

Stable Method of Attaching Thin Films to Torsionally Compliant Space Structures

Thesis by
George Arthur Popov

In Partial Fulfillment of the Requirements for the
Degree of
Doctor of Philosophy in Space Engineering



CALIFORNIA INSTITUTE OF TECHNOLOGY
Pasadena, California

2026
Defended May 21, 2026

© 2026

George Arthur Popov

ORCID: <https://orcid.org/0000-0001-5938-0528>

All rights reserved

ACKNOWLEDGEMENTS

There are many people who have left a lasting impression on me during my academic journey, and neither I nor my work would be the same without their unique, individual impacts on my life. I hold dear all of the positive examples those around me continue to set forth. Those at the beginning matter as much as those at the end, and I am deeply thankful and appreciative for all of the guidance, encouragement, and support I have received along the way, no matter how small or brief it might have been.

First I would like to thank my advisor, Sergio Pellegrino, who has done his patient best to guide me and raise the level of my work at all times. He has ensured that I am able to achieve the goals I set out to do, both during my time at Caltech and after, as well as provided an environment with many other marvelous people. Similarly, I owe my success to my committees, namely Professors Daniel Meiron (the chair for both my candidacy and defense), Kaushik Bhattacharya, Ares Rosakis, and Angkur Shaikeea, who all have given me many insightful ways to think about and approach my dissertation, and have helped shape its direction.

Similarly, I would like to thank the grants supporting my work, namely the first year Caltech Graduate Fellowship I received, the Caltech Space Solar Power Project which supported me for the majority of my time here, and the 2024 R&TD Innovative Spontaneous Concept program (Task #R24231) for the work related to the deployable toroid done at JPL. I would also like to thank Rachel Willick, who during her SURF helped lay the groundwork for the characterization of the double s-springs, and left behind for me an incredible framework from which to continue exploring parameters. Finally, I would like to thank Juan Mejia-Ariza for his guidance and mentorship during my brief stint at JPL making the deployable toroid.

A department doesn't run without a solid administration and group of support for its members. As such, I extend my gratitude to Christine Ramirez, Paula Mark, Liza Bradulina, Jamie Meighen-Sei, Sarah Pontes, Donna Mojahedi, Petros Arakelian, and our custodial staff, who all kept everything around me organized and running, whether it was a mentorship program, conference travel, or a random task that I wasn't sure who else to direct to.

My family has always supported my education to the fullest. My mom and grandmother taught and imparted as much onto me as they could, and laid the foundations for my success. I thank them as well as my other grandmother, dad, and brother for

all of their support over the years.

My favorite part about the space structures lab has always been its others members, past and current, visiting or long term. I am proud to have been surrounded by some of the kindest and compassionate people at Caltech. Thank you, Afonso, Alan, Alex, Armanj, Brayden, Brecht, Charles, Chinthaka, Divesh, Eyal, Fabio, Federico, Flavia, Gianfranco, Gulzhan, Harsha, Jaden, John, Josef, Maria, Matteo, Meital, Riccardo, Sahangi, Seiji, Tara, Tommaso, Uba, and Wen. In particular I have to underline the staff in our lab, Alan and Charles, who besides being great personal friends also are the backbone of every student project in one way or another. I will forever cherish the memories and interactions I have had with all of you, from sharing snacks and Lego magazines to late night spacecraft assembly. I would not have made it to the finish line without you all.

I also extend my gratitude and sincerity to everyone else in Pasadena who has been a friend over the years, on campus, in GALCIT, my roommates, PARSEC, or around town. There are too many of you to name, so I deeply apologize for anyone not mentioned explicitly. In particular I have to recognize the members of my masters cohort, without whom I would still not understand half of what I had to pass qualifying exams.

I thank my friends outside of the LA area as well as much as those close by. Thank you all for keeping in touch despite the physical distances, and supporting and entertaining me when I needed a reprieve from the constant sunshine. I look forward to visiting all of you from Penn and VIPER, crashing in Vancouver again (thank you Louis), and those still by Philadelphia; I should have more time to do so now! I also have to offer my gratitude to my research mentors in high school and undergrad, especially in the Bargatin Group. Igor's and Sam's mentorship left lasting impressions on me, and it is always my goal to try to repay their support to others.

There are many other incredible and kind mentors and people I have encountered along my journey thus far, and I offer my utmost appreciation to you all, and my apologies for not including more specifically dedicated expressions of gratitude and honor.

ABSTRACT

Ultralight space structures utilize composite structural elements supporting active thin films in order to achieve deployed configurations with lower areal densities. However, as these space structures get larger, they get increasingly less stiff, leaving them susceptible to adverse effects, such as torsional buckling. Simultaneously, this conflicts with the requirements that more ambitious space missions impose on the surface accuracy to make technologies like large phased arrays viable.

This thesis presents a novel method of continuously attaching thin films to deployable thin shell structures, allowing for materials with widely different coefficients of thermal expansion. It proposes a double s-spring border that exhibits a local post-buckling behavior and provides a tunable continuous edge attachment method that can maintain constant preload under large thermal strains. The mechanical behavior of the double s-spring under different mechanical loading conditions is studied both numerically and experimentally. Additionally, a reduced order model that greatly reduces the computational cost of optimizing double s-springs for various applications is presented.

In parallel, the torsional buckling of elastic composite frames is studied to understand the fundamental limits of attaching thin films without incurring torsional buckling. This study analytically calculates the critical prestress of torsionally soft square frames supporting an internal thin film. The study highlights the role of the attachment scheme, which has a very significant impact on the critical prestress. It is shown that the average of the compression load on a frame caused by a prestress is an invariant buckling load. The analytical calculation is verified via numerical finite element analyses and an experiment, which characterizes the post-buckling behavior of the torsionally soft frames as well. It is concluded that distributed edge attachments, such as the double s-spring, significantly increase the stability of space structures against torsional buckling. The findings for the torsionally soft square frames are validated against a high fidelity orthotropic material model in order to justify the assumptions made in the analytical study.

In the final section of the thesis, it is shown that the double s-spring continuous attachment scheme enables consistent deployment and compact packaging. The scheme is also shown to be versatile for additional applications, enabling novel deployment and folding schemes, such as a doubly-curved composite foldable toroid.

PUBLISHED CONTENT AND CONTRIBUTIONS

- [1] George A. Popov and Sergio Pellegrino. Thermally stable method of attaching precision thin films to deployable structures using high strain kirigami borders. *AIAA SCITECH 2026 Forum*, page 1830, 2026. doi: 10.2514/6.2026-1830.
G.A.P. participated in the conceptualization of the project, performed the experiments and numerical simulations, analyzed the results, and participated in the writing of the manuscript.
- [2] George A. Popov and Sergio Pellegrino. Torsional stability of deployable space structures supporting prestressed thin films. *AIAA SCITECH 2025 Forum*, page 0186, 2025. doi: 10.2514/6.2025-0186.
G.A.P. participated in the conceptualization of the project, performed the experiments and numerical simulations, analyzed the results, and participated in the writing of the manuscript.
- [3] Federico Benazzo, George A. Popov, Juan Mejia-Ariza, and Sergio Pellegrino. Thermally stable method of attaching precision thin films to deployable structures using high strain kirigami borders. *AIAA SCITECH 2026 Forum*, page 1040, 2026. doi: 10.2514/6.2026-1040.
G.A.P. participated in the conceptualization of the project, performed some experiments and numerical simulations, analyzed some results, and participated in the writing of the manuscript in relation to the thin films used.
- [4] George A. Popov and Sergio Pellegrino. Torsional stability of compliant deployable space structure frames supporting prestressed thin films. *[In Preparation]*, 2026.
G.A.P. participated in the conceptualization of the project, performed the experiments and numerical simulations, analyzed the results, and participated in the writing of the manuscript.

TABLE OF CONTENTS

Acknowledgements	iii
Abstract	v
Published Content and Contributions	vi
Table of Contents	vi
List of Illustrations	ix
List of Tables	xiv
Chapter I: Introduction	1
1.1 Research Outline	7
Chapter II: Double S-Springs	10
2.1 Introduction	10
2.2 Double S-Spring Pattern	12
2.3 Numerical Characterization of Double S-Springs	14
2.4 Influence of Design Parameters	17
2.5 Toughness	18
2.6 Further Numerical Model Optimization	23
2.7 Reduced Order Modeling of Double S-Springs	26
2.8 Double S-Spring Conclusion	33
Chapter III: Buckling of Rigid Frame Unit Cell	35
3.1 Introduction	35
3.2 Analytic Derivation of Buckling Loads via Adjacent Equilibrium	36
3.3 Analytic Derivation of Buckling Loads via an Energy Method	41
3.4 Buckling Load Numerical Simulations	50
3.5 Experimental Confirmation of Case C	54
3.6 Discussion	59
Chapter IV: Buckling of Elastic Frame Unit Cell	61
4.1 Introduction	61
4.2 Experimental Observations	64
4.3 Numerical Simulations and Comparison to Simplified Analytical Model	68
4.4 Discussion	73
Chapter V: Additional Experiments and Applications of Double S-Springs	76
5.1 Introduction	76
5.2 Coiling and Deployment	78
5.3 High Strain Foldable Toroid	81
5.4 Discussions	88
Chapter VI: Discussion and Conclusions	91
6.1 Summary of Contributions	91
6.2 Future Work	93
6.3 Closing Remarks	98
Bibliography	100

Appendix A: Double S-Spring Manufacturing Methods	107
Appendix B: Kirigami Spring Bending Study	111

LIST OF ILLUSTRATIONS

<i>Number</i>	<i>Page</i>
1.1 SSPD-1 in its deployed configuration in LEO.	2
1.2 Top and perspective views of the torsional buckling of trapezoidal strips due to changes in humidity inducing internal stresses.	4
1.3 The Hubble Space Telescope's second set of solar arrays undergoing torsional buckling due to thermal changes (Copyright ESA).	6
2.1 SSPP square unit cell showing a thin film mounted with double s-springs.	11
2.2 Common kirigami patterns.	12
2.3 Kirigami sheet spring (left) and square (right) cutting patterns under small uniaxial tension.	12
2.4 Single double s-spring unit cell.	13
2.5 Mechanics of the double s-spring pattern with the "S" highlighted in blue. A tessellated series of unit cells (left) is stretched by various amounts (right), with the different states corresponding to the regions illustrated in Figure 2.8.	14
2.6 Double s-spring design for numerical characterization example.	15
2.7 Force response for the example double s-spring design, under x extension and y shear.	16
2.8 Numerically calculated elastic force response of uniaxial stretching of the unit cell of double s-springs in Figure 2.6. Three roughly linear regions are identified corresponding to pre-buckling, post-buckling, and fully extended behavior of the pattern.	17
2.9 Effect of shifting the control point location of the example double s-spring on the uniaxial force response.	18
2.10 Effect of changing the thin film thickness of the example double s-spring on the uniaxial force response.	19
2.11 Effect of changing the ribbon width of the example double s-spring on the uniaxial force response. The results from both the purely elastic and plastic numerical material models are shown.	19
2.12 Effect of changing the ribbon width of the example double s-spring on the toughness and yield load.	20

2.13	Averaged experimental uniaxial loading response of the double s-spring compared to the numerical plastic model. Snapshots of the spring are shown for displacements of 0 mm, 1 mm, 2 mm, 3 mm, and 3.5 mm. The last value is right before total failure occurs, and the pattern tears in the middle.	21
2.14	Experimental measurements showing the effect of small and large amounts of localized plastic yielding on the uniaxial loading curve of the double s-spring.	22
2.15	Surface level plot obtained via DIC of a pretensioned film bonded to a composite frame. The slight saddle shape observed is due to the film being bonded on top of the left and right edges, but below the top and bottom edges.	23
2.16	Local image of a pretensioned film bonded to an Invar frame. Note the mild stretching and local buckling of the double s-springs. . . .	24
2.17	Self-intersection of geometry producing an import error and meshing difficulties for a higher control point coordinate and width combination.	25
2.18	519378 element locally refined mesh from a 20 cm $\times$ 20 cm unit cell film used in Abaqus via an input file generated in GMSH. The right image is a magnified view of the border.	26
2.19	Reduced order model mesh from a 20 cm $\times$ 20 cm unit cell film used in Abaqus. The right image is a magnified view of the border. In contrast to Figure 2.18, each spring pair is modeled by a single CONN3D2 connector element (dashed line), and the mesh of the film contains less elements. The external solid line is comprised of beam elements to provide nodes to attach the connector elements.	28
2.20	Maximum in-plane principal stresses; a) scale, b) high fidelity model, c) reduced order model (ROM) with coupled nonlinear CONN3D2 elements, d) ROM with coupled nonlinear CONN3D2 elements in opposing directions, e) ROM with uncoupled nonlinear CONN3D2 elements, f) ROM with unidirectional nonlinear CONN3D2 elements	29
2.21	Closer look at the stress distributions of the maximum in-plane principal stresses for the reduced order models; a) scale, b) reduced order model (ROM) with coupled nonlinear CONN3D2 elements, c) ROM with uncoupled nonlinear CONN3D2 elements, d) ROM with unidirectional nonlinear CONN3D2 elements	30

2.22	Minimum in-plane principal stresses; a) scale, b) high fidelity model, c) reduced order model (ROM) with coupled nonlinear CONN3D2 elements, d) ROM with coupled nonlinear CONN3D2 elements in opposing directions, e) ROM with uncoupled nonlinear CONN3D2 elements, f) ROM with unidirectional nonlinear CONN3D2 elements	31
3.1	Unit Cell Outline ([1]).	36
3.2	Four loading cases that are studied. A: Fixed direction corner forces. B: Fixed direction distributed load. C: Corner prestress applied by internal rods. D: Distributed prestress applied by biaxially-stressed membrane.	37
3.3	Torsional buckling mode.	38
3.4	Angles needed to calculate hinge rotation.	38
3.5	Vertical forces required to hold the frame in the buckled configuration.	39
3.6	Vertical forces required to hold a prestressed rod of the square frame in the buckled configuration.	39
3.7	Vertical forces required to hold a prestressed diagonal rod in the buckled configuration.	40
3.8	Vertical forces required to hold a prestressed membrane in the buckled configuration.	41
3.9	i. "Axial folding" mode and ii. "Corner displacement" mode.	42
3.10	Direction of forces in axial folding mode.	45
3.11	Angles needed to calculate hinge rotation.	46
3.12	Fixed force P applied in the "corner displacement" mode.	46
3.13	Follower force pair P applied in the "corner displacement" mode.	48
3.14	Diagram of a rectangular frame version of the buckling calculation. The other three forces and corner springs are omitted for visual clarity.	50
3.15	Case C post-buckling analysis. Figure a) shows the displacement-force response of the frame; figure b) shows the temperature vs. the displacement of the frame.	52
3.16	Case D post-buckling analysis.	53
3.17	Case D nonlinear buckling loads.	55
3.18	Case C experimental setup.	55
3.19	Case C experiment frame corner.	56
3.20	Case C initial experimental setup.	57
3.21	Case C experiment frame corner using 1095 steel and one strain gauge.	57
3.22	Frame in buckled state after being lifted.	58

3.23	Case C experimental comparison to numerical prediction.	59
3.24	Membrane thickness comparison of critical buckling temperature. . .	60
4.1	Top view of 20 cm $\times$ 20 cm carbon fiber reinforced plastic TRAC longeron and batten frame used in the Caltech Space Solar Power Project.	62
4.2	Previously manufactured rectangular frame with pretensioned pho- tovoltaic film from Ascent Solar Technologies attached via simple kirigami springs.	64
4.3	Illustration of using a hotplate to create a pretensioned film bonded to an Invar frame. First, a laser cut Kapton sheet (c) is placed on a silicone (a) and peel ply (b) layer. The Kapton is covered with an aluminum square (d), and the parts heated to equilibrium. The exposed tabs are epoxied, and the frame is placed on top (e). Once cured, the layers are removed in reverse (f).	66
4.4	A frame made of four battens pretensioned by pulleys. The film surface can be observed to be smooth, and the double s-springs along the edges displaying post-buckling behavior.	68
4.5	Side view of a batten and longeron frame with a manufacturing imperfection (top). After implementing a film with very low prestress (<1 N/m on each 20 cm edge), this effect did not get exacerbated (bottom).	69
4.6	Torsionally buckled frame made from four battens with a film sup- ported by double s-springs. Internal prestress was 6.6 N/m along each 20 cm edge.	69
4.7	Zoomed view of batten to flange overlap. The corner clamping and opposite corner force application boundary conditions are also illustrated.	71
4.8	Lowest buckling mode observed numerically for Case C in a linear elastic frame.	73
5.1	Packaging concept for SSPP, where the structure is first Z-folded and then coiled around four mandrills.	77
5.2	Equilibrium configuration resulting in longerons being unable to properly latch and complete deployment during the unfolding of the Z-folds[2].	78

5.3	Experimental setup used to gravity offload the frame structure and demonstrate deployment and latching of the composite frame unit cell[2].	79
5.4	Deployment test of the composite unit cell with double s-springs used to mount a pretensioned membrane. The top image shows the string holding the partially folded structure being cut, and the bottom image shows it successfully deployed shortly after.	80
5.5	Packaging test of the composite unit cell with double s-springs using a 0.1 m mandrill. The top left image shows the initially deployed unit cell frame, while the bottom right shows the fully smoothly coiled final form.	82
5.6	Geometry of toroidal tape-spring. The principal radii are R and r . . .	83
5.7	Toroidal tape-spring folding scheme.	83
5.8	Diagram of slit attachment for Kapton membrane.	84
5.9	Demonstration that with only an s-spring as opposed to double s-spring, local forces are not distributed adequately, leading to even small strains causing failure.	85
5.10	Toroid structure with attached membrane.	86
5.11	Folding and coiling sequence of toroid with membrane (visual). . . .	87
5.12	Folding and coiling sequence of toroid with membrane (experiment). .	88
5.13	Solar cells mounted on a film supported by double s-springs.	90
6.1	Top and side view of a prestressed film supported by double s-springs on a composite frame that has no torsional buckle.	94
A.1	Microscope view of laser cut double s-spring border. The actual width, as opposed to the nominal ribbon width of 0.6 mm, is shown labeled at various locations.	108
A.2	Small sample of double s-spring cut from Kapton using a Zünd cutter; the majority of sections were not able to be fully separated.	109
B.1	Force response graph of a kirigami spring with analogous force regions to Figure 2.8 highlighted. This spring achieves a higher displacement range than the ones used in SSPP due to additional segments added.	111
B.2	Kirigami spring prestressed via fixture under four-point bending test. .	112
B.3	Force probe measurement of kirigami spring under tension and orthogonal rotational boundary conditions.	113

LIST OF TABLES

<i>Number</i>	<i>Page</i>
2.1 Selected design for numerical characterization example.	15
3.1 Properties for Abaqus analysis.	50
4.1 Linear elastic frame dimensional properties.	63
4.2 Material properties for items used in making a batten and longeron frame.	63
4.3 Linear elastic frame composite orthotropic material properties.	70
4.4 Isotropic material property study results.	72
4.5 Orthotropic material property study results.	72
5.1 Diameter of structure during folding and deployment.	87

Chapter 1

INTRODUCTION

Deployable space structures have emerged as a fundamental requirement for many modern day space missions, enabling increasingly ambitious objectives to be achieved while under constraints imposed by launch vehicle dimensions, mass budgets, and operational efficiencies of mission critical components. Deployable space structures transition from a compact stowed configuration during launch to larger, more functional configurations once in orbit. Their use encompasses a wide spectrum of applications that could not be addressed with traditional single-configuration structures, such as telecommunication dishes, solar arrays, large telescopes, booms, solar sails, etc[3–5]. As humanity expands its activities in space ranging from low-earth orbits (LEO) to deep-space exploration, the demand for lightweight, reliable, and highly efficient deployable systems continues to grow.

Since the start of space exploration, deployable structures have evolved significantly. Initial systems primarily consisted of simple hinged appendages and extendable booms used for antennas and solar panels[6]. Nowadays, many space structures utilize flexible lightweight composite materials, origami-inspired geometries, and functional thin films that enable increasingly adaptive and efficient architectures[7]. Composite based architectures offer advantages such as higher stiffness, dimensional stability, precision, and durability under harsh space conditions such as extreme temperature cycling, radiation exposure, micro-meteoroid impacts, and vacuum-induced material degradation[8, 9].

Examples of large cutting edge deployable concepts include projects such as the James Webb Space Telescope (JWST), the Advanced Composite Solar Sail System (ACS3), and the Caltech Space Solar Power Project (SSPP)[10–12]. An early prototype of the SSPP, SSPD-1, which will be discussed more in Chapter 5, is shown in Figure 1.1. While these three projects do not share particularly similar mission objectives, or flight trajectories, there are some key similarities to highlight in the spacecraft. First, all three concepts must be sufficiently stiff to maintain their shape throughout maneuvers, which all of them require in order to meet their goals. Second, the three spacecraft are able to meet their stowage and mass requirements by taking advantage of a composite backbone structure, coupled with

tensioned thin films in between elements. Out of the three, JWST is the most similar to a classic space-frame truss-based system that provides high stiffness and load-bearing capability, while the other two structures rely on composites more to achieve exceptional packing efficiency and low areal density. Finally, while already massive, there are larger versions of each of these concepts already being imagined such as the Habitable Worlds Observatory (HWO), and other space based solar power and solar concepts for the latter projects, underlining the need to further evolve the limitations of the techniques used to construct them[13–15].

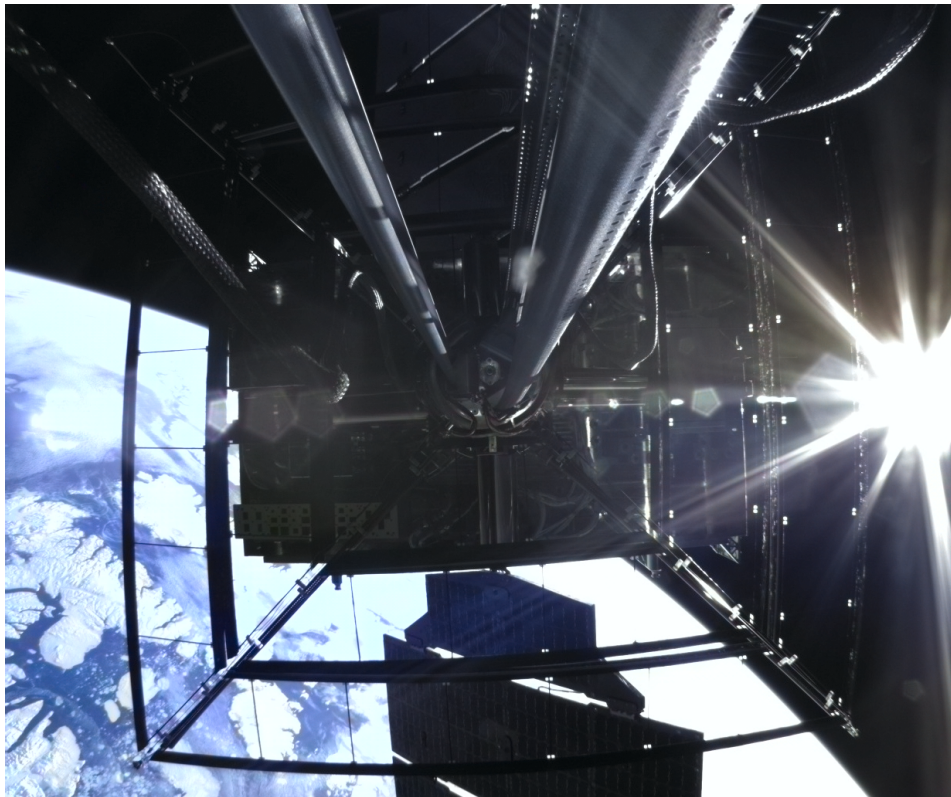


Figure 1.1: SSPD-1 in its deployed configuration in LEO.

In particular, this thesis will focus significantly on furthering the viability of the Caltech SSPP concept, since this is a primary project of the Space Structures Laboratory at Caltech. The SSPP mission concept envisions collecting solar power on-orbit in order to be able to send it down to Earth via phased arrays[12]. The main advantages of this are as follows. On the planet surface, the sun is eclipsed by the Earth itself for about half of the day (i.e. nighttime). In space, specifically LEO where the project is envisioned, this time is significantly less. Additionally, the sunlight available is not filtered by the atmosphere, meaning that solar power is not affected adversely by weather patterns. Similarly, the power transmitted down is

imagined to be at frequencies such as 10 GHz, meaning that the atmosphere will be mostly transparent to it. Lastly, a final advantage of the concept is that it is possible to direct this power wherever it is most needed, such as to ground stations following a natural disaster or to cover power-grid outages in major cities. As will be elaborated further in later chapters, a few things are necessary in order for this transmission to be successful. The most notable is a stable and predictable surface accuracy of this thin, lightweight structure, so that the phased arrays may interfere predictably and coherently and provide optimal power delivery[16]. Currently, prototypes on the scale of a couple of meters exist for the SSPP concept. When scaling to the final goal of a larger 60 m $\times$ 60 m spacecraft, this requirement becomes even more critical since deflections of the structure become more difficult to avoid.

The SSPP structure has been studied previously to ensure that it can maintain sufficient structural shape accuracies under vibrational and bending loads on the spacecraft, given its flat and relatively compliant geometry[17]. However, the SSPP architecture is still limited by another classic problem at the level of its strips of unit cells: torsional buckling. Torsional buckling in the SSPP structure is somewhat similar to the classic cases of lateral torsional buckling seen in I-beams, in civil engineering applications. I-beams consist of mass distributed mostly in their top and bottom flanges in order to raise the moment of inertia and resist bending loads, with a thin web region that can resist shear loads. However, this leaves them susceptible to torsion, resulting in torsional buckling from external loads or even self-weight being a potential failure mode, even if plastic deformation does not occur. Similarly, the SSPP structure's long trapezoidal ladder sections of unit cells (see Chapters 3-5 for more in-depth information) are made of open cross section Triangular Rollable and Collapsible (TRAC) longerons connected by battens, resulting in a geometry with flanges and a web region. These ladders of unit cells have been previously studied extensively against bending, localized and Euler buckling, and vibrational modes of failure. However, as can be seen in Figure 1.2, this type of design is susceptible to torsional buckling.

There are several challenges that make torsional buckling difficult to solve in in-space applications. While I-beams in terrestrial applications can be braced against torsional buckling, this is not typically an option for deployable structures[18]. The source of buckling in space is also different, where stresses are typically induced by changing environments. In an environment like LEO, the temperature a spacecraft experiences typically varies by a couple hundred degrees Celsius as it orbits the

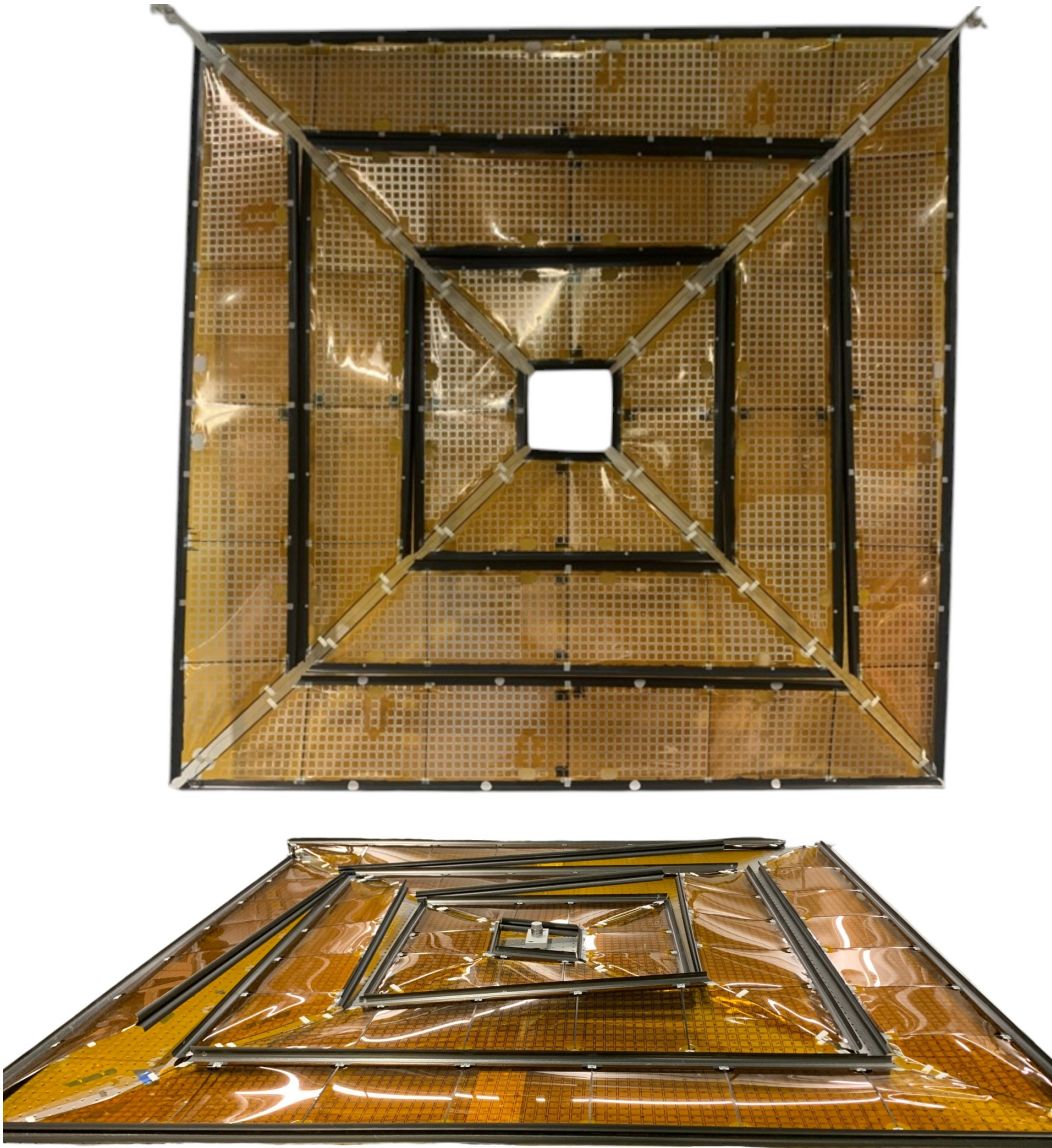


Figure 1.2: Top and perspective views of the torsional buckling of trapezoidal strips due to changes in humidity inducing internal stresses.

Earth[9]. Additionally, the vacuum of space introduces other sources of internal stresses, such as a change in humidity from when the spacecraft was manufactured on Earth (this is the effect seen in Figure 1.2). These effects cause strain, especially between components that expand or contract at different rates due to differences in their coefficients of thermal expansion (CTE) or other properties. In some material components stresses may occur as well due to effects such as thermal gradients across the spacecraft. A famous case study of thermal buckling in space induced by changes in temperature is illustrated in Figure 1.3, which shows the Hubble Space

Telescope with buckled solar arrays. The Hubble Space Telescope experienced bending and torsional buckling in its first and second generation of solar arrays from its launch in 1990 to when the second generation arrays were replaced by rigid panels in 2002[19, 20]. The buckled arrays were made of a rollable photovoltaic blanket supported by steel booms. These solar arrays suffered from not only a twist due to the thermally induced buckling, but even affected the pointing of the spacecraft due to the transient heating cycles causing a jitter. This was significant enough to prevent it from being able to make accurate Deep Field measurements.

Torsional buckling has been examined extensively for specific cases such as cylinders and trusses, but not extensively for flat structures [21]. General cases of in-space buckling have been studied since the 1950s, and focused increasingly more on composite structures since the 1970s[22]. Composites, such as carbon and glass fiber reinforced polymers, can have nearly zero CTEs, and offer high stiffnesses, which are two key methods of reducing buckling. However, as mentioned, deflections and torsional buckling still occur in these composite structures particularly due to their integration with other materials and thermal gradients, especially as thinner composite layups are utilized. For example, the Roll Out Solar Array (ROSA), which was deployed on the International Space Station (ISS), consists of a deployable photovoltaic blanket supported by composite booms has been previously studied to have thermally induced deflection and twist[23]. As mentioned already, the SSPP structure similarly suffers from thermally induced deflections, due to CTE differences. The torsional buckling of the SSPP concept has previously been investigated by both Luo and Pederson, but this problem has proved particularly challenging, since the films incorporated in the SSPP structure require biaxial prestressing across a range of temperatures to maintain good surface accuracy, which further increases internal forces and creates instability[24–26].

In addition to torsional buckling, CTE mismatch creates problems leading to surface inaccuracy in the film itself. Even if the frame itself is stable, internal stresses can lead to wrinkling in thin films that also leads to significant surface inaccuracies. A project focused on creating a doubly curved parabolic membrane reflector, Electrostatically Controlled Deployable Membrane Antenna (ECDMA), uses electrostatic forces to actively shape and tension a thin, conducting membrane into a precise reflecting surface[27]. This successfully increases surface accuracy, but relies on two curved surfaces as opposed to a single one, and even then exhibits visible areas of wrinkling. Other similar exotic solutions exist, such as taking advantage of multi-



Figure 1.3: The Hubble Space Telescope's second set of solar arrays undergoing torsional buckling due to thermal changes (Copyright ESA).

stable structures, but these are ill suited for flat projects like SSPP, especially when the expansion in both lateral and normal directions relative to the support structure has to be considered. Low or negative coefficients of thermal expansion materials could be used to "cancel out" the CTE mismatch and help mount films to prevent loss of sufficient pretension. However, bulk materials, such as zirconium tungstate are not able to be practically incorporated into the structure, and more advanced metamaterials typically rely on complicated lattices that are also difficult to manufacture and incorporate, and are more effective in 3-dimensional applications rather than flat structures[28].

Currently, the SSPP structure's composite backbone and thin films are interfaced via a kirigami spring, whose nonlinearity allows it to maintain a relatively constant force across large thermal strains. More information on this spring is provided in Appendix B and Chapter 2. This spring helps mitigate some of the issues presented, but does not optimally address some of the surface accuracy, integration, and surface area efficiency issues that must be solved to make the space based solar power concept viable.

Thus, this thesis aims to contribute to the development of efficient and reliable deployable systems by developing an effective method of attaching thin films to composite structures, and further understanding the torsional buckling of such structures.

Since many aspects of this study are difficult to capture purely analytically or experimentally, approaches will primarily focus on numerical analyses, verifying them, and then continuing to iterate them. These analyses are primarily done in Abaqus, a software suite for finite element analysis (FEA)[29]. FEA is a numerical procedure that allows the solving of physics effects across complex geometries by discretizing them into smaller components. While particularly complicated geometries may be computationally expensive, FEA opens the door for a more time efficient pattern of progressing research goals.

1.1 Research Outline

This thesis consists of a total of six chapters, including this Introduction. The main research goal for this thesis is to demonstrate how pretensioned thin films can be attached to thermally loaded deployable space structures with high surface stability and accuracy. This necessitates study of both the film attachment style that is outlined and designed in this thesis, as well as the composite frames to which they are attached to. This is done mostly via numerical analysis as stated, with simplifying analytic theory incorporated where appropriate and verification experiments where possible. Each chapter has a brief introduction and discussion attached to it, focusing in more detail on the necessary background to contextualize the research presented.

The second chapter in this thesis introduces the double s-spring. The double s-spring is the mechanism invented in the course of this work to be able to achieve the main research goal of stable attachment of thin films to compliant composite frames. This chapter first discusses how this spring was inspired and its parameters, and then numerically characterizes the double s-spring as a function of its main parameters. This includes a discussion of the tradeoffs present among the design parameters, focusing on the strength (or rather toughness) of the attachment, and experimental verification of the double s-spring's performance. It then takes the characterization obtained, further optimizes the numerical model pipeline, and uses it to build a reduced order model that allows for the quick and computationally efficient iteration of double s-springs in various applications.

Chapter 3 then focuses on a rigid rod and compliant hinge model of the composite frame to which films are primarily mounted to in the applications discussed, and discusses what is generally the best way to attach something to such a frame. An analytical solution for buckling load is derived in two different flavors, and it is

shown that for this simplified frame model, the average compression in its rigid elements can be treated as an invariant buckling load, meaning that it is agnostic to the manner in which loads are distributed for both fixed direction loads and loads that follow the deformation of the model. This general result is verified via a series of numerical studies, and an experiment.

In Chapter 4, a fully elastic high fidelity model of the composite frame, consisting of pultruded battens and TRAC longerons, is analyzed to determine whether or not the simpler model in Chapter 3 can adequately capture the behavior observed. Experimental observations of composite frames are also discussed to further supplement and contextualize the numerical analysis.

Chapter 5 focuses on additional experimental models created and their practicality for in-space applications. Primarily, the coilability and deployment consistency of films mounted via double s-springs are debuted experimentally. Another application of double s-springs, a foldable toroid that takes advantage of their high strain capabilities, is also discussed.

Finally, Chapter 6 summarizes the total outcomes presented in the thesis. It outlines the main contributions of the work to the field, tools created, and limitations presented. It underlines the main research directions remaining and the vectors for future development of thin film attachment and composite frame architectures.

This thesis also includes two appendices. Appendix A provides more details on the physical manufacturing and cutting of double s-springs. It focuses primarily on why they are made the way they are, and the shortcomings of this approach. Appendix B presents a previous design of kirigami spring that was done during the course of this work, focusing on showing that despite out of plane deformations occurring during the post-buckling regime, the bending stiffness is unchanged.

While the attachment style is chosen to be presented first since it is the main innovation, focus, and topic of interest in the thesis, there is an alternative logical framework in which this work could have been organized. By understanding and predicting torsional buckling via a rigid rod and elastic hinge framework, and then validating this simplified model to a higher fidelity model of the unit cell frame, we can elucidate the optimal style of attachment. Additionally, design parameters for this attachment style, which happens to be the double s-spring, are obtained. Then, a model is developed such that double s-spring geometry may be optimized subject to these constraints. Finally, the experimental applications for double s-springs

can be examined to show that it achieves the in-space requirements imagined and maximizes surface accuracy.

Chapter 2

DOUBLE S-SPRINGS

Part of this chapter was adapted from the following publication:

George A. Popov and Sergio Pellegrino. Thermally stable method of attaching precision thin films to deployable structures using high strain kirigami borders. *AIAA SCITECH 2026 Forum*, page 1830, 2026. doi: 10.2514/6.2026-1830.

2.1 Introduction

Thin films made of Kapton, Mylar, or multilayer membranes perform necessary functions as photovoltaic, structural, reflector, insulation, and transmission devices. Examples include deployable solar sails such as IKAROS and LightSail, space telescope sunshades such as in JWST, and space based solar power satellites [30–35]. A specific focus of this research is the Caltech Space Solar Power Project (SSPP), which uses thin films to achieve a multifunctional large area space structure with solar cells and phased RF transmitters that require high surface accuracy [12].

Previously, thin films have largely been mounted using different types of direct attachments, constant force cable tensioners, or corner localized springs ([2, 14, 32, 36]). These methods lead to observable disadvantages, such as wrinkling, which can lead to structural damage and/or the loss of surface accuracy ([37, 38]). Mounting films by utilizing springs only in the corners of a film also results in areal loss if the film edges have to be shaped to prevent wrinkling. Also, there is a high failure potential due to a low number of attachment points. An established way of providing a continuous attachment is a shear compliant border as described by Leifer, which involves cutting thin, parallel rectangular segments out of the edge of a film ([39, 40]).

In order to maintain surface accuracy and address issues exacerbated by large differences in coefficients of thermal expansion (CTE), thin films must be prestressed. This issue is especially challenging when trying to directly attach films to a supporting structure in order to maximize the surface area. For ultra lightweight structures, such as the Caltech SSPP structure that utilizes a ladder of carbon fiber square frames shown in Figure 2.1, this leads to manufacturing challenges. Thus, in order

to develop large scale deployable space structures, it is necessary to have more effective methods of continuously attaching thin films.

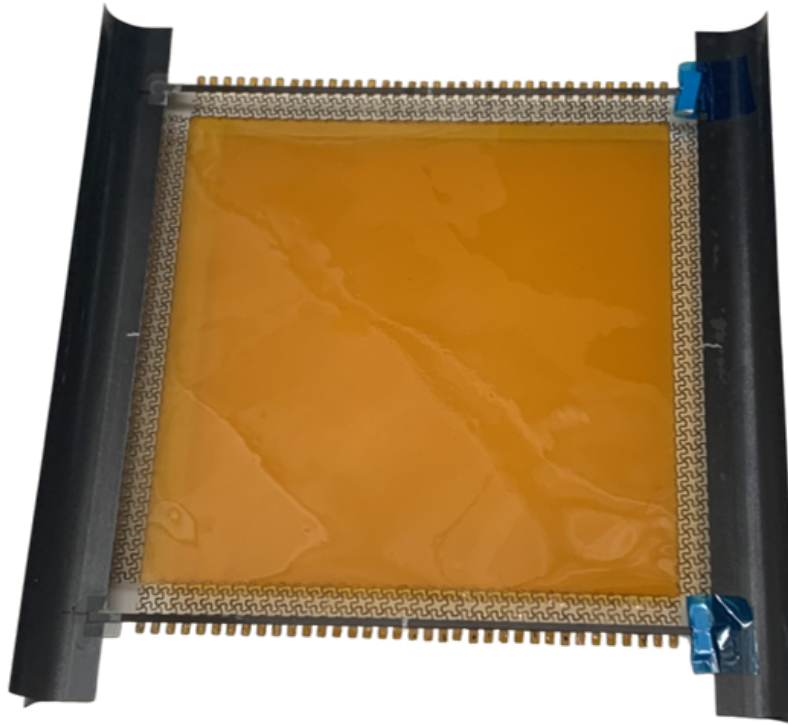


Figure 2.1: SSPP square unit cell showing a thin film mounted with double s-springs.

The inspiration to solve this challenge was found by looking at recent studies of kirigami and other styles of cutting patterns to make structured two-dimensional metamaterials with desirable properties. A variety of such patterns is shown in Figure 2.2. For our application, kirigami sheets (used as springs) and Miura-Ori inspired square cutting patterns (Figure 2.3) are of particular interest ([41, 42]). They exhibit interesting force responses under uniaxial tension: an initial buckling load is reached, the film buckles locally and deforms in a soft manner until fully extended, and then resumes further elastic deformation. This is a desirable behavior as a relatively constant force can be maintained for large deformation after buckling.

However, in the case of these two existing cutting patterns, there are a few disadvantages that make them unsuitable for mounting thin films on space structures. The local buckling phenomena oftentimes result in out of plane offsets for the film, which lead to surface inaccuracies and can initiate wrinkling. Also, these patterns only maintain this behavior under uniaxial stretching, whereas accommodating thermal expansion also necessitates lateral motion. Therefore, a kirigami spring, as shown

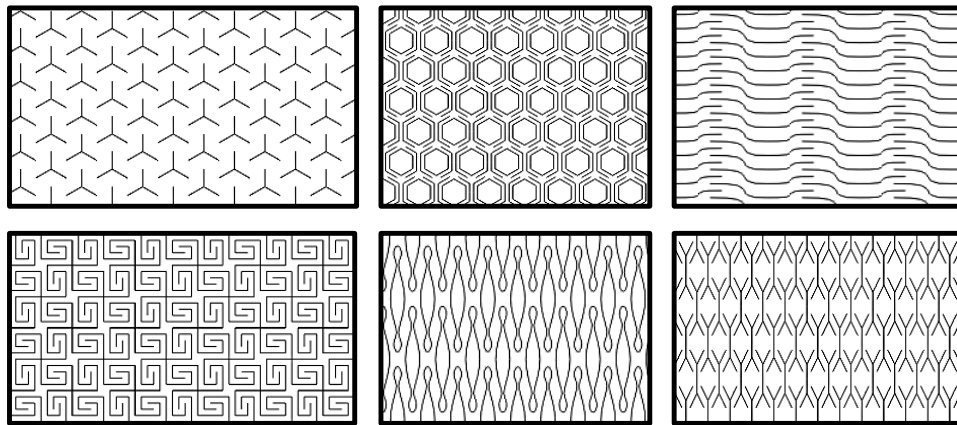


Figure 2.2: Common kirigami patterns.

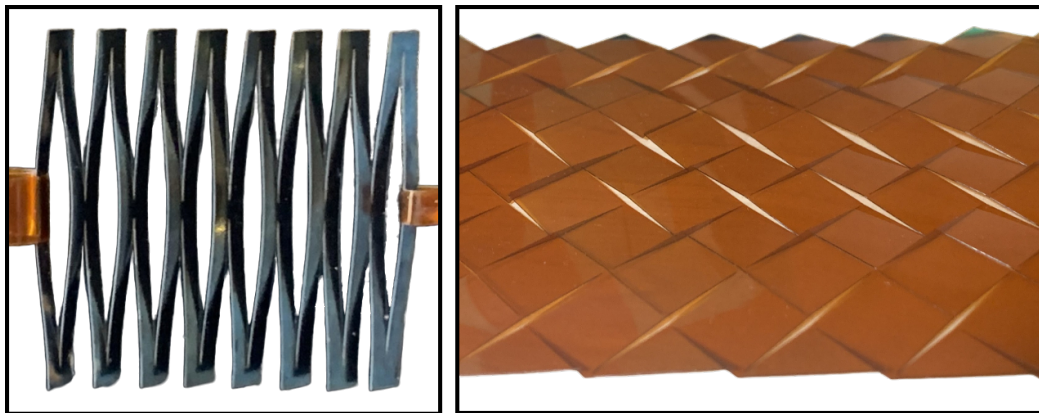


Figure 2.3: Kirigami sheet spring (left) and square (right) cutting patterns under small uniaxial tension.

in Figure 2.3, is only effective as a corner spring, and not as a continuous support for an entire thin film edge. Additionally, in the case of the kirigami springs, the cutting pattern requires a high modulus material such as steel to carry adequate levels of prestress, but the square pattern is too stiff even for commonly used thin films such as Kapton and Mylar. Finally, these patterns make sharp cuts in the film, which can easily tear under load. Hence, a new cutting pattern was envisioned for this application.

2.2 Double S-Spring Pattern

In order to overcome the limitations described above, the "double s-spring" pattern pictured in Figure 2.4 is proposed. It consists of a single B-spline, rotated 4

times to define a unit cell of two intersected "S"'s. In practice, corner fillets are introduced where the splines meet in order to prevent stress concentrations that lead to localized tearing. These proposed springs are sufficiently stiff to support the pretension of membranes, although tolerant of stretch variations, and robust enough to survive space based applications since their design prevents stress concentrations and distributes loads effectively.

The spring geometry is controlled by three main parameters: the coordinate of the B-spline's control point, the width of the ribbon, and the overall length of the unit cells. This makes it easy to tune and adapt to local features of the thin film, such as electronic components that may emit more heat but require lower prestress. It is also simple to manufacture via laser cutting.

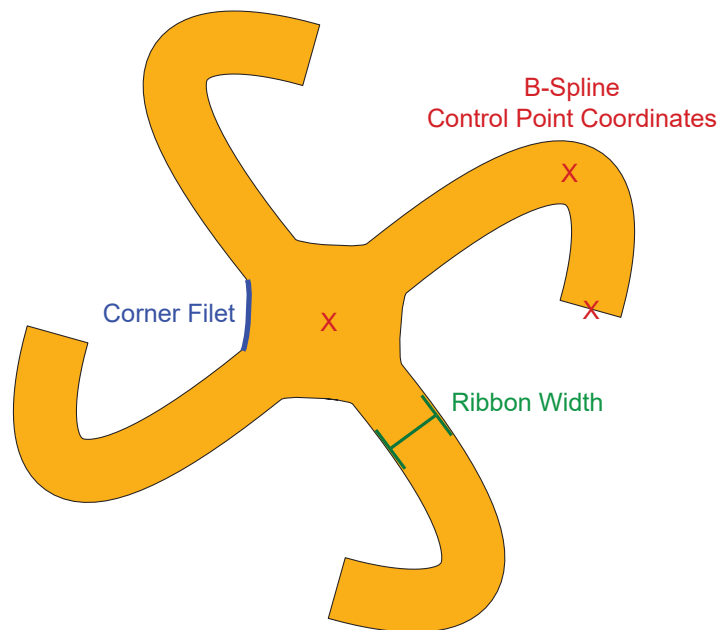


Figure 2.4: Single double s-spring unit cell.

The double s-spring works by mechanically stretching until there is enough tension to form a local buckle, which then begins to twist the legs of the "S" out of plane. This is in contrast to the patterns in Figure 2.3, which bend and rotate out of plane. Once this motion is initiated, the spring can sustain large displacements with relatively negligible force increases as it continues to deform elastically, similar to a ribbon twisting. The edge of the pattern remains approximately planar. Once the "S" has been fully extended, the pattern begins to stretch once again. In Figure 2.5, regions associated with these behaviors have been highlighted. The force vs. displacement graph in Figure 2.8 shows the overall response.

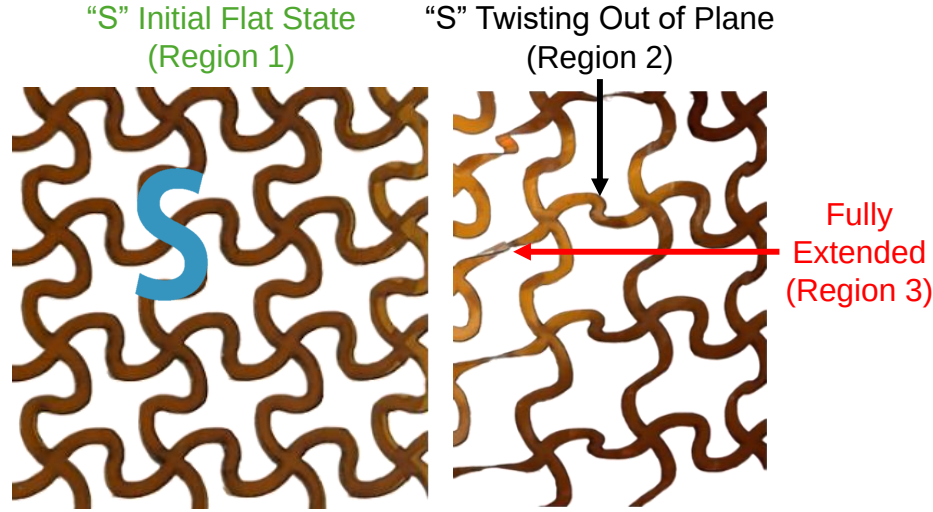


Figure 2.5: Mechanics of the double s-spring pattern with the "S" highlighted in blue. A tessellated series of unit cells (left) is stretched by various amounts (right), with the different states corresponding to the regions illustrated in Figure 2.8.

The experimental and numerical results presented in the next section indicate that, due to the large displacement capacity of the localized post-buckling behavior, the double s-spring cutting pattern can be used to achieve relatively constant and uniform biaxial stress states in thin films.

2.3 Numerical Characterization of Double S-Springs

To investigate the double s-spring's nonlinear and coupled behavior, its in-plane force response was characterized as a function of both x and y deformations for a series of parameters. Since behavior is coupled between several unit cells, a single unit cell model of the pattern with periodic boundary conditions is not sufficient to capture the full behavior. Thus, a multi-unit cell pattern was created for analysis. This analysis was done by writing a Matlab script to generate the geometry, importing it into Abaqus 2024, and conducting a post-buckling response analysis. For the example presented, the parameters in Table 2.1 were used, with the thin film modeled as Kapton, and the cutting pattern design shown in Figure 2.6.

The pattern uses a width of two unit cells in the x -direction. Two unit cells were chosen to keep the areal loss minimal, but still retain full functionality of the attachment scheme. Once a .dxf file was generated and imported into Abaqus, a planar geometry was generated in 3D using S4R reduced integration thin-shell elements. A buckling eigenvalue analysis was carried out with a small load initially applied along

Parameter	Value
Unit Cell Size	5 mm
Control Point Coordinate	(3.6, 3.2) mm
Ribbon Width	0.6 mm
Thin Film Modulus	2.67 GPa
Poisson Ratio	0.34
Thin Film Thickness	0.05 mm

Table 2.1: Selected design for numerical characterization example.

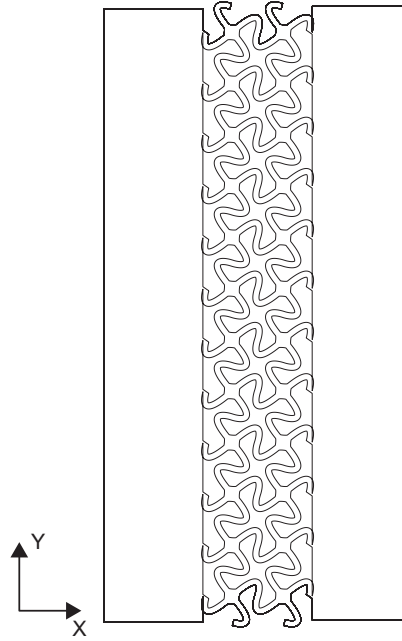


Figure 2.6: Double s-spring design for numerical characterization example.

the edge in the x -direction. The edge region adjacent to the double s-spring region was held fixed against translations in the x - and y -directions. The resulting first 10 buckling eigenmodes were then seeded into the model as imperfections using the *IMPERFECTION command to carry out a post-buckling study, with a maximum imperfection magnitude of half the thickness of the thin film. A mesh convergence study was done for both of these steps, and a mesh element size of 0.1 mm was chosen for the double s-springs and a 0.5 mm element size was chosen globally. An imperfection sensitivity analysis was also conducted, with <3% maximum deviation in the force response observed in the post-buckling analysis between all different types of imperfections. Hence, a set of imperfections that yielded post-buckling

behavior closest to experimental observations was chosen. As part of the characterization, it was also observed that the average force per unit cell converges for a length of 12 unit cells in the y -direction, which is the pattern shown in Figure 2.6.

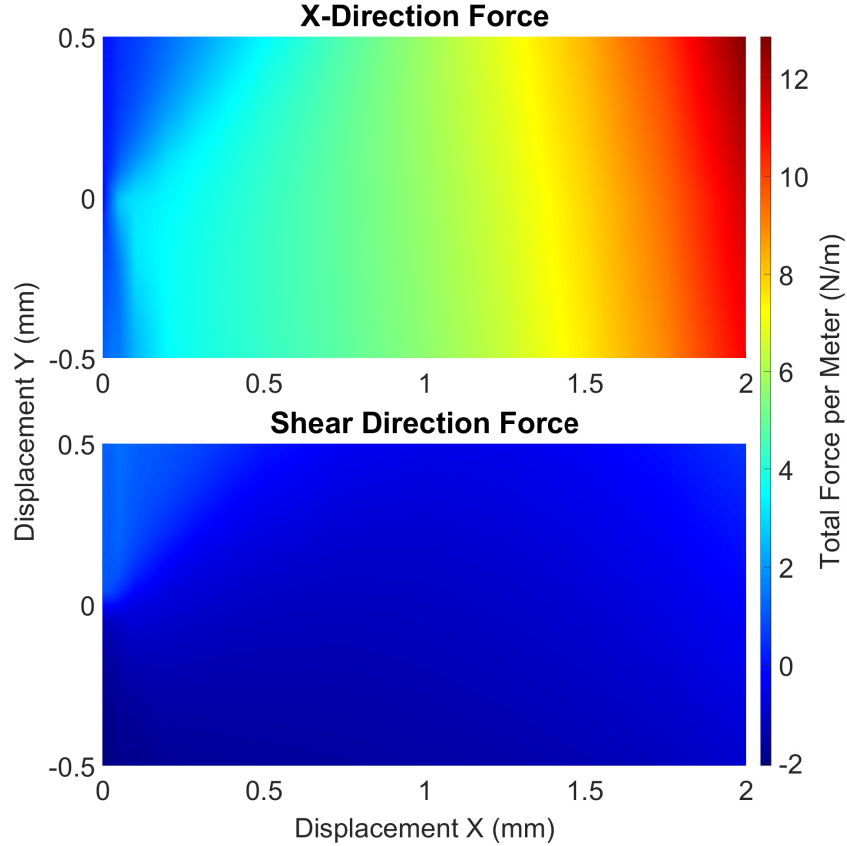


Figure 2.7: Force response for the example double s-spring design, under x extension and y shear.

The post-buckling analysis consisted of multiple repeated steps. First, the thin shell springs were stretched in the x -direction by 2 mm (20% stretch), since this is approximately where the spring begins to enter the third elastic regime described in Section 2.2. Additionally, assuming a 20 ppm/ $^{\circ}\text{C}$ coefficient of thermal expansion, this the maximum amount of displacement a 1 meter film would need to accommodate at each edge, assuming a 200 $^{\circ}\text{C}$ temperature change. Then, the spring was sheared by ± 0.5 mm in the perpendicular direction, and returned to a no shear condition. After this, the x -direction stretch was decreased, and the process was repeated. This was done to ensure that the loading and unloading curves match. In the case of the double s-springs, no significant hysteresis was observed in any load cycle in this numerical simulation.

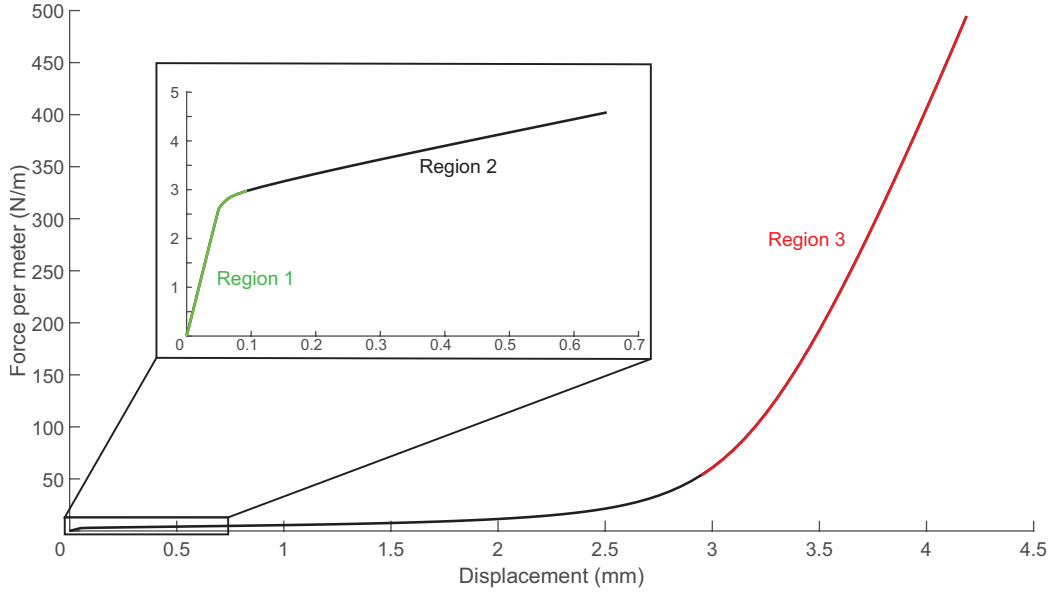


Figure 2.8: Numerically calculated elastic force response of uniaxial stretching of the unit cell of double s-springs in Figure 2.6. Three roughly linear regions are identified corresponding to pre-buckling, post-buckling, and fully extended behavior of the pattern.

The x -direction and y -direction force responses as a function of the edge displacement are shown in Figure 2.7. As desired, the force has little variation after the initial buckling load until $\sim 30\%$ displacement, where it begins to sharply increase, as shown in Figure 2.8. Due to the chirality of the double s-springs, there is a slight asymmetry in the force behavior in the shear direction.

2.4 Influence of Design Parameters

Since the y -direction force response is an order of magnitude lower than the x -direction forces, the effects of altering the geometry above of the double s-spring can be seen most prominently in the uniaxial behavior in the x -direction. Three desirable characteristics can be examined: the force per meter magnitude and force-displacement slope in region 2, the displacement range of region 2, and the toughness of the double s-spring. Figure 2.9 shows the general effect of moving the control point on the uniaxial response, where decreasing the x -coordinate of the control point can be weakly correlated to increasing the slope of the force response and decreasing the displacement range. This is due to it increasing the relative "sharpness" of the s-spring bends while keeping the total relative length of the bend the same. The y control point affects the force response more strongly, where a lower y control

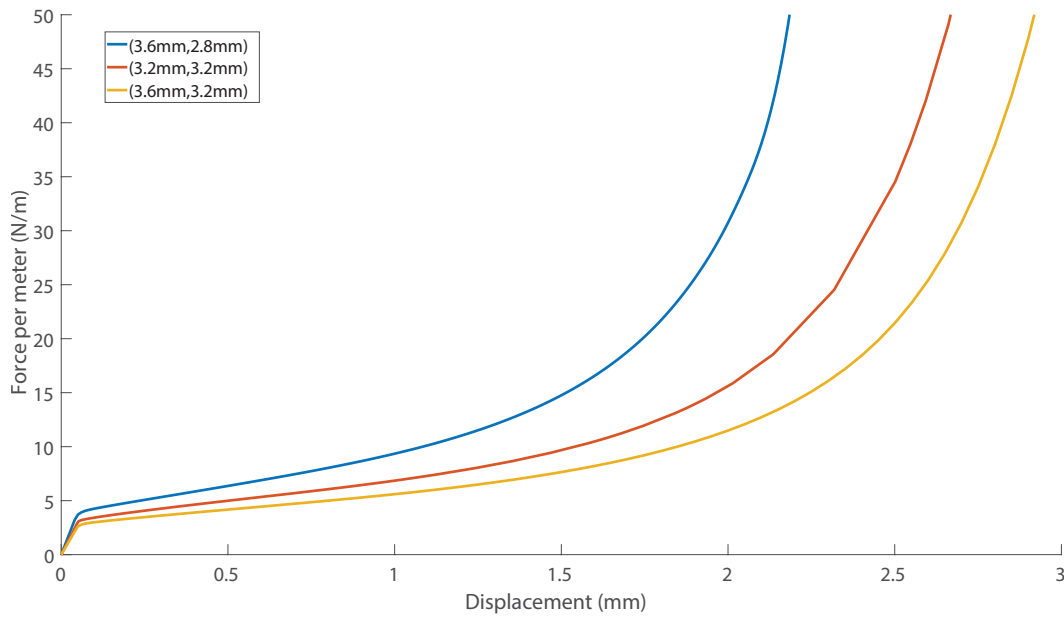


Figure 2.9: Effect of shifting the control point location of the example double s-spring on the uniaxial force response.

point value also increases the slope and magnitude of the force in region 2, due to it having a greater effect on the total arc length of the "S."

The uniaxial force response is quite sensitive to the thickness of the film used for a fixed unit cell size. Increasing the thickness even by a small amount results in a large increase in the slope and magnitude of the force response, and also drastically decreases the displacement range, as shown in Figure 2.10.

The effect of varying the width of the double s-spring ribbon is shown in Figure 2.11: the thicker the ribbon, the higher the force per meter and the lower the displacement range. However, the width of the ribbon also has the greatest effect on the toughness of the double s-springs, as discussed in the following section.

2.5 Toughness

The toughness of the double s-spring was evaluated with the *Plastic option in Abaqus, and verified experimentally. In this study, toughness is defined as the total amount of strain energy the double s-springs undergo before plastic yielding is observed on any part of the film, since after this point, the loading curves will change, and additional stretching will cause permanent deformation. Polymers like Kapton are ductile, and the yielding behavior can be difficult to analyze since the

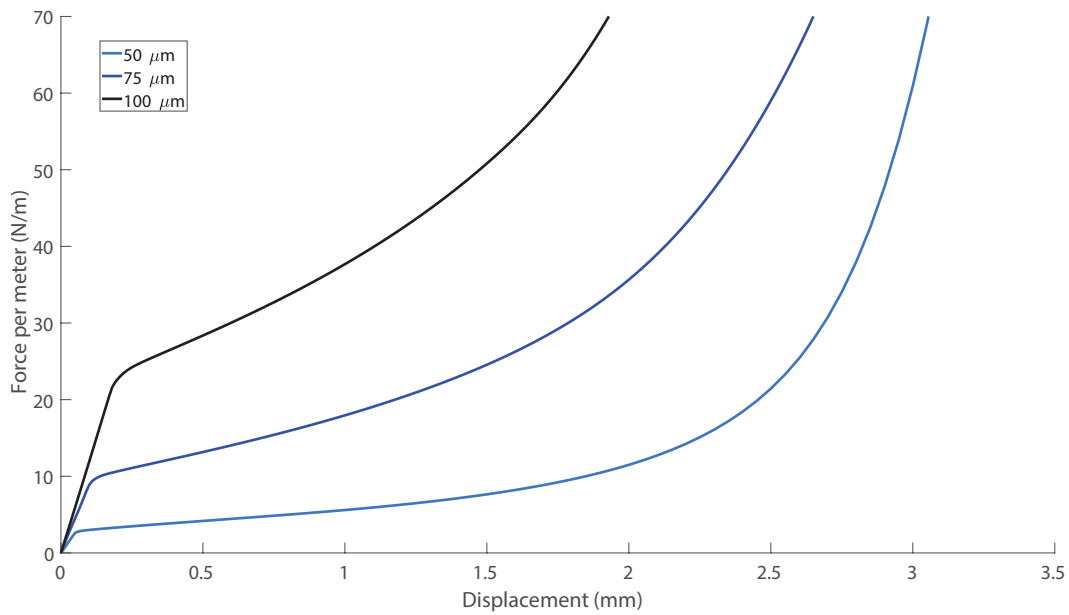


Figure 2.10: Effect of changing the thin film thickness of the example double s-spring on the uniaxial force response.

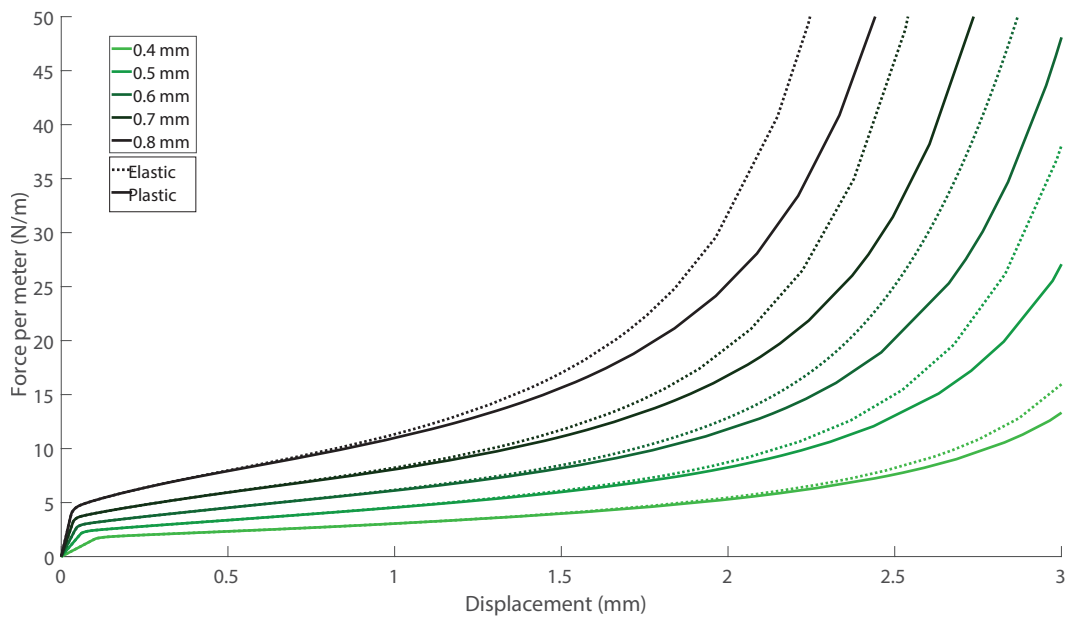


Figure 2.11: Effect of changing the ribbon width of the example double s-spring on the uniaxial force response. The results from both the purely elastic and plastic numerical material models are shown.

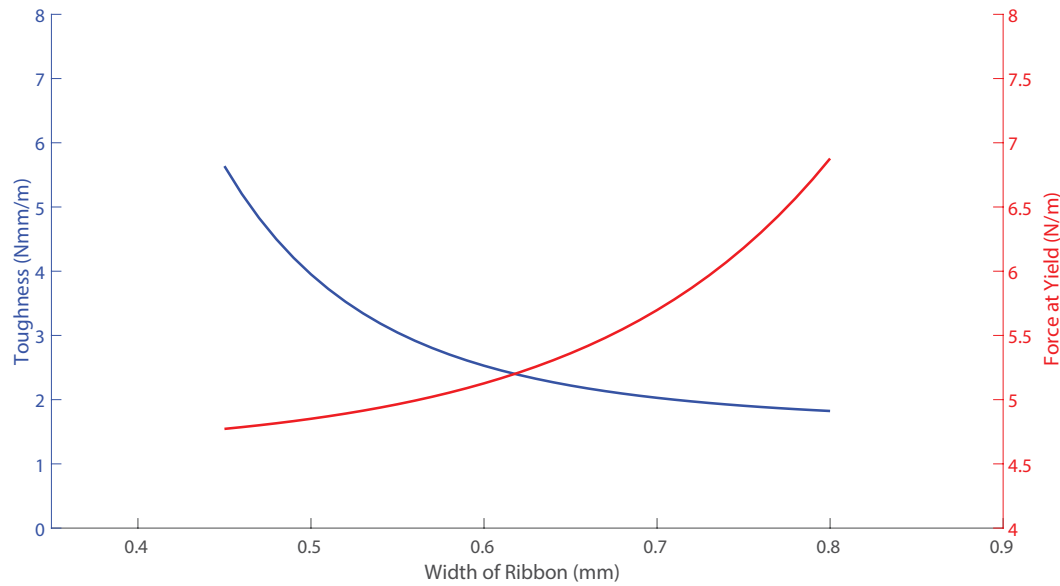


Figure 2.12: Effect of changing the ribbon width of the example double s-spring on the toughness and yield load.

material may only visibly fail at very high displacements, despite suffering local plastic deformation. Material properties for the Kapton 3% yield point were used ([43]). The comparison for the uniaxial loading curves between the plastic and the elastic models in Abaqus for a series of various spring ribbon widths is shown in Figure 2.11. Figure 2.12 shows that despite lower spring widths having more desirable force response curves and higher toughness, a lower maximum load can be sustained. This higher toughness is somewhat counterintuitive as more material is cut away for lower width springs. However, smaller widths behave less stiffly and more like ribbons, resulting in less localized strains.

Two types of experiments were done to verify the double s-spring behavior based on the parameters in Table 2.1. Samples were manufactured via a laser cutter from Kapton. Due to the thinness of the Kapton and the curved geometry of the springs, there were sectors where the dwell time of the laser was relatively longer, more material was ablated, and a slightly lower width than specified occurred (see Appendix A for more details on manufacturing). In the first experiment, ten samples were stretched on an Instron material testing machine continuously to failure. The averaged measured response is shown in Figure 2.13. The measured behavior falls between the 0.5 mm and 0.6 mm width parameters due to the imperfections introduced in the laser cutting process. The second type of experiment stretched the

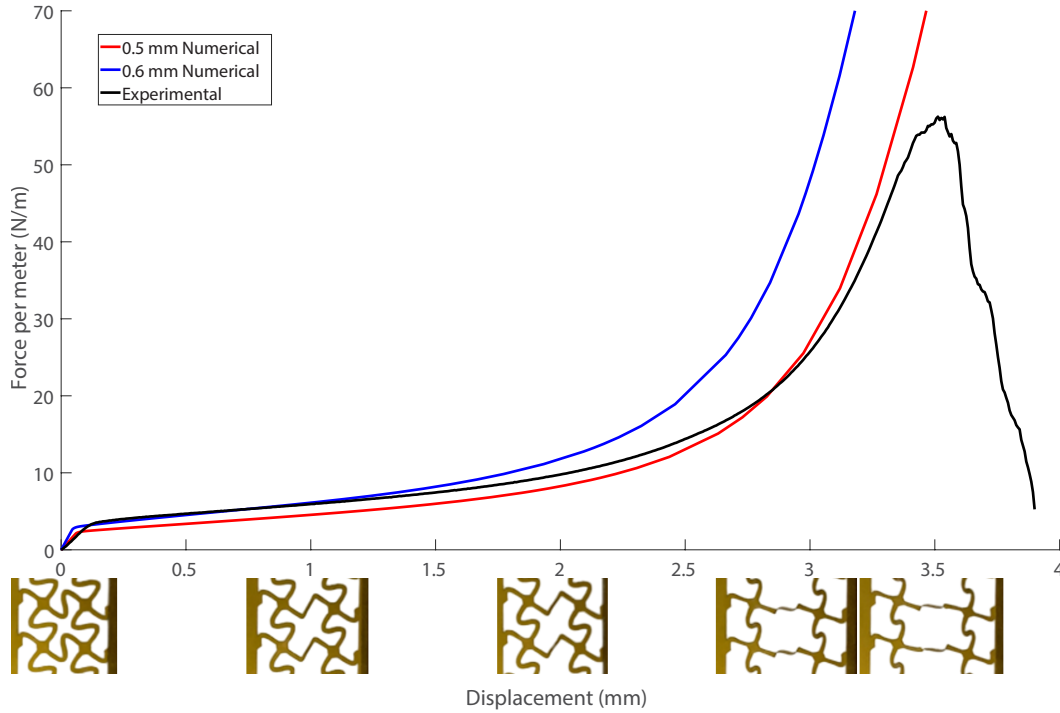


Figure 2.13: Averaged experimental uniaxial loading response of the double s-spring compared to the numerical plastic model. Snapshots of the spring are shown for displacements of 0 mm, 1 mm, 2 mm, 3 mm, and 3.5 mm. The last value is right before total failure occurs, and the pattern tears in the middle.

samples by increasing amounts to see the effect of plastic yielding on the uniaxial force response, and verify where plastic deformation starts. Figure 2.14 shows how the loading path has changed for a locally yielded spring after increasingly larger previous displacements. Changes to the loading curve are observable after ~ 0.8 mm of displacement, similar to what is predicted by the Abaqus plastic model for a 0.6 mm width ribbon. The effect of the localized plastic deformation is especially evident once the spring has been stretched into region 3. The localized deformation acts as a large imperfection, eroding the sharpness of the transition between region 1 and region 2. However, the tail end of each curve falls onto the curve of the first experiment, as expected, suggesting that additional cycles that reach the same maximum displacement do not immediately cause further localized deformation. Additionally, the slope of region 2 is not significantly affected, meaning that the performance of the double s-spring is also not significantly degraded if the spring

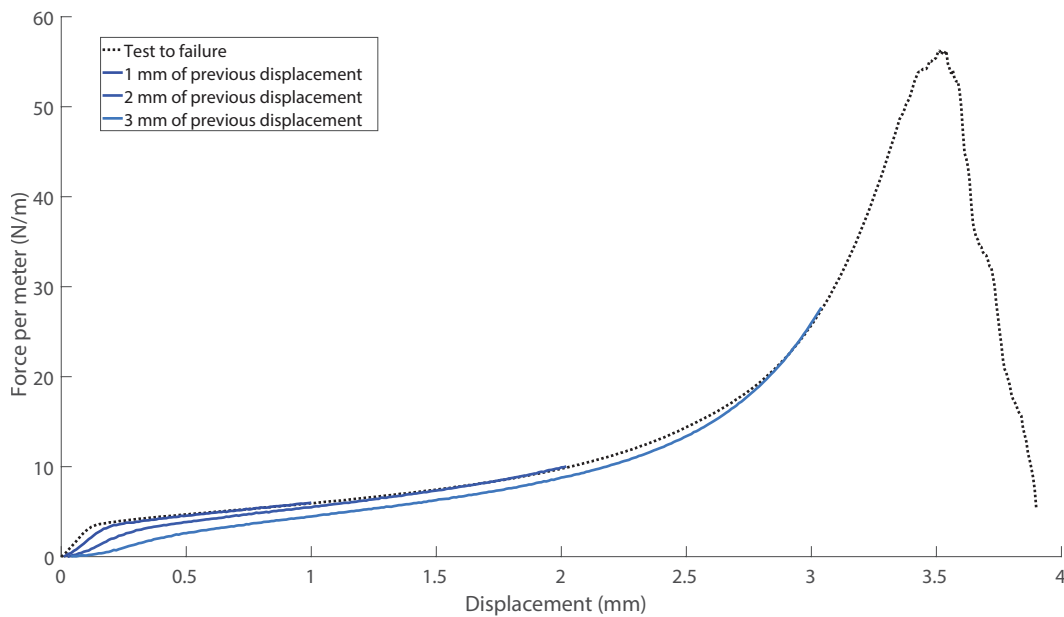


Figure 2.14: Experimental measurements showing the effect of small and large amounts of localized plastic yielding on the uniaxial loading curve of the double s-spring.

is stretched to prestress a structure.

Finally, an experiment was done to demonstrate that pretensioned films attached with double s-spring connections minimize wrinkling in mounted thin films. See Chapter 4 for more details on this process. A 200 mm by 200 mm square of 50 μm thick Kapton had a 10 mm wide border of double s-springs cut into each edge, speckled with a permanent marker using a Zünd cutter, pretensioned to a force of 6.6 N/m on each side using a series of weights and pulleys, and then bonded to a carbon fiber composite structure as shown in Figure 2.15. Digital image correlation (DIC) was then used to get a precise measurement of the surface of the membrane. The maximum variation in the uncut membrane surface was observed along the edges, with amplitudes of $<100 \mu\text{m}$, or less than twice the thickness of the film, demonstrating that a thin film prestressed with double s-springs undergoes no visible wrinkling. The slight saddle shape observed in the image is due to the fact that the battens and longerons of the frame are bonded on top of each other. Hence, to minimize the offset created by this, the film was bonded on top of the left and right edge, but below the top and bottom edge. This offset can be further corrected in future designs by utilizing a spacer layer if necessitated by mission requirements.

Similar experiments were also done using a hotplate to control the pretension while bonding the film to an Invar frame, as shown in Figure 2.16, which also showcases the local buckling of the double s-springs. These films had lower pretension, but very similar surface accuracy results without additional local variations, such as the saddle shape observed in Figure 2.15.

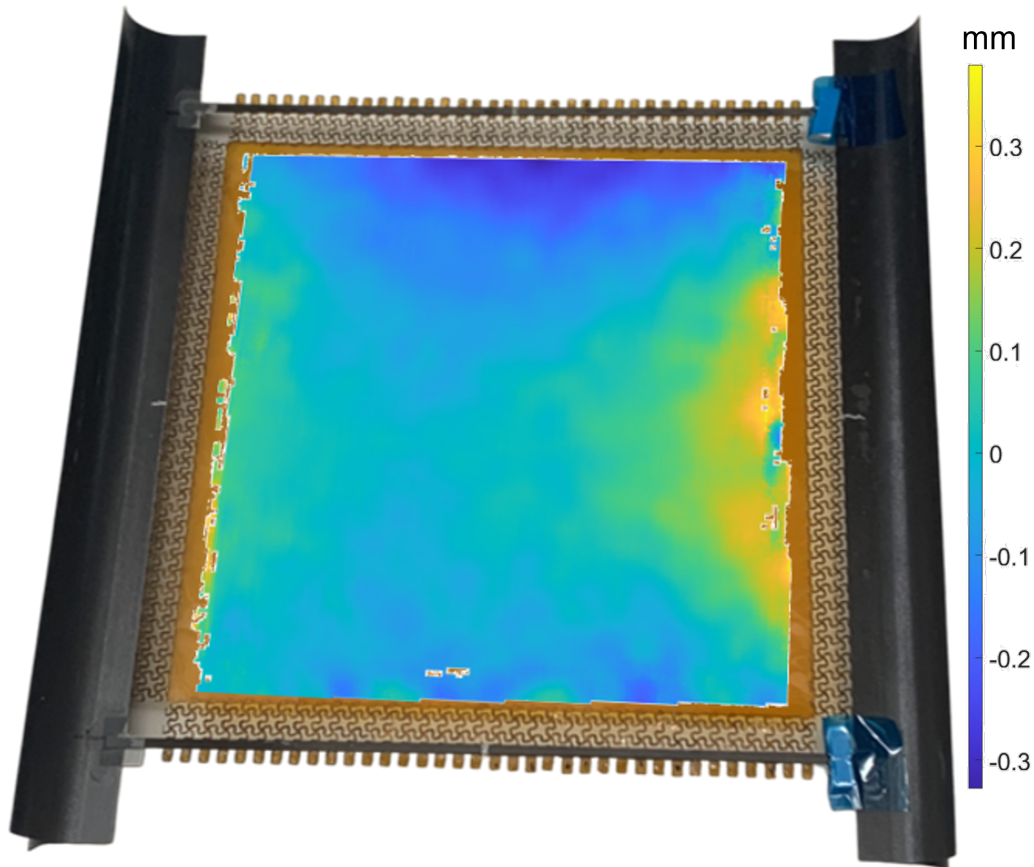


Figure 2.15: Surface level plot obtained via DIC of a pretensioned film bonded to a composite frame. The slight saddle shape observed is due to the film being bonded on top of the left and right edges, but below the top and bottom edges.

2.6 Further Numerical Model Optimization

In Section 2.3, which describes the numerical simulations for a series of double s-springs, models were created in MATLAB, and then imported into Abaqus 2024. This import into Abaqus can be automated using Python scripting, which can also be used to apply boundary conditions, mesh the assembly, and run a simulation. This approach is appropriate for simulating small strips of double s-springs. However,

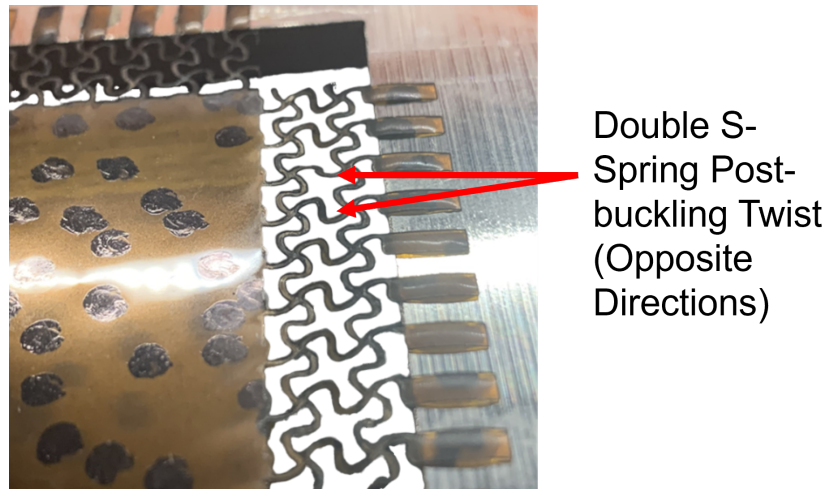


Figure 2.16: Local image of a pretensioned film bonded to an Invar frame. Note the mild stretching and local buckling of the double s-springs.

a more robust approach is preferred for a more general set of spring geometry parameters. For a larger set of control point coordinates and width parameter combinations, there becomes a sharper bend in the elbow of the double s-spring, resulting in a self-intersecting geometry, as shown in Figure 2.17 among other phenomena that make it difficult to parse the imported geometry in Abaqus, and requires manual intervention in this otherwise automated pipeline. This type of geometric error can oftentimes be fixed using SolidWorks to extrude the DXF and merge the separated domains. However, this additional step can be time-consuming due to the large amount of points used to define the geometry regardless of hardware resources. Even if the geometry is successfully imported afterwards, it still has to be meshed in Abaqus, which is not always successful, and is once again time intensive. Additionally, if a set of geometric parameters is known to work well for a given mesh size, occasionally additional issues arise during mesh refinement studies as well due to Abaqus's meshing methodology. Finally, for larger geometries, like the ones used to verify the results of the reduced order model described in the following section, it is computationally inefficient to design and import them into Abaqus. Hence, in order to efficiently optimize the double s-springs parameters for a given design goal, and be able to effectively iterate through simulations, it is computationally necessary to further streamline the methods used to generate and evaluate double s-springs.

The approach developed to tackle the above concerns is to form and mesh the geometry in a single software. An open source software, GMSH was chosen for this. GMSH allows for the creation of a manifold double s-spring geometry and

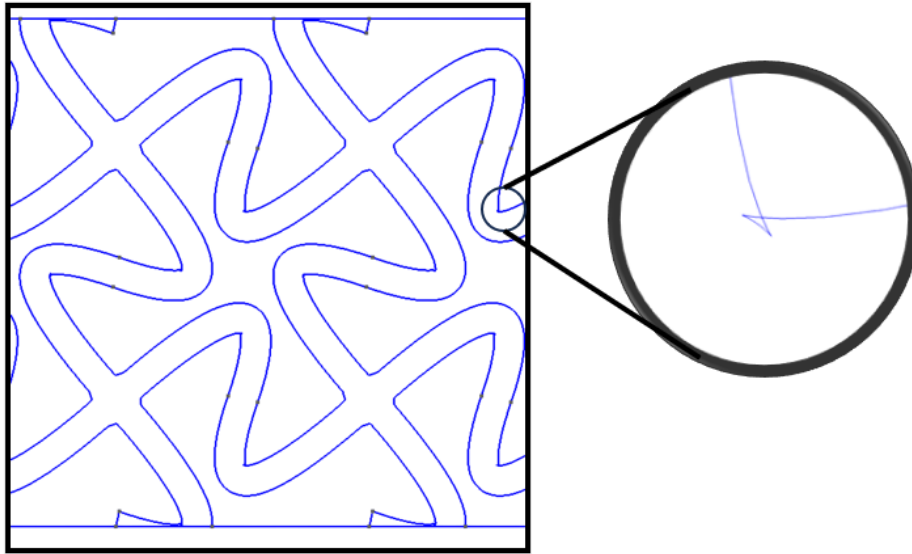


Figure 2.17: Self-intersection of geometry producing an import error and meshing difficulties for a higher control point coordinate and width combination.

mesh, and exports it with predefined node and element sets as an Abaqus input file (.inp) (among a plethora of other potential options). Predefining the sets allows for an easy assignment of boundary conditions and material properties, relieving some of the difficulties associated with handling orphan meshes in Abaqus. GMSH generates the geometry natively and meshes it using its own geometric code (.geo). GMSH can also alternatively be scripted via Python, C++, Julia, and FORTRAN APIs for more advanced operations. By creating and meshing the geometry in GMSH as opposed to MATLAB, many errors such as the self intersection case in Figure 2.17 can be resolved outside of Abaqus without the use of additional tools such as SolidWorks. GMSH iterates geometric parameters and associated meshes much faster, since the DXF export from MATLAB generates curves via a series of discretized points, as opposed to encoding information as a series of both points and curves, resulting in extremely unoptimized files and computationally intense meshing requirements. Exporting these point based geometries in Abaqus also results in the meshing algorithm attempting to create a mesh node at each point, meaning a careful balance is needed between enough points to properly represent a small radius (such as the fillets in the model), but not create an excessive number of elements or overly constrain the mesh. By meshing the native geometry immediately, the generation and meshing of Figure 2.6 is reduced to ~30 seconds from a couple hours for an inefficient set of parameters for a $20\text{ cm} \times 20\text{ cm}$ film. Figure 2.18 shows

such a relatively well defined and meshed geometry as imported into Abaqus from GMSH which would normally suffer the self-intersection shown in Figure 2.17.

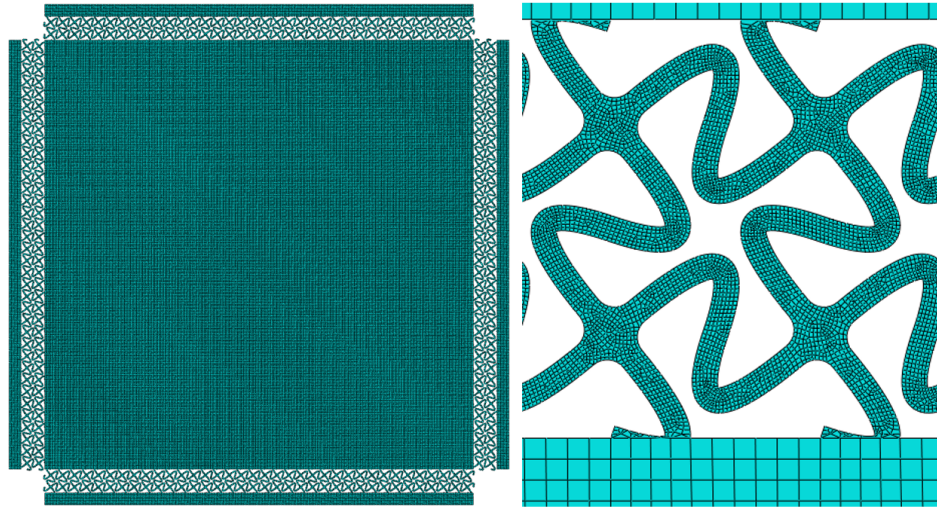


Figure 2.18: 519378 element locally refined mesh from a 20 cm $\times$ 20 cm unit cell film used in Abaqus via an input file generated in GMSH. The right image is a magnified view of the border.

The input file can then be directly run in Abaqus once simulation steps and material domains/properties are properly defined in it. This can be done using a simple Python script automating the modification and execution of these files. Following these definitions, it is simple to seed imperfections, evaluate surface accuracy, or run thermal-displacement simulations to evaluate and iterate the behavior of the springs either as done previously, or characterize their applied performance in larger structures, such as the one illustrated in Figure 2.18 and described in the following sections.

2.7 Reduced Order Modeling of Double S-Springs

While the optimization of the model creation in the preceding section greatly reduces computational effort in preparing simulations to evaluate double s-springs, such simulations still can be computationally expensive. Additionally, while for small strips double s-springs are not particularly sensitive to the chosen imperfections, for larger groups of double s-springs poorly chosen imperfections face convergence difficulties. This can be addressed by employing a reduced-order model to further investigate the effects of double s-springs on supported films and mounting structures. Due to the thinness of the ribbons that make up the springs, they require many mesh elements to define a full-resolution model, even when using 2D shell

elements. In the example presented in section 2.3, each unit cell shown required 999 elements. For a $1\text{ m} \times 1\text{ m}$ structure, this means that 1.6 million elements would be needed just to model the double s-spring pattern (ignoring the film or frame), and large scale simulations would be unreasonably expensive. CONN3D2 elements in Abaqus provide a solution to this, while also removing the need for seeding imperfections to obtain spring behavior. CONN3D2 elements can be defined to capture the coupled, nonlinear, multidimensional behavior of double s-springs in the space between two nodes, as well as be assigned reference lengths to quickly apply initial prestress to a structure. Reducing the elements necessary and removing the need for a series of simulations to evaluate parameters greatly increases computational efficiency, and thus helps further guide the optimization of the parameters presented.

A CONN3D2 element can be used to define double s-spring behavior at several different levels of detail. By taking the information displayed in Figure 2.7 as sets of tabulated values and assigning it to a CONN3D2 element with CARTESIAN type behavior, and assigning a nominal bending stiffness of the membrane in the z direction based on the film thickness and width to constrain out of plane motion, the full behavior of the double s-spring is captured at a unit cell pair level. This is verified simply by creating two nodes assigning a local coordinate system, and applying forces and displacements to one of them, and measuring corresponding reaction forces at the other node. Optionally, it is also possible to constrain the rotation of these two nodes to follow each other using either the ALIGN type connection definition, or prescribing a rotational stiffness corresponding to the torsional and bending stiffnesses of the film. This was found to not significantly affect the output in the evaluation below. For less detail, uncoupled behavior or single direction definition of the double s-spring can be used. AXIAL connector types were found to be an inappropriate choice for this model, as they yield stress distributions inconsistent with analytical estimates and the high fidelity model below.

However, there are a couple of caveats that need to be addressed when utilizing multidimensionally coupled nonlinear CONN3D2 elements. First, as with most implicit analyses, even static ones, Abaqus utilizes a time step increment to solve the underlying matrices and PDEs. For parts connected with only CONN3D2 elements, such as illustrated in Figure 2.19, this solution is prone to struggling with rigid body modes and calculating the resulting forces, since it attempts to obtain a solution by solving them as a quasistatic spring mass system with some velocity. Hence, in order to obtain a convergent solution, the connectors also are required to have a

damping value prescribed to them. The magnitude of this value in this section was chosen to be 0.01 Ns/m in all directions. This value's magnitude does not affect the solution as long as it is high enough to converge the solution, but not excessive as to over damp the solver for the time step increments prescribed. For values up to 1000 Ns/m, the solution obtained was identical, hence the smallest value that consistently converged the simulation was used.

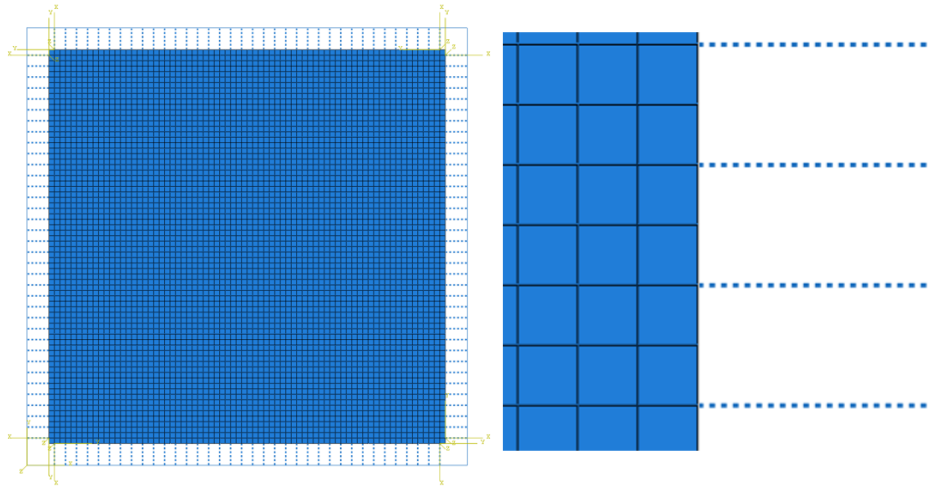


Figure 2.19: Reduced order model mesh from a 20 cm $\times$ 20 cm unit cell film used in Abaqus. The right image is a magnified view of the border. In contrast to Figure 2.18, each spring pair is modeled by a single CONN3D2 connector element (dashed line), and the mesh of the film contains less elements. The external solid line is comprised of beam elements to provide nodes to attach the connector elements.

Another caveat is that Node A (local orientation node) is much easier computationally to move than Node B, since the local orientation at node B is ignored. Hence, for the model in Figure 2.19, Node A is both preferred logically and computationally to be the node on the film. In fact, despite that the node on the defined frame is fully constrained in this set of simulations, choosing it as Node A would result in a failure to converge. Using PROJECTION CARTESIAN instead of CARTESIAN or removing the coupling both solve this. Due to the way that PROJECTION CARTESIAN calculates the local directions and the additional computational expense, the latter method is preferred. As seen below in Figures 2.20, 2.21, and 2.22, and can be predicted by the fact that the asymmetry in Figure 2.7 is rather low, the effect of removing the coupling is minimal on the stress magnitude prediction, but is occasionally necessary for predicting more precisely the overall stress distribution.

Having acknowledged the potential limitations of the reduced order model, its

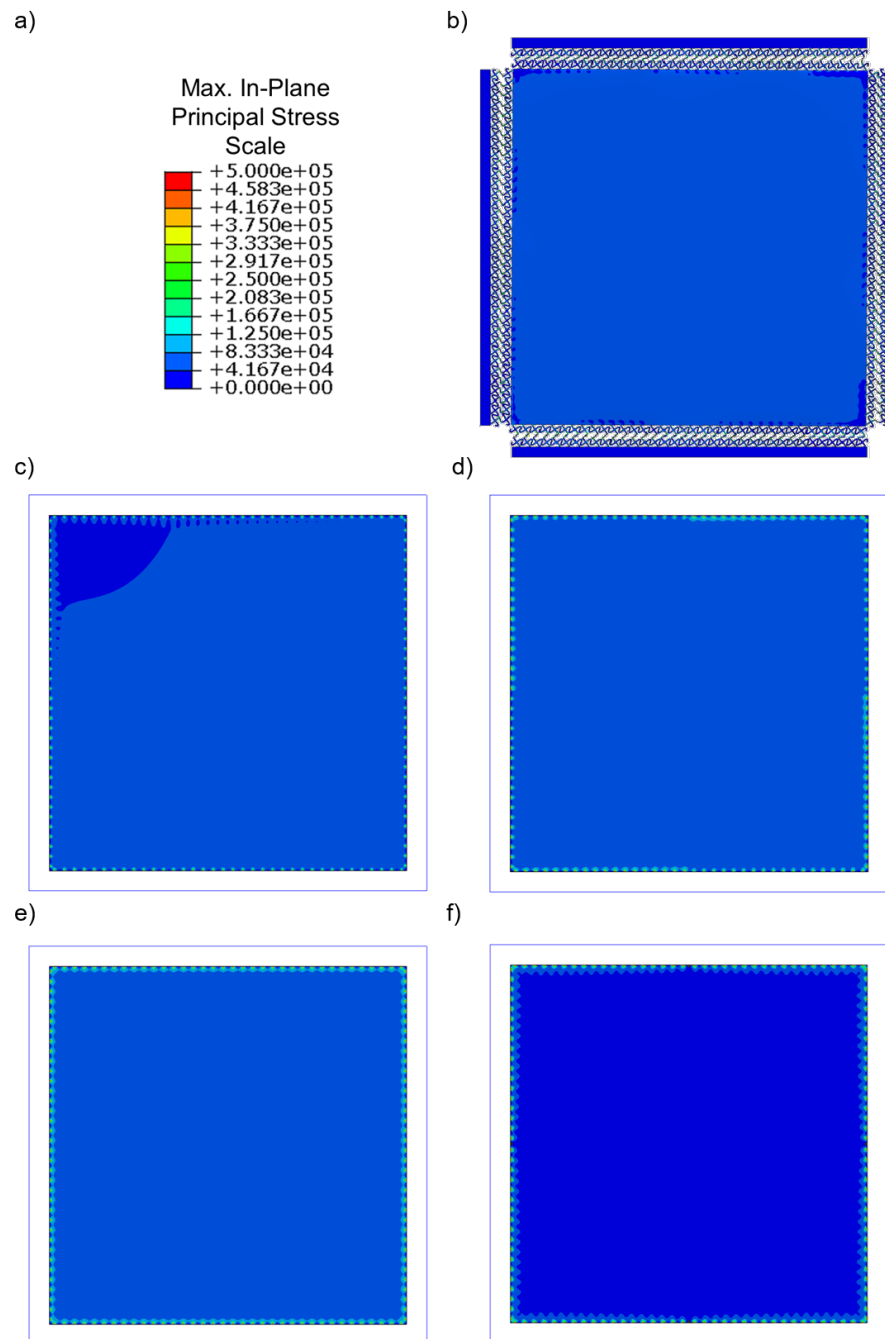


Figure 2.20: Maximum in-plane principal stresses; a) scale, b) high fidelity model, c) reduced order model (ROM) with coupled nonlinear CONN3D2 elements, d) ROM with coupled nonlinear CONN3D2 elements in opposing directions, e) ROM with uncoupled nonlinear CONN3D2 elements, f) ROM with unidirectional nonlinear CONN3D2 elements

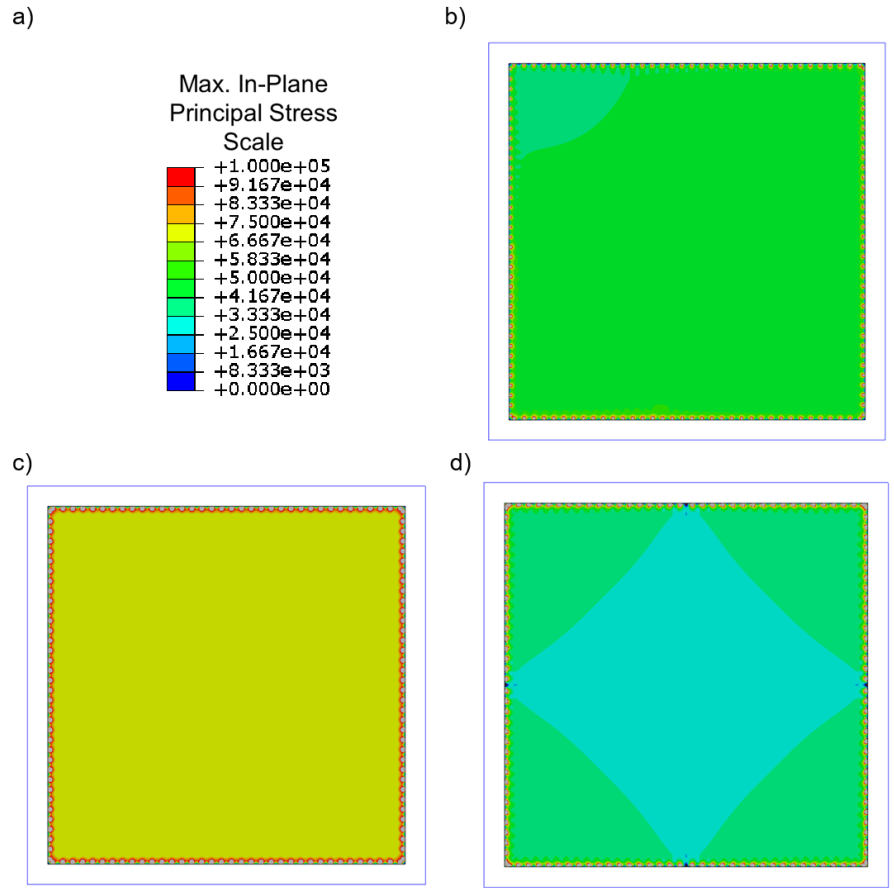


Figure 2.21: Closer look at the stress distributions of the maximum in-plane principal stresses for the reduced order models; a) scale, b) reduced order model (ROM) with coupled nonlinear CONN3D2 elements, c) ROM with uncoupled nonlinear CONN3D2 elements, d) ROM with unidirectional nonlinear CONN3D2 elements

performance can be evaluated in comparison to a high fidelity model. The high fidelity model was generated using GMSH as described in the previous section, and consists purely of S4RT elements. Instead of seeding geometric imperfections as described for the 2×12 unit cell samples, this time an initial *STATIC displacement step was done with moments applied in the spring region to stimulate the post-buckling behavior along the edges. While geometric imperfections would have provided similar results, this method allows for an analytic set of functions to be used (such as a sine series), allowing for iterations of different imperfections to be run more efficiently for this specific model. A mesh convergence was also performed. The edge regions outside of the springs were fully constrained. The center region, the film itself, then has a temperature applied to it (-100 °C) in order to induce a uniform thermal strain throughout the whole model, and displace the springs,

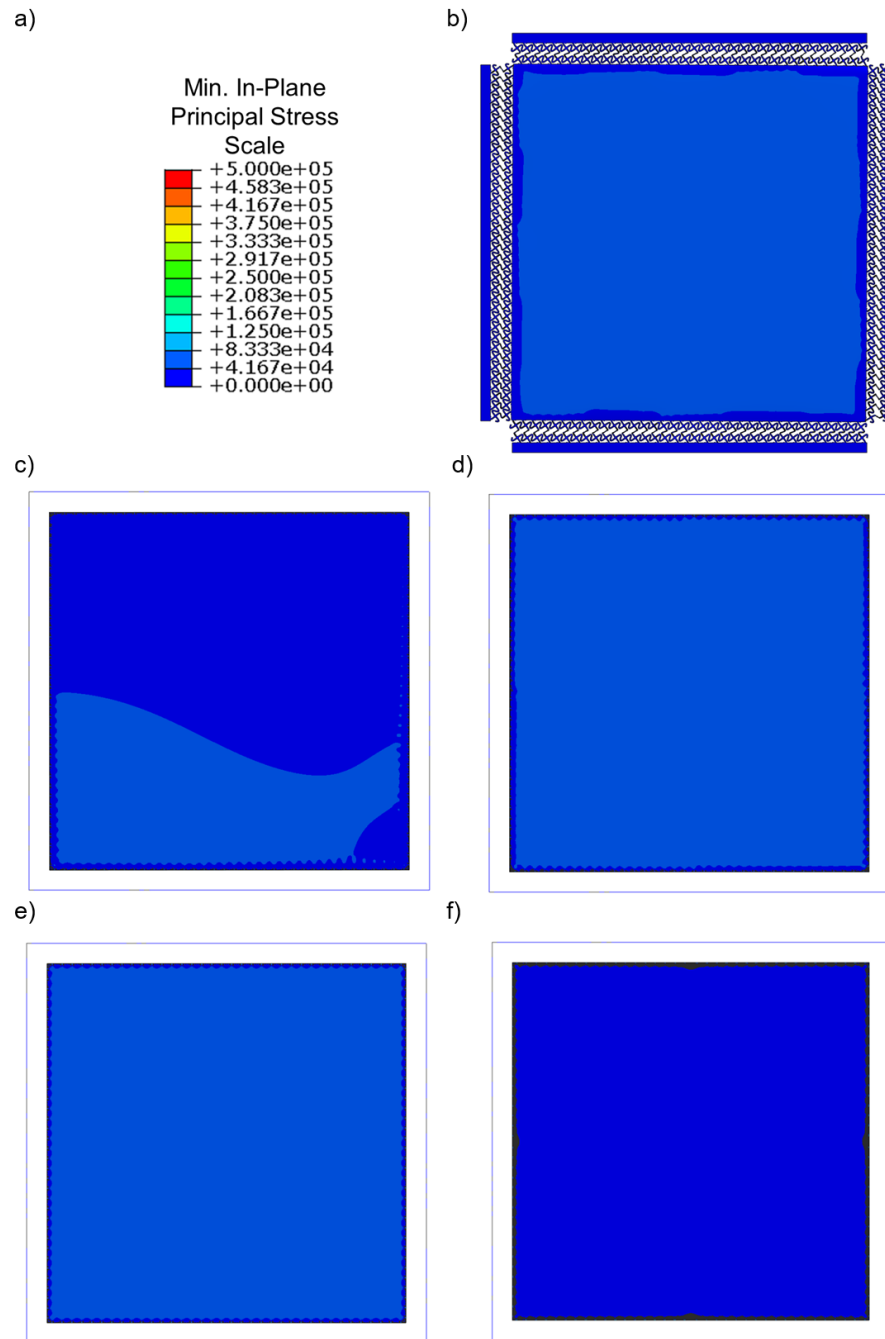


Figure 2.22: Minimum in-plane principal stresses; a) scale, b) high fidelity model, c) reduced order model (ROM) with coupled nonlinear CONN3D2 elements, d) ROM with coupled nonlinear CONN3D2 elements in opposing directions, e) ROM with uncoupled nonlinear CONN3D2 elements, f) ROM with unidirectional nonlinear CONN3D2 elements

while the applied moment boundary condition is removed. This is equivalent to an in-space application with no initial prestress, and results in a similar pretension to the start of region 2 described in Figure 2.8.

For the reduced order model, shown in Figure 2.19, a similar procedure was applied. This assembly consists of a grid of 288×288 (82944) S4T shell elements, 36×4 (144) CONN3D2 connector elements, and 320 B21 beam elements for the frame edges. The beam elements are fully constrained, similar to the edges of the high fidelity model, and the S4T elements simply have a thermal load of -100°C applied to them. The film has slight geometric imperfections (maximum amplitude is approximately the film's thickness) based on the first 20 eigenmodes applied to it as well in order to permit wrinkling if it should occur. This size model was also found to be convergent for the majority of the studies conducted, but under much higher stress states, it is necessary to also distribute the nodal forces of the connectors to nearby nodes to obtain accurate results. The time to run the reduced order model is typically under 30 minutes, whereas the high fidelity model takes approximately 12 hours.

Figures 2.20, 2.21, and 2.22 show the maximum and minimal principal stress distributions. As can be seen, 4 cases besides the high fidelity model were examined: fully-coupled connector elements, fully-coupled connector elements with half of the elements orientated in the opposite direction along each edge starting from the middle of the edge, uncoupled connector elements, and uncoupled connector elements with only the dominant (x -direction) behavior modeled. The maximum and minimal principal stresses were chosen as the metrics of interest as examining them provides information on both the presence of biaxial prestress and its uniformity. If compression is observed anywhere in the film, it is also a sign that it is likely to form wrinkles. The out of plane displacement was also measured to see if any wrinkles did indeed occur, and if so, how large their magnitudes are.

As can be seen in the graphs, the reduced order model with the coupled connector elements provides the best analog for the high fidelity model, as expected. It does a particularly useful job at modeling the maximum principal stress, and as such effectively allows for numerical experimentation on how to utilize the asymmetry of the double s-springs to more uniformly pretension the suspended membrane, such as by flipping the orientation of some elements. This results in a more uniform and equal distribution of the minimum principal stress as well, resulting in a more biaxial stress state. The uncoupled models do an adequate job of roughly estimating the

stress, but especially in the unidirectional case, fail to match the exact distribution well, further underlining the benefit of the multidimensional nature of the double s-springs. No wrinkling in any of the models was observed, and outside of the out of plane displacement in the actuated springs of the high fidelity model (and for a very small amount of the film adjacent to them), all out of plane displacements were less than the thickness of the film.

2.8 Double S-Spring Conclusion

Double s-springs, inspired by kirigami cutting patterns, offer a unique, thermally stable solution for continuously attaching thin films onto space structures. A numerical characterization and analysis of their multidimensional behavior was conducted. Unlike previous patterns found in the literature, they avoid sharp cuts and large out of plane buckling. This leads to them being able to handle large local strains, global displacements, and prestressing without tearing, as well as maintaining biaxial prestress and high surface accuracy under thermal strains, as shown by experimental and numerical studies. They are shown to minimize areal loss, be easily manufacturable, robust, and widely adaptable for a variety of potential scenarios through a small amount of tunable parameters. The y -direction behavior is beneficial, as it is a relatively low force compared to the x -direction, and two sets of opposing patterns can be used on a single edge to help better maintain constant biaxial prestress and deter subsequent wrinkling. The effect of various tunable parameters on the dominant behavior of the double s-springs was also discussed, and they were shown to have a high toughness. The study of these parameters showed clear design optimization trade-offs, such as stiffer springs typically having lower maximum displacements, and springs that were able to withstand higher strain energy before plastically yielding having lower yield forces. Experimental performance of the double s-springs was found to be well predicted by numerical simulation, and used to further investigate the effect of localized plastic yielding, showing that double s-springs remain functional even despite local deformations. It was also shown experimentally that their multidimensional behavior successfully minimizes the formation of wrinkles under prestressing.

Besides providing a thermally stable continuous edge connection for pretensioning, double s-springs show merit in other space based applications. Their ability to facilitate high displacements and strains lends itself well to folding structures, such as the composite toroid, where traditional methods would either be too bulky or fail during packaging ([44]). This application is discussed further in Chapter 5. Their

tunability also allows for thermally dissimilar or active elements to be accommodated by varying the double s-spring unit cells adjacent to them, such that stresses are distributed evenly, or in the case of more fragile components, diminished. Furthermore, double s-springs can be placed not just along the edges of a membrane, but utilized throughout to help further support and decouple components.

Finally, there are several future research vectors that have merit for double s-springs. Further experimental testing of the double s-spring under thermal loading may be done to determine the cyclic durability under extreme conditions, for which a creep model may also be developed, since a deployment for just a 2 year mission in low Earth orbit would likely require 10,000+ thermal cycles. Alternative materials could be investigated for their durability under space conditions, as well implementing elements such as wiring to pass through the ribbons. Similarly, it is worthwhile to investigate other parameters of the double s-spring such as the effects of the unit cell size, distribution of the chirality of elements, and material modulus in order to further optimize their characteristics/implementation. For the reduced order model developed, it would also be beneficial to validate it for loads higher than $-100\text{ }^{\circ}\text{C}$, and in particular large displacements or pretensions.

Chapter 3

BUCKLING OF RIGID FRAME UNIT CELL

Part of this chapter was adapted from the following publications:

George A. Popov and Sergio Pellegrino. Torsional stability of deployable space structures supporting prestressed thin films. *AIAA SCITECH 2025 Forum*, page 0186, 2025. doi: 10.2514/6.2025-0186.

George A. Popov and Sergio Pellegrino. Torsional stability of compliant deployable space structure frames supporting prestressed thin films. *[In Preparation]*, 2026.

3.1 Introduction

Ultralight space structures for applications such as space based solar power require very large planar surfaces to be deployed in orbit. Current deployable space structures technologies ([5, 14, 45]) do not meet the mass, precision, and volume launch constraints to make these applications economically and physically viable. Hence, recent missions such as the Caltech Space Solar Power Project (SSPP) ([46]) envisage prestressed thin films mounted onto flexible deployable structures.

To achieve high surface accuracy and stability, thin films require sufficient biaxial prestress levels and attachment to supporting deployable structures. This prestress must be maintained across the range of temperatures and thermal gradients that occur in orbit. This is difficult to implement since the films and structures are made from materials with dissimilar coefficients of thermal expansion (CTE). A fundamental limitation is that the prestressing loads applied by the film to the deployable structure have the effect of reducing its stability, and torsional buckling occurs at relatively low prestress levels, even if the structural members are designed with a sufficient margin against the Euler buckling load.

Thus, it is crucial to investigate the stability of thin films attached to compliant deployable space structures. Currently, membranes are either directly attached to frames or integrated via soft corner springs. Directly attaching membranes to non-stiff frames results in overall buckling ([25]). This is advantageous for creating multi-stable periodic structures, but it is unacceptable when flat surfaces are required ([47]). Corner springs provide a limited number of connection points, which increases the design and integration complexity, and hinders deployment

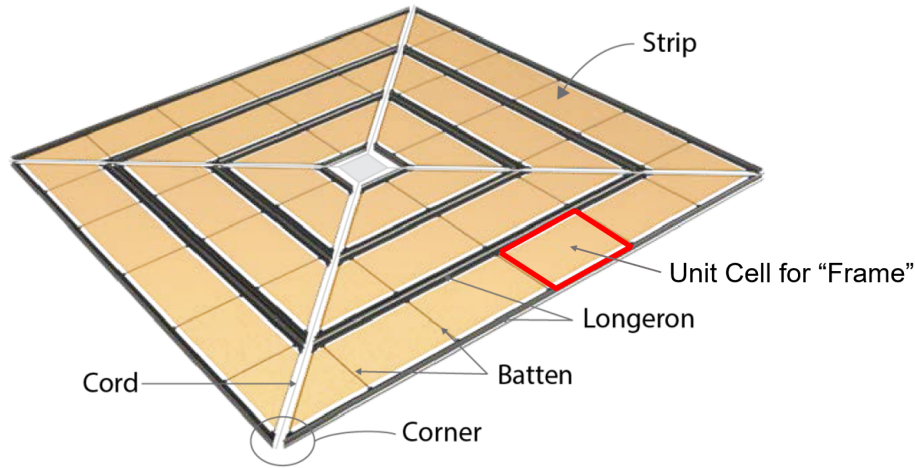


Figure 3.1: Unit Cell Outline ([1]).

([2]). Hence, it is advantageous to consider mounting thin films using a series of distributed edge springs as opposed to corner springs.

Figure 3.1 shows the deployable SSPP prototype concept, from which a representative single square unit cell frame has been extracted ([12]). It is critical to develop a conceptual understanding of the fundamental implications of different attachment schemes. As this unit cell is loaded by internal forces, its critical buckling load and post-buckling behavior depend on how the loading condition “follows” the deformation of the structure.

In this chapter, the unit cell of the SSPP will be modeled as a square frame of four rigid rods connected by frictionless hinges with linear-elastic torsional springs at the corners, attached to an internal, prestressed film that applies either concentrated corner forces or uniformly distributed loads on the frame. Both fixed-direction forces and “follower” forces will be considered.

The specific four cases that are focused on in this chapter are shown in Figure 3.2, where in cases A and B the prestress acts in fixed directions, and in cases C and D the prestress follows the deformation of the frame.

3.2 Analytic Derivation of Buckling Loads via Adjacent Equilibrium

There are several methods to solving for the critical buckling load P_{crit} of a structure knowing its initial and deformed configurations, such as the (adjacent) equilibrium method, using energy, or creating an eigenvalue problem. This section will derive the critical buckling value of the prestress for the four cases using the simplest

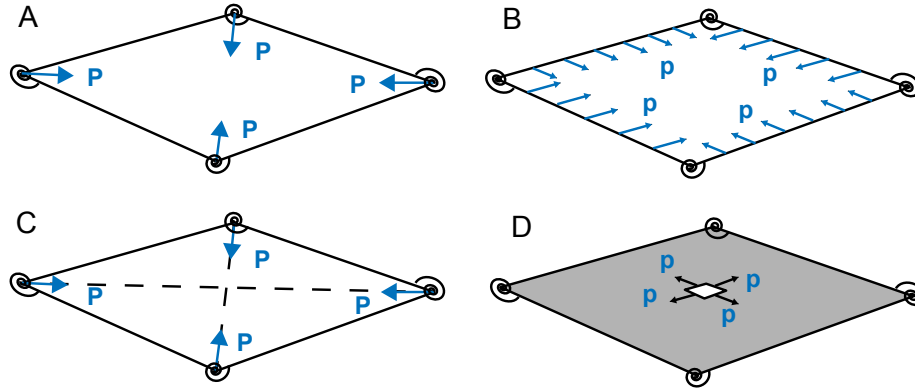


Figure 3.2: Four loading cases that are studied. A: Fixed direction corner forces. B: Fixed direction distributed load. C: Corner prestress applied by internal rods. D: Distributed prestress applied by biaxially-stressed membrane.

method, which is the aforementioned (*adjacent equilibrium method*) ([48, 49]), which considers the equilibrium of the prestressed structure in an infinitesimally buckled configuration and then imposes the condition that the structure should be in equilibrium without additional loads. The first step in this study is the derivation of the critical buckling load for the four different loading cases in Figure 3.2.

The structure consists of four rods of equal length L , connected by cylindrical joints and torsional springs of stiffness k at the corners, as shown in Figure 3.3. Note that this structure has a single degree of freedom which, for infinitesimally small deformation, allows it to be deformed in the twisted shape shown in Figure 3.4. A further detailed discussion of this buckling mode will be presented in Chapter 3.3.

Note that in cases A and C, P is the value of the corner prestressing forces and the corresponding axial compression, C , in the rods can be calculated as:

$$C = \frac{P}{\sqrt{2}} \quad (3.1)$$

In cases B and D the axial compression in the rods that corresponds to the distributed prestress p can be calculated as:

$$C = \frac{pL}{2} \quad (3.2)$$

The following preliminary results will enable the derivations for the four cases to proceed more quickly.

First, the square frame admits the aforementioned torsional buckling mode shown in Figure 3.3. Here the corner rotations, θ , can be obtained from Figure 3.4. The

analysis is quite subtle and is based on noticing that the hinge axes through the center point of the frame, O , when the frame is buckled. From the figure, it can be shown that:

$$\alpha = \frac{\delta}{L/2} \quad (3.3)$$

$$\beta = \sqrt{2}\alpha = \frac{2\sqrt{2}\delta}{L} \quad (3.4)$$

and, most importantly:

$$\theta = \sqrt{2}\alpha = \frac{2\sqrt{2}\delta}{L} \quad (3.5)$$

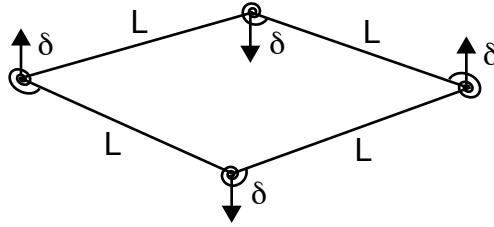


Figure 3.3: Torsional buckling mode.

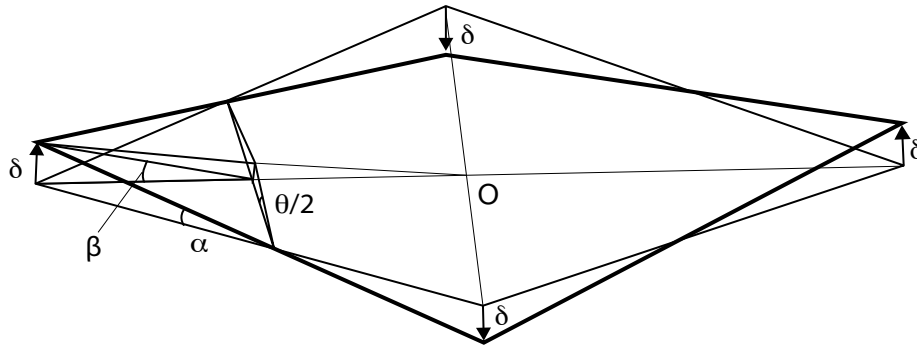


Figure 3.4: Angles needed to calculate hinge rotation.

Second, the vertical forces, F_1 , required to hold the square frame in the deformed configuration shown in Figure 3.5, which are related to the hinge rotations and the spring stiffness, can be calculated using the equation of virtual work. They have magnitude:

$$F_1 = 8k \frac{\delta}{L^2} \quad (3.6)$$

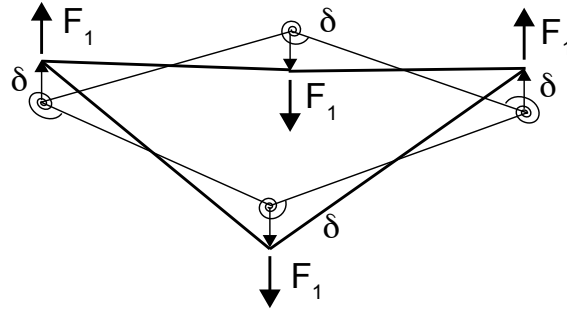


Figure 3.5: Vertical forces required to hold the frame in the buckled configuration.

Third, the vertical forces, F_2 , required to hold a rod of the square frame, carrying an axial prestress C , in the buckled configuration shown in Figure 3.6 are given by

$$F_2 = 2C \frac{\delta}{L} \quad (3.7)$$

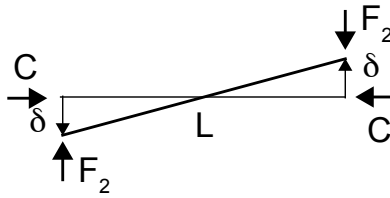


Figure 3.6: Vertical forces required to hold a prestressed rod of the square frame in the buckled configuration.

Fixed Direction Corner Force (Case A)

In this case, the prestressing forces P on the frame do not change direction when the frame buckles and hence do not apply a vertical force component to the joint. Hence, the vertical equilibrium equation for a joint in the buckled configuration is:

$$F_1 - 2F_2 = 0 \quad (3.8)$$

Substituting Eq. 3.6 and Eq. 3.7 into Eq. 3.8 gives:

$$8k \frac{\delta}{L^2} - 4C \frac{\delta}{L} = 0 \quad (3.9)$$

and substituting Eq. 3.1 gives

$$8k \frac{\delta}{L^2} - 2\sqrt{2}P \frac{\delta}{L} = 0 \quad (3.10)$$

which can be solved for P to find the critical value of P :

$$P_{crit} = 2\sqrt{2}\frac{k}{L} \quad (3.11)$$

The corresponding critical compression in the rods of the square frame is

$$C_{crit} = 2\frac{k}{L} \quad (3.12)$$

Fixed Direction Distributed Load (Case B)

In this case, the prestressing distributed along each edge also does not change direction when the frame buckles. Hence, the derivation in the previous section is still valid with the only difference that Eq. 3.2 is used instead of Eq. 3.1. Accounting for this difference, the critical distributed prestress is

$$p_{crit} = 4\frac{k}{L^2} \quad (3.13)$$

and the corresponding critical compression in the rods of the square frame matches and is

$$C_{crit} = 2\frac{k}{L} \quad (3.14)$$

Follower Corner Prestress Applied by Diagonal Rods (Case C)

In this case, the diagonal rods connected via spherical joints apply follower forces on the frame, which have non-zero vertical components that contribute to the equation of equilibrium in the buckled configuration. This case serves to approximate a corner spring attachment scheme. The vertical force component, F_3 , has been analyzed in Figure 3.7 and the following expression was obtained:

$$F_3 = \sqrt{2}P\frac{\delta}{L} \quad (3.15)$$

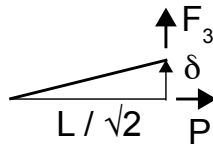


Figure 3.7: Vertical forces required to hold a prestressed diagonal rod in the buckled configuration.

In this case the vertical equilibrium equation for a joint in the buckled configuration is:

$$F_1 - 2F_2 + F_3 = 0 \quad (3.16)$$

which, proceeding as above, leads to the critical prestress:

$$P_{crit} = 4\sqrt{2}\frac{k}{L} \quad (3.17)$$

and the corresponding critical compression in the rods of the square frame is

$$C_{crit} = 4\frac{k}{L} \quad (3.18)$$

Follower Distributed Prestress Applied by Membrane (Case D)

In this case, the internal membrane applies follower distributed forces on the frame, which have non-zero vertical components that contribute to the equation of equilibrium in the buckled configuration. The vertical force component, F_3 , shown in Figure 3.8 is given by:

$$F_3 = p\frac{\delta}{2} \quad (3.19)$$

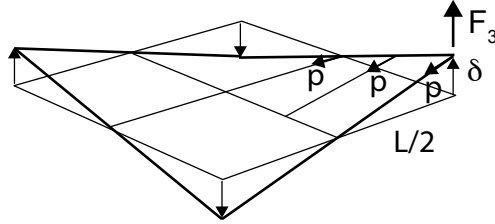


Figure 3.8: Vertical forces required to hold a prestressed membrane in the buckled configuration.

The vertical equilibrium equation for a joint in the buckled configuration is:

$$F_1 - 2F_2 + F_3 = 0 \quad (3.20)$$

which leads to the critical prestress:

$$p_{crit} = 8\frac{k}{L^2} \quad (3.21)$$

and the corresponding critical compression in the rods of the square frame is

$$C_{crit} = 4\frac{k}{L} \quad (3.22)$$

3.3 Analytic Derivation of Buckling Loads via an Energy Method

For this analytical analysis, the first step is the derivation of the critical buckling load for the four different loading cases in Figure 3.2, and then generalizing to an

arbitrary mixed force loading scenario. As before, the structure consists of four rods of equal length L , connected by cylindrical joints at 45 degrees to the edges and torsional springs of stiffness k at the corners, as shown in Figure 3.9. There are two primary instability mode shapes in the simplified structure presented with the same lowest eigenvalue regardless of how in-plane forces are applied, assuming that the edge rods are significantly rigid compared to the corner connections represented by the torsional springs. The first mode is a simple "axial folding" mode, where the structure folds in half across diagonally through either of the opposing hinge pairs, but not simultaneously both. The second mode is a similar but slightly more complicated infinitesimally small deformation mode, used in the previous section, which can be visualized as two opposing corners raising and the other two corners lowering, as shown in Figure 3.9. Upon careful inspection, when linearized for infinitesimal displacements, the "axial folding" and "corner displacement" modes are actually mathematically identical. The post-buckling loading path follows the former "axial folding" mode, since both pair of opposing hinges cannot rotate simultaneously, but the second infinitesimal "corner displacement" representation is useful for calculating a generalized buckling load as will be shown in the latter part of this section. It is also helpful in creating a general theory that shows that the critical buckling load can be expressed as a constant critical compression value for a given structure that is agnostic of the exact loading distribution, dependent only on the type of loading applied.

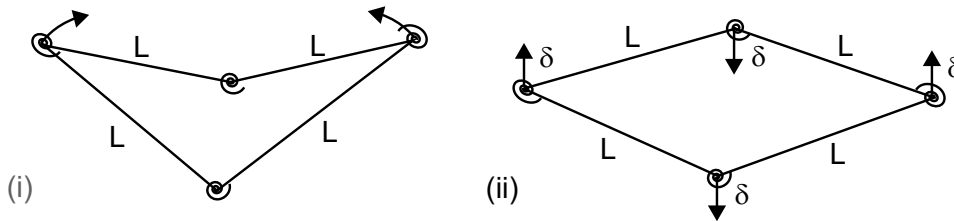


Figure 3.9: i. "Axial folding" mode and ii. "Corner displacement" mode.

The following preliminary results hold true in general for the four cases, independent of whether a "fixed" or "follower" load is applied. In this study, a "fixed" load is used to refer to the type of applied force in cases A and B, where its vector direction does not change in reference to the global frame as the structure deforms. A "follower" load is used to describe cases C and D, where the direction of the forces applied by an internal pretension change direction as the structure deforms. This vector is assumed to always point along the saddle formed by the twisting of the structure.

Note that in cases A and C, P is the value of the corner prestressing forces and the corresponding axial compression, C , in the rods can be calculated as:

$$C = \frac{P}{\sqrt{2}} \quad (3.23)$$

In cases B and D the axial compression in the rods that corresponds to the distributed prestress p can be calculated as:

$$C = \frac{pL}{2} \quad (3.24)$$

This formulation will allow us to quickly calculate the buckling critical compression C_{crit} in the loading cases presented, the same as in the previous section.

Derivation of Buckling Loads for the Four Simple Cases

There are several methods to solving for the critical buckling load P_{crit} of a structure knowing its initial and deformed configurations, such as the (adjacent) equilibrium method or creating an eigenvalue problem. In this section, the most classic methodology, an energy approach, is used, where the total potential energy of the structure is calculated, and then the second derivative is examined in order to find an inflection point that signifies a buckling load being reached, and the structure transitioning from a stable to unstable configuration. The formula for this is as follows:

$$\Pi = U - W \quad (3.25)$$

In the axial folding case of motion of the simplified model, the only strain energy is due to moments in the springs, and the work applied by the in-plane forces, which can be represented by a force " P ," traveling a distance u leading to:

$$\Pi = \frac{1}{2}M\theta^2 - Pu \quad (3.26)$$

Which is the classic formulation for the case of an axially loaded rigid bar mounted on a torsional spring. Hence, by looking at just half of the structure along its diagonal and taking that the work displacement is equal to $u = l(1 - \cos(\theta))$ and linearizing u as $u \cong l\frac{\theta^2}{2}$ due to the small displacement, we arrive at:

$$\Pi = \frac{1}{2}K\theta^2 - \frac{1}{2}Pl\theta^2 \quad (3.27)$$

Where taking the second derivative and setting it to zero to find the inflection point of the total energy yields:

$$\frac{d^2\Pi}{d\theta^2} = K - Pl = 0 \quad (3.28)$$

$$P_{crit} = \frac{K}{l} \quad (3.29)$$

Where for all cases $K = 2k$ (the two springs lying on the diagonal) and $l = \frac{L}{\sqrt{2}}$ (location of the applied force) for case A. Hence for just the corner loading of the frame:

$$P_{crit} = \frac{2\sqrt{2}k}{L} \quad (3.30)$$

and plugging in Eq. 3.23:

$$C_{crit} = \frac{2k}{L} \quad (3.31)$$

Similarly, in the case of the distributed fixed direction edge load, we can integrate the load along the edge to find the corresponding work term:

$$W = \int_0^{L/\sqrt{2}} pl(1 - \cos(\theta)) dl = \frac{pL^2}{4}(1 - \cos(\theta)) \quad (3.32)$$

And plugging it into Eq. 3.27 and then linearizing and taking the second derivative as before we can find that:

$$P_{crit} = \frac{4k}{L^2} \quad (3.33)$$

and plugging in Eq. 3.24 we again get the same critical compression as Case A:

$$C_{crit} = \frac{2k}{L} \quad (3.34)$$

For the next set of cases (C and D), a follower force is used instead of a fixed force. The structure can be examined exactly as in the previous two cases A and B, but with one small caveat, which can be observed by looking at the symmetry of the buckled structure, shown in Figure 3.10, assuming equal contraction of the interior rods. For the same amount of infinitesimal displacement u as before, the work done is now a smaller component, $u = l(1 - \cos(\frac{\theta}{\sqrt{2}})) \cong l\frac{\theta^2}{4}$ due to the rotation of the force vector P . Hence, Eq. 3.27 now becomes instead:

$$\Pi = \frac{1}{2}K\theta^2 - \frac{1}{4}Pl\theta^2 \quad (3.35)$$

However, the forces applied and their locations remain the same, as does the work integral besides the angle dependent term. Hence, after finding the second derivatives for the follower load cases (C and D) the critical buckling loads are simply half of the ones for the fixed direction loading cases (A and B), as is the critical compression (which is the same for both, and next will be shown to be the same for all follower loads):

$$P_{crit} = \frac{4\sqrt{2}k}{L} \quad (3.36)$$

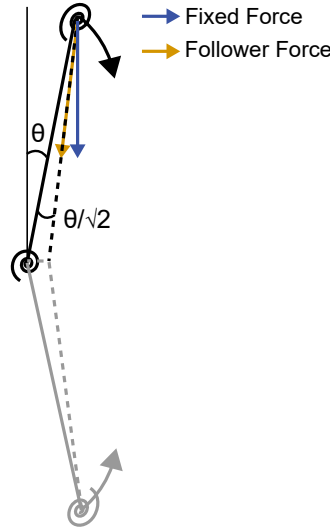


Figure 3.10: Direction of forces in axial folding mode.

$$p_{crit} = \frac{8k}{L^2} \quad (3.37)$$

$$C_{crit} = \frac{4k}{L} \quad (3.38)$$

Generalized Critical Buckling and Critical Compressive Loads

One particular advantage of using the energy method is that it allows for a generalized nonsymmetric load case, which can be used to prove the fact that the critical compression value is that same regardless of how it is achieved. As discussed previously, there are two possible buckling mode shapes that lead to the same lowest critical buckling value for the structure presented. The second infinitesimal "corner displacement" mode shape, labeled in Figure 3.11 is particularly useful as it allows the calculation of the buckling load for a set of arbitrarily applied in-plane loads. Here the whole structure is examined instead of just half. Eq. 3.25 is once again used, and the strain energy term is defined as just the energy stored in the corner springs:

$$U = 4\left(\frac{1}{2}k\theta^2\right) \quad (3.39)$$

By looking at Figure 3.12, and assuming small displacements δ , we can observe that i arbitrary forces $P \sum_i c_i (\cos\phi_i + \mu_i \sin\phi_i)$ (where c_i are relative scaling factors between the forces, ϕ_i are angles of the force from the normal of the edge, and μ_i

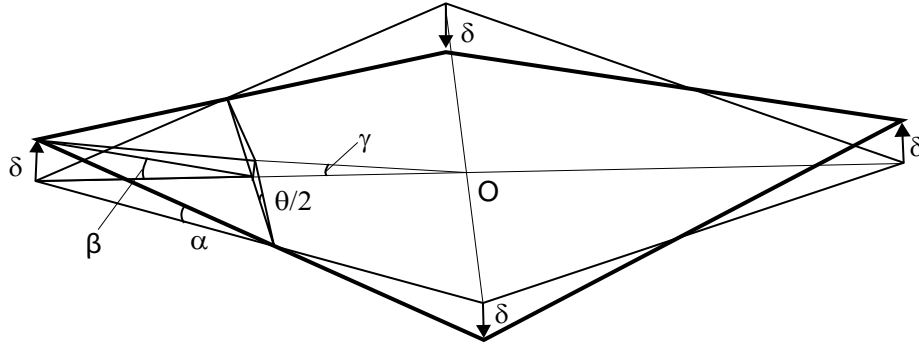


Figure 3.11: Angles needed to calculate hinge rotation.

are the scaling fractions of how far that the force is applied from an edge center) do work in-plane as defined for the fixed loading case:

$$W_{fixed} = P \sum_i c_i (\cos \phi_i + \mu_i \sin \phi_i) L (1 - \cos \frac{\beta}{\sqrt{2}}) \quad (3.40)$$

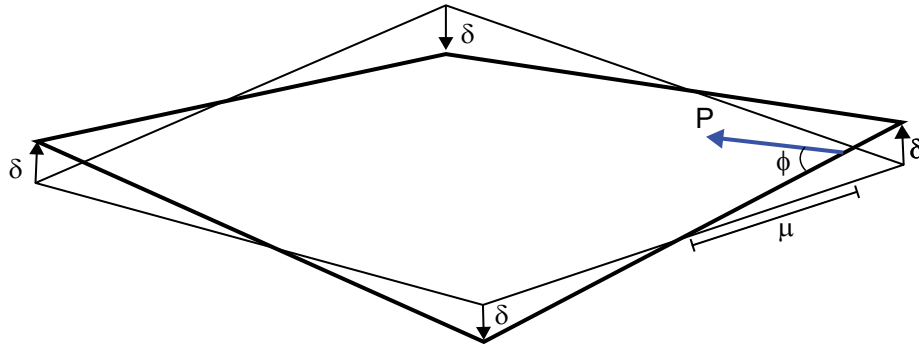


Figure 3.12: Fixed force P applied in the "corner displacement" mode.

From Figure 3.11 we can also note that since the rigid rods provide a kinematic constraint, the angle which the torsion springs bend, θ , can be related to the other angles in the figure as $\theta = \sqrt{2}\alpha = \beta = 2\gamma$, which allows us to take the derivative. Taking the difference between equations 3.39 and 3.40 and once again performing the second derivative, we obtain that:

$$\frac{d^2\Pi}{d\theta^2} = 4k - \frac{1}{2}P \sum_i c_i (\cos \phi_i + \mu_i \sin \phi_i) L \cos(\frac{\beta}{\sqrt{2}}) \quad (3.41)$$

and linearizing $\cos(\frac{\beta}{\sqrt{2}}) \cong 1$ and solving for P_{crit} :

$$P_{crit} = \frac{k}{L} \frac{8}{\sum_i c_i (\cos\phi_i + \mu_i \sin\phi_i)} \quad (3.42)$$

There are two important caveats to be noted in this result. If, for example, only a single force is being applied, the structure will be subject to just a rigid body motion, and will obviously not buckle. Hence, the forces being applied must be balanced with each other, or else they will not contribute to the buckling load. Secondly, forces can also cancel out a buckling effect. If compressive loads are applied in some locations, and tensile loads elsewhere, such as if two balanced moment force pairs are applied on either side of the structure, the structure will not buckle in this mode. Additionally, by taking $i = 4$, $c_i = 1$, and $\phi_i = \frac{\pi}{4}$, we recover the result for case A of $P_{crit} = 2\sqrt{2}\frac{k}{L}$. Similarly, replacing the sum with an integral can be used to recover the result for case B.

In order to derive the compression critical load, we can make the assumption that each force in the structure $Pc_i(\cos\phi_i + \mu_i \sin\phi_i)$ is balanced by another force located somewhere else in the structure to prevent rigid body motions. Looking at a sum of such forces, the average compression within the structure due to this force pair can be quickly derived, yielding the same result as before (Eq. 3.31 and Eq. 3.34):

$$C = \frac{\text{forces in each side} \times \text{side length}}{\text{total length of sides}} \quad (3.43)$$

$$C_{crit} = P_{crit} \frac{L \sum_i c_i (\cos\phi_i + \mu_i \sin\phi_i)}{4L} = \frac{2k}{L} \quad (3.44)$$

The case of the follower force can be derived using the same procedure. Here there is one additional component in the out of plane direction as the vector rotates. In the fixed force case, the force vector does not rotate, and thus $P = P_{in-plane}$ by default. The out of plane component can also be assumed to be negligible in the derivation for the follower as well, and P can be approximated as just $P_{in-plane}$. For the sake of generality, the following derivation will not make this assumption. However, since a follower force necessarily acts inside the structure, as illustrated in Figure 3.13, we will assume that the forces act as a pair:

$$W_{follower} = P \sum_i c_i (2\cos\phi_i + (\mu_i - \lambda_i)\sin\phi_i) \left(\sin\left(\frac{\gamma}{\sqrt{2}}\right) \times L \sin\left(\frac{\gamma}{\sqrt{2}}\right) - \cos\left(\frac{\gamma}{\sqrt{2}}\right) \times L(1 - \cos\left(\frac{\gamma}{\sqrt{2}}\right)) \right) \quad (3.45)$$

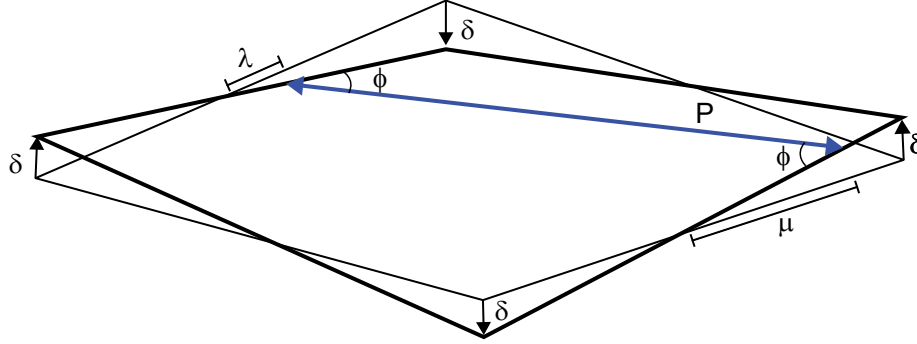


Figure 3.13: Follower force pair P applied in the "corner displacement" mode.

and once again combining eq. 3.39 with this result and taking the second derivative:

$$\frac{d^2\Pi}{d\theta^2} = 4k - \frac{1}{8}P \sum_i c_i (2\cos\phi_i + (\mu_i - \lambda_i)\sin\phi_i) L \cos\left(\frac{\gamma}{\sqrt{2}}\right) \quad (3.46)$$

which leads to:

$$P_{crit} = \frac{k}{L} \frac{32}{\sum_i c_i (2\cos\phi_i + (\mu_i - \lambda_i)\sin\phi_i)} \quad (3.47)$$

The result for case C can be derived by having $i = 2$, $c_i = 1$, and $\phi_i = \frac{\pi}{4}$:

$$P_{crit} = \frac{k}{L} \frac{32}{(2\sqrt{2} + (2)\sqrt{2})} = 4\sqrt{2} \frac{k}{L} \quad (3.48)$$

and for case D by integrating over the sides. The derivation for the critical compression for a follower force can be done the same as in eq. 3.44, but this time noting a minor difference in that force P as drawn in Figure 3.13 is applied to two places on the structure. Taking care of this, we demonstrate that for any set of internal forces applied, the critical load of the average compression in the structure is invariant:

$$C_{crit} = P_{crit} \frac{\frac{L}{2} \sum_i (2\cos\phi_i + (\mu_i - \lambda_i)\sin\phi_i)}{4L} = \frac{4k}{L} \quad (3.49)$$

One detail to note in the derivation for the "corner displacement" variation presented is the paradox of when a force is applied at the middle of an edge. In this case, calculating the displacement for the work with β versus γ may initially seem to be a bit arbitrary, since the force moves identically as function of θ in this case. In order to resolve this potential discrepancy, we can either move the applied for an infinitesimal amount away from the center, or refer back to the actual post-buckling path the structure assumes, and do a similar derivation with the "axial folding" mode as in the previous subsection just for this unique case.

Finally, the derivation utilizing the energy method is not limited to a square frame geometry. The same derivation steps used can be extended to rectangular frames for example, or frames with unequal internal angles, as long as the buckling mode remains the same and the assumption that opposing hinges are collinear holds, such that this buckling mode can be achieved to at least the first order approximation. If the ratio of the side lengths becomes exceedingly large, this derivation will also be an unreasonable estimate of the buckling mode as well. A simple example via the "axial folding" version of the buckling mode (Figure 3.10) for corner forces in a rectangle with side lengths a and b is illustrated in Figure 3.14, where the fold line is illustrated in the blue dashed line, and only one of the four forces applied is pictured. $l = ab/\sqrt{a^2 + b^2}$ in this scenario, and P is scaled by the cosine of the angle between the line of l and the diagonal of the rectangle. This scalar can also be calculated as $\sin(\arctan(a/b)) = \frac{2ab}{a^2+b^2}$. Hence, the results for fixed and follower forces are in Eq. 3.50 and Eq. 3.51, respectively:

$$P_{crit} = \frac{2k}{\frac{ab}{\sqrt{a^2+b^2}} \frac{2ab}{a^2+b^2}} = \frac{k(a^2 + b^2)^{\frac{3}{2}}}{a^2b^2} \quad (3.50)$$

$$P_{crit} = \frac{4k}{\frac{ab}{\sqrt{a^2+b^2}} \frac{2ab}{a^2+b^2}} = \frac{2k(a^2 + b^2)^{\frac{3}{2}}}{a^2b^2} \quad (3.51)$$

Which, for the case of $a = b = L$, recover the results for the case of the square frame.

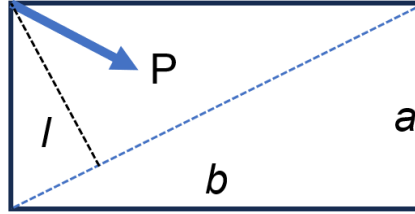


Figure 3.14: Diagram of a rectangular frame version of the buckling calculation. The other three forces and corner springs are omitted for visual clarity.

3.4 Buckling Load Numerical Simulations

In order to verify the analytic results for square frames, the finite element software Abaqus 2024 was used to conduct a series of numerical buckling and post-buckling analyses to match the four cases considered in the previous section. A 0.343 m by 0.343 m square frame was modeled with 2, 0.1715 m long B32 beam elements per side (8 elements in total). Four diagonal wires, modeled with CONN3D2 connector elements of 0 length, were used to connect the beam element nodes within each of the corners to define multi-point hinge constraints (MPC hinge) between them. Local CSYS datums were then used to orientate these wires such that the axis of each hinge points to its opposite corner. The beams of the square frame were assigned the properties of aluminum 6061 and a tubular cross section with the properties listed in Table 3.1 was assumed.

Property	Value
Edge Length (L)	0.343 m
Cross Section (OD/Thickness)	0.00635 m / 0.0015 m
Young's Modulus	68 GPa
Poisson's Ratio	0.33
Density	2,700 kg/m ³
Hinge Spring Constant	0.303 Nm

Table 3.1: Properties for Abaqus analysis.

The dimensions and stiffnesses were chosen to match the experiment in Section 3.5, as well as to aid numerical convergence.

Fixed-Direction Forces

The first pair of analyses were conducted using the "Buckle" solver in Abaqus. Both the "axial folding" and "corner displacement" modes were simulated in order to further verify that they admit the same eigenvalue. For the "axial folding" mode, one edge was held fully constrained. For the "corner displacement" mode, the

midpoints of all of the edge beams were each fixed in the out of plane and tangential directions, such that rotations of the edges and sliding towards the center of the structure was unconstrained. Then, external forces as described in the analytic analysis were assigned, relative to a global coordinate system. Both corner forces (case A) and forces distributed along the edges (case B) were considered. The first buckling mode of the structure with both sets of constraints and forces showed the expected twist and was examined to determine the eigenvalue of interest.

The resulting buckling eigenvalues closely matched their corresponding analytic predictions. For the corner loaded "case A" the numerically simulated eigenvalue is 2.495 N (analytical prediction 2.499 N), while the edge loading "case B" is 10.29 N/m (analytical prediction 10.30 N/m). The distributed load result thus has equal compression loads in the rods to the corner force case as predicted by the C_{crit} formulation. The results were the same with both sets of boundary conditions used. Using a stiffer beam, such as steel for the material or modeling the rods as solid, brings these eigenvalues even closer to the analytical solutions, whereas using a softer material or lowering the cross section for the edge beams can lower their Euler buckling loads, and switches the dominant mode, leading to a lower buckling eigenvalue, and possibly alternative buckling modes.

A mesh convergence analysis revealed no difference in the eigenvalue when increasing the number of elements for this set of simulations, thus the minimal amount as described above was used. Similarly, increasing the interpolation in the numerical beam formulation or enabling hybridization did not affect the eigenvalues obtained.

Internal Prestress

The buckling prestress limit for Case C is not as simple to calculate numerically since the force cannot be applied as an external boundary condition. Two sets of analyses were conducted where four additional elements were defined inside the frame and connected via spherical joints at their intersections with each other and the frame in the undeformed configuration. For the first set of analyses, the additional elements were modeled with T3D2 truss elements to allow for the "Buckle" solver to be used. A critical temperature eigenvalue was determined via the *Buckle command propagating a negative *Temperature load only in the internal strips, inducing a thermal strain. The internal rods were assigned Kapton strip material properties ($E=2.76$ GPa, $w=0.01$ m, $t=0.0001$ m) to match a typical space application and the experiment. The first buckling mode was once again the expected deformation mode

with both types of boundary conditions. Thus, the resulting critical temperature buckling eigenvalue in Abaqus was then converted to a strain and used to calculate the critical buckling force P_{crit} by solving the resulting statically indeterminate problem. The force result for the Kapton internal strip and other materials simulated is the same, with a P_{crit} of 4.990 N, once again closely matching the analytical prediction of 4.997 N, and approaches this number even closer if stiffer rods are defined.

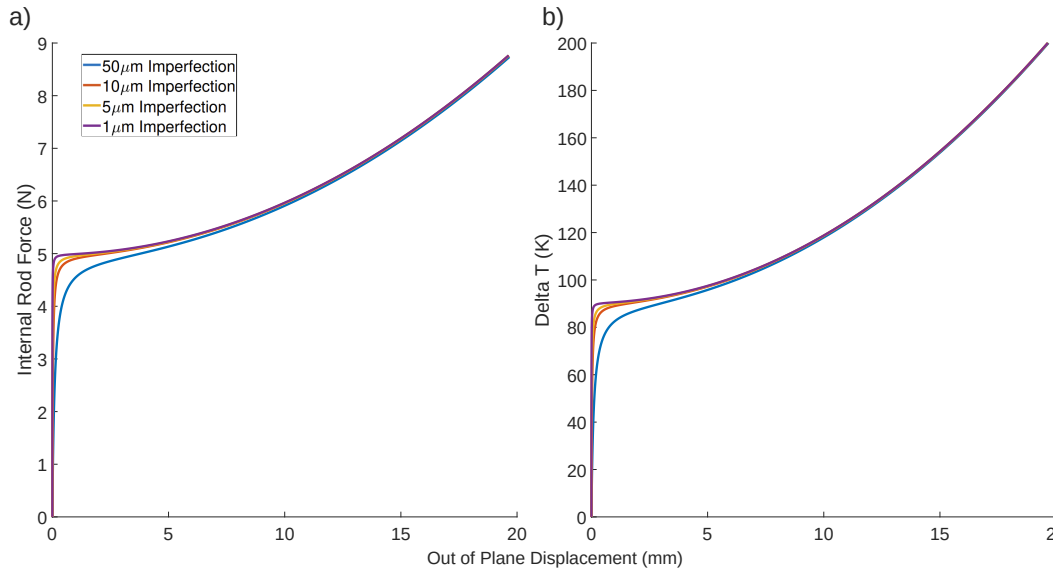


Figure 3.15: Case C post-buckling analysis. Figure a) shows the displacement-force response of the frame; figure b) shows the temperature vs. the displacement of the frame.

Then, this result was further verified by examining the post-buckling solution using T3D2T elements for the internal rods, which were controlled using a temperature field during a “Coupled Temperature-Displacement” solver step with geometric non-linearity enabled. Kapton is used again as the strip material for the internal rods. The external aluminum rigid rods had no temperature change applied and a 0 CTE. This is also an analogous model for low CTE composite structures in space applications, since the integrated components typically have a much higher CTE. Since the frame geometry and deformation mode remain constant, the *IMPERFECTION command can be used to seed the desired buckling mode from the previous analysis. Several different imperfections were considered in predicting the post-buckling response, ranging from 50% of the strip thickness down to 1%. As the magnitude of the imperfection is decreased and prestress is applied by lowering the temperature, the bifurcation point in the force displacement graph becomes sharper, and both the critical buckling load and post-buckling behaviors can be observed and calculated

from the temperature change and internal tension of the truss elements. Both the force and temperature change are plotted in Figure 3.15 as a function of the out of plane post-buckling response for the "axial folding" mode. The "corner displacement" mode has a similar pattern, but is not plotted here since the post-buckling path is not the one followed in the experiment as the deformation mode is just an infinitesimal representation.

Once again, a mesh and element type convergence was performed for case C and no significant change in the eigenvalues or post-buckling displacement was observed.

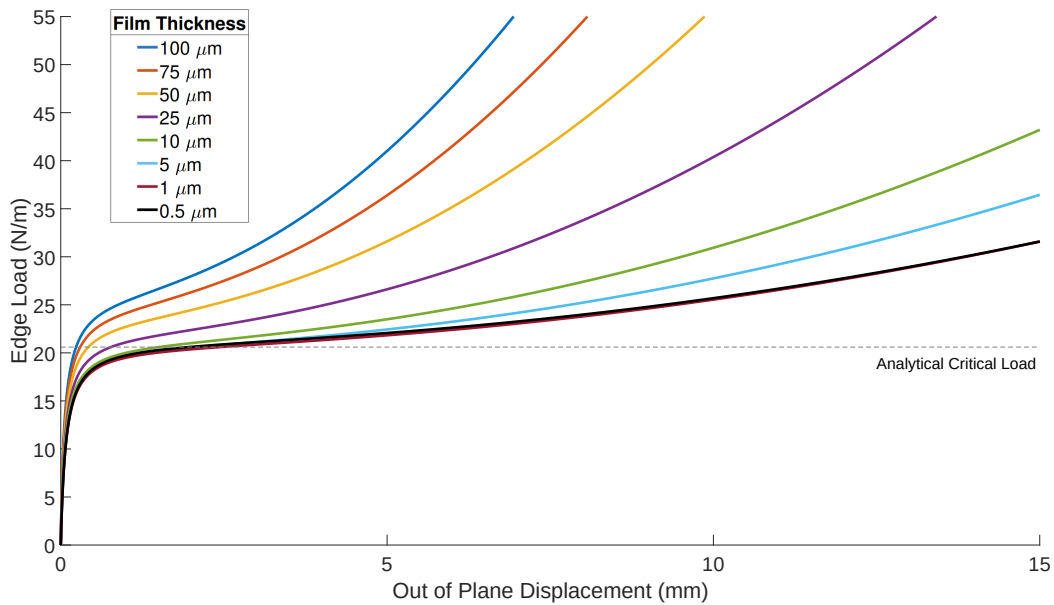


Figure 3.16: Case D post-buckling analysis.

The membrane (case D) can be modeled similarly in both bifurcation and post-buckling analyses by implementing a temperature dependent membrane inside of the frame, finding the deformation mode, seeding the expected mode shape, and then lowering the temperature to apply a prestress on the frame. A Kapton membrane made of S4 shell elements was used for the eigenvalue analysis and S4T elements were used for the post-buckling analysis. The membrane edge was pinned to the beam elements at each node except for the corners via a tie constraint. An MPC pin constraint was used in addition to the existing MPC hinge constraint at the corner nodes in order to avoid joining the beams to each other. A mesh convergence study determined that increasing the total number of elements per edge from 2 to 20 for both the beams and the film was sufficient to achieve mesh convergence for this

study. In this loading case, it was observed that the buckling load is sensitive to the thickness of the membrane as well, since it provides a torsional stiffness, as seen in Figure 3.16, where the effective stiffness of the frame increases for thicker films, leading to a higher buckling load than predicted analytically. A 50 μm imperfection was used to calculate the post-buckling response for all membrane thicknesses, since that was the minimum required for convergence of all film thicknesses. Thinner films were observed to have more uniform prestress distributions. Due to the fact that the stress is not uniform along the edge, the edge load was calculated by an integrated average of the total stress along the edge. For films thinner than 5 μm , the added stiffness is negligible, and the theoretical critical value of 20.6 N/m calculated analytically is matched to an accuracy better than 0.05%, similar to the previous cases.

The bifurcation analysis in case D also required a further nonlinear analysis to obtain the exact critical loads. This is due to a stiffness variation in the unit cell under loading. The *Buckle command outputs a linearized buckling load around the current stiffness, hence the structure must be prestressed incrementally and the load recalculated to obtain a more accurate estimate. To accomplish this, a predefined temperature field was applied to the S4 elements in the model, generating a prestress and calculating geometrically nonlinear configuration, and then the *Buckle operation was run again to determine the new buckling temperature load relative to this prestress. This sequence was repeated until the temperature prestress converged to the critical load, where the prestressed state itself is found to be an unstable configuration. The critical buckling loads as a function of the membrane thickness are plotted in Figure 3.17. As expected and observed in the post-buckling analysis, for thinner membranes the additional stiffness added is negligible and the critical load converges to the analytical solution.

3.5 Experimental Confirmation of Case C

In order to further verify the analytic and numerical predictions of Case C, particularly the numerically predicted post-buckling behavior, a frame was constructed with the dimensions listed in Table 3.1. The corners of the frame include machined hinges made from aluminum 6061, fitted with thermoplastic flanged bearings designed for low friction radial and thrust applications. These bearings were in turn fitted onto opposite ends of a short, hollow aluminum shaft, and a 180 degree steel torsional spring was mounted around the center of this shaft at each corner. Grooves were cut into the machined hinges for the legs of the spring, which were then fixed

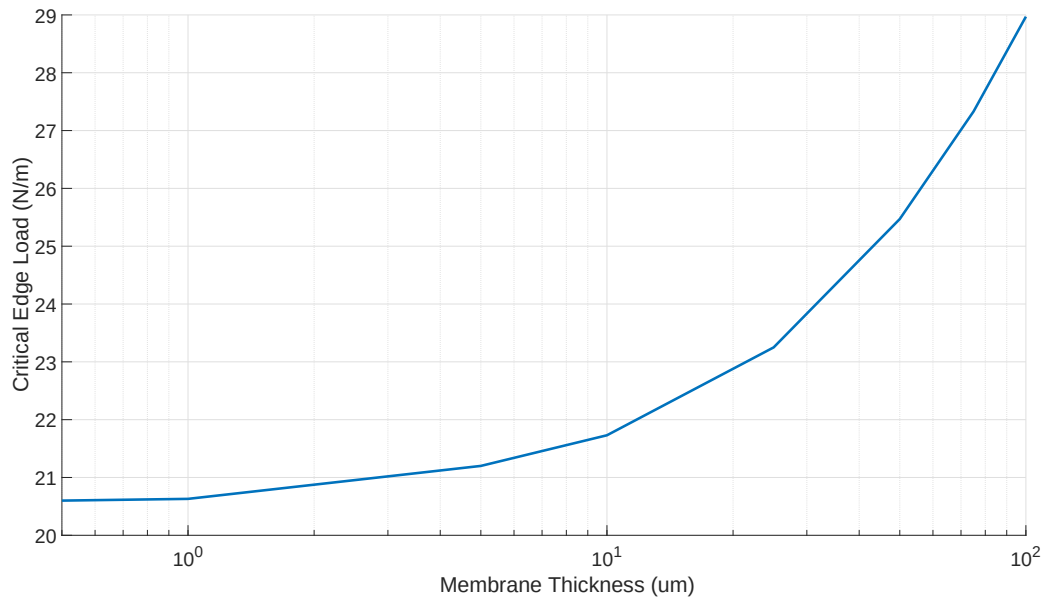


Figure 3.17: Case D nonlinear buckling loads.

using epoxy and a set screw, allowing for an angle adjustment in each corner if necessary to zero out the moments in the unbuckled state. The adjacent corners of the frame were then connected using an aluminum tube, whose length and rotation were locked by machining a flat surface at the ends and using a pair of set screws at each connection. An OptiTrack marker was placed on each corner as well to be able to accurately measure the out of plane deformation during the experiment. A corner of the frame along with the overall experimental setup are shown in Figure 3.19 and Figure 3.18, respectively.

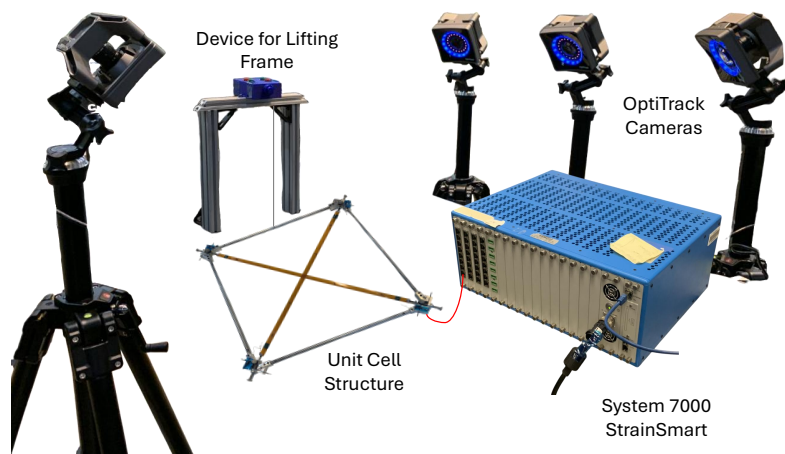


Figure 3.18: Case C experimental setup.

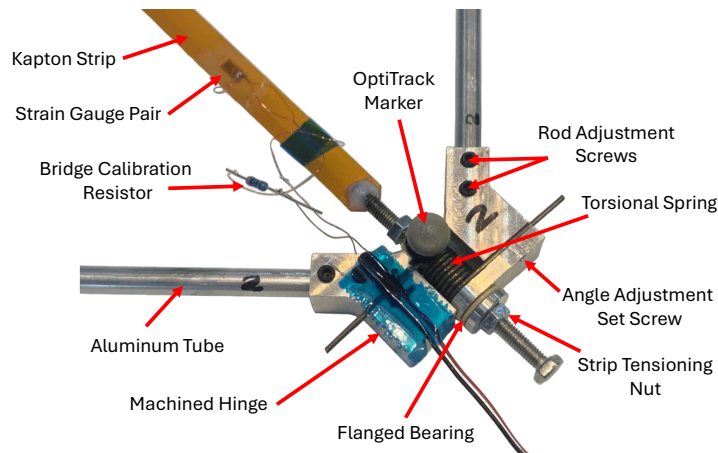


Figure 3.19: Case C experiment frame corner.

Initially, the internal rods were constructed by laser cutting a $100\ \mu\text{m}$ thick 1095 tempered steel sheet into a $0.01\ \text{m}$ thick strips. These strips were then joined using a brass rivet, and epoxied into M4 screws that were slotted and placed into each corner through the hollow shaft. A nut on each screw could be turned in order to apply a prestrain at each of the four corners. Four 125BZ Micro Measurements general purpose strain gauges were then mounted (one on each strip) to calculate the strain in each strip during the experiment. This strain was read using Micro Measurements's P3 Strain Indicator and Recorder. The strain to force relationship of the internal steel strips was determined by vertically suspending a strip and loading it with masses up to $1\ \text{kg}$ in $25\ \text{g}$ increments. This relation was found to be $5.51\ \mu\text{e}/\text{N}$. This version of the experiment can be seen in Figure 3.20 and Figure 3.21. However, this experiment was constrained due to the resolution of the recording unit, the relatively high modulus of the steel resulting in low strain magnitudes at the critical buckling load, and sensitivity to bending strains in the strips.

To improve the initial experiment, the internal "rods" were constructed by laser cutting a $100\ \mu\text{m}$ thick Kapton sheet instead into two $0.01\ \text{m}$ wide strips. These strips were then joined in the center using a thin adhesive, and epoxied into slotted M4 screws placed into each corner through the hollow shaft. A nut on each screw can once again be turned in order to pull each screw out against the hollow shaft and apply a prestrain from each of the four corners. Eight 125BZ Micro Measurements general purpose strain gauges were then mounted (one on each side of each "rod") to calculate the strain in each "rod" during the experiment. The gauges were wired together in an opposite-leg active half bridge configuration in order to cancel out



Figure 3.20: Case C initial experimental setup.

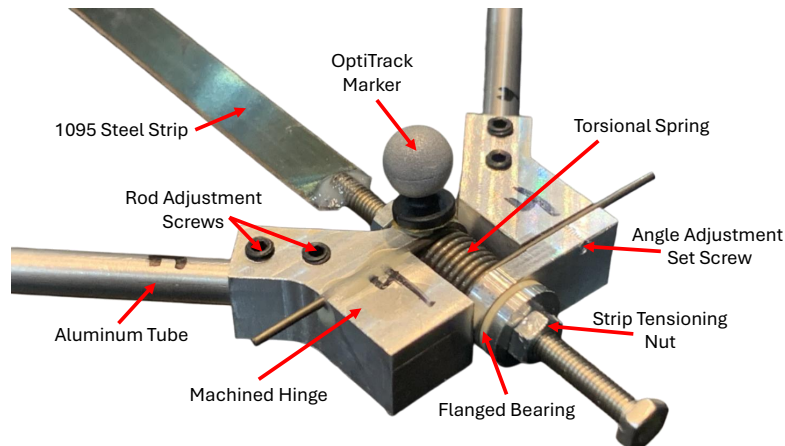


Figure 3.21: Case C experiment frame corner using 1095 steel and one strain gauge.

any bending strains and read purely the tension in each "rod." A resistor was added for each pair for balancing and calibration purposes. The strain to force relationship was calibrated for each pair of gauges before the strips were integrated into the frame by vertically suspending a strip and loading it with precision masses up to 2 kg in 50 g increments. This relation was found to be $67 \mu\epsilon/\text{N}$. The tensile strain was read using Micro Measurements's System 7000 StrainSmart Data Acquisition System for both the calibration and the experiment, allowing for proper gauge balancing, compensation, and a live data read out.

Four PrimeX 22 motion capture cameras were utilized to track the corner displacements during the experiment. The experiment consisted of adjusting the prestrain so that each of the internal rods has the same strain in small increments, and then lifting one edge of the frame by its center of moment, and measuring the out of

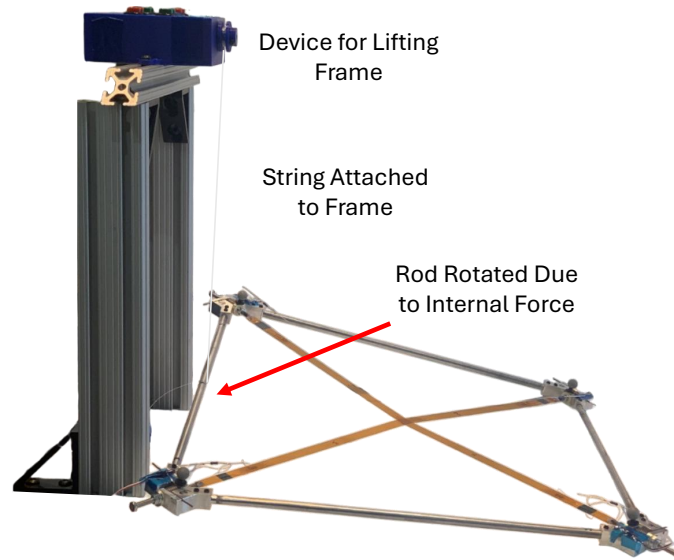


Figure 3.22: Frame in buckled state after being lifted.

plane displacement of one corner relative to the plane of the other three corners. The frame was slowly lifted using a spooled string to minimize disturbances during this process, as it was necessary to offload the self-weight quasistatically to allow for the post-buckling displacements to be observed. The lifted frame is shown in Figure 3.22, where a prestrain greatly exceeding the buckling load was applied for the purpose of illustrating the buckled configuration. The frame was found to have a 2.3 mm initial imperfection, and as such, a matching post-buckling analysis was repeated in Abaqus, similar to the one in Figure 3.15. The empirical data is plotted against this analysis in Figure 3.23, and matches well with the numerical prediction. It should be noted that in contrast to the study done in Figure 3.15, the internal rod force here is defined as the force measured in the prebuckled configuration, not the actual force in the internal members once the self-weight is offloaded, since the experimental data points were chosen via strain control. Hence, the bifurcation is significantly less observable in this plot, especially with the increased imperfection. An additional simulation was done to obtain the corresponding force data in Abaqus, and the results matched appropriately via the thermal strain. Additionally, despite low friction bearings, in a few experimental runs the structure did not buckle when lifted. These points are omitted from the graph.

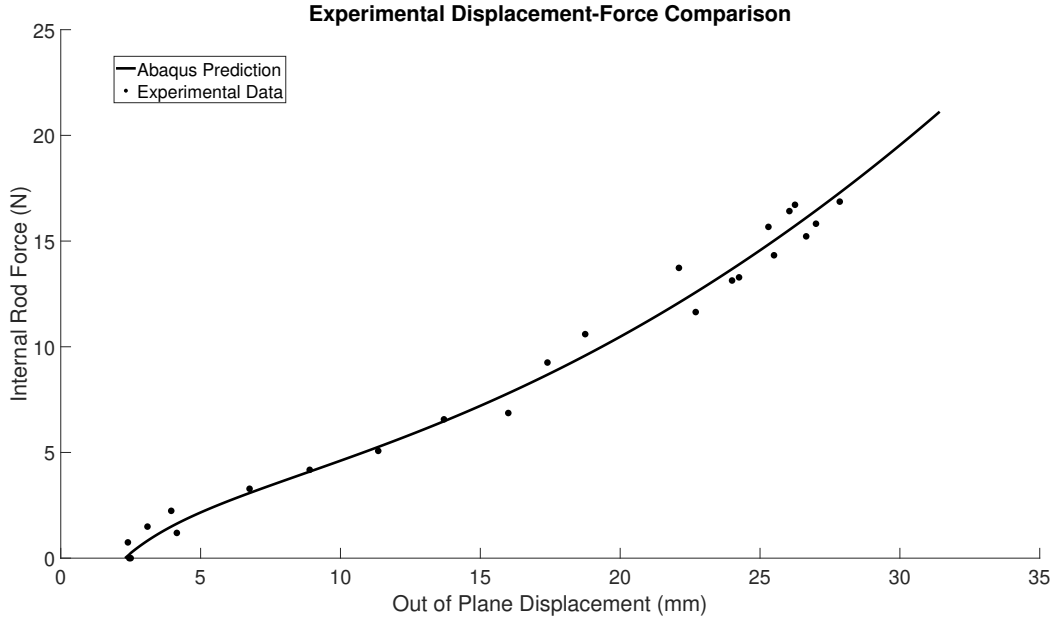


Figure 3.23: Case C experimental comparison to numerical prediction.

3.6 Discussion

The analytical theories produced in this chapter predict the behavior of the frame unit cell structure in both the numerical simulations and experiments. It is shown that follower forces always sustain twice as high of a prestress and thus are more structurally stable than what is predicted in a fixed direction case. Additionally, it is shown that the average critical compression load on the frame is invariant. This leads to the conclusion that higher local prestress values can be achieved in attached films without further impacting the stability of the structure, as long as the net total force on the frame does not increase. Shear forces also affect the compression in a frame along its edge, meaning that their orientation can stabilize or destabilize the structure.

For case D, increasing the membrane thickness further increases the critical load over the traditional corner connections such as case C. However, the caveat here is that the critical buckling temperature quickly decreases in magnitude for a membrane directly connected to the frame. While a $100\ \mu\text{m}$ strip of Kapton in case C buckles the frame at about $90\ \text{C}$ temperature change, the same thickness membrane in case D has an temperature eigenvalue of $\Delta T \approx 3\ \text{C}$. As shown in Figure 3.24, this disparity in the critical temperature between the two attachment schemes continues regardless of the thin film thickness despite the force increasing for the membrane. However,

this eigenvalue is inversely related to the thickness as well, meaning that thinner films can withstand much higher temperature changes in a space-based application before causing an instability. Also, this shows that if a novel constant force attachment style is used to distribute the load along the edge, there is a significant increase in structural stability, as well as the potential for a much higher prestress for the film, leading to additional advantages such as greater surface accuracy of the film in deployable structures as detailed earlier.

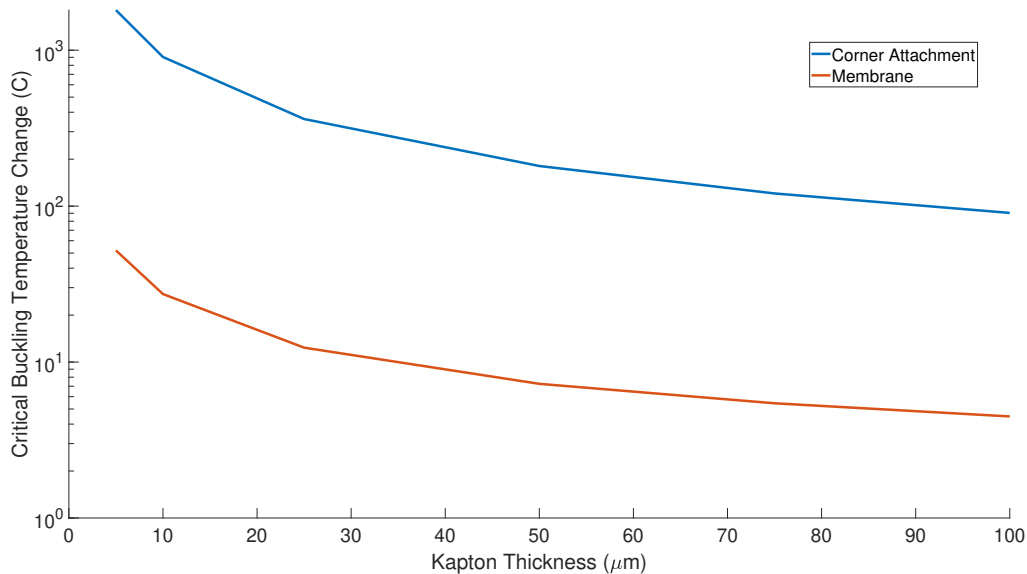


Figure 3.24: Membrane thickness comparison of critical buckling temperature.

Future work entails the design of such constant force distributed connections, as well as designing an experiment where a membrane can be accurately prestressed to measure the buckling load and post-buckling responses of case D experimentally, as well as characterizing the stiffness increase of thicker films. Furthermore, the experiment in case C may be repeated using piezoelectric actuators to more accurately control the prestrain of the frame, and better match the analytical prediction. Finally, a more generalized theory for developing structures with noncontinuous edge attachments may be derived, as well as for non-square or repeated unit cells.

Chapter 4

BUCKLING OF ELASTIC FRAME UNIT CELL

4.1 Introduction

In the previous chapter, the analytical buckling loads for a simplified frame made out of four rigid rods and four corner torsional springs were calculated and verified via numerical and experimental methods. However, a frame made out of ultralight weight and ultra-thin composites may not necessarily behave similarly, even if it appears to exhibit a similar primary buckling mode[26]. A few key assumptions need to be discussed and verified about the simplified frame model to ensure accurate predictions. As such, this chapter will describe the experimental observations that led to the development of a simplified model, and then focus on comparing a higher fidelity linear elastic model to the simplified frame model that was previously evaluated.

The frame unit cell used in the Caltech Space Solar Power Project concept currently consists of two pultruded battens and two Triangular Rollable and Collapsible (TRAC) longerons[50]. This unit cell is shown in Figure 4.1. The battens and TRAC longerons used to construct this unit cell are overlapped slightly and permanently joined with a scrim glass connector which serves to bond them locally but does not appreciably affect the local stiffness of the connection. The battens and TRAC longerons are assumed to be able to withstand the local traditional buckling loads that may be imposed (i.e. Euler buckling)[51, 52]. The design of the longerons is also driven by two additional factors: areal efficiency (larger flanges require larger gaps between the trapezoids of unit cells in order to have space to be folded before deployment, and thus reduce the effective area of the photovoltaics and transmitters needed) and local buckling of the flange region[15]. This type of highly imperfection sensitive phenomena has been extensively studied by our group and others[51, 53–55].

Based on these previous studies, the parameters chosen for the study of the linear elastic frame model are listed in Table 4.1. An internal size of 20 cm $\times$ 20 cm was chosen as this is the largest square unit cell that can fit in the thermal test chamber in the lab, but in the future a larger unit cell of 1 m $\times$ 1 m is envisioned. The longerons were constructed using a [+45 GF / 0 UD CF / -45 GF][+45 GF][+45 GF / 0

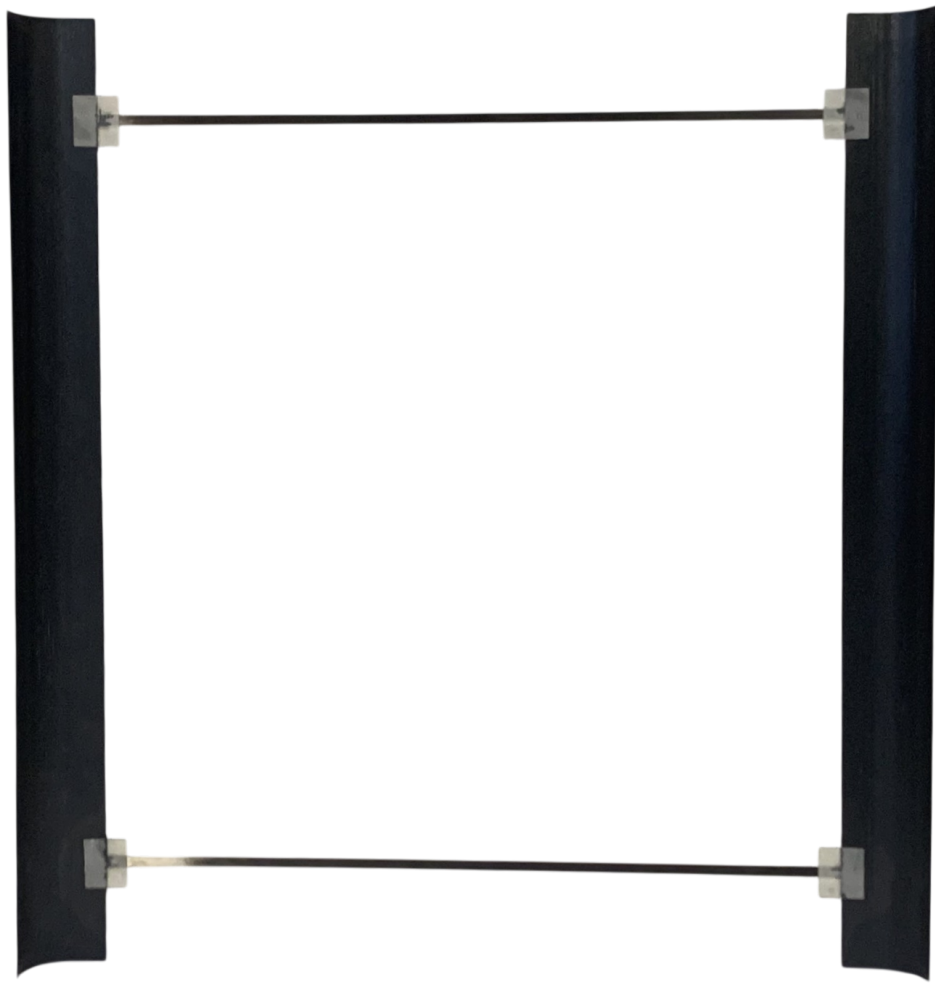


Figure 4.1: Top view of 20 cm $\times$ 20 cm carbon fiber reinforced plastic TRAC longeron and batten frame used in the Caltech Space Solar Power Project.

UD CF / ± 45 GF] layup at the web, with each [± 45 GF / 0 UD CF / ± 45 GF] flange joined by a middle glass fiber layer. More details about the manufacturing of longerons can be found in [56]. Table 4.2 lists the key material parameters of the pultruded DPP battens made from Torayca T700 and Bisphenol A Epoxy Resin, and the MR70 carbon fiber, the E-glass Fiber layers, and the NTPT 415 resin polymer that are used in the manufacturing process for the TRAC longeron layup [57–60]. For the samples discussed in this section, a two cure process was used to help minimize imperfections [61].

The open cross section of the TRAC longerons makes them particularly susceptible to torsional buckling. Hence, the unit cell frame does indeed easily exhibit a similar buckling mode to the one studied in the previous chapter, both due to limited

Parameter	Value
Longeron flange radius	13.6 mm
Web width	8 mm
Longeron flange arc	105°
Total longeron length	250 mm
Batten length	210 mm
Batten width	3 mm
Batten thickness	0.6 mm
Internal side length	200 mm
Batten/Longeron web overlap	3 mm × 5 mm

Table 4.1: Linear elastic frame dimensional properties.

Batten Properties	Value
Young's Modulus (Primary direction)	140 GPa
Shear Modulus	7.6 GPa
Poisson's Ratio	0.35
Density	1.9 g/cc
Thickness	0.6 mm
Glass Fiber Properties	
Young's Modulus	65-75 GPa
Shear Modulus	28-32 GPa
Poisson's Ratio	0.2
Density	2.54-2.60 g/cc
Thickness	25 gsm
Carbon Fiber Properties	
Young's Modulus	324 GPa
Shear Modulus	N/A
Poisson's Ratio	0.39
Density	1.8 g/cc
Thickness	30 gsm
Polymer Properties	
Young's Modulus	3.01 GPa
Shear Modulus	3.3 GPa
Poisson's Ratio	N/A
Density	1.21 g/cc
Thickness	15 gsm

Table 4.2: Material properties for items used in making a batten and longeron frame.

torsional stiffnesses of the components, and the low stiffness at the connection region. Previous studies have also observed this and looked at sensitivities of similar structures to parameters such as force offsets, multiple unit cells, and post-buckling out of plane displacements [25]. In these studies, it is generally observed

that the less desired effects are exacerbated as the structures are scaled. Therefore, by validating the model developed in Chapter 3, it would allow the design focus to shift to solely the torsional stiffness of the frame, and give a more clear path forward as to how to improve the architecture (and attachment scheme).

4.2 Experimental Observations

Several models of unit cell frames with films attached were manufactured for a variety of purposes during the course of this thesis, such as studying the effects of pretension on the film (see Chapter 2) and deployment tests (see Chapter 5). Most of them had the same $20\text{ cm} \times 20\text{ cm}$ internal dimension. For a successfully embedded film, it must have some pretension after the manufacturing process is complete. For previous methods, such as the frame depicted in Figure 4.2, where multiple components are bonded at multiple interfaces, this process can be difficult and arduous, and often can lead to unequal pretensioning as well[62]. Hence, for the double s-springs developed in Chapter 2, two main methods were developed and utilized in order to attempt to mitigate these past difficulties.

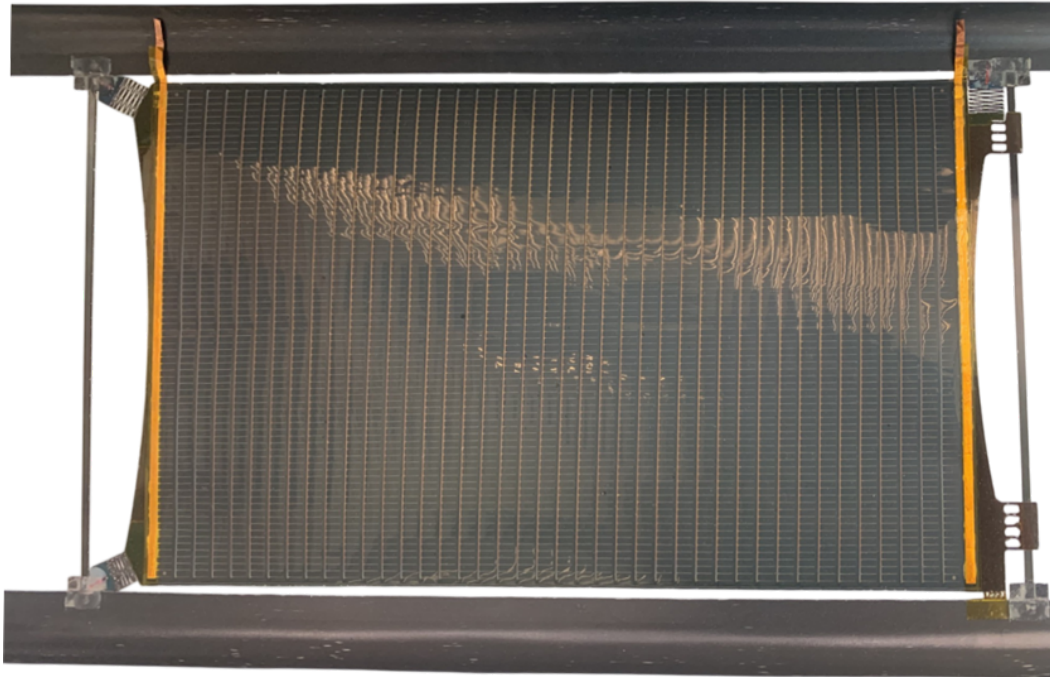


Figure 4.2: Previously manufactured rectangular frame with pretensioned photovoltaic film from Ascent Solar Technologies attached via simple kirigami springs.

The process for the first method is shown in Figure 4.3. This method involves applying a thermal strain to the Kapton, and exploiting the CTE mismatch between

the frame and the film before bonding. This method is particularly effective for bonding a pretensioned frame to an Invar frame, but since composite frames have a CTE much lower than the Kapton, this method was also useful for generating lower prestresses in composite frames. First, a Kapton sheet (typically 50 microns thick), is speckled with a permanent marker using a randomly generated dot pattern via a Zünd cutter. This pattern is useful for conducting digital image correlation (DIC) for future measurements. Paint was not used for speckling, as it would be in most typical DIC applications, since it would have a significant thickness and comparable modulus to the Kapton, resulting in measurements being significantly affected. Instead, additional lighting to minimize shadows and a uniform white background is used to ensure high enough contrast. The film is then laser cut with the double s-spring pattern. More information on this process can be found in Chapter 2 and Appendix A.

Films in this case were made with one tab per each spring pair or with a joined continuous edge tab. The thermal expansion mismatch between the Kapton surface at the bonding location to the frame was not large enough to necessitate the tabs for a 20 cm edge, but using tabs is likely necessary if this method is to be scaled to prevent stress build up. These tabs can be seen in Figure 2.16. At this step, a DIC baseline is conducted on the Kapton to measure an initial stress free reference state for strain measurements. Next, a thin layer of silicone is placed on the hotplate to allow for even pressure to be applied to the film. A layer of peel ply is also placed between the Kapton and the silicone to prevent accidental bonding between the two in subsequent steps. The internal Kapton membrane is then covered by an aluminum square plate. This ensures good thermal contact with the hotplate, as well as preventing any curling and ensuring flatness of the film. Aluminum was the material of choice due to it having a nearly identical CTE to Kapton, preventing any shear loads between the two as they heat up. The hotplate was then set to its maximum temperature, typically 220-240 °C, with time given to allow the system to equilibrate and all components to heat up. A two part cure low out-gassing epoxy (LOCTITE® EA E-120HP) is then applied to the exposed tabs not covered by the aluminum square. While typically this epoxy takes 24 hours to fully set, its work life is greatly reduced at elevated temperatures, and as such this step was completed very quickly, and then the frame unit cell placed on the epoxy. The aluminum square was created to be the same thickness as the frame unit cell for Invar. For a composite frame, an additional aluminum rectangular frame was used to ensure good contact and keep the longerons flattened during the adhesive cure. To further ensure good

thermal and bonding contact, additional mass was placed on top of the hotplate as the epoxy cured. After 24 hours, the layers were removed in reverse order.

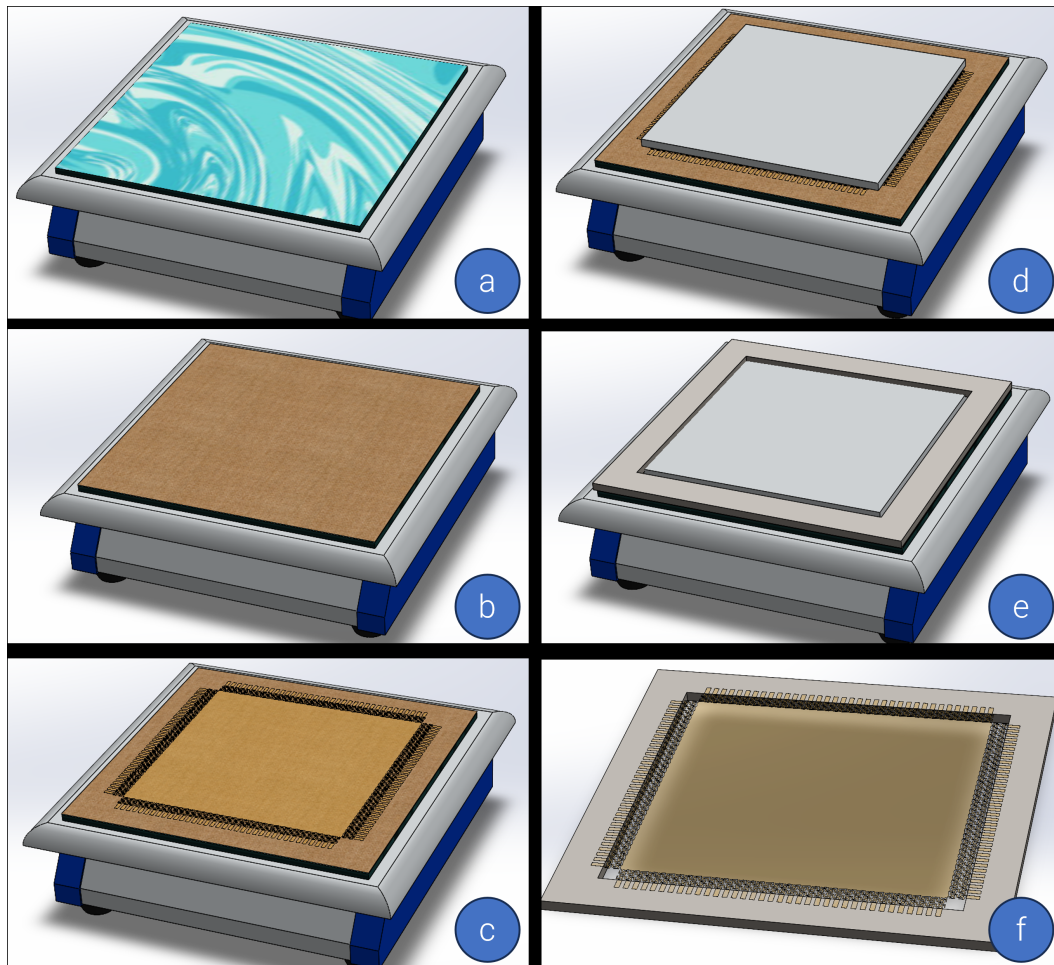


Figure 4.3: Illustration of using a hotplate to create a pretensioned film bonded to an Invar frame. First, a laser cut Kapton sheet (c) is placed on a silicone (a) and peel ply (b) layer. The Kapton is covered with an aluminum square (d), and the parts heated to equilibrium. The exposed tabs are epoxied, and the frame is placed on top (e). Once cured, the layers are removed in reverse (f).

The hotplate method has five main limitations: scaling, pretension limit, surface offsetting, uniformity, and moisture control. For a $20\text{ cm} \times 20\text{ cm}$ structure a hotplate is the perfect size, but for a $1\text{ m} \times 1\text{ m}$ unit cell, a very large hotplate would be needed, and is quite unreasonable. Additionally, this method relies on exploiting the difference in CTE, which means it is also limited by the temperature of the hotplate. Going much higher in temperature is difficult, and would also result in damage to the structure. The film can also only be attached on one side of the structure. As shown in Figure 2.15, it is preferable to attach the film above and below adjacent

sides, since the composite thickness is larger than the film thickness. Mounting any functional components could be problematic too when attempting to weigh the film down, as they may create areas of concentrated stress or suffer damage. Moreover, the hotplate itself already does not perfectly prestress the film, as when measured with an IR camera, it was observed that variations of nearly 50 °C exist across its surface. Finally, Kapton is a hygrothermally sensitive material. While the film can be desiccated before manufacturing, this method provides less room for a humidity controlled environment. It was observed that about a day after manufacturing, some prestress is lost due to the Kapton absorbing additional moisture.

The alternative method used allows for most of these issues to be mitigated or solved. The setup for this method is illustrated in Figure 4.4. Here, the pretension is achieved by using a series of pulleys and weights along the edge. The first steps of cutting and speckling the membrane are the same as the first method. Once again, one continuous edge tab or several smaller ones can be used. For convenience, additional tabs for string attachments can also be used as seen in Figure 4.4. Each edge of the film is then bonded to a batten or longeron. Then, each edge is carefully pretensioned with a series of masses. The two masses at each corner should each be half of the mass of ones along the center to ensure equal pretensioning along the edge. Once each edge is pretensioned, the corners where the frame edges intersect can be bonded, with pressure applied using small clamps if necessary. This method has a much more precise and even resulting pretension, along with greater magnitude potential for pretensioning. Since the double springs are well characterized, a displacement control version of this method is also possible, and currently being developed as seen in Chapter 6. Additionally, this method is not limited by the size of something like the hotplate, and can be utilized in a moisture controlled environment, allowing for higher manufacturing consistency. The only main drawback of this method is that it too suffers pretension loss from relaxation. Once the external prestress from the attached masses and pulleys is removed, the structure is free to deform elastically, and thus some pretension may be lost. However, this can be amended by solving the statically indeterminate problem ahead of time to calculate the initial prestress required for the system. Due to the nonlinear nature of the double s-springs, this effect is also minimized as the force changes minimally with even relatively large displacements as well assuming a significant initial prestrain.

Similar to and driven largely by the imperfections observed in the longerons, physically manufactured frames tend to have imperfections as well. An example of



Figure 4.4: A frame made of four battens pretensioned by pulleys. The film surface can be observed to be smooth, and the double s-springs along the edges displaying post-buckling behavior.

this can be observed in Figure 4.5. In this pair of images, the top image shows a manufactured frame, which does not have any film yet attached. Using the hotplate method to prestress a film to <1 N/m on each edge, a film was attached to it in the bottom image. In this case, since the prestress was relatively low, the frame did not undergo additional out of plane deformation. In both of these cases, the frame was gravity offloaded, as in terrestrial conditions its self weight is enough to not exhibit this imperfection. However, in Figure 4.6, a much higher prestress is applied to a frame made out of just the less torsionally stiff battens. While this frame had less of an initial imperfection had it not been prestressed, there is a significant torsional buckle present due to the internal prestress, even when sitting on a flat surface as shown. This particular specimen was similarly gravity offloaded and measured with a FaroArm LiDAR to characterize its deformed shape. This revealed a 16 mm total deflection of the frame, stimulating and informing the studies presented in the previous chapters.

4.3 Numerical Simulations and Comparison to Simplified Analytical Model

While high quality composite frames can be manufactured, their ultralight nature makes it difficult to conduct precise buckling load studies on them experimentally. Hence, a numerical model was created to simulate the buckling load and torsional stiffness of the frame created experimentally, and compare it to the analytical model developed in Chapter 3. This model was created using Abaqus 2024, and is dimensioned according to the parameters listed in Table 4.1. While imperfections



Figure 4.5: Side view of a batten and longeron frame with a manufacturing imperfection (top). After implementing a film with very low prestress (<1 N/m on each 20 cm edge), this effect did not get exacerbated (bottom).



Figure 4.6: Torsionally buckled frame made from four battens with a film supported by double s-springs. Internal prestress was 6.6 N/m along each 20 cm edge.

were observed in the experimental models, and as seen in the previous chapter can significantly affect both the buckling and post-buckling behavior, the numerical model studied in this section does not feature torsional imperfections, and assumes that the batten and TRAC longerons are joined in-plane. Furthermore, it also does not assume any local imperfections, which as discussed in the introduction, may be significant to the behavior of TRAC longerons.

This study was done first for a frame made of an isotropic material for validation purposes, and then adapted to an orthotropic material to better model the experimental models previously produced. The material properties for the battens are listed in Table 4.2, with an estimated 6.5 GPa tangent modulus, while the effective layup properties for the orthotropic model are in Table 4.3. Some of the engineering constant properties, particularly those related to the out of plane direction for the Poisson's ratio and shear moduli, are poorly estimated but were not found to

dominate the behavior observed, and hence left as is. For the preliminary isotropic material model, E1 was used. The numerical model consists of 30,597 S4R elements for the frame, and was found to be convergent for a global element size of 1 mm. A thickness of 0.1665 mm, 0.072 mm, and 0.7665 mm were used for the web, flange, and intersection of the web with the batten, respectively, as calculated from the layup, and verified using a micrometer (omitting the scrim glass connector).

Parameter	Value
E1	134 GPa
E2	134 GPa
E3	6.5 GPa
ν_{12}	0.35
ν_{13}	0.35
ν_{23}	0.35
G12	7.6 GPa
G13	7.6 GPa
G23	7.6 GPa

Table 4.3: Linear elastic frame composite orthotropic material properties.

The first part of the study clamped (fully constrained) one corner of the frame in the region of overlap between the batten and TRAC longeron web. After this, a small force (1 mN), was applied to the opposite corner. This pair of boundary conditions mimics the onset of the "axial folding" observed and modeled in the previous chapter. The force applied was validated to be small enough to be agnostic to nodal rotations, ensure a linear elastic deformation regime, and that nonlinear geometric effects were not observed. Examining the corner displacement, an effective corner spring constant, " k ," was calculated as $k=0.0128$ Nm for the isotropic frame and as $k=0.00344$ Nm for the orthotropic frame. This is as expected, as the bending/torsional stiffnesses of the components used to construct the frame were significantly reduced.

Now that both the " k " torsional spring constant and " L " side lengths are known, the buckling loads expected can be calculated analytically for a variety of loading scenarios, similar to the previous chapter. These results, based on the loading conditions of Figure 3.2, are tabulated in Table 4.4 and Table 4.5 for the isotropic and orthotropic material studies, respectively, alongside their numerical counterparts. To obtain the buckling loads numerically, the *Buckle command was used for Cases A and B, where either a point load or distributed edge load boundary condition was applied, and the smallest eigenmode was examined. In all cases this eigenmode

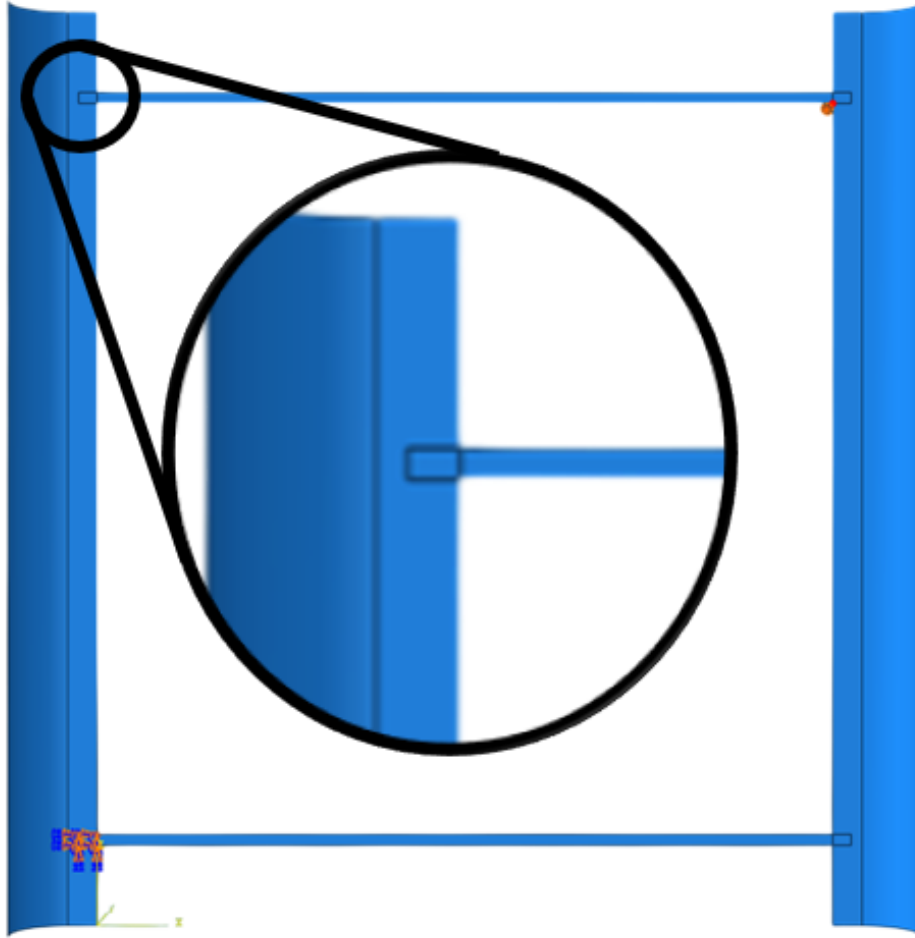


Figure 4.7: Zoomed view of batten to flange overlap. The corner clamping and opposite corner force application boundary conditions are also illustrated.

was observed to be the expected torsional buckling mode. For Case C, similar to the previous chapter, a thermal buckling load approach was used. For the isotropic material version of the simulation, the same approach of four T3D2T elements was used to create the internal rods. By applying this thermal strain in the rods, it is then possible to extract a critical temperature eigenvalue, and solve the statically indeterminate problem. Since in this case the model is higher fidelity than the previous chapter, this is difficult to do analytically, and thus Abaqus was used to calculate the effective critical buckling load based on the critical temperature obtained. Using *Static, the frame was constrained to prevent rigid body motions, and the thermal load applied to the internal rods, and the resulting stress state used to calculate the effective force. In this case however, using just four truss elements created a significant nodal stress concentration where they were attached to the

linear elastic frame, leading to a higher buckling load than expected. Thus, for the orthotropic model, this simulation was changed to include internal Kapton strips modeled as shell elements instead, with a width of 0.01 m and a thickness of 50 microns. The properties for Kapton can be found in Section 3.4. In both approaches for Case C, the expected buckling mode was the lowest mode obtained, and is shown in Figure 4.8. This set of simulations was similarly verified for mesh convergence, as well as for nonlinear effects. It was determined that accounting for nonlinear geometric effects did not affect the stresses, meaning that using this linear model is sufficient to fully capture the buckling behavior.

Case	Analytical	Numerical
A	0.181 N	0.215 N
B	1.28 N/m	1.72 N/m
C	0.362 N	1.33 N

Table 4.4: Isotropic material property study results.

Case	Analytical	Numerical
A	0.049 N	0.054 N
B	0.34 N/m	0.44 N/m
C	0.097 N	0.12 N

Table 4.5: Orthotropic material property study results.

One aspect that did have a significant effect on the calculated critical buckling load was how rigid body modes were restricted during the buckling value calculation. Typically, one node or one area of the overlap region was fixed. Rigid body modes must be restricted particularly when using the Lanczos eigensolver. In this simulation, the Lanczos eigensolver is preferred to avoid a large amount of negative eigenmodes from being calculated, since it allows for a minimum and maximum desired value to be prescribed. Alternatively, the structure can also be prestressed to avoid a near zero initial stiffness, but this comes at the cost of an additional computational step, and can still yield some negative eigenvalues, thus requiring more total values to be calculated. If the structure is instead held by the flange, or rather the flange edge of one of the longerons is constrained at its end, the calculated buckling value rises, since the effective stiffness is greatly increased. For example, in the isotropic material case, the buckling prediction calculated numerically rises to 0.24 N in Case A from 0.215 N, and for the orthotropic material, this value nearly doubles from 0.054 N to 0.094 N. Similar trends were observed for the other cases studied as well.

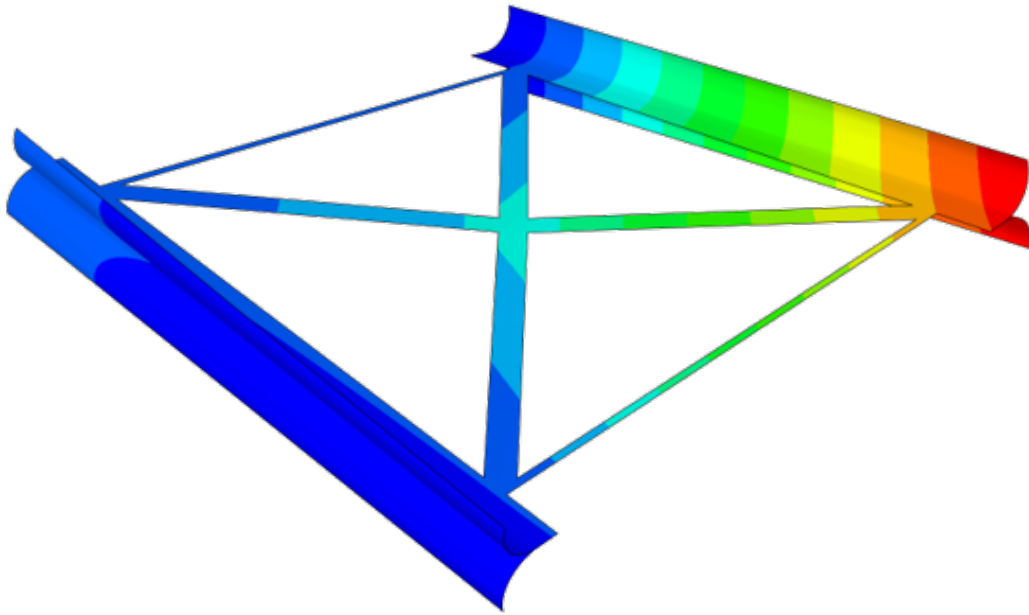


Figure 4.8: Lowest buckling mode observed numerically for Case C in a linear elastic frame.

4.4 Discussion

By calculating the effective torsional stiffness " k " of the linear elastic frame, we are able to obtain a very good estimate of the buckling load of the frame. For the cases studied in this section, A-C, it was shown that this analytical estimate from the formulas derived in the previous chapter consistently provides a close upper bound to the real load that was calculated using the higher fidelity linear elastic model of the frame unit cell in the previous section. Conversely, this also means that the four rigid rod reduced order model used in the previous chapter is an extremely computationally efficient and relatively accurate model, and can be used to further study attachment schemes and buckling of the unit cell structure. The remaining discrepancies between the analytical model and linear elastic model studied in this chapter are likely to do with exactly how the linear elastic frame buckles: there is both a displacement in the flanges and a deformation in the web of the TRAC longeron. Thus, when simulating to find the torsional stiffness " k " for the frame, the overall deformation is slightly different than the deformation induced by the forces that are applied internally. Particularly the distributed load estimate is a bit higher than the point load, likely due to it effectively slightly stiffening the web. This is similar to how in the previous chapter, if the "rigid" rods used in simulation were a lower modulus, they would be closer to their Euler buckling loads, and thus deviate

from the analytical estimate.

The linear elastic model also paves the way for several further future studies that will help analyze and improve the SSPP and other similar frame unit cell based structures. Currently, the primary focus is to examine the effect of the material overlap section, and how to best optimize it for weight, folding capacity, and torsional stiffness. This model also provides a simple numerical twin to study other methods of stiffening the frame structure, such as changing dimensions of the battens and longerons used, or alternatively examining closed cross section longeron designs (resulting in higher torsional stiffnesses and potentially reducing manufacturing imperfections), such as small Omega booms in order to further stiffen the structure against its current primary failure mode.

Other future studies will focus on expanding the capabilities and accuracy of this model. First, as briefly discussed in the introduction, TRAC longerons are particularly sensitive to imperfections, largely due to their open cross section. The imperfections in the TRAC longerons contribute significantly to their stability as studied previously by others [53]. This chapter did not yet study the effect of these types of imperfections on the stability of the frame, or of more general imperfections based on its primary buckling mode. Similarly, previous studies have looked at other factors as well for analogous TRAC longeron and batten structures, such as the one in Figure 4.2, and revealed additional sensitivities to the effects of having multiple unit cells. For these studies and the studies done in the previous chapter, the post-buckling behavior of the frame has also been studied. While relatively simple to do, the post-buckling behavior of the linear elastic frame model has not yet been thoroughly studied or compared to the simplified model. This is quite important, especially since the post-buckling behavior changes significantly when implementing a membrane, such as in Case D, which also needs to be studied in correlation with this frame model. Case D has also not been examined yet for the linear elastic model, nor how the presence of a membrane potentially stiffens or destabilizes the structure.

Once these additional studies characterize the limitations of the simple analytical frame model, the next step will be to combine it with the reduced order model developed in Section 2.7. This will allow for computationally efficient design of both the frame and the attachment scheme. Likewise, this combined model can then be validated by the experimental models and observations presented in Section 4.2 of this chapter, and serve to inform future experimental designs, as well as the

eventual scaling of the unit cell and fabrication of larger structures.

Chapter 5

ADDITIONAL EXPERIMENTS AND APPLICATIONS OF DOUBLE S-SPRINGS

Part of this chapter was adapted from the following publication:

Federico Benazzo, George A. Popov, Juan Mejia-Ariza, and Sergio Pellegrino. Thermally stable method of attaching precision thin films to deployable structures using high strain kirigami borders. *AIAA SCITECH 2026 Forum*, page 1040, 2026. doi: 10.2514/6.2026-1040.

5.1 Introduction

The Caltech Space Solar Power Project (SSPP) has several key steps in its deployment. All of these steps have to be performed successfully and reliably in order for the structure to be able to fulfill its missions critical requirements of collecting and then delivering power on orbit. In early 2023, a prototype of the SSPP concept, the Deployable on-Orbit ultra-Light Composite Experiment (DOLCE) flew on the Caltech Space Solar Power Demonstration 1 (SSPD-1, Figure 1.1) mission in Low Earth orbit[63–65]. The purpose of this mission was primarily to demonstrate the scaled deployment concept designed by Caltech, where the batten and TRAC longeron frames discussed in the previous chapters are first uncoiled (unwrapped) and then released (unfolded) into a flat state. The reverse (packaging) process of this deployment is show in Figure 5.1[50]. For the purposes of the SSPD-1 mission, DOLCE did not include any films attached to the composite frame that deployed, due to the torsional instabilities observed when implementing said films discussed in the Introduction Chapter of this thesis, which could have potentially jeopardized the deployment demonstration.

The primary mechanism for keeping the SSPD-1 structure stowed was pressure wrapping. Once the structure was Z-folded into a four-point star, it is held in place by four cylinders that keep the star from deploying. A Kapton membrane, connected to additional cylinders that actively tension this additional membrane, is used to keep the structure tightly packaged and under constant tension while stowed and during uncoiling. More details about this packaging scheme can be found in the cited literature[66].

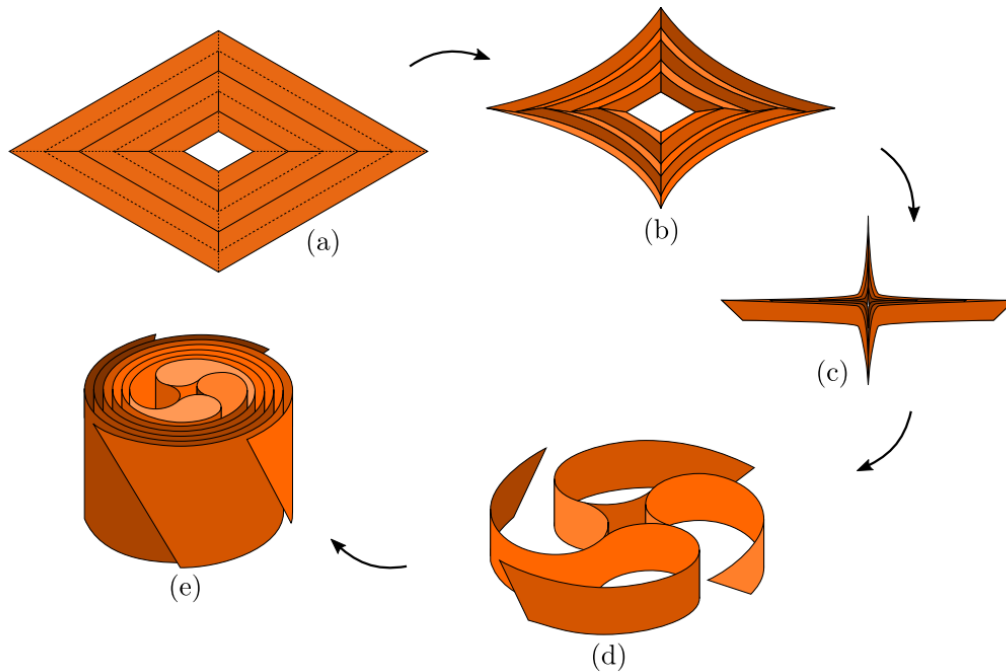


Figure 5.1: Packaging concept for SSPP, where the structure is first Z-folded and then coiled around four mandrills.

During the packaging process, the composite frame and its attached membranes thus undergo stresses and compaction. As such, whatever attachment scheme is used must be able to withstand these forces, as well as maintain the original pretension after a successful deployment. Packaging must also be done for a large length, meaning that the coiling has to be smooth and repeatable, or else imperfections such as bumps may form and damage the structure. Local effects and such as these as well as global effects such as changes in temperature or humidity may affect the packaging process. Additionally, during the release of the star pattern, the trapezoid segments undergo a highly dynamic process. More information about this process can be found in the associated literature[67]. During the SSPD-1 mission, despite membranes being omitted, there were still difficulties encountered with the latching of the longeron strips due to the ill pretensioning of the structure via the support booms[63]. Given the complicated nature of the deployment, these types of difficulties must be mitigated as much as possible.

In addition to SSPD-1, there are a number of other deployable space structure concepts that utilize high strain components, or require film prestressing in order to successfully complete mission objectives. Examples of these concepts include the composite deployable telescopes, multi-arm mesh reflectors, deployable hub-centric

solar sails, foldable cylinders, pantographs, tensegrity structures, etc[68–72]. When attaching or bonding functional thin films to these concepts, they must also be able to survive deployment, be supported post deployment, and not hinder deployment. As will be demonstrated in this section, double s-springs have the capability to be packaged efficiently and smoothly, deploy consistently in scenarios where other methods have previously failed, and also offer merit in undergoing high strains to enable further novel deployable concepts.

5.2 Coiling and Deployment

A previous study in the Space Structures Lab has demonstrated that deployment of pretensioned films inside of a TRAC longeron frame can prevent TRAC longerons from locking[2]. As can be visualized in Figure 5.2, when attached at the corners via previous constant force springs, the film inside of the frame creates an equilibrium configuration. If the deployment lacks sufficient kinetic energy to surpass this point, the structure will become stuck. This is a fundamental concern with this type of deployable concept.

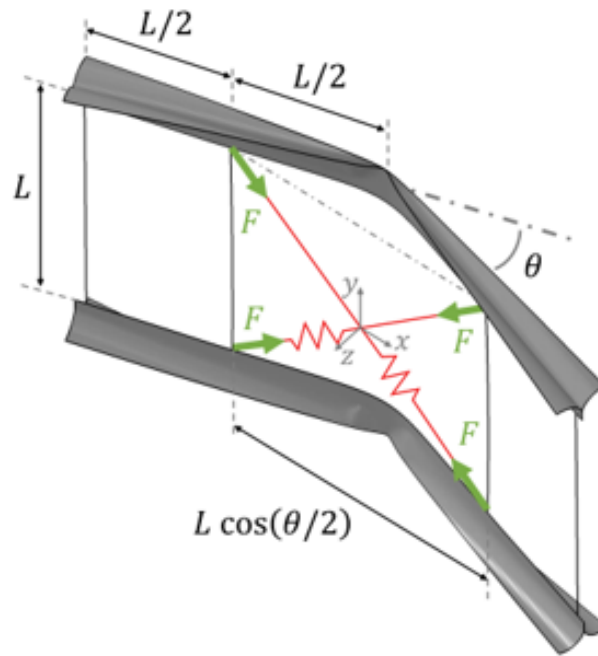


Figure 5.2: Equilibrium configuration resulting in longerons being unable to properly latch and complete deployment during the unfolding of the Z-folds[2].

Double s-spring attachments, being a distributed edge connector and not just a corner connection, are able to overcome this shortcoming of previous corner style

attachments. The experimental setup from the previous study, where the frame was suspended vertically to facilitate gravity offloading, was utilized for this demonstration as well, illustrated in Figure 5.3. This experimental setup allows for the longerons to move and latch without potentially interfering with other parts of the setup, and the strings used to suspend the frame are long enough to not apply a significant torque as the structure deploys that could potentially affect the experiment.

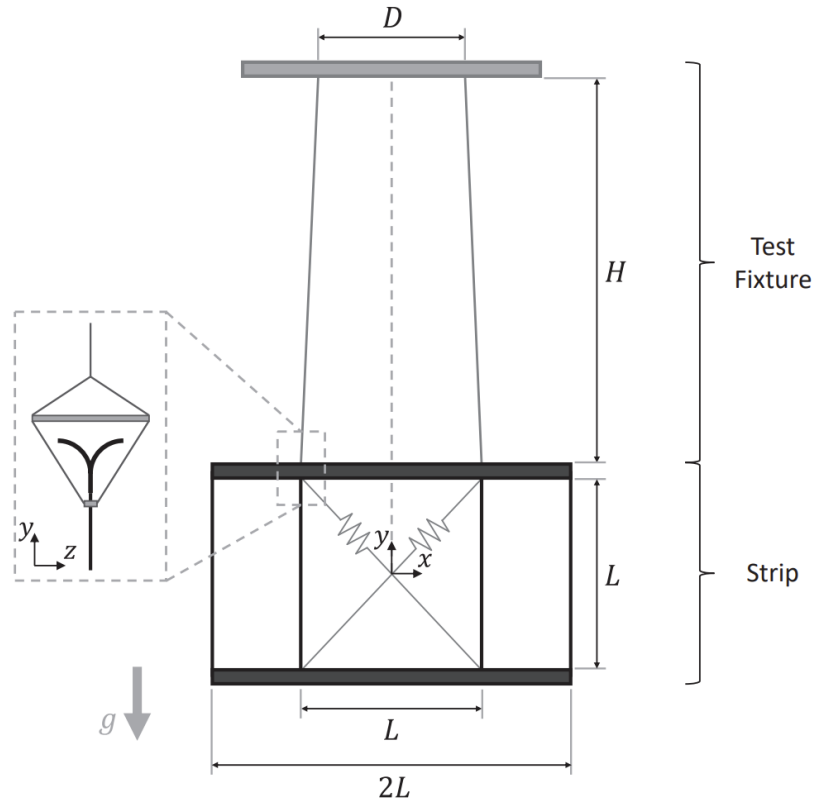


Figure 5.3: Experimental setup used to gravity offload the frame structure and demonstrate deployment and latching of the composite frame unit cell[2].

The results of the deployment experiment with double s-springs are illustrated in Figure 5.4 below. The film in this experiment was pretensioned to a value of 6.6 N/m. The experimental steps were as follows. First, the TRAC longerons were carefully flattened uniformly to prevent a stress concentration that could locally damage the composite. Then, the TRAC longerons were fixed in this flattened configuration using hairpins. Once secured, the frame unit cell can be given a curvature by bending the TRAC longerons. The closing angle of the frame is defined based on the angle between the ends of either longeron, subtracted from 180 degrees, as shown in Figure 5.2. This angle was set from 10 degrees to 180 degrees in 5 degree

increments and then secured using a piece of string tying the opposing battens together at their center. Once the battens were secured, the partially folded frame was suspended by the pair of strings, and the hairpins securing the longerons in their flattened configuration removed. The ends of the longerons would then return to their original configuration, whereas the center would remain flattened due to the curvature. After this step, the string holding the opposing battens would be severed, thus releasing the structure. As can be seen in Figure 5.4, in the configuration with the double s-springs, the longerons would then latch, and return to their original deployed configuration. Unlike the previous corner style connections, with the double s-spring mounting style the longerons latch regardless of the initial angle.

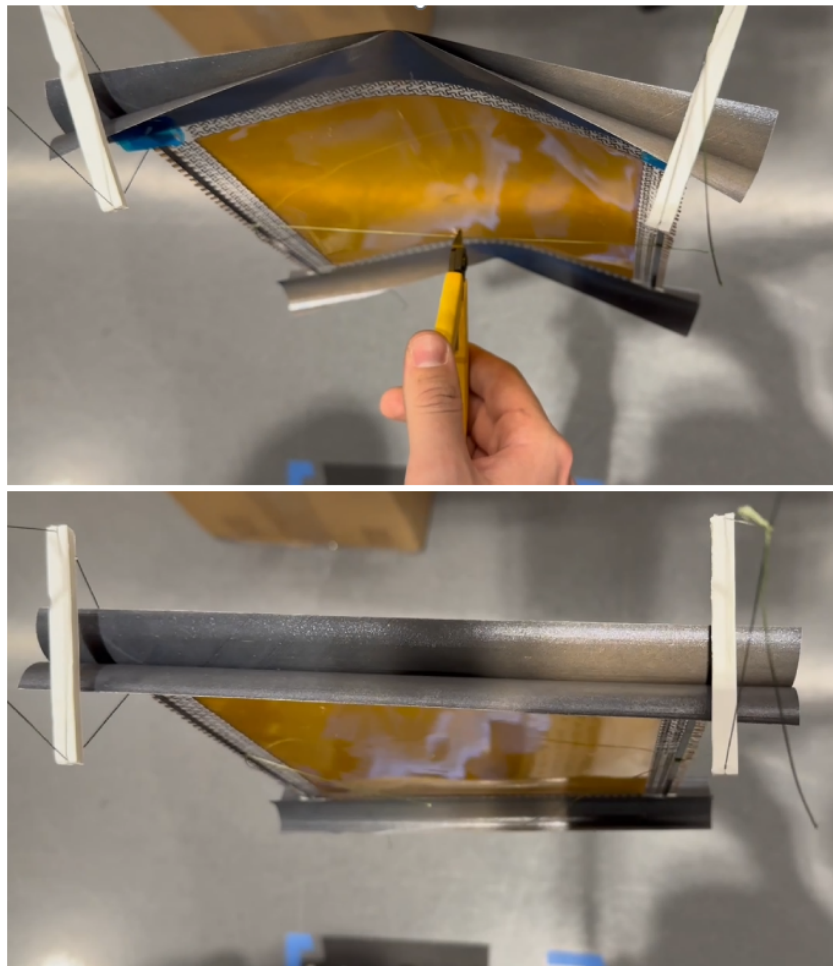


Figure 5.4: Deployment test of the composite unit cell with double s-springs used to mount a pretensioned membrane. The top image shows the string holding the partially folded structure being cut, and the bottom image shows it successfully deployed shortly after.

As described in Chapter 5.1, unit cell frames in SSPP are typically packaged using pressure wrapping before the deployment analyzed above. For the TRAC longerons used to make the unit cell frames simulated and made in Chapter 4, the minimum radius of curvature before damage is likely has been previously studied to be about 0.05 m. Hence, these structures can be wrapped around at most a 0.1 m mandrill. As shown in Figure 5.5 below, the packaging process can be done in a smooth, controlled manner, where the final form is compact, repeatable, and free of topological local effects. Similarly, the frame unit cell can be unwound by reversing the packaging process. Unlike previous kirigami spring designs made from 1095 spring steel, which had thin, brittle connections and difficulty packaging without local effects, the ductility of the Kapton double s-springs allow them to be deployed out of the mandrill without any damage to the frame, attachment scheme, or pretensioned film, and maintain this pretension through multiple deployments.

5.3 High Strain Foldable Toroid

A novel deployable concept enabled by double s-springs consists of a foldable toroid. A toroid of this variety is difficult to both manufacture and fold, requiring a doubly-curved composite structure. The toroid itself is created by implementing a curved tape-spring as the supporting outer ring for a circular deployable structure. The cross section of the tape-spring is the arc of a circle defined by the radius r and angle θ , shown in Figure 5.6, which is then rotated around a vertical axis at a distance R to define an axisymmetric toroidal shell. In this study, $R=266.5$, $r=12.7$ mm, and $\theta=120^\circ$.

The folding scheme for the composite ring is as follows. First, the shell is pinched at eight equidistant locations along the circumference. Unlike the pinching of traditional singly curved tape-spring, here the material is not fully flat as it is non-developable. Yet, the flattening is sufficient to allow for localized bending of the structure at all eight points to obtain a cross shape, as shown in Figure 5.7. Folding needs to be in both directions about the pinched region: to create both convex and concave folds. These are referred to as opposite-sense and equal-sense bending in traditional tape-springs [4], respectively. The last folding stage consists of deforming each side of the resulting cross around the center to thus further reduce the overall packaged size. This scheme is highly scalable, as the number of pinching locations and spiral folds around the central hub can be increased to accommodate larger architectures, without increasing the curvature changes experienced by the toroidal shell. To demonstrate the feasibility of this scheme, the folding of the curved tape-

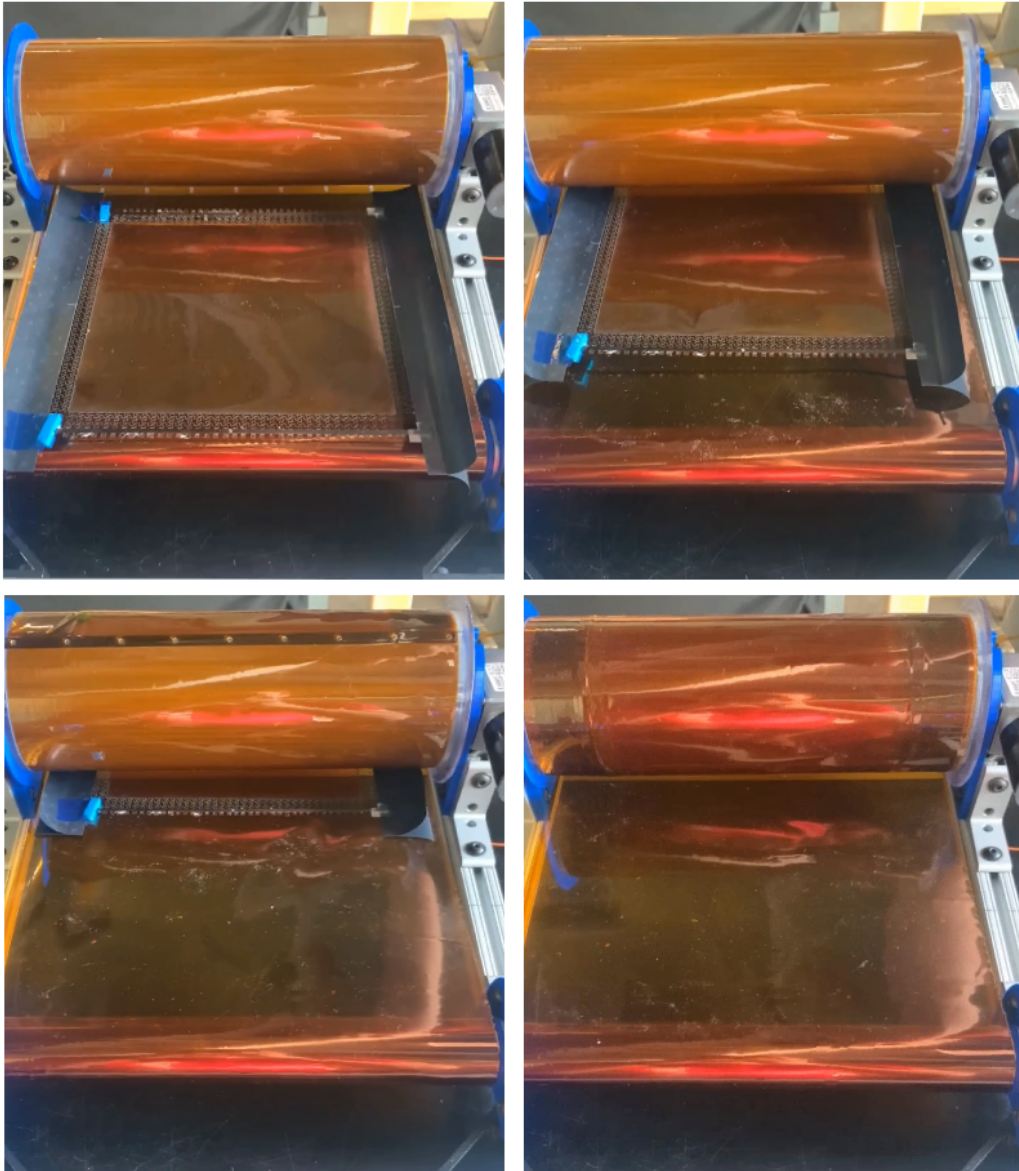


Figure 5.5: Packaging test of the composite unit cell with double s-springs using a 0.1 m mandrill. The top left image shows the initially deployed unit cell frame, while the bottom right shows the fully smoothly coiled final form.

spring was simulated numerically, as presented in the full paper along with other information, such as the folding of a physical prototype in greater detail[44].

The applications of this foldable structure range from antennas to solar arrays, and hence attaching to the toroidal shell to a flexible membrane to provide a surface for functional elements is crucial. In addition, prestressing the internal membrane is required to increase the overall structural stiffness, as well as to increase the surface

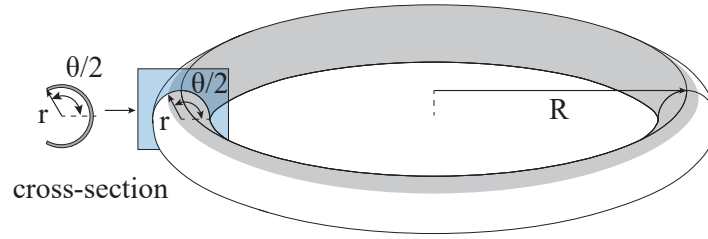


Figure 5.6: Geometry of toroidal tape-spring. The principal radii are R and r .

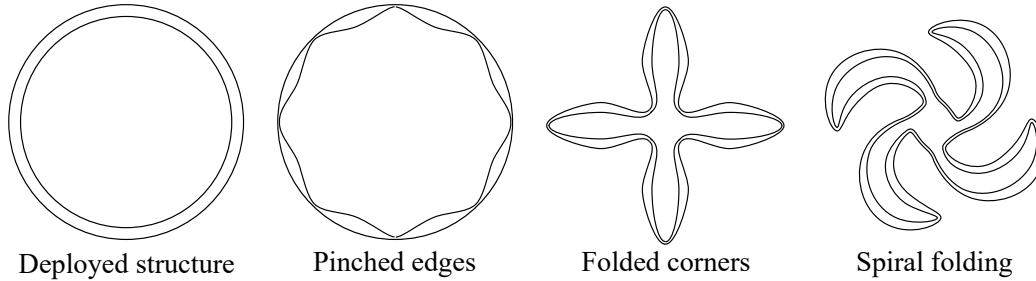


Figure 5.7: Toroidal tape-spring folding scheme.

accuracy. The selected material for this membrane was a sheet of $50 \mu m$ thick Kapton due to its availability and extensive use in space applications.

As described in the full paper, the folding and deployment sequences involve considerable deformation of the perimeter structure, along with whatever it supports (i.e. the membrane). Thus, attaching the membrane directly to the shell would result in either debonding between the two parts due to excessive strain or hindering of the folding due to an excessively stiff epoxy-based adhesive. Therefore, a compliant yet resilient attachment method is necessary. The solution was found by creating the two dimensional perforation in the Kapton sheet previously studied (double s-springs), which allows coupling between in-plane tensile strains, essentially acting as a multidirectional spring to distribute an even and consistent edge pretension, even under thermal loading. This pattern is obtained by laser cutting the Kapton sheet with the repeated double s-spring geometry shown in Figure 5.8, on the left.

This pattern is extremely similar in nature to the one presented in Chapter 2, with a slightly diminished size for the double s-spring unit cells farther from the composite edge such that the tessellated unit cells of the pattern form a circular arc instead of following the previously straight edge. As before, this pattern consists of a basic repeating unit where two curves with S shapes are intersecting at right angles. The unit cell is repeated both radially and circumferentially to construct a tessellation that

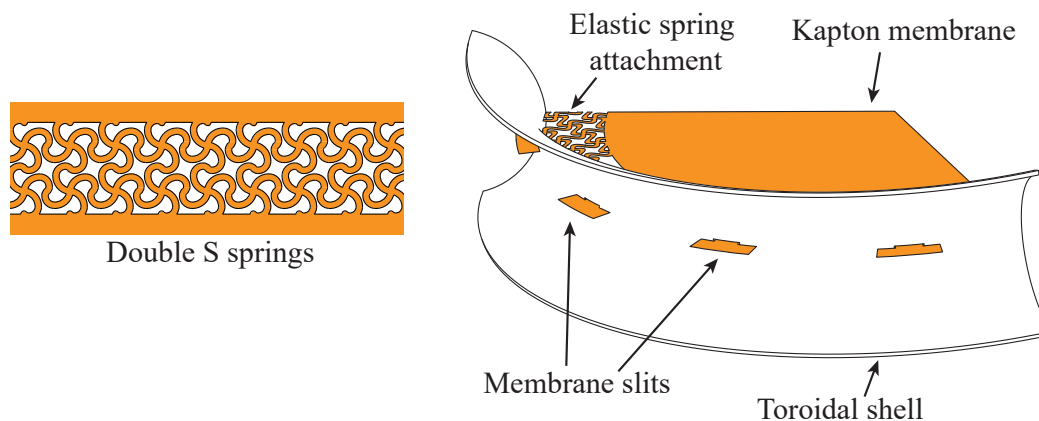


Figure 5.8: Diagram of slit attachment for Kapton membrane.

lies within the plane of the Kapton, and yet can accommodate large elastic strains in a post-buckling regime due to the in-plane bending and out-of-plane twisting of the ribbon-like elements. This applies a significant initial preload that stays approximately constant with increasing stretching. The S curves for the unit cell and size of the springs can be parametrized in multiple ways, thus changing the stiffness and maximum allowable stretch of this attachment system. The mechanical coupling between unit cells ensures a robust and durable geometry by distributing localized loads across multiple elements, preventing them from snapping, and maintaining uniform prestress even in the case of damage. The diagram in Figure 5.8 shows the double s-springs between two parallel lines, however as mentioned the unit cell is again modified to a curved boundary, as required in this case. The exact width and height of the outermost unit cell is 5 mm, and a total of five unit cells were cut in the radial direction to achieve sufficient elastic elongation in the membrane to survive the stretch incurred in the folding process. It should be noted that, in this case, there is not a significant amount of lateral shear forces imparted on the film relative to the edge, such as the ones that might be anticipated for the previous square unit cells, where the double s-spring's coupled 2-dimensional behavior creates additional advantages for using a pattern that handles strains in two dimensions. However, as can be seen in Figure 5.9, when a regular s-spring as opposed to double s-spring is used, even just the minor strain of flattening the composite rim prior to folding is enough to cause a tear. This highlights that the coupled nature of the double s-spring serves to distribute local force concentrations and act as a more robust and reliable method of attachment.

The Kapton membrane was cut to a total diameter of 505.5 mm, which is 2.1 mm

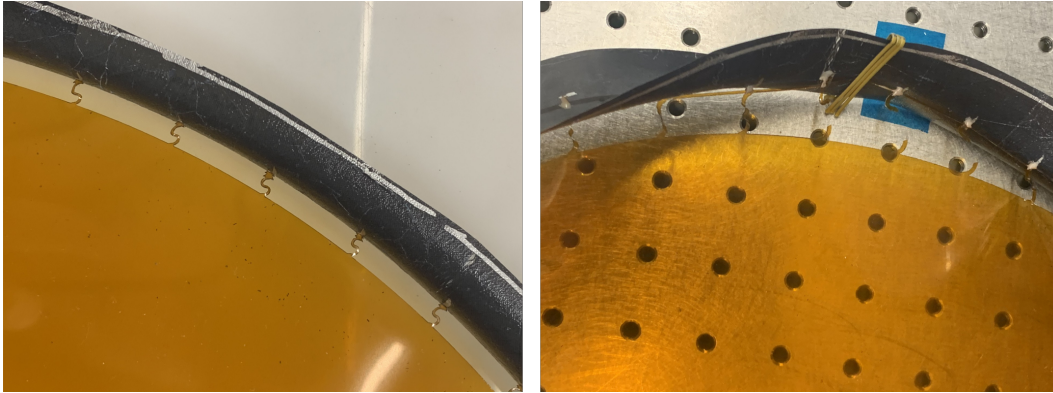


Figure 5.9: Demonstration that with only an s-spring as opposed to double s-spring, local forces are not distributed adequately, leading to even small strains causing failure.

smaller than the inner diameter of the toroidal ring. This was done to achieve a small stretch in the double s-springs, thus providing the aforementioned prestress in the fully deployed configuration. The membrane was then attached to the toroid by cutting small slots along the midplane of the toroid ($\theta = 0^\circ$) at intervals of 8 degrees, as shown in Figure 5.8, resulting in 7 spring elements per T-tab. The attachment is purely mechanical, and no adhesive is required. These slots could marginally hinder the performance of the composite laminate, and possibly act as crack initiators, but as they are located at the midplane of the toroid, which does not undergo any curvature change along the y-axis during flattening or folding and also has the least strain due to the negative Gaussian curvature of the tape-spring, this concern is largely mitigated. Thus, the influence of these slots is minimal in terms of acting as failure initiation points. No apparent reduction in stiffness was measured in the toroid after the slotting process.

The resulting prototype structure with the trimmed composite rim and the Kapton membrane is shown in Figure 5.10. The lack of wrinkles in the membrane shows the efficacy of the double S springs in applying a uniform prestress to the membrane and achieving a high surface accuracy, despite relatively large deformations in the springs themselves.

Folding of Structure

The deployment sequence of the complete structure was tested experimentally as well. The pinching at the 8 fold points was performed using hair pins and, to maintain the composite flat throughout the folding, 3D printed cylinders with a vertical rigid

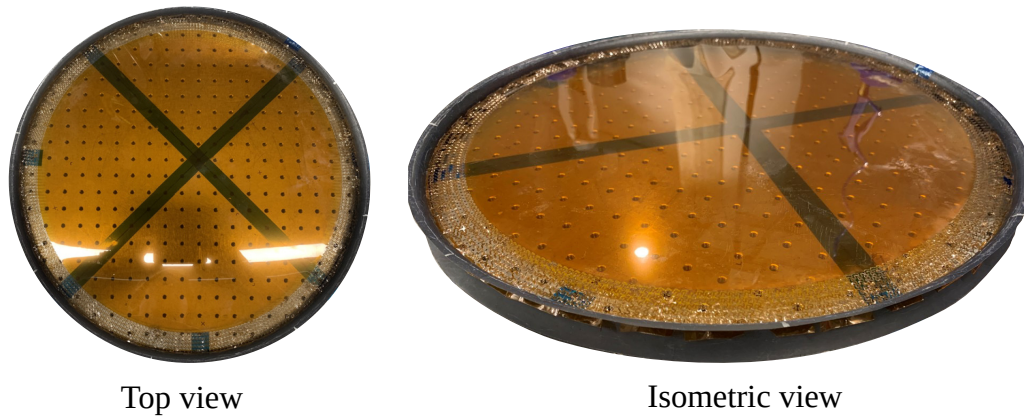


Figure 5.10: Toroid structure with attached membrane.

beam were placed at the pinched locations. The structure after pinching is depicted in the first snapshot in Figure 5.12 labeled as "Pinched". Next, the concave fold locations were pushed towards the center of the structure to create the cross shape.

This step had to be performed one fold at a time due to the general deformation of the toroidal shell. From a kinematic analysis, it is clear that if all four pinched locations are moved towards the center at once, the convex fold locations will move outward. This would cause a large extension of the membrane which, despite the large elastic strain capacity of the double s-springs, could result in damage of the membrane. Instead, moving one edge at a time ensures that this extension is small. In fact, when the concave folds are moved past their radial location shown at stage 3 folding, the convex fold locations had to be moved towards the center as well to accommodate the new cross shape.

After stage 4 folding, the convex folds were moved radially towards the center as well as circumferentially to achieve the spiral folding mentioned in Figure 5.7. At this point the circular envelope of the structure was 284 mm, as summarized in Table 5.1. This result, compared to the unfolded diameter of 533 mm, indicates a packaging efficiency of 53.2%, which for a small scale prototype is encouraging. In particular, the spiral folding was limited to approximately 45° due to the half meter size of the toroid. If the diameter were increased without changing other parameters of the composite laminate, namely thickness and cross section, the spiral folding efficiency could be increased by performing more turns of the folded arms around the center of the folded structure.

Observing the deployment sequence, after removing the cylinders at the convex folds

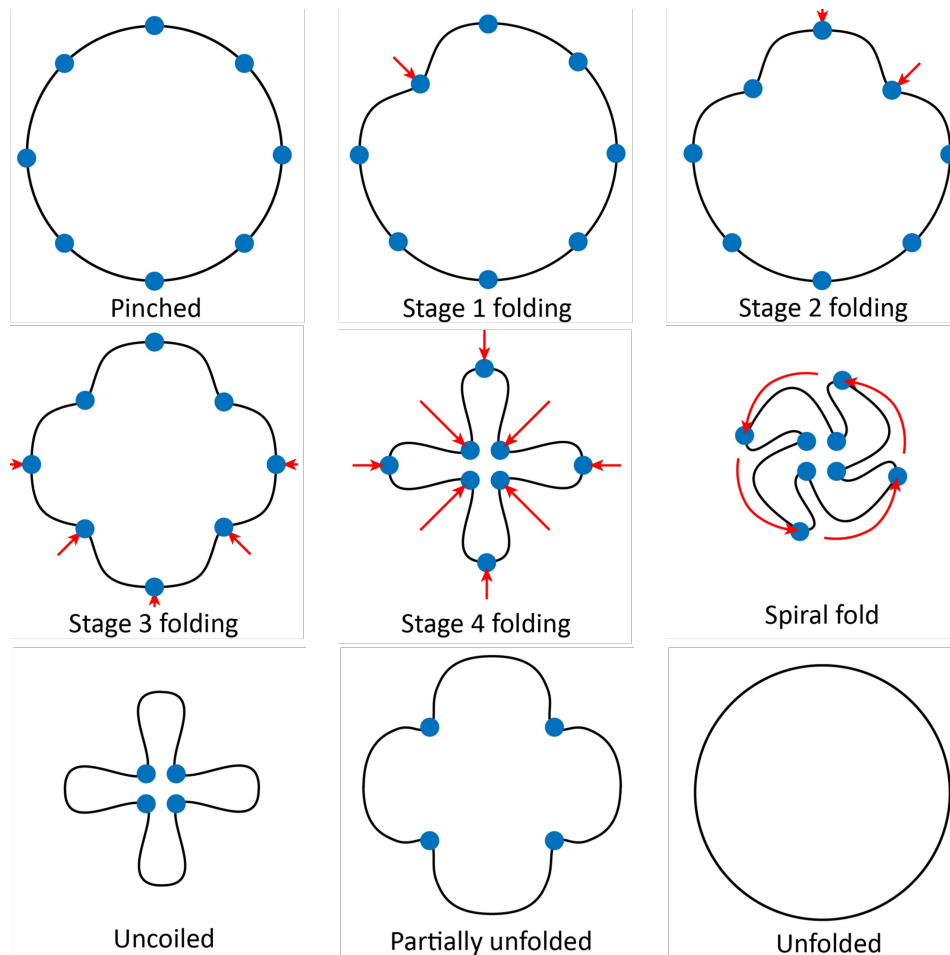


Figure 5.11: Folding and coiling sequence of toroid with membrane (visual).

Stage	Envelope diameter (mm)	Packaging efficiency (%)
Pinched	533	53.2
Stage 1 folding	533	
Stage 2 folding	477	
Stage 3 folding	477	
Stage 4 folding	387	
Spiral fold	284	
Uncoiled	428	
Partially unfolded	482	
Unfolded	533	

Table 5.1: Diameter of structure during folding and deployment.

the toroid returns to a shape similar to its stage 4 folding, despite an asymmetry due to slipping of the structure about one of the supporting blocks during the spiral fold. Unlike the stowing process, deployment can be done spontaneously by simply

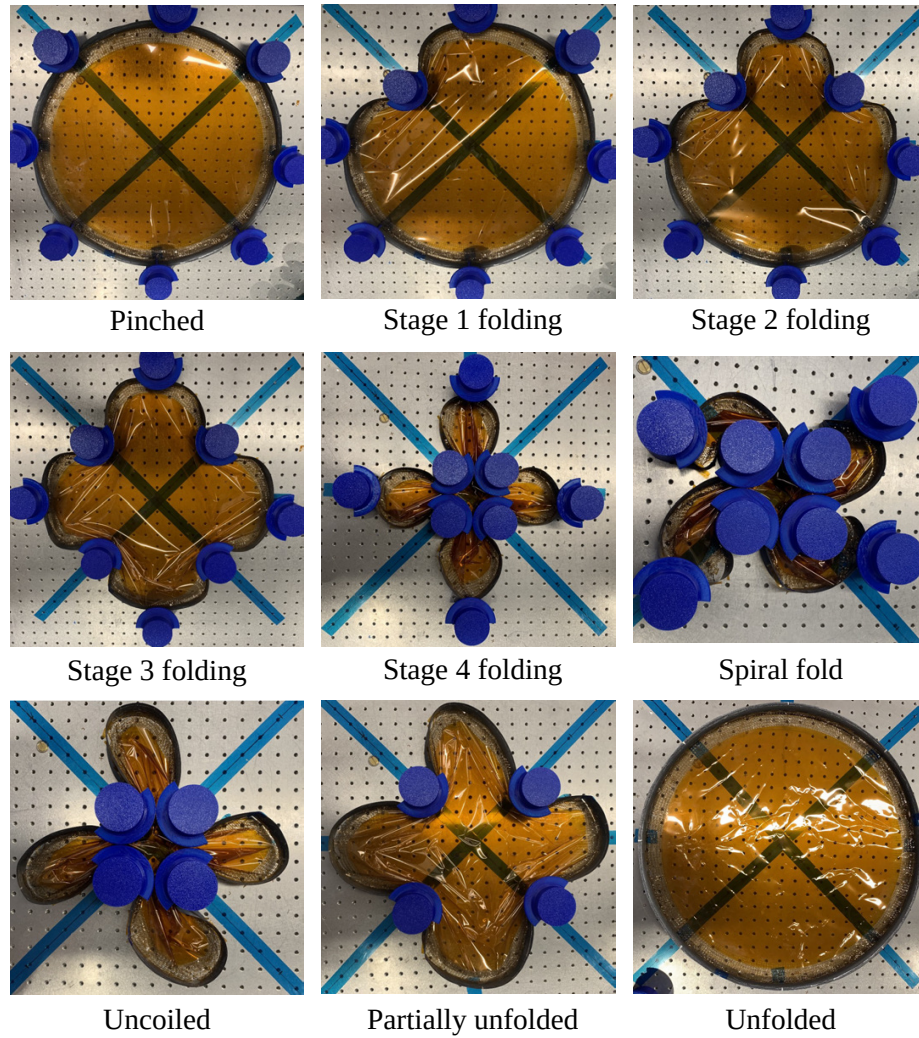


Figure 5.12: Folding and coiling sequence of toroid with membrane (experiment).

removing the cylinders. In addition, there appear to be localized buckles at the opposite-sense folds, possibly due to imperfections in the composite laminate that led to plastic buckles during the application of the large curvature changes. Finally, it is clear that random creases form throughout the membrane. Therefore, pre-creasing the membrane with an origami pattern that can adapt to the folding deformations is a crucial next step, which is studied further in the full paper[44].

5.4 Discussions

The work in this section successfully experimentally showcases that implementing double s-springs not only aids in solving existing deployment and packaging issues, but also opens the door for novel new deployable concepts that would have previously

been difficult to implement. Further optimization can also be done, similar to that mentioned in Chapter 2. For example, in the deployable toroid, the edges of the membrane that are pinched by the cylinders see much less local strain than the parts of the membrane along the edge that is in between them. Hence, less springs may be used in those areas, and some additional functional area recovered. Further experiments can also be constructed to verify the packaging efficiencies for both the toroid concept as well as the square unit cell as it is scaled in both size and number.

Several other experimental frames have been made for other qualitative investigations during this thesis as well. Two examples are using thinner membranes and a humidity experiment. The thinner membrane experiment showed qualitatively that double s-springs can work very well for a 25 micron thick film, but unit cell size needs to be further optimized in order to have a sufficiently high level of prestress. The humidity experiment consisted of placing a unit cell frame with a pretensioned Kapton membrane, similar to that described in Chapter 4, in ambient environments, and a box full of desiccant. Since Kapton is relatively hygroscopic, a loss or gain of pretension has been observed as mentioned before. In this experiment, it was qualitatively observed that the double s-springs are able to maintain adequate pretension in different humidity settings given sufficient initial prestrain.

Further prototypes are currently being innovated in the Space Structures Laboratory as well. Current work is focused on integration with the photovoltaic cells, and in the future with other electronics and the phased array transmitters. An image of a membrane with photovoltaic cells can be seen in Figure 5.13. These types of integrations will help further guide the optimization of how much prestress is necessary, as well as guide the development of double s-spring mounted membrane designs that may be locally adapted for functional features and testing of additional practical constraints.

All of the brief case studies in this section show scaling potential as well for future space flight projects. Aside from their versatility for other potential deployable designs, and given their reliability and advantages over previous methods, it is highly likely that the next version of a Space Solar Power Demonstrator will seek to test a structure with thin films present in both the deployment and active feature functionality. Additionally, the goal of the project necessitates larger surface areas than currently designed, hence, it is likely that double s-springs' capabilities will need to continue being expanded and tested with this purpose in mind. Thus, future prototypes will also be employed to investigate the behavior and performance of

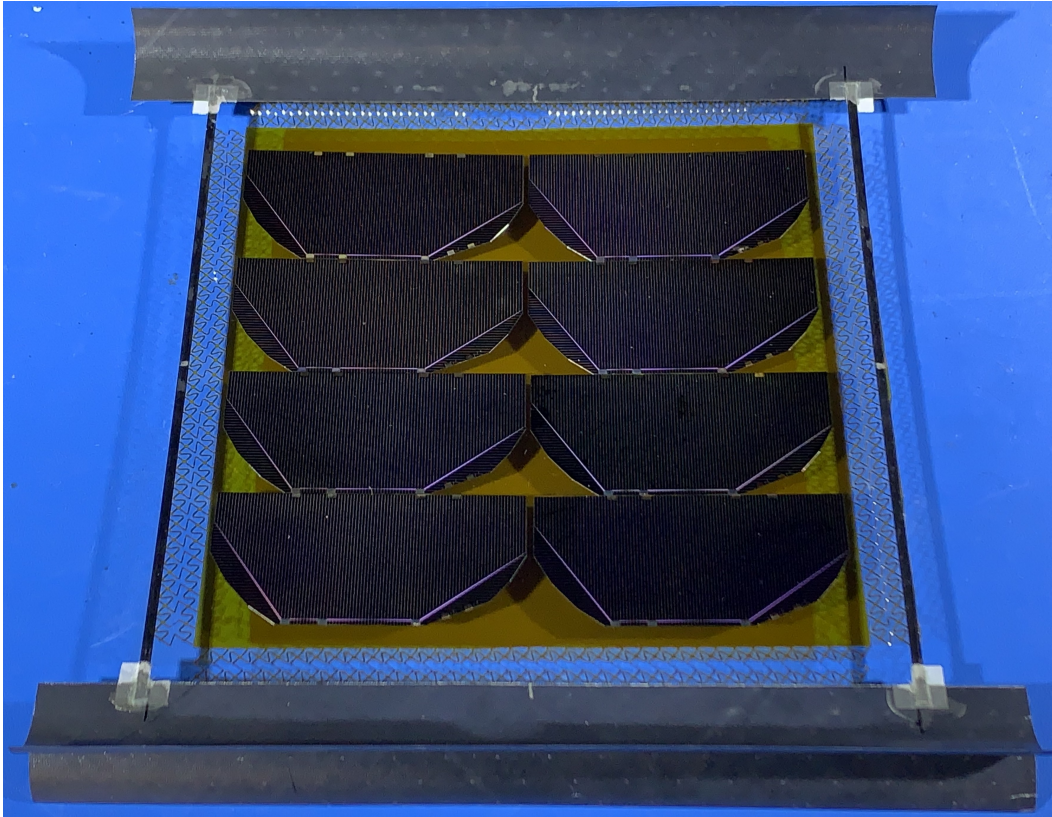


Figure 5.13: Solar cells mounted on a film supported by double s-springs.

double s-spring supported thin films in extreme environment scenarios, such as in radiation and more extreme thermal loading, as well as for larger unit cells.

Chapter 6

DISCUSSION AND CONCLUSIONS

6.1 Summary of Contributions

This thesis encompasses a relatively broad range of interconnected topics. The goal of this section is to outline how the research objective of demonstrating how to attach pretensioned thin films to thermally loaded deployable space structures with high surface stability and accuracy was met, connect the individual conclusions of each chapter, and in the next section explain what is still to be done to perfect the topics developed. For this, it is logically best to start with the conclusions of Chapter 3, where an analytic model of a rigid rod frame with elastic hinges was studied. In this section, two key conclusions were drawn. The first key conclusion is that for arbitrarily applied forces, the torsional buckling load can be calculated for the unit cell frame as a function of its side length and corner stiffness. This load will always be twice as high for internal follower forces as opposed to forces applied in a fixed direction, and moreover, this load is more practically defined as a critical average compressive load, which is also dependent only on the two variables, meaning that knowing this information fully defines the torsional buckling limit. This compressive load's invariance also means that the buckling of the frame depends solely on the frame and the manner in which it experiences loads, and not on how something is attached to it. Thus, it is beneficial to distribute the attachment on the frame evenly to comprehensively support the thin film, and reduce areal loss. However, the other key conclusion in Chapter 3 is as follows: attaching a membrane along an edge, as opposed to just the corners, applies significantly more force per temperature change, leading to thermally induced buckling to occur at much lower thermal strains, even if the membrane offers additional stiffness to the structure in torsion. This is of course, assuming a linear elastic material attached directly to the frame. Hence, this second conclusion highlights the need for a nonlinear, relatively constant force attachment method in order to optimize both the structure's capacity to avoid torsional buckling and areal efficiency.

Chapter 4 expanded the conclusions derived in Chapter 3. It first showed experimentally the need for careful integration of thin films to practical structures, and how these structures can be created. It then transitioned to showing numerically

that the lessons taken from studying a simplified structure made of rigid rods and elastic hinges could be transferred to a more realistic orthotropic high fidelity representation of the unit cell frame. By examining the bending stiffness of the structure diagonally, it is possible to calculate a close bound on the torsional buckling load for the various force application cases previously studied in detail in Chapter 3. This means that the model developed in Chapter 3 offers relatively accurate predictions of the buckling load, and both the simplified structure and high fidelity studies can be used in tandem to inform an optimal construction.

Having validated the lessons presented in Chapter 3, it is logical to return now to the double s-spring pattern studied in Chapter 2. Having found a method to determine the level of pretension acceptable in a structure, as well as the best way to support thin films from the perspective of the frame, double s-springs offer a realistic tunable solution. In this chapter, the main focus was not just on the design and influence of parameters on the performance of the spring, but also on creating robust and efficient simulation tools to be able to iterate and design the double s-spring for general application cases. Specifically, the double s-spring was simulated using CONN3D2 connectors that were able to sufficiently capture its nonlinear behavior, and allow for the optimization of the springs' orientations locally. This model was validated to a high fidelity model as well. Additionally, this section contained a practical demonstration of the double s-spring's ability to maintain high surface accuracy and avoid wrinkling by measuring the surface accuracy using DIC. These results showed that this attachment style kept the surface variability to under 100 microns, even close to the attachments. This was also later validated using the numerical model developed.

Chapter 5 then completes the story contained in this thesis by demonstrating that not only do double s-springs work in specific, theoretical, or controlled scenarios, but they are also a practical way to implement thin films into a spacecraft structure. This chapter primarily demonstrated their ability for reliable deployment where previous methods had potential failures, as well as their durability and efficiency in packaging scenarios. It also explored the fact that their uses are not limited to just square unit cell architectures. Their unique high strain capacity (and coupled nature that distributes local force concentrations) allows them to permit highly desirable novel deployment architectures, such as a folding toroid. One significant advantage of the toroid is that due to its symmetry, effects like potential torsional buckling of the frame are circumvented. The folding of the toroid is demonstrated experimentally,

and it is discussed as well that as this architecture scales up, it becomes a significantly efficient packaging method.

Putting the lessons learned together, Figure 6.1 shows the *pièce de résistance* of this thesis. This frame looks completely normal, and that is the point, and what makes it special. Even though the film has been adequately prestressed, there are no wrinkles in it, or torsional twists in the frame. This is the most important contribution found in this thesis, since it culminates all of the previous studies into a single practical demonstration. This frame unit cell was assembled using a highly efficient and repeatable technique. Since the force-displacement was well categorized previously, the film was mounted using a displacement technique (pins in-between the spring elements) to hold the edges, and sandwiched to the composite frame. This displacement control method results in more even prestressing, not requiring prior bonding to the frame components, and allows for more points to control the film with. This method is also less time consuming than the pulley approach, and easier to scale. The biaxial prestress for this film is about 2 N/m, which is just below the follower buckling load estimated for this frame.

The understanding of the unit cell buckling developed in this thesis, as well as the contribution of the double s-springs and their modeling, lays the foundation for ambitious and novel projects like SSPP to not only be achievable, but viable. Without this work, key functional components such as phased arrays simply cannot be utilized. This technology also provides potential benefits in other applications, such as preventing effects like jittering in highly sensitive telescopes, increasing the area of deployable structures, and thermally decoupling locally active components from adjacent ones.

6.2 Future Work

While many things were achieved and demonstrated in this thesis, there is even more work left to be done to perfect and fully understand the content addressed before everything is optimized. This section will discuss future work as it pertains to several different key areas. This section both reiterates some previously discussed topics, and discusses needs that arise when synthesizing the individual sections into one big picture.

Reduced Order Modeling

Starting with the concept of optimization in this discussion is logical, since it is the natural next step to combine all of the tools that were developed as part of this

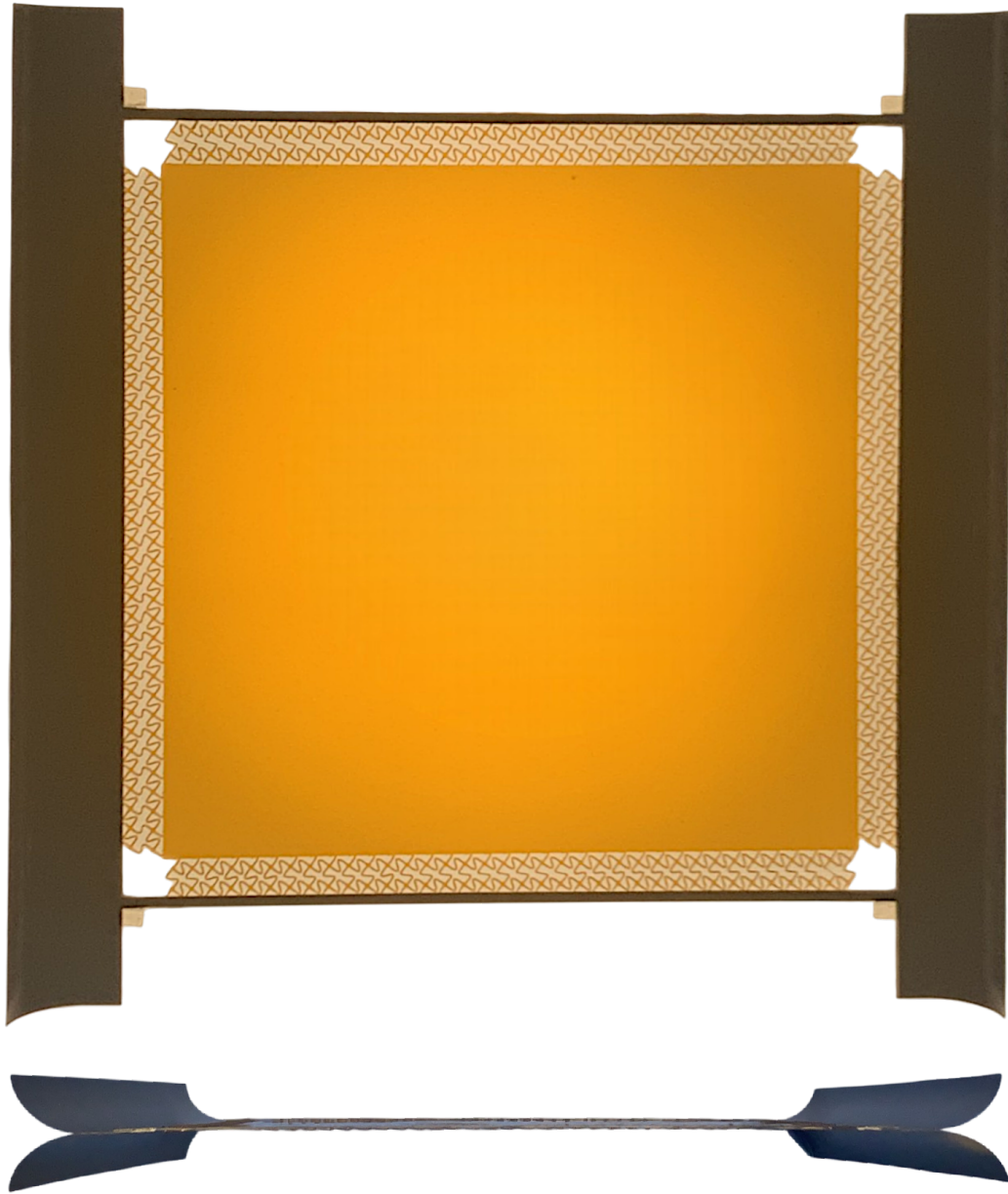


Figure 6.1: Top and side view of a prestressed film supported by double s-springs on a composite frame that has no torsional buckle.

thesis. Several simplified and reduced order models were created; the two primary ones being the beam and MPC hinge numerical model for the frame unit cell, and the connector element model used to represent the double s-springs. The former is used to analyze just the composite frames, while the latter is intended to help model the effects of the connection scheme on the mounted film. Combining these two models into one would allow the study of the complete system in a much more

computationally efficient fashion. As mentioned in Chapter 2, using the full coupled multidimensional behavior of the double s-springs is potentially problematic with CONN3D2 elements from a standpoint of convergence. Hence, the combined model would also have to be analyzed and validated carefully in order to be trusted. Similarly, the longerons and battens of the frame model have thicknesses and offsets in the physical world, and are not simply one dimensional. Hence, these affects have to be studied before they can omitted fully. As seen in Chapter 4 when examining the effects of different boundary conditions, mounting the film in different ways may also potentially affect the way the longerons in particular behave, and thus change the effective stiffness of the frame.

Following a robust and verified model, the next step is to actually utilize this model. While a flat, non-buckled frame was achieved experimentally using some of the results obtained, there has not yet been further work showing that this geometry is actually the optimal one. Perhaps the strain range, toughness, or some other element can be drastically improved still, especially considering the factors like unit cell size not having been fully studied yet. This also entails determining the constraints for optimization, since there are tradeoffs as stated before. It is not yet clear for the SSPP project or other possible applications which elements are the most important, or which can perhaps be sacrificed further (e.g. less strain capacity in exchange for a smaller unit cell that allows for more area for functional components).

Lastly, the current reduced order models are based primarily on set material properties and quasistatic force conditions. Properties of soft polymers like Kapton are subject to change with temperature and other external factors, which the model does not yet account for. While some thermal strain and loading scenarios were studied, the reduced order models were not rigorously studied and compared to full fidelity models under temperature changes, or with varying amounts of prestrain. More research needs to be done to finish validating these models under mixed loading conditions.

Unit Cell Frame

The unit cell frame, made of TRAC longerons and battens, is perhaps the most well studied part of the SSPP project in the literature in terms of its components and their individual peculiarities. However, this thesis did not investigate specifically how to ensure that these components provide the maximum torsional stability, and at what point another failure mode might occur (another eigenmode of the frame becomes

the new lowest eigenvalue). Thus, the overall architecture would benefit incredibly from utilizing the high fidelity model of the fully linear elastic frame to investigate what adjustments can be made to improve the torsional stiffness, or if a different approach, such as using hollow deployable battens or closed cross section longerons offers worthwhile tradeoffs.

Additionally, compared to the simplified frame model, two key elements were not verified using the high fidelity model. First, Case D was not addressed due to the additional computational time this would have taken. Similarly, some analytic results, like the buckling of a rectangular frame, have not been studied using numerical methods yet either. Second, a significant point of the rigid rod and elastic hinge model was the study of post-buckling deformations, both numerically and experimentally. This was not done with the high fidelity model, and is necessary in order to adequately combine the reduced order models presented as described above.

As stated in the Chapter 4 Discussion, the unit cell frames have also not been studied extensively for imperfections. Imperfections are difficult to avoid in composite manufacturing, and have been previously shown to significantly affect buckling behavior, specifically in TRAC longerons. As such, imperfections, for both an initial twist, and local effects on the flanges should be well understood in order to develop a robust model.

Finally, the affects of additional structural elements should be studied for their impact on torsional buckling of the unit cell frame. As mentioned previously, multiple unit cell frames tend to exacerbate post-buckling displacements. In earlier studies, it was also determined that a thicker membrane can significantly raise the buckling force. However, this was assuming a direct bond. It is possible that for using an edge attachment scheme that is discretized locally, such as double s-springs, diminishes this effect's potency.

Scaling

One of the key visions for making the SSPP concept viable is creating a $60\text{ m} \times 60\text{ m}$ structure. This means that all aspects of the structure must be scaled from where they are now, starting with the unit cells. Currently, the strain range of region 2 of the double s-springs ($\sim 2\text{ mm}$), is adequate for a $1\text{ m} \times 1\text{ m}$ unit cell with a Kapton film to undergo $\sim 200^\circ\text{C}$ of temperature change. Hence, for larger structures, which will necessitate larger unit cells, this value will need to be tuned further. Likewise, the models and theories developed that have been validated for smaller unit cells,

will require additional validation as they are scaled up, and elements such as the Euler buckling load of the frame components are approached.

The physical construction of the attachment and manufacturing of the double s-spring film also needs to be further innovated and scaled. In Figure 6.1, the attachment of the springs to the frame is not visible, but consists of a single strip in order to more easily stretch and pretension the edge prior to bonding. A single strip like this in a larger structure may result in causing a distributed stress due to the CTE mismatch between it and the composite web of the TRAC longeron. This could lead to debonding, or induce curvatures, similar to a bimetallic strip. The current manufacturing process for the patterned film also involves laser cutting the pattern. While this is an efficient method from the perspective of prototyping lab scale models, for patterning a single $1\text{ m} \times 1\text{ m}$ film edge, this would take several hours. Hence, developing a more scalable manufacturing method, such as stamping the pattern through the film with a steel hole punch, would be advantageous.

Double S-Spring Pattern and Film Attachment

The double s-spring pattern and film attachment in general is the area with the broadest amount of future work remaining. Similar to the unit cell frames, thermal strain remains the major concern of the SSPP project, and an area requiring more rigorous study in the film itself. This applies for both numerical studies, and experimental validation. Specifically, the biaxial prestraining of the film needs to be validated experimentally. This is a difficult process, since DIC on the transparent film itself is challenging, the large size of the film requires techniques such as scanning DIC to properly study the full area undergoing prestrain, and unlike measuring the surface offsets, it also requires a reference measurement before the film is mounted. Additionally, observing phenomena such as thermal buckling also necessitates a large enough thermal chamber, and a method to properly gravity offload the frame, which adds a tradeoff between over-constraining the motion of the unit cell and making DIC easier. Kapton is also a relatively hygrothermal material as mentioned, meaning that in addition to thermal effects, humidity should also be considered and examined.

In the previous studies conducted, the film was not shown to wrinkle. However, as the structure is scaled up, there is no guarantee that this remains the case, and as such, the limitation of wrinkling, specifically due to potential shear effects, should be determined, and mitigated using the chirality of the springs if possible. The

chirality is an additional point of remaining optimization, as it has already been shown in Chapter 2 that using at least two different spring directions leads to a more uniform and biaxial prestress distribution.

One major concern of this attachment style is its longevity and durability. This was a major consideration in the original design of the double s-spring. While double s-springs have lasted hundreds of stretch cycles without accumulating cyclic damage, a mission to LEO would incur tens of thousands of thermal cycles over the period of just a few years. Thus, it is vital to ensure that the plasticity effects studied in Chapter 2 do not lead to a physical failure or loss of pretension overtime.

While double s-springs are a particularly effective and simple design, it is certainly not the only design possible. Other cut patterns can be created that also result in twist like and ribbon like behavior. Additional parameters, such as perhaps a triple s-spring or a double s-spring with unequal S 's (i.e. if there is a bias in thermal expansion to one principal axis), can also be envisioned. In the foldable toroid, a slight change in the aspect ratio of each unit cell allowed it to be tessellated in a circle. Analogously, this could also be done for local features in a structure, such as an active component situated along the edge. Studying the performance of the system after all components are implemented, such as the solar cells in Figure 5.13, is key to gaining a complete understanding.

As a final note, the material used, Kapton, is primarily a surrogate to model a thin film that many space structures might use or mount components to. Other variants of it, such as CP1 polyamide, exist with properties such as low moisture absorption, better uniformity, and high transparency that may improve the performance of the film and reduce the requirements placed on the double s-springs[73]. Especially for alternative applications outside of the SSPP project, other materials in general such as Mylar may also be preferred or required. As these materials and applications shift, the attachment style used must adapt too.

6.3 Closing Remarks

Simplicity and efficiency are two key ingredients for elegance. As such, this thesis hopes to provide an elegant understanding of and solution to a problem in pushing the fundamental limits of ultralight deployable space structures. Utilizing numerical, analytical, and experimental models to predict and eliminate thermal effects and buckling in composite structures is the only way that larger structures can continue to scale. While many details remain to be further developed, a practical solution to

a problem that will become more and more common has been shown to exist, and this thesis hopefully provides a step in the right direction in making next generation space structures not only viable, but practical.

BIBLIOGRAPHY

- [1] D.A. Türk and S. Pellegrino. Parametric design of conforming joints for thin-shell coilable structures. *AIAA Scitech 2019 Forum*, page 1259, 2019. doi: <https://doi.org/10.2514/6.2019-1259>.
- [2] A Haraszti, N.H. Reddy, and S. Pellegrino. Dynamic deployment of foldable composite structures with pre-tensioned springs. *AIAA SCITECH 2024 Forum*, page 1842, 2024. doi: <https://doi.org/10.2514/6.2024-1842>.
- [3] Gokhan Kiper and Eres Soylemez. Deployable space structures. In *2009 4th International conference on recent advances in space technologies*, pages 131–138. IEEE, 2009.
- [4] Koryo Miura and Sergio Pellegrino. *Forms and Concepts for Lightweight Structures*. Cambridge University Press, Cambridge, 2020. ISBN 978-0-521-43274-0. doi: 10.1017/9781139048569.
- [5] X Ma, T Li, J Ma, Z Wang, C Shi, S Zheng, Q Cui, X Li, F Liu, H Guo, and L Liu. Recent advances in space-deployable structures in china. *Engineering*, 17:207–219, 2022. doi: <https://doi.org/10.1016/j.eng.2022.04.013>.
- [6] M Card and W Boyer. Large space structures-fantasies and facts. In *21st Structures, structural dynamics, and materials conference*, page 674, 1980.
- [7] Brandi N McPherson and Jeffrey L Kauffman. Dynamics and estimation of origami-inspired deployable space structures: A review. In *AIAA Scitech 2019 forum*, page 0480, 2019.
- [8] Suraj P Rawal, Mohan S Misra, and Robert G Wendt. Composite materials for space applications. Technical report, National Aeronautics and Space Administration, 1990.
- [9] E Grossman and I Gouzman. Space environment effects on polymers in low earth orbit. *Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms*, 208:48–57, 2003.
- [10] Charles Atkinson, Jonathan Arenberg, Larry Gilman, Ted Messer, Patty May, Dave Moon, Kevin Patton, Jim York, Stan Backovsky, Jim Tucker, et al. Technology demonstration of large stable cryogenic composite structures for jwst. In *UV/Optical/IR Space Telescopes: Innovative Technologies and Concepts III*, volume 6687, pages 13–23. SPIE, 2007.
- [11] William K Wilkie. Overview of the nasa advanced composite solar sail system (acs3) technology demonstration project. In *AIAA scitech 2021 forum*, page 1260, 2021.

- [12] A Fikes, M Gal-Karziri, E Gdoutos, M Kelzenberg, E Warmann, R Madonna, H Atwater, A. Hajimiri, and S. Pellegrino. The caltech space solar power project: Design, progress, and future direction. *Proc. IEEE WiSEE Space Sol. Power Workshop*, pages 1–5, 2022.
- [13] Lee Feinberg, John Ziemer, Megan Ansdell, Julie Crooke, Courtney Dressing, Bertrand Mennesson, John O’Meara, Joshua Pepper, and Aki Roberge. The habitable worlds observatory engineering view: status, plans, and opportunities. In *Space Telescopes and Instrumentation 2024: Optical, Infrared, and Millimeter Wave*, volume 13092, pages 548–559. SPIE, 2024.
- [14] Les Johnson and Frank Curran. Solar cruiser technology maturation plans. *NASA Report*, 2020.
- [15] Oren S Mizrahi, Phillip Jahelka, Eleftherios Gdoutos, Jesse Brunet, Alex Ayling, Austin Fikes, Ailec Wu, Richard Madonna, Harry A Atwater, Sergio Pellegrino, et al. Space solar power generation: A viable system proposal and technoeconomic analysis. *Joule*, 9(6), 2025.
- [16] Volkan Gurses, Samantha I Davis, Raju Valivarthi, Neil Sinclair, Maria Spiropulu, and Ali Hajimiri. An on-chip phased array for non-classical light. *Nature Communications*, 16(1):6849, 2025.
- [17] H. M. Y. Chinthaka Mallikarachchi and Sergio Pellegrino. Dynamic Analysis of Large Flat-foldable Arrays – Natural Frequency Bounds and Influence of Prestress. In *AIAA SCITECH 2025 Forum*. American Institute of Aeronautics and Astronautics, 2025. doi: 10.2514/6.2025-1194. _eprint: <https://arc.aiaa.org/doi/pdf/10.2514/6.2025-1194>.
- [18] Todd Helwig. Lateral torsional buckling and torsional bracing of steel girder systems. *Proceedings of the Annual Stability Conference*, 2025.
- [19] Masahiko Murozono and Earl A Thornton. Buckling and quasistatic thermal-structural response of asymmetric rolled-up solar array. *Journal of spacecraft and rockets*, 35(2):147–155, 1998.
- [20] Steven A Hawley. Hubble space telescope solar array concerns and consequences for servicing mission 2. *Journal of Spacecraft and Rockets*, 53(1): 15–24, 2016.
- [21] Bernard Budiansky. Post-buckling behavior of cylinders in torsion. In *IUTAM SYMP. ON THE THEORY OF THIN SHELLS*, number NASA-CR-88232, 1967.
- [22] Earl A Thornton. Thermal buckling of plates and shells. *American Society of Mechanical Engineers*, 1993.

- [23] Matthew K Chamberlain, Steven H Kiefer, and Jeremy Banik. On-orbit structural dynamics performance of the roll-out solar array. In *2018 AIAA spacecraft structures conference*, page 1942, 2018.
- [24] John M Pederson and Sergio Pellegrino. Thermal deformation of ultrathin composite structures in a vacuum environment. In *AIAA SCITECH 2024 Forum*, page 0412, 2024.
- [25] W. Luo and S. Pellegrino. Buckling analysis of a ladder deployable structure supporting a prestressed film. *AIAA SCITECH 2023 Forum*, page 0366, 2023. doi: <https://doi.org/10.2514/6.2023-0366>.
- [26] Wen Luo and Sergio Pellegrino. Stability of torsionally soft, deployable structures supporting prestressed membranes. *Journal of Aerospace Engineering*, 38(5):04025069, 2025.
- [27] Baoyan Duan, Yiqun Zhang, and Jingli Du. *Large deployable satellite antennas*. Springer, 2020.
- [28] Devashish Dubey, Anooshe Sadat Mirhakimi, and Mohamed A Elbestawi. Negative thermal expansion metamaterials: a review of design, fabrication, and applications. *Journal of Manufacturing and Materials Processing*, 8(1): 40, 2024.
- [29] Abaqus User Manual. Abaqus user manual. *Abacus*, 1:1, 2020.
- [30] Osamu Mori, Hirotaka Sawada, Ryu Funase, Mutsuko Morimoto, Tatsuya Endo, Takayuki Yamamoto, Yuichi Tsuda, Yasuhiro Kawakatsu, Jun'ichiro Kawaguchi, Yasuyuki Miyazaki, et al. First solar power sail demonstration by ikaros. *Transactions of the Japan Society for Aeronautical and Space Sciences, Aerospace Technology Japan*, 8(ists27):To_4_25–To_4_31, 2010.
- [31] David A Spencer, Bruce Betts, John M Bellardo, Alex Diaz, Barbara Plante, and Justin R Mansell. The lightsail 2 solar sailing technology demonstration. *Advances in Space Research*, 67(9):2878–2889, 2021.
- [32] John Moery. Development of robust thermo-optical thin-film membranes for the james webb space telescope sunshield. In *Space Telescopes and Instrumentation I: Optical, Infrared, and Millimeter*, volume 6265, pages 1086–1095. SPIE, 2006.
- [33] Robert J Walters, JC Garner, SN Lam, JA Vasquez, WR Braun, RE Ruth, SR Messenger, JR Lorentzen, R Bruninga, PP Jenkins, et al. Materials on the international space station—forward technology solar cell experiment. *Materials Science and Engineering: B*, 116(3):257–263, 2005.

- [34] Rory Barrett, Doug Campbell, Mark Lake, Larry Adams, Robert Taylor, Erik Abrahamson, Mark Scherbarth, Jeffry Welsh, Gregg Freebury, Neal Beidleman, et al. Development of a passively deployed roll-out solar array. In *4th International Energy Conversion Engineering Conference and Exhibit (IECEC)*, page 4011, 2006.
- [35] James Pearson and Robert Romanofsky. Thin film antenna development and optimization. In *47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference 14th AIAA/ASME/AHS Adaptive Structures Conference 7th*, page 2229, 2006.
- [36] Michael Piszczor. Thin film photovoltaic blanket & array technology development within nasa. In *1st International Energy Conversion Engineering Conference (IECEC)*, page 5921, 2003.
- [37] Diogo Galhofo, Nuno Silvestre, Augusto Moita de Deus, Luis Reis, Antonio PC Duarte, and Rita Carvalho. Structural behaviour of pre-tensioned solar sails. *Thin-Walled Structures*, 181:110007, 2022.
- [38] Harry A Atwater, Artur R Davoyan, Ognjen Ilic, Deep Jariwala, Michelle C Sherrott, Cora M Went, William S Whitney, and Joeson Wong. Materials challenges for the starshot lightsail. *Nature materials*, 17(10):861–867, 2018.
- [39] Jack Leifer, Jonathan Black, W Belvin, and Vaughn Behun. Evaluation of shear compliant borders for wrinkle reduction in thin film membrane structures. In *44th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, page 1984, 2003.
- [40] Jack Leifer. Simplified computational models for shear compliant borders in solar sails. *Journal of Spacecraft and Rockets*, 44(3):571–581, 2007.
- [41] Midori Isobe and Ko Okumura. Initial rigid response and softening transition of highly stretchable kirigami sheet materials. *Scientific reports*, 6(1):24758, 2016.
- [42] Ahmad Rafsanjani and Katia Bertoldi. Buckling-induced kirigami. *Physical review letters*, 118(8):084301, 2017.
- [43] DuPont. *DuPont Kapton Summary of Properties*, 2022.
- [44] Federico Benazzo, George A Popov, Juan Mejia-Ariza, and Sergio Pellegrino. Deployable composite toroid holding a circular prestressed membrane. In *AIAA SCITECH 2026 Forum*, 2026.
- [45] J. Banik and P. Hausgen. Roll-out solar arrays (rosa): Next generation flexible solar array technology. *AIAA SPACE and Astronautics Forum and Exposition*, page 5307, 2017. doi: <https://doi.org/10.2514/6.2017-5307>.

- [46] E Rodgers, E Gertsen, J Sotudeh, C Mullins, A Hernandez, H Le, P. Smith, and N. Joseph. Space based solar power. *NASA Report*, 2024.
- [47] A Mukherjee, G. Risso, and 2023. Ermanni, P. Quantifying the strength of stability of multi-stable structures: A new design perspective. *Thin-Walled Structures*, 189, 2023. doi: <https://doi.org/10.1016/j.tws.2023.110921>.
- [48] Don O. Brush and Bo O. Almroth. *Buckling of Bars, Plates, and Shells*. McGraw-Hill Book Company, US, 1 edition, 1975.
- [49] Robert Millard Jones. *Buckling of Bars, Plates, and Shells*. Bull Ridge Corporation, Blacksburg, Virginia, illustrated edition, 2006.
- [50] Manan Arya, Nicolas Lee, and Sergio Pellegrino. Ultralight structures for space solar power satellites. In *3rd AIAA Spacecraft Structures Conference*, page 1950, 2016.
- [51] Christophe Leclerc and Sergio Pellegrino. Nonlinear elastic buckling of ultra-thin coilable booms. *International Journal of Solids and Structures*, 203: 46–56, 2020.
- [52] Christophe Leclerc, Lee L Wilson, Miguel A Bessa, and Sergio Pellegrino. Characterization of ultra-thin composite triangular rollable and collapsible booms. In *4th AIAA spacecraft structures conference*, page 0172, 2017.
- [53] Fabien Royer, John W Hutchinson, and Sergio Pellegrino. Probing the stability of thin-shell space structures under bending. *International Journal of Solids and Structures*, 257:111806, 2022.
- [54] Chao Xie, Zhiyi Wang, Yu Liu, Feng Gao, Li Qin, Jinping Yang, Fujun Peng, and Wujun Chen. Design and demonstration of a roll-out membrane antenna based on thin-walled deployable composite booms. *Thin-Walled Structures*, 205:112486, 2024.
- [55] Xiang Liu and Guoping Cai. Review of dynamics and active control of large-scale space membrane antenna. *Astrodynamics*, 8(1):1–26, 2024.
- [56] Sergio Pellegrino, Harry A Atwater, Seyed Ali Hajimiri, Eleftherios E Gdoutos, Christophe Leclerc, Fabien A Royer, and Antonio Pedivellano. Coilable thin-walled longerons and coilable structures implementing longerons and methods for their manufacture and coiling, April 25 2023. US Patent 11,634,240.
- [57] The Composites Store. Comparison data for pultruded shapes, Jan 2011.
- [58] Mitsubishi Chemical Carbon Fiber and Composites. Mitsubishi pyrofil mr, Jan 2023.
- [59] JPS Composite Materials. Physical properties of e-glass fiber, Jul 2023.
- [60] North Thin Ply Technology. Ntpt tp415 data sheet, Oct 2019.

- [61] Meital O Carmi and Sergio Pellegrino. Manufacturing imperfections of open cross-section deployable thin-shell composite structures. In *AIAA SCITECH 2025 Forum*, page 0189, 2025.
- [62] Tristan Faschinger. Integration of thin-film photovoltaics into ultralight deployable space structures. Master’s thesis, ETH Zurich, 2022.
- [63] Charles Sommer, Eleftherios Gdoutos, Alan Truong, Alexander Wen, Jonathan F Sauder, Richard Madonna, and Sergio Pellegrino. Deployable on-orbit ultra-light composite experiment (dolce) on the caltech space solar power demonstration 1 (sspd-1) mission. In *AIAA SCITECH 2025 Forum*, page 1007, 2025.
- [64] Eleftherios Gdoutos, Charles F Sommer, Alan Truong, Alexander Wen, Antonio Pedivellano, Kanthasamy Ubamanyu, Richard G Madonna, and Sergio Pellegrino. Development of the deployable on-orbit ultralight composite experiment (dolce) for the space solar power project demonstration mission. In *AIAA SciTech 2022 Forum*, page 1266, 2022.
- [65] Austin Fikes, E Gdoutos, M Klezenberg, Alex Ayling, O Mizrahi, Jonathan Sauder, C Sommer, A Truong, Alexander Wen, A Wu, et al. The caltech space solar power demonstration one mission. In *2022 IEEE International Conference on Wireless for Space and Extreme Environments (WiSEE)*, pages 18–22. IEEE, 2022.
- [66] Eleftherios Gdoutos, Alan Truong, Antonio Pedivellano, Fabien Royer, and Sergio Pellegrino. Ultralight deployable space structure prototype. In *AIAA Scitech 2020 Forum*, page 0692, 2020.
- [67] Narravula Harshavardhan Reddy and Sergio Pellegrino. Dynamics of the Caltech SSPP deployable structures: structure–mechanism interaction and deployment envelope. In *AIAA SCITECH 2023 Forum*. American Institute of Aeronautics and Astronautics, 2023. doi: 10.2514/6.2023-2065. _eprint: <https://arc.aiaa.org/doi/pdf/10.2514/6.2023-2065>.
- [68] Thomas W Murphey, William Francis, Bruce Davis, and Juan M Mejia-Ariza. High strain composites. In *2nd AIAA spacecraft structures conference*, page 0942, 2015.
- [69] Simon David Guest. *Deployable structures: concepts and analysis*. PhD thesis, University of Cambridge Cambridge, UK, 1994.
- [70] Sergio Pellegrino. Deployable structures in engineering. In *Deployable structures*, pages 1–35. Springer, 2001.
- [71] Giulia E Fenci and Neil GR Currie. Deployable structures classification: A review. *International journal of space structures*, 32(2):112–130, 2017.

- [72] Takaomi Chubachi, Hiroshi Furuya, and Hiraku Sakamoto. Boom-membrane space structure's incomplete deployment after storage in origamisat-1 ground demonstrator. *AIAA Journal*, 62(7):2401–2413, 2024.
- [73] Spinoff 1998. Thin film...large payoff. *NASA Report*, 1998.

*Appendix A***DOUBLE S-SPRING MANUFACTURING METHODS**

This section discusses the manufacturing method used in creating double s-spring patterned films, including the tolerances of such methods. Double s-springs during this thesis were created primarily using a laser cutter from Universal Laser Systems. This is a relatively high precision method of manufacturing that allows for rapid iteration of multiple designs, and features as small as a few hundred microns (unit cells as small as 3 mm have been successfully manufactured), as opposed to techniques such as creating cutting molds. However, this technique requires a few caveats to be addressed.

The first such caveat of using laser cutting to manufacture patterns in Kapton is that the polymer is sensitive to parameters such as the wavelength, the intensity, and the speed of the laser. 9.3 μm , 10.6 μm , and 1.06 μm lasers were used, of which the 9.3 μm laser provided the best absorption to properly and consistently ablate the Kapton film. When cutting the Kapton, it is best to cut it at high speed and medium power, with multiple passes if necessary. Excess energy absorbed into the Kapton results in two effects: denaturing of the polymer (bubbling and noticeable color change) near the cut area, and charring of the cut edge. The material ablated usually results in 100-120 μm of material removed, which is corrected by applying a kerf offset. Melting of the edge, and especially charring, results in a minor "dog bone" effect, where the material at the very edge of the film has a slight increase in thickness, typically 10-20%. Charring also results in more brittle edges of the Kapton film. Lowering the PPI of the laser cutter in order to prevent energy oversaturation and minimize charring can also result in a jagged or "sawtooth" edge, which perhaps may lead to easier crack propagation and tearing of the film. Hence, a parameter study was done in order to help minimize the presence of these adverse effects.

Another difficulty created by the laser cutter manufacturing technique is that the precision of the laser cutter varies, mostly based on the trajectory taken by the laser. As the laser cutter moves throughout the pattern, the gantry has to accelerate and decelerate in order to achieve the geometry designed. This results in a distribution of dwell times of the laser, and thus in some areas more material is ablated than

in others. This is visible in particular in Figure A.1, where it can be observed that different parts of the pattern have slightly different ribbon widths despite a constant 0.6 mm nominal width prescribed throughout the pattern. As discussed in the experimental section of Chapter 2, this results in performance of the double s-springs in real life being slightly modified from numerical prediction.

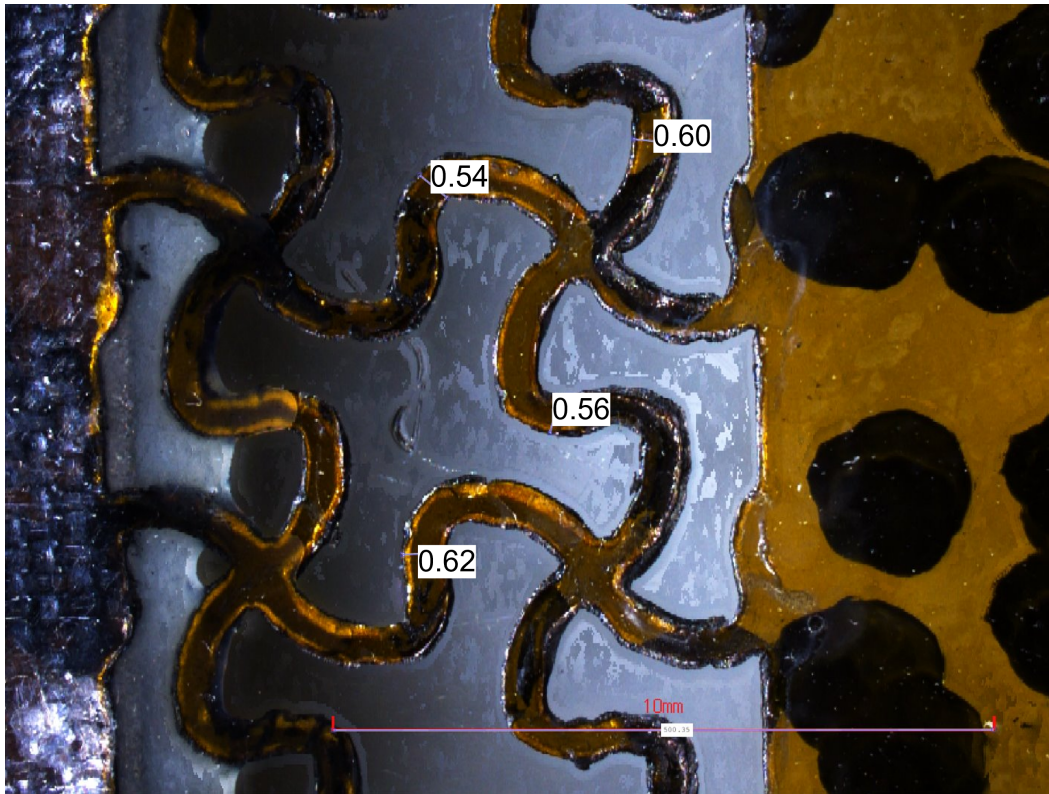


Figure A.1: Microscope view of laser cut double s-spring border. The actual width, as opposed to the nominal ribbon width of 0.6 mm, is shown labeled at various locations.

In order to attempt to mitigate some difficulties associated with the laser cutting, several different over and underlayer materials were also experimented with the Kapton. In general it was determined that additional materials such as paper, Teflon, and other films underneath the Kapton were not necessary. When cutting other thin film patterns, these materials can assist with the lift-off of the film after a pattern is etched. However, the double s-springs are relatively robust, and as such do not require such precautions. They can be cut on common materials such as an acrylic plate. Such a plate is necessary to help keep the film flat during cutting, and prevent scattering off of the honeycomb mesh below the film from ablating excess material. A metal plate, such as steel, is preferred for this process, as sometimes the

Kapton can fuse to the polymers underneath, and metal plate undergoes less etching itself, and thus can be reused more times. Care must be taken with metal plates so that any scratches on them do not reflect the laser in undesired locations. Similarly, over-layers were found to generally be unnecessary as well. The main benefit from over-layers on the Kapton is that they help remove soot that is redeposited on the film from the ablated material. However, due to the quantity of soot produced in this process, films must still be cleaned before being used, and as such over-layers serve more as a barrier to clean surface cuts and require additional cutting passes more than they aid in post-processing of the film.

Kapton has also been cut using a Zünd cutter, which uses a similar gantry to a laser cutter, but instead cuts the film with a razor blade. This method has a significant benefit in post-processing of the film, as well as precision in terms of consistent cutting widths and kerf offsets. However, it fails due to the time required and final product quality. In order to make the sharp bends of the springs, the blade must either rotate as it cuts, or "peck" the film to create the cut. The prior method is effective, but oftentimes results in the film snagging on the blade, and thus being dragged with it and tearing. The pecking method requires significantly more time, and as seen in Figure A.2, oftentimes fails to fully pierce every part of the film.



Figure A.2: Small sample of double s-spring cut from Kapton using a Zünd cutter; the majority of sections were not able to be fully separated.

While the mechanical cutting method is not fully functional, it does offer a proof

of concept for an alternative once a pattern's parameters are fully prototyped. As alluded in the start of this appendix, a hole punch method with a cutting die, seems to be a likely easy solution for quickly and consistently making large amounts of patterned film, in which the pattern is simply stamped. This type of punch would require a relatively cost intensive and precisely machined die, but would be able to be reusable and scalable, and a reasonable method as long as changes to the design are not necessary.

Appendix B

KIRIGAMI SPRING BENDING STUDY

This section primarily discusses the behavior of kirigami springs, specifically the kind previously used briefly in the Caltech SSPP, shown in the left of Figure 2.3. These are corner spring connections that offer a relatively constant force with large displacements via a bending type post-buckling behavior, as seen in Figure B.1, similar to Figure 2.8. These springs are similarly tunable to double s-springs, but are practical for the application discussed in only one direction. Additionally, they incur large out of plane offsets due to the rotation of their segments in their post-buckling behavior. While initially it may seem possible that this out-of-plane behavior may be alternatively advantageous for the sake of increasing bending and/or torsional stiffness of the structure that they are incorporated in, this was not observed experimentally. In fact, this style of kirigami spring has extremely low bending stiffness due to the extremely thin connection points that the segments share between each other.

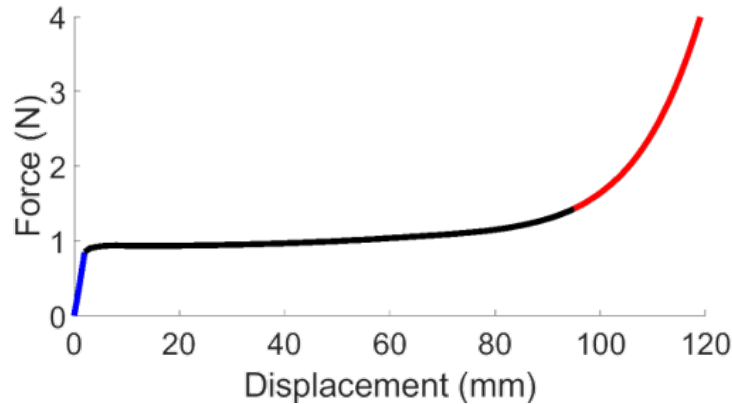


Figure B.1: Force response graph of a kirigami spring with analogous force regions to Figure 2.8 highlighted. This spring achieves a higher displacement range than the ones used in SSPP due to additional segments added.

To verify these findings, thirty kirigami springs were laser cut (similar to the methods discussed in Appendix A) from 100 μm thick 1095 spring steel. Spring steel is necessary as opposed to a softer polymer like Kapton, since the bending regime for this cut pattern results in very low forces otherwise. Three experiments were conducted with the springs: a stretch test, a four-point bending test (Figure B.2),

and a probing test (Figure B.3). The four-point bending and probing test had the kirigami springs held in an adjustable fixture, such that the prestrain of the springs could be varied throughout the experiments.

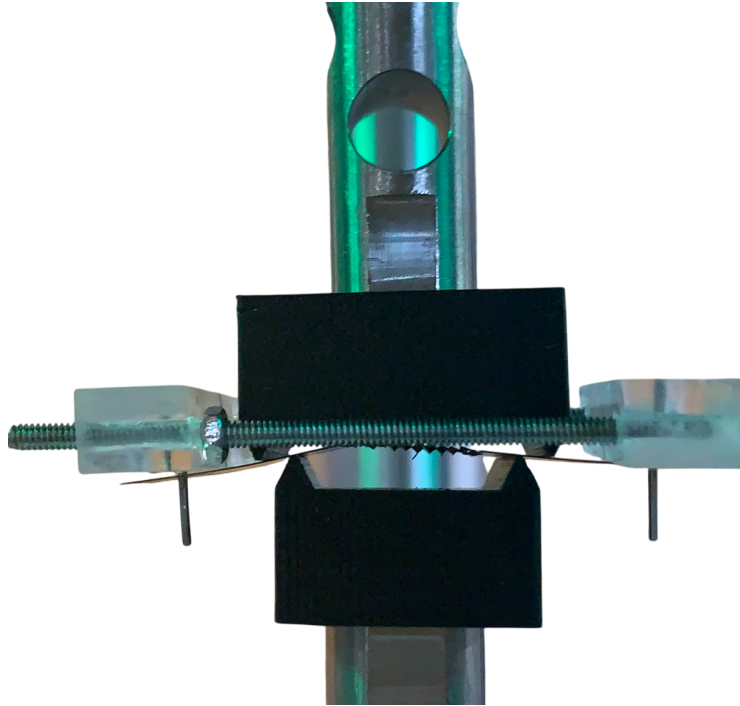


Figure B.2: Kirigami spring prestressed via fixture under four-point bending test.

The stretch tests showed that due to laser cutting based imperfections, some of the springs' second region force varied by as much as 20% (0.8 vs. 1 N). Those with small manufacturing imperfections matched numerical (Abaqus 2024) simulations relatively well. Aside from the force-displacement relation distribution, the results for the second two bending tests were much more consistent. For the four-point bending test, 0, 6, 12, and 18 mm of pretension was applied for springs with a patterned region of 5 cm, similar to the one illustrated in the left of Figure 2.3. In this test, the spring edges were held by a loop of Kapton, so that their ends could rotate freely with the buckling of the pattern. For the single point probing test, displacements in increments of 2 mm were used up to 20 mm. In this test, the ends of the spring were instead held fixed either coplanar to each other either vertically or horizontally. In all cases, the bending stiffness was calculated based on measuring a load cell force, and was independent of any applied pretension, even for cases of manufacturing imperfections. This value was determined to be 0.0696 N/mm, which is consistent with the bending stiffness that can be analytically estimated for

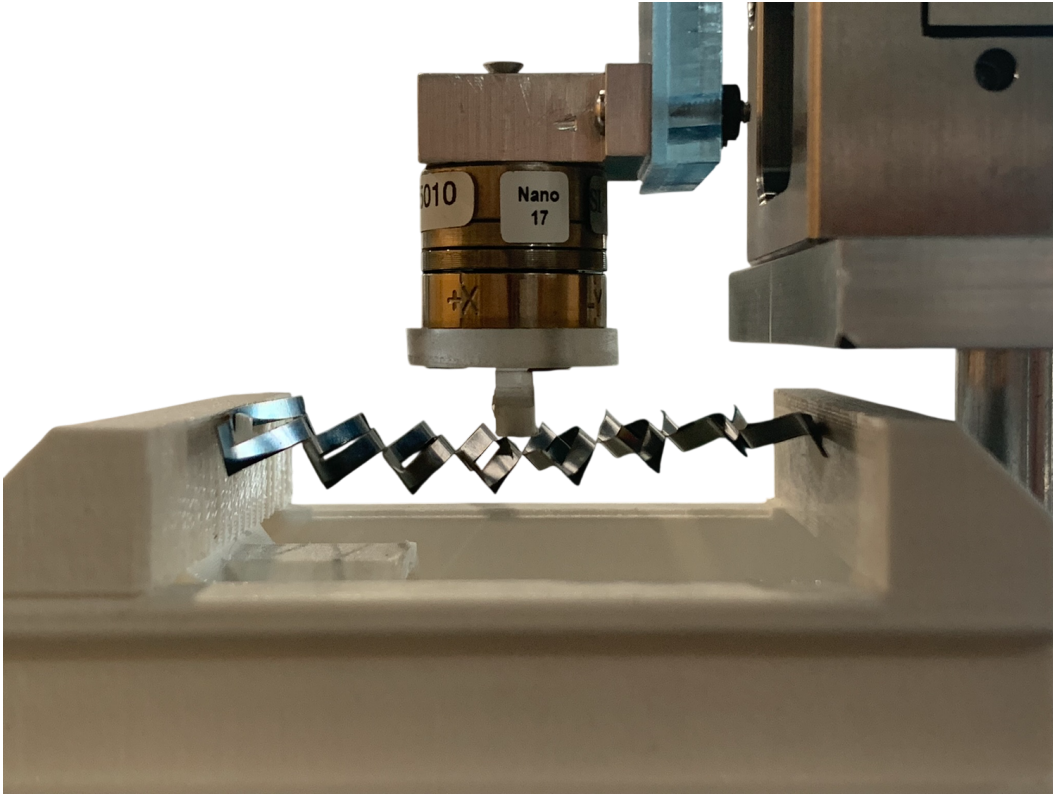


Figure B.3: Force probe measurement of kirigami spring under tension and orthogonal rotational boundary conditions.

the pattern, as well as the experimental findings for an unstressed sample. Hence, despite out of plane motions, the bending stiffnesses of kirigami springs do not increase in the post-buckling regime of the springs.