

Studies of Magneto-Optically and Magnetostatically Trapped Cesium in a Cryogenic Vacuum Apparatus

Thesis by

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Abstract

The use of a cryogenic apparatus has several advantages for the magneto-optical and magnetostatic trapping of atoms. We have measured the lifetimes of magneto-optical and magnetostatic traps for cesium to be greatly increased in a cryogenic vacuum environment relative to a room-temperature vacuum environment, due to reduced background pressure and reduced mass and kinetic energy of background gas atoms. Another advantage of a cryogenic apparatus is that superconducting magnetic field coils can easily be used to generate very high magnetic field gradients for stronger confinement of the trapped atoms. We report the first measurements of the stability of MOT's at high magnetic field gradients and laser intensities, and compare the results to a model based on evaporation of atoms from the MOT using a Fokker-Planck equation approach. This model is more consistent with the data than an alternative model based on ejection of atoms by magnetic dipole forces. We also measure the dynamical parameters for magneto-optically trapped atoms at both high and low magnetic field gradients and with trapping laser intensities above saturation and compare the data to a simple physical model based on polarization gradient cooling. Consistent with measurements by other groups, the measured friction is consistent with the polarization gradient model, but the measured spring constant is drastically reduced and more consistent with the predictions of the Doppler theory.

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Chapter 1 Introduction

The field of optical cooling and trapping of neutral atoms has evolved rapidly in the last ten years, from a few isolated experiments in the mid-1980's to one of the largest communities in atomic physics, culminating in the production of the gaseous Bose-Einstein condensate in 1995. Herein we describe experiments using magneto-optical and magnetostatic traps under extreme experimental conditions that are not widely used by researchers in this field, including very high magnetic field gradients, large trapping laser intensities, and cryogenic vacuum environments. We will show that the use of a cryogenic vacuum greatly extends the lifetime of atomic traps by reducing the rate and energy of background gas collisions and enables the use of superconducting magnetic field coils to easily generate adjustable high magnetic field gradients. These high field gradients are well beyond those available to most other laboratories, and are used to study the limits of optical trap stability in the highly confined regime as a function of laser detuning and intensity. We also study the dependence of the optical trap dynamical parameters (spring constant and velocity damping) on magnetic field gradient and laser detuning and intensity.

The basic systems studied in these experiments are magneto-optical and magnetostatic neutral-atom traps, and so we briefly describe how they work. A more detailed description can be found in Metcalf [1] and the references therein.

1.1 Theory of Magnetostatic Trapping

We start with the magnetostatic trap (or MST) because it is the simpler of the two traps used, both conceptually and experimentally. All that is required is to generate a magnetic field around the atom such that the magnetic potential $\vec{\mu} \cdot \vec{B}$ of the atom has a local minimum at some point. There are many ways to do this.

One technique would be to generate a magnetic field geometry with a local maxi-

imum of amplitude at some point. Unfortunately for atom trappers, this is forbidden by Maxwell's equations in free space. However, magnetic field geometries with a local minimum of amplitude are allowed. These can be further subdivided into two classes: those with zero gradient of the magnetic field at the center of force, such as the Ioffe configuration, and those with non-zero gradient of the magnetic field, such as the quadrupole configuration.

The Ioffe configuration, pictured in Figure 1.1, is the simpler to understand. If an atom is in an atomic sublevel for which energy increases with increasing magnetic field (a weak-field-seeking state), then when in an Ioffe trap it will roll down the potential hill to the trap center. If the atomic energy is less than the trap depth, it cannot escape.

The quadrupole configuration is more subtle, although in practice easier to achieve. As the two-dimensional slice of the magnetic field configuration in Figure 1.2(a) shows, the magnetic field amplitude increases linearly with distance from the trap center, but the direction varies with angular position around the trap and reverses sign along any path through the trap center. The magnetic field strength along the x axis is shown along with the corresponding energy levels for a $J = 1/2$ atom in Figure 1.2(b).

The problem for trapping can be readily seen by considering a path on a straight line through the center (Fig. 1.2(b)). An atom starting at rest at the right of the trap in the $m = +1/2$ state will slide down the magnetic potential to the center of the trap; without any mechanism to change to the $m = -1/2$ state, it will continue through and be lost out of the left side of the trap. The same problem will eject an atom that passes anywhere in a region too near the trap center, as in the path shown in Figure 1.2(a). In this case the atom passes through a region where the magnetic field direction changes rapidly compared to the local Larmor frequency, and its final state will be a mixture of trapped and untrapped states. These atomic transitions are known as Majorana transitions, and the region in the trap where these transitions lead to losses is commonly referred to as the Majorana region.

The quadrupole MST works because there are many orbital trajectories which do not enter the Majorana region. One such path is marked in Figure 1.2(a). An atom

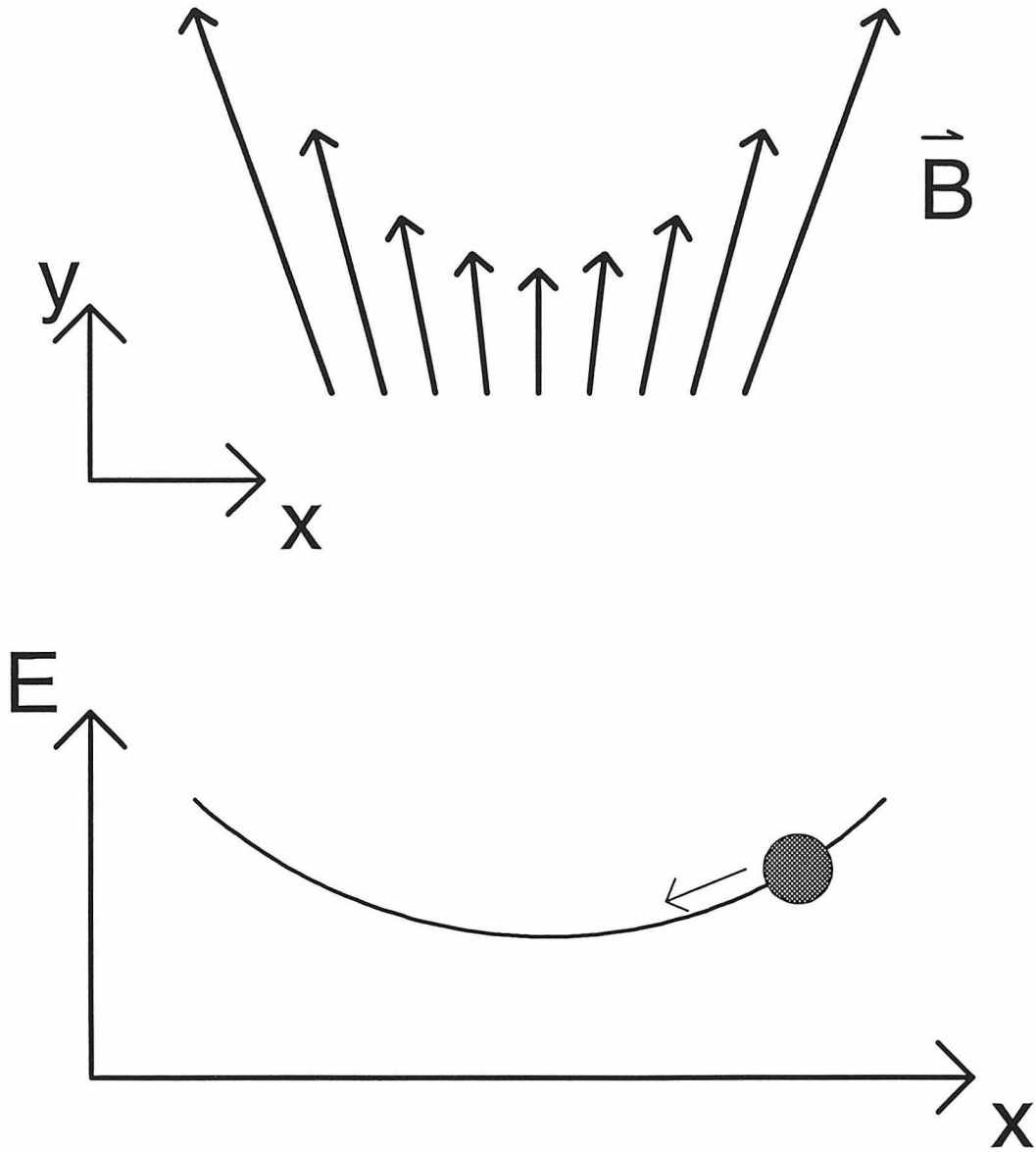


Figure 1.1: The magnetic field configuration for a two-dimensional slice through an Ioffe trap, with the corresponding atomic magnetic potential energy variation leading to a stable trap.

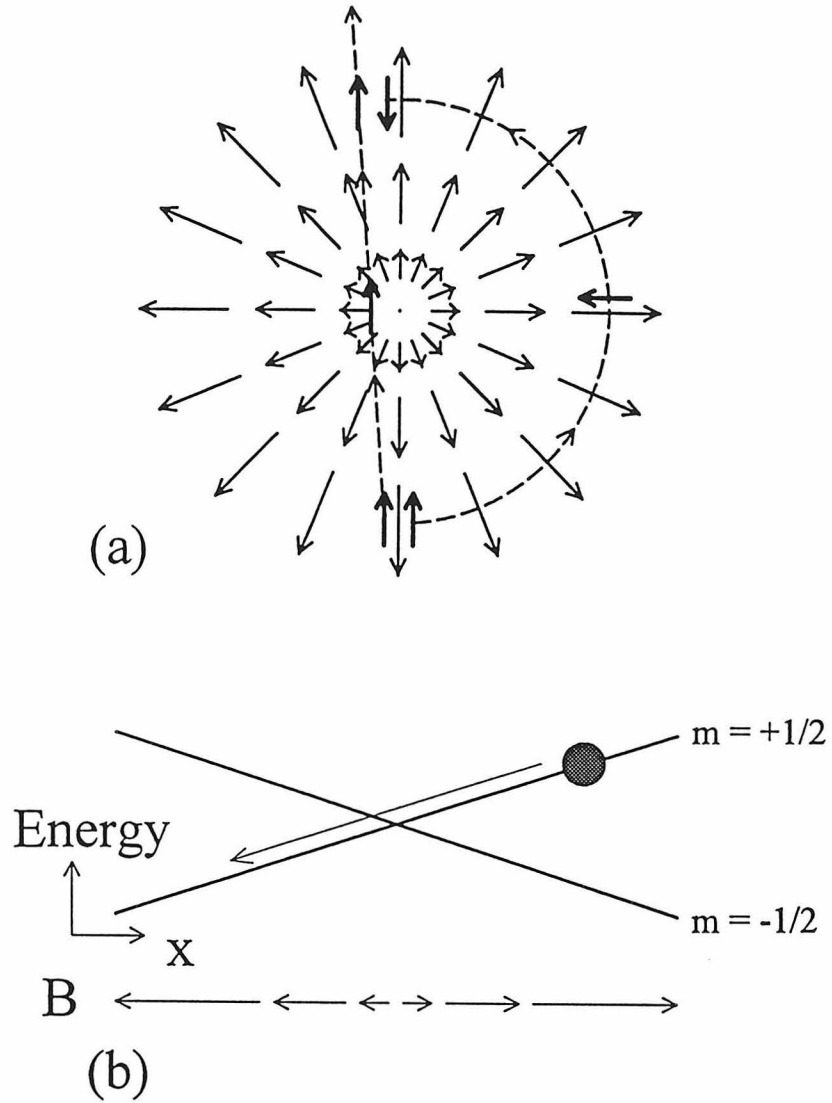


Figure 1.2: Aspects of the quadrupole MST. (a) A two-dimensional section of a quadrupole MST magnetic field showing trapped and untrapped trajectories. (b) The energy levels along a path through the MST center.

initially in the trapped $m = +1/2$ state at the bottom of the trap and moving at a right angle to the trap center will feel a force toward the trap center, curving its orbit. Meanwhile, Larmor precession around the relatively slowly changing magnetic field direction causes the atomic spin to adiabatically follow the local magnetic field. As a result, when the atom reaches the top of the trap, it is now in the $m = -1/2$ state as defined along the initial quantization axis, and is still trapped.

Both quadrupole traps and Ioffe-style traps are in common use for trapping neutral atoms. Quadrupole traps have the advantages of ease of implementation (they are generated with the simple “anti-Helmholtz” coil pair ubiquitous in optical trapping) and relatively greater trapping force due to the nonzero gradient of magnetic field at the trap center. These advantages are sizable enough to lead researchers to devise clever tricks to prevent Majorana losses, including the TOP trap [2] and the combined FORT/MST [3]. Ioffe-style traps, on the other hand, are inherently free of Majorana losses and are spin-polarized, which can be useful in the study of certain scattering processes; however, they require more complicated coils or permanent magnet configurations, and are much less deep for a given coil size and current. We will show in a subsequent chapter how cryotrapping improves the performance of both types of trap. It should be pointed out that there are no dissipative forces in either type of MST.

1.2 Doppler Cooling and Zeeman Trapping

The basic magneto-optical trap (MOT) uses scattered nearly resonant laser light to provide the trapping force on the atom, with an inhomogeneous magnetic field to spatially tune the atomic transition frequency, thus providing a center of force. The detuning of the laser also provides a velocity dependence to the atom-laser interaction, which is exploited to viscously damp the atomic velocity, providing cooling. We first describe the cooling force, called *Doppler cooling*, making reference to Figure 1.3.

Imagine an atom at rest, irradiated by two counterpropagating laser beams of frequency ω_L less than the atomic frequency ω_A . Also, let the laser beam intensities

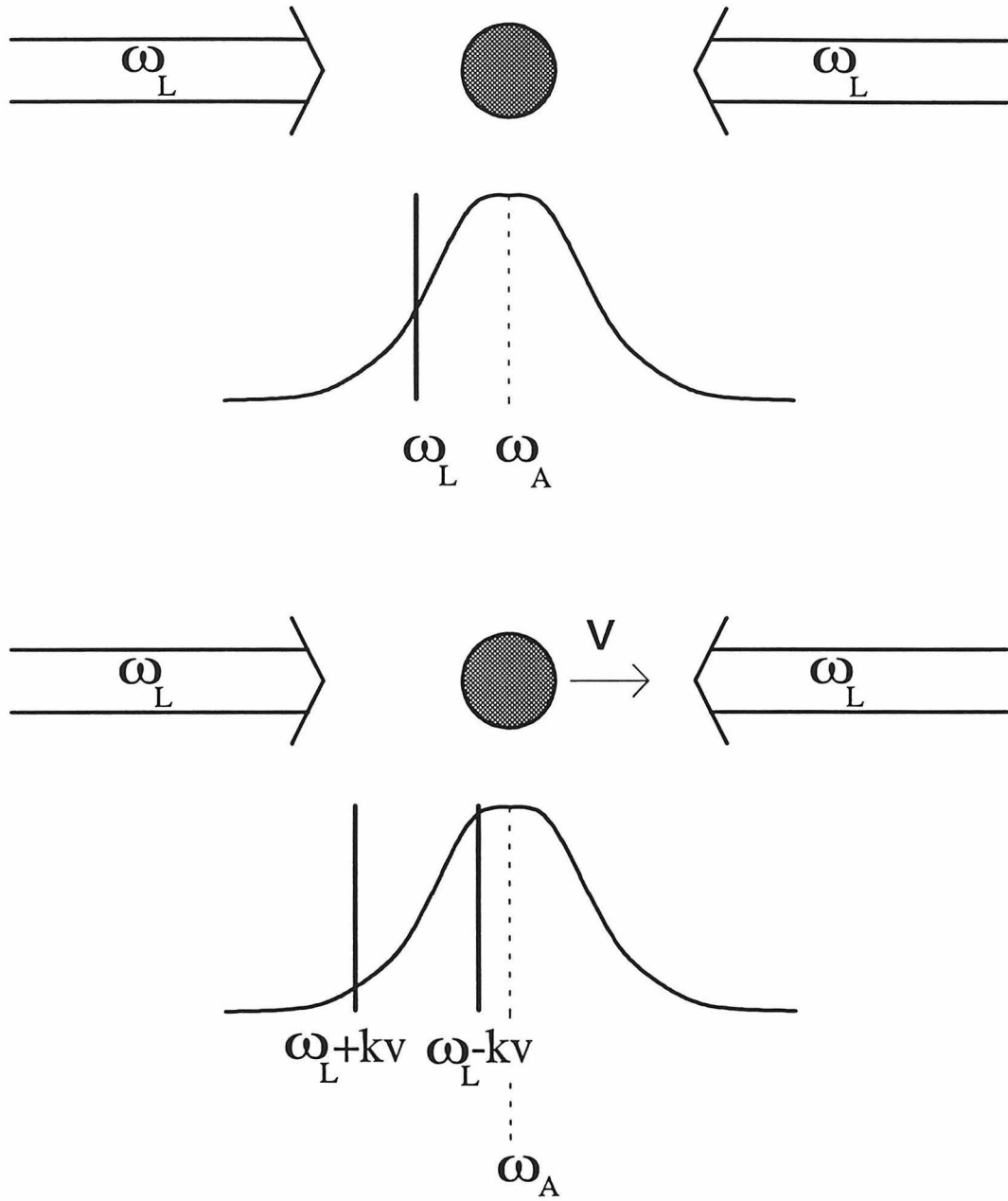


Figure 1.3: Doppler cooling: in the upper configuration, with the atom at rest, the counterpropagating lasers, at ω_L , are equally detuned from the atomic resonance ω_A . In the lower configuration, where the atom has velocity v , the two lasers acquire equal and opposite detunings of kv as seen by the atom.

be equal and low enough that conservative electric dipole forces are not present¹. The atom will scatter photons equally from both beams, and the net force on the atom due to photon scattering is zero.

Now give the atom some velocity v along the laser beam propagation axis. The atom sees the laser beam propagating against its motion as Doppler shifted to the higher frequency $\omega_L + kv$, and for small atomic velocity the frequency of that beam is closer to resonance with the atom, so the atom scatters more photons from that beam, feeling a force

$$F_+ = \hbar k \frac{\Gamma}{2} \frac{I}{I_{sat}} \frac{1}{1 + I/I_{sat} + [2(\delta - kv)/\Gamma]^2}, \quad (1.1)$$

where $\delta = \omega_L - \omega_A$ is the laser detuning from resonance. In contrast, the atom sees the copropagating laser beam as Doppler shifted to the lower frequency $\omega_L - kv$, detuning that beam even further from resonance with the atom, so the atom scatters fewer photons from that beam, feeling a force

$$F_- = -\hbar k \frac{\Gamma}{2} \frac{I}{I_{sat}} \frac{1}{1 + I/I_{sat} + [2(\delta + kv)/\Gamma]^2}. \quad (1.2)$$

Thus, the moving atom tends to scatter more photons from the laser opposing its motion, and is slowed. For small atomic velocity $kv \ll \Gamma$, the slowing force is proportional to the atomic velocity:

$$F = F_+ + F_- = -\alpha v = 4\hbar k \frac{I}{I_{sat}} \frac{(2\delta/\Gamma)kv}{[1 + (2\delta/\Gamma)^2]^2}. \quad (1.3)$$

This model has used a one-dimensional cooling laser configuration, and the random recoil direction of the emitting atom will cause it to be heated in the directions orthogonal to the laser propagation axis. However, there is nothing to prevent one from adding counterpropagating laser beam pairs to the orthogonal directions, yielding cooling in all three dimensions. Such a configuration is referred to as *optical molasses*. In the low intensity limit, the presence of the orthogonal lasers alters the

¹this latter constraint is not necessary to achieve Doppler cooling but simplifies the understanding.

force only by increasing the saturation of the transition; in three dimensions the damping coefficient becomes

$$\alpha = -4\hbar k^2 \frac{I}{I_{sat}} \frac{2\delta/\Gamma}{[1 + 6I/I_{sat} + (2\delta/\Gamma)^2]^2}. \quad (1.4)$$

So far we have shown how to bring an atom to rest in three dimensions using the viscous force of suitably tuned laser beams, but have not yet provided a trapping mechanism to confine them to a region of space. This is achieved by the same quadrupole magnetic field configuration used for the quadrupole MST, along with suitably chosen laser polarizations.

A one-dimensional model with a $J = 0 \rightarrow J' = 1$ atom is the simplest model that contains all the essential physics (see Figure 1.4). A quadrupole magnetic field provides the same position-dependent splitting of the energy levels seen in the quadrupole MST. This time, however, two counterpropagating laser beams of σ_+ and σ_- polarization and negative detuning irradiate the atom. An atom at rest at the center of the trap has its $m = \pm 1$ transitions equally resonant with the two laser beams, so that it scatters photons equally from the two beams and feels no net force. However, an atom at rest to the right of the trap center will be more resonant with the leftward-propagating beam than with the rightward-propagating beam, due to the different Zeeman shifts of the atomic sublevels and the transition selection rules. The atom scatters more photons from the leftward-propagating laser and is pushed toward the trap center. A similar analysis shows an atom to the left of center will also be pushed toward center. The atom feels a restoring force, and this force can be shown to be linear for displacements from center small enough that the Zeeman shift of the levels is much less than the atomic linewidth. By a derivation analogous to that of the damping coefficient with the Zeeman detuning $g\mu B'x/\hbar$ replacing the Doppler detuning kv , we obtain:

$$F_+ = \hbar k \frac{\Gamma}{2} \frac{I}{I_{sat}} \frac{1}{1 + 6I/I_{sat} + [2(\delta - g\mu B'x/\hbar)/\Gamma]^2}. \quad (1.5)$$

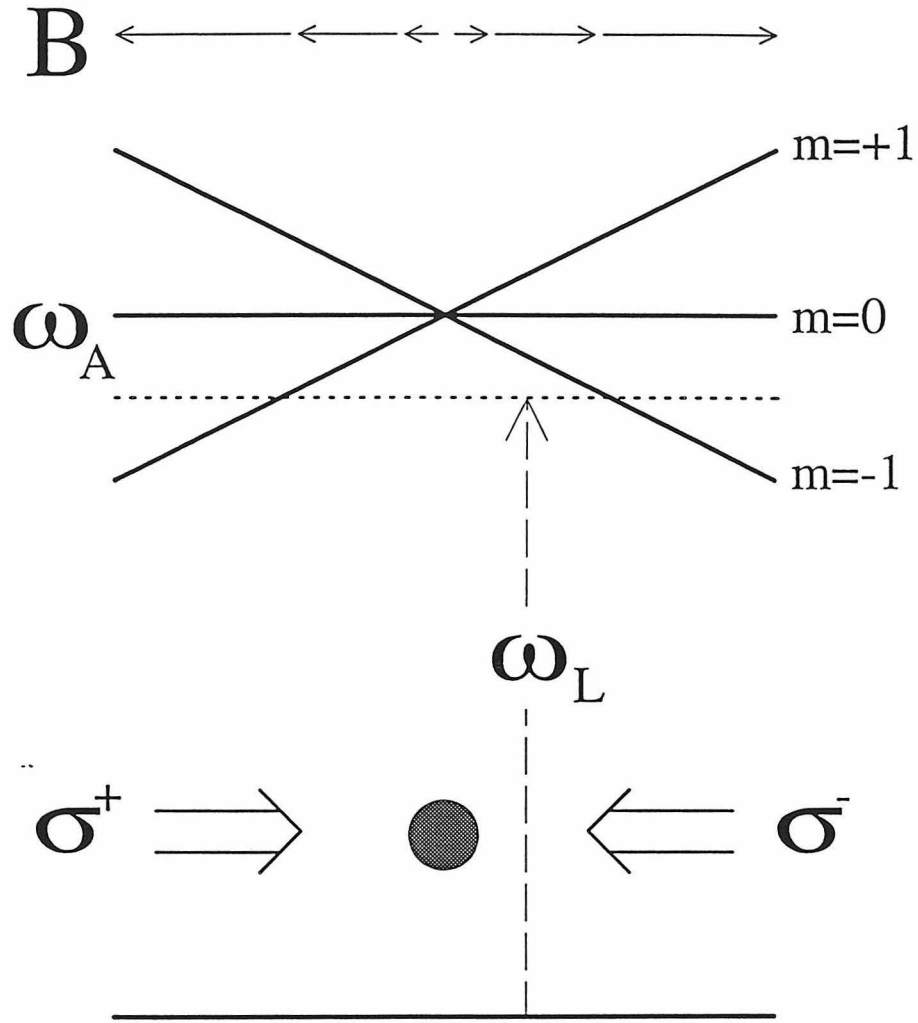


Figure 1.4: The Zeeman force: in the magnetic field shown, the Zeeman sublevels will have spatially-dependent shifts. These shifts bring the atom closer into resonance with the laser driving it towards the center of the trap.

$$F_- = -\hbar k \frac{\Gamma}{2 I_{sat}} \frac{1}{1 + 6I/I_{sat} + [2(\delta + g\mu B'x/\hbar)/\Gamma]^2}. \quad (1.6)$$

By summing these two forces $F = F_+ + F_- = -kx$ and letting $g\mu B'x/\hbar \ll \Gamma$ we arrive at the equation

$$\kappa = -4g\mu k B' \frac{I}{I_{sat}} \frac{2\delta/\Gamma}{[1 + 6I/I_{sat} + (2\delta/\Gamma)^2]^2}. \quad (1.7)$$

Observe that the configuration and detuning of the lasers is identical to that for Doppler cooling. Doppler cooling is in fact at work in a MOT, so the trapped atom will have an average motion described by a damped harmonic oscillator.

1.3 Polarization Gradient Cooling and Trapping Mechanisms

The trapping of atoms in MOT's had not yet celebrated its first birthday when experiments began to indicate that other mechanisms were cooling the atoms in addition to the Doppler mechanism. Trap temperatures were lower than predicted by the Doppler theory [4], and observation of gain in a probe laser beam propagating through the trap provided evidence of orientation of the atomic ground sublevels, with consequent dynamical implications [5, 6].

It was soon apparent that these sub-Doppler cooling mechanisms relied on the multi-level structure of the trapped atom. It was also apparent that the sub-Doppler cooling was strongly dependent on the type of polarization gradient present in the laser field, although very different types of polarization gradient could cool with different dependences on laser detuning, some with and some without an additional magnetic field [7, 8, 9]. Two of them are of primary relevance to MOT's: induced orientation cooling and ellipticity gradient Sisyphus cooling. The Sisyphus cooling is conceptually simpler, and will be treated first.

1.3.1 Ellipticity Gradient Sisyphus Cooling

There are several types of Sisyphus cooling [7, 10, 11]; they all cool atoms by forcing them to use kinetic energy to climb a potential hill, then sending them to the bottom of a potential hill by means of some radiative transition which carries away the excess potential energy. Of particular interest in cesium MOT's is that of Dalibard and Cohen-Tannoudji [7], which considers the case of a $J = 1/2 \rightarrow J' = 3/2$ atom irradiated by counterpropagating laser beams with orthogonal linear polarization in one dimension; that is, where the electric field is given in the laboratory frame by

$$\vec{E} = E_0[\hat{x} \exp(ikz) + \hat{y} \exp(-ikz)]e^{i\omega t} + \text{c.c.}, \quad (1.8)$$

which we have pictured in Figure 1.5(b). It is easy to show that the laser polarization varies from right circular to linear to left circular to orthogonal linear over an optical wavelength; that is to say, there is a gradient of ellipticity of the laser polarization. If we consider the transition diagram for the $J = 1/2 \rightarrow J' = 3/2$ atom with its Clebsch-Gordan coefficients (Fig. 1.5(a)), we can see two effects of the ellipticity gradient for an atom at rest. One is that the ground state sublevels will have different spatially dependent light shifts, due to the different strengths of the optical transitions (refer to Fig. 1.5(c)). In the low-intensity regime, the light shifts of the $|m = -1/2\rangle$, $|m = 1/2\rangle$ states will be

$$\Delta E_{-1/2} = \frac{\hbar\delta s}{3}(2 + \cos(2kz)), \quad (1.9)$$

$$\Delta E_{+1/2} = \frac{\hbar\delta s}{3}(2 - \cos(2kz)), \quad (1.10)$$

where s is the generalized saturation parameter

$$s = \frac{\Omega^2/2}{\delta^2 + \Gamma^2/4} = \frac{2I/I_{sat}}{\delta^2 + \Gamma^2/4}. \quad (1.11)$$

Thus, at a region of right circular polarization, the $m = 1/2$ ground state will have 3 times the light shift of the $m = -1/2$ ground state. The opposite will be the case at

the regions of left circular polarization. In the regions of linear polarization, the two ground state sublevels will have the same light shift. The second effect is that the local light field will tend to optically pump the atoms into the ground state sublevel with the greater light shift: in regions of pure circular polarization, all the atoms will be in the lower ground state, and in regions of linear polarization, the two ground state sublevels will be equally populated. The time frame for this optical pumping is of order

$$\tau_P \sim \left(\frac{\Omega^2 \Gamma}{\delta^2 + \Gamma^2/4} \right)^{-1}, \quad (1.12)$$

and will tend to be longer for an atom near the potential minimum to be pumped up than for an atom near a potential maximum to be pumped down.

In the absence of this optical pumping, an atom moving along the laser propagation axis would simply roll up and down in the sinusoidal potential of the light shift, and the force would be purely conservative. However, due to the finite time taken to optically pump the atom into the local equilibrium distribution of ground states, an atom starting in the lower ground state will tend to stay there as it climbs the light shift potential, as shown by the path of the open circle in Figure 1.5(c). As it nears the top of the light shift potential, it has a relatively high probability to undergo a Raman-antiStokes transition to the new lower ground state. As a result, there are more atoms rolling up potential hills than down them, and averaged over an optical wavelength the atom gives up energy to the optical field. The energy is carried away by the Raman transitions that optically pump the atoms. The damping coefficient α can be estimated by considering the situation when $kv = 1/\tau_P$, when the force can be expected to be at a maximum. The energy lost by the atom rolling up the light shift potential well is

$$E = \frac{2}{3} \hbar \delta s = \frac{2 \hbar \delta}{3} \frac{\Omega^2/2}{\delta^2 + \Gamma^2/4}, \quad (1.13)$$

and this occurs in a time τ_P . This cooling rate is related to the force by

$$\frac{E}{\tau_P} \sim \frac{dE}{dt} = Fv = -\alpha v^2. \quad (1.14)$$

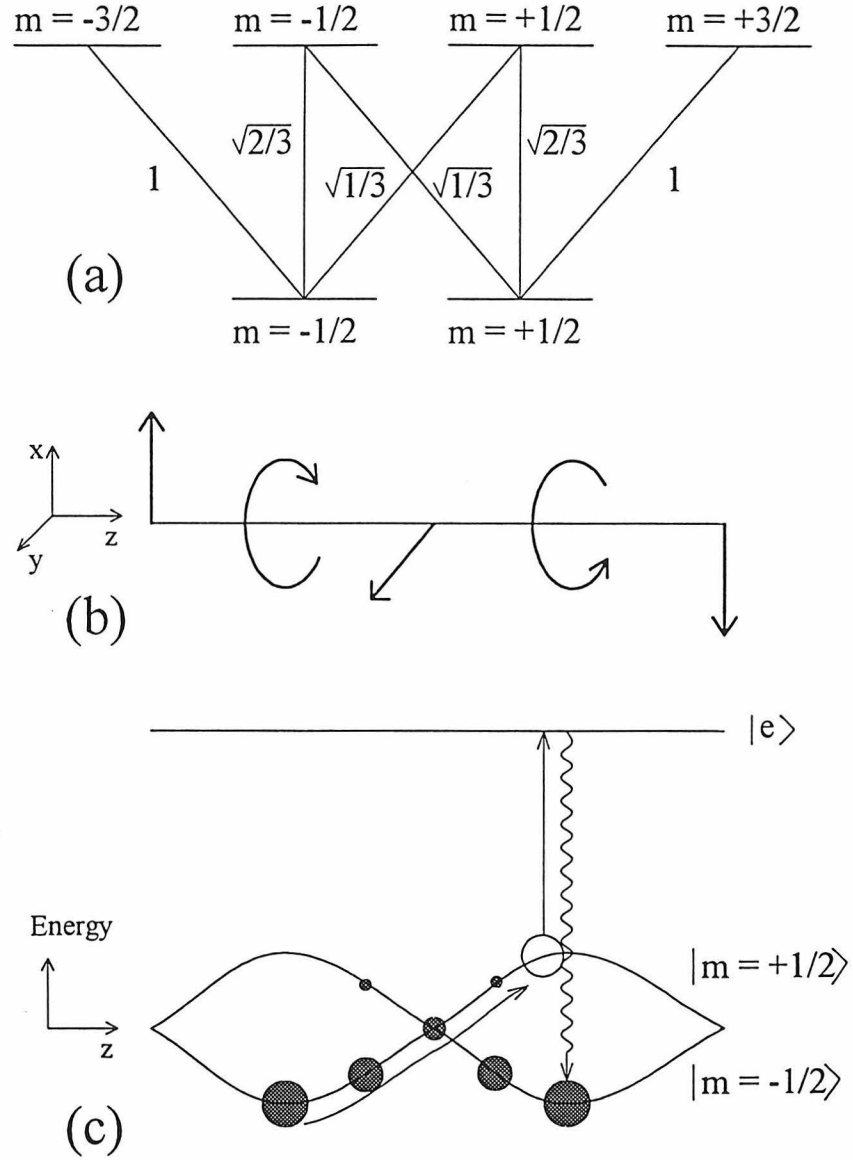


Figure 1.5: Aspects of Sisyphus cooling: (a) Clebsch-Gordan coefficients for the $J = 1/2 \rightarrow J' = 3/2$ atom. (b) Spatial variation of the local laser polarization. (c) Steady-state populations of the ground states. The mechanism for energy loss by a moving atom is depicted by the path of the open circle.

Using $v^2 = 1/k^2\tau_P^2$, we get the approximate result

$$\alpha \sim -k^2 E \tau_P \sim -\frac{\hbar k^2 \delta}{\Gamma}. \quad (1.15)$$

A more detailed calculation using the optical Bloch equations in one dimension gives

$$\alpha = -3\hbar k^2 \delta / \Gamma. \quad (1.16)$$

An important feature of this cooling mechanism is that it is independent of the laser intensity in the low-intensity limit.

This analysis can readily be shown to apply in general to any $J \rightarrow J' = J + 1$ transition. It also applies in three dimensions whenever a gradient of the ellipticity of the laser polarization occurs. Notice that in this laser configuration, stimulated redistribution of photons from one laser beam to the other can occur even in one dimension. This is why the ground state light shifts can vary spatially, yielding a momentum transfer from the light field to the atom.

1.3.2 Induced Orientation Cooling and Trapping

If the atom is irradiated by counterpropagating $\sigma_+ - \sigma_-$ laser beams in one dimension, stimulated redistribution of photons can no longer occur, and there can be no gradient of the light shift potential. This is the configuration for induced orientation cooling [7]. The simplest system that undergoes induced orientation cooling is the $J = 1 \rightarrow J' = 2$ atom. This is because this cooling mechanism relies on an orientation of the atomic ground state, and the lowest spin that can have non-trivial orientation is $J = 1$.

In this case, the electric field has the form

$$\vec{E} = -\frac{E_0}{\sqrt{2}}(\hat{x} + i\hat{y})e^{i(kz - \omega t)} + \frac{E_0}{\sqrt{2}}(\hat{x} - i\hat{y})e^{-i(kz + \omega t)} + c.c. \quad (1.17)$$

$$= -\sqrt{2}iE_0[\sin(kz)\hat{x} + \cos(kz)\hat{y}]e^{-i\omega t} + c.c. \quad (1.18)$$

The polarization of the resultant laser field is linear, but rotates about the z axis as

one moves along the z axis. Thus, there is no gradient of the ellipticity, but rather of the polarization axis. If we adopt a local quantization axis that points along the local electric field (and label this axis y), and make use of the Clebsch-Gordan coefficients for this atomic level structure, we find that the relative populations of the ground state sublevels $|m = -1\rangle_y, |m = 0\rangle_y, |m = +1\rangle_y$ are equal to 4/17, 9/17, 4/17, respectively, for an atom at rest. The different ground states will have different light shifts as well: the $|m = \pm 1\rangle_y$ states will have 3/4 of the light shift of the $|m = 0\rangle_y$ state, which is

$$\Delta E_0 = \hbar \delta s. \quad (1.19)$$

This will be true at any position along the laser propagation direction; only the quantization axis along which we define J varies.

If we then let the atom have a small velocity v , the atom will find itself in a electric field polarization that rotates with angular velocity kv . By transforming to a reference frame that rotates at this frequency, we can then model the system as one with a fixed laser polarization and an inertial rotation of the atomic polarization due to a potential kvJ_z , which can be modelled by a fictitious magnetic field kvJ_z/μ_z .

The effect of this fictitious potential, for small velocity, is to perturb the zero-velocity eigenstates of the optical potential. In particular, the perturbed ground state of the atom will have different values of the populations of the $|m = 1\rangle_z$ and $|m = -1\rangle_z$ states. This difference leads to an imbalance in the scattering from counterpropagating laser beam as compared to the copropagating beam. The result of the perturbation calculation gives the value of the damping coefficient

$$\alpha = -\frac{120}{17} \frac{\hbar k^2 \delta \Gamma}{5\Gamma^2 + 4\delta^2}. \quad (1.20)$$

It can be easily seen that this cooling mechanism will also produce a trapping mechanism in the presence of a magnetic field along the z axis; in this case the fictitious magnetic field kvJ_z/μ_z is joined by a real magnetic field B_z . As a result, the atomic velocity will be damped not to zero but to the value $v_B = \mu B_z/kJ_z$. In the linearly increasing magnetic field of a MOT, this leads to a spring constant equal

to

$$\kappa = -\frac{120}{17} \frac{dB}{dz} \frac{g\mu_B k \delta \Gamma}{5\Gamma^2 + 4\delta^2}. \quad (1.21)$$

1.4 Number of Trapped Atoms

The population of a MOT is usually determined by three basic processes: the loading rate of atoms into the trap, ejection of atoms by collisions with background gas atoms, and inelastic collisions between trapped atoms. These processes are included in the trap density equation

$$\dot{n} = R - (1/\tau + E)n - \beta n^2 \quad (1.22)$$

where n is the density of the trapped atoms, R is the loading rate, $1/\tau$ is the background gas scattering rate, and β is the inelastic scattering rate between trapped atoms. The study of β in MOT's has been a very productive field of atomic physics in recent years [12], giving researchers the ability to sensitively study very low energy scattering [13], photo-ionization processes [14], light-assisted collisions [15, 16], and intermolecular potentials. The presence of the term $-En$ in Eq. 1.22 has not been considered in the literature prior to the experiments reported here. Later in this thesis we report the first measurements of this parameter when we discuss the stability of MOT's containing one atom at very high magnetic field gradients in very low background gas pressures. In such a system the atom escapes by evaporating over the relatively low trap potential barrier. This is a special experimental configuration that emphasizes a process that is normally insignificant in nearly all MOT's, due to relatively low magnetic field gradients and large number of atoms. We will show later that βn^2 greatly exceeds En for any trap with more than one atom.

In many experiments, what is measured is not the trap density n , which varies within the MOT, but the total number of trapped atoms N . Before deriving an equation for the trap population, the density profile must be known. For traps of

sufficiently low density, the spatial profile is a three-dimensional Gaussian:

$$n(x, y, z) = n_0 \left(\frac{1}{2\pi\sigma_x^2} \right)^{3/2} \exp\left(-\frac{(x^2 + y^2)}{2\sigma_x^2} - \frac{z^2}{\sigma_z^2} \right) \quad (1.23)$$

In this formula the z axis is chosen along the symmetry axis of the quadrupole field. The field gradient along the z axis is twice that along the x and y axes because $\vec{\nabla} \cdot \vec{B} = 0$, so the half-width along the z axis is one-half that along the x or y axis in the limit that the density is sufficiently low that the trapped atoms are noninteracting. If there are enough atoms that interatomic forces limit the maximum density, the trap temperature will rise and the half widths in the z , x , and y directions will become equal, although the profile remains Gaussian [17].

In an MST, there are no laser beams present, and so no atoms are in the excited state. Since the excitation energy is the primary energy source for inelastic collisions that eject atoms from the trap, in MST's the β term of the trap population equation is due to hyperfine state-changing collisions, which have a much lower cross section. Experimentally, the $1/\tau$ term is always dominant in the absence of some externally imposed forced evaporation.

Chapter 2 Introduction to Cryotrapping

The research described here studies the physics of optically and magnetically cooled and trapped cesium atoms. The essential physics are in the interactions of atoms with resonant light and magnetic potentials, and in the interactions of the atoms with each other. However, there are properties of atomic traps that can be greatly enhanced by creating them in a cryogenic vacuum environment. These advantages are significant enough, and the experimental techniques unique enough, that we refer to such research specifically as *cryotrapping*.

A significant problem faced by many researchers is the short lifetimes of optical and magnetic traps. The lifetime of a trap is determined by two factors: trap loss due to collisions between trapped atoms, and loss due to collisions with background (untrapped) gas atoms; these are the $1/\tau$ and β terms in Eq. 1.22, respectively. At typical experimental magnetic field gradients ($\sim 1 - 50$ G/cm), the lifetime of a MOT is limited by the background gas collisions for low MOT atomic density. As mentioned earlier, MST lifetimes are nearly always limited by background collisions, regardless of atomic density. In most ultrahigh vacuum systems the background gas pressure limits the lifetime of a MOT to about 100 seconds or less and that of an MST to about 10 seconds or less. This time limit can be a serious hurdle to experiments such as measurements of small collision cross sections between trapped atoms, evaporative cooling to Bose-Einstein condensation, or observation of individual atoms in MOT. As we will show, in a cryogenic vacuum apparatus the background pressure is reduced by orders of magnitude below that of room-temperature vacua. The expected background gas density in a sealed dark chamber cooled to 4 K can be extremely low, assuming the walls of the chamber are not saturated with helium. Pressures as low as 5×10^{-17} Torr have been reported [18, 19], and under ideal conditions at 4 K one theoretically expects an essentially perfect vacuum [20]. Light incident on the inner walls of the chamber can significantly increase the background

pressure, due to light-induced desorption of helium from the cryogenic surfaces [21]. Because of this, we may expect pressures below $\sim 10^{-12}$ Torr will be difficult to attain in the presence of scattered light from atom-trapping laser beams.

Another significant advantage to cryotrapping is the ability to incorporate superconducting magnetic coils into the experiment. This allows for the generation of magnetic fields and magnetic field gradients far in excess of those normally available to room temperature experiments. We will show in the next chapter how these extremely high field gradients were exploited to study the stability of MOT's.

2.1 Version 1

2.1.1 Apparatus

Our first experiments in the trapping of neutral cesium in a cryogenic vacuum apparatus were conducted in the chamber schematically displayed in Figure 2.1, which we call *Version 1* [22].

The apparatus was essentially a pair of independent nested vacua: an inner vacuum chamber in the form of an octagonal copper block cooled by a liquid helium (LHe) cryostat, where the trapped cesium atoms were stored in ultra-high vacuum conditions, and an outer vacuum chamber, held at about 10^{-3} Torr by a roughing pump, which reduced the conductive and convective heat load on the inner chamber and slowed the condensation of atmospheric gases on the windows of the inner chamber. A shield surrounding the inner chamber and cooled to a temperature between those of the inner and outer chambers (typically 77 K) served to reduce the radiative heating of the inner chamber and also acted as a thermal anchor for the current-carrying wires leading out from the inner chamber. At first this shield was made of copper and cooled by an internal flow of liquid nitrogen, but later it consisted of a simpler aluminum shield covered with Superinsulation and cooled by contact with the radiative shield of the cryostat.

The cesium beam entered the inner vacuum chamber from a thin-walled stainless

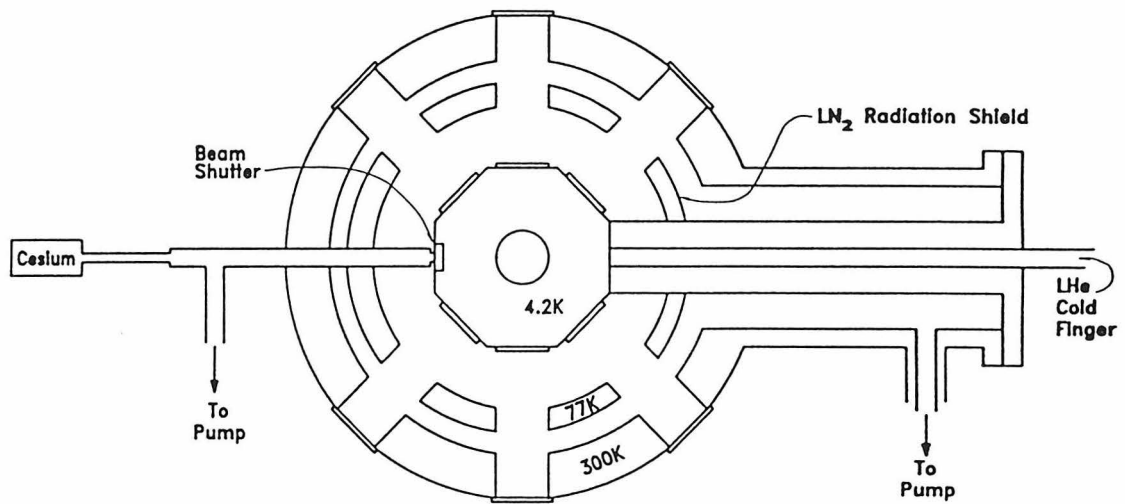


Figure 2.1: Cryotrap Version 1. The cooling and trapping laser beams entered the cryotrap through the windows that are at 45 degrees to the Cs beam axis and through windows on the top and bottom of the cryotrap. The magnetic field coils (not shown) were mounted with their symmetry axis vertical. The diameter of the outer vacuum chamber is 20.3 cm.

steel bellows traversing the outer vacuum chamber. The cesium beam source was an ion-pumped effusive oven similar to that of Tompa *et al* [23]. A cesium ampule was stored in a thin-walled stainless steel half-nipple which could be isolated from the rest of the vacuum system by a valve. After the ampule was pumped and baked to remove impurities, the half-nipple was compressed by a clamp, crushing the ampule and releasing the cesium. The beam flux was switched on and off by the valve, and the flux was varied by heating and cooling the half-nipple. Though primitive, this oven provided a large cesium flux. Too large, in fact; ion pumps saturate quickly when pumping cesium, and ion pump failure was a frequent problem.

The inner chamber was evacuated through the bellows by the ion pump on the cesium oven to a typical pressure of 10^{-7} Torr. By then cooling the inner chamber with the LHe cryostat, its entire surface became a cryopump, and all remaining gases were condensed onto the walls with very high efficiency save helium and hydrogen, which were pumped only moderately. At this point the inner chamber pressure was dominated by gas instreaming from the cesium oven through the bellows. To eliminate this, a Teflon shutter was installed at the mouth of the bellows inside the inner chamber. The shutter was mounted to the chamber by a spring so that it rested in the open position. The shutter was pulled closed using a nylon wire fastened to a pulley on a rotary-motion feedthrough in the cesium beam source.

Optical access into the inner chamber was through 1" glass windows, four equidistant around its circumference and one each in its top and bottom for the trapping and repumping lasers, and two in the sides at right angles to the cesium beam for fluorescence detection. Matching windows were mounted in the outer vacuum chamber. All window vacuum seals as well as all cryogenic vacuum seals used indium wire gaskets.

The quadrupole magnetic field was generated either by normal coils external to the vacuum system or by superconducting coils inside the inner vacuum chamber. The normal coils were wound from 16 gauge copper magnet wire onto aluminum forms with inner radius 2.3 cm, outer radius 5.7 cm, and thickness 2.9 cm, and their centers were separated by 11.1 cm. By calibrating the magnetic field of a single coil along the

coil axis with a Hall probe, we measured the trapping gradient to be 6.7 gauss/cm per Ampere of current through the coils. The superconducting coils were wound from copper-clad monofilamentary niobium-titanium wire with Formvar insulation (provided by Supercon Inc., serial number 372M-27K1B1A, total diameter .006") onto coil forms made from 303 stainless steel of inner radius .33 cm, outer radius .67 cm, and thickness .27 cm. The coils were separated by .95 cm, measured from their centers. Each coil has 400 turns of wire.

Magnetostatic traps with long lifetimes have previously been demonstrated with sodium [24] and hydrogen [25], both using cryogenic chambers to achieve very low background gas densities. Our cryotrapping apparatus differed from these others in having optical access along more than a single axis in order to fully exploit laser cooling and trapping technologies.

2.1.2 Experiments

In an investigation of long-lived magneto-optical traps, we used the external magnetic field coils to generate a weak quadrupole field in the inner chamber, with a typical axial gradient $B' \approx 10$ G/cm, and loaded the trap directly from the cesium beam. Once the trap was loaded, the beam shutter was closed and the trap fluorescence monitored as a function of time. Figure 2.2 shows data with a fairly high initial trap population, where the decay is clearly dominated by collisions between trapped cesium atoms. After correcting for a measured residual trap loading rate (see below), we modeled the trap loss with $\dot{N} = -N/\tau - \beta' N^2$, where N is the number of trapped atoms (assumed proportional to fluorescence) and β' can be derived from β by integrating Eq. 1.22 over the measured MOT density distribution. We measured $1/\tau \approx 0$ and values of β consistent with those reported elsewhere [13]. With very low trap populations we measured $N/\dot{N}$ as high as 50 minutes, still consistent with no losses from background collisions. From these data we can place a lower limit of $\tau \gtrsim 60$ minutes for a fairly deep MOT. We expect $1/\tau$ to be extremely small not only because of the low number of background gas molecules, but also because these

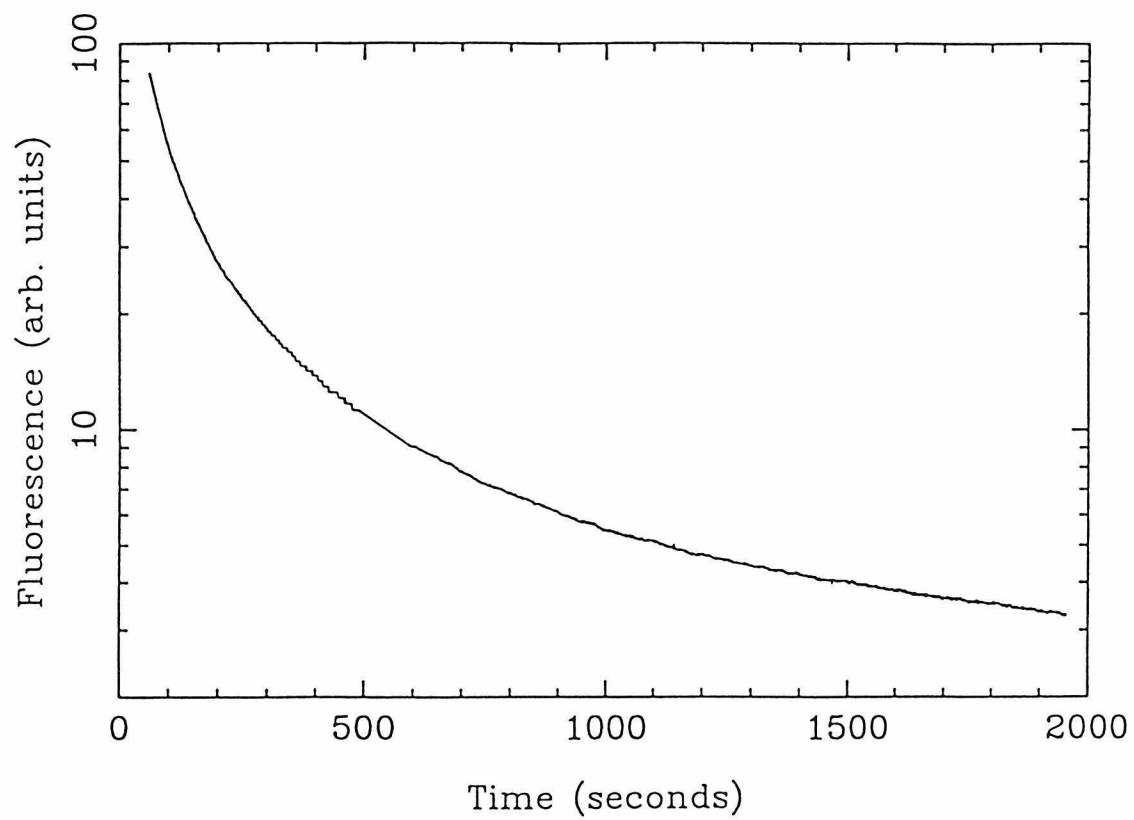


Figure 2.2: Fluorescence vs. time for a MOT with a high initial density after the Cs beam is blocked.

molecules (predominantly hydrogen and helium) are light and moving slowly; thus a typical He-Cs or H₂-Cs elastic collision does not impart enough kinetic energy to a cesium atom to eject it from the MOT.

With low trap populations we also observed an unusual trap loading rate with the cesium beam blocked, of typically ~ 0.2 atoms/sec at magnetic field gradients of 10 G/cm. This was measured by observations of MOT fluorescence with no initial trap loading from the thermal beam ($N(t=0) = 0$). The loading rate was not well correlated with the tightness of the beam shutter seal, suggesting that the loading was not due to leakage around the beam shutter. We believe the loading is due to cesium atoms weakly bound to the inner surface of the cryogenic chamber along with adsorbed helium, which are subsequently released by light-induced desorption. A brief “bake” of the inner chamber at 25 K, to release the adsorbed helium atoms, resulted in a reduction of the trap loading rate by a factor of approximately five.

We also investigated the lifetimes of quadrupole MST’s in our apparatus, using the superconducting coils to generate the required magnetic fields. After first loading a MOT with an axial field gradient of 50 G/cm, the cesium beam shutter was closed and the repumping laser (tuned to the $F = 3 \rightarrow F' = 4$ transition) was blocked, so the MOT trapping lasers (tuned just below $F = 4 \rightarrow F' = 5$) subsequently pumped all the atoms into the $F = 3$ ground state manifold. A few milliseconds later, the trapping lasers were also blocked. Approximately 10 percent of the initial MOT population was thus trapped in a quadrupole MST. After a waiting period T with both lasers blocked, the trapping and repumping lasers were then unblocked in reverse order and the remaining atoms were recaptured into the MOT. By measuring the recaptured MOT fluorescence as a function of T we determined the MST decay time. MST decays at higher field gradients were measured by first forming the MST at 50 G/cm, then ramping up the field gradient (while the lasers were blocked), waiting a time T , ramping the field gradient back down to 50 G/cm, and reforming the MOT. Figure 2.3(a) shows MST decay measurements at 50 G/cm, and Figure 2.3(b) shows MST decay times as a function of axial field gradient. We found that MST decay times depended to some degree on the recent history of the vacuum system, and the

best results were obtained only with frequent 25 K bakes to remove adsorbed helium gas. From this it appears that our MST lifetimes are limited in part by the pressure of the room-temperature part of the inner vacuum chamber, typically 10^{-7} Torr.

We expect the MST losses to be dominated by collisions with background gas atoms, which we assume are predominantly helium. This gives a conservative estimate of the vacuum quality, since He is heavier than H_2 and has a slightly stronger van der Waals interaction with Cs. For small-angle scattering we can write the He-Cs collisional rate equation

$$\frac{1}{N} \frac{dN}{dt} \simeq 2\pi \int \sigma(\theta, v) \theta d\theta dn(v) v \quad (2.1)$$

$$\equiv n_0 \int \tilde{\sigma}(E) dE \quad (2.2)$$

where N is the number of trapped Cs atoms, $\sigma(\theta, v)$ is the van der Waals cross section [26, 27] as a function of the He recoil angle θ and He initial velocity v , $E \simeq m_{He}^2 v^2 \theta^2 / 2m_{Cs}$ is the Cs kinetic energy after a collision, n_0 is the total He density, and $dn(v)$ is the Boltzmann velocity distribution with $\int dn = n_0$. Figure 2.4(a) shows calculated $\tilde{\sigma}(E)$ for background He at temperatures of 4 K and 300 K, using a He-Cs van der Waals coefficient [28] $C_6 = 3.2 \times 10^{-78}$ J m⁶. Figure 2.4(b) shows $\int_{E_0}^{\infty} \tilde{\sigma}(E) dE$, which gives the trap loss rate as a function of trap depth E_0 .

We estimate the trap depth for our coil configuration to be $\sim 20B' \mu\text{K}$, where B' is the axial field gradient in G/cm. This gives a trap decay which depends only weakly on B' , in contrast to the observations in Figure 2.3(b). This discrepancy may be due to additional MST losses arising from scattered laser light in the trapping region. Even a very low level of scattered light, primarily from the repumping laser, would be sufficient to optically pump atoms into untrapped magnetic sublevels at the observed rates. Further, the optical pumping losses are expected to have a strong (albeit somewhat complex) dependence on B' , which may explain the observations. Additional discussion of the dependence of the MST decay rate on trap depth will follow in section 2.2. Using the measured MST decay time at 1 kG/cm to set an upper limit on the background pressure gives $P < 2 \times 10^{-12}$ Torr. It should be noted that the

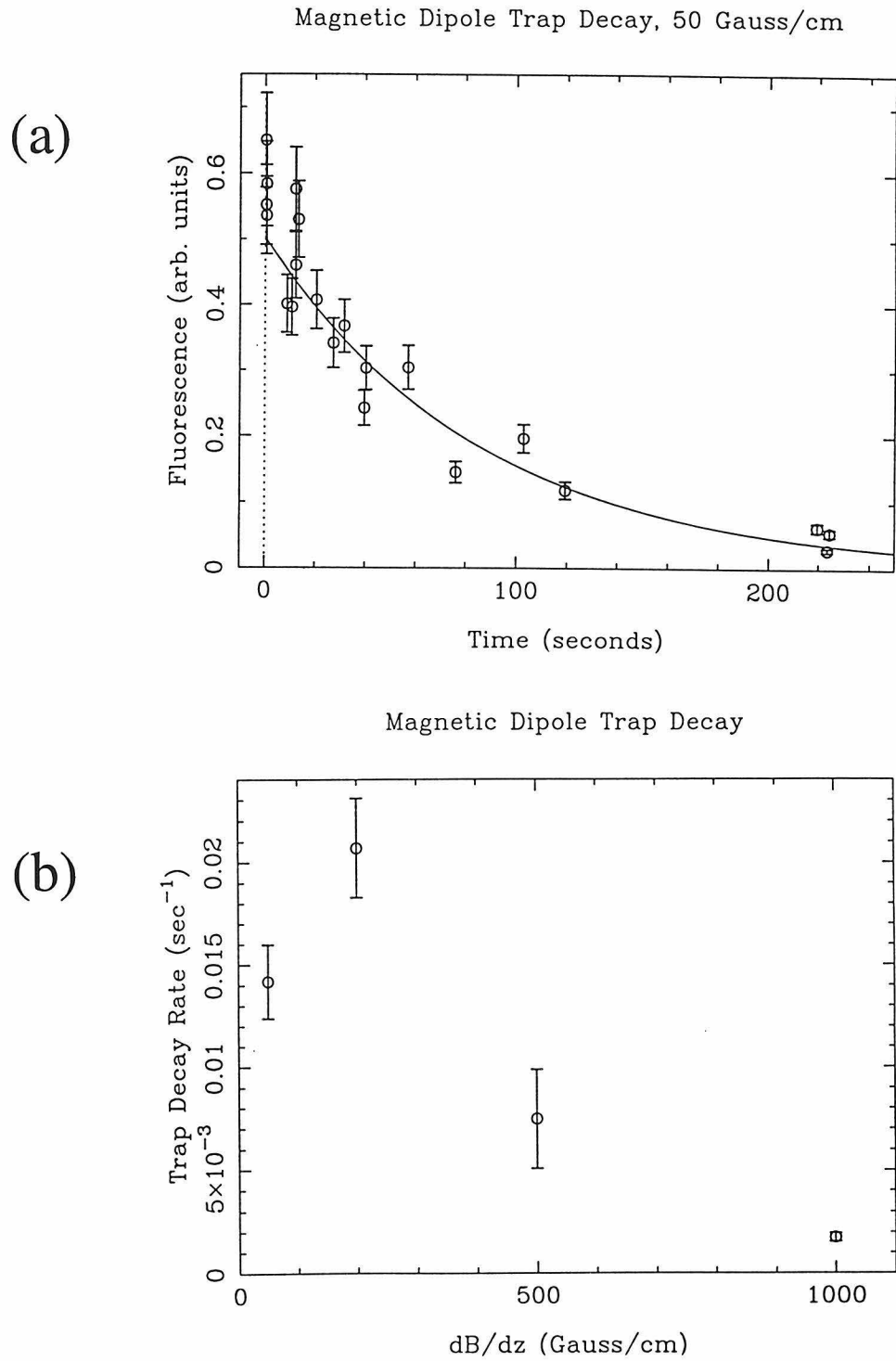
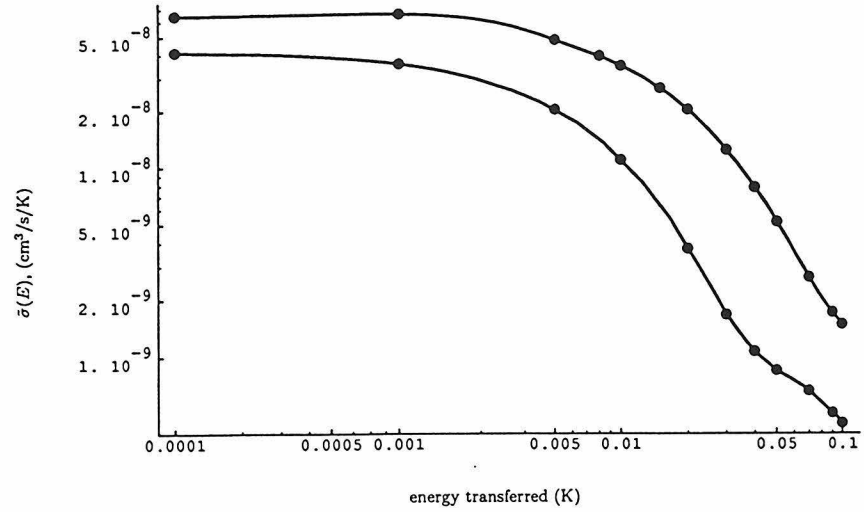


Figure 2.3: MST decay curves. (a) Decay of a quadrupole MST at 50 Gauss/cm. Note that each point corresponds to a decay measurement of a individual MST as described in the text. (b) MST decay rates as a function of trapping field gradient.

Cs-He Differential Scattering Cross Section at Low Energy Transfers for 4K and 300K

(a)



Trap Loss Rate Coefficient for Cs-He Scattering at 4K and 300K

(b)

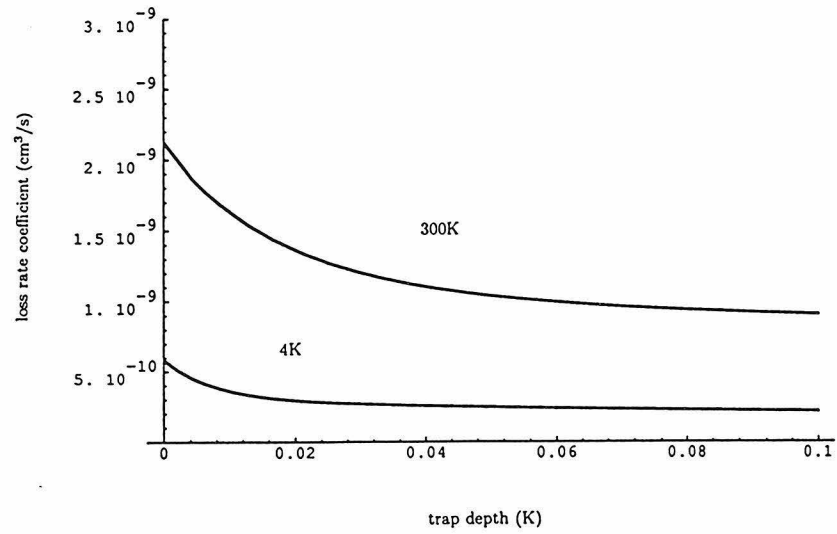


Figure 2.4: Elastic scattering cross sections of He with trapped Cs. (a) The differential cross section $\tilde{\sigma}(E) = \int \sigma(v, E) v P(v) dv$ as a function of the energy transferred to the Cs in K. The lower curve is for 4 K helium, the upper for 300 K helium. (b) The trap loss rate coefficient vs. trap depth due to collisions with helium at 4 K and 300 K. The decay rate of an MST is obtained by multiplying the loss rate coefficient by the background helium density in cm^{-3} .

construction of the cryogenic chamber and the difficulty in determining pressure at cryogenic temperatures made a measurement of the vacuum pressure by other means impractical.

2.2 Version 2

The Version 1 prototype cryotrapping apparatus proved that the technique of cryotrapping is sound, but it had several deficiencies that were serious enough to warrant building a new apparatus. The frequent ion pump failure and high vacuum pressure of the cesium beam oven have already been mentioned. In addition, budgetary constraints had forced the design to have the cryostat mounted in a horizontal position, rather than vertically; thus the liquid helium did not pool around the cold finger but instead flowed past it into the exhaust, wasting expensive liquid helium. The joint between the inner vacuum chamber and the cold finger had a low thermal conductivity, giving an inner vacuum temperature probably no lower than 6-9 K.

The inner chamber of Version 1 was small and compact, which made it difficult to install magnetic field coils and placed drastic constraints on what experiments could be done.

We found that the vacuum pressure in the outer vacuum ($\sim 10^{-3}$ Torr) would only delay frost condensation on the inner vacuum chamber windows by about two days. This condensation could easily be driven off by allowing the system to warm to near room temperature, something which was done periodically anyway to conserve liquid helium, but this required significant down time between experiments, and the constant thermal cycling of the system necessitated continual adjustment of the optics around the chamber.

Finally, since the inner chamber and the cesium oven were one integral vacuum system, changes to the inner vacuum apparatus required exposing the entire cesium-coated apparatus to atmosphere, which inevitably meant dismantling and cleaning it.

All of these problems are resolved in Version 2 of the cryotrap, which is depicted

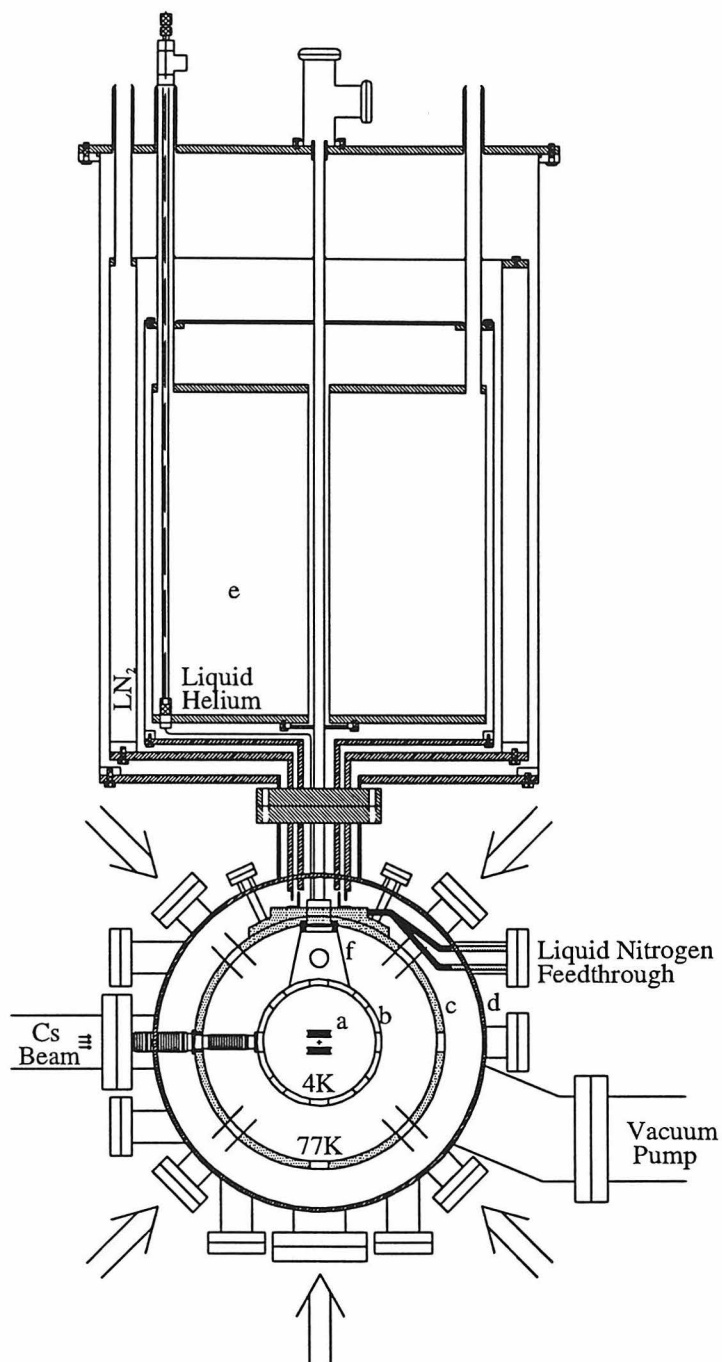


Figure 2.5: Cryotrap Version 2: a) the superconducting magnetic field coils, b) the inner vacuum chamber, c) the radiation shield, d) the outer vacuum chamber, e) the liquid helium dewar. The horizontal trapping laser beams and radiation shield tubes are shown rotated up 90° from the horizontal plane. The downward trapping laser beam enters horizontally and is reflected downward through the trap by the mirror at f.

in Figure 2.5. In this system, the spherical OFHC copper inner vacuum chamber of 6" inner diameter is mounted beneath a vertically mounted cryostat which is fed liquid helium by gravity from a 50 liter stainless steel dewar. The mating surfaces between the cryostat and inner chamber are gold-coated and sandwich an indium foil at high bolting pressure to maximize the thermal conductivity [29, 30, 31]. A stainless steel outer vacuum chamber surrounds both inner chamber and cryostat, again to reduce conductive and convective heating of the inner chamber and dewar. By pumping directly on the outer vacuum chamber with a large-bore turbopump to less than 10^{-7} Torr, the condensation problem is essentially eliminated (we have not seen condensation yet in Version 2, despite months-long cooldowns). Surrounding the inner chamber within the outer chamber is a copper radiation shield that is directly cooled by contact with a tube through which flows liquid nitrogen fed by gravity from a 30 liter dewar. This radiation shield also serves as a thermal anchor for current-carrying leads to the inner chamber and for the bellows from the cesium oven. Both the inner vacuum chamber and the radiation shield are coated with gold to reduce their emissivity, and the outside of the radiation shield in addition is covered with several layers of Superinsulation. The radiation shield is skewered with tubes to provide optical access to the inner chamber while minimizing the total solid angle of the outer chamber seen by the inner chamber. To minimize reflection of 300 K blackbody radiation onto the inner chamber, the inner surfaces of the tubes are covered with black Teflon heat shrink tubing. Thanks to the careful attention to the heat flow into the inner vacuum chamber, it can be kept cool for two weeks with 50 liters of liquid helium, at a temperature of 4.2 K with no further effort or as low as 3.1 K by pumping on the helium exhaust. The radiation shield requires 20 liters of LN_2 daily. The temperature of the inner chamber and radiation shield are monitored with diode thermometers. The mechanical stability of the inner chamber is guaranteed without serious heat loading by a set of guy wires tying it to the radiation shield, which in turn is rigidly mounted to the outer vacuum chamber.

The improved cesium beam apparatus is depicted in Figure 2.6. Every effort was made to allow the beam apparatus to remain under vacuum indefinitely if possible.

The whole of it is connected to the inner vacuum chamber through a gate valve, which can be closed to maintain vacuum in the beam source while the inner chamber is brought to atmosphere for modifications. After roughing the beam source through the still (described later), the source is pumped by a 20 liter/sec Perkin-Elmer ion pump through a right-angle valve, which can be closed to isolate the beam source under vacuum briefly while the ion pump is removed and replaced, if need be. A second roughing port behind the right-angle valve allows the isolated ion pump section to be roughed independently.

The beam source proper consists of a cesium oven section, where a sample of cesium is held at a temperature from 25°C for no beam to up to 125°C depending on the beam intensity desired. The cesium atoms effuse from the oven through a multichannel glass capillary array of 2 mm thickness, 1 cm diameter open aperture, and 10 micron pore diameter (Galileo Electro-Optics Corp., part number C13S20M10), as first demonstrated by Gilbert and Weiman [32]. The open area of the array is 50% nominal and it is made of soda lime glass. The use of a wide array of very narrow channels allows for high collimation of the atomic beam with large beam flux. It also maintains a high pressure differential between the oven and the rest of the vacuum without an additional pump on the oven side. The glass capillary is maintained at a constant 170°C. This temperature, elevated with respect to the rest of the cesium oven, prevents the glass capillary from clogging and also serves to reduce the fraction of Cs_2 in the beam [32]. The cesium is loaded into the oven through a still that allows the fresh cesium to be independently pumped and baked before it is added to the oven, all without breaking the vacuum of the oven. The still is a thin-walled stainless steel half-nipple similar to the one used in the Cs oven of Version 1.

The glass capillary array is followed by a pair of copper baffles. These baffles are each cooled to about -30°C through a high current vacuum feedthrough by a pair of Peltier coolers mounted to a chilled water line. To prevent thermal shorting of the Peltier coolers by condensed water ice, the Peltier coolers and feedthroughs were encased in airtight Lucite canisters containing Drierite. The first baffle doubles as a recirculator designed to recycle waste cesium beam flux [33, 34, 35]. It has a 1 cm

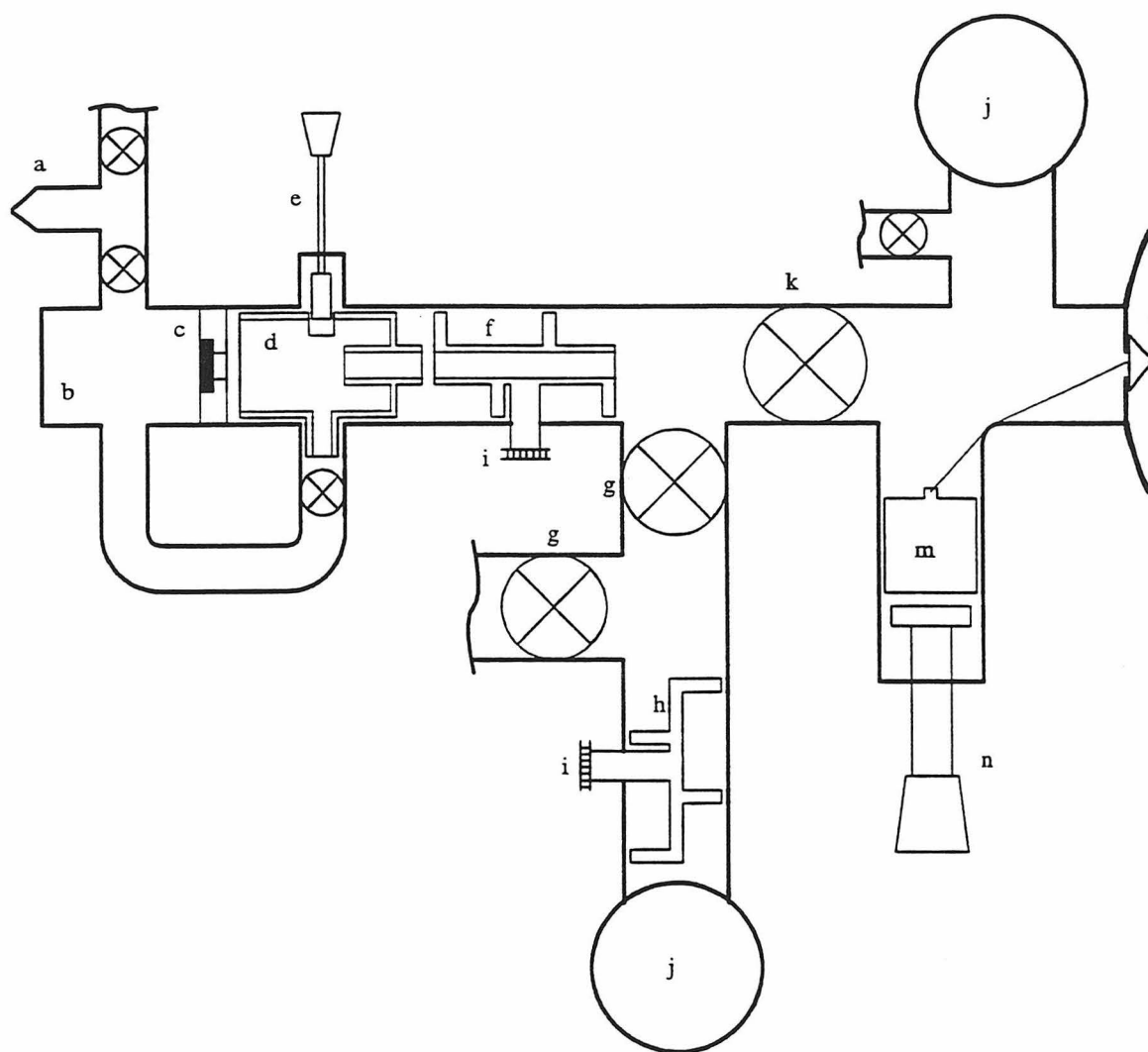


Figure 2.6: The cesium beam apparatus: a) still, b) Cs oven, c) multichannel glass capillary array, d) recirculator/baffle, e) Teflon shutter and linear motion feedthrough, f) collimating baffle, g) right angle valves, h) baffle, i) Peltier cooler and high current feedthrough (not shown for recirculator), j) 20 l/sec ion pump, k) gate valve, l) copper shutter, m) pulley weight, n) linear motion feedthrough. The stainless steel bellows between the interlock chamber and copper shutter are not explicitly shown.

round aperture to pass the collimated part of the cesium beam on to the inner vacuum and intercepts a large fraction of the remaining cesium flux. A Teflon shutter on a linear feedthrough can be used to block the aperture, shutting off the atomic beam. The recirculator is designed such that when it is heated, cesium collected inside it will liquify and drip out from a spout at the bottom into a bellows leading back into the cesium oven. This bellows is shut off with a valve during beam operation. As of this writing the initial charge of cesium to the oven has not run out, and so this feature of the beam apparatus has yet to be tested. The six-way cross that contains the recirculator is bolted to the heated flange holding the glass capillary array and so it is cooled by several wraps of copper tubing containing chilled water.

The second baffle is a central tube to pass the cesium beam to the inner chamber with staggered outer vanes to capture the uncollimated cesium vapor. A third baffle not in the cesium beam path captures cesium vapor before it can reach the beam apparatus ion pump, and is a series of staggered vanes without the central aperture.

The cesium beam apparatus was connected to the inner vacuum chamber via an interlock chamber. This interlock chamber serves two purposes. It contains a second ion pump (20 liter/sec Perkin-Elmer) and roughing port, which allow the inner chamber to be pumped down independently of the cesium beam apparatus after modifications and which also help maintain high vacuum in the inner chamber when the gate valve is closed for ultra-low pressure long trap lifetime experiments. It also contains a shutter to isolate the inner chamber from the interlock chamber. This shutter consists of a polished copper plate that rests in a holder below the cesium beam when in the open position. When pulled into the closed position, it rests against a polished surface surrounding the cesium beam inlet. The shutter is tied to a stainless steel weight in the interlock chamber by a Kevlar thread. The weight pulls the shutter closed by means of a pulley; a linear actuator lifts the weight to allow the shutter to fall into the open position.

The improved vacuum in Version 2 is demonstrated by the quadrupole MST decay curves in Figure 2.7. The lower curve is the decay curve for a field gradient of 50 gauss/cm, while the upper curve is that for 1500 gauss/cm. This data was collected

in the same way as that in Figure 2.3. It is immediately obvious that the decay rate decreases with time, with the decay at long times approaching pure exponential decay with the decay time equal to 4000 sec for both magnetic field gradients, in contrast to the measured dependence on field gradient in Version 1. However, initially the 50 gauss/cm MST decays about three times faster than the 1500 gauss/cm MST, in rough agreement with the Version 1 data. Using a similar analysis to that for Version 1, we find the upper limit to the pressure in the inner chamber to be 3×10^{-13} Torr.

It seems clear that, since the long-time behavior of both traps' decay is consistent with a 4000-second decay time, the interpretation of the field gradient dependence as being solely due to differences in trap depth is not correct. We currently have several hypotheses regarding the nonexponential decay of quadrupole MST's in our cryotrap. One is that the pressure in the inner chamber decays to its equilibrium value after the shutter is closed with a ~ 500 second time scale, and the observed loss rate accurately reflects the pressure dependence on time. The initial differences in the loss rate then reflect the different trap depths for the different field gradients. However, this theory predicts different limiting decay rates at long times for the two field gradients. Another is that when an MST is loaded from a MOT, many of the atoms are in states that are unbound, but require many orbits to escape. A third possibility is that the hotter atoms are evaporating from the MST, the remaining atoms rethermalizing to a lower temperature and thus evaporating more slowly later on. The evaporation rate is expected to be much more strongly dependent on the trap depth for a given trap temperature than the loss rate due to background collisions, and in combination with a 4000-second background collision loss rate would explain the data well. The investigation of the decay of MST's is currently underway in connection with evaporative cooling to Bose-Einstein condensation.

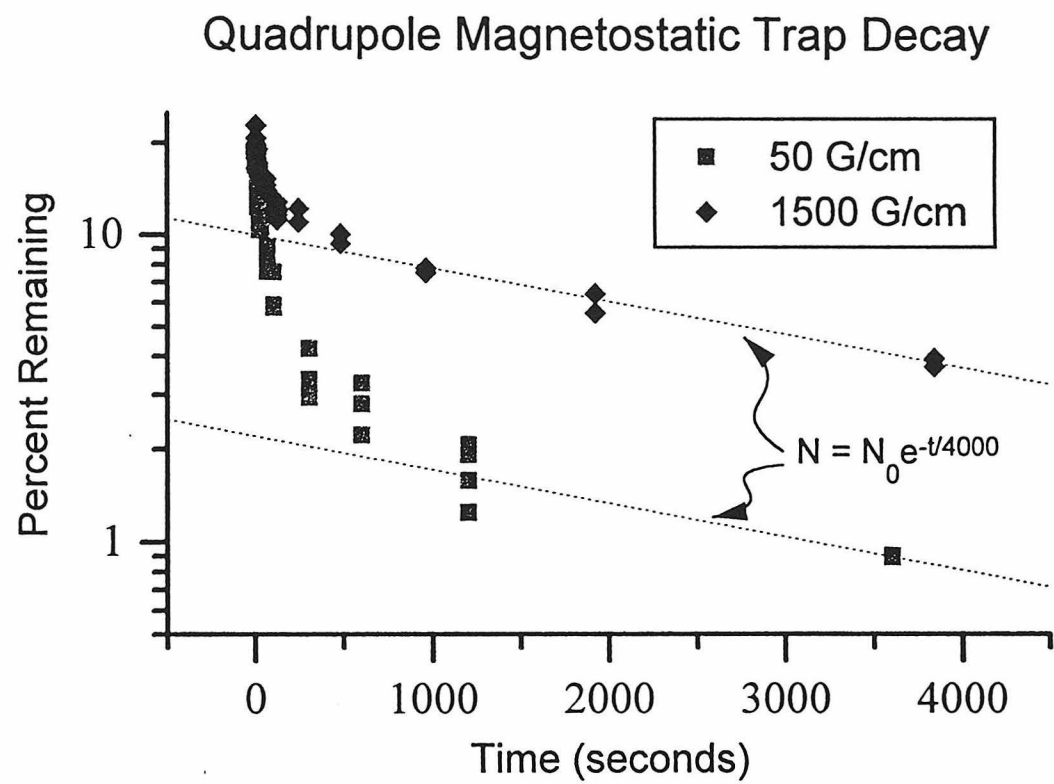


Figure 2.7: The Version 2 MST decay curves.

Chapter 3 Studies of Magneto-optical Trap Stability at High Magnetic Field Gradients with Single Trapped Atoms

3.1 Imaging of Single Trapped Atoms

One serious hurdle in the study of MOT's at high magnetic field gradients is the instability of the MOT to inelastic collisions between trapped atoms. Such collisions are usually sufficiently exothermic to eject both atoms from the trap, and lead to a trap density loss rate that varies as the square of the trap density. At conventional MOT magnetic field gradients, the atomic density is low enough that the lifetime of MOT's against this loss mechanism is of the order of minutes for traps of thousands of atoms and hours for traps of hundreds of atoms. However, at magnetic field gradients of 300-3000 G/cm, the MOT is sufficiently compressed that two atoms will collide and be ejected within a few seconds. Therefore, in order to have sufficient time to study the properties of high magnetic field gradient traps, single atom detection sensitivity is essential.

The dominance of losses due to inelastic collisions in high magnetic field gradient MOT's is shown in Fig 3.1. The data are the lifetimes of one- and two-atom MOT's at various magnetic field gradients. The data were collected by first loading the MOT at a low magnetic field gradient (~ 10 G/cm), ramping up to an intermediate field gradient and waiting for the trap to decay to one or two atoms, and then ramping up to the final field gradient and measuring the lifetime of the MOT. Each data point is derived from several such measurements. The curve through the two-atom data points is the expected trap lifetime from Equation 1.22 with $\beta = 2 \times 10^{-11}$ cm³/s, where the trap radius, too small to be resolved directly, is scaled from the radius of

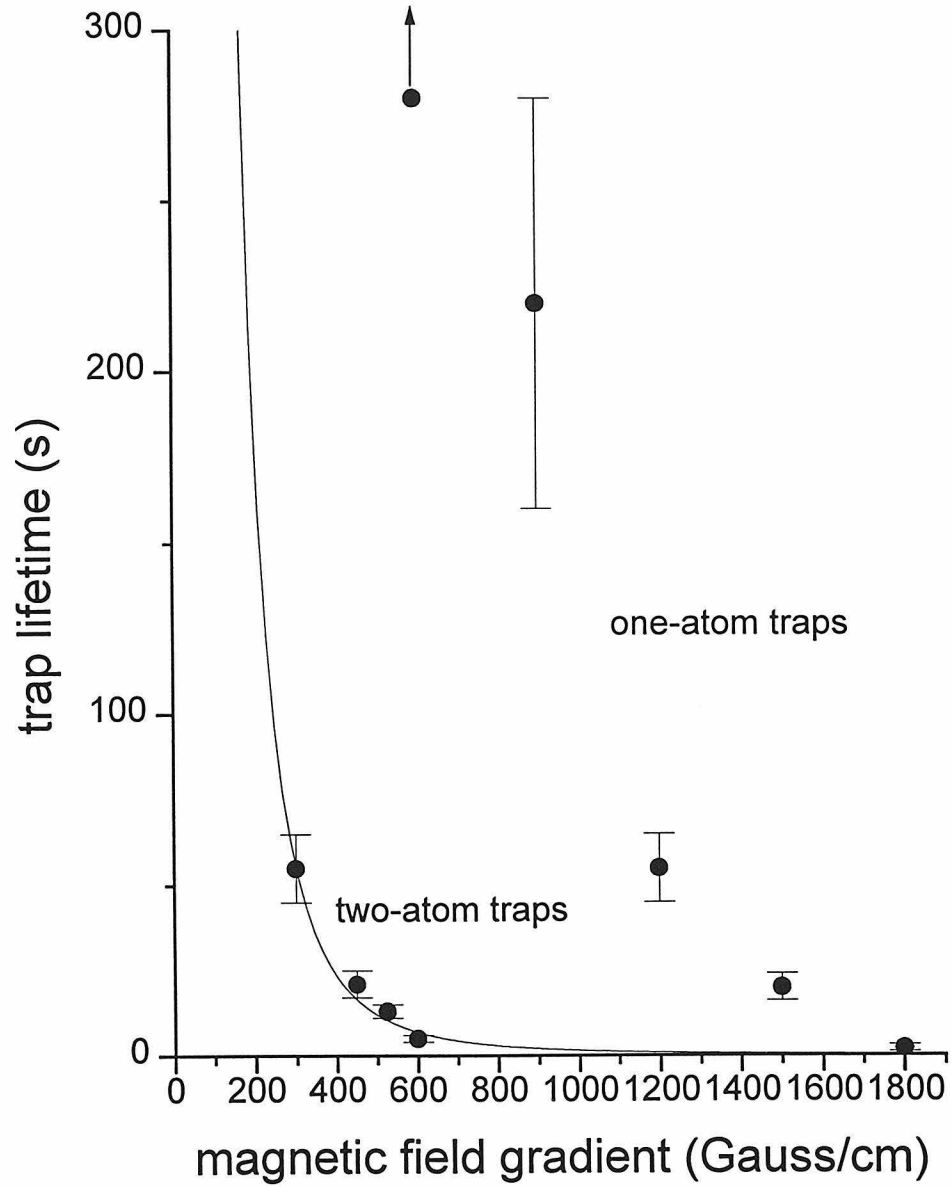


Figure 3.1: Lifetimes of one- and two-atom MOT's as a function of magnetic field gradient. The curve through the two-atom data is the predicted lifetime against inelastic trap-trap collisions with $\beta = 2 \times 10^{-11} \text{ cm}^3/\text{s}$. The trapping laser detuning was 7.1 MHz and the laser intensity was $6.1 \text{ mW}/\text{cm}^2$ per beam.

the MOT at the low magnetic field gradient (we show in Chapter 4 that the trap spring constant does scale linearly with field gradient over these high field gradients). The value of β used is typical for the laser parameters of the measurement. While it is admittedly questionable to assign a density to two atoms, this plot confirms that inelastic trap-trap collisions are indeed responsible for the rapid decay of high magnetic field gradient multi-atom traps. The dependence of the single-atom MOT lifetime on magnetic field gradient will be discussed shortly in connection with measurements of the highest stable trapping magnetic field gradient; we will show that the decreasing trap lifetime versus magnetic field gradient is due to the increasing rate of evaporation of atoms over the shrinking trap potential barrier. First we describe the technique used to detect single trapped atoms.

Individual neutral atoms adsorbed on a solid substrate are routinely resolved by sensitive scanning tunnelling microscopes [36] and individual ions in a Penning trap have been resolved in several experiments [37, 38, 39]. Individual trapped neutral atoms were first observed by Hu *et al.* [40] using an avalanche photodetector (APD) in a room-temperature vapor cell trap. While successful at detecting the quantized jumps in fluorescence associated with individual atoms falling into and out of the trap, Hu's method has several drawbacks for our work. In a vapor cell trap, there is no good way to control the loading rate of the trap independently of the laser and magnetic parameters, as there is with a shuttered atomic beam. Also, the vapor fluorescence creates a background light level that can obscure the trapped atom fluorescence. An APD yields only an integrated fluorescence, rather than an image, so that focussing and background light baffling are tricky. Finally, in anticipation of the MOT dynamics experiments to be described in the next chapter, the APD technique has the disadvantage that the atom's motion cannot be observed directly, but must be inferred by partially masking the trap image as the trap moves, discarding precious photons.

Our approach instead has been to use a highly sensitive intensified charge-coupled device (ICCD) camera to image the trap. Essentially, an ICCD consists of an intensifier, which multiplies the incident photon flux by several orders of magnitude

while preserving the image resolution, followed by charge-coupled device pixel array, which detects the image for conversion to a video signal. The intensifier is an imaging multichannel electron multiplier with an input photocathode to convert incident photons to electrons prior to multiplication and an output phosphor to convert the multiplied electrons back to photons prior to photodetection by the CCD. The intensifier is optically coupled the CCD by a fiber optic minifier. Our specific camera is a Xybion ISG-350 camera with a GEN-III intensifier sensitive at 852 nm.

The drawback of an ICCD camera is that its sensitivity depends on a number of factors, most notably the intensifier gating duty cycle, the CCD gain and black level, and the gain of the intensifier tube. It is the intensifier tube gain which is most problematic. With its very high gain, the intensifier tube can be damaged by exposure to bright light, and so is built with protection circuitry that automatically reduces the gain if it senses too much incident light. This makes reproducible control of the camera sensitivity difficult. Nonetheless, as Figure 3.2 shows, the difference between one atom, two atoms, and no atoms is clearly visible. Furthermore, in the experiments described here, absolute fluorescence is less important than trap position, which can be calibrated precisely.

Figure 3.3(a) is a time series plot of the observed trap fluorescence made by integrating the outputs of the pixels exposed to the MOT. The distinct levels associated with no, one, and two atoms are clearly visible, and there is a hint of the level associated with three atoms, although the very short lifetime of multi-atom traps tends to broaden this level, since each point corresponds to 2 seconds of ICCD frame averaging. The histogram plot (Fig. 3.3(b)) shows the existence of no, one, two, and three atom traps even more clearly. The relative time spent at each level is of no direct physical consequence, because the trap was loaded and reloaded several times by shuttering and unshuttering a small continuous loading rate from an atomic beam. However, it is significant that two atoms frequently decay directly to no atoms, and that a single atom frequently does not decay to no atoms until a second atom enters the trap. This is further evidence for the high instability of the MOT to internal inelastic collisions. The background trap loss rate is due either to collisions with background

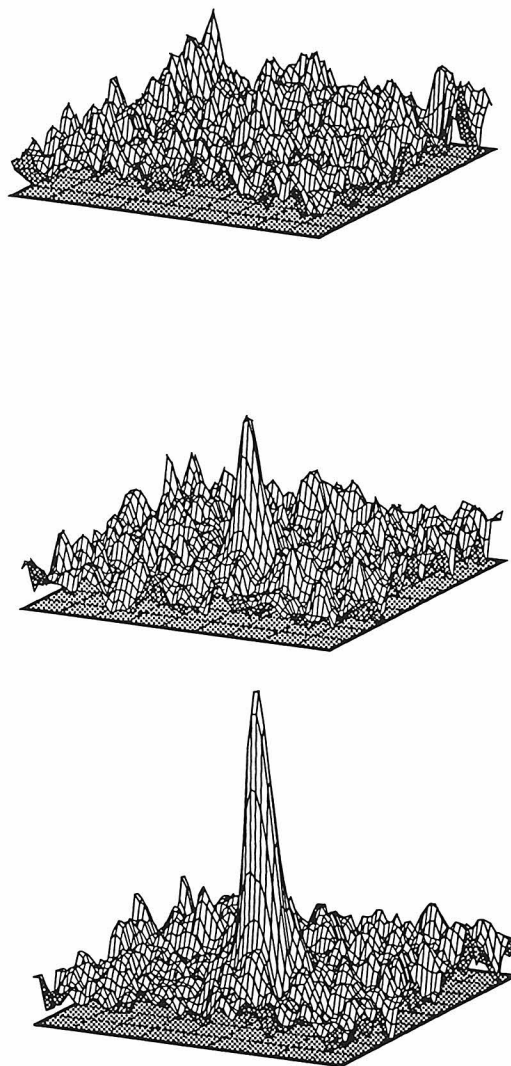


Figure 3.2: ICCD images showing no atoms, one atom, and two atoms.

gas atoms, which of course are rare in our cryogenic UHV system, or evaporation, which we discuss in detail in the remainder of this chapter. Single atom traps have been measured to have a lifetime of at least five minutes, as Figure 3.1 shows.

3.2 Measurements of the Stability of Single-Atom MOT's

Our first application of single-atom photosensitivity was the measurement of the limits of stability for MOT's with respect to laser intensity, laser detuning, and magnetic field gradient. The technique was fairly simple. First the laser detuning and intensity were set, and the magnetic field gradient was taken to a low value in order to load the MOT. Höpe *et al.* [41] have shown that for vapor-loaded MOT's, the trap loading efficiency is drastically reduced at high magnetic field gradients due the Zeeman force being active over a smaller volume, and a similar argument is valid for MOT's loaded from a thermal atomic beam. Then, once the MOT was loaded, the magnetic field gradient was ramped to an intermediate value until the trap decayed to a single atom. Finally, the magnetic field was ramped upward until the atom disappeared from the trap. Because there were other loss mechanisms in the trap besides instability, such as occasional transient laser noise, and because the single atom lifetime is a function of field gradient, the measurement was repeated several times for each laser setting. The data points would typically cluster around a high field gradient value, with outlier points at lower gradients. These outliers were discarded, and the cluster points averaged, to obtain the estimated maximum field gradient. We take "maximum" field gradient to mean that field gradient for which the trap lifetime is about one second, the shortest-lived trap which we can detect. For very large laser detunings where the trap loading rate was low even at low magnetic field gradients, it was necessary to load the MOT at a lower laser detuning, and then ramp up the detuning to the final value before ramping up the magnetic field gradient.

The data are presented in Figure 3.4. The laser intensity per trapping beam was

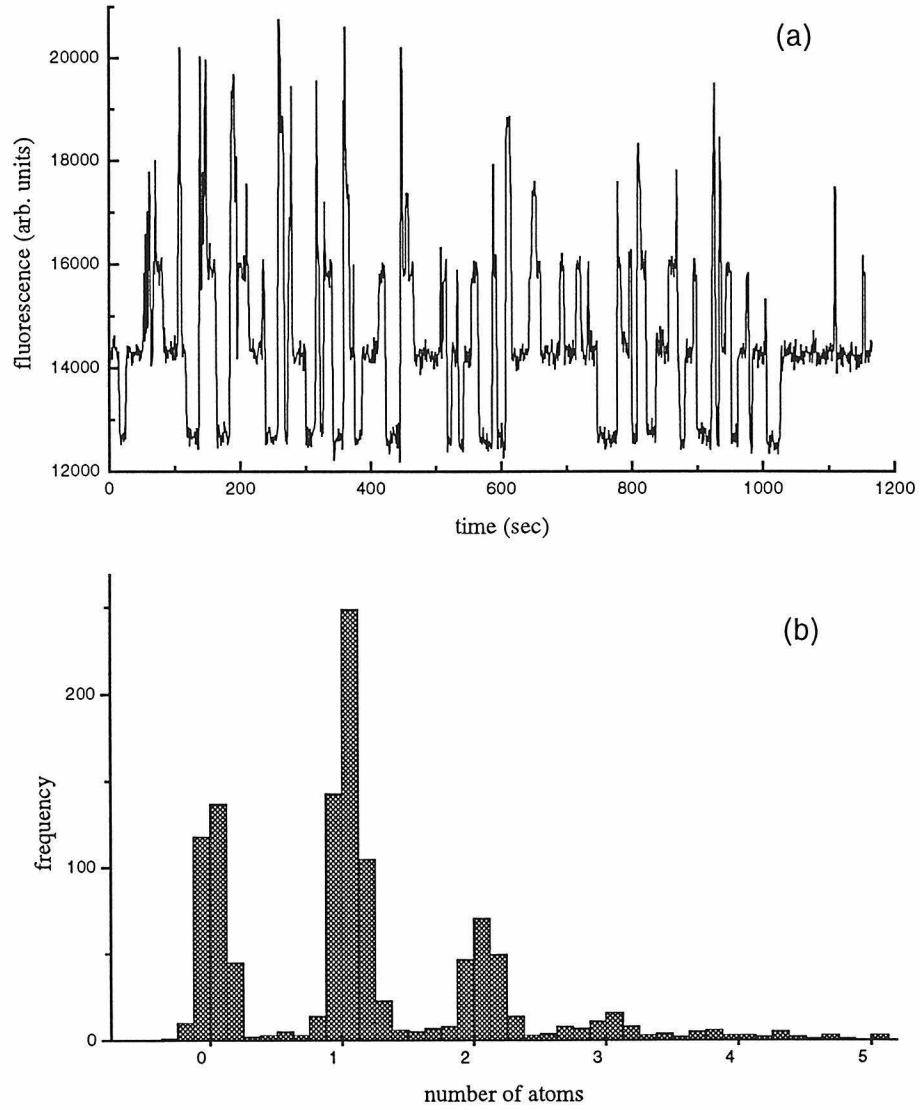


Figure 3.3: (a) Time series of fluorescence from a high magnetic field gradient MOT. Contributions to the fluorescence from individual atoms are clearly visible. (b) Histogram plot of fluorescence levels in the time series plot, showing resolution of zero, one, two, and three atom MOT's.

varied from a minimum value of $.39 I_{\text{sat}}$, below which the detected atomic fluorescence was too low to resolve individual atoms, to a maximum value of $104 I_{\text{sat}}$, the limit set by the power available from the trapping diode laser. The laser detuning was varied from the minimum value that would make a MOT to the maximum that would make a MOT or 100 MHz, whichever was less. The 100 MHz detuning limit was set by the trapping laser heterodyne locking circuit, which is described in Appendix A. Based on the scatter in the measurements of the maximum field gradient and on the observed variation with such parameters as the laser alignment, we estimate that the error in the quoted values is 30%.

Our magnetic field gradients were at first limited to less than about 3 kG/cm, or 2 A coil current. At higher currents, the magnetic field coils were driven into the normal state and could not be operated without large voltages and heating. This limit was much lower than we expected based on calculations and on measurements of the critical current of the superconducting wire in a liquid helium dewar. Analysis of the apparatus indicated that the copper leads to the superconducting coils were being heated by the high current and that this heat was what drove the coils normal. This information was used in the design of the subsequent set of leads, which have higher maximum currents. This limiting magnetic field gradient prevented stability measurement for only a few parameters and so was not a significant inconvenience. All data that were limited by this effect were remeasured.

The repumping laser sweep was a potential source of trap instability, due to its finite sweep frequency through resonance with the $F = 3 \rightarrow F' = 4$ transition (typically 10 MHz sweep amplitude at 2 kHz sweep frequency). This effect was checked by varying the sweep amplitude and observing the effect on the maximum stable magnetic field gradient. The effect of reducing the sweep to nearly zero from a higher value was only a 10% increase in the maximum field gradient. We took all data at the lowest possible sweep amplitude. The data at high laser intensities was obtained by telescoping the trapping laser beam to a smaller radius (6 mm, from 10 mm) in order to focus the laser power. Comparing data with identical laser parameters for the two beam radii yields no observable effect of beam radius.

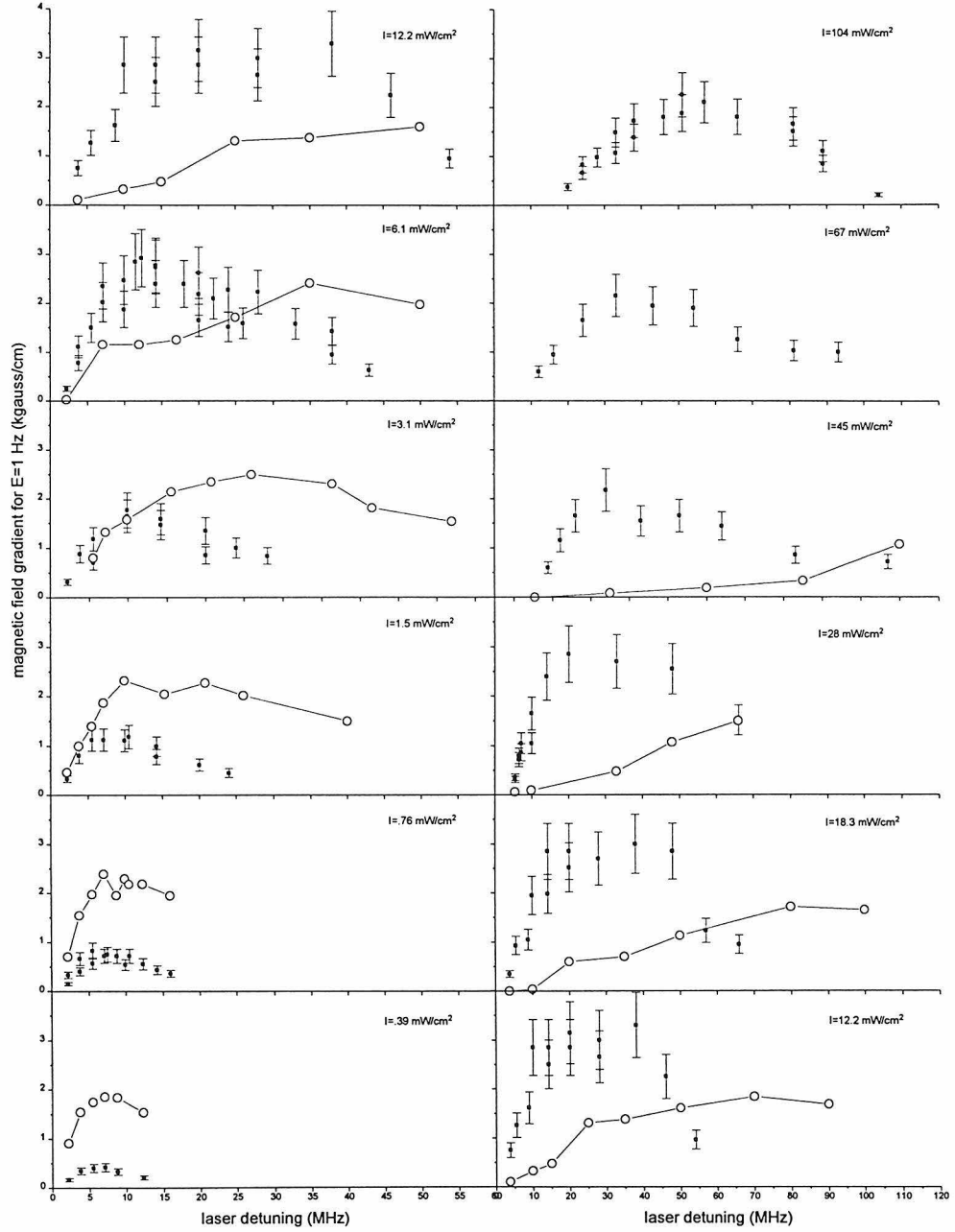


Figure 3.4: The maximum stable magnetic field gradients for magneto-optical traps. The error in each point is estimated at 30%. The data joined by the straight lines are the results of the numerical model.

These data, along with those plotted in Figure 3.1, are the first evidence for evaporation of atoms from magneto-optical traps, which we denoted in the trap population equation (Eq. 1.22) as $\dot{n} = -En$, the other terms being insignificant. The data as plotted in Figure 3.4 may be considered to be contour plots of E as a function of the magnetic field gradient, laser intensity, and laser detuning, with the data tracing the curve $E = 1$ Hz.

3.3 Analysis of Stability Data; A Fokker-Planck Equation Approach

3.3.1 Overview

The equation of motion for an atom in a MOT can be approximated by a damped harmonic oscillator, if the velocity and displacement of the atom are sufficiently small. Because the sources of the cooling and trapping forces are spontaneous optical processes, the damping is accompanied by a stochastic force which leads to a diffusion of the atomic momentum. Thus, the atomic motion is actually Brownian motion in an external potential. It is the competition between this stochastic heating and the Doppler or polarization-gradient cooling that determines the MOT temperature. The stochastic part of the atomic motion will be ignored in the dynamical studies of trapped atoms described in the next chapter, since what was measured there was the motion of the mean particle displacement.

However, the escape of an atom from a MOT is driven entirely by this diffusion, and so the stochastic nature of the force must be more fully considered in this case. This is the problem of diffusion of particles over a potential barrier, which has been often treated in the literature [42, 43, 44]. We begin by deriving the stochastic equation of motion for the probability density of the atomic position and momentum, and show that it reduces to the Smoluchowski equation by virtue of the overdamping of the harmonic oscillator equation. We then derive a conservative quasipotential from calculations of the Zeeman force based on the optical Bloch equations in a magnetic

field, and use the formula for the diffusion constant to find the rate of diffusion of the atom over the potential barrier by use of the Kramers rate equation.

3.3.2 Derivation of the Trap Loss Rate Equation

The stochastic partial differential equation for diffusion of particles in one dimension in the presence of an external force $F(z)$ is the Kramers equation:

$$\frac{\partial W}{\partial t} = \left[-\frac{\partial}{\partial z}v + \frac{\partial}{\partial v} \left(\alpha v - \frac{F(z)}{m} \right) + \frac{\alpha k_B T}{m} \frac{\partial^2}{\partial v^2} \right] W. \quad (3.1)$$

Here W is the probability density and α is the damping coefficient in the trap (see Eq. 1.4). The modelling of the atomic motion as one-dimensional is justified in the next subsection. If the motion of atoms in the trap is heavily damped, this allows a considerable simplification of the stochastic equation, which reduces to the Smoluchowski equation:

$$\frac{\partial W}{\partial t} = \frac{1}{m\alpha} \left[-\frac{\partial}{\partial z} F(z) + k_B T \frac{\partial^2}{\partial z^2} \right] W. \quad (3.2)$$

The stochastic equation is now of a single variable z . If this force is derived from a potential $V(z)$, then the stationary solution of Equation 3.2 ($\partial W/\partial t = 0$) implies the existence of the conserved diffusion current j , where

$$j = -\frac{k_B T}{\alpha} \exp(-V(z)/k_B T) \frac{d}{dz} (W \exp(V(z)/k_B T)). \quad (3.3)$$

This equation can be integrated between two points a and b to yield the formula

$$j = \frac{k_B T}{\alpha} \frac{W e^{V(z)/k_B T} \Big|_b^a}{\int_a^b e^{V(z)/k_B T} dz}. \quad (3.4)$$

The standard approach to diffusion over a potential barrier proceeds from Eq. 3.4 by assuming that the particles are initially trapped in a local minimum of the potential in a quasiequilibrium state, with a high potential barrier separating them from a nearby global minimum, and calculates the rate of particle diffusion over the barrier

into the global minimum. This model is not applicable to the MOT, which has a potential as shown in Figure 3.5(a). In this case, the trap center is the global minimum, and the atoms diffuse from there to unbound free states at the maximum of the potential. Therefore, the derivation for escape from the MOT will differ from the typical approach, and we will obtain a variation of the standard Kramers rate equation. We make the *ad hoc* assumption that an atom which ventures up the trap potential to a point one trap temperature below the maximum potential will be lost, and so the trap loss rate is given by the rate at which atoms reach this point. We label this point b , and the center of the trap as a . Since the potential in the vicinity of a is harmonic ($V(z) = \frac{1}{2}\kappa z^2$), and we assume a Maxwellian quasistatic equilibrium around a , the number of particles ν_a near a is related to W at a by

$$d\nu_a = W_a \exp(-V(z)/k_B T) dz, \quad (3.5)$$

$$\nu_a = W_a \int_{-\infty}^{+\infty} \exp(-\kappa z^2/k_B T) dz, \quad (3.6)$$

where the integral is taken over $\pm\infty$ since the main contribution comes from the region near $z = 0$. This has the solution

$$\nu_a = W_a \sqrt{\frac{2\pi k_B T}{\kappa}}. \quad (3.7)$$

If we now assume that the trap depth is much greater than the trap temperature, we can evaluate j . In the numerator of j the contribution at b can then be treated as negligible, and j can be approximated as

$$j = \frac{k_B T}{\alpha} \frac{W_a}{\int_a^b e^{V(z)/k_B T} dz}. \quad (3.8)$$

The atom escape rate from the trap P is then given by the diffusion current from a to b divided by the atom number at a , or

$$P = \frac{j}{\nu_a} = \frac{\sqrt{\kappa k_B T / 2\pi \alpha^2}}{\int_a^b \exp(V(z)/k_B T) dz} \quad (3.9)$$

$$= \frac{\sqrt{\kappa D_{zz}/2\pi\alpha^3}}{\int_a^b \exp(V(z)\alpha/D_{zz})dz}, \quad (3.10)$$

where we have expressed the trap temperature in terms of the diffusion and damping coefficients:

$$k_B T = D_{zz}/\alpha. \quad (3.11)$$

Having established the formalism with which the trap loss rate can be related to the trap potential and damping and diffusion coefficients, the next step is to calculate these dynamical parameters.

3.3.3 Assumptions in the Trap Model

We first assume that no polarization gradient forces are active in the MOT, only Doppler and Zeeman forces. This assumption is assumed to be valid because at high field gradients, the atoms spend sufficient time at high enough magnetic fields to turn off the polarization gradient (P-G) mechanisms. We can justify this assumption in the following way. The radius r_l within which polarization gradient mechanisms are effective is given by [17]

$$\mu_B B' r_l = C_l \frac{\hbar \Omega^2}{\delta}, \quad (3.12)$$

which represents the condition that the Zeeman shifts exceed the light shifts of the ground states (Ω is the Rabi frequency and δ is the laser detuning). The coefficient C_l is a numerically evaluated parameter of order unity that accounts for the atomic level structure, and is ~ 0.9 for cesium. Combined with the restoring force due to induced orientation effects, this formula can be used to estimate the depth of the P-G part of the trap. Although the formula is accurate for low intensity, measurements described in the next chapter indicate that the force saturates, giving a maximum P-G depth. Furthermore, those measurements indicate that the spring constant is roughly consistent with the Zeeman model, implying that polarization gradient trapping is not very effective. The temperature of atoms in the trap can be calculated to be much greater than this depth, but much less than the depth due to Zeeman forces,

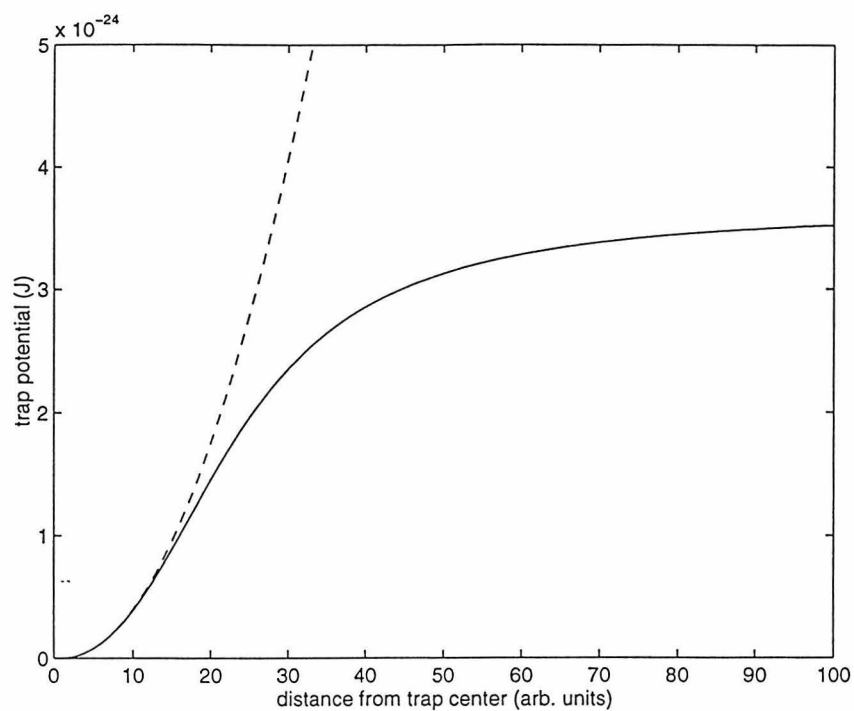
so a simple Zeeman model is appropriate.

We have chosen to exclude polarization gradient effects from the model by considering a $J = 0 \rightarrow J' = 1$ atom with atomic parameters appropriate for cesium (mass, resonant frequency and radiative lifetime). Since the $J = 0$ ground state has no Zeeman sublevels, the polarization gradient effects described in the Introduction do not act upon it. This reduction of the number of levels shortens the computational time considerably, at the cost of ignoring possible modifications to the momentum distribution due to multilevel effects other than polarization gradient cooling. Most importantly, the differing Clebsch-Gordan coefficients for $\Delta J = \pm 1$ transitions for states with large magnetic quantum numbers can lead to strong correlations between successive momentum steps in a one-dimensional $\sigma_+ - \sigma_-$ molasses [45, 46]. We will discuss the implications of this effect when comparing the data to our model.

We assume that the diffusion coefficient is independent of the atom's velocity and of the magnetic field. The first assumption seems to be approximately confirmed by experiments by Hoogerland *et al.* [47] in metastable neon in a 1-D $\sigma_+ - \sigma_-$ molasses at high laser intensity. It should be noted that their experiment was in a molasses, which has no magnetic field to shut off the polarization gradient cooling. The effect of the magnetic field on the diffusion coefficient is more complicated. Walhout *et al.* [48] showed that if the Landé factors of the ground state g_g differs from that of the excited state g_e , then the addition of a magnetic field will both shift the velocity dependence of the diffusion coefficient to the equilibrium velocity and change its shape (if $g_g = g_e$ then only the velocity shift occurs). In cesium the ground and excited states have different Landé shifts. Our use of a $J = 0 \rightarrow J' = 1$ atom in the calculations precludes the inclusion of this effect.

We also assume that the damping coefficient is uniform throughout the MOT, and that the trap motion is overdamped. The presence of the magnetic field will have a negligible effect on the Doppler cooling near the trap center, but as the atom moves out to larger magnetic fields, the cooling force will be modified. The approximation that leads to Eq. 3.2 requires that the motion of the atom be strongly overdamped. This is not the case near the center of a trap with high magnetic field gradients

(a)



(b)

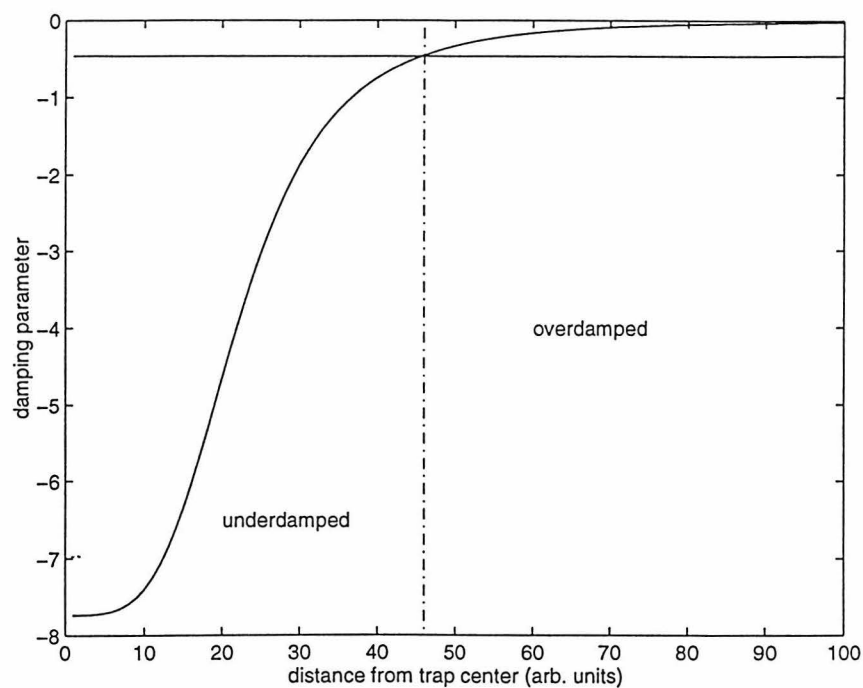


Figure 3.5: (a) A typical trap potential, calculated using the optical Bloch equations. The harmonic potential at the center of the trap is included for comparison. (b) A theoretical plot showing the trap behavior goes from underdamped near the center to overdamped away from the center.

and without polarization gradient cooling, but it is easy to show that as the atom moves out to regions of large magnetic field, the trapping force drops off much more quickly than the cooling force, yielding overdamped motion at the outer regions of the MOT. The cooling force and trapping force in the Doppler theory were given in Eqs. 1.4 and 1.7, in the limit that the Doppler shift and Zeeman shifts are small compared to the natural linewidth. If we generalize the $J = 0 \rightarrow J' = 1$ model used to derive the Zeeman force (Fig. 1.4) to include a small velocity but arbitrary magnetic field, we get for the force from the leftward going beam

$$F_+ = \hbar k \frac{\Gamma}{2} \frac{I}{I_0} \frac{1}{1 + 6I/I_0 + [2(\delta - kv + g\mu B'x/\hbar)/\Gamma]^2}, \quad (3.13)$$

while the force due to the rightward going beam is

$$F_- = -\hbar k \frac{\Gamma}{2} \frac{I}{I_0} \frac{1}{1 + 6I/I_0 + [2(\delta + kv - g\mu B'x/\hbar)/\Gamma]^2}. \quad (3.14)$$

It is easy to see that the resultant force from these two beams will be the same as the Doppler force derived before, but with v replaced by $v' = v - g\mu B'x/\hbar k$, so long as $v' \ll \Gamma$ (compare Eqs. 3.13 and 3.14 with Eqs. 1.1 and 1.2). Thus, at every value of $B'x$ there is an equilibrium value of v , and for small deviations around this velocity the damping coefficient has the same value it has for small values near the trap center. Meanwhile, the trapping potential typically has the form shown in Figure 3.5(a), which is diminished with respect to the harmonic potential at the center of the trap as the displacement is increased. Figure 3.5(b) shows how the trap behavior changes from underdamped to overdamped with increasing displacement for a laser intensity per beam of $.39I_{\text{sat}}$, a laser detuning of 2.2 MHz, and the magnetic field gradient near the stability limit. The plotted damping parameter is the effective spring constant $\kappa'(z) = F(z)/z$ in arbitrary units versus distance from the center of the trap. The distance scale is the same as that for Fig. 3.5a, which was calculated using the same physical parameters. The horizontal line represents the value of κ' for critical damping. The dashed line shows the position in the trap where the atomic

motion becomes overdamped. Clearly, much of the inner portion of the trap is not in the regime which justifies the use of Eq. 3.2 if only Doppler forces are considered. Of course, this model also ignores the fact that polarization gradient cooling at the center of the trap makes the atomic motion there overdamped. In the interest of having a clear and simple model to which the data can be compared, the question of how to most accurately model the damping coefficient in all regions of the trap was dropped in favor of using Eq. 3.2 with the calculated value of α due to Doppler forces.

Although we model a fully three-dimensional MOT, we assume that only the motion along the z axis (trap axial symmetry axis) need be considered. It is along the z axis that the magnetic field gradient has its largest value. This is therefore the direction along which the trap depth is a minimum, and our Monte Carlo simulations of very high magnetic field gradient traps have confirmed that the atoms always escape along this axis.

We assume that the laser intensity is spatially uniform. This is not the case in this experiment, which uses laser beams of Gaussian profile that fall off in intensity away from the trap center. However, it is a good approximation to model the laser intensity as constant, for two reasons. First, since the atoms escape along the z axis, we may consider the trapping force as being due primarily to the z beams, whose intensity of course does not vary along their propagation axis. The effect of the orthogonal lasers is primarily through the saturation of the transition, which drops to a minimum of $2I/I_0$ far from the trap center. Second, at the very high magnetic field gradients used in this experiment, the atoms are Zeeman-detuned well out of resonance with the trapping laser before the laser intensity has dropped significantly. As mentioned before, we have found no variation of maximum stable magnetic field gradient with laser beam waist size, suggesting that the spatial variation of laser beam intensity is not important.

It should be clear at this point that an exact calculation of the optical forces everywhere in a cesium MOT is not feasible. The spring constant and damping and momentum diffusion coefficients in a MOT in general will depend on the laser intensity

and detuning, the dimensionality of the trap, the allowed transitions of the atom, the polarization of the laser field, the magnetic field gradient, and whether there are interactions between trapped atoms, and in addition can vary spatially within the trap. For example, in one dimension and at high laser intensity, the momentum diffusion will differ for a $J = 0 \rightarrow J' = 1$ atom irradiated by counterpropagating linearly polarized laser beams and for such an atom irradiated by counterpropagating $\sigma_+ - \sigma_-$ laser beams. The reason for this is that in the former case the atom can absorb a photon from one laser beam and decay via stimulated emission due to the other beam, thus transferring two photon momenta to the atom, while in the latter case this process is not possible. A MOT uses the $\sigma_+ - \sigma_-$ configuration, and so in one dimension will not have the stimulated momentum diffusion. However, in three dimensional MOT's there is nothing forbidding redistribution of photons between laser beams of orthogonal propagation vectors, and so this diffusion will occur. For this reason, the momentum diffusion does not saturate at high intensity for a 3-D MOT.

The diffusion coefficient also varies over a cubic optical wavelength as the local laser polarization and intensity changes, so we average both the force and the diffusion over a cubic laser wavelength. Since we do not control the relative phases of the trapping lasers in our experiment and they may vary significantly over the lifetime of the MOT, we randomly choose a set of relative phases and average the optical forces with respect to them.

3.3.4 Calculations of the Force and Diffusion Using the Optical Bloch Equations

We calculated the optical forces on an atom in a three-dimensional laser field and magnetic field gradient using a Pentium PC computer running MATLAB. Our calculations of the forces and of the diffusion coefficients of the atom utilize the technique of Mølmer [49], which we now describe. The Hamiltonian for the internal energy of

the atom is

$$H_{\text{atom}} = \hbar\omega_0 \sum_m \sigma_+^{(m)} \sigma_-^{(m)} \quad (3.15)$$

where $\sigma_{\pm}^{(m)}$ is the raising/lowering operator for the m^{th} excited level. The Hamiltonian for the interaction of the atom with the (classical) optical field is

$$\begin{aligned} H_{\text{int}} = & \hbar\Omega(\sigma_+^{(1)} e^{i(kz+\phi(1)-\omega_L t)} + \sigma_-^{(1)} e^{-i(kz+\phi(1)-\omega_L t)} \\ & + \sigma_+^{(-1)} e^{i(kz+\phi(2)+\omega_L t)} + \sigma_-^{(-1)} e^{-i(kz+\phi(2)+\omega_L t)} + \dots), \end{aligned} \quad (3.16)$$

$$(3.17)$$

where ϕ is the relative phase of each laser beam. For clarity, we present the 1-D formula here; the full 3-D formula as implemented in the MATLAB code is given in Appendix B. We can suppress the explicit time dependence of H_{int} and simplify the ensuing equation of motion for the density matrix ρ by going to an interaction picture with an unperturbed Hamiltonian given by

$$H_0 = \hbar\omega_L \sum_m \sigma_+^{(m)} \sigma_-^{(m)}. \quad (3.18)$$

The interaction Hamiltonian in this new picture is given by the transformation

$$\begin{aligned} H_I = & e^{iH_0 t/\hbar} (\hbar\omega_0 \sum_m \sigma_+^{(m)} \sigma_-^{(m)} + \\ & \hbar\Omega(\sigma_+^{(1)} e^{i(kz+\phi(1)-\omega_L t)} + \sigma_-^{(1)} e^{-i(kz+\phi(1)-\omega_L t)} + \dots)) e^{-iH_0 t/\hbar} \end{aligned} \quad (3.19)$$

$$= \hbar\delta \sum_m \sigma_+^{(m)} \sigma_-^{(m)} + \hbar\Omega(\sigma_+^{(1)} e^{i(kz+\phi(1))} + \sigma_-^{(1)} e^{-i(kz+\phi(1))}) \quad (3.20)$$

where we have used the commutation relations $[\sigma_-^{(m)}, \sigma_+^{(n)}] = \delta_{mn}$. The equation of motion for ρ_I can now be given in the interaction picture as

$$\dot{\rho}_I = \frac{1}{i\hbar} [H_I, \rho_I] + \Gamma \sum_m (2\sigma_-^{(m)} \rho_I \sigma_+^{(m)} - \rho_I \sigma_+^{(m)} \sigma_-^{(m)} - \sigma_+^{(m)} \sigma_-^{(m)} \rho_I) \quad (3.21)$$

where the spontaneous decay has been added outside the commutator as given by Walls [50]. We are not interested in the transient behavior of the atoms, but in their

behavior at long times when they have reached a steady state in their interactions with the optical field. Therefore, in the calculations we have re-expressed the equation of motion of ρ_I as a matrix equation

$$\dot{\rho}_I = L\rho_I \quad (3.22)$$

where ρ_I is now a column vector, and evaluate the steady-state solution for ρ_I by finding the nullspace vector ρ_{ss} of L , normalized so that the populations of the atomic levels sum to 1. This is done by solving the equation

$$e_p = L_p \rho_{ss}, \quad (3.23)$$

where L_p is obtained by choosing a row p of L corresponding to a population element in ρ and changing that row's elements into 1's if they multiply a population element in ρ or 0's if they multiply a coherence element in ρ , and e_p is column vector of the same size as ρ with all zero elements except for a 1 in row p (shown here for a two-level system with states labelled a and b):

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} \\ L_{41} & L_{42} & L_{43} & L_{44} \end{pmatrix} \begin{pmatrix} \rho_{aa} \\ \rho_{ab} \\ \rho_{ba} \\ \rho_{bb} \end{pmatrix} \quad (3.24)$$

The force felt by the atom is then given in the Schrödinger picture by

$$\vec{f} = \langle \vec{F} \rangle = \left\langle \frac{dH_{\text{int}}}{dx} \right\rangle \quad (3.25)$$

$$= \text{Tr} \left[\frac{dH_{\text{int}}}{dx} \rho_{ss} \right] \quad (3.26)$$

where the spatial derivative is taken only on the electric field and not on the density matrix elements. As seen from the form of H_{int} , this requires finding the expectation values of $\sigma_+^{(m)}$ and $\sigma_-^{(m)}$. The diffusion coefficient in momentum space is given by the

formula

$$D_p = \int_0^\infty d\tau [\langle F(\tau)F(0) \rangle - \langle F \rangle^2] \quad (3.27)$$

and is a tensor, although we have omitted the coordinate indices since we are interested in D_{zz} only. If we represent the force in the form

$$F(\vec{R}) = \sum_{i,j} F_{ij} |j\rangle \langle i| \quad (3.28)$$

and define the Heisenberg operator

$$\Phi_{ij}(\vec{R}, \tau) = |j\rangle \langle i| (\tau) [F(\vec{R}, 0) - \langle F(\vec{R}) \rangle], \quad (3.29)$$

then the diffusion coefficient has the form

$$D(\vec{R}) = \text{Re} \sum_{i,j} F_{ij}(\vec{R}) \int_0^\infty d\tau \phi_{ij}(\vec{R}, \tau), \quad (3.30)$$

where $\phi_{ij}(\vec{R}, \tau) = \langle \Phi_{ij}(\vec{R}, \tau) \rangle$. The time evolution of $\phi_{ij}(\vec{R}, \tau)$ follows the same equations of motion as ρ_{ij} , due to the quantum regression theorem [51, 52]. This allows for the analytic solution of the integral if we express the ϕ_{ij} in a column vector ϕ :

$$\dot{\phi}(\vec{R}, \tau) = L(\vec{R})\phi(\vec{R}, \tau) \quad (3.31)$$

$$L(\vec{R}) \int_0^\infty d\tau \phi(\vec{R}, \tau) = \phi(\vec{R}, \infty) - \phi(\vec{R}, 0) = -\phi(\vec{R}, 0) \quad (3.32)$$

where the last equation follows because the atomic operators are not correlated at infinite times. This equation is inverted to yield the integral, with the additional condition that ϕ has zero trace imposed by replacing L with L_p and $\phi(\vec{R}, 0)$ with $\phi(\vec{R}, 0)_p$, where the appropriate element of ϕ has been set to zero:

$$\int_0^\infty d\tau \phi(\vec{R}, \tau) = -L_p^{-1}(\vec{R})\phi(\vec{R}, 0)_p. \quad (3.33)$$

This calculation gives the part of the diffusion coefficient due to fluctuations in the dipole force. The part of the diffusion coefficient due to radiation pressure is given by the formula

$$D_{spont} = \frac{\Gamma}{2} \sum_m \langle m | \rho_{ss} | m \rangle \langle (\hbar k)^2 \rangle_m, \quad (3.34)$$

where m labels the excited sublevels and $\langle (\hbar k)^2 \rangle_m$ is the projection of the fluorescence photon momentum in the z direction. The momentum projection values were found by Javanainen *et al.* [53] to be $3/5(\hbar k)^2$ for $m = 0$ and $7/10(\hbar k)^2$ for $m = \pm 1$. As mentioned above, we average the diffusion coefficient over a cubic optical wavelength to get the final value.

Since the force felt by the atom is a function of the magnetic field, and not of the magnetic field gradient, we calculated the force on a grid of equally spaced magnetic field values up to a value where the force was negligible. We then approximated the force by a Chebyshev polynomial, and differentiated the polynomial to obtain κ (or α , by its relation to κ) and integrated it to obtain the quasipotential. It was in the integration and differentiation of the Chebyshev polynomial that the field gradient was incorporated. The diffusion coefficient is calculated only for $B = 0$.

The results of the stability calculation are plotted along with the data in Figure 3.4. The model calculations assume that the Landé factor g_L for the transition is 1. This corresponds to the Zeeman splitting of the $m_F = 4 \rightarrow m_{F'} = 5$ transition in cesium. The other trapping transitions $m_F \rightarrow m_{F'} = m_F \pm 1$ have smaller Zeeman splittings and so the average value of g_L in the trap is somewhat less than 1. Therefore, the results of the model represent a maximum prediction and should be multiplied by a scaling factor less than but of order 1 for comparison to the data. A more detailed model that accounted for the full multilevel structure of cesium could calculate this scaling factor, but such a calculation is not accessible to our $J = 0 \rightarrow J' = 1$ model. Despite the presence of this scaling factor, it seems clear that the model presents only a rough agreement with the experimental data; the model consistently predicts a higher value for the maximum attainable field gradient than is actually measured, especially at the lowest values of the laser intensity. Although both experiment and

theory show a trend towards optimum stability at higher detunings for higher laser intensities, the theory shows this effect to a much higher degree. This appears to be a consequence of the $J = 0$ ground state in the model, which prevents the atom from being optically pumped into a dark ground state at very large Zeeman shifts, as can happen in the real atom.

The comparison of the model with the one-atom trap decay data of Figure 3.1 is presented in Figure 3.6. The rightmost curve is the prediction of the decay model without any corrections. The middle curve is the prediction of the decay model with the temperature set to twice the calculated Doppler value, to give a sense of how the temperature relates to the trap stability.

Another potential source of MOT instability at high magnetic field gradients is the $\vec{\mu} \cdot \vec{B}'$ force. We showed in earlier chapters how this force leads to stable confinement of atoms in a spherical quadrupole magnetic field if the atom is in the proper ground state. Of course, this force expels atoms in the wrong ground state from the trap. Similar considerations hold for the excited atomic states, which are also present in the MOT. Since the atoms in a MOT have negligible polarization, this force will average to a very small value compared to the optical force, and in any case does not increase more rapidly with magnetic field gradient than the optical trapping force. However, it is not so clear that the fluctuations in this force do not lead to significant heating. A heuristic argument leads us to expect the magnetic forces to be insignificant for the field gradients in our experiment. Since the magnetic dipole force is $F = g\mu_B B'$, and g differs between connected sublevels by factors of order unity, we can estimate the variance in the force as being of order $\mu_B B'$. This force will be fluctuating on time scales of order Γ^{-1} for saturated transitions. The momentum change accumulated by the atom in this time is just $p = Ft \sim \mu_B B'/\Gamma \sim 10^{-29}$ kg m/s for $B' = 1000$ gauss/cm. However, the recoil momentum of a single photon is $p_{recoil} = \hbar k = 7.7 \times 10^{-28}$ kg m/s, and so we expect optical momentum transfer to dominate magnetic dipole forces in the saturated regime, especially since the optical diffusion is in part due to stimulated processes that do not saturate at high laser intensity. Given the measured maximum stable magnetic field gradients, the momentum diffusion due to magnetic

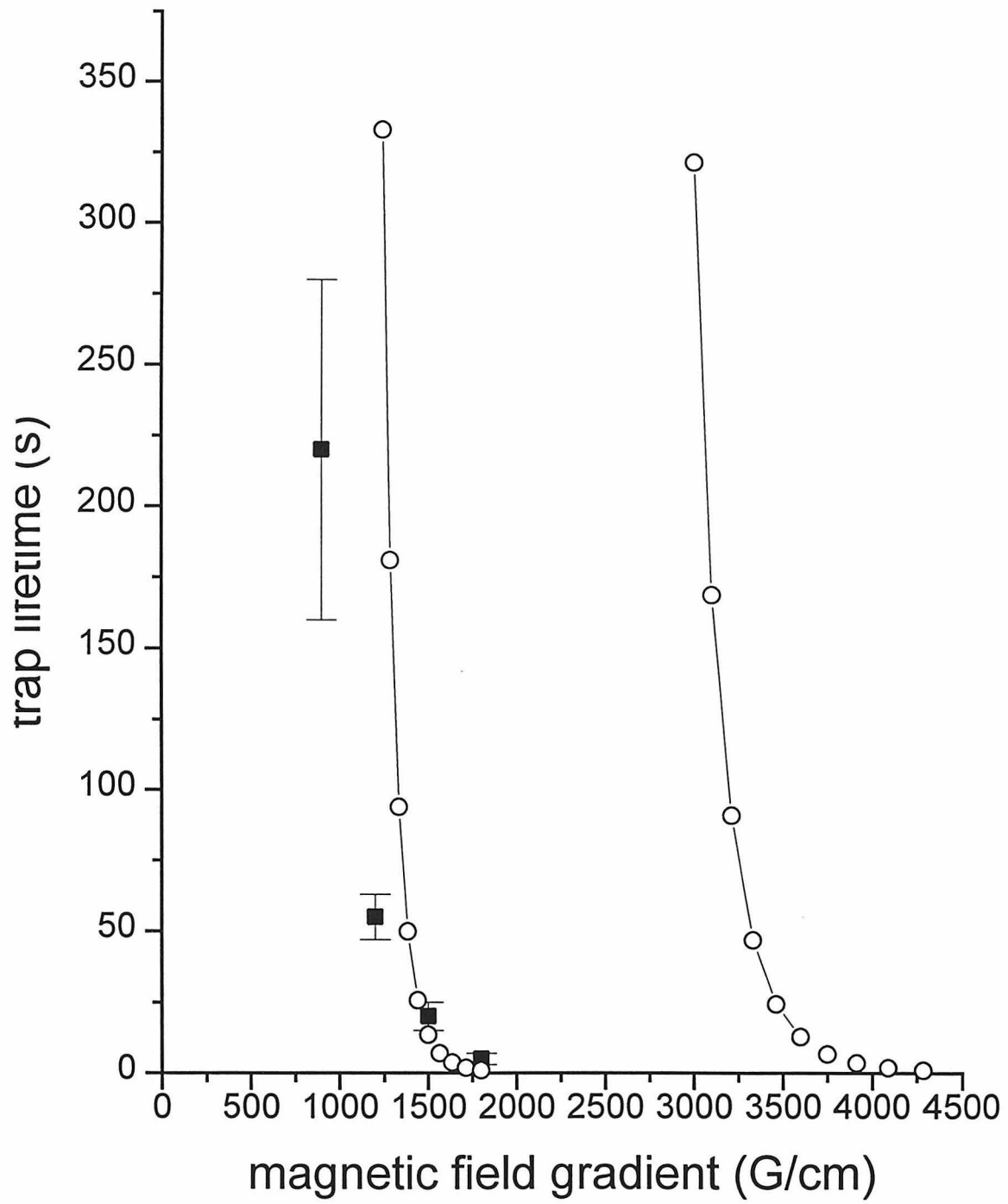


Figure 3.6: The one-atom trap decay data of Figure 3.1, with the predictions of the evaporation model.

dipole forces will be two orders of magnitude smaller than that due to optical forces and so is unlikely to be responsible for trap decay. For this reason, we believe that we have measured the diffusive evaporation of atoms from MOT's.

Future studies of the stability of MOT's at high field gradients could extend this work in several ways. An experiment that has control over the relative phases of the trapping laser beams could be designed to investigate the dependence of the maximum field gradient on that parameter. It would also be interesting to test the stability of traps using atoms with a less complicated level scheme to see if the stability depends on level structure. Most useful in this regard would be to trap an atom with a $J = 0 \rightarrow J' = 1$ transition, for which polarization gradient effects would be entirely absent. Better theoretical understanding of the trap loss rate would come from a model that accounts for the full level structure of cesium, especially insofar as it can predict the influence of polarization gradient effects in the trap. Such a model does not currently exist, however, since the low spring constants in MOT's reported in the next chapter and by other groups has yet to be explained by the P-G theory. A model of the trap decay based on the Kramers equation (Eq. 3.1) rather than the Smoluchowski equation (Eq. 3.2) would better account for the stochastic motion of the atoms in the underdamped regions of the trap. However, these models are likely to be numerically intensive.

Chapter 4 Dynamics of Magneto-Optical Traps at High Magnetic Field Gradients and Laser Intensities

4.1 Introduction

As we showed in the Introduction, the force laws which describe the cooling and trapping of atoms in the simple theory of a MOT lead to the equation of motion of a damped harmonic oscillator, where the damping coefficient α is a function of the trapping laser intensity and detuning and the spring constant κ is a function of the trapping magnetic field gradient as well.

Detailed knowledge of the values of the trap dynamical parameters are useful for a variety of reasons. Many experiments, including evaporative cooling of atoms into a Bose condensate, require as their starting point a collection of atoms of the highest possible phase space density (i.e. high density and low temperature). This involves an optimization of the spring constant, damping coefficient and diffusion coefficient [17]. For example, one such technique used to maximize the phase space density is the rapid compression of a MOT by rapidly increasing the magnetic field gradient [54]. The response of the trapped atoms to rapid changes in the magnetic field gradient will depend on the relative strengths of the spring constant and damping coefficient. Also, the temperature of the atoms in a trap is directly related to the dynamical parameters by the formula $k_B T = D/\alpha$, where D is the momentum diffusion coefficient and α is the velocity damping coefficient. The quest for lower temperatures in a MOT often involves attempts to maximize α . Finally, the dynamical parameters of a MOT are interesting in their own right, as a probe of the interaction between light and atoms. It is in this spirit that we undertook the experiments described here.

Several techniques have been used by other groups to measure these parameters. The earliest method was that used by Raab *et al.* [55] in the very first experiment demonstrating optical trapping, in which a pushing laser beam displaced the atoms from the trap center. This displacement was related to the force calculated for such a beam. This technique requires confidence in the value of the pushing force, which is not independently measured, and in addition the presence of the pushing laser may alter the trap parameters by optically pumping the atoms. If the pushing laser is turned off, the atoms re-equilibrate with a timescale given by the ratio of the damping coefficient to the spring constant. This measurement of equilibration time does not rely on knowledge of the pushing beam force.

A second technique is to introduce an intensity imbalance between a counterpropagating pair of trapping laser beams and measure the shift in the MOT position [56, 57]. While the expected trap shift is well understood theoretically for small laser intensity, at or above saturation the relationship is less well understood. Also, the amount of displacement varies with the relative phase of the imbalanced lasers [57], an effect which may or may not reflect an actual change in the spring constant.

Another technique is to independently measure the temperature of the trapped atoms, for example by a time-of-flight technique, and use observations of the trap size to determine the spring constant [55, 45]. This technique, unlike the previous two, does not introduce any perturbations into the MOT. However, it is subject to the errors in temperature measurement and trap size measurement. Both this technique and the previous one were also used in the landmark paper by Raab *et al.*

Kohns *et al.* [58] introduced another technique, that of using an homogeneous oscillating external magnetic field to vary the position of the center of force sinusoidally. This alters the law of motion for the atoms to that of a forced damped harmonic oscillator, and the dynamical parameters can be determined by observing the phase and amplitude of the atomic response. This technique introduces no forces not already present in the trap, and the motion of the trap can be measured with high precision, yielding better measurements of the forces in the trap. In combination with measurement of the trap size, this technique also allows for measurement

of atomic temperature in the MOT if the atomic density is kept low enough that the atoms are noninteracting. The Kohns experiment studied trapped rubidium; a similar experiment by Höpe *et al.* [41] studied trapped cesium at relatively elevated magnetic field gradients.

Our experiment uses the same technique used in the Kohns experiment. We begin by describing how the cryotrapping apparatus for the stability measurements described in the previous chapter was adapted for the dynamics measurements. We then show how the data were collected and analyzed to extract the dynamical data and exclude competing effects such as eddy currents in the cryotrap. We then show how the dynamical parameters vary with magnetic field gradient, laser detuning, and laser intensity, and conclude by comparing our results to the theoretical predictions given in the Introduction and the results of other experiments that have been done already.

4.2 The Experimental Setup

This experiment was conducted in the same cryogenic trapping apparatus used for the trap stability measurements, with the addition of a third superconducting magnetic field coil which supplied the oscillating external magnetic field along the symmetry axis of the trap (See Figure 4.1). The magnetic field away from the center of a single coil is not strictly homogeneous, unlike the case in Kohns' experiment, where a pair of Helmholtz coils were used. However, the amplitude of the atomic motion was sufficiently small that in the vicinity of the trapped atoms the external field was essentially homogeneous. This was verified in an experiment designed to test the linearity of the atomic response to the external field, which we describe later.

The light collecting optics and ICCD camera used in the trap stability experiment were also used in this experiment for the high magnetic field gradient data (the fast lens was unnecessary for the multi-atom low field gradient traps and had too narrow a field of view, so that data was collected with the ICCD camera on the opposing viewport). In order to collect dynamical data, we made a 'movie' of the trap motion

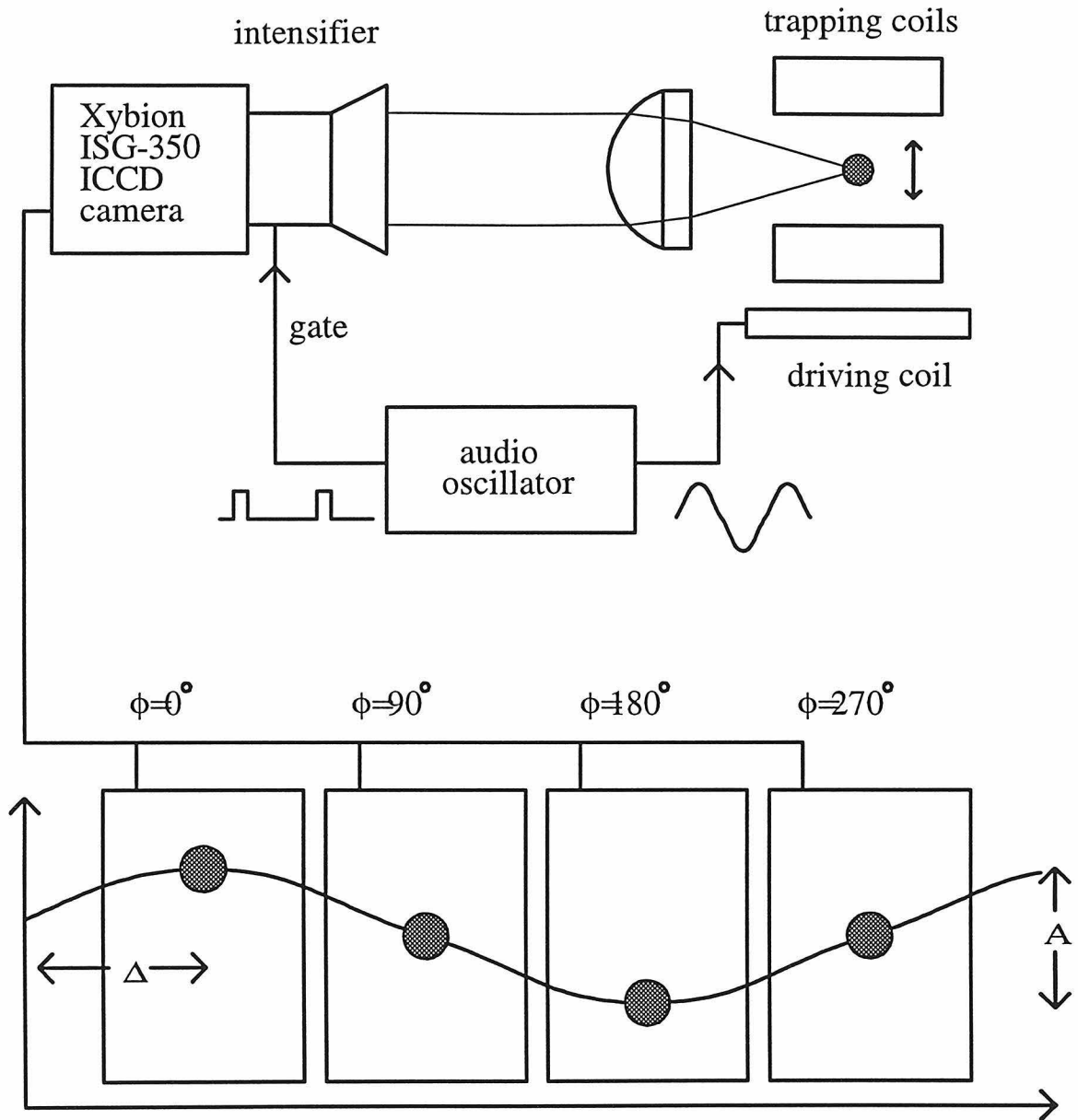


Figure 4.1: The experimental setup for the dynamics measurements. For fixed laser intensity and detuning and fixed magnetic field, the external homogeneous magnetic field was modulated, causing the MOT to oscillate. The intensifier tube of the ICCD camera was gated on synchronously with the modulation to collect images of the MOT corresponding to the four quarters of the oscillation period. These images were then analyzed to determine the amplitude and phase lag of the MOT oscillation.

by collecting the images in the following manner. The sinusoidal voltage driving the external field coil was converted to a TTL square wave by a comparator and AND-ed with the TTL square wave reference signal from the signal generator, which was synchronized to the sine output such that the positive edge of the reference coincided with the peak of the sine wave. By inverting either, both, or neither of the square waves prior to AND-ing, pulses were generated with 25% duty cycle at 0° , 90° , 180° and 270° relative to the sine wave. These pulses were used to gate the intensifier of the ICCD camera, thus strobing it on for precisely one quarter of the trap oscillation. The video output of the camera was integrated on the frame grabber for a time sufficient to collect a good image (~ 0.5 -3 sec) for each phase. In this way we generated a data set of four images, each image corresponding to a different phase of the oscillation. By analyzing the set of four images corresponding to a given run the amplitude and phase of the trap oscillation for that run was deduced.

The most significant systematic error present in this experiment is the frequency-dependent phase and amplitude shift of the oscillating magnetic field relative to the driving voltage. This shift is due to the resonant behavior of the RL circuit formed by the coil and driver source impedance, as well as by eddy currents in the copper of the cryogenic vacuum system. This phase and amplitude shift was measured using a differentially amplified split photodetector to monitor the fluorescence of the Cs beam as it traversed a single resonant laser beam propagating along the trap symmetry axis. By applying a small trapping magnetic field, the beam fluorescence was spatially limited to a small stripe at the trap center. Adding the oscillating magnetic field made the fluorescent region oscillate spatially with the exact phase and amplitude of the driving magnetic field, since there are no significant atomic dynamical effects in this configuration. The differential output of the split photodetector was compared to the coil driving voltage with a lock-in amplifier to yield the phase lag and amplitude drop, which are plotted in Figure 4.2 for the cryotrap at 3.1 K. Cooling the chamber from 77 K to 3.1 K made a significant difference in the measured phase and amplitude shift, due to the influence of eddy currents in the copper in the system. At 77 K, the phase and amplitude shifts were consistent with the resonant nature of the RL

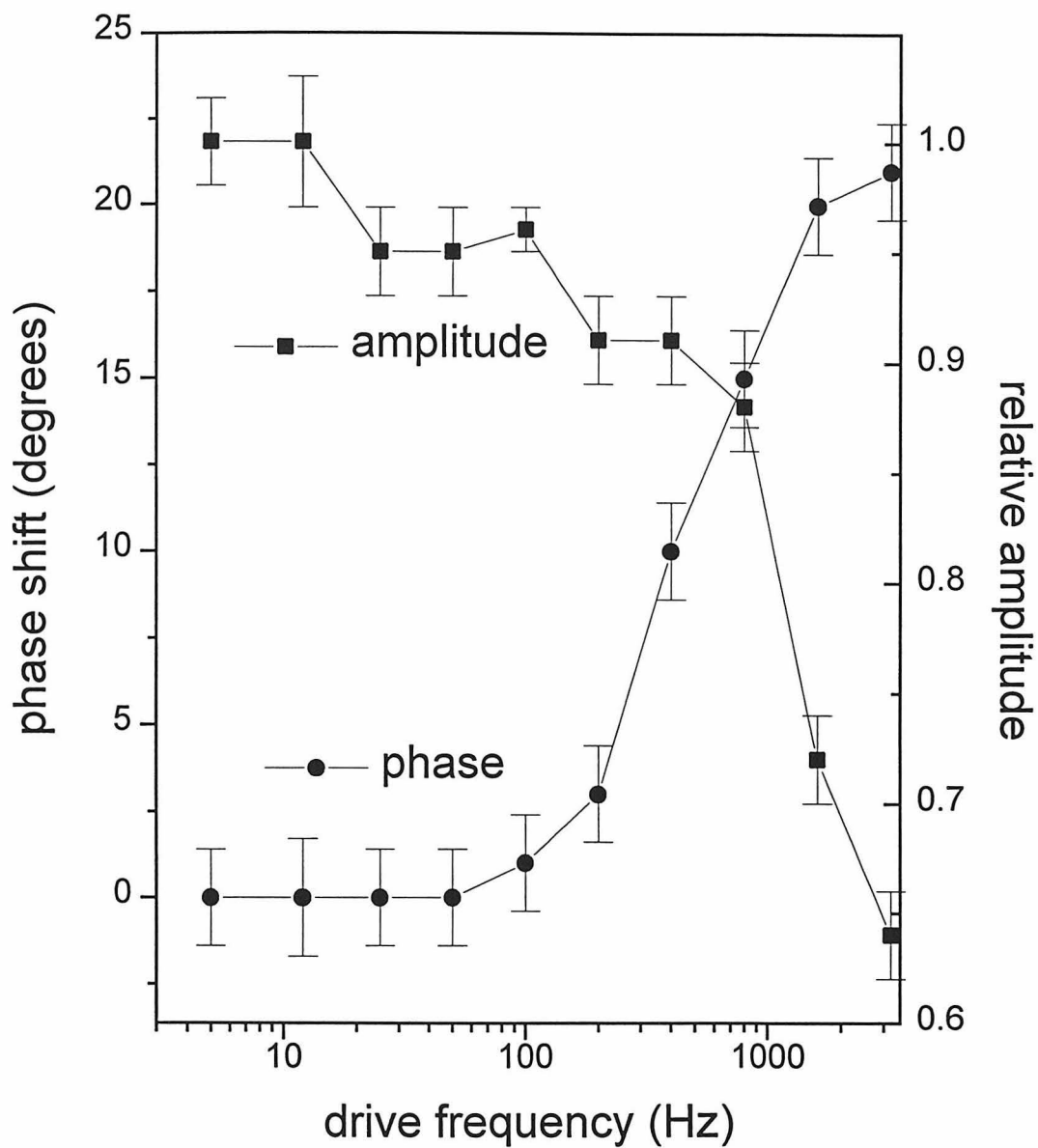


Figure 4.2: A plot of the phase shift and amplitude drop of the oscillating magnetic field due to eddy currents in the cryotrap at $T = 3.1$ K. The lines joining the points are meant only to guide the eye.

circuit of the coil, and led to only a few degrees of phase correction at the highest drive frequencies.

4.3 The Data Collection

Two sets of dynamical data were collected, one at conventional magnetic field gradients of ~ 20 G/cm to measure the dynamical parameters α and κ , and another at high magnetic field gradients (150-1800 G/cm) to examine the behavior of dynamical parameters as the spring constant is varied.

For each driving frequency, a background image was collected and subtracted from the dynamics images, to separate the trap signal from the background fluorescence and scattered light (the atomic beam was left on during data collection for convenience in loading, since ultra-low background pressure is not needed for these experiments). For multiple-atom traps at low field gradients, the trap fluorescence signal was large enough that background subtraction was unnecessary, and so it was omitted. The images, background-corrected or not, were then fit by two-dimensional gaussians with adjustable widths along both axes.

The choice of gaussian trap profile for the fitting function is an approximation borne of necessity, since the integrated trap profile is strictly gaussian only if it is not accelerating. An exact profile of the integrated trap fluorescence can be easily derived but only numerically evaluated, making its use in a fitting routine unfeasible on small computers. Since it is the centroid of the trap position which is the desired fit parameter, simply measuring the center of gravity of the trap fluorescence would be theoretically accurate as well as computationally easier. However, this evaluation is easily skewed by background noise due to incomplete background correction. The gaussian fit is the more robust estimator of the trap centroid.

To verify that the oscillations of the MOT were small enough to be considered in the linear regime and that astigmatism in the fast lens was not a problem, we measured the linearity of the oscillation amplitude as a function of the amplitude of the voltage signal across the driving coil. Figure 4.3 is a plot of the linearity data taken

with the laser detuning at 12 MHz and the intensity at 6.1 mW/cm^2 , the trapping magnetic field gradient at 300 Gauss/cm, and the oscillation frequency at 200 Hz. The drive voltages measured encompass the range used in the entire experiment.

4.4 The Data Analysis

The trap images were digitized and stored on a computer, along with background images. After the backgrounds were subtracted, gaussians were fit to the trap images to locate their centroids. The fit of the gaussian to the trap shape was very good except in those cases where the trap motion was several times the trap radius, i.e. for low trapping magnetic field gradients. In these cases the trap image was elongated and assymetrical. However, the centroid of the fit was an good estimation of the center of gravity of the trap position, as based on tests using computer-synthesized data.

The results of the test of the gaussian fit program using the synthesized data have shown that the fit program overestimates the displacement of the trap centroid from the center of oscillation by a small factor that increases to 8% in the limit of large oscillation amplitudes. Since the data are symmetrically distributed around the center of oscillation, the mean of the data remain a good estimator of the center, and so the displacement of the trap centroid from this center can be inferred and the correction factor applied. All the data have been corrected in this way.

The statistical errors in the data were estimated by collecting several image data sets under identical trapping and driving conditions and comparing the fitted centroid locations for identical phases. Directly comparing the centroid locations for images of similar phase from different data sets gave a distribution of halfwidth 0.2 pixels around the mean. However, this estimate led to uniformly better than expected fits of the centroids to the oscillation curve. Therefore, the centroid value for each phase in a data set was expressed relative to the mean over all the phases in the data set, under the assumption that the center of oscillation was not fixed, but rather slowly moving compared to the time to collect a data set. This reduced the halfwidth of

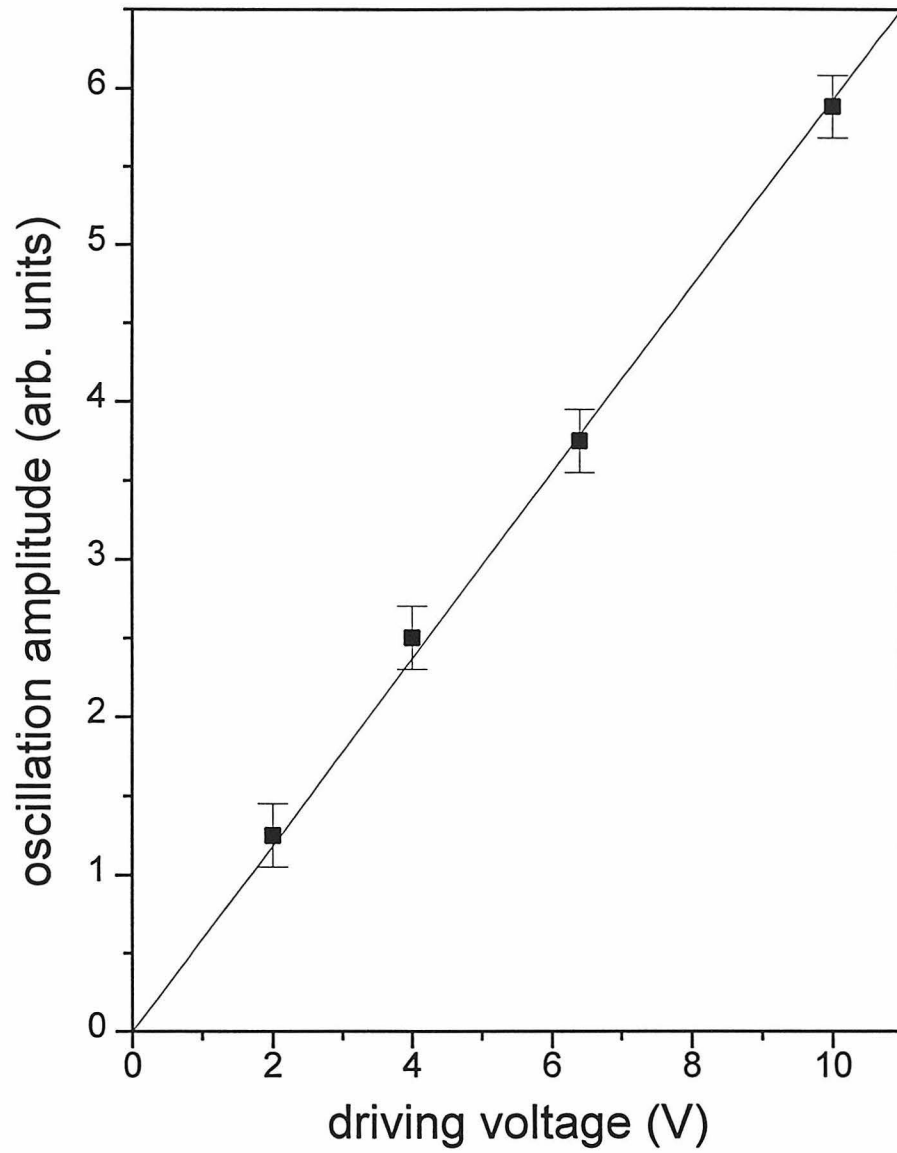


Figure 4.3: A measurement of the linearity of the MOT response to the oscillating magnetic field. The laser parameters are 12 MHz detuning and 6.1 mW/cm^2 intensity per beam, the trapping magnetic field is 300 Gauss/cm, and the oscillation frequency is 200 Hz.

the distribution to 0.05 pixels, a much greater reduction than would be expected if the center of oscillation were in fact fixed. A slow drift in the center of oscillation is to be expected due to drift in the bucking magnet power supplies used to null the background magnetic field.

Because the trap image was gated over one-quarter of the oscillation period, and not instantaneously at specific phases, the functional variation of the trap centroid location is not a simple sinusoid. If the actual motion of the centroid of the trap is given by $y(x) = A + B \cos(x + C)$, where x is time as measured in degrees, then the mean pixel location for the n^{th} quarter of the oscillation period is

$$y_n = \frac{2}{\pi} \int_{n\pi/2}^{(n+1)\pi/2} [A + B \cos(x + C)] dx \quad (4.1)$$

$$= A + \frac{2}{\pi} B [\cos(n\pi/2 + C) - \sin(n\pi/2 + C)]. \quad (4.2)$$

This is the function that is fit to the trap centroid locations. This complication is tolerated because if the gating time were reduced sufficiently to “freeze” the trap motion, the trap image would have required integration over proportionally more oscillation periods to obtain the same level of signal. This would have increased the background noise level, and prevented measurements at the highest field gradients, where the trap lifetimes are short.

The oscillation parameters were fit to the data using the Origin plotting package, which uses the Levenberg-Marquardt algorithm to perform a least-squares fit. Because each fitting session modelled four data points using a three-parameter function, the minimized χ^2 for any single session will vary significantly from 1 much of the time. For this reason goodness of fit was estimated collectively for many fitting sessions by comparing the distribution of χ^2 to that expected.

The oscillation parameters B and C in the oscillation equation above correspond to the amplitude and phase shift of the atomic motion relative to the driving force, respectively (A is just the undriven trap centroid location and not a dynamical parameter). After correcting them for the phase and amplitude shifts of the oscillating magnetic field due to eddy currents and resonant effects, this data can then be plotted

against driving frequency and used to determine the dynamical parameters α and κ , using the equations

$$B(\omega) = \frac{\kappa B_0}{\sqrt{(\kappa - m\omega^2)^2 + \alpha^2\omega^2}}$$

$$C(\omega) = \arctan\left(\frac{\alpha\omega}{\kappa - m\omega^2}\right).$$

Figure 4.4 plots a typical curve of phase shift versus frequency and oscillation amplitude versus frequency.

It turns out that for oscillation frequencies well below the resonance frequency ω_0 the formulas for $B(\omega)$ and $C(\omega)$ are more dependent on the ratio $\epsilon = \alpha/\kappa$ than on their absolute values:

$$B(\omega \ll \omega_0) \approx \frac{B_0}{\sqrt{1 + \epsilon^2/\omega^2}}$$

$$C(\omega \ll \omega_0) \approx \arctan(\epsilon\omega).$$

For this reason, the standard deviations in the fit values of α and κ tend to have a large degree of correlation, and so ϵ and κ were the parameters we fit to the data. For a given value of ϵ , variations in κ lead to changes in the phase and amplitude curves only near or above the resonant frequency, i.e. that frequency where the phase shift is 90 degrees. For an overdamped trap (as we shall see is the case here), this is precisely the region where the oscillation amplitude is small and where the relative errors in the oscillation amplitude and phase are large. For this reason, ϵ is measured with much better precision than κ (or, by extension, α).

If the model of the trap motion as being that of a damped harmonic oscillator is correct, then the amplitude data should be consistent with the phase data, i.e. they should correspond to the same values of ϵ and κ . Figure 4.4 shows that this is the case. It is also expected that the spring constant is directly proportional to the magnetic field gradient, while the damping coefficient is independent of the magnetic field gradient since the atoms in a MOT collect in a region of very small magnetic field. The position damping time ϵ , being equal to α/κ , should be inversely proportional to the magnetic field gradient. Figure 4.5 shows that this expected behavior is indeed

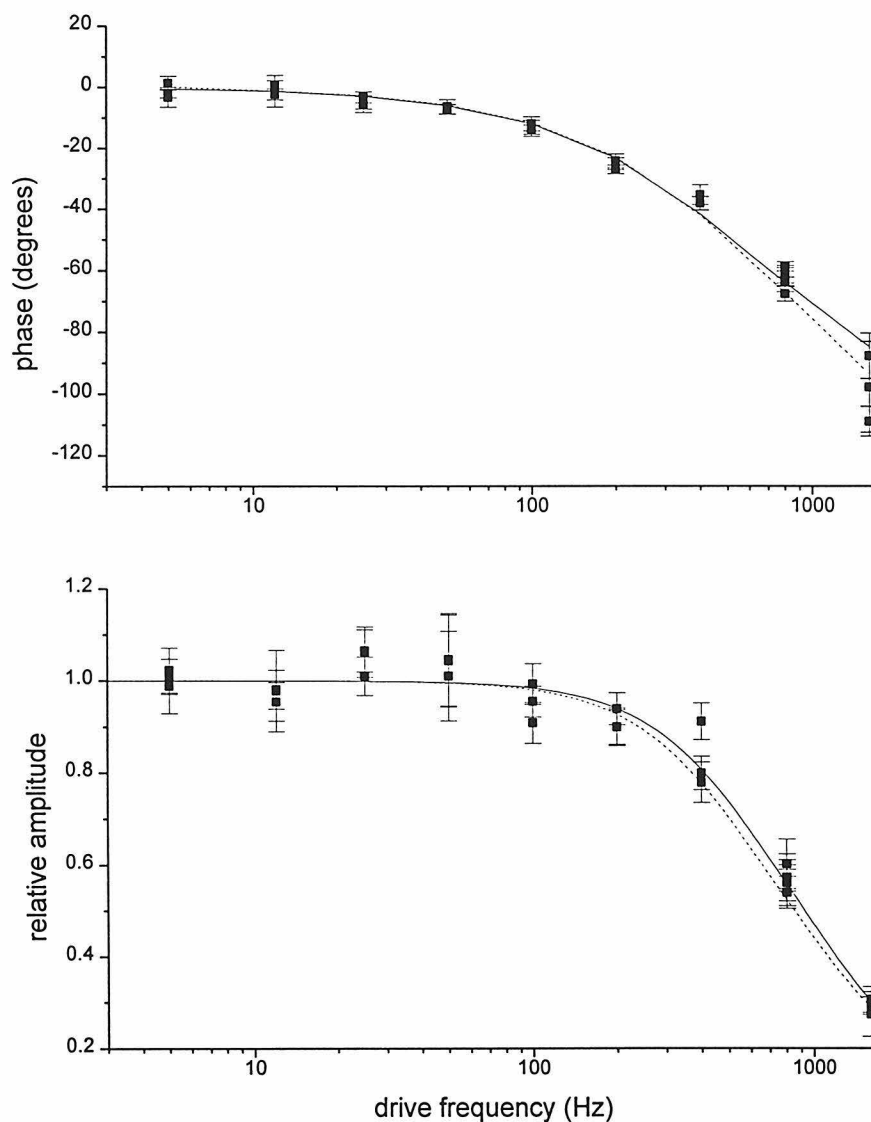


Figure 4.4: Phase and amplitude data from one data set. The solid lines in each graph are the oscillation curves fit to the data in that graph. The dashed lines in each graph are the oscillation curves derived from the fit to the data in the alternate graph. These data were collected at $B' = 1050$ G/cm, with the laser detuning at 20 MHz and the intensity at 6.1 mW/cm².

seen in actual MOT's even at very high magnetic field gradients.

The measured values of α are plotted in Figure 4.6. The most prominent feature of the data are their elevated values compared to the Doppler theory, which is shown for comparison as the bottom curve on the graph. This alone is clear evidence for sub-Doppler cooling processes in the MOT. Also shown for comparison to the data are the curves expected for the two polarization-gradient cooling mechanisms at low intensity and in one dimension. As pointed out in the previous chapter, in a three-dimensional trapping geometry the polarization field is fairly complex, and the type of polarization gradient seen by a travelling atom will depend on its path through the optical field and the relative phases of the trapping laser beams. In such a field both ellipticity gradients and polarization rotations can be expected, with the result that the net cooling mechanism will be some average of Sisyphus cooling and induced orientation cooling. In addition, the presence in the 3-D case of trapping laser beams orthogonal to the atomic motion will lead to optical pumping of the atomic ground states in competition with that of the beams involved in cooling, and so lead to an overall reduction of the cooling. Mølmer [49] has made numerical calculations based on three-dimensional optical molasses for an atom with a $J = 1 \rightarrow J' = 2$ transition. His calculation for the $\sigma_+ - \sigma_-$ geometry used in a MOT assumes the fairly special case of no relative phase difference between the lasers at the trap. Such a configuration is special in that the laser polarization is everywhere linear and varies only in direction. The result of this calculation is shown by the open circles in Figure 4.6. A more general phase relationship between the lasers leads to circular polarization components in the local laser field. For this reason, Mølmer's calculation for the case of 3-D LPL cooling, which does involve local ellipticity gradients, is also included in Figure 4.6. The actual damping coefficient in a MOT may be expected to lie somewhere between the two curves. The data show that this expectation is largely confirmed. Another feature of the data worth noting is the lack of any clear dependence of α on the laser intensity. This is a feature that is expected of polarization gradient cooling in the low intensity limit, and seems to be roughly confirmed in the data.

Our results for the spring constant κ are presented in Figure 4.7. In contrast to

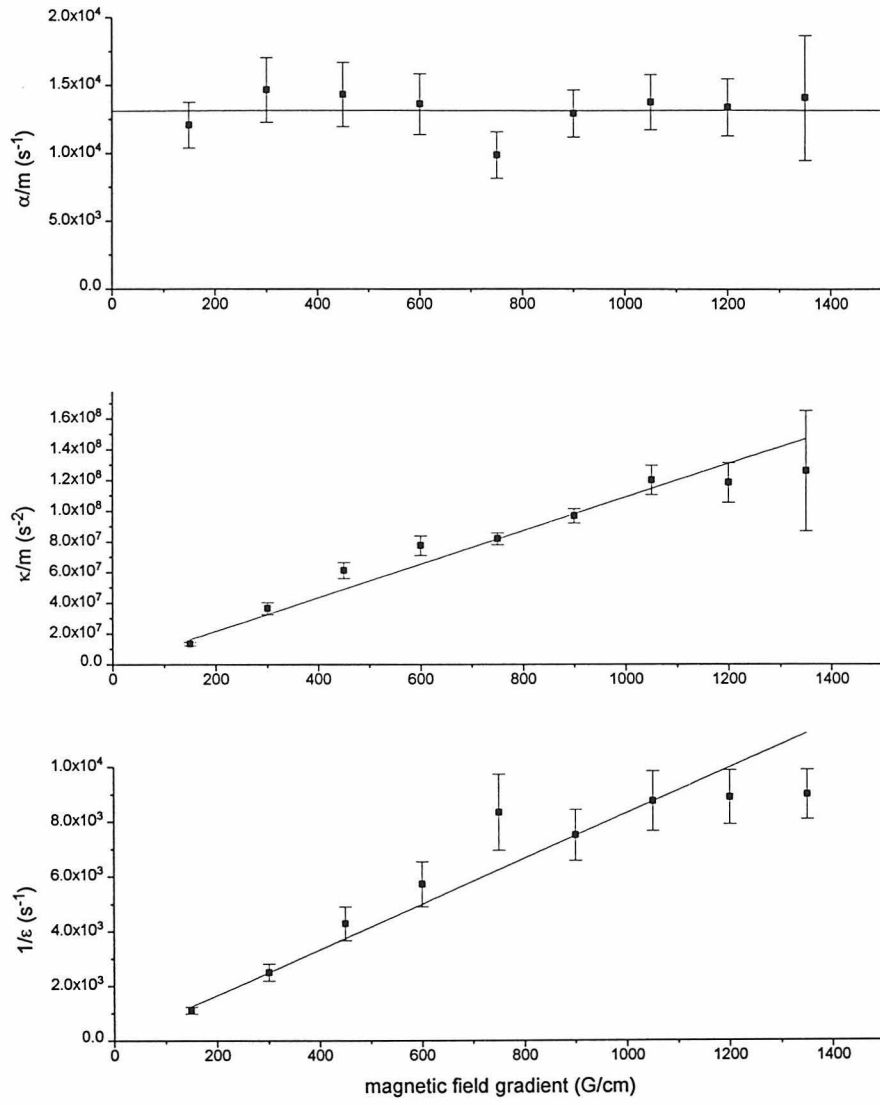


Figure 4.5: Behavior of the MOT dynamical parameters with changing magnetic field gradient. As expected, the damping coefficient α is independent of B' , while the spring constant κ varies linearly with B' and the position damping time ϵ is inversely proportional to B' . The data were collected with the laser detuning at 9.9 MHz and the laser intensity at 3.1 mW/cm².

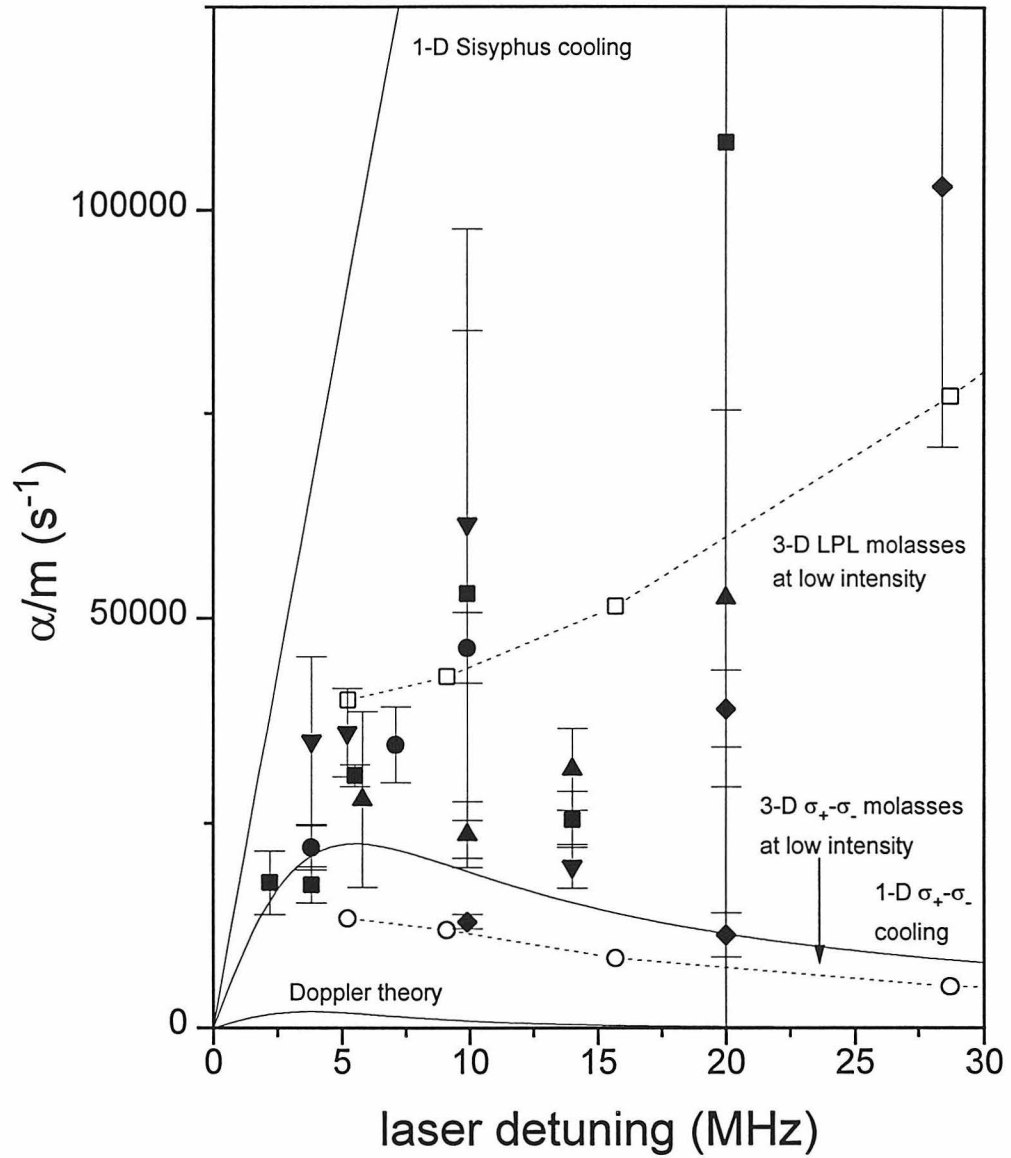


Figure 4.6: Measured values of the damping coefficient α as a function of laser detuning. The symbols distinguish the data as follows: circles, 1.7 mW/cm^2 ; down triangles, 4.0 mW/cm^2 ; squares, 7.8 mW/cm^2 ; up triangles, 16 mW/cm^2 . The open circles and open squares are results from Mølmer's 3-D calculations; the lines joining the points are meant to guide the eye. The Doppler curve is for $I = I_{\text{sat}}$, a close estimate of the largest value it can take. The Sisyphus and $\sigma_+ - \sigma_-$ curves are from Equations 1.16 and 1.20, respectively.

α , which is measured to be in good agreement with the sub-Doppler cooling theory and well above the values predicted by the Doppler theory (Eq. 1.4), κ is measured to be at least a factor of 10 lower than is expected by the sub-Doppler theory and a factor of four lower than the Zeeman theory (Eq. 1.7). Still, if the two theories are scaled by constants to match the data, it is seen that the dependence of κ on laser detuning is consistent with both induced orientation cooling and the Doppler theory, to within the precision of the measurements.

Our results for the position damping time ϵ are presented in Figure 4.8. As explained earlier, the accuracy with which ϵ is measured is much better than that of α or κ independently, since it is ϵ which dominates the low-frequency dynamical response of the MOT. The upper graph in Figure 4.8 contains the data collected at low magnetic field gradients, while the lower curve represents data collected with single-atom traps at high magnetic field gradients. As expected from the behavior of α (high compared to the Doppler theory) and κ (low compared to the polarization gradient theory), ϵ is much larger than predicted by either theory. What is not expected in light of the earlier data is the apparent linear dependence of ϵ on the laser detuning. Such a dependence has in fact been previously observed by Drewson *et al.* [45]. The present data confirm their results and in addition show that ϵ has no intensity dependence to within $\sim 20\%$ over an intensity range of almost a factor of 10, from 1.7 mW/cm² to 16 mW/cm². The best explanation for this intensity insensitivity in ϵ is that both α and κ are also independent of intensity. This is consistent with our data, even considering the uncertainties in those measured values. Others have measured α and κ under different situations and confirmed this intensity independence for high saturation [57, 45]. The model for the linear dependence of ϵ on laser detuning assumes that both Sisyphus and induced orientation cooling are active in the MOT, and so α at a given detuning is an average of the two, yielding a value that is nearly constant over the laser detunings considered here. This is consistent with our and others' measurements [45, 59]. According to this theory, since κ is caused only by induced orientation effects, it varies inversely with laser detuning for detunings $\geq 2\Gamma$, and so ϵ must be linear in laser detuning. This inverse relationship of κ to laser detuning is

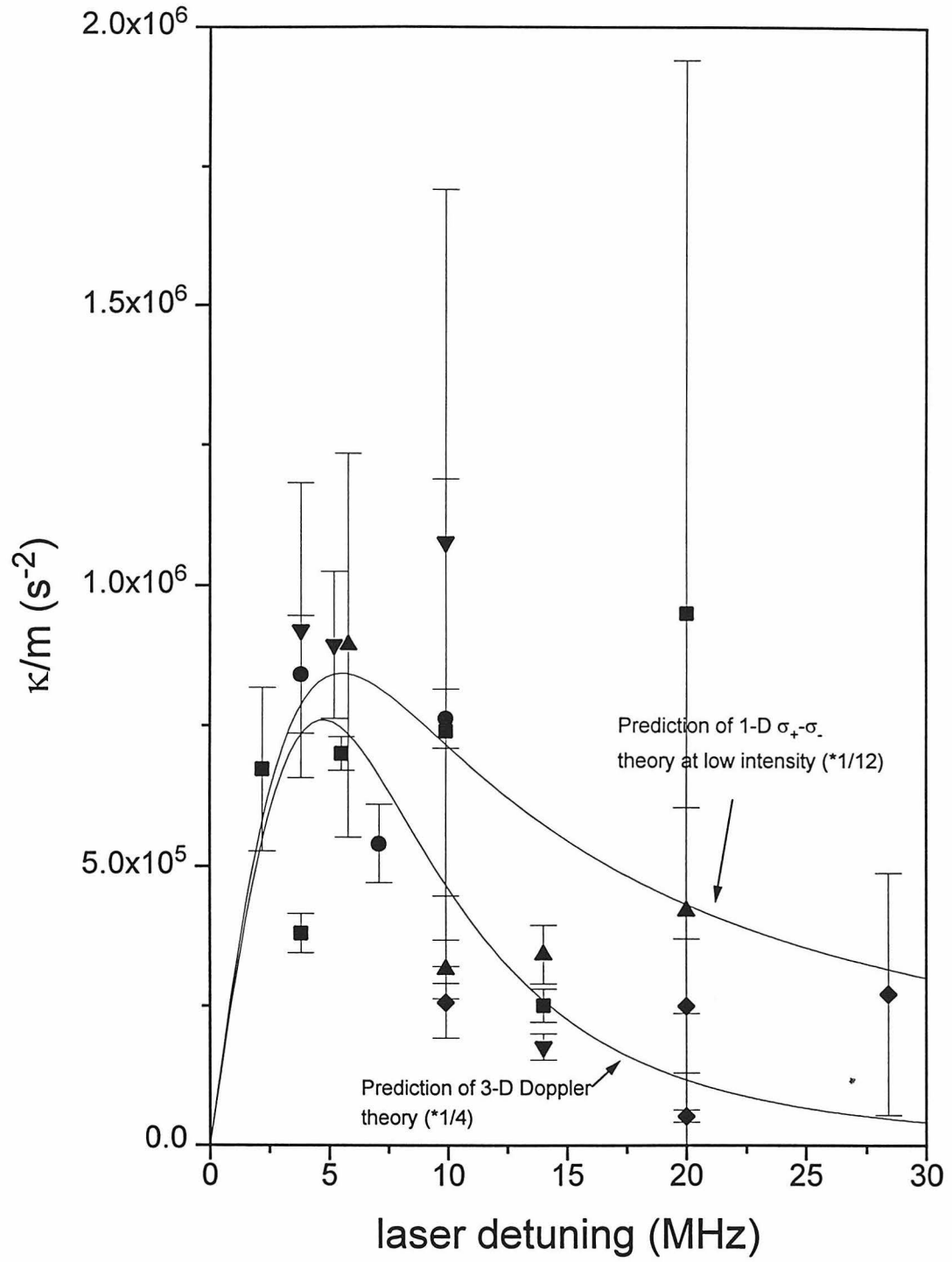


Figure 4.7: The spring constant data. The symbols have the same meaning as in Fig. 4.6.

certainly present in our data and elsewhere [60, 57, 45]; however, the magnitude of κ is far below that expected for the induced orientation theory, especially given that α fits so well with the theory, and so this model for ϵ remains incomplete.

The values of ϵ taken at high magnetic field gradients differ significantly from those taken at low magnetic field gradients, as shown by a comparison of the two graphs in Figure 4.8. However, the general features of linearity with respect to laser detuning and intensity independence remain correct (the two data points at 20 MHz detuning were collected with 3.1 mW/cm² intensity/beam for the upper point and 6.1 mW/cm² for the lower point). It is the slope of the laser detuning dependence and its intercept that differ between the two sets of data. In general, the values of κ measured at high field gradients are lower relative to those at low field gradients than the values of α are, so the larger values of ϵ seem to reflect a reduction of the spring constant at high field gradients. We can think of two causes for this. One is that at the highest magnetic field gradients the atoms venture into the nonlinear region of the force. The linearity measurement and the independence of κ on B' shown in Figure 4.5 would seem to exclude this possible cause, however. The other is that the high field gradient traps had a different “mix” of Sisyphus and induced orientation cooling than the low field gradient traps due to slight misalignments of the polarizing optics. Drewson *et al.* [45] have noted that the position time constant is very sensitive to the ellipticity of the trapping lasers beams along the trap motion, and set an overall scale uncertainty of 2 for their data. The trapping laser optics were realigned between the collection of the two data sets, possibly changing the trapping laser polarization. Scaling down the high field gradient data by a factor of two brings the two data sets into close agreement.

4.5 Discussion

While the results obtained by this work are a significant extension to prior measurements, they unfortunately do little to resolve the outstanding questions. Most significantly, the unexpected linear dependence of the position damping time on laser

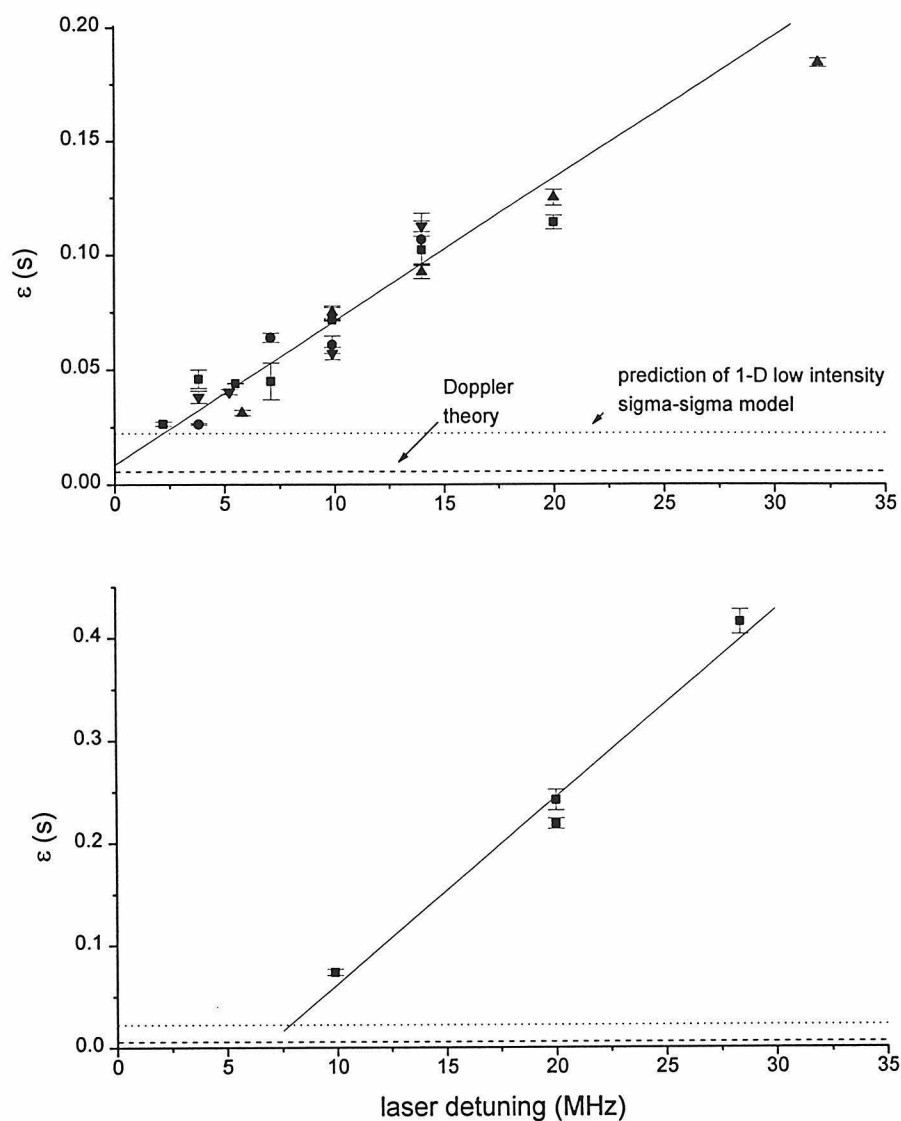


Figure 4.8: The position damping time data. The upper graph contains data collected at low magnetic field gradients with multi-atom traps, and the lower graph contains the data collected at high magnetic field gradients using single-atom traps. In both graphs the prediction of the Doppler theory (multiplied 10x) is shown by the dashed line, and the prediction of the 1-D induced orientation theory (also multiplied 10x) is shown by the dotted line. The different symbols in the upper graph have the same meaning as in Fig. 4.6.

detuning found by Drewson *et al.* [45] has been confirmed here, with the additional knowledge that there is no dependence on laser intensity over the range of intensities studied here, all above saturation. This is also consistent with our observation that neither α nor κ show any clear dependence on intensity above saturation. This insensitivity of κ to laser intensity is also observed in the intensity imbalance measurements of Steane *et al.* on cesium [56] and the intensity imbalance and position damping measurements of Wallace *et al.* on rubidium [57]. However, it is the relatively precise intensity insensitivity of the position damping time that is most surprising.

A review of the published measurements of κ reveals another interesting trend: κ is measured to have the largest values when the intensity imbalance technique is used, and to have the smallest values when the oscillating magnetic field technique is used, with measurements based on the trap size and temperature tend to range in between. This is reflected even in the data of Wallace *et al.*, who use both intensity imbalance and temperature/trap size techniques. Townsend *et al.* [17] have pointed out that when measuring κ using techniques that move the trapped atoms into regions of non-zero magnetic field, care must be taken that the atoms do not enter the nonlinear region of the trapping force. The linearity test performed in our experiment rules out that possibility in our work. Townsend also warns that multiple scattering of photons can lead to underestimates of κ in dense traps. This seems to us to be a matter of perspective (a reduction of κ at high atomic density is interesting in its own right), but the point is well taken: we do not control the trap population (except in the single-atom traps), and it seems clear that our low-field-gradient traps are in the multiple scattering regime, based on observations of the spatial profile. The somewhat larger values for κ observed in our single-atom MOT's tend to contradict this interpretation, although the single-atom data are within the scatter of the multiple-atom data, and we have already noted that κ is known to have a strong dependence on trapping laser polarization. Interestingly, the data of Höpe *et al.* [41], which were collected using oscillating MOT's at a relatively high magnetic field gradient (63 G/cm), were taken at even lower atomic densities than the Townsend data, and still show a smaller value of κ than Townsend reports.

Appendix A How We Lock Lasers

A.1 The Lasers Used in the Experiments

All of the experiments described above were done using diode lasers. Diode lasers have a number of distinct advantages for cesium spectroscopy [61]. They are more or less readily available at 852 nm, with output powers up to 150 mW or higher. They are relatively inexpensive at \$700 apiece. They are easily tuned by a variety of simple techniques such as temperature control, current control, and external feedback. Sidebands are generated simply by modulating the injection current. They are also compact; several of them can be easily installed on a single optical bench.

The lasers used in these experiments were STC type LT50A03u lasers, mounted in the apparatus depicted in Figure A.1. The laser is mounted within an aluminum block temperature-stabilized to 1 mK/hour maximum drift. A fast collimating compound lens (Melles Griot type 06 GLC 001 with 6.5 mm focal length) collimates the output beam into an elliptical beam with a 4:1 aspect ratio; there is slight vignetting of the long axis of the beam by the lens. The main cavity feedback element is a holographic grating (Edmund Scientific type J43,224 with 2400 grooves/mm) mounted in the Littrow configuration as described by MacAdam *et al.* [62], in which the first order diffracted beam is retroreflected into the laser. In this way the grating together with the back face of the laser diode forms the laser cavity. As a feedback element, the cavity transmission profile can be adjusted both by the grating's tilt with respect to the laser and by its spacing from the laser. In practice the tilt is adjusted by means of set screws and then held fixed, and the laser is tuned by varying the grating distance by means of a piezotransducer on the grating's flexure mount. The whole of this apparatus is mounted within a Lucite box with an additional layer of temperature stabilization. Both temperature servos use Peltiers as the heating/cooling elements.

The electronics for the temperature servos are adapted from a design used by

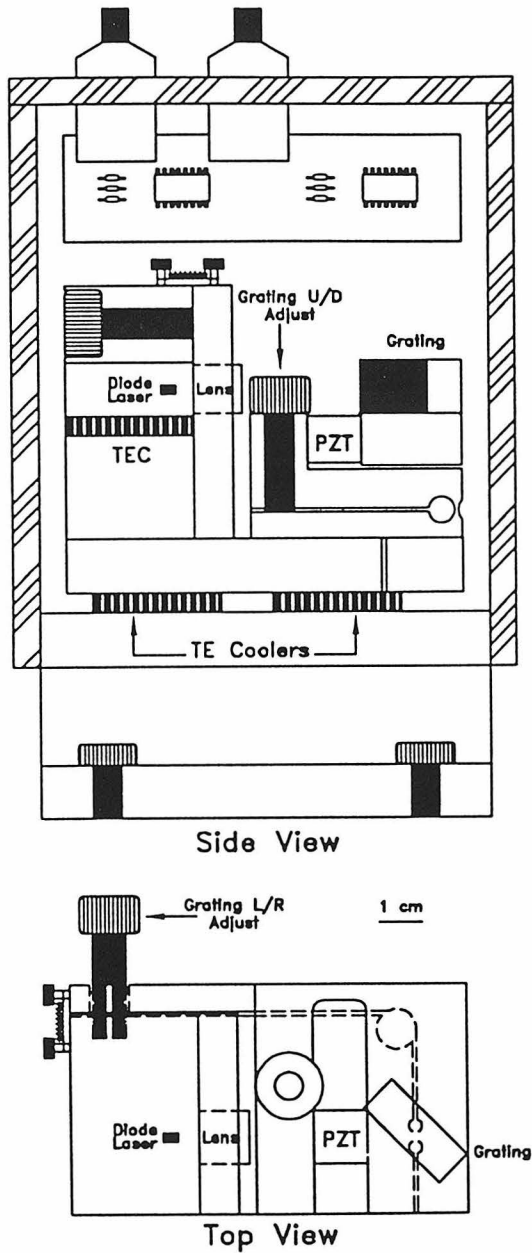


Figure A.1: The diode laser assembly. The laser diode output is collimated by the lens before striking the grating, which with the back facet of the diode forms a feedback cavity. The grating left/right and up/down tilts are controlled by the screws shown. The grating/laser distance is varied using the PZT.

Bradley *et al.* [63] and are essentially a PID control system to heat or cool a thermistor mounted near the laser diode to be stabilized so that the thermistor voltage remains fixed compared to a stable Zener reference.

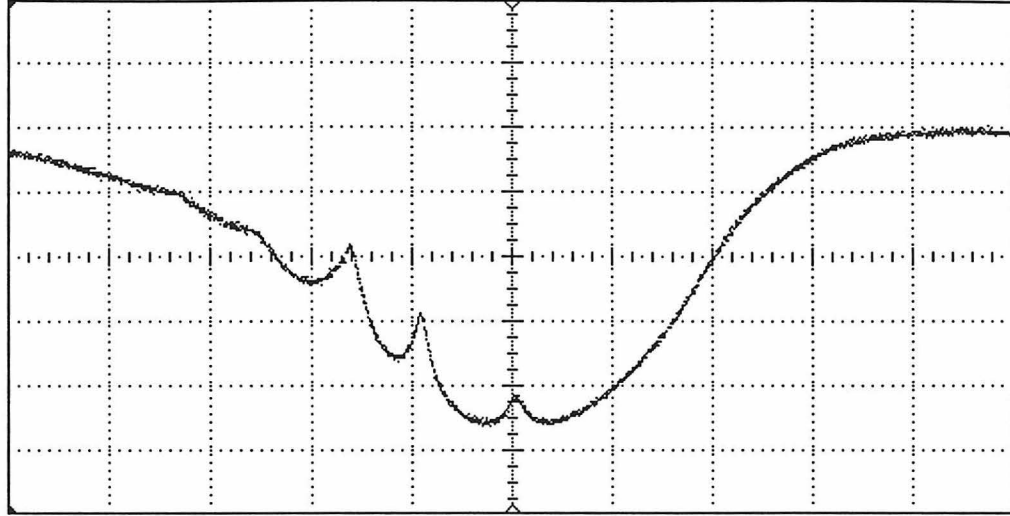
The injection current to the diode lasers is provided by the low-noise high speed diode laser current controller of Libbrecht and Hall [64]. It provides a highly stable current supply to the laser diode that can be independently and simultaneously modulated at RF for adding sidebands and at audio frequencies for frequency stabilization feedback control.

A.2 The Repumping Laser

Of all the lasers in our experiments, the repumping laser has the least stringent stability requirements, and therefore has the simplest locking arrangement. Its job is to optically pump atoms out of the $F = 3$ ground state before they escape from the trap, and since the escape time is of the order of milliseconds or greater, and only a few photons are required to optically pump atoms into the $F = 4$ ground level using the $F = 3 \rightarrow F' = 4$ transition, it is sufficient to have the repumping laser sweep through resonance at a few kHz at saturation intensity.

As both this laser and the master laser (described later) use the cesium saturation absorption spectrum as a frequency reference, we briefly describe saturation absorption spectroscopy here. A more detailed description can be found in the excellent review by Schmidt *et al* [65]. We typically divert a fraction of the laser beam through a cesium cell using a beamsplitter. This pump beam excites the atomic transition to a degree dependent on the beam intensity and detuning, and in particular on the velocity of the atom along the beam axis. The beam is then retroreflected through the cell, acting as a probe beam that is collected by a photodetector (PD). Since the pump beam intensity is sufficient to saturate the transition, it bleaches the medium for the probe beam if the probe is resonant with the same set of atoms in the cell as the pump, thus decreasing the overall absorption. This occurs only at a few specific laser frequencies. Since the Doppler shifts for atoms with a given velocity are equal

(a)



(b)

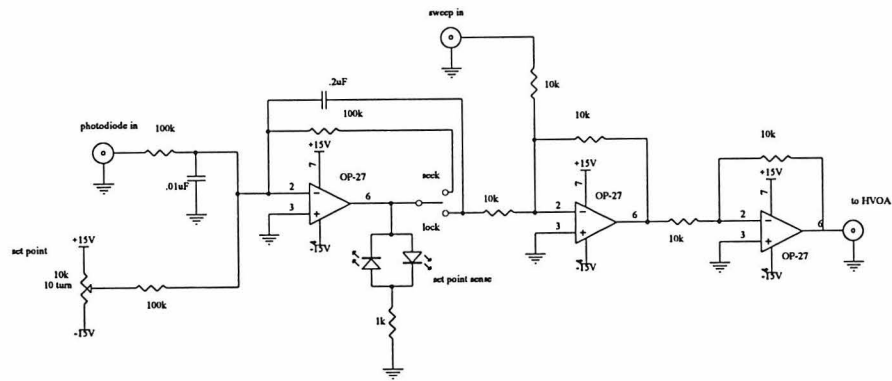


Figure A.2: The repumping laser servo: (a) a picture of a saturation spectrum. (b) The repumping laser servo electronics.

and opposite for the two counterpropagating laser beams, the atoms with zero velocity along the beam axis will be bleached if the laser is on a resonance frequency, thus causing a dip in the absorption spectrum. If the laser frequency is exactly halfway between two transition frequencies, there will be a set of atoms with velocity such that the pump beam will be Doppler-shifted into resonance with one transition and the probe beam with the other. Dips in the absorption spectrum at these frequencies are called crossover resonances. Figure A.2(a) shows one such saturation absorption spectrum.

This locking circuit is shown in Figure A.2(b). The voltage to the piezotransducer (PZT) is swept at a frequency between 2 and 5 kHz, typically at a resonant frequency of the PZT. The amplitude of the sweep is such as to vary the laser frequency over a few atomic linewidths. This results in an oscillating voltage from the PD which is averaged and compared to a DC set point by the first amplifier. This setpoint is chosen so that the atomic resonance is somewhere in the sweep range. Any differences between the average and the setpoint are amplified by the integral gain circuit and added to the the PZT sweep voltage by the second amplifier. To lock the circuit, one simply views the PD signal on an oscilloscope until the desired resonance is visible, adjusts the setpoint to equal the PD average, as indicated by equal brightness of the indicator LED's, and switches on the lock. The setpoint then controls the sweep center frequency.

A.3 The Trapping Laser

The trapping laser stability requirements are much more stringent. The laser width must be kept well below the atomic linewidth and its frequency must be held stable to comparable precision. Its frequency must also be continuously tunable over a range of more than 100 MHz. In addition, its beam profile must be accurately gaussian in order for the MOT shape to be regular.

A.3.1 The Master Laser

We heterodyne-lock our trapping laser to a master laser which is accurately and stably locked to the $F = 4 \rightarrow F' = 5$ transition using a scheme related to the Pound-Drever method [66] for locking a laser to a cavity. In our system, sidebands at about 9 MHz are added to the master laser by modulating its injection current. When the master laser beam passes through the saturation absorption cell, the sidebands are differentially absorbed by the $F = 4 \rightarrow F' = 5$ peak, the degree of absorption depending on the detuning of the center frequency from resonance. In the absence of differential absorption, the beat note between one sideband and the central peak is of equal amplitude and opposite phase to the beat note between the other sideband and the central peak, and therefore the two beat notes cancel. In the presence of differential absorption, the two beat notes have unequal amplitude and no longer exactly cancel out, and so a residual beat note at the modulation frequency remains. For small detunings, the amplitude of the residual beat note is linear with detuning, and its phase shifts 180 degrees at resonance. The residual beat note is detected by a fast photodetector and mixed with the modulation signal to yield a DC error signal that is fed to the laser injection current and to the grating PZT to tune the laser.

The major source of systematic error for this locking scheme is modulation of the laser power due to causes other than dispersion by the atomic vapor. One such cause is laser amplitude modulation. Since the laser power increases with injection current, current modulation causes amplitude modulation as well as frequency modulation. (In fact, part of the frequency change is induced by the amplitude change via the phase-amplitude coupling coefficient α [67].) The beat note caused by this effect has no laser frequency dependence over the scan width of the laser (~ 100 MHz), and thus gives a constant offset to the DC error signal. In practice, no such offset was observed in the error signal to within 100 kHz, so its presence is assumed to be ignorably small.

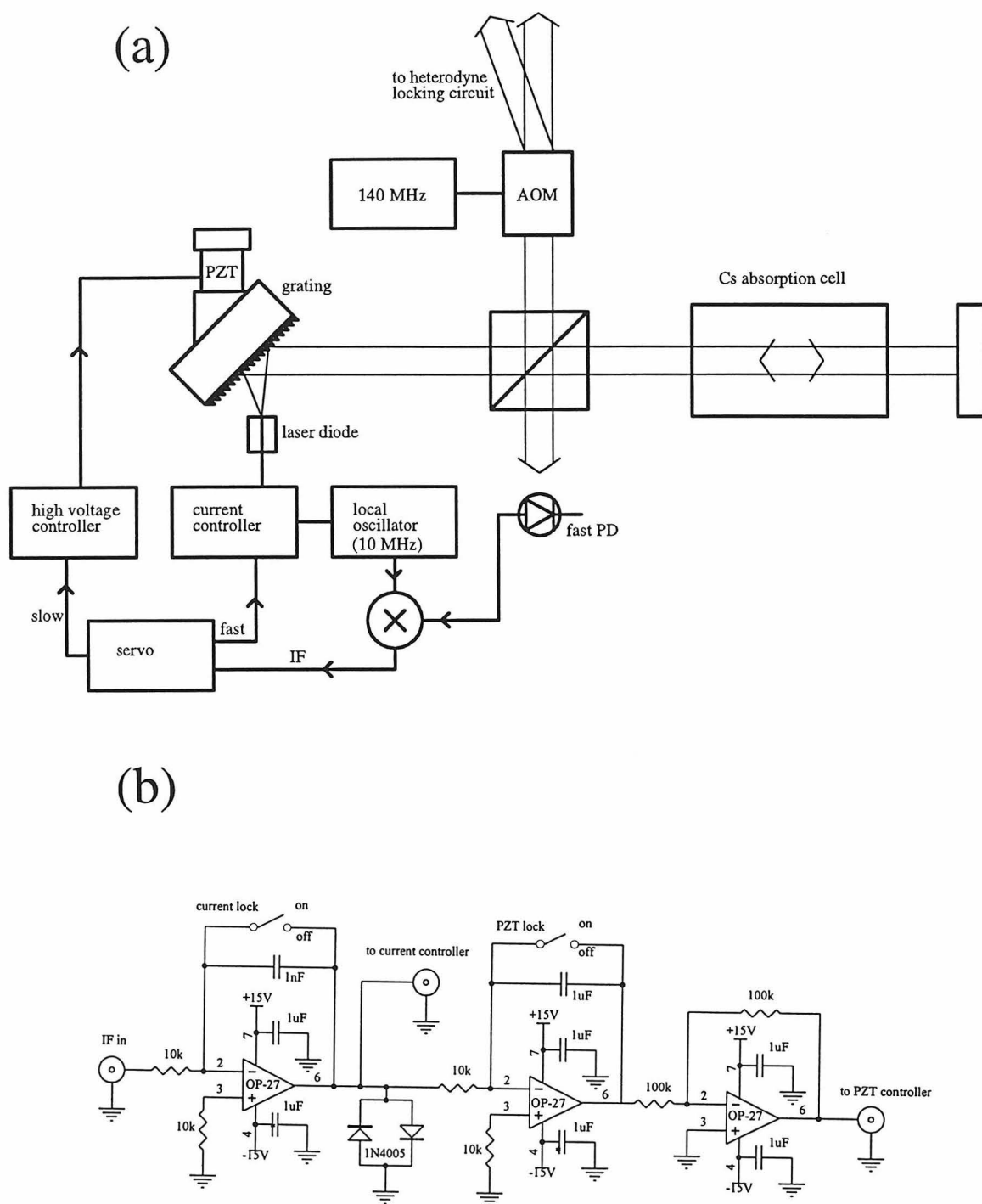


Figure A.3: The master laser servo. (a) The master laser optics. (b) The master laser servo electronics.

A.3.2 Master Laser Servo

The locking schematic is shown in Figure A.3. It is a twin-servo system that isolates the high-frequency error signal to the servo controlling the injection current, which has a bandwidth of 100 kHz, and leaves the low-frequency error signal to the servo controlling the PZT, which has only 1 kHz bandwidth but can tune the laser over a much wider range without mode-hopping. The circuit does this by using an integral control servo to lock the IF signal of the mixer to zero volts. This servo is sensitive from DC to 100 kHz. The second integral control servo locks the output of the first servo to zero volts, but with a bandwidth of only 100 Hz. The output of the second servo is allowed to control the PZT over its entire voltage range. The diode clamp on the output of the first servo improves the robustness of the locking circuit against large transients by keeping the first servo output within a diode drop of ground. The stability of the servo was measured by beating a laser locked in this manner with a laser locked by a simple servo that compared the Cs absorption spectrum voltage with a voltage setpoint. The beat note had a width of 500 ± 100 kHz, which sets an absolute upper limit on the locked laser width if the second laser is assumed to have zero width. Assuming the second laser to have the same width, the master laser lock stability becomes 300 kHz. This latter scenario is more likely as the laser width is limited by the characteristics of the cavity formed by the grating and the laser.

The mixer used in this and the following circuits is the Mini-Circuits ZAD-1, a Level 7 mixer with 500 MHz bandwidth. The LO signal to a Level 7 mixer must be +7 dBm for optimum performance, so in practice the LO is set at +7 dBm for the mixer and then attenuated about 20 dB at the laser current controller RF input to get the cleanest mixed-down dispersion signal. The fast PD output is amplified, typically by 40 dB, prior to the mixer to increase the servo gain.

A.3.3 Heterodyne Lock Servo

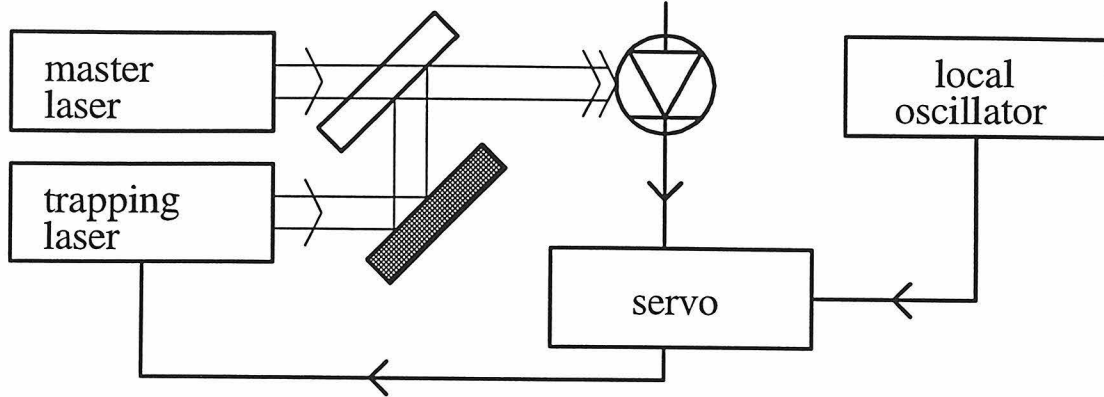
The trapping laser is heterodyne locked to the master laser using a phase locked loop (PLL). Good descriptions of PLL technique abound in the literature [68] and will

not be presented here. About 4% of the trapping laser power is diverted from the main beam and beat with the master laser onto a fast photodetector of 250 MHz bandwidth. This signal is then amplified and sent to the phase locked loop circuit, which is shown in Figure A.4 and which we now describe in some detail. The input analog signal is first converted to a digital signal by the input comparator, which is a Plessey SP9685, chosen for its high speed. To reduce the frequency of the beat note to a range more readily accessible to pulse generators, the digital signal is then divided by 64 by the prescaler. A second comparator converts the slow digital signal from MECL logic to TTL logic. The trimpot on its noninverting input is adjusted to setpoint midway between MECL high and MECL low. The converted signal is sent to the phase/frequency detector, which compares the phase of the divided beat note to that of a local oscillator and outputs a voltage proportional to the phase difference. This voltage, which is the error signal, is amplified and directed to the modulation current and grating PZT voltage of the trapping laser in the same way as is done for the master laser. The fast part of the servo (up to the phase/frequency detector) is built on a printed circuit board with a ground plane for high bandwidth, and the power leads have series chokes to guard the power supply from RF ripple. An EH1560 programmable pulse generator, which can be programmed to switch rapidly between pulse rates, is used as the local oscillator, and its frequency controls the trapping laser detuning.

As a test of the trapping laser stability, we measured the beat note used in the heterodyne lock, and found it to be 500 ± 100 kHz. This is consistent with the earlier measurement and again suggests that the laser width is limited not by the locking servoes but by the laser/grating cavity. Since feedback of the trapping laser into the master laser from the fast PD, or vice-versa, will render the lasers somewhat unstable and increase their widths dramatically, isolation of the lasers from each other was carefully maintained.

The trapping laser detuning to the red of the $F = 4 \rightarrow F' = 5$ resonance can range from a few MHz to 100 MHz. For certain experiments, it may be necessary to tune to the blue as well as to the red, or even to exact resonance. It is difficult to

(a)



(b)

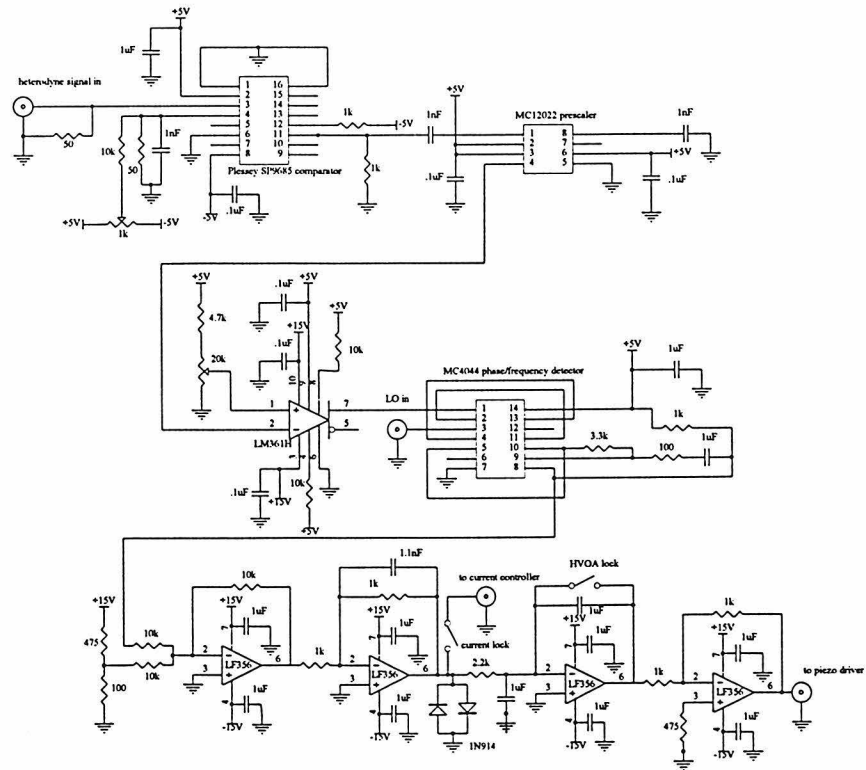


Figure A.4: The trapping laser servo. (a) The heterodyne optics. (b) The heterodyne servo electronics.

design phase locked loops that operate well over orders of magnitude of frequency, and they do not work at DC at all. For this reason, the master laser is frequency shifted 140 MHz to the blue by an acousto-optical modulator (AOM) before it is beat with the master laser. In this way, the entire tuning range of the trapping laser varies the beat note by only a factor of 2, and the error signal need not change sign to lock the trapping laser a few MHz to the blue of resonance. The AOM frequency is very stable and does not measurably change the lock stability.

A.4 The Cleaning Cavity

To give the trapping laser a smooth gaussian profile, we pass it through a non-confocal cleaning cavity. The cleaning cavity is mounted with one mirror fixed and the other mirror mounted on a PZT, which controls the spacing of the cavity. The cavity must have its spacing locked at a value such that the laser is resonant with the lowest order transverse mode of the cavity. To do this, sidebands are added to the trapping laser via modulation of the injection current. The sideband spacing is somewhat larger than the cavity mode width, about 9 MHz. To the degree that the central peak and sidebands are not resonant with the cavity they will be reflected, and as the cavity resonance differs from the laser frequency the reflection coefficients will differ. The reflected laser beam will have a beat note, and just as in the case for the master laser, the amplitude and phase of the beat note are a measure of the cavity detuning. This technique is like an “inverse” Pound-Drever method, in which the cavity is locked to the laser frequency rather than vice-versa. Unlike the master laser, the cavity resonance frequency is controlled only by a PZT, and so the control circuit is only a single integral gain servo, similar to those controlling the PZT’s in the master laser servo and trapping laser servo. The trapping laser optics up to and including the cleaning cavity are shown in Figure A.5.

The optics required to align and lock the cleaning cavity are worth mentioning. Alignment of the cavity for optimum transmission of the main trapping laser peak leads to retroreflection of the sidebands and the unwanted transverse modes into the

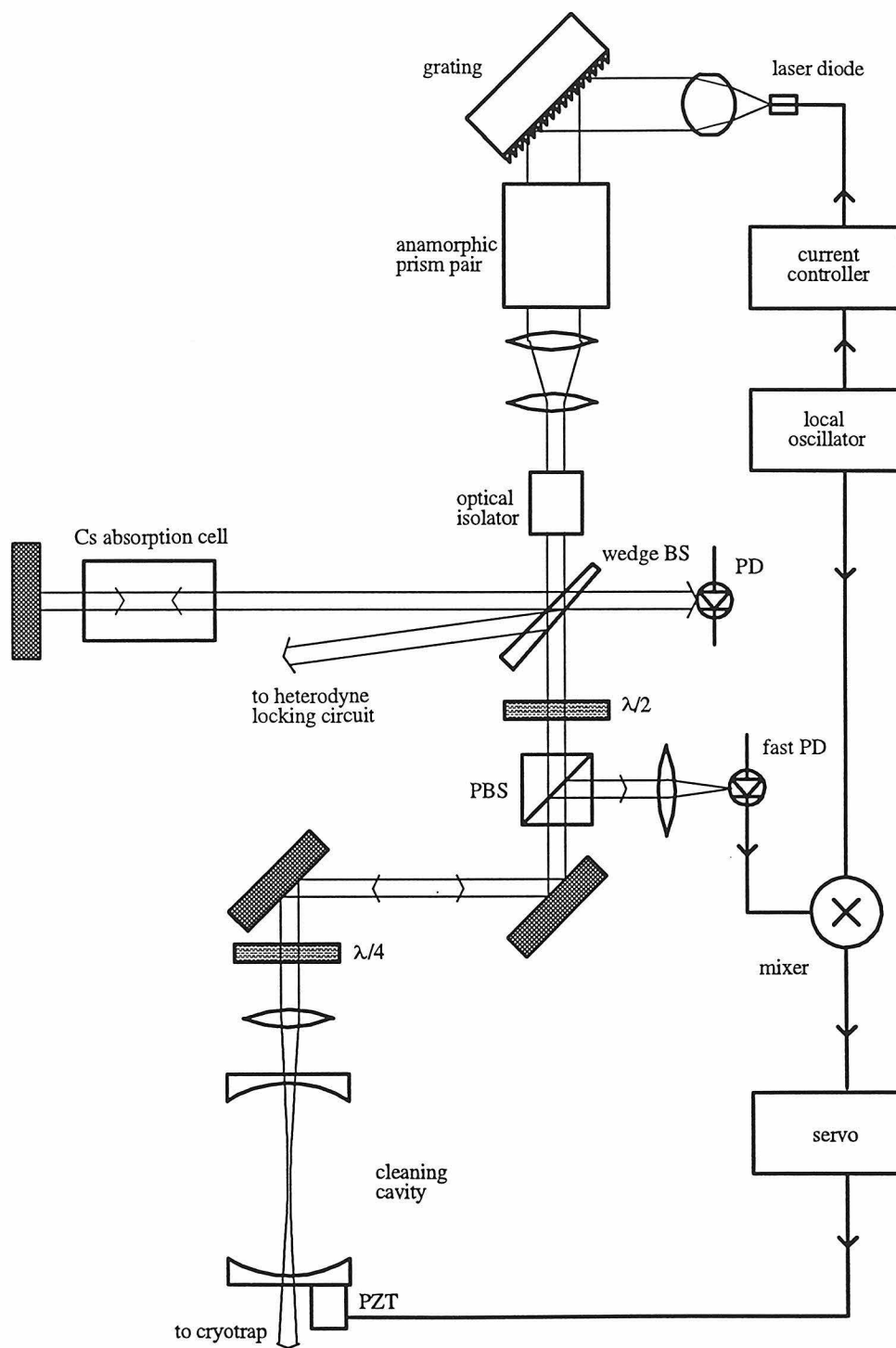


Figure A.5: The trapping laser optics, including the cleaning cavity.

laser diode. Also, the setup of the MOT optics itself leads to retroreflection. This feedback of laser power can overcome the locking circuits and increase the laser noise, so several steps are taken to prevent it. First, a Faraday isolator is used to attenuate the retroreflected laser beam by about 40 dB. This is alone not sufficient, though, so retarders are used to give the forward and retroreflected laser beams perpendicular linear polarizations, and a polarizing beamsplitter separates the retroreflected beam from the forward beam with about 20 dB efficiency and directs it to the fast PD.

A.5 The Fast Photodetector

The RF dither lock of the master laser requires photodetection of optical amplitude modulation in the neighborhood of 10 MHz, and the heterodyne lock of the trapping laser requires photodetection of laser beat frequencies from 140 MHz to 240 MHz. To accomplish this we designed a fast photodetector with a bandwidth of 250 MHz and detectable response out to 700 MHz, as determined by observing the beat note between two lasers, one locked and one free-running.

The schematic is shown in Figure A.6. The photodiode is an EG&G FND-100, chosen for high speed and sensitivity at 852 nm. This high speed is obtained via the large -90V bias. The SCR circuitry in parallel with the photodiode is an overvoltage protection circuit, included because the photodiode is damaged by a bias in excess of 125V. If the input exceeds 100V, the SCR then clamps it to zero and the LED indicator lights. The bias must then be removed and reapplied to restart the circuit. It is important to remember that this protective circuit requires the bias voltage supply to have a current limiting feature set just above the normal photodiode operating current if the overvoltage protection circuit is to be effective. The fast portion of the photodetection signal is AC-coupled to the NE5205 preamplifier, which boosts it 20 dB. The LM317 generates the 8V supply voltage for the preamplifier, the value chosen to maximize the bandwidth at the expense of some flatness of response. The inductor decouples the rapid load fluctuations from the power supply. The slow part of the photodetection signal is amplified 10x.

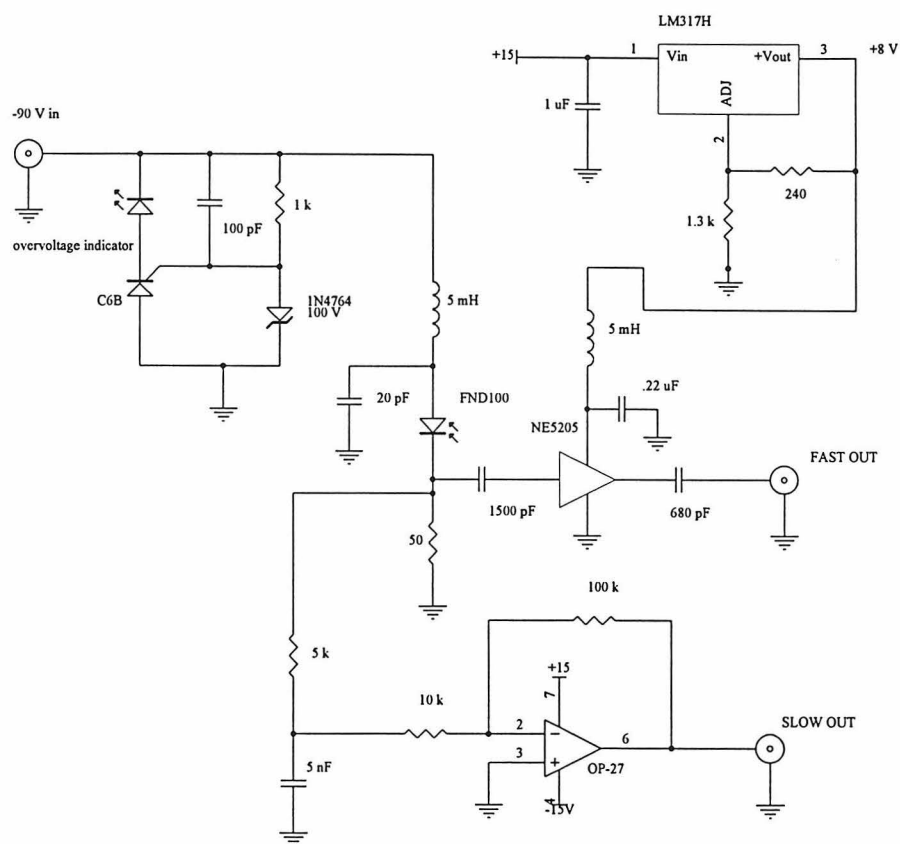


Figure A.6: The fast photodetector schematic. The SCR circuit at the -90V bias input provides overvoltage protection at -100V input. The NE5205 amplifier provides 20dB gain to the fast output.

Certain construction techniques are used for best results. The circuit resides on a printed circuit board with a ground plane, and all high-frequency signal paths are kept short. The edges of the board are shielded with copper tape. Lastly, the RF signal is launched from the printed circuit board with a short length of BNC coax, rather than with simple wires.

Appendix B The Optical Bloch Equation Program

```
% KAPPA2.M solves the optical Bloch equations for a J=0-J'=1 atom
% in a 3-D sigma+ sigma- laser field in MATLAB.
% Written by Phil Willems 3-29-96

% some constants

hbar=1.05e-34;
gamma=3.1e7; % gamma is the spontaneous decay rate
W=gamma*sqrt(.1/1.04/8); % W is the Rabi frequency, also in units of gamma
wa=1.4*gamma; % wa is the laser detuning in linewidths
k=2*pi/852e-9; % k is the laser wavenumber
B=9.27e-24/1.05e-34*Bp*dist; % B is the magnetic detuning in linewidths;
                                % dist is a global variable in the MATLAB
                                % environment

% create the Hamiltonian
sm1=sigmam1; % the sigmamx macros define the raising and
sm2=sigmam2; % lowering operator matrices
sm3=sigmam3;
Ha =hbar*wa*((1+B/wa)*sm1'*sm1+sm2'*sm2...
+(1-B/wa)*sm3'*sm3); % interaction Hamiltonian
                        % in the rotating wave
                        % approximation
```

% the stimulated Hamiltonian Ha has spatial dependence thanks to the
 % explicit electric field

```

Hi1=hbar*W*(sm1*exp(i*(k*z+phi(1)))+sm1'*exp(-i*(k*z+phi(1))));
Hi2=hbar*W*(sm3*exp(-i*(k*z+phi(2)))+sm3'*exp(i*(k*z+phi(2))));
Hi3=hbar*W*((-i/2*sm1+1/sqrt(2)*sm2+i/2*sm3)*exp(-i*(k*y+phi(3)))...
+(i/2*sm1'+1/sqrt(2)*sm2'-i/2*sm3')*exp(i*(k*y+phi(3))));
Hi4=hbar*W*((-i/2*sm1-1/sqrt(2)*sm2+i/2*sm3)*exp(i*(k*y+phi(4)))...
+(i/2*sm1'-1/sqrt(2)*sm2'-i/2*sm3')*exp(-i*(k*y+phi(4))));
Hi5=hbar*W*((1/2*sm1+1/sqrt(2)*sm2+1/2*sm3)*exp(-i*(k*x+phi(5)))...
+(1/2*sm1'+1/sqrt(2)*sm2'+1/2*sm3')*exp(i*(k*x+phi(5))));
Hi6=hbar*W*((1/2*sm1-1/sqrt(2)*sm2+1/2*sm3)*exp(i*(k*x+phi(6)))...
+(1/2*sm1'-1/sqrt(2)*sm2'+1/2*sm3')*exp(-i*(k*x+phi(6))));
Hi=Hi1+Hi2+Hi3+Hi4+Hi5+Hi6;
H=Ha+Hi;
L0=spre(H/(i*hbar))-spost(H/(i*hbar)); % stimulated and internal
                                         % Hamiltonian expressed as a
                                         % superoperator

```

```

C1=sqrt(gamma)*sm1;
C2=sqrt(gamma)*sm2;
C3=sqrt(gamma)*sm3;
CdC1=C1'*C1;
CdC2=C2'*C2;
CdC3=C3'*C3;
L10=spre(C1)*spost(C1')+spre(C2)*spost(C2')+spre(C3)*spost(C3');
L11=-0.5*spre(CdC1)-0.5*spost(CdC1);
L12=-0.5*spre(CdC2)-0.5*spost(CdC2);
L13=-0.5*spre(CdC3)-0.5*spost(CdC3);

```

```

L1=L10+L11+L12+L13; % spontaneous decay 'Hamiltonian'
                        % expressed as a superoperator

L=L0+L1;

Lp=L;
Lp(1,:)= [1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1]; % make the Molmer matrix

ep=[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]';
rhoss=Lp\ep; % calculate the steady state
            % density matrix column vector
            % rhoss

% the dipole force is the expectation value of the dipole moment
% dotted into the gradient of the electric field. result3 is the dipole
% force.
dHi1=i*k*(hbar*W*(sm1*exp(i*(k*z+phi(1)))-sm1'*exp(-i*(k*z+phi(1)))));
dHi2=i*k*(hbar*W*(-sm3*exp(-i*(k*z+phi(2)))+sm3'*exp(i*(k*z+phi(2)))));

dHiz=dHi1+dHi2;

trfunc3=m2trace(dHiz); % m2trace() converts operator matrices
result3=trfunc3*rhoss; % into a form where the trace can be
                        % taken by multiplication on rhoss

% stimulated part of the diffusion coefficient in z direction

interz=dHiz'*reshape(rhoss,4,4);
psiz=interz(:)-result3(1)*rhoss;
psiz(1)=0;

```



```
integralz=Lp\psiz;
```

```
Dstim=-real(dHiz.'*reshape(integralz,4,4));
```

```
% spontaneous part of the diffusion coefficient in z direction
```

```
Dspon=real(.5*gamma*((rhoss(1)+rhoss(11))*.7*(hbar*k)^2+...  
rhoss(6)*.6*(hbar*k)^2));
```

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- [68] See, for example, the application notes for the MC4044 phase-frequency detector in the Motorola MECL data book, 5th edition.