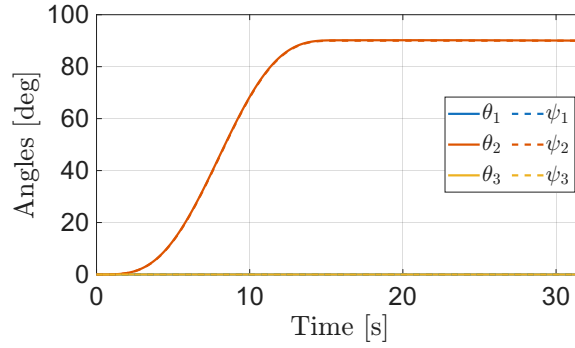


Figure 3.6: Hub root angles for a seventh degree smooth polynomial maneuver input torque. The ROM (solid) *predicts* well as compared to full fidelity model(dashed).

The hub angles are predicted well during the maneuver by this ROM as shown in Figure 3.6 but the error increases as the time progresses after the maneuver. The ROM struggles to capture magnitude of elastic displacements. For example at points P and Q on the structure, ROM under-predicts the elastic displacements in Figure 5.7 compared to the results from full fidelity numerical model in Figure 2.6, and 2.7. However, it does capture the oscillatory behavior post maneuver. These errors can be improved by tuning the ROM threshold for truncation of singular values.

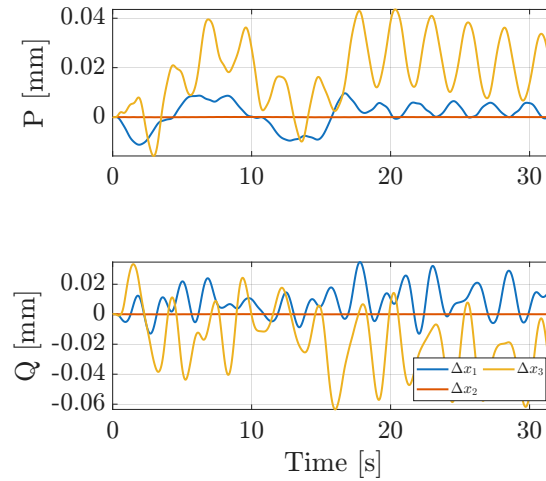


Figure 3.7: Elastic displacements on the structure as predicted by ROM.

3.4.3 ROM results - *optimization*

The baseline for slew maneuver experiments, the seventh degree polynomial $\mathbb{P}^7$ defined in Subsection 2.2.1 is used as a reference to compare results. A ninth degree polynomial is used to address residual deformation reduction in the flexible structure. Because of the initial and terminal conditions as mentioned in Equation 2.2, the terminal term in the objective function in Equation 3.34 is automatically satisfied. Consider a special case where $[\lambda, \beta] = [0, 0]$ and the post maneuver requirements are to minimize the residual deformation in the flexible structure (second term in the *post* terms in Equation 3.34).

The algorithm in section 3.3 is applied to the reduced order model as shown in Figure 3.5 and resulting optimal input (solid) in the form of actuator angular momentum is compared with baseline seventh degree polynomial solution(dashed). This overlay plot is shown in Figure 3.8. Corresponding hub angle comparison is shown in Figure 3.9 with resulting optimized maneuver exhibiting a slightly slower angle change in the first half and a faster variation in time in the second half. Elastic displacements are compared between the optimal (solid) and baseline (dashed) maneuver excitations at two different locations P and Q on the structure in Figure 3.10. Optimal maneuver show better performance in significantly decreasing residual excitations for same duration baseline maneuver. However, the peak deformations during the maneuver are slightly increased. This behavior can be improved with a combination of objective function and maneuver basis modifications. Penalizing peak deformations more than the residual ones and using a higher order polynomial for optimization basis will enable even better maneuvers. Modal amplitudes and velocities are reduced by a factor of five post maneuver by means of these optimal excitations compared to baseline as shown in Figure 3.11. Another interesting observation is that the optimal maneuver suppresses the structural modes in a non-uniform fashion in the post maneuver phase.

3.5 Discussion

This chapter has described a new data-driven identification methodology for CMG-flexible spacecraft system. Unlike commonly used data-centric schemes where the major focus is towards equation-free modeling, the present work utilizes the equations of motion to establish new observables addressing nonlinearities and enabling robustness to variation of dynamic properties of the system. A good quality *reconstruction* of the training data from the numerical model is achieved. More importantly, the algorithm is able to *predict* hub angles quite well even for a

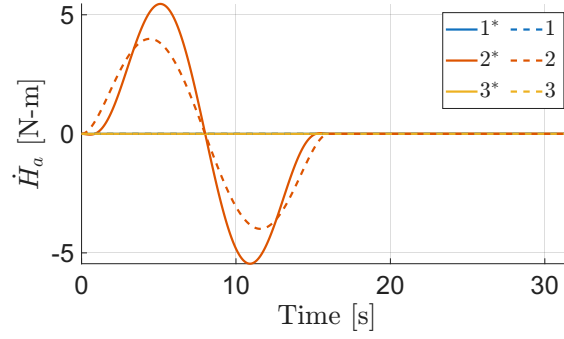


Figure 3.8: CMG angular momentum derivative resulted from optimization on ROM shown in solid lines compared to baseline seventh degree polynomial based curve in dashed lines.

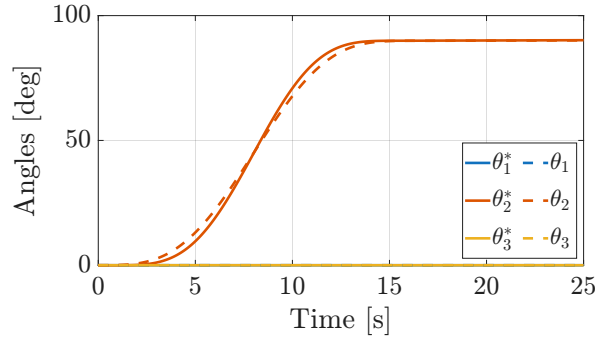


Figure 3.9: Hub root angles from full numerical model (absolute truth). Baseline torque (dashed) and optimal polynomial torque (solid).

larger hub angle change inputs compared to the training data. On the other hand, flexible body elastic displacements are under-predicted and could be improved with reduced order model parameter adjustments like singular value thresholds or number of stacks in the augmented measurement vector. Finally, the optimized maneuver using the reduced order model shows promising results in terms of reducing residual elastic displacements for a maneuver of same duration with a simplest basis of ninth degree polynomial without any assumptions on the dynamic properties of the flexible structure.

Although a motivating example of SSPP spacecraft is analyzed in the present work, the proposed algorithm can be adjusted depending on system of choice by implementing different observables to produce a reduced-order-model accounting for various nonlinear behaviors. This technique could prove very beneficial for applications involving multi-physics interaction. For example, ablative shield design for a re-entry spacecraft with a combination of thermal, aerodynamics and mate-

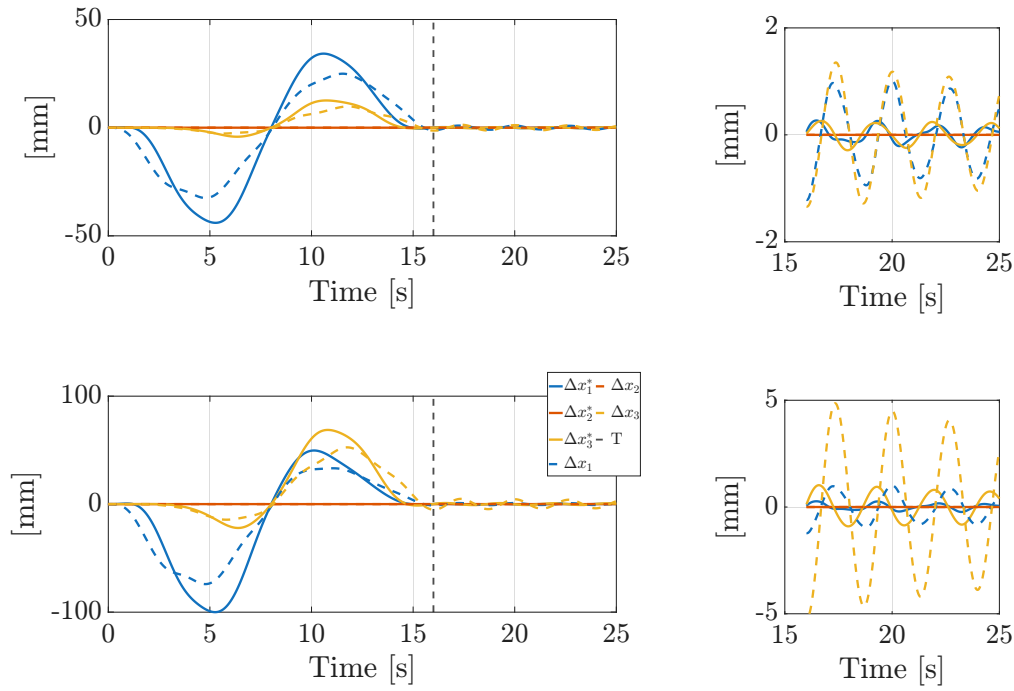


Figure 3.10: Elastic displacements at points P (top) and Q (bottom) from full numerical model on the left with baseline torque (dashed) and optimal polynomial torque (solid). Plot on the right shows the same data post maneuver in the form of residual excitations.

rial science aspects; a morphed aircraft wing design driven through aerodynamic effects coming from control surfaces, elasticity of the base structure wing; landing dynamics of reusable launch vehicle with coupled effects through control schemes, aerodynamic regime changes, ground effects and structural modes, etc.

The ideas proposed in this chapter will be validated in Chapter 5 through small-scale slew maneuver experiments.

Limitations of this technique in our application include sensitivity to noise in the observations, repeated eigenvalue of flexible modes for symmetric structures and Ill-conditioned control matrix B due to insufficient excitation. Another caveat is choosing enough observables so as to fully define system dynamics or choice of dimensionality of the reduced order system. Moreover, in case of implementation of this technique on real spacecraft computers, low fidelity versions of the model need to be estimated and numerically easier SVD algorithms need to be applied. All of these require careful attention and further research.

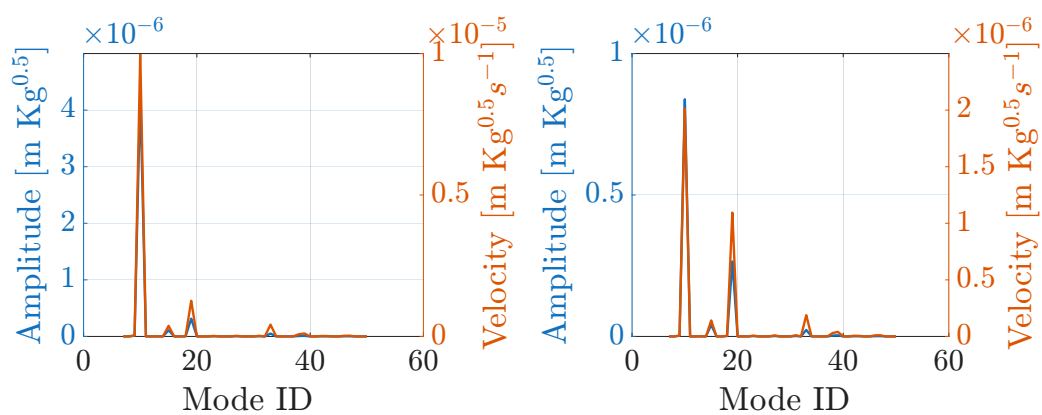


Figure 3.11: Peak modal amplitudes post maneuver with baseline torque (left) and optimal polynomial torque (right).

Chapter 4

PHYSICS-BASED SMALL SCALE SLEW MANEUVER EXPERIMENTS

Ground testing [27] of momentum control devices is quite challenging. A major difficulty is compensation for gravity. Experiments must be done in a way that approaches a closed system, i.e. without external torques. There are several techniques to create zero-g like conditions for experimental purposes, for example buoyant supports, offloading mechanisms etc. However, the use of air bearings is the most popular choice because they can provide very low friction suspension. Based on the number of degrees of freedom, one can either choose planar, cylindrical, spherical or other combinations of air bearings. The majority of previous work [35],[36] has focused on air bearings that allow only one degree freedom motion which decreases the complexity of dealing with the center of mass imbalance of the structure. Others [37],[30],[38] presented methods with 3 DOF capability with a variety of gravity compensation techniques, but only for rigid body experiments. There is no mention of 3 DOF experiments with CMGs that address flexible body deformation during slew maneuvers.

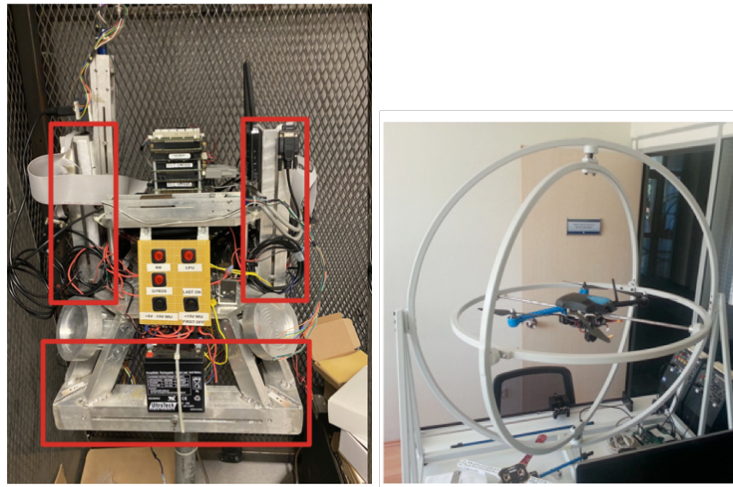


Figure 4.1: State of the art gravity compensation techniques. Sliding masses with the system on an air-bearing (left)[37]. Passive adjustment between system and an outside truss (right)[38].

This chapter focuses on experiments with three rotational degrees of freedom using a

commercially available spherical air bearing. The goal is to develop a ground test set up to demonstrate slew maneuvers with a flexible structure attached to an actuator. We will perform rotations around a general axis without any active center of mass control. This chapter will describe design of the pyramid CMG (equations described in Section 2.5) and a small-scale flexible structure for slew maneuver experiments. Structural response for the seventh degree zero-jerk smooth polynomial maneuver (Section 2.2.1) and input-shaped polynomial maneuver (Section 2.2.3) is presented and compared.

4.1 System overview

A pyramid configuration of CMG is selected for the experiments. The theoretical model is discussed in Section 2.5. The material for the rotors is selected as Brass 360. Each rotor is designed with mass moment of inertia $2.374E-4 \text{ kgm}^2$. Rotors are supported with high speed ball bearings on either side enclosed within metal bracket assembly made from stainless steel 410 as shown in Figure 4.2. Axial preload is provided to ensure any unwanted vibrations using a Belleville disc spring. The wires from the spin motor are carefully swept towards a slipring allowing for continuous power supply to the spin motor throughout the motion of the gimbal motor.

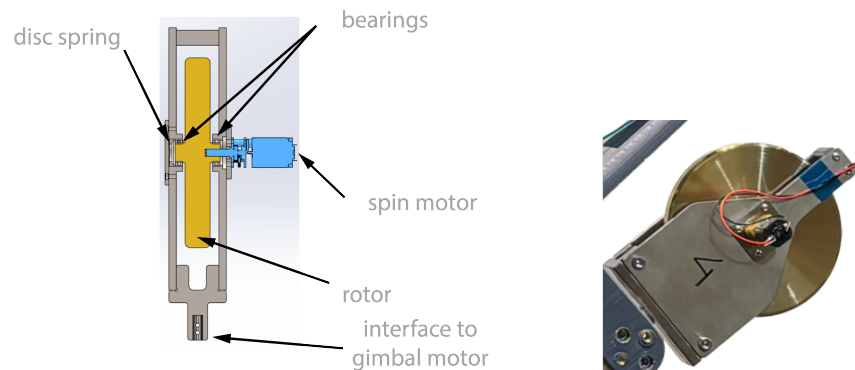


Figure 4.2: Rotor assembly schematic on left, and actual hardware on the right.

A 6V 10:1 micro metal high power carbon brush gear-motor is selected for spinning the each rotor. For each gimbal, a high gear ratio 227:1 Metal Gearmotor 25Dx71L mm low power 6V with 48 counts per revolution encoder is selected. Each spin motor/gimbal motor pair is controlled via a DRV8833 Dual Motor Driver Carrier. Teensy 4.1 controller is selected due to its large number of I/O pins. The motor control electronic architecture shares similarity with[36]. The chosen architecture is shown in Figure 4.3.

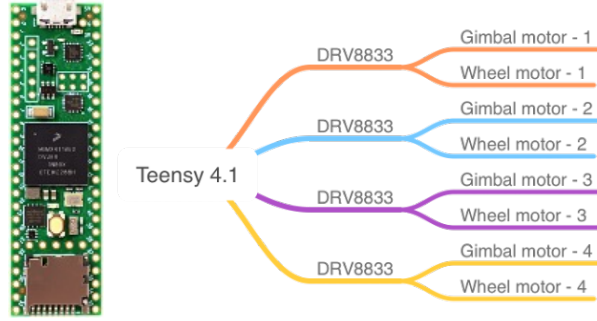


Figure 4.3: Motor-driver architecture.

The assembly is constructed with combination of custom build Aluminum 6061 parts and commercially available t-slotted frames.

A schematic of the assembly is shown in Figure 4.4, left.

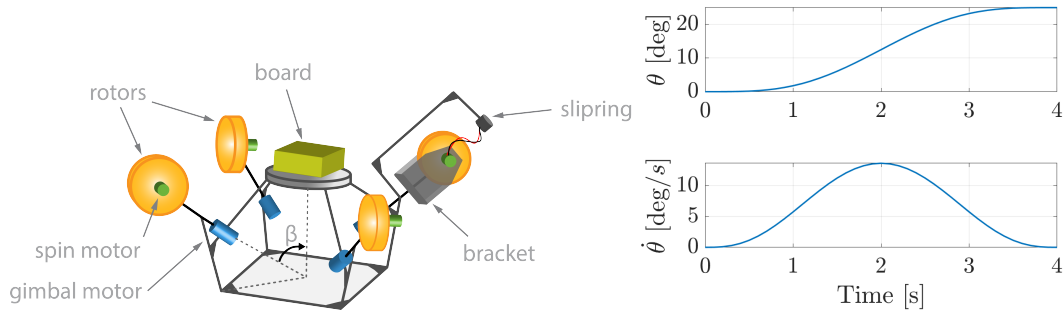


Figure 4.4: Pyramid CMG prototype schematic (left), smooth polynomial maneuver (right).

4.2 State estimation

Consider the measurement vector y with gimbal angles δ , assembly angular velocity Ω and quaternion describing passive rotation from body fixed frame to local frame p .

$$y = [\delta, \Omega, p] \quad (4.1)$$

Gimbal angles δ are measured using encoder counts in gimbal motors. The orientation of the assembly is estimated by Inertial Measurement Unit(IMU) chip MPU6050 which has a combination of three axis gyro rate sensors and three axis linear accelerometer sensors. Angular velocity of the assembly Ω is measured from gyro rate sensor. The quaternion p representing rotation from body frame to inertial fixed frame requires estimation. Madgwick algorithm[61] is selected to get an approximate quaternion with data fused from rate-gyro and accelerometer sensors.

Both gyro and accelerometer type sensors have measurement uncertainties and especially rate sensors have bias drifts on top of the noise. Gyro-rate sensor data while being placed flat on the table is shown in Figure 4.5.

The position of the sensor in the body-fixed frame (Figure 4.4, left) is $r_s = (1.18mm, 69.24mm, 90.33mm)$ based on CAD.

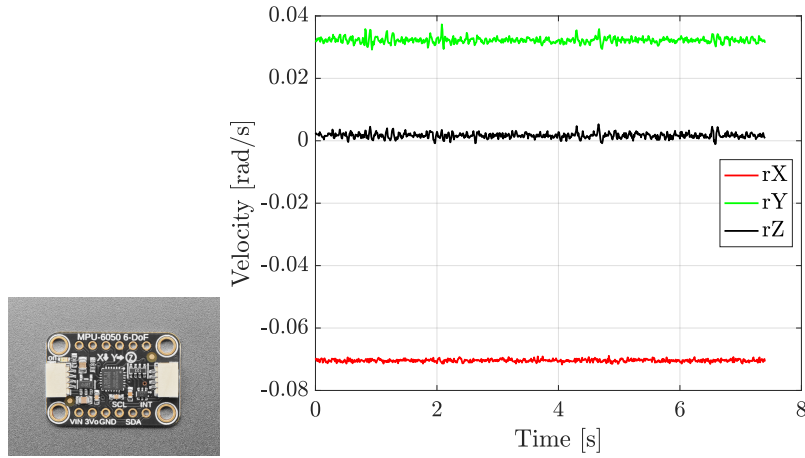


Figure 4.5: Adafruit MPU6050 (left), Gyro rate sensor data with MPU6050 flat on table.

4.2.1 Estimating quaternion from gyro sensor

Updating from angular velocity measurements(Ω)

$$\dot{p}(t) = 0.5 \begin{bmatrix} 0 & \Omega_3 & -\Omega_2 & \Omega_1 \\ -\Omega_3 & 0 & \Omega_1 & \Omega_2 \\ \Omega_2 & -\Omega_1 & 0 & \Omega_3 \\ -\Omega_1 & -\Omega_2 & -\Omega_3 & 0 \end{bmatrix} p = [D(\Omega(t))]p(t) \quad (4.2)$$

Estimate in time discrete form

$$p_{t+1}^{\Omega} = ([D(\Omega_t)]\Delta t + 1) p_t^{\Omega} \quad (4.3)$$

4.2.2 Estimating quaternion from accelerometer

Accelerometer sensor reads contributions from Coriolis, angular acceleration, gravitational acceleration in local frame B . The relationship between accelerometer reading a_s and the corresponding quaternion p^G is defined below:

$$a_s = \Omega \times (\Omega \times r_s) + \alpha \times r_s + [p^G \otimes \hat{G} \otimes p^{G*}]_{1:3} \quad (4.4)$$

$$\hat{G} = [0, 0, -g_0, 0] \quad (4.5)$$

For a fastest 3 s, 25° maneuver possible with the CMG, maximum contributions from Coriolis and angular acceleration are as follows

$$\Omega \times (\Omega \times r_s) \approx [0.0, -0.007 - 0.009] \quad (4.6)$$

$$\alpha \times r_s \approx [0.0, -0.03, 0.02] \quad (4.7)$$

Therefore it is safe to assume

$$a_s \approx [p^G \otimes TG \otimes p^{G*}]_{1:3} \quad (4.8)$$

Expanding Equation 4.8 we get a set of three nonlinear equations shown in 4.9.

$$\frac{a_s}{g_0} = \begin{bmatrix} 2(p_1 p_3 - p_2 p_4) \\ 2(p_2 p_3 + p_1 p_4) \\ p_4^2 + p_3^2 - p_1^2 - p_2^2 \end{bmatrix} \quad (4.9)$$

Solution of non linear system of accelerometer equations (4.9)

- System of equations

$$\begin{bmatrix} 2(p_1 p_3 - p_2 p_4) \\ 2(p_2 p_3 + p_1 p_4) \\ p_4^2 + p_3^2 - p_1^2 - p_2^2 \end{bmatrix} - \frac{a_s}{g_0} = f(p^G) \quad (4.10)$$

- Jacobian with respect to quaternion on unit sphere can be calculated

$$\nabla f = J(p^G) = \begin{bmatrix} 2p_3 & -2p_4 & 2p_1 & -2p_2 \\ 2p_4 & 2p_3 & 2p_2 & 2p_1 \\ -4p_1 & -4p_2 & 0 & 0 \end{bmatrix} \quad (4.11)$$

- Use steepest gradient with objective $g(p) = (f^T f)/2 \implies \nabla g = (\nabla f)^T f$

$$^{(k+1)}p^G = ^{(k)}p^G - \mu \frac{\nabla g}{\|\nabla g\|}; \quad k \in \{0, 1, 2, 3, \dots, n\} \quad (4.12)$$

- A conventional approach to optimisation would require multiple iterations of equation (4.12) to be computed for each new orientation and corresponding sensor measurements
- However, it is acceptable to compute one iteration per time sample provided that the convergence rate governed by μ is equal or greater than the physical rate of change of orientation

$$p_{t+1}^G = p_t^G - \mu^* \frac{\nabla g}{\|\nabla g\|} \quad (4.13)$$

$$\mu^* = \alpha \|\dot{p}^\Omega\| \Delta t \quad ; \quad \alpha > 1 \quad (4.14)$$

4.2.3 Fusing two estimations

Fusing two measurements to estimate quaternion

$$\hat{p} = \gamma p^G + (1 - \gamma) p^\Omega \quad (4.15)$$

An optimal value[61] of γ is related to gyro sensor rate error $\Delta\omega_{gs}$:

$$\gamma = \frac{\beta}{\frac{m\mu^*}{\Delta t} + \beta} ; \quad \beta = \sqrt{\frac{3}{4}} \Delta\omega_{gs} \quad (4.16)$$

After several assumptions a good estimation of the quaternion $\hat{p}_t$ is the following

$$\hat{p}_t = \frac{\beta\Delta t}{\mu^*} \left(-\mu^* \left(\frac{\nabla g}{\|\nabla g\|} \right)_t \right) + (1 - 0) \left(\hat{p}_{t-1} + \Delta t \dot{p}_t^\Omega \right) \quad (4.17)$$

With this formulation , the estimator takes around 20 s to converge to its steady state values at the beginning of the experiment.

4.3 Flexible structure

A four-fold symmetric structure was designed for the experiment. Its four sides form a square and four diagonals are connected to the CMG at the root. Each diagonal member joins two side members using brackets of mass m . The members are pultruded carbon fiber strips. The diagonals are circular cross-section tubes with outer radius r and thickness t . The side members have a rectangular cross-section of width b and height a . The assembly is performed using 3D-printed PLA brackets, which hold the members in place with epoxy glue.

A planar structure would impose several difficulties, because it has a finite state of self stress (Figure 4.6, left) and hence a small change in member length due to bonding or cutting can result in buckling of the structure.

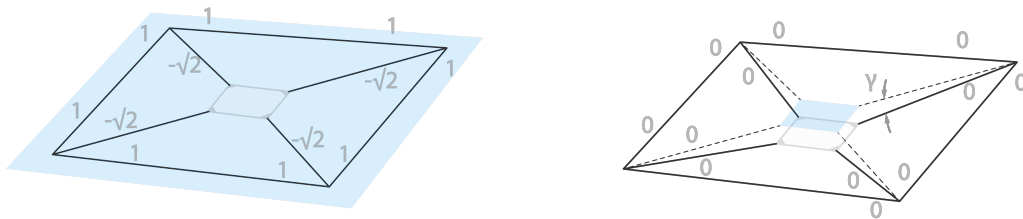


Figure 4.6: Finite state of self stress in the planar structure (left) and non planar structure (right).

To remove the state of self-stress, the geometry of the structure was modified by moving the center connections slightly out of plane so that the diagonals make an

angle γ with the plane of outer vertices, shown in Figure 4.6, right. These changes make the structure statically determinate and also reduce the out-of-plane deflection due to self-weight significantly from 12.4 cm in the planar configuration to 1.01 cm in case of the frustum geometry for the same boundary conditions and material properties. The planar version has the first mode at 0.98 Hz while the frustum version has first frequency of 1.41 Hz. The mode shapes of the final structure are shown in Figure 4.8.

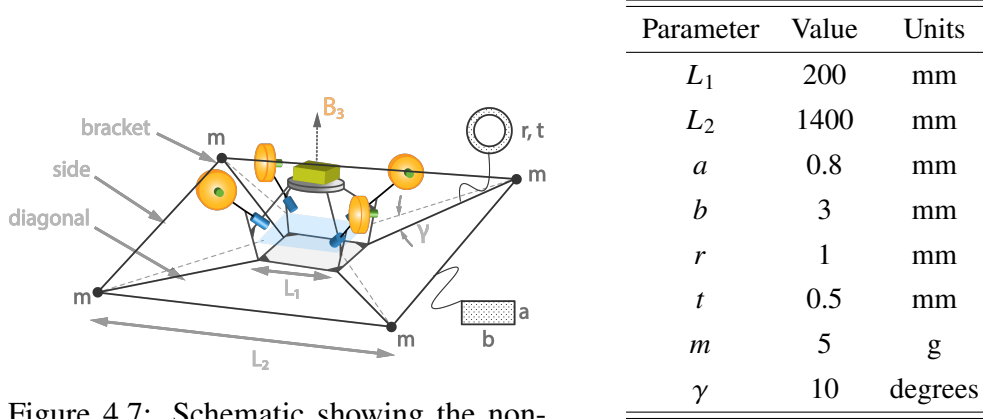


Figure 4.7: Schematic showing the non-planar structure attached to CMG.

Table 4.1: Flexible structure details.

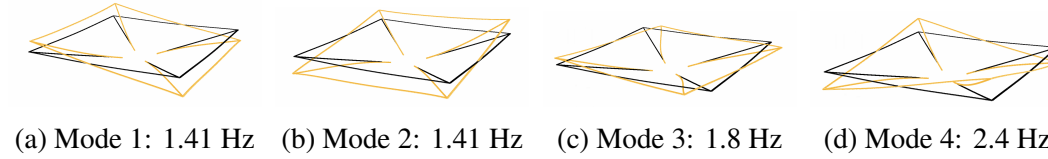


Figure 4.8: Mode shapes for fixed boundary condition at the center.

Details about the geometry and member cross sections are provided in table 4.1. An experimental validation of the frequencies was done with a laser vibrometer setup. The structure is attached to a rigid frame fixed to a shaking table as shown in Figure 4.9, left. A periodic chirp excitation of the table with frequency changing from 1 Hz to 2.1 Hz in a time of 128 s was imposed. Three peaks were observed, but the phase data was too noisy to be certain that the peaks corresponding to natural vibration modes. As a result, we proceed with another experiment to confirm the peaks to be mode shapes.

An OptiTrack motion capture system with a network of four cameras at 120 Hz was used to capture the time history of vibrations. The structure was lightly tapped to excite it. For example, modes-1 and 2 were excited by tapping in phase adjacent vertices. A similar technique was applied to excite the in-plane twisting mode 3. The fourth

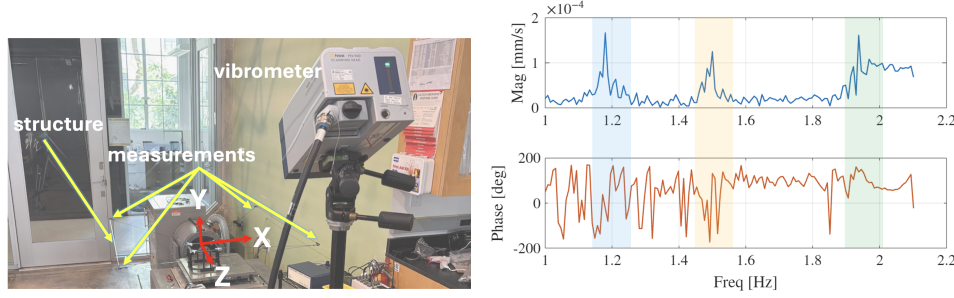


Figure 4.9: Vibrometer experimental setup (left), velocity and phase measurements (right).

mode was excited by moving two opposing vertices out of plane upwards and other downwards. Discrete Fourier transform of the time data were used to identify the major frequency content and compare them with the previous experiment. There is good agreement in results for both experiments (table 4.2). These results show that the structure is slightly softer compared to Abaqus model, for e.g. the first mode as measured from experiments appears at 1.18 Hz, in contrast to 1.4 Hz estimated by finite element analysis(Figure 4.8a). This likely due to the addition of extra mass of epoxy while bonding the members to the PLA brackets. Upon adjusting the point masses at the corners from 5 g to 7.4 g, good agreement between the experiments and the FEA model was achieved.

Mode	Abaqus [Hz]	Vibrometer [Hz]	OptiTrack [Hz]
1	1.14	1.19	1.18
2	1.14	1.19	1.18
3	1.6	1.5	1.49
4	2.2	1.92	1.93

Table 4.2: Frequencies of flexible structure with fixed BC.

4.4 Design of a maneuver experiment

It is important that the flexible structure exhibits observable elastic displacements during the maneuver. The Optitrack motion capture system is capable of position detection within 0.2 – 0.25 mm accuracy. The elastic deformations depend on the flexibility of the chosen structure relative to the maneuver α . A agility of $\alpha^{(S)} = 5$ is selected for structure design with respect to a 4 s duration maneuver. This agility ratio is same as the results for numerical slew maneuvers of a 10 m $\times$ 10 m SSPP structure in Chapter 2. This helps design a maneuver duration.

From table 4.2, the first fundamental period for the prototype structure is $T_n^{(P)} = 0.84s$. We can develop a minimum bound on our maneuver agility which gives a maximum maneuver duration of $T^{(P)} \leq 4.23s$ for the prototype. Once the maneuver agility is established, an actuator able to perform the constant eigen axis smooth polynomial maneuvers of the chosen flexible structure is designed. By changing the gimbal angles δ_i over time, the actuator is able to perform a rotation maneuver about the center of the spherical air bearing, along the direction shown in Fig. 4.10(right). The gimbal angles and the corresponding gimbal rates for a smooth polynomial maneuver of duration of $T^{(p)} = 4s$ are shown below in Figure 4.12 (left,center). The maneuver angle was selected as 10° due to constraints on air bearing range and actuator saturation. A second maneuver of equal magnitude yet opposite direction was performed from this extreme position to bring the orientation back to zero. This was done so that the structure would orient itself back perpendicular to gravity. A rest time of $T_r = 5s$ was introduced between the two maneuvers to observe transient vibration. This composite maneuver is called Go-Rest-Return (GRR) maneuver. The angle and velocity changes with time for GRR maneuver are shown in Figure 4.10, left.

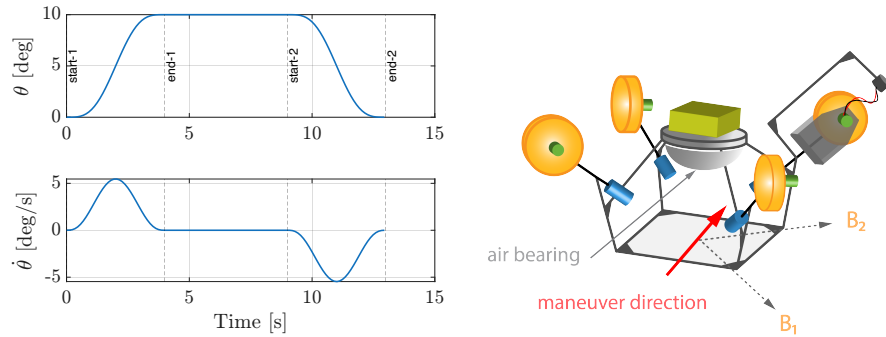


Figure 4.10: Go-Rest-Return (GRR) smooth polynomial maneuver (left), and maneuver direction relative to prototype (right).

4.4.1 Design details

A key component of a CMG is the spinning rotors. They are constrained to rotate along their axis using ball bearings on either side with stainless-steel enclosure. The rotor mass m_r , radius r and spin velocity ω can be related to the max torque output τ_g from each gimbal rotor subassembly along maneuver axis in the system using the maximum possible gimbal rate $\dot{\delta}$:

$$\tau_g = \frac{1}{2} m_r r^2 \omega \dot{\delta} \sin(\beta) \quad (4.18)$$

Assuming equal torque contributions from each CMG, one can arrive at a relationship between CMG parameters and the maximum maneuver acceleration $\ddot{\theta}$:

$$4 \times \tau_g \geq I \ddot{\theta} \quad (4.19)$$

Apart from torque requirements, the stored angular momentum should also be sufficient for exchange with the body based on desired maneuver. When the CMG reaches the limits of its angular momentum capability, it exhibits *external or elliptic singularities*[27]. One can visualize these surfaces based on CMG array placement. Following [62], the outermost singular surface $4H\{++++\}$ is obtained (Figure 4.11) for the chosen configuration. If h_o is the momentum stored in each wheel, the maximum possible momentum exchanges along the body axes B_1 , B_2 , B_3 directions are $3.36h_o$, $3.36h_o$, and $3.06h_o$ respectively. These values were used to size the CMG rotors with respect to maneuver momentum $I\dot{\psi}$.

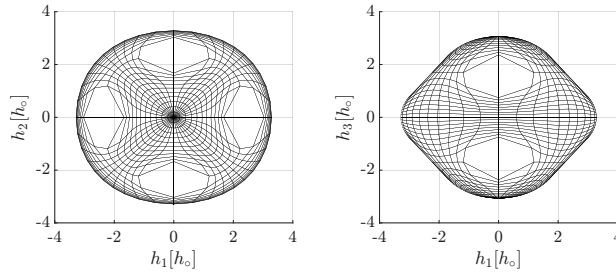


Figure 4.11: External singular surface for pyramid CMG with $\beta = 50^\circ$.

A maneuver requiring high gimbal rates would result in larger gimbal excursions which would increase the likelihood of the CMG reaching a singularity. For example, the present design results in gimbal excursions within 10° as shown in Figure 4.12. Nonetheless, a singularity-robust steering algorithm [27] was implemented to make sure that the solutions to Equation 2.44 exist as the singular values are close to zero. This technique works well for internal singularities. However, external singularities are impassable and require careful planning. In the present experiments, singularities were prevented by minimizing the external gravity torque on the system and performing maneuvers with momentum exchange within the surface shown in Figure 4.11.

The CMG frame design accommodates coarse adjustments in the Center of Mass (CoM) position in the vertical direction as shown in blue in Figure 4.13, right. Finer adjustments were performed using slotted weights placed on each side of the CMG. The square base of the pyramid was designed to physically restrict the in-plane

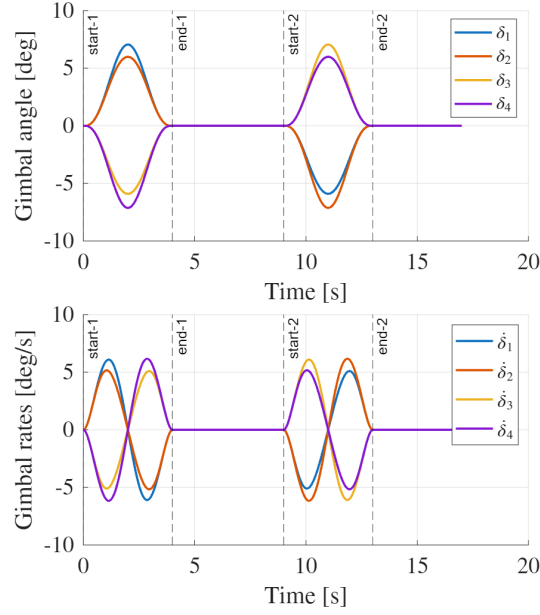


Figure 4.12: Gimbal angle (left) and rate (right) solution for pyramid CMG.

rotations through contact with the air bearing pedestal. The prototype placed on the air bearing can perform complete 360° rotations about the B_3 axis perpendicular to the table, and in-plane rotations ranging from $\pm 21^\circ$ upto $\pm 30^\circ$, depending on height adjustment of the assembly, before the square base comes in contact with the air bearing pedestal (Figure 4.13, left). Table 4.3 gives a summary of the manufactured design.

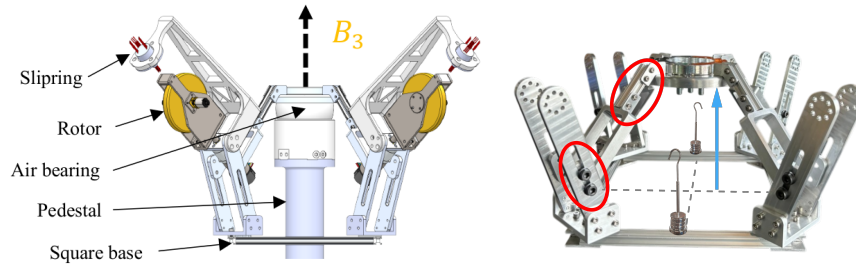


Figure 4.13: Schematic of prototype on air bearing (left) and, assembled truss (right) with adjustable points shown in red for performing center of mass adjustments along B_3 .

4.5 Experimental procedure

The designed flexible structure was attached to the CMG at four points using 3D printed PLA brackets. The assembly was then placed on the air bearing powered with a 45 psi compressed air line. The whole system was balanced by positioning slotted weights on all four sides. This was followed by careful placement of three

Table 4.3: CMG prototype details.

Property	Value	Units
Total mass	5.2	Kg
Dimensions	$0.43 \times 0.43 \times 0.29$	m
Total inertia	diag(0.1085, 0.1038, 0.144)	kg·m ²
Rotor inertia about spin axis	2.374E−4	kg·m ²
Pyramid skew angle (β)	50	degrees
Rotor nominal spin velocity	2000	RPM

support blocks at the bottom of the assembly restricting the movement relative to the surface of the table. Once the wheels reached the desired RPM, the support blocks were carefully removed. The closed loop control was activated relative to the quaternion at this orientation (gravitationally stable point). The advantages of using the gravitational equilibrium point (trim state) for closed-loop are two fold. Firstly, it saves control effort during non maneuver times, which leads to delayed occurrence of singularities. Secondly, it desensitizes the effect of error in mounting of the Inertial Measurement Unit (IMU) sensor on maneuver dynamics.

Four OptiTrack motion capture cameras were placed above the structure, looking down to have line of sight at all times. For the purpose of measurements, a body fixed cartesian coordinate system B was placed on the CMG such that the B_1 axis is along the maneuver axis, see Figure 4.14. Position measurements are performed in a global cartesian coordinate system O , calibrated to have O_3 along the local gravity. The maneuvers were planned with a change in angle $\Delta\theta = 10^\circ$ in $T = 4$ s according to the smooth polynomial. After the first maneuver had been completed, the CMG was commanded to hold the same orientation for $T_r = 5$ s. Finally, it was reorientated back by the same polynomial to its original configuration and commanded to stabilize.

A control law was developed for closed loop quaternion trajectory tracking in attitude by constructing error dynamics of the spacecraft attitude as a second order system:

$$\tau = \mathbf{I}\dot{\Omega}_d + \mathbf{I}[\Omega_d]_{\times}\Omega + [\Omega]_{\times}\mathbf{I}\Omega - (\mathbf{K}_p p_e + \mathbf{K}_w(\Omega - \Omega_d)) \quad (4.20)$$

$$\mathbf{K}_p \in \mathbb{M}_{3 \times 4}, \mathbf{K}_w \in \mathbb{M}_{3 \times 3} \quad (4.21)$$

where $[\cdot]_{\times}$ represents skew symmetric tensor of the vector inside. The error in the

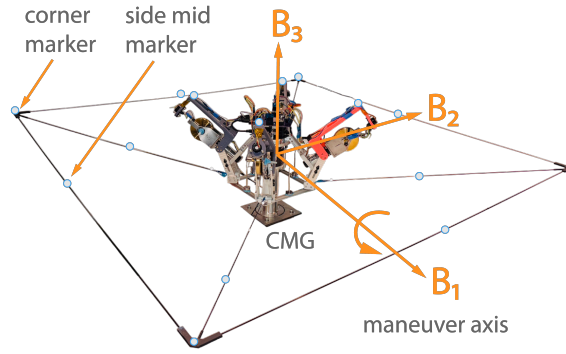


Figure 4.14: Experimental set up of CMG with the flexible structure.

quaternion state p_e relative to desired quaternion p_d given by:

$$p_e = p_d \otimes p^* = \begin{bmatrix} p_4 & p_3 & -p_2 & -p_1 \\ -p_3 & p_4 & p_1 & p_2 \\ p_2 & p_1 & p_4 & p_3 \\ p_1 & p_2 & p_3 & p_4 \end{bmatrix} \begin{bmatrix} p_{d1} \\ p_{d2} \\ p_{d3} \\ p_{d4} \end{bmatrix} \quad (4.22)$$

where $()^*$ represents quaternion conjugate and $() \otimes ()$ represents quaternion product. Quaternions p , p_e , $p_d \in \mathbb{H}$ follow the right-handed Hamiltonian convention[23]. The quaternion p defines a passive rotation from the local (body fixed) to global (inertial) frame for the CMG hub. It is important to note that the control law does not contain any terms related to strain or strain rates in the flexible structure over time and is purely controlled from the orientation of the central rigid body or CMG hub.

4.6 Experimental results for smooth polynomial maneuvers

The orientation of the CMG was measured as a function of time, comparing the OptiTrack measurements (absolute truth) with micro-controller values. Since quaternion data lacks intuitive insight, they were converted into a vector-angle $[\mathbf{n}, \psi]$ combination to present the transformation from body frame to inertial frame. This vector angle pair is the Rodriguez parameter notation[50]. A maximum positional error of 0.15 mm was achieved during the OptiTrack system calibration.

Tracking was performed throughout the maneuver with the control law. The state estimation with the micro-controller shows an attitude error within $|\psi - \psi_d|_{max} = 0.4^\circ$ in all three directions combined (Figure 4.15, left). However, from the OptiTrack measurements, this value was 1.33° (Figure 4.15, right). This minor inconsistency in the orientation error is explained by the limitations in the estimation algorithm

and the lack of magnetometer sensor in the IMU chip. We used the Madgwick algorithm[61] to filter and apply sensor fusion from gyro sensor and accelerometer sensor at 250 Hz sampling.

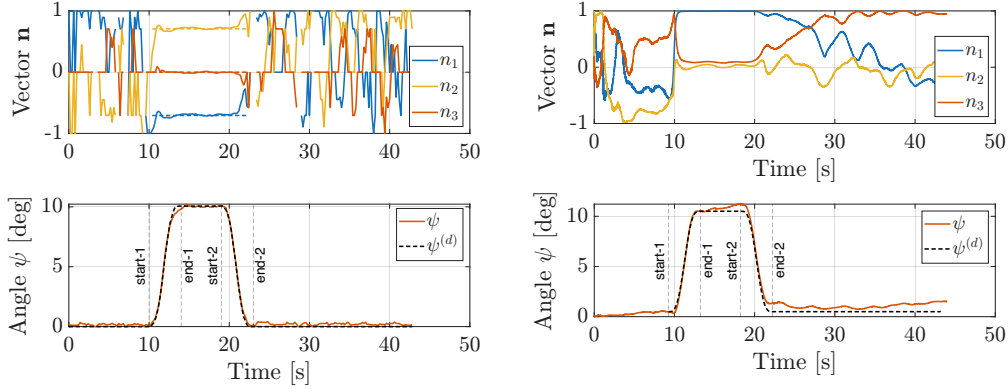


Figure 4.15: Rodrigues parameters from Madgwick filter (left) and absolute truth (right).

The excitations in the flexible structure during the maneuver was studied. The elastic displacements $\Delta x^{(B)}$ were calculated in the body frame B for side mid point and corner point as shown in Figure 4.14. The pure quaternion corresponding to the position of a point in inertial frame O is $\hat{X}(t) \in \mathbb{H}_p$

$$\hat{X} = \begin{bmatrix} \frac{x^{(O)}}{\|x^{(O)}\|} \\ 0 \end{bmatrix} \quad (4.23)$$

The position relative to CMG fixed frame B is given by

$$x^{(B)} = [p^* \otimes \hat{X} \otimes p]_{1:3} \quad (4.24)$$

Finally, elastic displacements need to be computed relative to the configuration at the beginning of the maneuver

$$\Delta x^{(B)} = x^{(B)}(t=0) - x^{(B)}(t) \quad (4.25)$$

As seen in Figure 4.16, in-plane oscillations $\Delta x_1^{(B)}$ and $\Delta x_2^{(B)}$ are limited to 1 mm w.r.t to the CMG fixed frame.

The out of plane component $\Delta x_3^{(B)}$ is rather interesting. Apart from the small period small amplitude oscillations, it shows a static component of deflection as well. This can be observed at the end of the first polynomial maneuver where the out of plane elastic displacements reach 10 mm and around 1.8 mm peak-to-peak post maneuver excluding the DC component for both the locations on the structure.

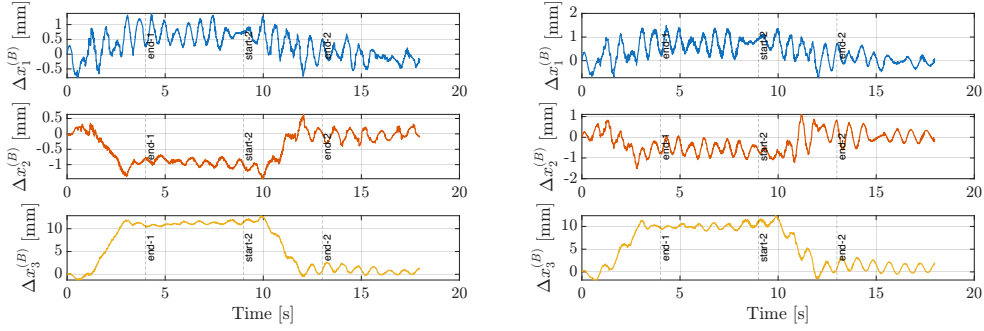


Figure 4.16: Elastic displacements for side mid point (left) and corner point (right).

A Discrete Fast Fourier Transform of the elastic displacements was used to establish dominant frequencies from the time data. Two major peaks were observed, one at 1.26 Hz and other at 1.611 Hz. Both are higher than their corresponding counterparts from the vibration table experiments, i.e. 1.18 Hz and 1.45 Hz. This is explainable in terms of change in boundary conditions, with a floating root i.e. rotations free yet translations fixed in these spherical air bearing tests as compared to the fixed root in the vibrometer experiments.

Spatially, at side mid point and corner point, the elastic displacements in the frequency domain are quite similar as shown in Figure 4.17. They both exhibit same peak frequencies except for $\Delta x_2^{(B)}$, where the corner point has an additional peak at 1.61 Hz.

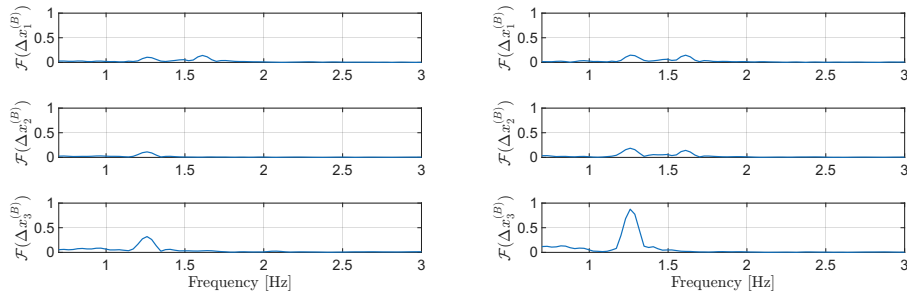


Figure 4.17: Fourier Transform of elastic displacements in body frame for side mid point (left) and corner point (right).

4.7 Experiments with reduced excitations

Input shaping technique with two impulses is utilized to get new polynomial maneuvers as discussed in Section 2.2.3. For demonstrating input shaping in the experiments, a slightly more aggressive polynomial acceleration with $\Delta\theta = 10^\circ$ and $T = 2$ s was chosen, to facilitate the observations of the elastic displacements of the

structure

$$A_1 = 0.507 \quad ; \quad t_1 = 0 \quad (4.26)$$

$$A_2 = 0.492 \quad ; \quad t_2 = 0.789 \quad (4.27)$$

Input-shaped maneuver plots are shown in Figure 4.18. All derivatives up to the jerk $\ddot{\theta}$ exhibit piecewise continuity. The only penalty is the additional increase in time of maneuver from $\Delta T = 2 \text{ s}$ to $\Delta T = 2 + t_2 = 2.789 \text{ s}$.

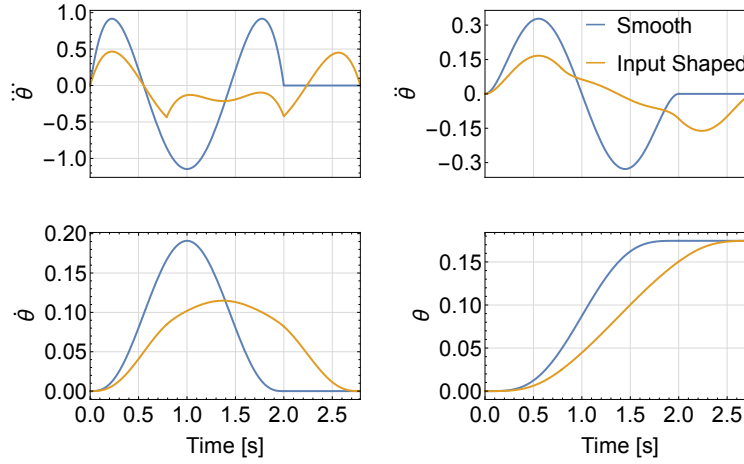


Figure 4.18: Resulting maneuver jerk, velocity and angle from input shaping acceleration.

4.7.1 Results

The elastic deformation in the case of the 2 s maneuver is higher than the 4 s maneuver at all locations. For example, for the mid-point on the side, the out-of-plane oscillations $\Delta x_3^{(B)}$ without the DC component increases from 1.8 mm (Figure 4.16) to 5.89 mm peak-to-peak as shown in Figure 4.19. However, after the application of input shaping to the baseline maneuver, the same oscillations reduce to 1.15 mm within 4 s post maneuver and 0.47 mm in the next 4 s (Figure 4.20). An interesting observation is that the elastic deformation magnitude along the maneuver axis remains unaffected by this technique, at least in the experiments. A comparison between elastic displacements in the frequency domain for the two polynomial GRR maneuver and the input-shaped polynomial maneuver is shown through Figures 4.21 and 4.22 respectively. A reduction of amplitude of the frequency peak for out of plane deformation $\Delta x_3^{(B)}$ is observed, which is evidence of vibration suppression for that mode.

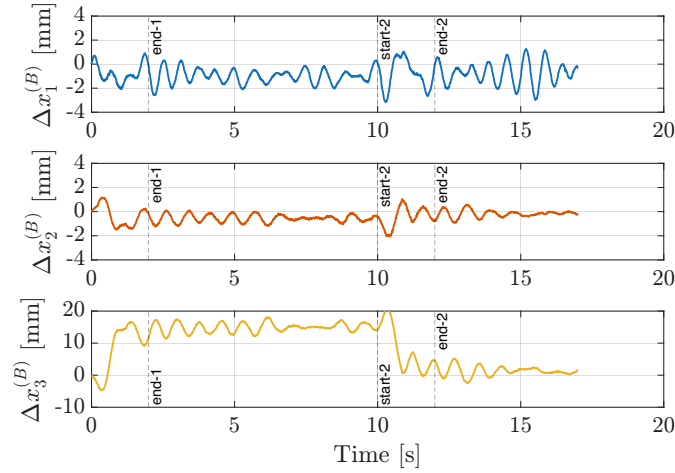


Figure 4.19: Elastic displacements for side mid point with seventh degree smooth polynomial maneuver.

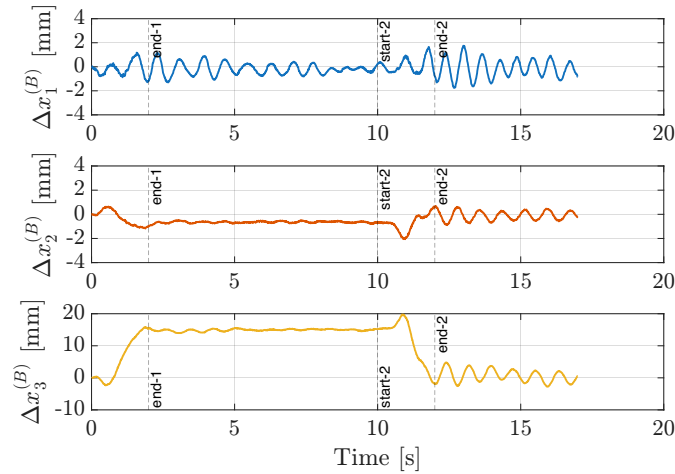


Figure 4.20: Elastic displacements for side mid point with input shaped polynomial maneuver.

The maximum pointing error for the aggressive 2 s polynomial maneuver is 3.03° , which is worse than its 4 s counterpart. A high pointing error of 3.39° is observed in the input shaped maneuver as well. This is due to the limitations of the current Inertial Measurement Unit (IMU) chip MPU6050 which lacks magnetometer sensor, which provides rather poor estimation along body axis B_3 . The accelerometer data is not reliable for estimating rotations along B_3 axis, which forces the control to rely heavily on the gyro sensor in the algorithm, especially in case of dynamically slow events, which eventually results in small errors in state estimation due to inherent drift in these sensors.

In a further extension of the study above, slightly incorrect variants of shaped maneuvers were performed to understand the effects due to model uncertainty. The target frequency for input shaping is changed to 1% and 5% of the original value in two separate experiments. The elastic out of plane deformations on the side mid points were observed to be 3.28 mm and 5.79 mm respectively peak-to-peak. A formal way of comparing the effectiveness of input shaping is to plot residual vibrations as a function of uncertainty as originally shown by Singer et. al.[43]. A similar version of this plot is shown in Figure 4.23.

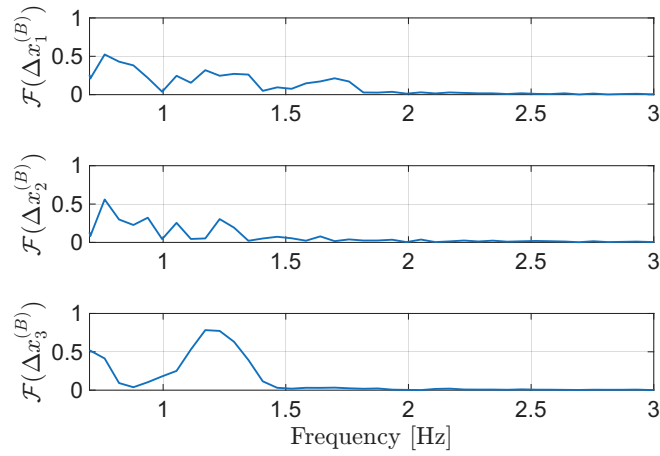


Figure 4.21: Fourier Transform of elastic displacements for side mid point with seventh degree smooth polynomial maneuver.

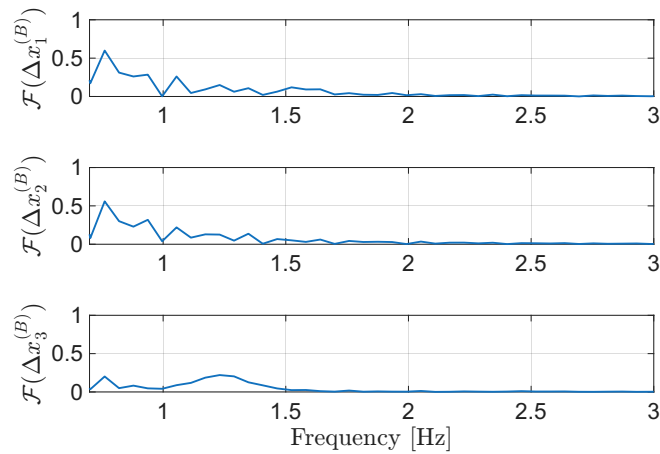


Figure 4.22: Fourier Transform of elastic displacements for side mid point with input shaped polynomial maneuver.

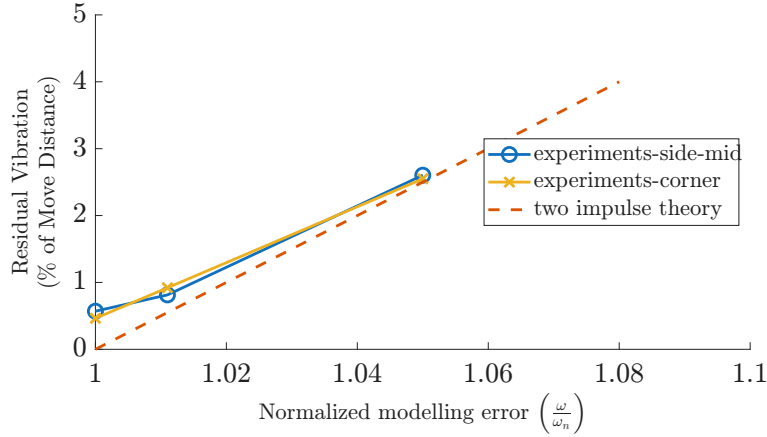


Figure 4.23: Sensitivity of experimental residual vibrations to modeling errors in terms of frequency uncertainty and their comparison to theoretical two impulse sensitivity[43].

4.8 Discussion

This chapter has presented the experimental results of closed-loop smooth slew maneuvers performed on a flexible structure with an in-house pyramid CMG. Flexible structure strains were not utilized for closing the loop, but only central body orientation and angular rate estimations were used to perform tracking in attitude. A 10° smooth polynomial maneuver with 4 s maneuver time is performed within the measured 1.33° combined error in all directions. The measured elastic displacements and their frequency content provide information of the modes excited during the maneuver. These results could be utilized to improve mathematical models in future studies.

With the second set of experiments, the results for maneuvers modified for vibration suppression are presented, observing significant reduction in elastic deformation of a continuous system by input shaping of the original maneuver accounting for only one mode. Based on the experimental application of this technique, it seems to be ineffective in reducing excitation in the structure along the direction of the maneuver axis. However, apart from this limitation, if applied with correct model information, input shaping is demonstrated to be strong candidate for reducing flexible structure excitation in out of plane direction with little impact on attitude error.

Another limitation based on the construction of ZV input shaping is its sensitivity to modeling errors. Very large spacecraft structures are difficult to test in their deployed configuration and show uncertainty in terms of mode shapes and frequency relative

to their respective finite element models. And hence, data-driven methods of system identification on board appear to be an attractive solution for this problem and its implementation is discussed in Chapter 5.

Chapter 5

DATA-BASED SYSTEM IDENTIFICATION EXPERIMENTS FOR OPTIMAL SLEW MANEUVERS

The purpose of this chapter is to demonstrate the nonlinear system identification framework extension based on DMDc discussed in Chapter 3 which can enable discovery of new maneuvers that not only are agile ($\alpha < 10$) but also decrease the residual vibrations in the system without accurate knowledge of the dynamic properties. The discussion begins with the proposed reduced order model (Section 3.2) applied to a numerical model of the experimental CMG-flexible structure system similar to formulation developed in Section 2.3. The intention is to identify the system dynamics from this numerical model via time discrete data to re-validate the chosen technique with numerical model of a smaller scale ~ 1 m. This is followed by an optimization formulation to explore better maneuvers. Compared to the input shaping technique discussed in Subsection 2.2.3, the proposed system identification framework addresses many different kinds of requirements whether it is reducing hub oscillations, or residual elastic deformations, or peak deformation during the maneuver etc.

Once successful optimization is achieved within the numerical model, the same technique is deployed in the CMG-flexible body experiment described in Chapter 4. First set of experiments involve disturbing the closed-loop system and corresponding ROM fitted using the measurements and inputs. This is followed by offline optimization subject to the ROM obtained. And finally, these candidate optimal maneuvers are applied in the experimental setup and deformation measured and compared to the seventh degree zero-jerk polynomial baseline response.

5.1 Validation of identification methodology through flexible body-CMG numerical model

Traditionally, DMDc method is developed to analyze measurement data collected from complex systems where closed loop force has been applied during the observation period. However, in our system to develop sufficiency of excitation, we utilize an external disturbance source other than the CMG i.e. tapping the structure at a point to excite structural modes. We perform CMG-flexible structure stabilization in our numerical model with respect to a single reference orientation.

For better disambiguation between the dynamics and the control, we choose special conditions for our numerical experiment by combining both open-loop phase and a closed loop phase. Open loop at the beginning helps identify the matrix $\mathbf{A}$ by the algorithm without any control from the CMG. A time of 10 s after the tapping excitation is chosen for open loop period and then the closed-loop begins.

We observe elastic the deformations Δx of the 12 different points on the structure relative to the hub fixed body frame highlighted in orange in Figure 5.1, hub angular velocities Ω_h , and hub quaternion p_h to form linear measurements. The nonlinear observables comprise of cross term between angular velocity and angular momentum of the actuator $\Omega_h \times H_a$, quadratic angular velocities at hub $\bar{\Omega}_h$, and quadratic quaternion at hub $\bar{p}_h$ (see Section 3.2). The corresponding CMG actuator angular momentum time derivative $\dot{H}_a$ at discrete times forms the input snapshot matrix $\mathbf{\Gamma}$. The linear observables $\mathbf{Y}$, input snapshot matrix $\mathbf{\Gamma}$ and the non linear observables combined together to form the augmented snapshot matrix $\mathbf{S}$.

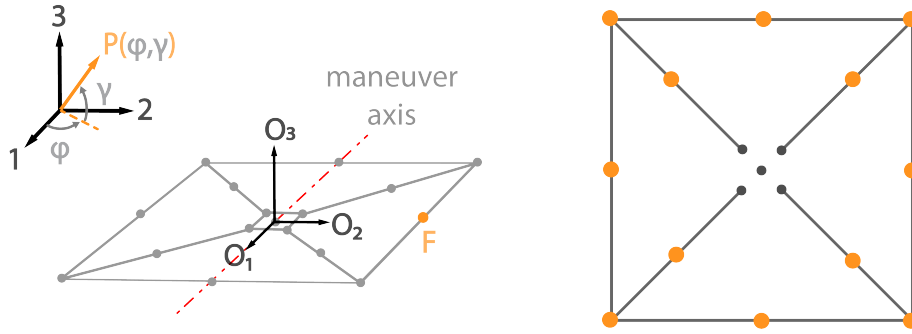


Figure 5.1: Prototype structure (1.2 m $\times$ 1.2 m) with disturbance location F (left), observation points on the structure (right).

As mentioned before following non linear reduced order model will be utilized for system identification

$$\mathbf{Y}' = \begin{bmatrix} \mathbf{A}_{n \times n} & \mathbf{B}_{n \times q} & \mathbf{P}_{n \times 3} & \mathbf{Q}_{n \times 6} & \mathbf{R}_{n \times 10} \end{bmatrix} \begin{bmatrix} \mathbf{Y} \\ \mathbf{\Gamma} \\ \Omega \times H_a \\ \bar{\Omega}_h \\ \bar{p}_h \end{bmatrix} \triangleq \mathbf{GS} \quad (5.1a)$$

$$\mathbf{Y} = \begin{bmatrix} \Delta x \\ H_a \\ \Omega_h \\ p_h \end{bmatrix}; \quad \mathbf{\Gamma} = \dot{H}_a \quad (5.1b)$$

Gravity is not considered for these numerical experiments. Excitation is applied using gentle tapping in the experiment which could be equivalent to an attitude thruster firing. Only the data corresponding to post excitation is utilized for developing an ROM. The tapping disturbance is assumed to be a gaussian pulse:

$$P(t) = \frac{A_o}{\sigma\sqrt{2\pi}} e^{-\frac{(t)^2}{2\sigma^2}} ; \quad 3\sigma = 0.5 \text{ s}, \quad A_o = 0.1 \quad (5.2)$$

5.1.1 ROM results - reconstruction

The truncated singular values are shown in Figure 5.2. The measurement-input augmented matrix has higher threshold than the measurement snapshot matrix. Reconstructed hub angles and elastic displacements are shown in Figure 5.3 and 5.4 respectively. The algorithm works well for both the open-loop($t < 10$) and closed-loop($t > 10$) phases of the simulation.

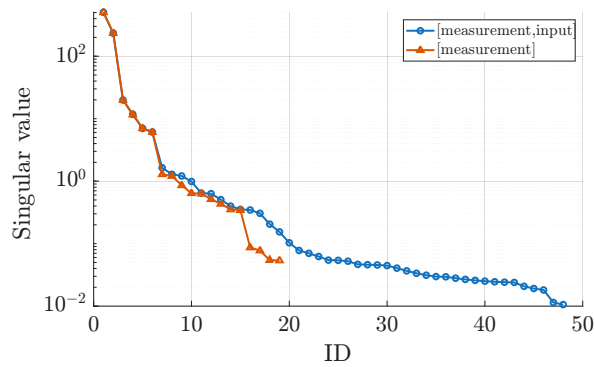


Figure 5.2: Reconstruction of hub root angles.

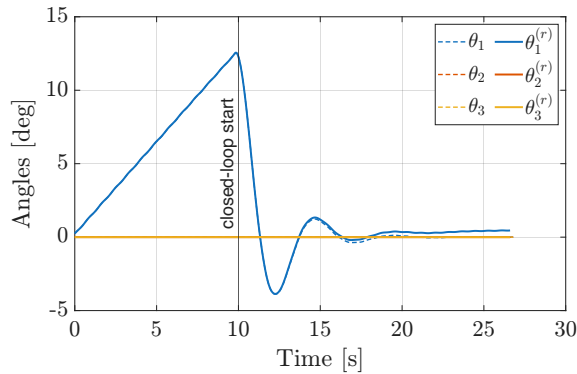


Figure 5.3: Reconstruction of hub root angles.

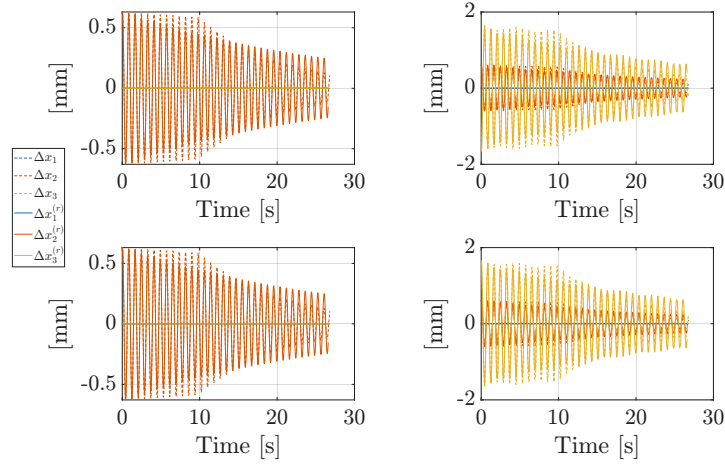


Figure 5.4: Reconstruction of elastic displacements of side mid points.

The estimated matrices for the ROM is shown in Figure 5.5. The measurements are arranged in an order spatially varying from edge of the structure towards the center or hub which is observable as larger values of a_{ij} towards top left of the $\mathbf{A}$ matrix.

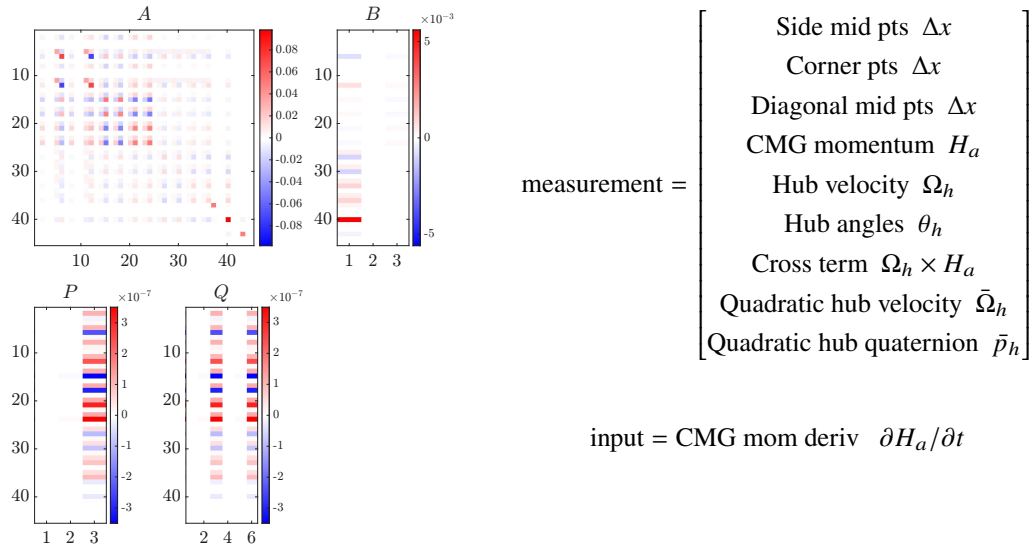


Figure 5.5: Estimated matrices.

5.1.2 ROM results - *prediction*

This ROM defined by matrices A, B, P and Q is then used for *prediction* with inputs corresponding to the baseline seventh degree polynomial slew maneuver about axis-1. Then, the results are compared with the behavior obtained from a full fidelity numerical model.

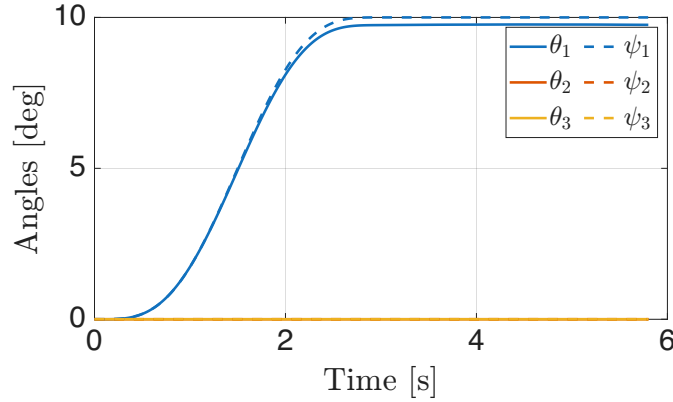


Figure 5.6: Hub root angles for a seventh degree smooth polynomial maneuver input torque. The ROM (solid) *predicts* well as compared to full fidelity model(dashed).

The hub angles are very well predicted by this ROM as shown in Figure 5.6. However, the ROM struggles to capture all elastic displacements. For example at side mid points on the structure, ROM underpredicts the magnitude of these elastic displacements but captures the oscillatory behavior post maneuver as shown in Figure 5.7 as compared to the results from the full fidelity numerical model as shown in Figure A.7. This inconsistency is overlooked for now and the ROM is used to explore optimal maneuvers next.

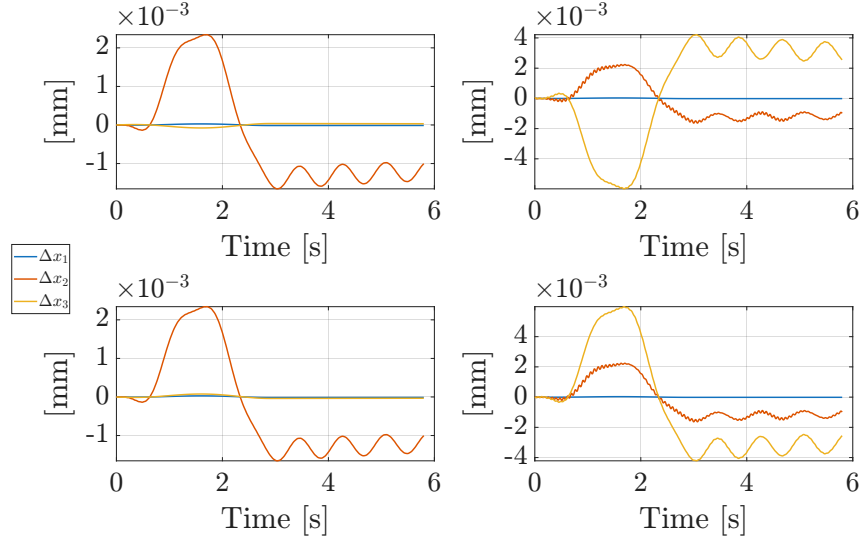


Figure 5.7: Elastic displacements at four points on the structure on the middle of each side as predicted by ROM.

5.1.3 ROM results - *optimization*

The baseline for slew maneuver experiments, the seventh degree polynomial $\mathbb{P}^7$ defined in Subsection 2.2.1 is used as a reference to compare results.

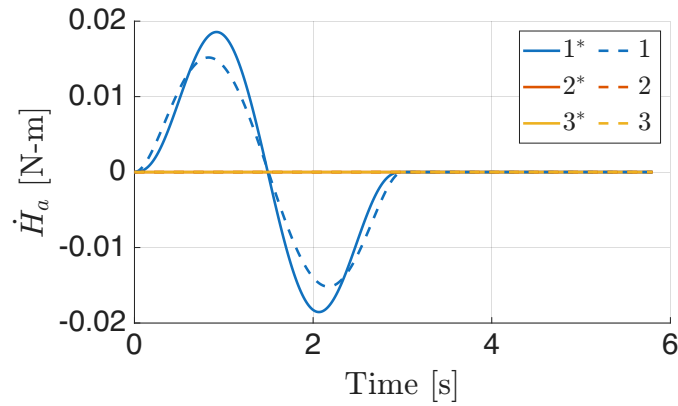


Figure 5.8: CMG angular momentum derivative resulting from optimization on ROM shown in solid lines, compared to baseline seventh degree polynomial based curve in dashed lines.

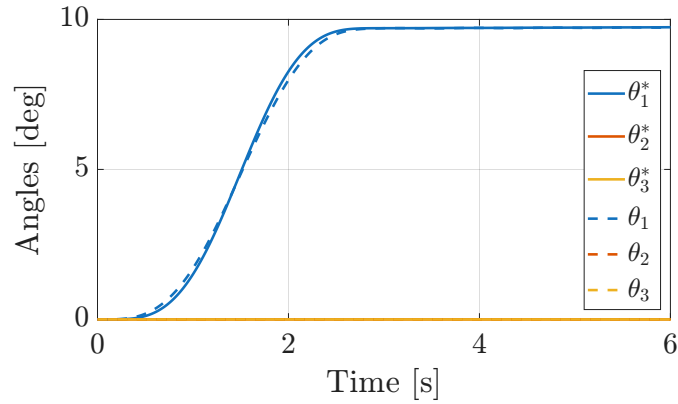


Figure 5.9: Hub root angles from full numerical model (absolute truth). Baseline torque (dashed) and optimal polynomial torque (solid).

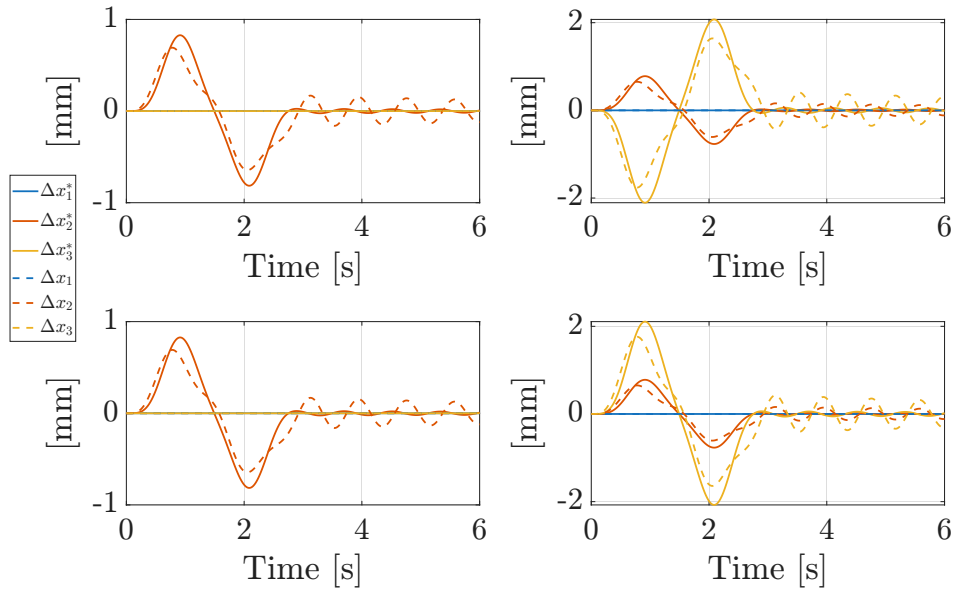


Figure 5.10: Elastic displacements at side mid points from full numerical model (absolute truth). Baseline torque (dashed) and optimal polynomial torque (solid).

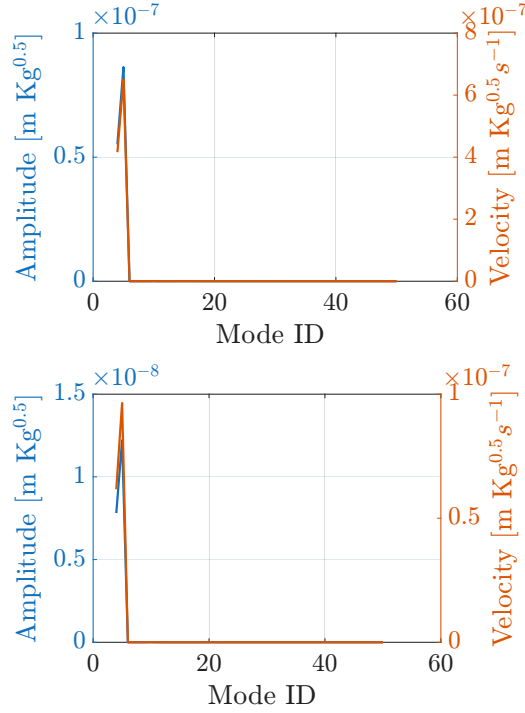


Figure 5.11: Peak modal amplitudes post maneuver with baseline torque (left) and optimal polynomial torque (right).

5.2 Nonlinear DMDC for experiments

Experimental identification is done similar to numerical model identification, with tapping excitation. Two different identification experiments were performed with different tapping locations as shown in Figure 5.12, left. A combination of open-loop, closed-loop experiment was not performed because of the restrictions on in-plane-rotation angles of the air bearing.

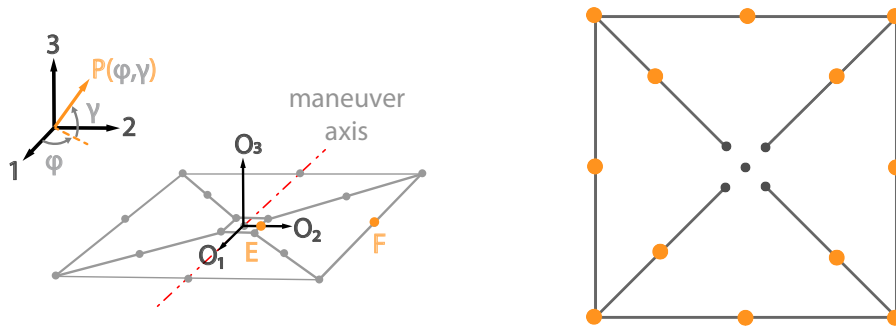


Figure 5.12: Prototype structure (1.2 m $\times$ 1.2 m) with disturbance locations E and F (left), observation points on the structure (right).

Experiments on ground are slightly different from attitude maneuvers in space. This is due to the presence of gravitational torque on the system. Several advances in

gravitational compensation for attitude experiments have been discussed in [37] and [38]. The current hardware setup utilizes combination of adjustable truss and slotted weights for minimizing the distance between Center-Of-Mass(COM) and Center-Of-Rotation(COR) as shown in Figure 4.13, right. However, there is still a small offset $\|X_{COM} - X_{COR}\| \approx 0.1 \text{ mm} \equiv T_s \approx 20 \text{ s}$ time period of air-bearing/CMG system is present which is a limitation of balancing technique and compensation procedure. This means that gravitational torque oscillates zero-peak-zero values from equilibrium orientation to extreme tilts and back with this time period T_s . To make sure that this is accounted for, the ROM with $\bar{p}$ basis as a nonlinear observable is required (see Section 3.2).

5.2.1 Identification results from tapping at root of the structure

The closed loop angles and angular velocities as measured from the OptiTrack system are shown in Figure 5.13, left and right respectively. Only a single stack of measurement vector is selected to capture the behavior in the CMG/flexible structure system due to noisy measurements of CMG angular momentum H_a from the micro-controller. Measurements are started at the peak value in the hub angle, when the closed-loop torque from the CMG starts to kick in. The singular value thresholds for measurement, input augmented matrix and shifted measurement matrix are shown in Figure 5.14:

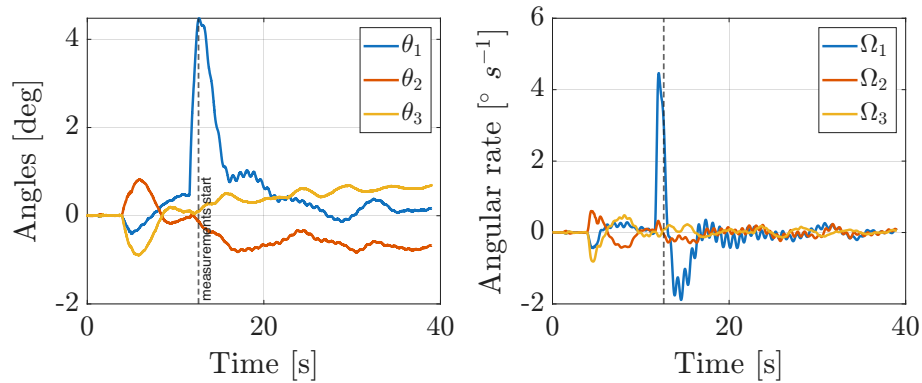


Figure 5.13: Hub angles and angular velocities due to tapping excitation.

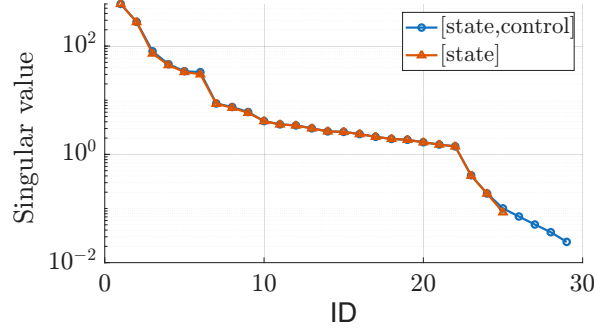


Figure 5.14: Singular values.

Experimental hub quaternion and the corresponding reconstructed components are shown in Figure 5.15 with exceptional performance. However, the algorithm results in partial reconstruction of the elastic deformations at the side mid points in the structure as shown in Figure 5.16. The oscillations are captured but the magnitude is underestimated for elastic displacements for long time horizon. This could be improved by utilizing by performing measurements with noise improvement and multiple stacking of measurement vector as augmented state.

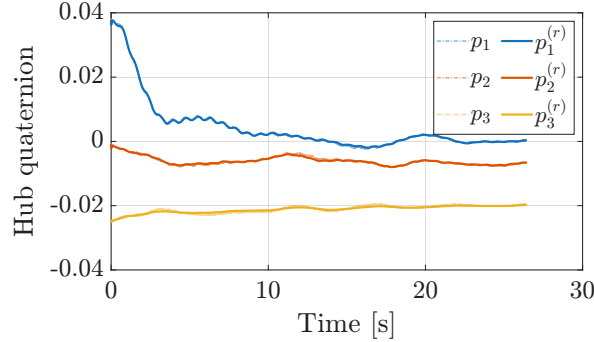


Figure 5.15: Reconstruction of hub root angles.

The estimated matrices for the ROM are shown in Figure 5.17. Similar to results for 10 m spacecraft (Figure 3.5), Matrix A is observed to have three diagonals be higher magnitude than the rest of the elements. This indicates that elastic displacements update on a point is predominantly dependent on the displacements of at least one other point on the structure. Moreover, higher values in several columns at the end of matrix A suggests strong dependency on CMG angular momentum H_a , hub angular velocity Ω_h and hub orientation quaternion p_h . A low magnitude in matrix P suggests small state-input coupling $\Omega_h \times H_a$ between hub angular velocity Ω_h and CMG angular momentum H_a . Similarly, magnitude of components in matrix

Q points to a small nonlinearity in the dynamics due to Coriolis torque $I \times I\Omega$. Higher values in matrix R means that the gravitational torque is still affecting the the dynamics which explains the DC component in elastic displacements. Within matrix R , higher values in columns four ($p_4 p_1$) and six ($p_2 p_4$) representing the basis $\bar{p}$ (see Equation 3.19) is explained by a higher out-of-plane component of the COM r_3 . (See Equation 3.17b).

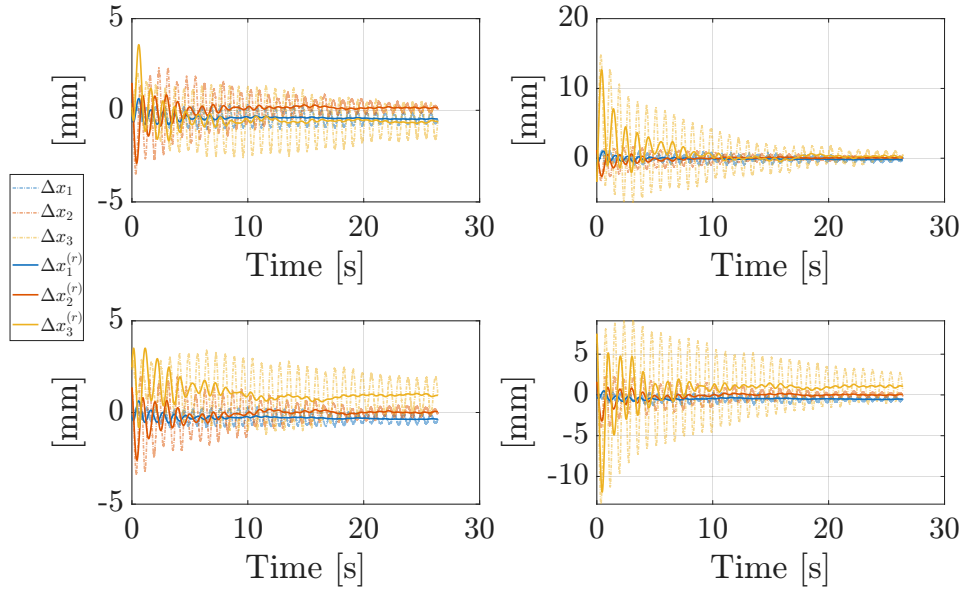


Figure 5.16: Reconstruction of elastic displacements of side mid points.

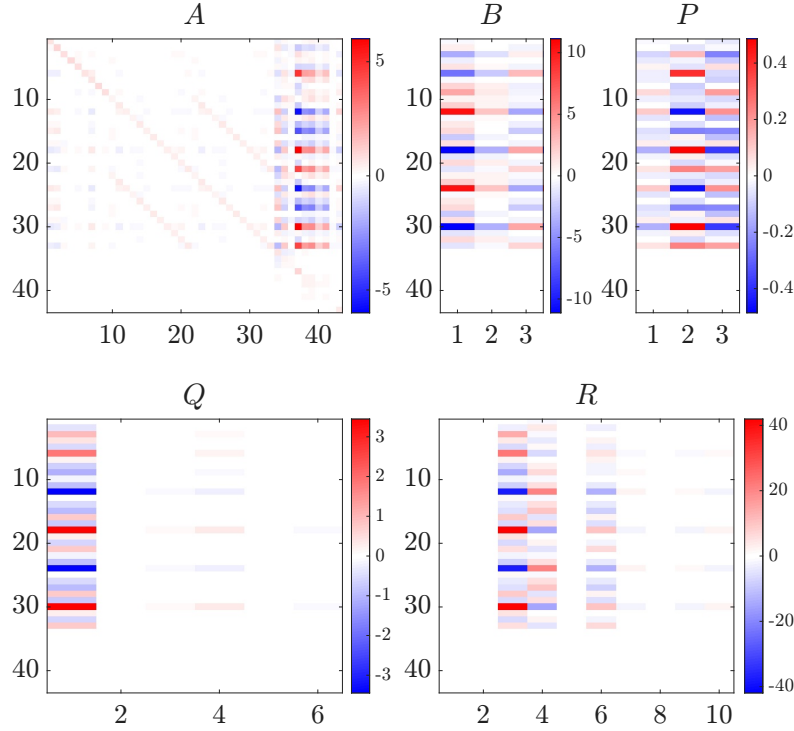


Figure 5.17: Estimated matrices with tapping at root of the structure.

5.2.2 Identification results from tapping at edge of the structure

This identification method results in more vibration modes visible in the data and smaller disturbance in Hub angles as compared to the previous identification measurements. The closed loop angles and angular velocities as shown in Figure 5.18, left and right respectively, have higher frequency content as compared to tapping at root (Figure 5.13). Measurements are started slightly after the tapping, when the closed-loop torque from the CMG starts to kick in. Singular value thresholds for measurement, input augmented matrix and shifted measurement matrix are shown in Figure 5.19

Experimental hub quaternion and the corresponding reconstructed components are shown in Figure 5.20 with exceptional performance. However, the algorithm results in partial reconstruction of the elastic deformations at the side mid points in the structure as shown in Figure 5.21. The frequencies are well matched but the amplitudes become incorrect at a later time in the training data.

The estimated matrices for the ROM are shown in Figure 5.22. Matrix A is observed to have higher values in several columns at the end, suggesting strong dependency

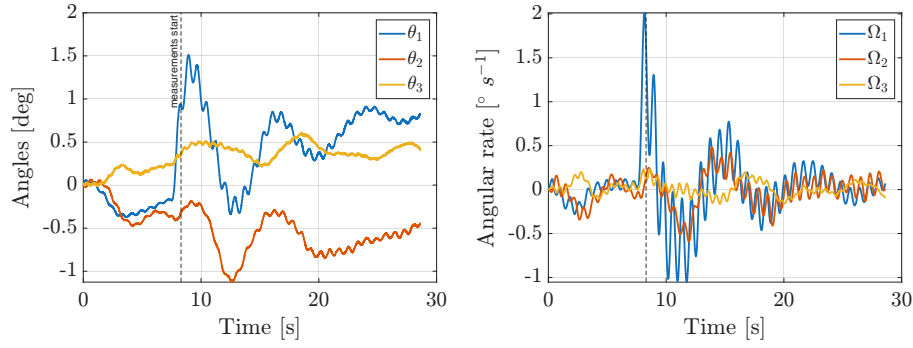


Figure 5.18: Hub angles and angular velocities due to tapping excitation.

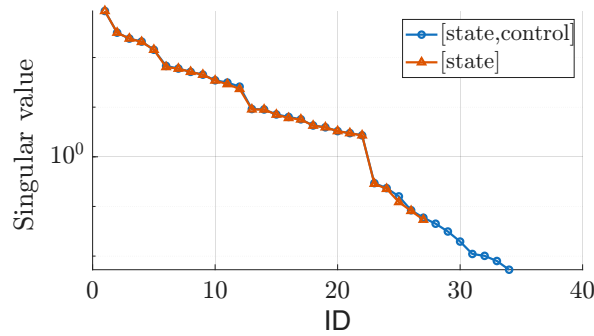


Figure 5.19: Singular values.

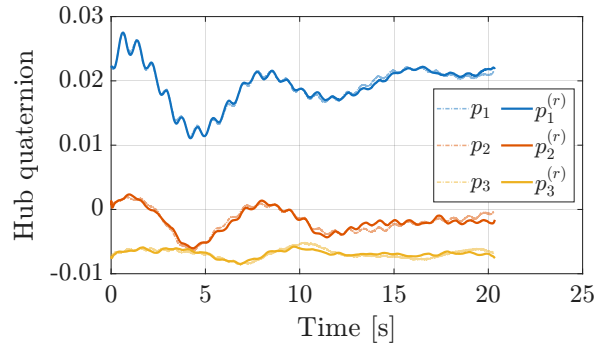


Figure 5.20: Reconstruction of hub root angles.

on CMG angular momentum H_a , hub angular velocity Ω_h and hub orientation quaternion p_h . A low magnitude in matrix P suggests small state-input coupling $\Omega_h \times H_a$ between hub angular velocity Ω_h and CMG angular momentum H_a . Similarly, magnitude of components in matrix Q points to a small nonlinearity in the dynamics due to Coriolis torque $I \times I \Omega$. Higher values in matrix R means that the gravitational torque is still affecting the the dynamics which explains the DC component in elastic displacements. Within matrix R , higher values in columns four $p_4 p_1$ and six $p_2 p_4$ representing the basis $\bar{p}$ (see Equation 3.19 is explained by a higher out-of-plane

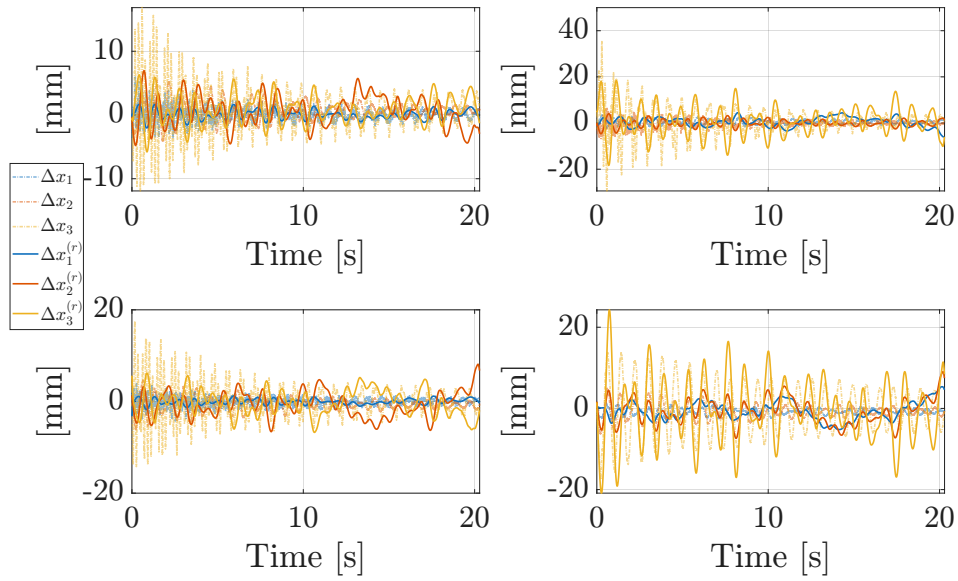


Figure 5.21: Reconstruction of elastic displacements of side mid points.

component of the COM r_3 , (see Equation 3.17b).

5.3 Maneuver optimization on experimental ROM and prediction results

The baseline for slew maneuver experiments, the seventh degree polynomial $\mathbb{P}^7$ defined in Subsection 2.2.1 is used as a reference to compare results. A ninth degree polynomial is used to address residual deformation reduction in the flexible structure. Similar to Section 3, a special case is considered where $[\lambda, \beta] = [0, 0]$ and the post maneuver requirements are to minimize the residual deformation in the flexible structure (second term in the *post* terms in Equation 3.34).

The algorithm in Section 3.3 is applied to the reduced order model as estimated in Subsections 5.2.1 and 5.2.2. The resulting optimal inputs (solid) in the form of actuator angular momentum for system identification with root disturbance (Figure 5.23, left) and edge disturbance (Figure 5.23, right) are compared with baseline seventh degree polynomial solution (dashed).

Corresponding hub angle comparison as *predicted* by ROM is shown in Figure 5.24 with resulting optimized maneuver exhibiting a slightly slower angle change in the first half and a faster variation in time in the second half in both these candidates. Elastic displacements as *predicted* by ROM are compared between the optimal (solid) and baseline (dashed) maneuver excitations at a single point on the middle of a side on the structure in Figure 5.25. Optimal maneuvers show better performance

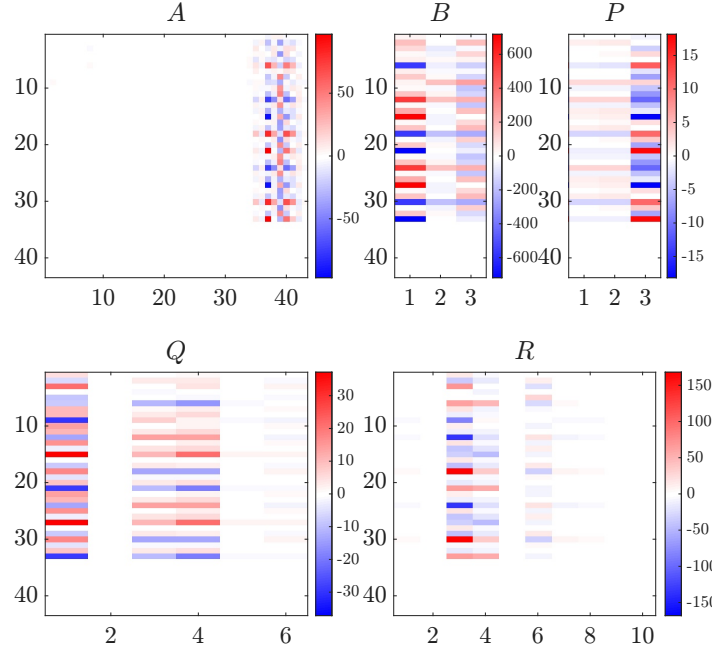


Figure 5.22: Estimated matrices with tapping at edge of the structure.

in significantly decreasing residual excitations for same duration baseline maneuver in ROM predictions. The magnitude of predicted displacement is accurate by the first candidate (Figure 5.25, left).

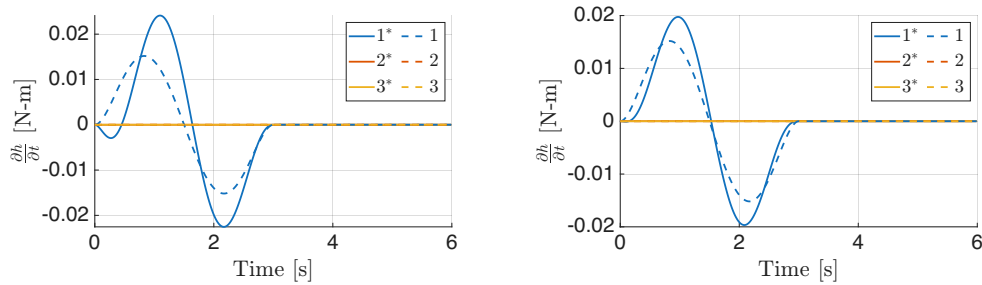


Figure 5.23: Momentum derivative candidates #1(left) and #2(right) from optimization with ROM based on experimental identification.

5.4 Experimental optimal slew maneuver results

These maneuvers are then selected to perform in the actual CMG experiment prototype and structural response measured. A comparison of results using the candidate maneuver #1 and #2 in the form of elastic displacement at a corner point in the structure is shown in Figures 5.26 and 5.27 , left and right respectively in solid lines. When compared to the baseline seventh degree maneuver in dashed line, both

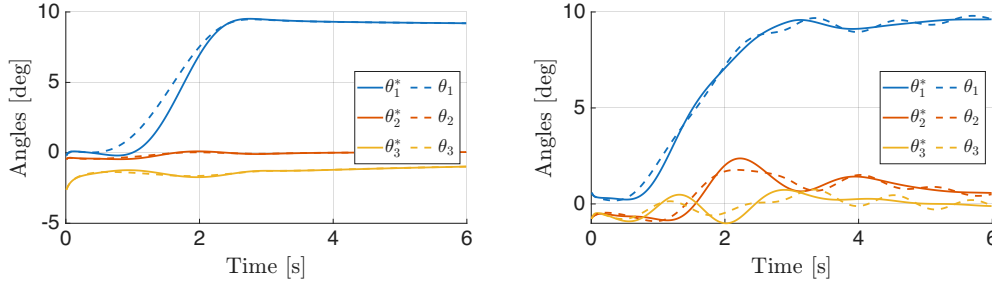


Figure 5.24: ROM hub angle predictions from candidates #1(left) and #2(right) shown in solid relative to baseline polynomial prediction in dashed lines.

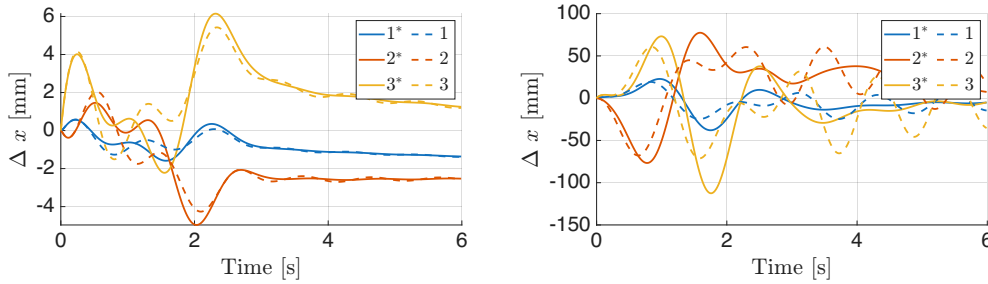


Figure 5.25: ROM elastic displacement prediction at a single side mid point from candidates #1(left) and #2(right) shown in solid relative to baseline polynomial prediction in dashed lines.

these candidates show promising reduction in residual vibrations in direction 2 and 3. Similar to input shaped maneuvers, peak vibrations in direction 1 are unchanged excluding the DC component.

In the case of side mid point on the structure, both optimal maneuver candidates do not show any decrease or increase in the residual excitations as seen in Figures 5.28 and 5.29.

5.5 Discussion

This chapter has presented results of the novel identification technique developed in Chapter 3 when applied to the experimental slew maneuver apparatus developed in Chapter 4. CMG/flexible structure system in closed-loop is provided a torque impulse by gentle tapping at a location and measurements performed some time after the tapping. Two Reduced-Order-Models(ROMs) are obtained in two separate experiments with change being the disturbance location, i.e. root vs. edge. Both these models successfully *reconstruct* the structural response within the training data. ROM established with root disturbance performed better for *predictions* in terms of hub angle prediction and magnitude of elastic deformations in the

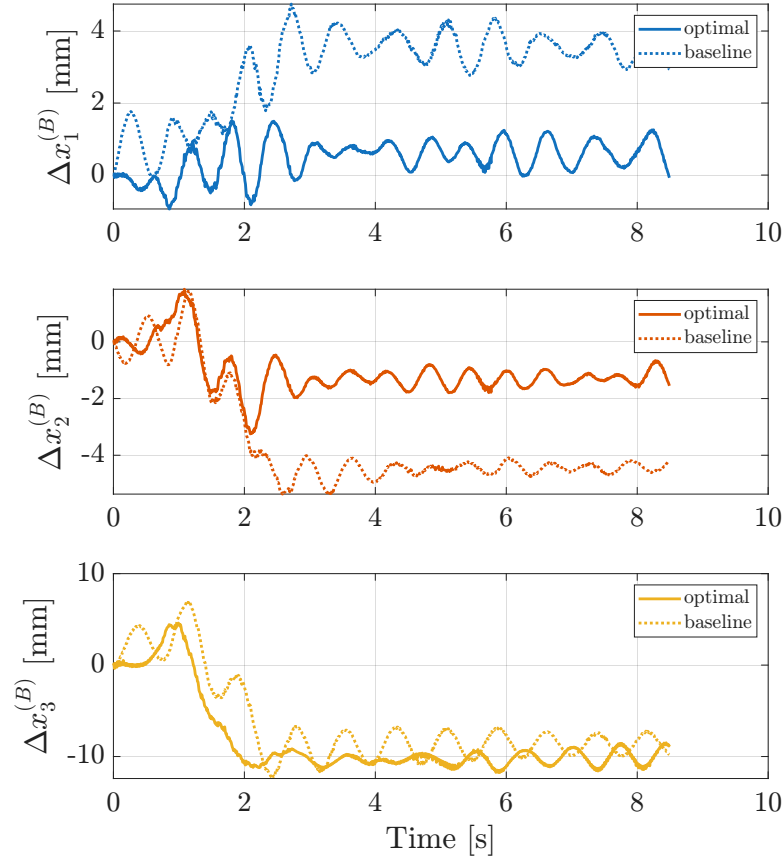


Figure 5.26: Experimental elastic displacements at a single corner point with torque candidates #1 shown in solid relative to baseline polynomial excitation in dashed lines.

structure. Maneuver optimization is performed in each of the two ROMs and this results in two vibration optimal maneuver candidates. Both these candidates show a promising of 60-70% reduction in elastic deformation. This is less effective as compared to input-shaped maneuver with 80% reduction in Chapter 4. Another observation in the structural response from both the optimized maneuvers is limited time vibration suppression. After about 4 s post maneuver, the elastic displacements begin to increase and reach a value similar to baseline maneuver response. This is undesirable and could be happening due to the following reasons.

- **Spillover:** Proposed DMDc considers only the modes excited during tapping. Higher order modes can make the system unstable in long term. Often the structural damping suppresses this phenomenon automatically but not always.

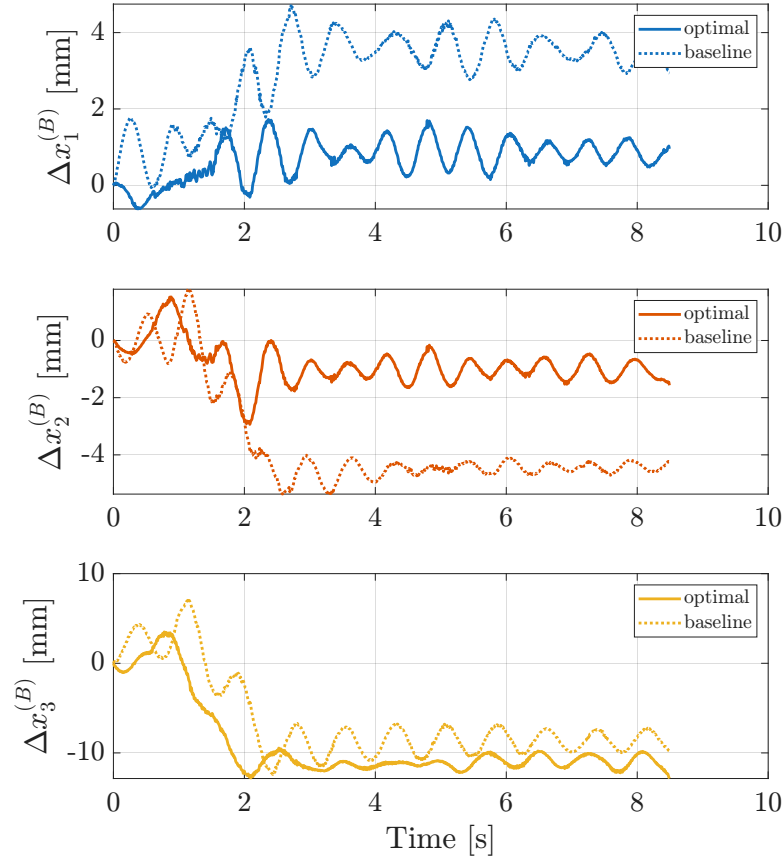


Figure 5.27: Experimental elastic displacements at a single corner point with torque candidates #2 shown in solid relative to baseline polynomial excitation in dashed lines.

This has been pointed out as one of the major issues in active control of large space structures[63] suggesting nonlinear control strategies [64] and [65].

- **Observability/Controllability loss due to repeated eigenvalues:** Symmetric observation points can lead to loss of controllability. Detailed conditions are discussed in [66]. Moreover, modes captured during identification have very little overlap between their observability and controllability gramians leading to unbalance identification[67, Ch 9].
- **Simplified basis for optimal maneuvers:** The ninth-degree polynomial in Sec 2.2.2 provides a simplest basis with two degrees of freedom to shape the polynomial for optimization purposes. It is certainly possible that this is not enough for suppressing higher modes.

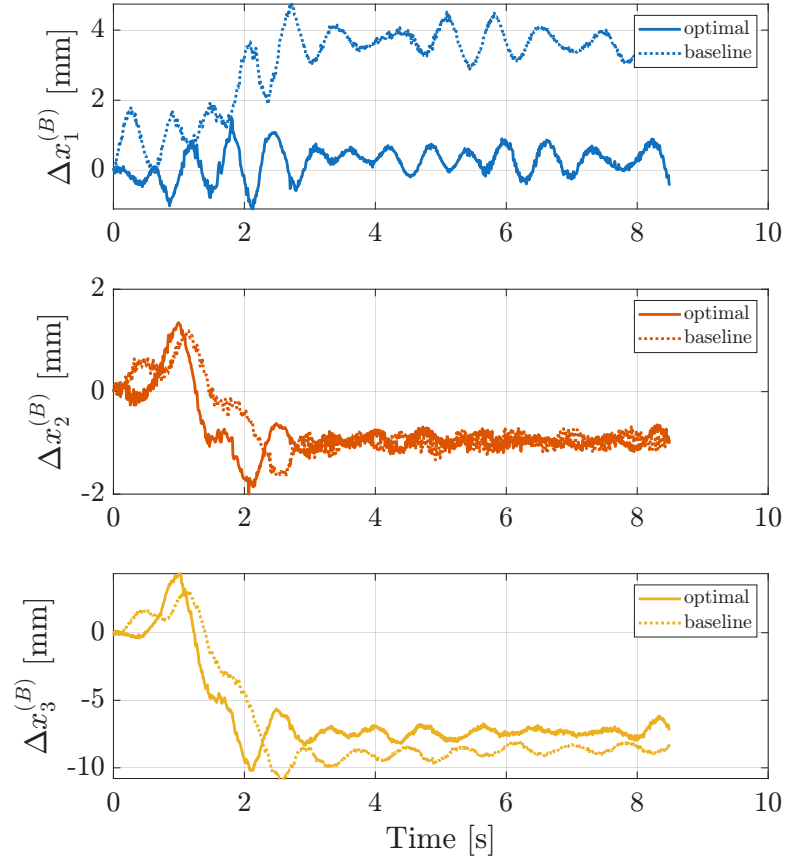


Figure 5.28: Experimental elastic displacements at a single side mid point with torque candidate #1 shown in solid relative to baseline polynomial excitation in dashed lines.

Regardless of these observations, the data-driven methodology has its strength in not requiring any assumption of the dominant mode shapes or frequencies. With future research towards the directions mentioned above, it is certainly applicable to real-world structures with uncertainties in stiffness and damping.

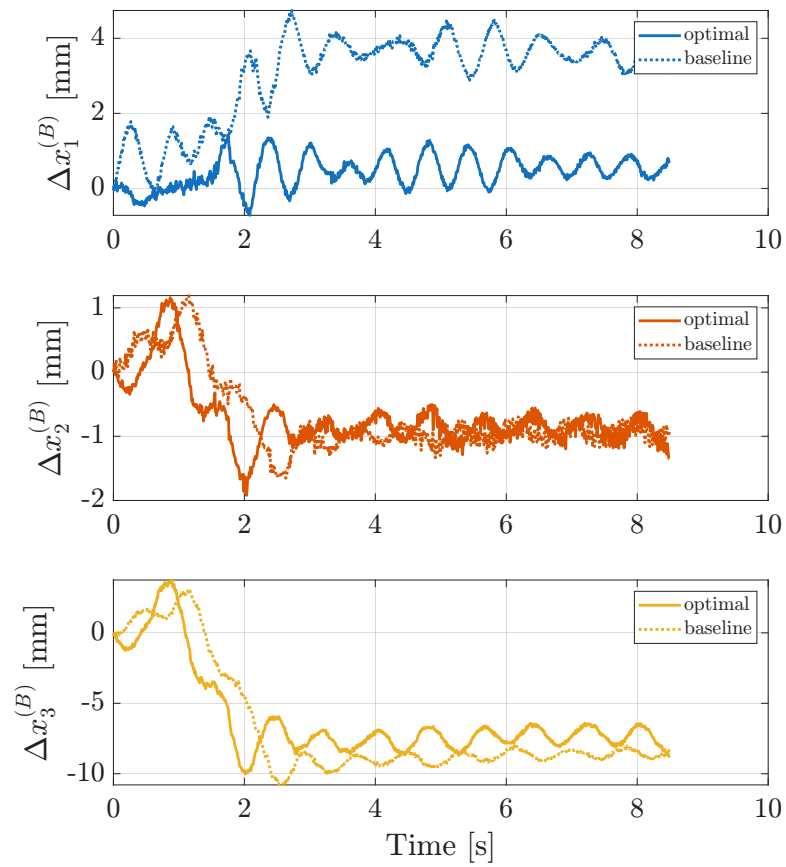


Figure 5.29: Experimental elastic displacements at a single side mid point with torque candidate #2 shown in solid relative to baseline polynomial excitation in dashed lines.

NEW ACTUATOR CONCEPT FOR ULTRALIGHT SPACECRAFT

This chapter presents the design of a novel orientation control actuator for SSPP architecture of size $10 \text{ m} \times 10 \text{ m}$. The work presented is motivated by utilizing the existing volume on the spacecraft for attitude actuator capability. Components of the SSPP deployment mechanism DOLCE[8] are used for this purpose. Analysis on sizing of the actuator is presented. This is followed by optimization of maneuver execution with regards to prevention of geometrical interferences and ensuring repeatability as required by the periodic maneuvers in the power optimal guidance[10]. Finally, closed-loop control strategy is motivated based on sensitivity studies performed in the open-loop control.

6.1 A new momentum control device

With the onset of large aperture spacecraft, attitude actuators need to be innovative and efficient. For example momentum flap on JWST telescope allows for counterbalance[68] of solar pressure from sun-shield, saving significant amount of propellant for the additional trim torque throughout mission life. Mass translators and reflectivity control devices on ACS3 spacecraft enable redundancy in control strategies [69]. The International Space Station (ISS) is controlled by CMGs[50, Ch-7], which are custom-designed for high-inertia applications. It has four parallel-mounted double-gimbal CMGs. Rotors have an RPM of 6600 with total angular momentum of 3500 ft-lb-s or 4745 N-m-s each about its spin axis. This value is a full order of magnitude higher compared to 100 N-m-s available commercially.

The work in this chapter uses the inverse dynamics of rigid spacecraft as developed in Chapter 2 to design a novel momentum actuator design governed by the seventh degree polynomial slew maneuver (Subsection 2.2.1). It has been shown that these maneuvers have several benefits compared to more traditional minimum time bang-bang maneuvers. Especially for a rigid/flexible body system with low damping, these maneuvers are effective in achieving less angular velocities in the residual vibrations [9, Section 3]. For the purpose of the work presented in this chapter, a maneuver with duration $T = 16 \text{ s}$ ($\alpha = 5$) and angle change of $\Delta\theta = 90^\circ$ (Figure 6.1) is selected.

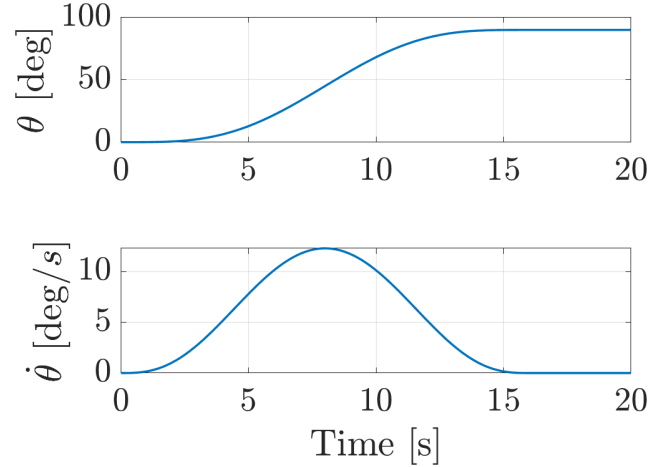


Figure 6.1: Smooth polynomial maneuver with 90° angle change in 16 s.

For the SSPS architecture, we estimate momentum requirements by a scaling study for different spacecraft sizes assuming a planar architecture with areal density 0.1 kg/m^2 [4]. We utilize peak angular velocities within the seventh-degree zero-jerk smooth polynomial maneuver[9] to estimate the maximum momentum exchange required. Different time duration of maneuvers relative to the spacecraft flexibility can change the required peak angular momentum. Maneuvers ranging from highly agile ($\alpha = 5$) to relatively slower maneuver ($\alpha = 20$) are considered. We use frequency of the flexible spacecraft from previous work[6] (Figure 6.2, top) to define agility relative to the maneuver duration.

Spacecraft of different sizes are considered ranging from 10 m to 60 m. Peak torque and peak angular momentum required for four different maneuvers of increasing agility are plotted in Figure 6.2, mid and bottom respectively. Spacecraft larger than 30 m x 30 m in size requires angular momentum above 100 Nms even for relatively slow state of the art maneuvers with $\alpha = 20$ as shown in Figure 6.2, bottom. Moreover, the target size of 60 m SSPP requires two to three orders of magnitude higher angular momentum than 10 m spacecraft. All these observations motivate our need to design a custom momentum exchange actuator.

The main goal of this chapter is to present a preliminary design of a momentum control device integrated with the existing deployment mechanism for the structure. Momentum device design is presented, followed by the governing equations of motion for the designed configuration. Next, we focus on actuator sizing based on selected maneuver requirements, assuming a rigid spacecraft with details shown in Table 6.1. Due to the system being over-actuated, there exist infinitely many

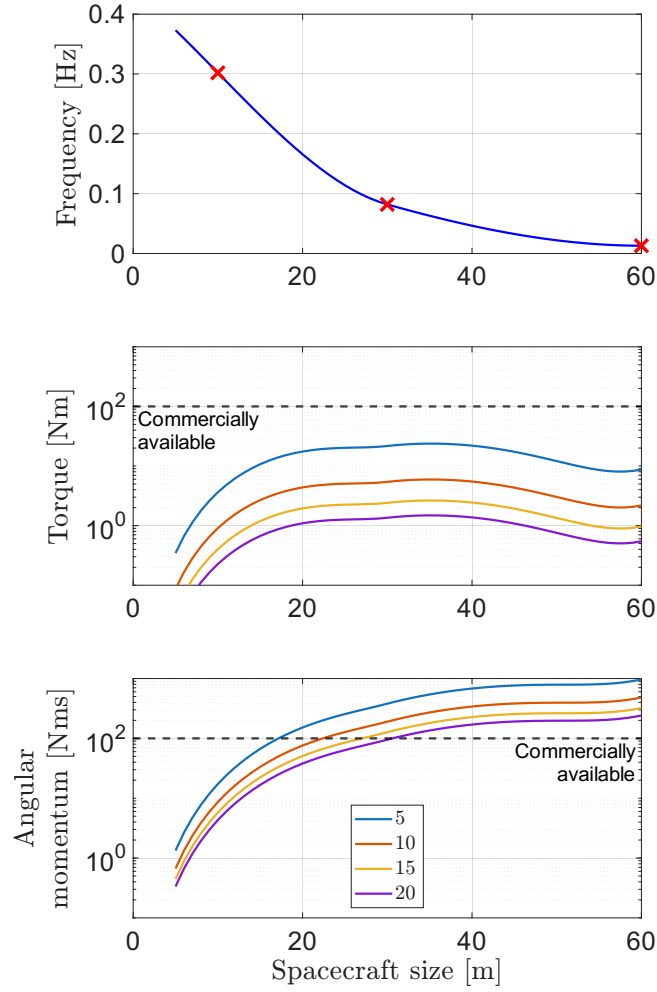


Figure 6.2: Fundamental frequency of SSPS structure FE model[6] (top), Torque required (middle), Momentum required at different α (bottom).

solutions to perform a single maneuver. This behavior is exploited by performing optimization on the solutions to develop efficient actuation of the device. Once the design is selected, we combine the developed framework with a local sensitivity analysis to understand the effect of parameters on spacecraft dynamics. Based on these studies, we then present results of rigid spacecraft maneuvers using the closed-loop quaternion feedback scheme developed in Section 2.4 based on inverse control.

Quantity	Magnitude	Units
I_{xx}	172.1	kgm^2
I_{yy}	86.5	kgm^2
I_{zz}	86.5	kgm^2
D	10	m

Table 6.1: Spacecraft parameters.

6.2 Mechanism concept

In the present work, we propose to develop an SGCMG cluster using components from the DOLCE deployment mechanism[8] itself. In the previous work, the cylinders act as a central support for the pressure wrapped structure and while deployment hold the folded strips at a higher strain energy state. Once deployed, these cylinders release the pressure wrapped structure which then achieves the flat deployed shape. However, once the deployment is over, these cylinders remain functionally inactive and become a dead mass.

The present work proposes an idea to add high inertia rotors in these cylinders thereby creating multi-functionality. These rotor components can spin about their individual axes to store angular momentum. These spin axes can be controlled independently using high torque motors. With four inputs in cylinder orientation, control of three rotational degrees of freedom of the spacecraft is achieved. A sketch of the idea is shown in Figure 6.3.

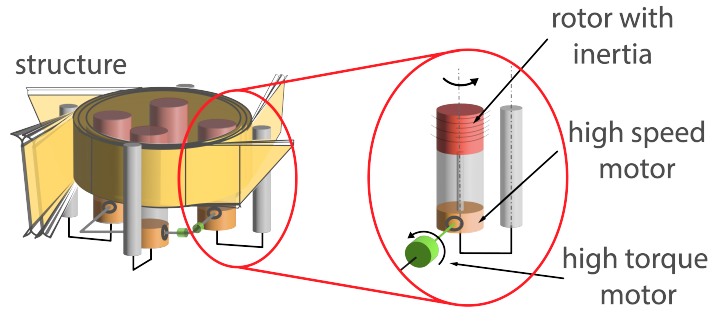


Figure 6.3: SSPS momentum control device with the spacecraft structure stowed.

6.2.1 System description and equations of motion

Consider a 10 m x 10 m SSPS architecture with the satellite hub at the center of the structure (Figure 6.5). For the purpose of designing the attitude control device, the whole hub-structure combination is considered rigid body with inertia tensor $\mathbf{I}$ and angular momentum $\mathbf{H}$. One can define a body fixed Cartesian coordinate frame

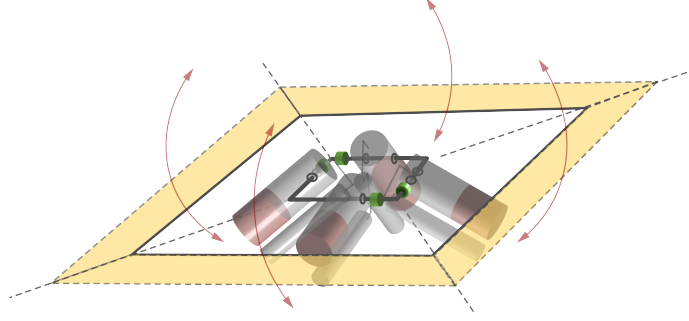


Figure 6.4: SSPS momentum control device with structure deployed and attitude control functionality active.

B at the center of mass of the spacecraft with B_2 and B_3 along the eigenaxes in plane and B_1 along the normal as shown in Figure 6.5. Consider another frame O coincident with frame B at time $t=0$ but is fixed to an inertial observer. Momentum device will have four cylinders spinning about their independent axes s_j determined by two angles (ψ_j, δ_j) . These axes can be independently controlled via high torque motors in the base of the mechanism. Each cylinder is assumed solid with inertia tensor with respect to a coordinate frame located at one of its ends as $J = \text{diag} \left(\frac{ML^2+3R^2}{12}, \frac{ML^2+3R^2}{12}, \frac{MR^2}{2} \right)$. Inertia components about the spin axes are used to estimate stored angular momentum while the other components are required to estimate the torque input in the high torque motors. Maneuver $\theta(t)$ about B_2 is achieved using independent motion of the cylinders as shown in Figure 6.5.

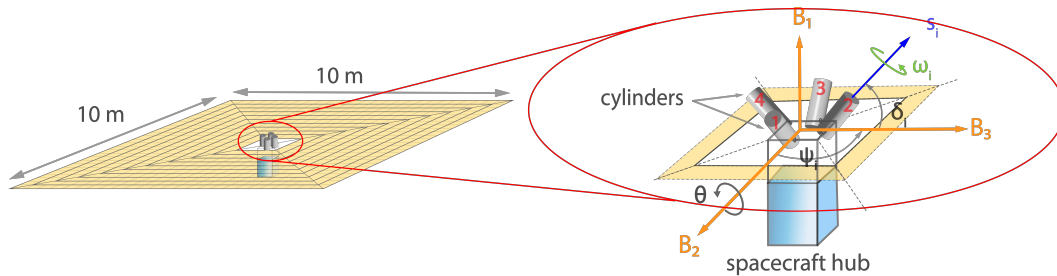


Figure 6.5: Momentum control device nomenclature.

The total angular momentum of the closed system is sum of angular momentum of spacecraft with structure plus that of the momentum device actuator

$$\mathbf{H}_T = [\mathbf{I}]\mathbf{\Omega} + H_a \quad (6.1)$$

where, $\mathbf{\Omega}$ is the angular velocity of rigid spacecraft/structure combination. If each rotor is spinning with the same angular velocity ω , one can write the total angular momentum stored in the momentum device actuator based on gimbal angle of each

cylinder δ_i as follows:

$$\mathbf{H}_a = \begin{bmatrix} \sin(\delta_1) & \sin(\delta_2) & \sin(\delta_3) & \sin(\delta_4) \\ \cos(\delta_1) & 0 & -\cos(\delta_3) & 0 \\ 0 & \cos(\delta_2) & 0 & -\cos(\delta_4) \end{bmatrix} \mathbf{h}_o = [\mathbf{B}] \mathbf{h}_o \quad (6.2)$$

where $\mathbf{h}_o = J_{33}\omega [1, 1, 1, 1]^T = h_o [1, 1, 1, 1]^T$ is the vector of scalar angular momentum of rotors. Matrix, $[\mathbf{B}]$ is called the configuration matrix. Assuming absence of external torques, total angular momentum is conserved:

$$\frac{d\mathbf{H}_T}{dt} = \mathbf{0} \quad (6.3)$$

Due to non-inertial rotating frame B above equation becomes,

$$\mathbf{H}_T + \boldsymbol{\Omega} \times \mathbf{H}_T = \mathbf{0} \quad (6.4)$$

Using equations 6.1, 6.2 in 6.4 we can get a system of equations between spacecraft angular velocity $\boldsymbol{\Omega}$ and actuator torque $\mathbf{N}_a$

$$[\mathbf{I}]\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \times [\mathbf{I}]\boldsymbol{\Omega} = \mathbf{N}_a \quad (6.5)$$

$$-\mathbf{N}_a = \mathbf{H}_a + \boldsymbol{\Omega} \times \mathbf{H}_a \quad (6.6)$$

Time derivative of actuator angular momentum depends on the Jacobian matrix $[\mathbf{A}]$

$$\dot{\mathbf{H}}_a = \begin{bmatrix} \cos(\delta_1) & \cos(\delta_2) & \cos(\delta_3) & \cos(\delta_4) \\ -\sin(\delta_1) & 0 & \sin(\delta_3) & 0 \\ 0 & -\sin(\delta_2) & 0 & \sin(\delta_4) \end{bmatrix} h_o \dot{\boldsymbol{\delta}} = [\mathbf{A}] \delta h_o \quad (6.7)$$

Using equations 6.5, 6.6, and 6.7, one can develop a solution for gimbal angles $\boldsymbol{\delta} \in \mathbb{R}^4$ given the maneuver $\boldsymbol{\Omega}$. The system of equations are nonlinear three ODEs with four states being the gimbal angles. A sample solution for initial value of $\boldsymbol{\delta}_o = \{45^\circ, -45^\circ, 45^\circ, -45^\circ\}$ is shown in Figure 6.6.

6.3 Design exploration

This section will include design study relevant to rotor sizing. This will be followed by optimal selection of gimbal angle solution for the chosen design based on geometric and periodic constraints.

6.3.1 Rotor design and sizing

Several parameters for the rotors are studied, including radius (r), mass (M), and angular spin velocity (ω). We generate design spaces by identifying whether the

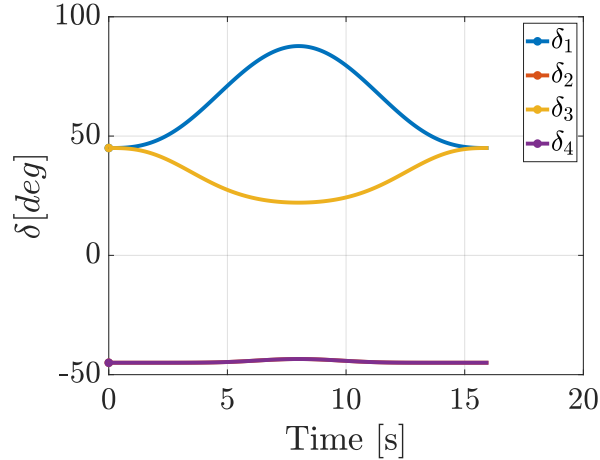


Figure 6.6: Gimbal angle solution for 90° polynomial slew with duration 16 s.

solution for the nonlinear system of equations exists. This is achieved by numerical integration of equations 6.5 and 6.6.

Let $(r, M, \omega) \in \mathbb{A} \subset \mathbb{R}^3$ be the *variable space* constrained with possible radius, mass and velocity of the cylinder. We can define a map $\mathcal{S} : \mathbb{A} \rightarrow \{0, 1\}$ where "1" represents the scenario when the non linear system (Eqns 6.5,6.6) has a solution for the polynomial smooth maneuver $\theta(t) \forall t \in [0, T]$ and "0" otherwise. It turns out that $\mathbb{A}$ space has multiple design points satisfying the solution requirements i.e. multiple design points with value "1". 10,000 designs points are explored in each of the two dimensional design spaces. In the solution zone, input torque in the high torque motor is calculated for each design. The peak torque required in the cylinders (N_g) to perform the maneuver is a function of radius, mass and RPM. It will be utilized to define the optimal set $\mathbb{A}^*$ as follows

$$N_g(r, M, \omega) = J_{11} \ddot{\delta}(t; r, M, \omega) \quad (6.8)$$

$$\mathbb{A}^* = \left\{ (r^*, M^*, \omega^*) : \arg \min_{(r, M, \omega) \in \mathbb{A}} [N_g] \right\} \quad (6.9)$$

Results

For the purpose of developing simple visualizations, only two variables are varied at a time and the third one is kept constant. All results below are for initial angle vector $\delta_o = \frac{\pi}{4}(1, -1, 1, -1)$. Solution variable equals zero is defined as non convergence at $(\Delta t)_{min} \approx 10^{-13}$ in the Runge-Kutta Four solver used for numerically solving Equations 6.5 and 6.6. Nominal cylinders are 1 m long. Results for (R, M) design space are shown in Figures 6.7 and 6.8. We can observe a clear separation boundary

between solution and no solution. Results for (R, ω) space are shown in Figures 6.9 and 6.10. It turns out that $\mathbb{A}^*$ or the solution boundary is not a good design point because of the no solution risk it accompanies if the practical uncertainty is variables is accounted for. Hence a design point (red star) is chosen away from the boundary.

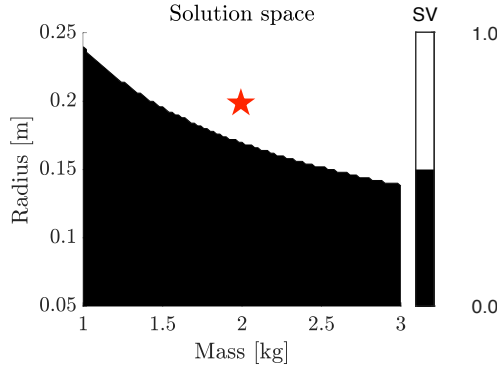


Figure 6.7: Solution space for mass-radius parameters for 5000 RPM in cylinders.

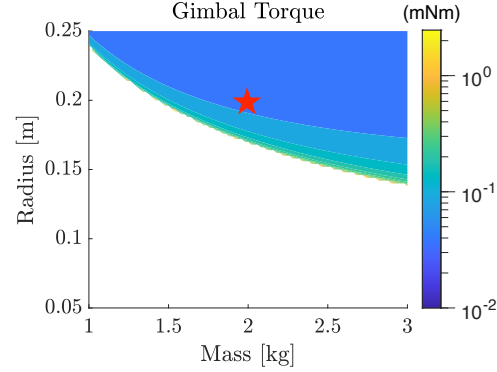


Figure 6.8: Torque contours for mass-radius solution space for 5000 RPM in cylinders.

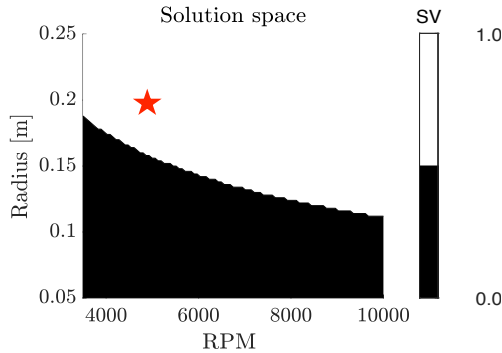


Figure 6.9: Solution space for radius-RPM parameters for 2 kg cylinder.

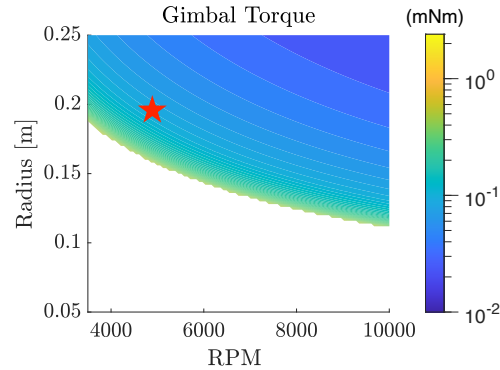


Figure 6.10: Torque contours for radius-RPM solution space for 2 kg cylinder.

6.3.2 Geometry-constrained maneuver execution.

The second study is focused on exploring optimal initial positions of the cylinders based on practical requirements faced during the maneuver execution. For e.g. , during the maneuver, a $|\delta_i| > 90^\circ$ would mean the cylinders interference with the payloads on top/bottom of the spacecraft hub. For solution shown in Figure 6.6 at $t = 5$ s, interference situation is shown in Figure 6.11. This is taken care off by exploring initial position space ($\mathbb{G} \in \mathbb{R}^4$) of the cylinders.

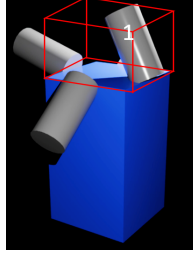


Figure 6.11: Interference with spacecraft components.

Finding the best initial positions of the cylinders is formulated as an optimization problem. Pattern search optimization routine is utilized. Several design parameters were frozen within variable space $\mathbb{A}$ to establish variation only over $\mathbb{G}$ space. 2 kg cylinders with radius 0.2 m and length 0.5 m spinning at 5,000 RPM are considered. Objective function to establish constrained movement of cylinders within $|\delta_i| \leq \delta_m = 90^\circ$ is as follows:

$$\text{Min}_{\delta_o \in \mathbb{G}} f_1(\delta_o) = \text{Min}_{\delta_o \in \mathbb{G}} \left\{ \text{Max}_{i \in \{1,2,3,4\}; t \in [0,T]} |\delta_i(t)| \right\} \quad (6.10)$$

Results

After 200 iterations of the algorithm, the optimal initial position vector is estimated to be $\delta_o^* = [0^\circ, -22.8.4^\circ, 56.2^\circ, 18.0^\circ]$ with $f_1^* = 56.2^\circ$. Cylinder position profiles for the smooth polynomial maneuver at the optimal initial position is plotted in Figure 6.12, left. Trajectories from the optimal solutions are well constrained within $\pm 74^\circ$.

6.3.3 Periodicity constrained maneuver execution

Another requirement in maneuver execution is repetition of same maneuver every new pass[10]. Periodicity in the cylinder gimbal angles (δ_i) ensures optimal initial positioning for the next maneuver occurrence. Based on geometry constrained execution only, as shown in Figure 6.12 (left), noticing cylinders 2 and 4 (δ_2 and δ_4), they do not return back to their initial positions. This substantiates a need for modification in the objective function enforcing periodicity with maneuver time T as follows

$$\text{Min}_{\delta_o \in \mathbb{G}} f_2(\delta_o) = \text{Min}_{\delta_o \in \mathbb{G}} \left\{ \text{Max}_{i \in \{1,2,3,4\}; t \in [0,T]} |\delta_i(t)| + \|\delta(T) - \delta(0)\| \right\} \quad (6.11)$$

With the updated objective function, the new optimal initial position is $\delta_o^* = [0.1^\circ, -24.6^\circ, 56.2^\circ, -24.6^\circ]$ with $f_2^* = 56.2^\circ$. Corresponding solution of gimbal angles is shown in Figure 6.12, right.

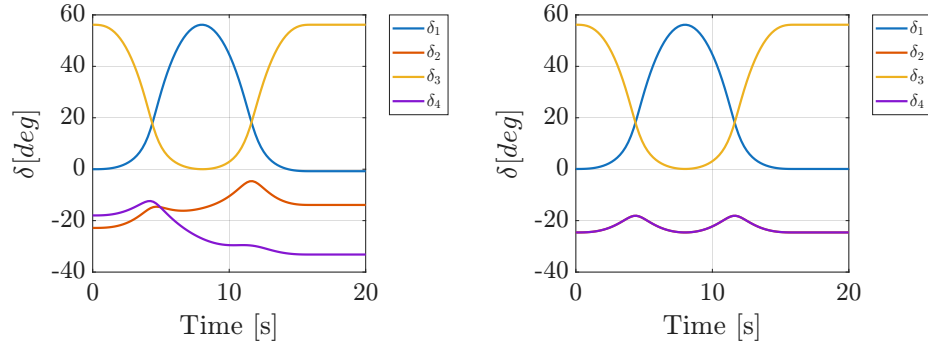


Figure 6.12: Resulting cylinder solution based on only constrained movement objective f_1 (left) and additional peridocity enforcement f_2 (right).

6.4 Sensitivity studies

The purpose of this section is to understand parameter sensitivities while designing the momentum control device. The designed system may not be perfect. First order sensitivity analysis is performed relative to the parameters of choice. This involves studying the effects of variations in the fixed coefficients or boundary conditions on the time varying states of the system. For a first order system such as the control moment gyro, one can develop sensitivity coefficient S_{ij} by calculating partial derivative of the system state y_i with model parameter a_j as per the algorithm in [70]

$$\mathbf{y}(t; \mathbf{a}) = \mathbf{f}(\mathbf{y}, t; \mathbf{a}); \quad \mathbf{y}(0) = \mathbf{y}_0; \quad \mathbf{y} \in \mathbb{R}^n, \mathbf{a} \in \mathbb{R}^m \quad (6.12)$$

The governing equations for the first-order sensitivity coefficients

$$[\dot{\mathbf{S}}] = [\mathbf{J}](t)[\mathbf{S}](t) + \frac{\partial \mathbf{f}(t)}{\partial \mathbf{a}} \quad (6.13)$$

$$S_{ij}(0) = \begin{cases} 1, & \text{if } a_j = y_i(0) \\ 0, & \text{otherwise} \end{cases} \quad (6.14)$$

where :

- $S_{ij} = \partial y_i / \partial a_j$ is the $n \times m$ sensitivity coefficient matrix
- $J_{ij} = \partial f_i / \partial y_j$ is the $n \times n$ Jacobian matrix

A major downside of this method is that it can only provide local sensitivities. It primarily focuses on understanding how our system's behavior changes in the vicinity of a chosen parameter configuration. Consequently it may fail to provide insight on

how combination of parameters could lead to higher/lower sensitivities. Nonetheless, it is a good starting point to develop intuitive understanding of parameter sensitivities.

One can select a variety of parameters for these studies including system inertia, initial conditions of the spacecraft, initial angles of the cylinders, rotor size etc. However, this chapter will discuss only the initial cylinder positions as parameters for maneuver sensitivity i.e. $\mathbf{a} = [\delta_1(0), \delta_2(0), \delta_3(0), \delta_4(0)]$. With this choice, we want to understand the implications of error in initial cylinder orientations ($\delta(0)$) onto the maneuver dynamics of the spacecraft undergoing the smooth polynomial maneuver $\theta(t)$. Assume that gimbal angle solution for no error in initial value is $\delta_p(t)$. In reality, these angles may have an error of δ_e at the beginning. The actual gimbal angles are sum of nominal and error values:

$$\delta(t) = \delta_p(t) + \delta_e \quad (6.15)$$

To obtain nominal solution, we use Equations 6.5, 6.6 and 6.7 which simplify to become Equation 6.16

$$[\mathbf{A}(\delta_p)]\delta_p h_o = [0, I_{22}\ddot{\theta}, 0]^T \quad (6.16)$$

Let the combined state $\mathbf{y}$ be composed of angular velocities and quaternion $\mathbf{y} = [\boldsymbol{\Omega}, \mathbf{p}]$. We use right handed Hamiltonian unitary quaternion[23] to define passive rotation from frame B to frame O . Time propagation is similar to approach in [71, Ch 2] as follows:

$$\mathbf{p} = 0.5 \begin{bmatrix} 0 & \Omega_3 & -\Omega_2 & \Omega_1 \\ -\Omega_3 & 0 & \Omega_1 & \Omega_2 \\ \Omega_2 & -\Omega_1 & 0 & \Omega_3 \\ -\Omega_1 & -\Omega_2 & -\Omega_3 & 0 \end{bmatrix} \mathbf{p} = [\mathbf{D}]\mathbf{p}; \mathbf{p} \in \mathbb{H}_1 \quad (6.17)$$

With the error in initial angles of gimbals, the forward dynamics of the spacecraft becomes

$$\mathbf{y} = \begin{bmatrix} \boldsymbol{\Omega} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -[\mathbf{I}]^{-1} [[\boldsymbol{\Omega}] \times [\mathbf{I}]\boldsymbol{\Omega} + [\mathbf{A}]\delta h_o + [\boldsymbol{\Omega}] \times [\mathbf{B}]h_o] \\ [\mathbf{D}]\mathbf{p} \end{bmatrix}; \delta(t) = \delta_p(t) + \delta_e \quad (6.18)$$

$$= \mathbf{f}(t, \mathbf{y}; \mathbf{a}) \quad ; \mathbf{a} = \delta_e \quad (6.19)$$

Open loop results

Using the results from constrained and periodic motion of the cylinders, the initial orientation is chosen to be: $[-4.8^\circ, 31.4^\circ, -73.2^\circ, 31.4^\circ]$. Sensitivity coefficient history is calculated with zero parameter values $\mathbf{a} = [0, 0, 0, 0]^T$ locally. Coefficients for angular velocities are shown in Figure 6.13. In the maneuver direction, Ω_2 is the most sensitive with a sensitivity of $-0.16^\circ/s$ per 1° of error in the initial value of the third gimbal. However, in the non-maneuver direction, Ω_3 is far more sensitive with $0.37^\circ/s$ per 1° of error in the initial value of the second gimbal. Similar plots can be created for quaternion sensitivities. However, they show an identical trend of increased sensitivity in the non-maneuver components.

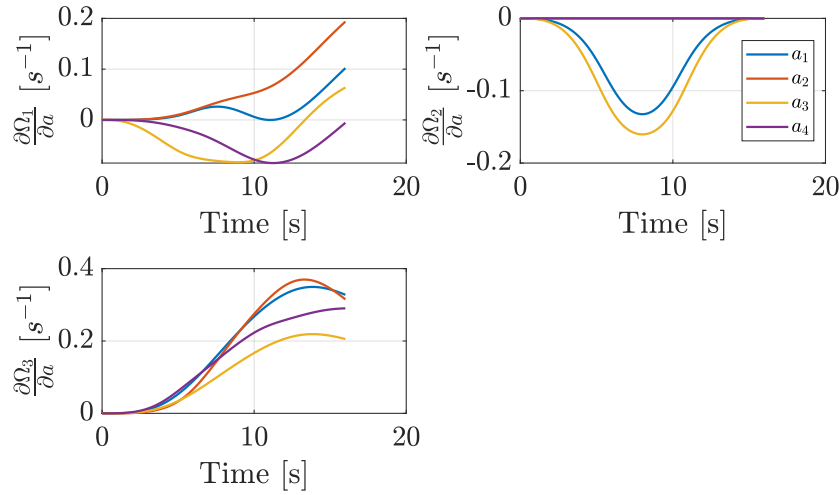


Figure 6.13: Angular velocity sensitivity coefficients.

To further increase understanding of the dynamics, one can purposefully add error in the initial values of the gimbals. An error of $+1^\circ$ is provided in initial values of all the gimbals. State history for maneuver dynamics in case of nominal and error in initial conditions is shown in Figure 6.14. Spacecraft Euler angles (ZYZ), in case of perturbed dynamics, are far from nominal at the end of maneuver. In the direction of maneuver, spacecraft orientation is off by -2.5° from the final 90° position and getting worse later on. In the other two directions, the results show non zero angles of -2.3° and 8.1° in the first and third Euler angle respectively.

6.4.1 Closed loop results

In this subsection, we show results using the control scheme proposed in Section 2.4. The inertia properties are taken from Table 6.1. We use the perturbation of $+1^\circ$

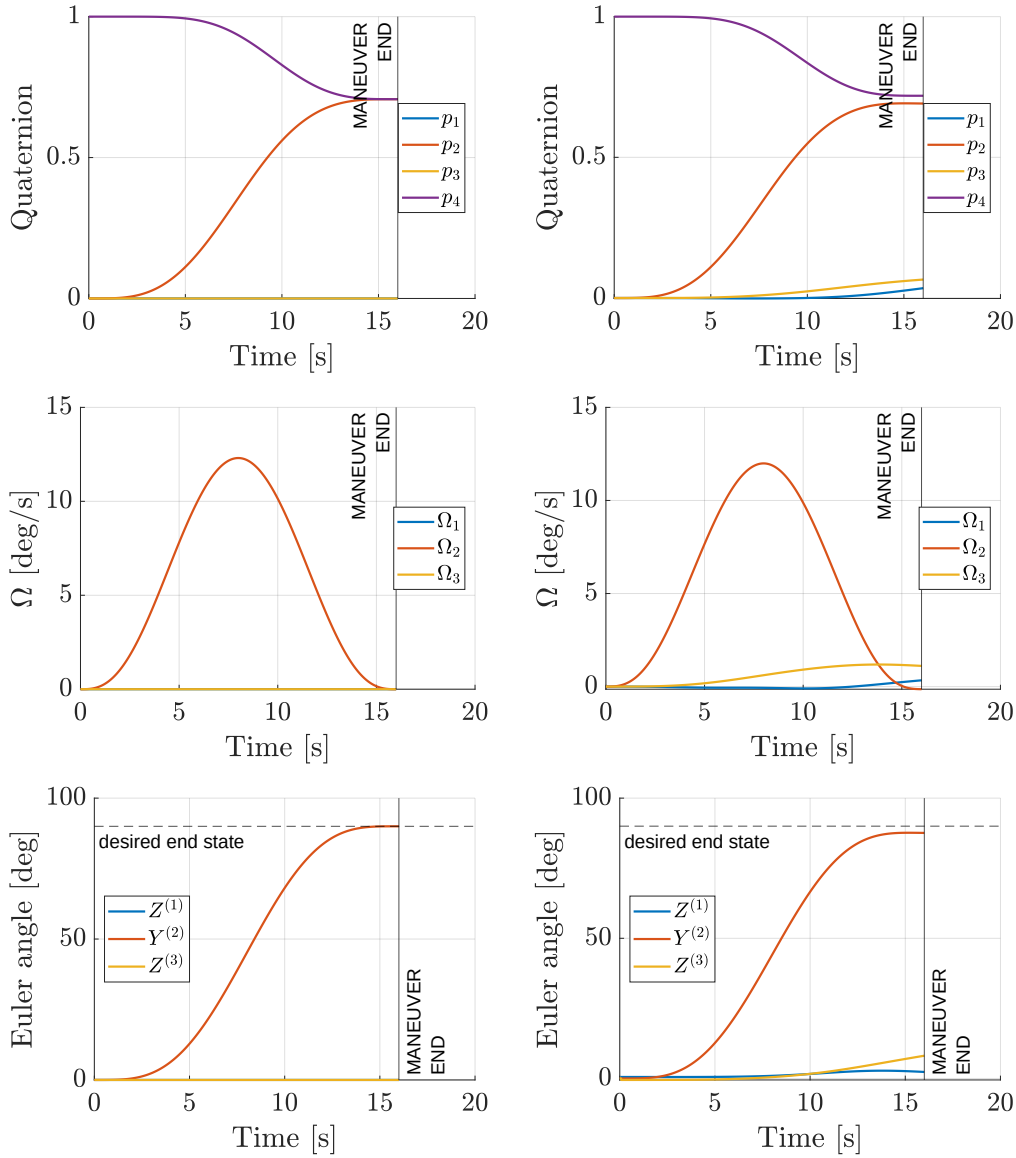


Figure 6.14: Spacecraft quaternion (top), angular velocities (middle) and ZYZ Euler angles (bottom) in open-loop control. Left three plots show nominal dynamics, right three show dynamics with a $+1^\circ$ error in initial angles of all gimbals.

error in initial gimbal angles. Thus, we keep same initial conditions corresponding to the open loop results in Figure 6.14. The reference trajectory is the same smooth polynomial as introduced in Equation 2.1 with time of maneuver 16 s and change in angle of 90° . The gains are chosen to be $(\lambda_1)_{kk} = 1$ and $(\lambda_2)_{kk} = 1$ with zero non diagonal terms. These values correspond to $\zeta_{cl} = 0.5$ and $\omega_{cl} = 1 \text{ rad/s}$ along each body axis. This simulation was performed in Simulink for a time span of 40 s.

Figure 6.15 shows time evolution of spacecraft quaternion, Euler angles and angular

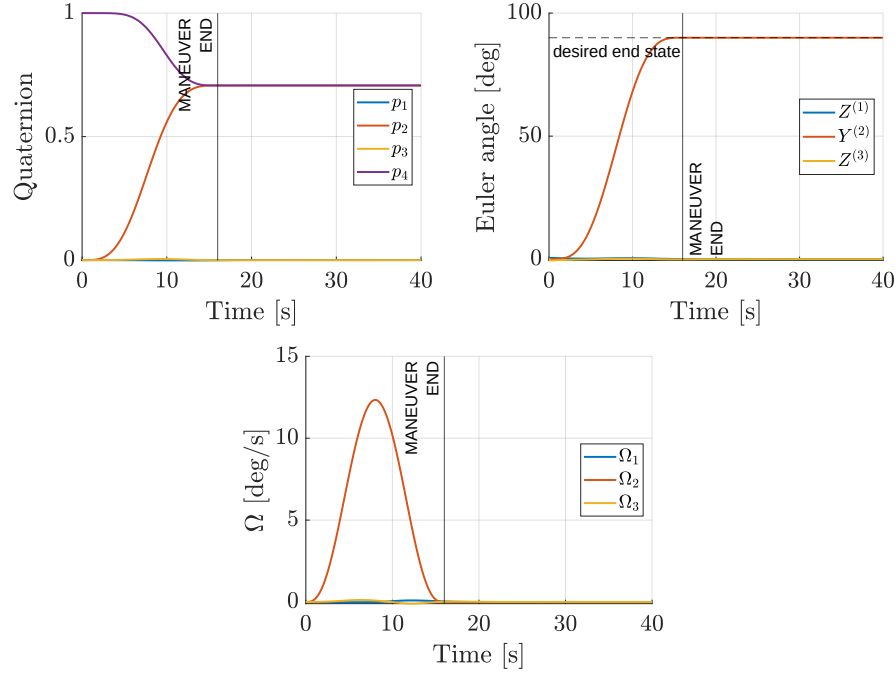


Figure 6.15: Spacecraft quaternion (left), angular velocities (middle) and ZYZ Euler angles (right) with closed-loop control.

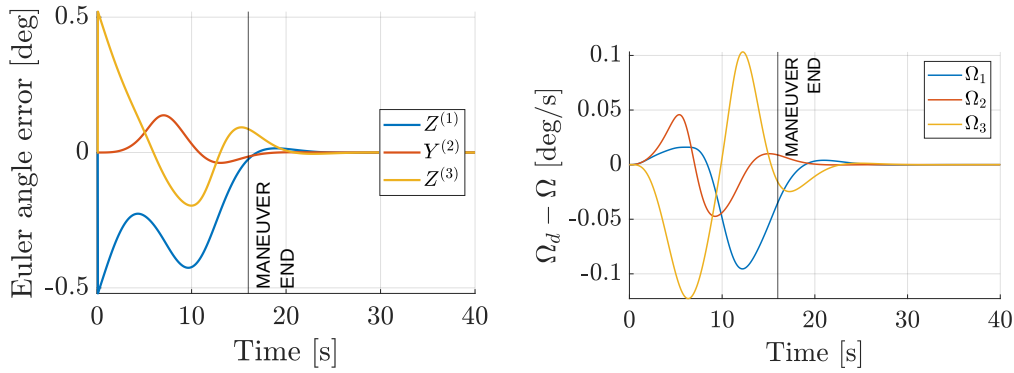


Figure 6.16: Error in Euler angles (left) and error in angular velocity components (right).

rates for close loop scheme proposed. As seen in Figure 6.16 (left), the error in Euler angles is bounded within 0.5° and converges to zero as the simulation goes forward. Similarly, in Figure 6.16 (right) the maximum error in angular velocity components are $[0.02, 0.05, 0.1]^\circ/s$ along respective body axis.

Time evolution of the components for constraint $\mathbf{s}(t)$ are shown in Figure 6.17. It takes about 10 s for the system to realize about the discrepancy in initial gimbal angles, which is roughly the same order as $T_{cl} = 2\pi\omega_{cl} = 6.3$ s. There is an

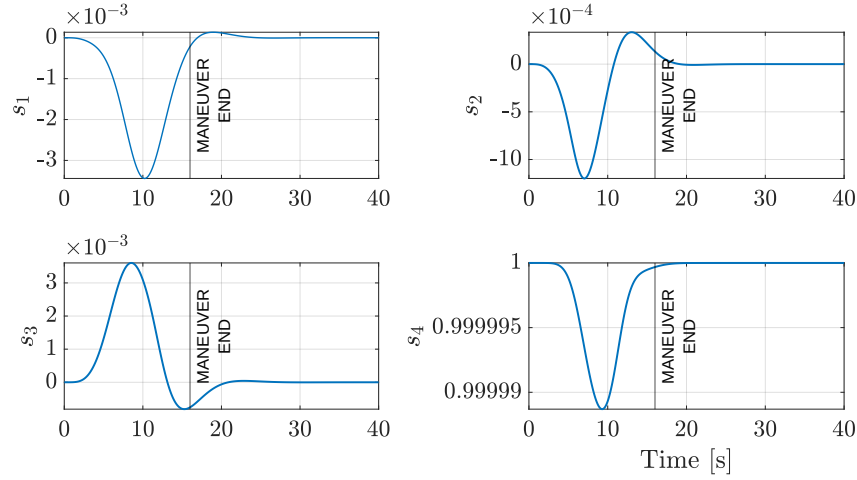


Figure 6.17: Time evolution of the constraint components.

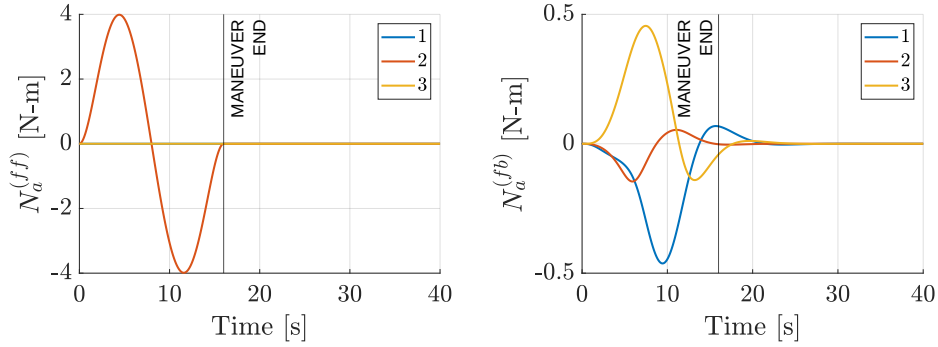


Figure 6.18: Actuator control effort terms, feedforward on left and feedback on right.

approximate 10 s settling time after the end of maneuver along each component which is very close to the approximate settling time for a second order system in a 2% tolerance band $t_s = \frac{4}{\zeta_{cl}\omega_{cl}} = 8 \text{ s}$

Control efforts are shown in Figure 6.18. Feedforward or the inverse component forms the majority of control effort with peak torque of 4 N-m. Feedback is 1/8 of the feedforward control, which points to the ability of closed loop system to handle the system uncertainty with ease. In case of an extreme perturbation in the system, the feedback will increase and might exceed the feedforward term.

6.5 Discussion

This chapter has described a novel concept for a momentum control device integrated with the existing deployment mechanism for a $10\text{ m} \times 10\text{ m}$ SSPS architecture with the goal of performing the required rest-to-rest maneuver. Sizing of the momentum device is performed by exploration of the design space of rotor parameters, size, spin speed, and mass. The resulting scatter plots contain all the information about the possible actuator sizes that are capable of performing the maneuver. After this, a single design is selected based on the practical implications of the actuator size and its accommodation together with the existing assembly without geometric interference. Once the rotor geometry is fixed, a methodology for performing the maneuver by identifying optimal initial cylinder positions is developed. A geometrically constrained motion is considered along with the periodicity in the cylinder motion as the criteria for the optimization scheme. These studies show that it is possible to achieve repeatable, constant eigen-axis rest-rest maneuvers for the SSPS architecture without relying on reorienting the cylinders each time. These changes in the maneuver execution improve the concept of operations of the spacecraft both in terms of power and complexity, considering a long mission life.

A sensitivity analysis for specific initial condition reveals that a higher state sensitivity is observed in the dynamics of non-maneuver directions. Open-loop simulations with a known error in initial conditions are performed. An error of 8.2° in Euler angles is estimated for an uncertainty as low as even 1° in initial cylinder angles. This could magnify into an alarming offset on the power transmitting beam when projected on the ground. Further, we also observed that the spacecraft enters into tumbling mode with rates as high as $1.5^\circ/\text{s}$ post maneuver. These results enforce the requirement of a closed-loop control scheme for attitude tacking. The developed framework resulted in convergence to the desired attitude within 8 seconds from the time of maneuver completion, for a very low 5% additional feedback effort compared to the open-loop maneuver.

Chapter 7

CONCLUSIONS

7.1 Summary and Contributions

This thesis has presented a feasible theory and experimental validation of a family of agile slew maneuvers for ultralight flexible spacecraft with natural frequencies several orders of magnitude lower than the conventional spacecraft. The traditional heuristic of designing maneuver duration based on lowest frequency of the architecture results in extremely slow maneuvers impacting efficiency of science objectives for the mission thereby creating economic implications. With the current research, a substantial improvement in maneuver times have been achieved in simulation and validated experimentally.

Chapter 2 provided a motivating example of Caltech's Space Solar Power Architecture [4] in the category Ultralight flexible spacecraft[9]. A theoretical framework for estimating structural response is presented with decoupling of equations using principal coordinate transformation (free-free mode shapes) taking into account the coupling effects from a momentum control device. Numerical simulation of slew maneuver with agility $\alpha = 5$ (much smaller than state of the art $\alpha \gg 10$) is performed and results are presented. Maximum out of plane deformation of 75 mm is observed for a 10 m $\times$ 10 m SSPP spacecraft undergoing a maneuver of duration 16 s and angle change of 90°. After the maneuver, the flexible structure results in a residual deformation along a set of mode shapes. These dominant modes are presented and found to be different than the lowest eigenvalue mode.

Chapter 3 presented a novel identification methodology for ultralight flexible spacecraft used to develop agile vibration optimal slew maneuvers. By using a small torque disturbance applied through thrusters in space, resulting flexible body deformations and hub orientation are proven to be useful measurements to develop a reduced order model. Additional functions of measurements have been proposed to capture nonlinearities. The algorithm has been applied to the numerical model of Caltech SSPP spacecraft developed in Chapter 2. A Reduced Order Model (ROM) has been developed by providing small torque disturbance with a finite time impulse in the form of a gaussian pulse to the hub of the spacecraft. The structural response due to the impulse has been perfectly *reconstructed* by means of the ROM

fit. Applying a seventh order polynomial zero-jerk maneuver torque to this ROM resulted in a good *prediction* of hub angles and angular velocities. Finally, this ROM is used to explore and minimize structural response within a family of maneuvers defined by a ninth degree zero-jerk polynomial. The resulting optimal maneuver when provided to the numerical model in Chapter 2 resulted in significant reduction in residual vibrations post maneuver.

Instead of applying an additional disturbance as proposed in this chapter for system identification technique, opportunistic identification seems to be more feasible in terms of fuel budget. For example, spacecraft constellation optimal trajectory requires several milli-Newtons of thrust per orbit for optimized orbit constellation trajectory as shown in [72]. This additional thrust firing could be utilized to periodically correct the spacecraft model, thus improving robustness.

Chapter 4 focused on addressing the challenges in performing slew maneuvers of a flexible body on earth while maintaining space-like conditions of a closed system (zero external force). Design of a Pyramid CMG is presented with details pertaining to the physical structure, electrical framework and center-of-mass balancing. Quaternion based closed-loop control to track hub orientation relative to a reference trajectory, as developed in Section 2.4, is implemented and demonstrated in the designed CMG. This chapter also developed new requirements for designing a small-scale flexible structure appropriate for slew maneuvers. The designed structure is sufficiently compliant, enabling agile slew maneuver tests, yet it is robust enough to not buckle under the influence of gravitational body loads. These properties allow experiments without the use of any gravity compensation like hanging springs, helium balloon supports etc. directly applied to the flexible structure. A seventh degree smooth polynomial zero jerk baseline maneuver (Section 2.2.1) with 4 s duration and 10° maneuver angle is performed within the measured 1.33° combined error in all directions. Even though demonstrated with a flexible structure attached, this error is very close to the *rigid body* published experimental results within the community [35], [73], [74]. Finally, structural response results from the baseline have been compared to the input-shaped smooth maneuver (Section 2.2.3) with the latter resulting in 80% reduction in elastic displacements for same maneuver agility.

Chapter 5 validated the system identification theory developed in Chapter 3 using experiments. Identification experiments are performed by tapping the CMG/flexible structure in closed-loop and responses measured. Then an ROM fit is created from

these measurements which enables discovery of vibration-optimal agile slew maneuvers using a simple two degree of freedom basis (Subsection 2.2.2). These candidate maneuvers when performed in the experiments result in significant reduction in residual vibrations for same maneuver durations when compared to baseline zero-jerk smooth polynomial maneuvers (Subsection 2.2.1). Apart from contributing to robustly identifying structural properties, the developed framework is also able to quantify the effect of gravitational body loads in the slew maneuver dynamics on ground using the observable $\bar{p}$ (see Equation 3.19).

Chapter 6 presented a new concept of an attitude control device based on a control moment gyroscope array embedded within the deployment mechanism DOLCE[8]. Contributing to the essence of stowed volume-efficient design of *ultralight spacecraft*, this work discusses actuator sizing and multiple methods of executing a single maneuver re-purposing specific components in DOLCE. Concepts like reducing actuator motion within a physical envelop and enforcing periodic movement in the gimbals are formulated which quite advantageous with the power optimal guidance schemes as presented by Marshall et. al. [10]. Finally, a sensitivity study on the rigid spacecraft-actuator system shows that a 1° uncertainty in the initial position of the gimbals results in substantial orientation and angular rate errors of 8.2° and $1.5^\circ/s$ respectively.

7.2 Future Research Directions

1. Formation flying of flexible spacecraft

Much of the work presented in this thesis assumes decoupling of rotational degrees of freedom with the position of spacecraft and absence of loads like gravity-gradients, radiation pressure, oblate earth etc. A recent study[72] showed optimal trajectory involving a combination of thrust firing and attitude change simultaneously for SSPP spacecraft constellation assuming rigid structure. Ideally, flexibility of the spacecraft needs to be accounted for precise constellation management. Hence, a possible avenue of for future work involves combining the orbital position dynamics with local thrust vector correction from attitude dynamics model for flexible spacecraft (Chapter 4). Ultimately, after adding in the translational-rotational coupling, the effectiveness of agile maneuvers discussed in this thesis need to be reevaluated with simultaneous orbit correction maneuvers and thereby its impact on the constellation shape.

2. Balanced system identification

Dynamic Mode Decomposition with control (DMDc) serves as a major backbone for the proposed system identification of flexible spacecraft in this thesis for performing a model-based control. One of the strengths of DMDc is ability to disambiguate contribution of dynamics driven by the previous state versus the previous input. In the work presented in this thesis, this was achieved by truncating the SVD modes after arranging them in ascending order of eigenvalues or descending order of energy. Several iterations in the truncation thresholds were performed to eventually develop reduced order models which were not only good at *reconstructing* the training data but also crisp enough to capturing the behavior of system to *predict* very different set of inputs.

These iterations in deciding truncation thresholds were required because energy based truncation results in unfaithful capturing of input-output dynamics, because it may be the case that the most energetic modes are nearly uncontrollable or unobservable, and therefore may not be relevant for control[67, Ch 9]. This makes the model infeasible for real-time applications.

However, a different hierarchy based decomposition of the system makes truncation much straightforward for model-based control. A *balanced system identification* only keeps the coherent structures that are most important for control. This would mean accurate long time horizon predictions and similar input-output characteristics with the ROM for a variety of inputs. Thus subsequent experiments with the CMG/flexible structure can study more complex ROMs based on combination of controllability and observability of the modes.

3. *Physics Informed Neural networks (PINNs)*

It would be interesting to compare PINNs with the proposed version of DMDc algorithm on two accounts: training data required vs. computational cost. as trade study in-line with real spacecraft implementation.

4. *Increased fidelity structural response*

The presented numerical model in Chapter 2 made several assumptions. Firstly, the free-free mode shapes of the flexible structure were assumed to be constant and the linear superposition valid, relative to any degree of deformation. This could be validated by comparing the results from the approach presented in this thesis with a finite element code accounting for geometric nonlinearity.

Secondly, the inertia of the actuator was assumed to remain approximately constant during the maneuvers. Although, for the experimental prototype in Chapter 4, the

proposed design resulted in a constant CMG inertia at any orientation in the gimbals, the SSPP momentum device developed in Chapter 6 can result in changes in inertia tensor as the cylinders move up or down. Cylinder gimbal axes do not pass through their individual center of mass (COM) leading to movement in the net COM as the cylinders move up or down. This not only changes system's COM by a small amount but also modifies inertia tensor of the central hub leading to over/under execution of maneuver angle change. Relaxing this assumption will improve orientation accuracy during maneuver. This can be accounted for by adding an extra term $\dot{I}\Omega$ in the Euler's equations presented in Equation 2.42 and reformulating the solution in the gimbals accounting for the change in inertia tensor.

5. Failed actuator components

The momentum device, as explained Chapter 6, has four inputs to control three outputs, which makes it an over-actuated system. An interesting future direction would be to study the effect of failure of one of these inputs leading to only three inputs for three rotational degrees of freedom to perform the required slew maneuvers.

6. Agile maneuvers with attitude thrusters

The techniques demonstrated in this thesis can be well applied with combination of attitude thrusters burns. Piecewise constant functions can be convolved with the input shaping impulses to achieve correct thrust duration and magnitude for suppressing vibrations. Similarly, data-driven approach with a basis comprised of "N" on-off impulses could be used for optimization framework constrained to the reduced order model resulting in sequential bang-off-bang type impulse maneuvers.

Identification of real world flexible spacecraft can be challenging. A lot of work needs to be done towards the kind of impulse that can be practically applied via attitude thrusters and its impact of the quality of identification. For example, one could tune the width and amplitude of these impulses to keep the oscillations within the safe limits, thus preventing spacecraft sensor saturation and other effects.

BIBLIOGRAPHY

- [1] Glaser, P. E., “Power from the sun: Its future,” *Science*, Vol. 162, No. 3856, 1968, pp. 857–861.
- [2] Woodcock, G., et al., “Solar power satellite system definition study, Phase II final report, volume II,” *Boeing Aerospace Co., Seattle, WA, Rep. D180-25461-2*, 1979.
- [3] Mankins, J. C., “A fresh look at space solar power: New architectures, concepts and technologies,” *Acta Astronautica*, Vol. 41, No. 4-10, 1997, pp. 347–359. [https://doi.org/10.1016/S0094-5765\(98\)00075-7](https://doi.org/10.1016/S0094-5765(98)00075-7).
- [4] Arya, M., Lee, N., and Pellegrino, S., “Ultralight structures for space solar power satellites,” *3rd AIAA Spacecraft Structures Conference*, 2016, p. 1950. <https://doi.org/10.2514/6.2016-1950>.
- [5] Mizrahi, O. S., Jahelka, P., Gdoutos, E., Brunet, J., Ayling, A., Fikes, A., Wu, A., Madonna, R., Atwater, H. A., Pellegrino, S., and Hajimiri, A., “Space solar power generation: A viable system proposal and technoeconomic analysis,” *Joule*, Vol. 9, No. 6, 2025, p. 101928. <https://doi.org/10.1016/j.joule.2025.101928>, URL <https://www.sciencedirect.com/science/article/pii/S2542435125001096>.
- [6] Mallikarachchi, H. C., and Pellegrino, S., “Dynamic Analysis of Large Flat-foldable Arrays – Natural Frequency Bounds and Influence of Prestress,” *AIAA SCITECH 2025 Forum*, American Institute of Aeronautics and Astronautics, 2025. <https://doi.org/10.2514/6.2025-1194>, URL <http://dx.doi.org/10.2514/6.2025-1194>.
- [7] Pedivellano, A., Gdoutos, E., and Pellegrino, S., “Sequentially controlled dynamic deployment of ultra-thin shell structures,” *AIAA Scitech 2020 forum*, 2020, p. 0690. <https://doi.org/10.2514/6.2020-0690>.
- [8] Gdoutos, E., Sommer, C. F., Truong, A., Wen, A., Pedivellano, A., Ubamanyu, K., Madonna, R. G., and Pellegrino, S., “Development of the Deployable on-Orbit ultraLight Composite Experiment (DOLCE) for the space solar power project demonstration mission,” *AIAA SciTech 2022 Forum*, 2022, p. 1266. <https://doi.org/10.2514/6.2022-1266>.
- [9] Marshall, M., “Dynamics of Ultralight Flexible Spacecraft during Slew Maneuvers,” Ph.D. thesis, California Institute of Technology, 2022. URL <https://resolver.caltech.edu/CaltechTHESIS:05262022-221946560>.
- [10] Marshall, M. A., Goel, A., and Pellegrino, S., “Power-optimal guidance for planar space solar power satellites,” *Journal of Guidance, Control, and Dynamics*, Vol. 43, No. 3, 2020, pp. 518–535. <https://doi.org/10.2514/1.G004643>.

- [11] KAKAD, Y., “Dynamics and control of slew maneuver of large flexible spacecraft,” *Astrodynamics Conference*, 1986, p. 2192. <https://doi.org/10.2514/6.1986-2192>.
- [12] Scrivener, S. L., and Thompson, R. C., “Survey of time-optimal attitude maneuvers,” *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 2, 1994, pp. 225–233. <https://doi.org/10.2514/3.21187>.
- [13] Bang, H., Junkins, J. L., and Fleming, P. J., “Lyapunov optimal-control laws for flexible structure maneuver and vibration control,” *Journal of the Astronautical Sciences*, Vol. 41, No. 1, 1993, pp. 91–118. URL <https://api.semanticscholar.org/CorpusID:221974051>.
- [14] Bilimoria, K., and Wie, B., “Minimum-time large-angle reorientation of a rigid spacecraft,” *AIAA Guidance, Navigation and Control Conference, Portland, Oregon, Paper*, 1990. <https://doi.org/10.2514/1.A35422>.
- [15] Bilimoria, K. D., and Wie, B., “Time-optimal three-axis reorientation of a rigid spacecraft,” *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 3, 1993, pp. 446–452. <https://doi.org/10.2514/3.21030>.
- [16] Wie, B., and Lu, J., “Feedback control logic for spacecraft eigenaxis rotations under slew rate and control constraints,” *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 6, 1995, pp. 1372–1379. <https://doi.org/10.2514/3.21555>.
- [17] Wie, B., Liu, Q., and Sunkel, J., “Robust stabilization of the space station in the presence of inertia matrix uncertainty,” *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 3, 1995, pp. 611–617. <https://doi.org/10.2514/6.1993-3803>.
- [18] McInnes, C., “Satellite attitude slew manoeuvres using inverse control,” *The Aeronautical Journal*, Vol. 102, No. 1015, 1998, pp. 259–266. <https://doi.org/10.1017/S0001924000065295>, URL https://www.cambridge.org/core/product/identifier/S0001924000065295/type/journal_article.
- [19] Caubet, A., Biggs, J., et al., “A motion planning method for spacecraft attitude maneuvers using single polynomials,” *AAS/AIAA Astrodynamics Specialist Conference*, Vol. 8, 2015.
- [20] Marshall, M., and Pellegrino, S., “Slew Maneuver Constraints for Agile Flexible Spacecraft,” *Journal of Guidance, Control, and Dynamics*, Vol. 46, 2023, pp. 1–15. <https://doi.org/10.2514/1.G007430>.
- [21] Wie, B., and Barba, P. M., “Quaternion feedback for spacecraft large angle maneuvers,” *Journal of guidance, Control, and Dynamics*, Vol. 8, No. 3, 1985, pp. 360–365. <https://doi.org/10.2514/3.19988>.

- [22] Margulies, G., “On real four-parameter representations of satellite attitude motions,” *Mathematical analysis Dept., Philco Western Lab., Palo Alto, CA, report*, Vol. 52, 1963.
- [23] Solà, J., “Quaternion kinematics for the error-state Kalman filter,” *CoRR*, Vol. abs/1711.02508, 2017. URL <http://arxiv.org/abs/1711.02508>.
- [24] Amiryar, M. E., and Pullen, K. R., “A Review of Flywheel Energy Storage System Technologies and Their Applications,” *Applied Sciences*, Vol. 7, No. 3, 2017. URL <https://www.mdpi.com/2076-3417/7/3/286>.
- [25] Chen, M. Z., Papageorgiou, C., Scheibe, F., Wang, F.-c., and Smith, M. C., “The missing mechanical circuit element,” *IEEE Circuits and Systems Magazine*, Vol. 9, No. 1, 2009, pp. 10–26. <https://doi.org/10.1109/MCAS.2008.931738>.
- [26] Gill, W. A., Howard, I., Mazhar, I., and McKee, K., “A Review of MEMS Vibrating Gyroscopes and Their Reliability Issues in Harsh Environments,” *Sensors*, Vol. 22, No. 19, 2022. <https://doi.org/10.3390/s22197405>, URL <https://www.mdpi.com/1424-8220/22/19/7405>.
- [27] Leve, F. A., Hamilton, B. J., and Peck, M. A., *Dynamics of Momentum-Control Systems*, Springer International Publishing, Cham, 2015. https://doi.org/10.1007/978-3-319-22563-0_4, URL 10.1007/978-3-319-22563-0_4.
- [28] Tafazoli, M., “A study of on-orbit spacecraft failures,” *Acta Astronautica*, Vol. 64, No. 2, 2009, pp. 195–205. <https://doi.org/https://doi.org/10.1016/j.actaastro.2008.07.019>, URL <https://www.sciencedirect.com/science/article/pii/S0094576508003019>.
- [29] Croomes, S., “Overview of the DART mishap investigation results,” *NASA Report*, 2006, pp. 1–10.
- [30] Schwartz, J. L., Peck, M. A., and Hall, C. D., “Historical review of air-bearing spacecraft simulators,” *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 4, 2003, pp. 513–522. <https://doi.org/https://doi.org/10.2514/2.5085>.
- [31] Rybus, T., Kolvenbach, H., and Yan, X. T., *Air-Bearing Microgravity Simulators for Space Robotics*, Springer Nature Switzerland, Cham, 2024, pp. 157–190. https://doi.org/10.1007/978-3-031-39214-6_8, URL https://doi.org/10.1007/978-3-031-39214-6_8.
- [32] Foust, R. C., Lupu, E. S., Nakka, Y. K., Chung, S.-J., and Hadaegh, F. Y., “Ultra-soft electromagnetic docking with applications to in-orbit assembly,” *69th International Astronautical Congress, Bremen, Germany*, 2018, pp. 1–5.
- [33] SUTTER, T., COOPER, P., YOUNG, J., and MCCUTCHEN, D., “Dynamic and attitude control characteristics of an International Space Station,” *28th*

- Structures, Structural Dynamics and Materials Conference*, 1987, p. 931. <https://doi.org/https://doi.org/10.2514/6.1987-931>.
- [34] Umland, J., and Eisen, H., “SRTM on-orbit structural dynamics,” *19th AIAA Applied Aerodynamics Conference*, 2001, p. 1588. <https://doi.org/https://doi.org/10.2514/6.2001-1588>.
- [35] Lappas, V., Steyn, W., and Underwood, C., “Design and testing of a control moment gyroscope cluster for small satellites,” *Journal of Spacecraft and Rockets*, Vol. 42, No. 4, 2005, pp. 729–739. <https://doi.org/https://doi.org/10.2514/1.7308>.
- [36] Papakonstantinou, C., Moraitis, G., Lappas, V., and Kostopoulos, V., “Design of a Low-Cost Air Bearing Testbed for Nano CMG Maneuvers,” *Aerospace*, Vol. 9, No. 2, 2022. <https://doi.org/10.3390/aerospace9020095>, URL <https://www.mdpi.com/2226-4310/9/2/95>.
- [37] Gilman, A., “Automatic Mass Balancing Of A Spacecraft Attitude Dynamics Simulator With Six Sliding Masses,” Master’s thesis, California Polytechnic State University, 2024. URL [https://digitalcommons.calpoly.edu/do/search/?q=\(author%3A%22Amelia%20J.%20Gilman%22%20AND%20-bp_author_id%3A%5B%20TO%20*%5D\)%20OR%20bp_author_id%3A\(%22aa4ee0f3-d5cb-48ac-aba6-8481dd369487%22\)&start=0&context=374206](https://digitalcommons.calpoly.edu/do/search/?q=(author%3A%22Amelia%20J.%20Gilman%22%20AND%20-bp_author_id%3A%5B%20TO%20*%5D)%20OR%20bp_author_id%3A(%22aa4ee0f3-d5cb-48ac-aba6-8481dd369487%22)&start=0&context=374206).
- [38] Hancer, M., Bitirgen, R., and Bayezit, I., “Designing 3-DOF Hardware-In-The-Loop Test Platform Controlling Multirotor Vehicles,” *IFAC-PapersOnLine*, Vol. 51, No. 4, 2018, pp. 119–124. <https://doi.org/https://doi.org/10.1016/j.ifacol.2018.06.058>, URL <https://www.sciencedirect.com/science/article/pii/S2405896318303549>, 3rd IFAC Conference on Advances in Proportional-Integral-Derivative Control PID 2018.
- [39] Smith, O. J., “Posicast control of damped oscillatory systems,” *Proceedings of the IRE*, Vol. 45, No. 9, 1957, pp. 1249–1255. <https://doi.org/10.1109/JRPROC.1957.278530>.
- [40] Gupta, N. K., “Frequency-shaped cost functionals-Extension of linear-quadratic-Gaussian design methods,” *Journal of Guidance and Control*, Vol. 3, No. 6, 1980, pp. 529–535. <https://doi.org/https://doi.org/10.2514/3.19722>.
- [41] “Chapter 10 - Frequency-Shaped Large-Angle Maneuvers of Flexible Spacecraft,” *Optimal Spacecraft Rotational Maneuvers*, Studies in Astronautics, Vol. 3, edited by J. L. JUNKINS and J. D. TURNER, Elsevier, 1986, pp. 358–403. <https://doi.org/https://doi.org/10.1016/B978-0-444-42619-2.50014-8>, URL <https://www.sciencedirect.com/science/article/pii/B9780444426192500148>.

- [42] Junkins, J., Rahman, Z., and Bang, H., “Near-minimum-time maneuvers of flexible vehicles-A Liapunov control law design method,” *28th Aerospace Sciences Meeting*, 1990, p. 663. <https://doi.org/https://doi.org/10.2514/6.1990-663>.
- [43] Singer, N. C., and Seering, W. P., “Preshaping Command Inputs to Reduce System Vibration,” *Journal of Dynamic Systems, Measurement, and Control*, Vol. 112, No. 1, 1990, pp. 76–82. <https://doi.org/10.1115/1.2894142>, URL 10.1115/1.2894142.
- [44] Craig, R., and Kurdila, A., *Fundamentals of Structural Dynamics*, Wiley, 2011. URL <https://www.wiley.com/en-us/shop/general-introductory-mechanical-engineering/fundamentals-of-structural-dynamics-2nd-edition-p-9781118174449>.
- [45] *ABAQUS Theory Manual*, Dassault, 2020.
- [46] Spring, K. W., “Euler parameters and the use of quaternion algebra in the manipulation of finite rotations: A review,” *Mechanism and Machine Theory*, Vol. 21, No. 5, 1986, pp. 365–373. [https://doi.org/10.1016/0094-114X\(86\)90084-4](https://doi.org/10.1016/0094-114X(86)90084-4), URL <https://www.sciencedirect.com/science/article/pii/0094114X86900844>.
- [47] Simo, J., “The (symmetric) Hessian for geometrically nonlinear models in solid mechanics: Intrinsic definition and geometric interpretation,” *Computer Methods in Applied Mechanics and Engineering*, Vol. 96, No. 2, 1992, pp. 189–200. [https://doi.org/10.1016/0045-7825\(92\)90131-3](https://doi.org/10.1016/0045-7825(92)90131-3), URL <https://www.sciencedirect.com/science/article/pii/0045782592901313>.
- [48] Craig, R. R., Su, T.-J., and Ni, Z., “State-variable models of structures having rigid-body modes,” *Journal of Guidance, Control, and Dynamics*, Vol. 13, 1990, p. 1157–1160. <https://doi.org/https://doi.org/10.2514/3.20594>.
- [49] McInnes, C., “Satellite attitude slew manoeuvres using inverse control,” *The Aeronautical Journal*, Vol. 102, No. 1015, 1998, pp. 259–266. URL <https://api.semanticscholar.org/CorpusID:114373847>.
- [50] Wie, B., *Space Vehicle Dynamics and Control, Second Edition*, American Institute of Aeronautics and Astronautics, 2008. <https://doi.org/10.2514/4.860119>, URL 10.2514/4.860119.
- [51] Defendini, A., Faucheux, P., Guay, P., Bangert, K., Heimel, H., Privat, M., and Seiler, R., “Control Moment GYRO CMG 15-45 S: a compact CMG product for agile satellites in the one ton class,” *10th European Space Mechanisms and Tribology Symposium*, Vol. 524, 2003, pp. 27–31. URL <https://api.semanticscholar.org/CorpusID:109885021>.

- [52] Kurokawa, H., “Survey of theory and steering laws of single-gimbal control moment gyros,” *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 5, 2007, pp. 1331–1340. <https://doi.org/https://doi.org/10.2514/1.27316>.
- [53] Bedrossian, N. S., Paradise, J., Bergmann, E. V., and Rowell, D., “Steering law design for redundant single-gimbal control moment gyroscopes,” *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 6, 1990, pp. 1083–1089. URL <http://hdl.handle.net/1721.1/14663>.
- [54] Palma, L., Peraza, A., Civantos-Prieto, D., Duarte, A., Materia, S., Muñoz, G., Peña-Izquierdo, J., Romero, L., Soret, A., and Donat, M. G., “Data-driven seasonal climate predictions via variational inference and transformers,” *npj Climate and Atmospheric Science*, Vol. 9, No. 1, 2026. <https://doi.org/10.1038/s41612-026-01320-z>, URL <http://dx.doi.org/10.1038/s41612-026-01320-z>.
- [55] Rodríguez, A., Kamarthi, H., Agarwal, P., Ho, J., Patel, M., Sapre, S., and Prakash, B. A., “Machine learning for data-centric epidemic forecasting,” *Nature Machine Intelligence*, Vol. 6, No. 10, 2024, p. 1122–1131. <https://doi.org/10.1038/s42256-024-00895-7>, URL <http://dx.doi.org/10.1038/s42256-024-00895-7>.
- [56] Adcock, C., and King, R. N., “Data-Driven Wind Farm Optimization Incorporating Effects of Turbulence Intensity,” *2018 Annual American Control Conference (ACC)*, IEEE, 2018, p. 695–700. <https://doi.org/10.23919/acc.2018.8431727>, URL <http://dx.doi.org/10.23919/ACC.2018.8431727>.
- [57] SCHMID, P. J., “Dynamic mode decomposition of numerical and experimental data,” *Journal of Fluid Mechanics*, Vol. 656, 2010, p. 5–28. <https://doi.org/10.1017/S0022112010001217>.
- [58] Tu, J. H., Rowley, C. W., Luchtenburg, D. M., Brunton, S. L., and Kutz, J. N., “On dynamic mode decomposition: Theory and applications,” *Journal of Computational Dynamics*, Vol. 1, No. 2, 2014, p. 391–421. <https://doi.org/10.3934/jcd.2014.1.391>, URL <http://dx.doi.org/10.3934/jcd.2014.1.391>.
- [59] Kutz, J. N., Brunton, S. L., Brunton, B. W., and Proctor, J. L., *Dynamic mode decomposition: data-driven modeling of complex systems*, SIAM, 2016. <https://doi.org/https://doi.org/10.1137/1.9781611974508>.
- [60] Shuster, M. D., “Survey of attitude representations,” *Journal of the Astronautical Sciences*, Vol. 41, No. 4, 1993, pp. 439–517. URL <https://ui.adsabs.harvard.edu/abs/1993JAnSc..41..439S>.
- [61] Madgwick, S. O. H., Harrison, A. J. L., and Vaidyanathan, R., “Estimation of IMU and MARG orientation using a gradient descent algorithm,” *2011 IEEE International Conference on Rehabilitation Robotics*, 2011, pp. 1–7. <https://doi.org/10.1109/ICORR.2011.5975346>.

- [62] Wie, B., “Singularity analysis and visualization for single-gimbal control moment gyro systems,” *Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 2, 2004, pp. 271–282. <https://doi.org/https://doi.org/10.2514/1.9167>.
- [63] Balas, M. J., “Reduced-order feedback control of distributed parameter systems via singular perturbation methods,” *Journal of Mathematical Analysis and Applications*, Vol. 87, No. 1, 1982, pp. 281–294. [https://doi.org/https://doi.org/10.1016/0022-247X\(82\)90167-6](https://doi.org/https://doi.org/10.1016/0022-247X(82)90167-6), URL <https://www.sciencedirect.com/science/article/pii/0022247X82901676>.
- [64] Preumont, A., Pay, A., and Decauwers, A., “Seismic analysis of a large FBR core Some SNR-2 preliminary calculations,” *Proc. Ninth Int. Conf. on Structural Mechanics in Reactor Technology, Lausanne, Switzerland, Vol. E*, 1987, p. 27. URL <https://repository.lib.ncsu.edu/server/api/core/bitstreams/dc41b522-5d1c-4fe0-a549-d1c0775698da/content>.
- [65] Balas, M., “Feedback control of flexible systems,” *IEEE Transactions on Automatic Control*, Vol. 23, No. 4, 1978, pp. 673–679. <https://doi.org/10.1109/TAC.1978.1101798>.
- [66] Yao, G., and Gao, X., “Controllability of regular systems and defective systems with repeated eigenvalues,” *Journal of Vibration and Control*, Vol. 17, No. 10, 2011, pp. 1574–1581. <https://doi.org/10.1177/1077546310382080>.
- [67] Brunton, S. L., and Kutz, J. N., *Data-Driven Science and Engineering: Machine Learning, Dynamical Systems, and Control*, Cambridge University Press, 2022. <https://doi.org/10.1017/9781009089517>, URL <http://dx.doi.org/10.1017/9781009089517>.
- [68] Arenberg, J., Flynn, J., Cohen, A., Lynch, R., and Cooper, J., “Status of the JWST sunshield and spacecraft,” *Space Telescopes and Instrumentation 2016: Optical, Infrared, and Millimeter Wave*, Vol. 9904, edited by H. A. MacEwen, G. G. Fazio, M. Lystrup, N. Batalha, N. Siegler, and E. C. Tong, International Society for Optics and Photonics, SPIE, 2016, p. 990405. <https://doi.org/10.1117/12.2234481>, URL <https://doi.org/10.1117/12.2234481>.
- [69] Lariviere, E., “Attitude Control of a Flexible Solar Sail using Translating Masses,” Master’s thesis, University of Toronto (Canada), 2021. URL <http://hdl.handle.net/1807/109272>.
- [70] Leis, J. R., and Kramer, M. A., “The simultaneous solution and sensitivity analysis of systems described by ordinary differential equations,” *ACM Trans. Math. Softw.*, Vol. 14, No. 1, 1988, p. 45–60. <https://doi.org/10.1145/42288.46156>, URL <https://doi.org/10.1145/42288.46156>.
- [71] Junkins, J. L., and Kim, Y., *Introduction to Dynamics and Control of Flexible Structures*, American Institute of Aeronautics and Astronautics, 1993. <https://doi.org/https://doi.org/10.2514/4.862076>.

- [72] Goel, A., Lee, N., and Pellegrino, S., “Trajectory design of formation flying constellation for space-based solar power,” *2017 IEEE Aerospace Conference*, 2017, pp. 1–11. <https://doi.org/10.1109/AERO.2017.7943711>.
- [73] KANATA, S., HAYASHIDA, F., and SHIMOMURA, T., “Experimental Verification for Attitude Control of Spacecraft Utilizing Double-Gimbal Variable-Speed Control Moment Gyroscopes,” *TRANSACTIONS OF THE JAPAN SOCIETY FOR AERONAUTICAL AND SPACE SCIENCES*, Vol. 66, No. 6, 2023, p. 209–216. <https://doi.org/10.2322/tjsass.66.209>, URL <http://dx.doi.org/10.2322/tjsass.66.209>.
- [74] Bahu, A., and Modenini, D., “On-ground experimental verification of magnetic attitude control for nanosatellites,” *2021 IEEE 8th International Workshop on Metrology for AeroSpace (MetroAeroSpace)*, IEEE, 2021, p. 568–573. <https://doi.org/10.1109/metroaerospace51421.2021.9511720>, URL <http://dx.doi.org/10.1109/MetroAeroSpace51421.2021.9511720>.

Appendix A

SYSTEM ID ON SMALL-SCALE SYSTEM NUMERICAL MODEL WITH CONVENTIONAL DMDC

The purpose of this section is to show how traditional DMDC fails to capture the dynamics of the small-scale CMG-flexible structure prototype as discussed in Chapter 4. DMDC algorithm is applied on the data measured from full fidelity numerical model. Number of stacks $ns = 4$ is chosen. The truncated singular values are shown in Figure A.1. Measurement reconstruction works quite well for both the hub angles (figureA.2 and elastic displacements (Figure A.3).

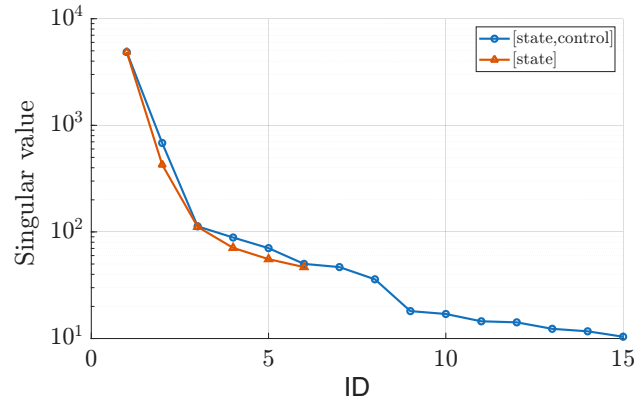


Figure A.1: Singular values for augmented matrix S and measurement shifted matrix Y' .

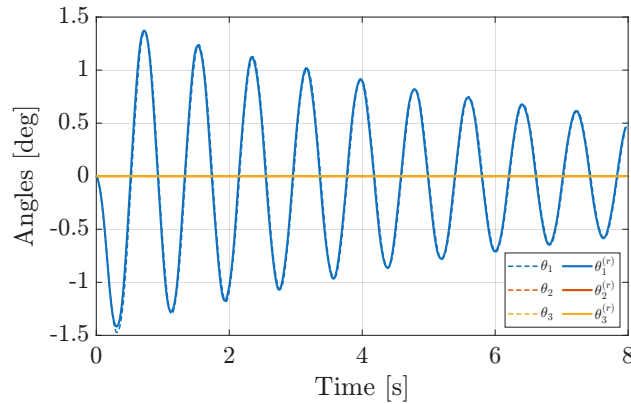


Figure A.2: Reconstruction of hub root angles.

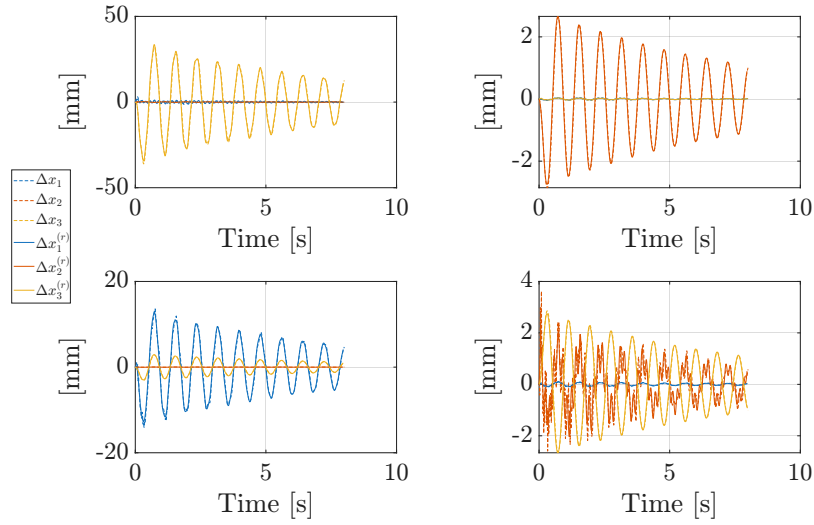


Figure A.3: Reconstruction of elastic displacements of side mid points.

The Reduced-Order Model (ROM) defined by matrices A and B as shown in Figure A.4 is then used to establish results for an input torque defined by a polynomial slew maneuver about axis-1 with maneuver duration $T = 3\text{ s}$ and maneuver angle $\Delta\psi = 10^\circ$. Then the results are compared with full fidelity numerical model. The ROM model is not able to provide good predictions of either the hub rotation angles (Figure A.5) or the elastic displacements (figures A.6,A.7) as compared to the ground truth from numerical model.

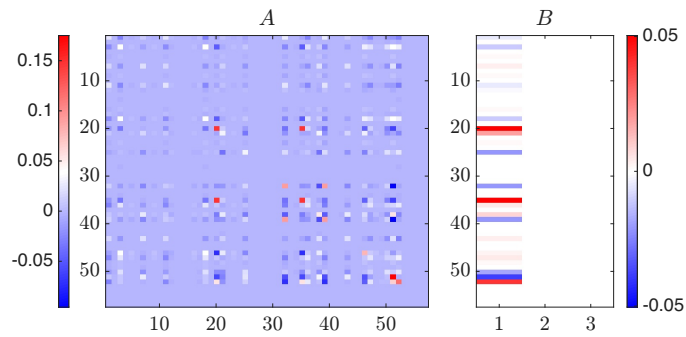


Figure A.4: Estimated matrices.

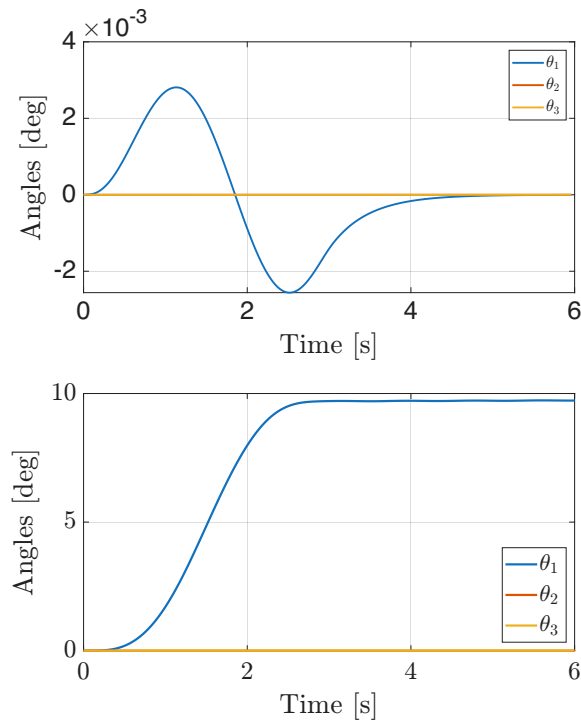


Figure A.5: Hub root angles for a smooth polynomial maneuver input torque. The ROM on the top does not perform well as compared to full fidelity model at the bottom.

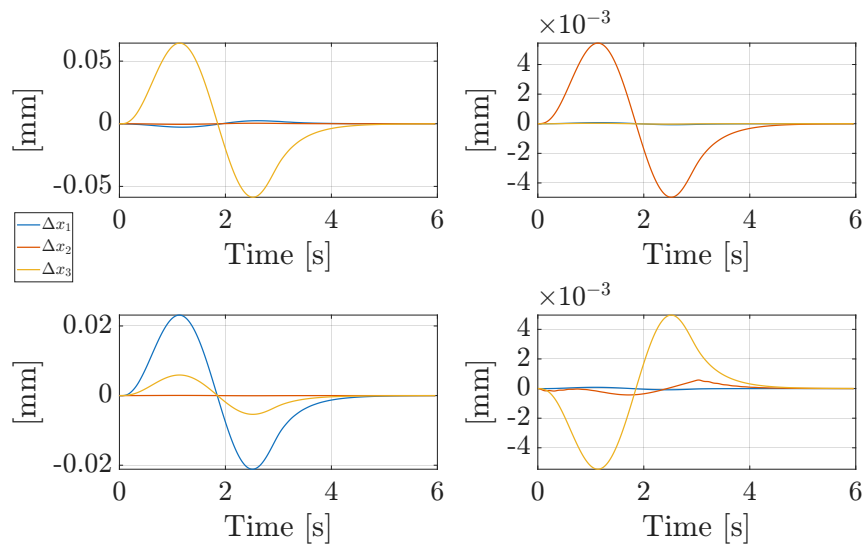


Figure A.6: Elastic displacements at side mid points for a smooth polynomial maneuver input torque for ROM.

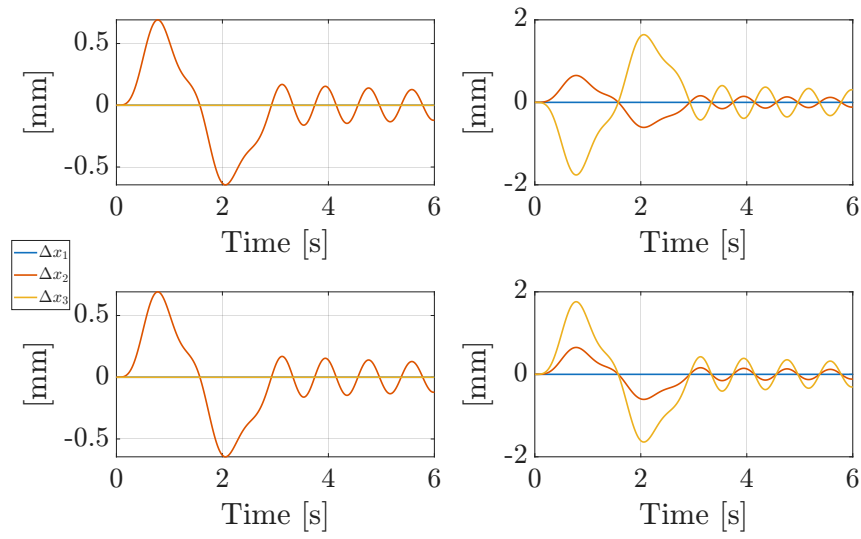


Figure A.7: Elastic displacements at side mid points for a smooth polynomial maneuver input torque for full fidelity model.

