INTERNAL FRICTION IN METALS

Thesis by

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TABLE OF CONTENTS

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The work was under the general direction of Dr. F. C. Lindvall, to whom thanks are due for the assistance and advice which he has given, and for the interest which he has shown throughout the project.

SUMMARY OF RESULTS AND CONCLUSIONS

(1) The mean damping capacity of steel over a stress range of several hundred pounds per square inch is a physical property of the material, the value of which can be checked accurately by successive tests on the same specimen, with the apparatus and method developed in the course of this investigation.

(2) The results of the damping capacity test can be duplicated accurately for successive specimens of the same material.

(3) There is a strong possibility that although the general trend of the stress-damping capacity relationship may follow a simple law, the true relationship for a material such as steel is extremely complex and somewhat erratic.

(4) The stress-damping capacity relationship indicates that for steel the damping does not follow the viscous friction law, although for low stresses the assumption of constant damping would not be far in error.

In the case of brass, the internal friction was practically independent of stress for the stress range covered in the tests.

(5) The damping capacity of the specimens tested was found to be a function of the maximum torsional stress set up in the specimen rather than a function of the maximum shear stress or of the maximum normal stress.

(6) In the case of materials whose damping capacity decreases with decreasing stress, the apparent damping capacity measured from a solid test specimen is lower than the true specific damping capacity of the material.

INTRODUCTION

A. Internal Friction in Solids

When a solid material is strained, there is, associated with the work done by the straining forces, a dissipation of energy in the form of heat. This dissipation of energy is due to the presence of internal frictional forces which tend to resist the motion of the material.

Imagine a specimen of a solid which has been subjected to a completely reversed stress cycle, as illustrated by the stress-strain curve of Figure 1. The area underneath the curve from 0 to A will represent the work done on the specimen per unit volume, by the straining forces. The area underneath the curve from A to B will represent the work per unit volume done by the elastic restoring forces. This work, however, will be less than that done by the straining forces by the amount which has been dissipated as heat by the frictional forces. From the figure, it will be seen that the area of the loop ABCDA represents this frictional loss for the complete cycle. This so-called "elastic hysteresis loop" is thus entirely analogous to the hysteresis loop obtained in the magnetization curves for various magnetic materials.

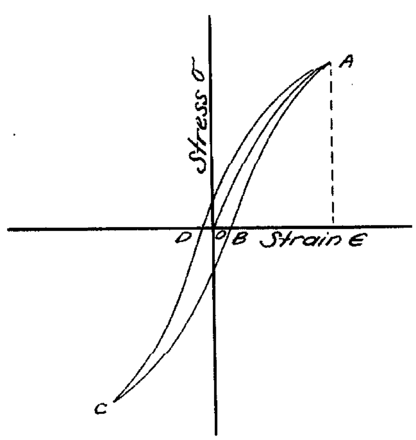

Figure 1.

The common structural metals, with the notable exception of cast-iron, have such a small dissipation of energy that the hysteresis loop cannot be detected by the usual methods for measuring stress-strain relationships. A consideration of the consequences of this internal loss in the case of vibration of the metal indicates that such a loss must be present, however, even for very small strains of the material. Suppose that a specimen of the material is made in the form of a cantilever spring, whose end is transversely deflected and then released, so that the spring executes free vibrations. During the vibration of the spring each element of the surface material passes through a completely reversed stress cycle of the type shown in Figure 1. The loss of energy in each cycle will be manifested in this case by a decrease in the amplitude of the vibration. Now, no matter how small the initial displacement or stress in the specimen may be, it seems unlikely that the spring would vibrate forever, even if the small amount of air-resistance present were completely eliminated. It is thus very probable that for any stress cycle, no matter how small, there is some energy dissipation.

Referring again to Figure 1, it will be seen that there cannot be an energy loss unless there is a permanent deformation of the material as represented by the strain OB in the diagram. We may thus conclude that such a stress as an "elastic limit", which corresponds to a stress at which there is no permanent deformation of the material, has no physical existence, and that similarly Hooke's Law is only relatively true, but can never be exact. There is, therefore, a simple explanation for the fact that measurements of increasing accuracy have always lowered the experimentally determined values of elastic limit and proportional limit for the various structural materials.

B. Practical Importance of Internal Friction

The practical importance of information as to the amount of internal friction in metals may be summarized under three headings:

(1) In many cases, the internal friction of the metal itself is the only factor which limits the amplitude of forced vibrations of machine parts or of structures. Examples are turbine blades, the hulls of ships, and transmission wires.

(2) There is reason to expect anomalous values of the total internal friction for entire parts or structures which contain internal defects such as cracks, inclusions, etc.. Hence, there is a strong possibility that the measurement of the internal friction might form a simple and quick non-destructive test of complete structures and of castings, forgings, and similar parts (Ref. 15)*.

(3) Since the amount of internal friction present is a physical property of the material, there is reason to expect a correlation between that amount of internal friction and other physical properties, especially other dynamic properties, of the material. In such a case the internal friction test might prove to be simpler, quicker, more accurate, or more sensitive than the tests available for the other properties.

C. Methods of Specifying Internal Friction

Two quantities have been widely used as measures of the internal friction loss in solids. The first defines the amount of internal friction, or the damping capacity, as the ratio of the energy lost per

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*Numbers in parentheses refer to the Bibliography.

cycle due to friction to the total strain energy of the material at its maximum strained position in the cycle. This quantity is called by Föppl the "specific damping capacity" and is generally given the symbol ψ. (Ref. 12). The second method imagines that a specimen of the material is executing free vibrations, and takes the logarithmic decrement, δ, of the vibration as a measure of the energy loss.

Both methods have the advantage of being dimensionless numbers, and of being easily obtainable from experimental data. There is no practical difference as to which number is used since, as is shown in Appendix I., the specific damping capacity is numerically just twice the logarithmic decrement.

Throughout the present paper, the specific damping capacity, ψ, usually referred to as "damping capacity", will be taken as the most useful means of presenting data for the following reasons:

(1) It is a dimensionless number.

(2) It has a physical significance, since it is an energy ratio.

(3) It can be found easily from experimental work.

(4) It can be compared easily with other measures commonly used for damping capacity, especially so for the logarithmic decrement. (See Appendix II.).

(5) It has significance for any method of testing, whether vibrations are involved or not.

Reason (5) is the advantage of the specific damping capacity over the logarithmic decrement.

D. Methods of Measuring Damping Capacity

There are five methods which have been used for the experimental determination of damping capacity:

(1) The stress-strain diagram can be determined experimentally, and the area of the hysteresis loop thus can be measured directly. (Ref. 2,10,21.).

(2) The amount of heat generated in the specimen as it is subjected to stress reversals can be measured. (Ref. 25).

(3) It can be shown that if a rotating cantilever beam is transversely loaded at the end, the effect of internal friction will be a small deflection of the end of the beam in a direction perpendicular to the transverse loading. This deflection of the material is thus a measure of the amount of internal friction in the material. (Ref. 29,30,31).

(4) The behavior of the material under conditions of forced vibration at varying frequencies can be shown to give a measure of the damping capacity. The forced vibrations may be longitudinal (Ref. 9,38,46), transverse (Ref. 5,15,17), or torsional (Ref. 5,6).

(5) From a curve showing the decay of free vibrations of a specimen, the logarithmic decrement and, hence, the damping capacity, can be computed directly. The vibrations may be transverse (Ref. 2,19,23, 24,35), or torsional (Ref. 11,12,19,26,40,43,45).

The first two methods mentioned involve considerable difficulties as to apparatus and accuracy of measurement. The greatest care must be taken in the construction and operation of the apparatus in order to attain results which compare at all in accuracy with the other methods.

The third method has the advantage that the relationship between damping capacity and frequency can be shown directly. The size and form of the specimen required, however, for sufficient accuracy in the determination, would seem to make this method impractical for any large-scale investigation.

Most of the work which has been done has been either of the forced or of the free vibration type. The usual method of conducting forced vibration measurements has been to excite the specimen by means of a constant energy, variable frequency source, and to plot the resulting resonance curve. The smaller the amount of internal friction, the narrower will be the resonant peak. The width of the resonance curve at some specified percentage of the maximum amplitude then is taken as a measure of the damping capacity.

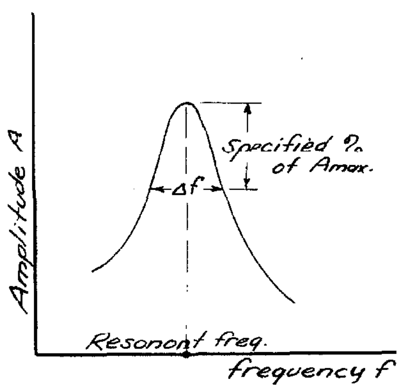


Figure 2.

This method has the advantage that the damping capacity can be measured accurately for very low stresses in the material.

There are two objections which can be raised in connection with this method:

(1) Since the magnitude of the stress cycle is continuously changing, it is not possible to study the variation in the damping capacity with stress. Since stress appears to be one of the most significant variables involved, this represents a serious limitation.

(2) The relationships between the shape of the resonance curve and the true damping capacity of the material depend on the type of relationship which exists between stress and damping capacity. Since this stress-damping capacity relationship is not known in general for a particular material, there is some question as to the physical interpretation of the resonance curve.

The first objection could be overcome by maintaining a constant amplitude of vibration at the various frequencies by changing the energy input to the specimen. From the energy-frequency resonance curve, shown in Figure 3, the damping capacity could be found.

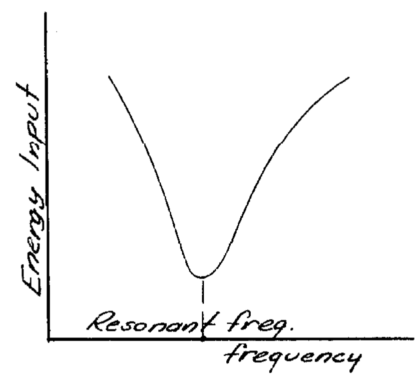


Figure 3.

The simplest of all of the methods consists of obtaining a record of the decay of free vibrations of a specimen of the material to be tested. Given the amplitude-time curve, the damping capacity at any stress may be quickly and accurately determined. Since this is the method chosen for the present investigation, complete details of the method and of its advantages will be taken up later.

E. The Results of Previous Investigations

Apparently the first mention of the phenomenon of internal friction is to be found in the works of Lord Kelvin. (Ref. 28). Kelvin's attention was directed to the subject by observations made by some of his students who were experimentally determining the periods of vibration of torsional pendulums, using as elastic members wires of various materials. The students noted that the rates of decay of the vibrations were markedly different for the various materials. They also noted that for the same material, different decay times would be obtained on Mondays than on Fridays, the "rest" which the material got over the weekend, after a busy week of testing, apparently having some effect on the physical properties of the material.

Kelvin saw the convenience of the torsional pendulum as a means of measuring internal friction, and performed several simple experiments from which he drew the following conclusions:

(1) The relationships between damping capacity and the frequency of vibration were found to be such that the internal damping forces could not be viscous in character, that is, the damping forces could not be assumed to be proportional to the velocity of strain.

(2) The previous stress history had a marked effect on the damping capacity.

(3) The general conclusion was reached that there can be no change in the shape of a solid, no matter how small, without some loss of energy as heat.

Since modern investigation has shown that Kelvin's conclusions were essentially correct, it is unfortunate that many of the later investigators were apparently unfamiliar with his work. The greater number of investigators from Kelvin's day down to several years ago, began their study by assuming that the internal friction was viscous. For the case of viscous friction the specific damping capacity is a constant of the material independent of the magnitude of the stress. Most of the early experiments, therefore, consisted of measurements of this "constant". Very unfortunately, some of the investigators adopted, as a means of recording their data, methods based on the assumption of viscous friction, so that not only are the conclusions which they drew from the data incorrect, but the data are of little use. It may be shown, for example, that if the damping is viscous in character, the time required for the amplitude of a free vibration to decrease to one-half of its original value is equal to $\frac{1.386}{\psi \times \text{frequency}}$. If the material itself did not follow the viscous friction law, however, data collected in the form of one-half amplitude times would not be very informative.

If viscous friction is assumed, it can be shown that the specific damping capacity should be directly proportional to the frequency. (Ref. 31). While Kelvin's simple experiments indicated that this was not the case, the matter was finally settled in a most convincing way by the work of A. L. Kimball. (Ref. 29,30,31). Using the ingenious rotating beam method devised by him, and outlined as method (3) above on page 5, Kimball found that there was a large frequency range in which the damping was essentially constant. The results of Kimball's tests may be summarized as follows:

(1) The specific damping capacity is independent of frequency over a large range. Thus internal friction is not viscous in character.

(2) While there was some variation in the measured damping capacity with the load on the beam, the damping capacity could be considered as essentially independent of the stress in the range of stress covered by the tests.

(3) The following empirical law was found to represent the facts for all materials tested:

$$\Delta W = \xi \sigma_m^2 \qquad \text{where:}$$

ΔW = loss in energy per cycle due to internal friction.

σ_m = maximum stress.

ξ = "solid friction coefficient" = $\frac{\psi}{2E}$, when E is the modulus of elasticity of the material.

For this solid friction law the specific damping capacity is a constant for any one material.

The work of the later investigators, done in all cases either by means of forced or free vibrations, has indicated that there is a definite change in the damping capacity with stress. The most common means of presenting data, therefore, at the present time, is to plot specific damping capacity versus stress.

There are few investigators who have reported the results of similar tests on similar materials. It is thus difficult to compare directly the results obtained by the various investigators. The type of agreement that is found is indicated in the following table. In each case the reported results have been converted to specific damping capacity, ψ, if necessary.

ψ = Specific Damping Capacity

Material	Forster & Köster (1938)	Kimball (1926)	Canfield (1928) Torsion	Canfield (1928) Bending	Iokibe & Sakai (1921)	Voigt (1892) Torsion	Voigt (1892) Bending
Aluminum	0.000092	0.0068	0.060	0.052	0.011	0.0121	0.00164
Copper	0.0071	0.0100	0.061	0.050	0.00152	0.00084	0.00143
Nickel	0.0144	0.0064			0.000212	0.00218	0.00354
Tin	0.0108	0.258				0.0220	0.0258
Zinc	0.0015	0.040				0.0116	0.0121
Iron	0.00112	0.0160	0.052	0.041	0.00256		
Brass		0.0096	0.048	0.038		0.00062	0.00135
Bronze		0.0064	0.048	0.054		0.00062	0.0020

In all cases the investigators have felt it possible to call the damping capacity of the material a constant, variations with stress apparently being small if detected. In most cases this is because the tests were made only in one narrow stress range.

It will be noted that not only does a wide variation exist between the values, but the variations themselves show little consistency. For example, copper is shown by Förster & Köster to have approximately one-half the damping capacity of nickel, while Kimball's results show just the reverse. In the comparison of torsion and bending carried out by both Canfield and Voigt, no consistent pattern of behavior can be discerned.

A more complete summary of the actual data obtained by the various investigators may be found in a recent paper by Kimball (Ref. 33).

The differences indicated in the above table, however, are not entirely inexplicable. It is now known that very slight changes in the structure, composition, state, and previous history of the specimen may have large effects on the damping capacity. It may well be that the larger portions of the discrepancies in the comparison of the table are actual differences in the physical properties of the materials tested. There are, in addition, many variable factors connected with the damping test, whose control is either not mentioned along with the experimental work, or which have been treated in entirely different ways by the different investigators. Some of these uncertain factors will be discussed later in connection with the present investigation.

The work of numerous investigators, however, has established some facts about the general behavior of damping capacity which it may be well to summarize here:-

(1) The damping capacity is independent of frequency over a large range.

(2) The damping capacity is dependent on the magnitude of the stress. In general the damping capacity increases with stress, although in some cases the opposite occurs. There is evidence to suppose that for very small stresses the damping capacity is essentially constant, but is still of an appreciable amount.

(3) The damping capacity increases with temperature. It has been suggested by Brophy that an apparent rise in damping capacity at high frequencies may be a temperature effect, since heat is being generated in the specimen by internal friction faster than it can be dissipated by the supports.

(4) The small amount of work which has been done on the problem of correlation of damping capacity with other physical properties seems to indicate that there is no general relationship between damping capacity and tensile strength, hardness, endurance limit, or impact resistance of the material. This negative conclusion should not be considered as firmly established.

(5) Some of the previous work indicates an increase of damping capacity with an increase in grain size, although this was not shown for all metals.

(6) High damping capacity in steel is accompanied by low notch sensitivity, and vice versa.

The experimental work on damping capacity has suffered in general from a number of defects which have very seriously limited the usefulness of damping capacity data to the engineer. These defects may be grouped under two headings.

In the first class are grouped the variable factors and the uncertainties in the measurement of damping capacity. Practically every damping investigation uses not only different types of instruments, but even entirely different methods. Each of these methods and each of the instruments has certain inherent variable factors and errors, which not only make it difficult to interpret the data from any one method, but make it practically impossible to compare the results of the various investigators.

In the second class we group uncertainties as to the nature of the material being tested. In no cases have the results of complete physical and metallographical tests been available to supplement damping tests. It must be required in all cases, in which the attempt to correlate damping with other properties is to be made, that complete tests of tensile strength, endurance limit, impact strength, structure, grain size, etc., be available.

The determination of the damping capacity of a metal, and the attempt to correlate it with other properties, is a process involving a very large number of variable factors, and unless these variable factors can be controlled properly, there can be very little confidence in conclusions drawn from the data.

From one point of view the large discrepancies in previously published work may be looked upon as fortunate. The damping capacity is apparently a very sensitive property of the material. For this reason, once it is understood, it becomes an even more interesting property to the engineer.

THE PRESENT INVESTIGATION

A. The Object of the Present Investigation

The present investigation was originally planned as a study of the correlations between damping capacity and other physical properties of metals. A study of previous work, however, indicated that so little was known about the damping test itself, that it would first be necessary to develop a reliable method of measuring the damping capacity of the metal.

The first problem, then, which it was considered should be solved, was the problem of establishing satisfactory techniques for the measurement of damping capacity. This involves both the design of an apparatus which would eliminate as many of the variables as possible, and a careful study of the variable factors which remain.

Accordingly, the following list of problems of a fundamental importance was compiled:-

(1) The selection of the best method of measurement for the type of investigation planned.

(2) An evaluation of the various instrumental errors:-

(a) The effects of clamping.

(b) Air resistance.

(c) Foundation losses.

(3) An experimental study of the reproducibility of results on the same specimen.

(4) A comparison of the results for different specimens of the same material.

(5) The effects of stress distribution in the specimen.

(6) The nature of the relationship between damping capacity and stress.

(7) An experimental study of the effects of combined stresses in the specimen.

(8) The effect of the previous history of the material on the results.

(a) Heat treatment.

(b) Cold-working.

(c) Fatigue.

(d) Corrosion Fatigue.

(e) Impact and repeated impact.

(9) The effect of temperature on results.

(10) The influence of the size and form of the specimen.

(11) A comparison of the results of the various methods of measurement.

The present investigation has been limited to a study of certain aspects of the first seven points mentioned.

B. The Selection of the Method of Measurement

Of the possible methods of measuring damping capacity, the only two which were seriously considered, for reasons mentioned above, were the forced vibration and the free vibration methods. Of these two methods, the free vibration method was considered to be the method which would permit the study of more of the variables involved. The particular advantages of the free vibration method for an investigation of this kind may be summarized as:-

(1) The damping capacity may be found as a function of stress.

(2) A relatively large range of stresses may be covered.

(3) The results may be interpreted easily by methods which make no assumptions as to the type of damping encountered.

(4) The apparatus is simple and can be designed for almost any degree of accuracy with little difficulty.

(5) The tests can be made quickly. This is an important advantage if any large scale attempt is to be made to correlate damping capacity with other properties.

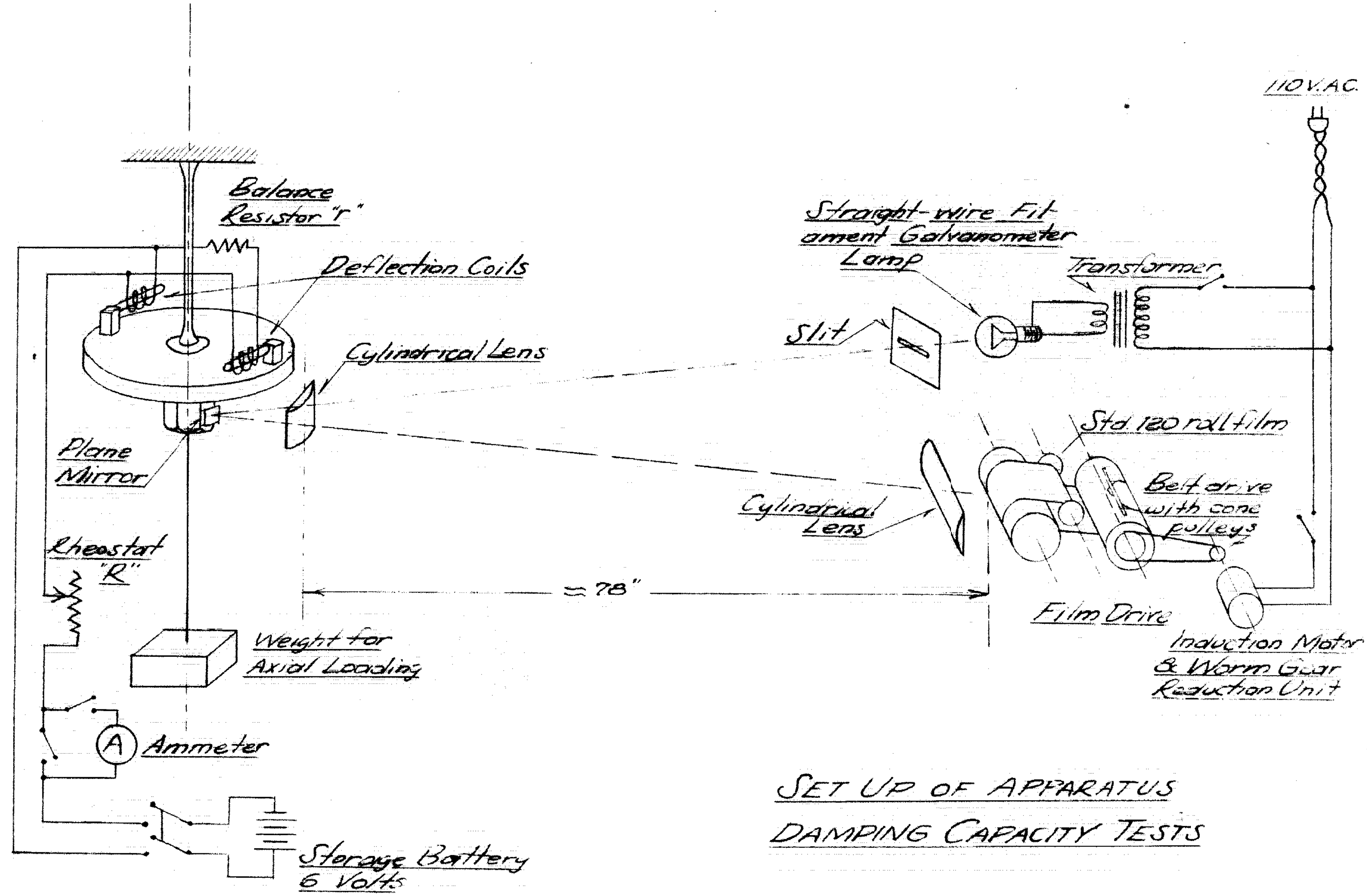
C. The Design of the Apparatus

In the design of the apparatus, the attempt was made to arrive at an instrument which would not only make it possible to study as many of the variables as possible, but also one which would be convenient enough in operation to be of practical use in an engineering sense. The degree of success in fulfilling these various requirements can be evaluated from the following discussion of the design features of the completed instrument. (See page 18 for the complete layout of the apparatus.)

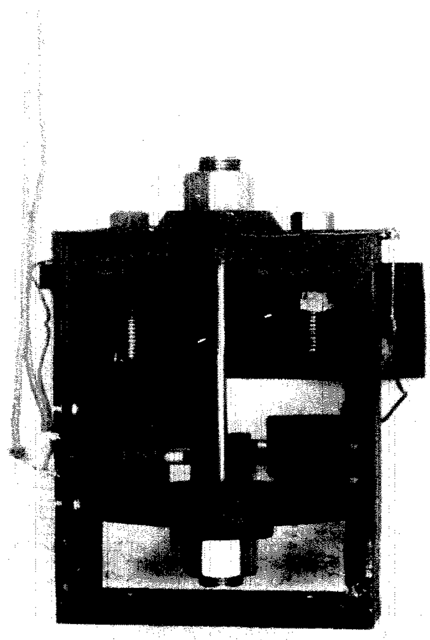
(1) It was decided that the free vibrations would be of the torsional type. This makes it possible to use cylindrical specimens of a type that can be accurately and inexpensively made. By combining axial and torsional stresses, a large variety of combined stress situations can be studied easily. (page 19.)

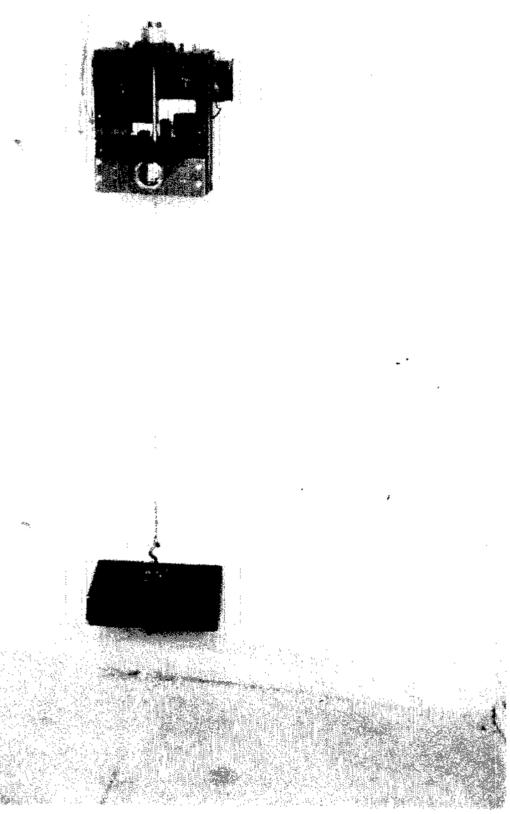
(2) The Design of the Specimen (pages 20 and 21.)

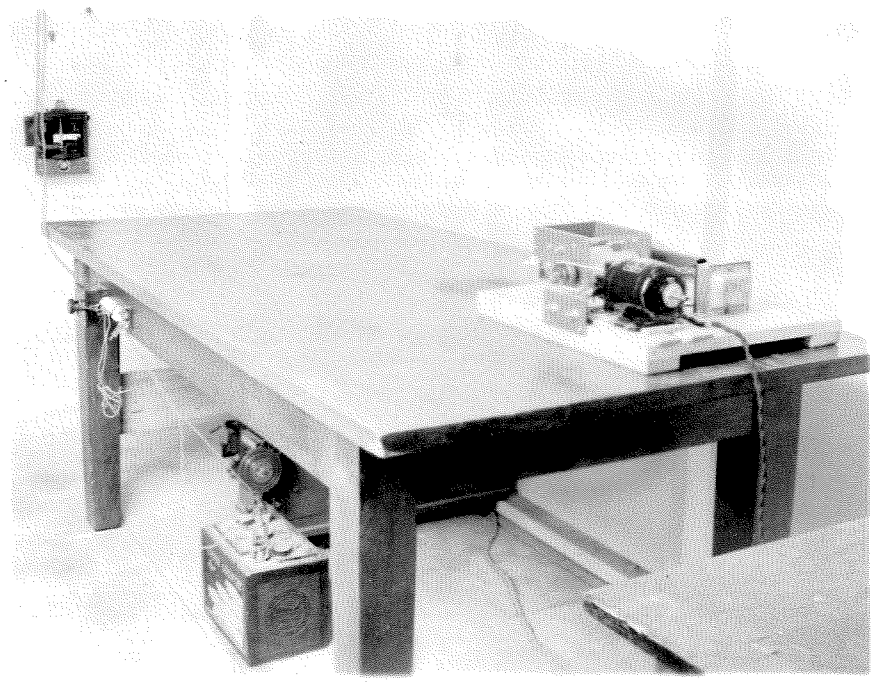
(a) The specimen was made large enough so that it represents a true sample of the material to be tested, and small enough so that it is inexpensive and easy to handle.

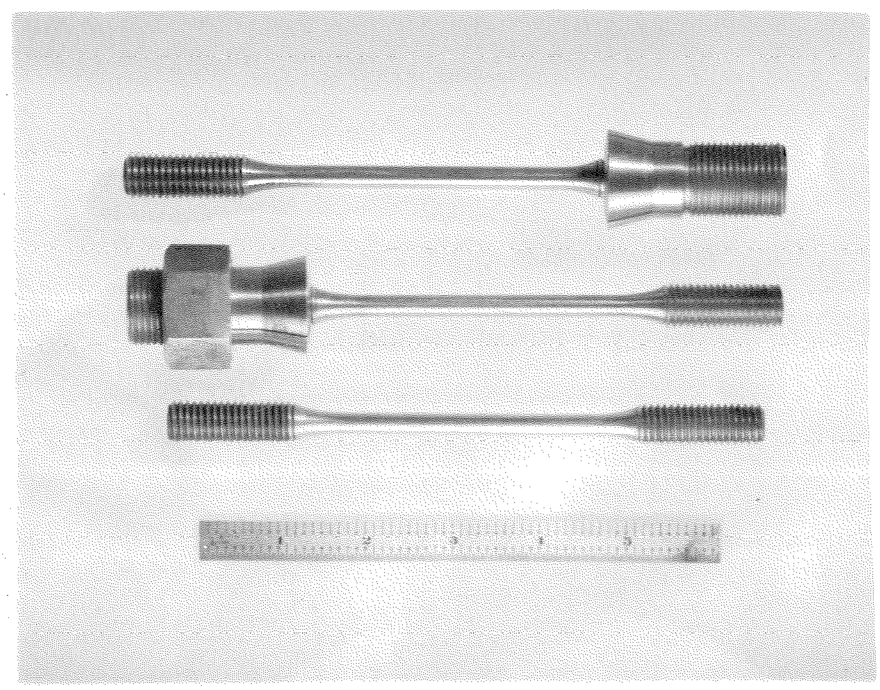


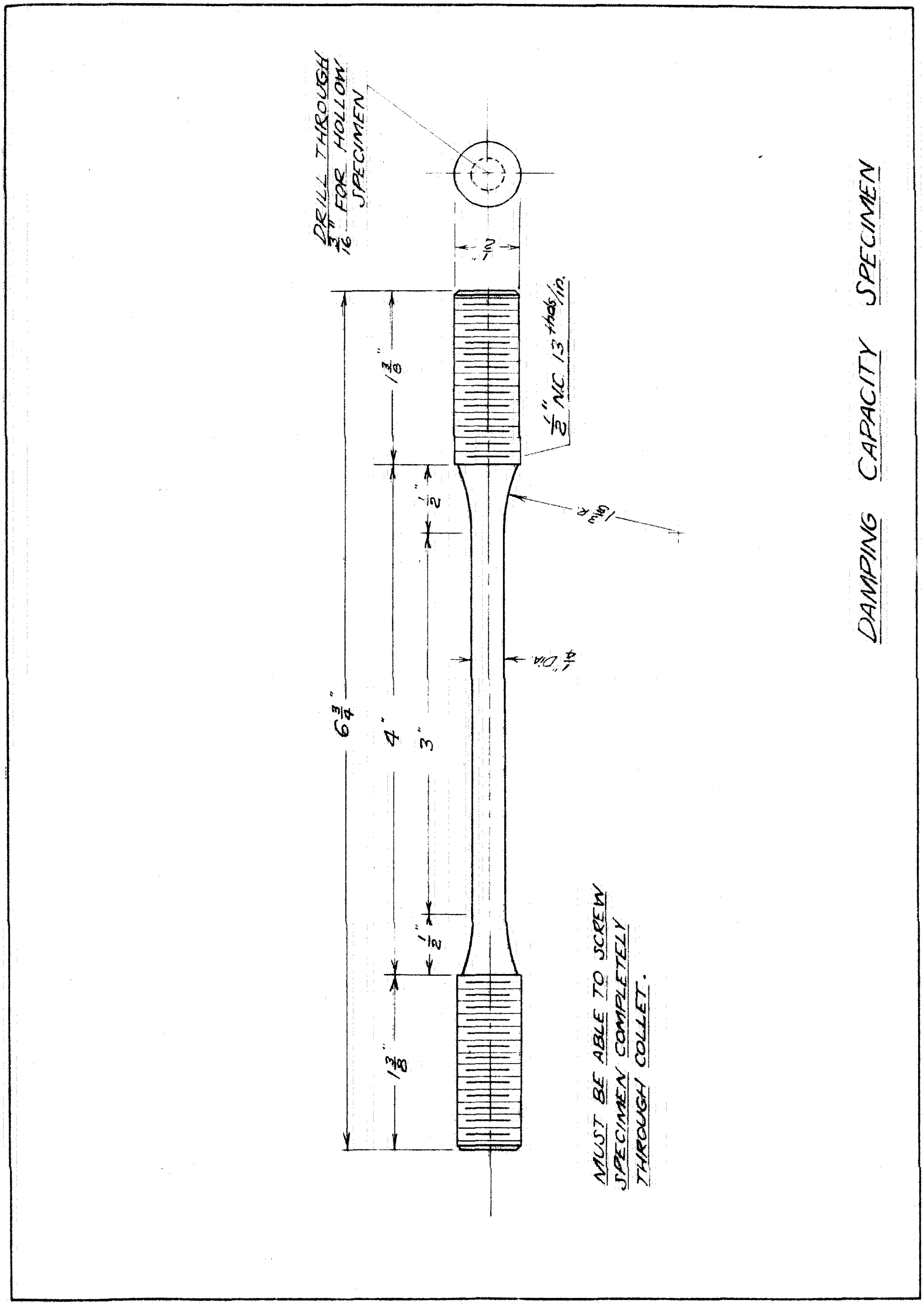
SET UP OF APPARATUS
DAMPING CAPACITY TESTS










DRILL THROUGH 3/16" FOR HOLLOW SPECIMEN
1/2"
1/2" N.C. 13 thds/in.
1 3/8"
1/2"
1/8 R
1/4" Dia.
6 3/4"
4"
3"
1/2"
1 3/8"
MUST BE ABLE TO SCREW SPECIMEN COMPLETELY THROUGH COLLET.
DAMPING CAPACITY SPECIMEN

(b) The specimen can be completely made in a lathe. Reasonable inaccuracies in the workmanship have no effect on the convenience of the test or on the results obtained.

(c) The dimensions of the specimen and of the apparatus give a frequency of vibration at which it is easy to make the required measurements, and which corresponds to frequencies often encountered in the operation of machine parts. It was considered that the independence of frequency and damping capacity in the range of frequencies in question had been so firmly established by the work of previous investigators, that the question would not have to be studied further. Hence, no effort was made to design the instrument for any one particular natural frequency. Similarly, the differences in frequency for various materials due to their different elasticities, were not considered to be of significance.

(d) The ends of the specimen were sufficiently enlarged so that the clamping device introduces no uncertain stresses at points where they might be objectionable. The stresses may be determined accurately in that portion of the specimen which is deformed under test. (pages 20 and 21.)

(e) The clamping device, a threaded collet chuck, (page 20) makes it possible to obtain positive clamping action over a considerable portion of the specimen by means of the wedging action of the threads. The method has the further advantage that the chucks and specimen are symmetrical with respect to the axis of torsional vibration. It was experimentally ascertained that the specimen would turn in the threads before the collet would turn in its tapered bearing surface, and that the force required to turn the specimen in the chuck was many times in excess of the maximum possible force which could be applied to the specimen in normal operation of the instrument.

(f) The specimen can be made either solid or hollow, with no other changes. Thus the effects of non-homogeneous stress distribution may be directly compared with the homogeneous case.

(3) The specimen can be subjected to any combination of axial tension or torsional stress without being disturbed in any way. The torsional stress set up is controlled entirely by the current in the solenoids, and the axial tension by the weights hung on the specimen by the flexible cable. (page 19.)

(4) The same instrument can be used for forced vibration studies, it being necessary only to supply the solenoids with an alternating current of variable frequency. Thus a direct comparison of free and forced vibrations can be obtained in the same instrument without the necessity of disturbing the specimen in any way.

(5) The optical system used measures the amplitudes of the vibration without introducing any appreciable damping into the system. This is a considerable advantage over systems using mechanical levers, friction scribers, induction bridges or any other type of electro-dynamical devices which may introduce more damping into the system than is to be measured in the specimen. The only other

possibility would seem to be a system which measures the change in the capacity of a condenser, since in this case the only damping forces would be the practically negligible electrostatic forces between the condenser plates.

The magnetic circuit used to apply the initial twist to the specimen does not introduce damping into the system. During free vibrations the solenoid circuit is open, so that even if a small amount of magnetization were retained by the core, no damping forces would result.

(6) The optical system used may be adjusted to give any magnification desired. By changing one lens the length of the optical path can be changed over very wide limits. For very large magnifications, provision is made for the use of a double or triple reflecting system. Specially ground flat mirrors of a high degree of accuracy are available for use in such a system.

(7) Although the stress range possible with the present equipment is large, it was thought desirable for the present investigation to limit the stress range to values of from about 400 lb. per sq. in. to 1500 lb. per sq. in. Most of the work which has been done with forced vibrations has been done in the low stress region, say from 0-100 lb. per sq. in., while most of the work done with free vibrations has been, largely because of inadequacies in the measuring equipment, in the high-stress range, from 5,000-30,000 lb. per sq. in. It was thought, therefore, that by conducting the present experiments in the medium stress range, not only the studies of the variables

involved would be of interest, but the actual values themselves would be of some significance.

(8) The time required to set up the specimen in the instrument and to make the test is reasonably short. Ordinarily 15 minutes should suffice for the complete test. The method thus compares in point of time required very favorably with other methods of physical testing, most of which require at least as much time, and many of them much longer. This point becomes of importance when the question of a large-scale investigation of the correlations between damping capacity and other physical properties is discussed.

With certain simple changes in the method of mounting the mirror on the specimen the convenience of operation may be further increased.

D. The Adjustment and Operation of the Apparatus

After the instrument was constructed, measurements were made to check upon the proper adjustment and operation of the apparatus. The method of balancing the torque applied to the specimen was as follows: The instrument was assembled with the specimen in place and two dial-gages, of the type which can be read to $\frac{1}{50,000}$ inch were adjusted on the frame of the instrument so that they indicated the displacements of the two solenoid plungers on diametrically opposed points of the vibrating disc. The solenoids were then energized, and the one which gave the larger force was noted. The solenoids were then connected in parallel, a small resistance r (page 18) being connected in series with the stronger coil. The value of this resistance was then changed until the dial gages indicated equal displacements. It was then assumed that the disc was being subjected to rotation only, with a negligible amount of translation due to electrical unbalance of the solenoids.

The optical system was checked against the dial-gage readings, agreement being obtained within the accuracy required.

A test run was made next on a specimen to check the general adjustment and operation of the instrument. The speed of the recording film was increased so that the wave shape of the vibration could be observed clearly, and the light beam was blocked out in its zero position, so that a zero line would appear on the record. The specimen was then twisted through a relatively large angle and released. The resulting record showed that the vibration was symmetrical about the zero position, and the wave form indicated no irregularities of any kind in the vibration. It was thus concluded that the instrument was in proper working order.

In making the actual tests on the specimens, the following precautions were taken:-

(1) In mounting the specimen in the instrument, care was taken not to stress the specimen any more than necessary. Two round pins were passed through holes in the side frame of the machine into diametrically opposed radial holes in the periphery of the vibrating disc, so that the lower chuck could be tightened without subjecting the specimen to torsional stress.

There were not enough pains taken, however, with the design of this clamping mechanism. It is probable that some of the specimens were subjected to higher torsional and bending stresses during insertion in the machine, than were applied later by the apparatus itself. This is a feature which should be improved in future designs, since a previous stressing of the material would be expected to have some effect on the damping capacity of the material.

(2) A first record was taken at a very low maximum stress, and other records were taken at successively higher initial displacements. This procedure enables one to study the effect of previous stressing of the material on the damping test.

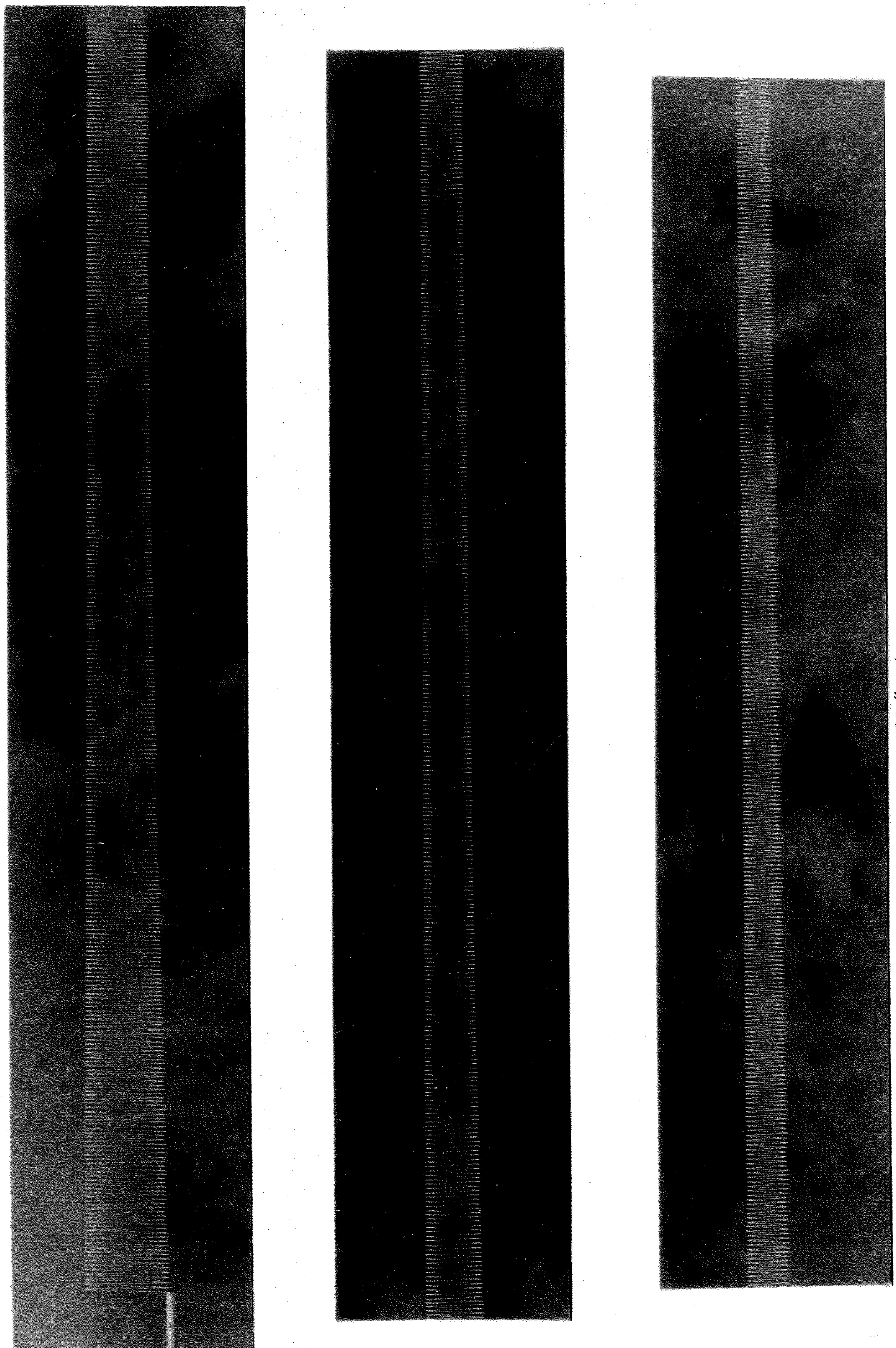
In the present investigation this procedure was followed only for those specimens which had been inserted without being overstressed.

(3) The initial twist was applied slowly to the specimen by gradually increasing the current in the solenoids by means of the rheostat (R on page 18). An increased stress due to a sudden initial loading of the specimen was thus avoided.

(4) In the combined stress study the axial loads were applied successively, without unloading the specimen at any time. The specimen was not subjected, therefore, to cycles of axial stress.

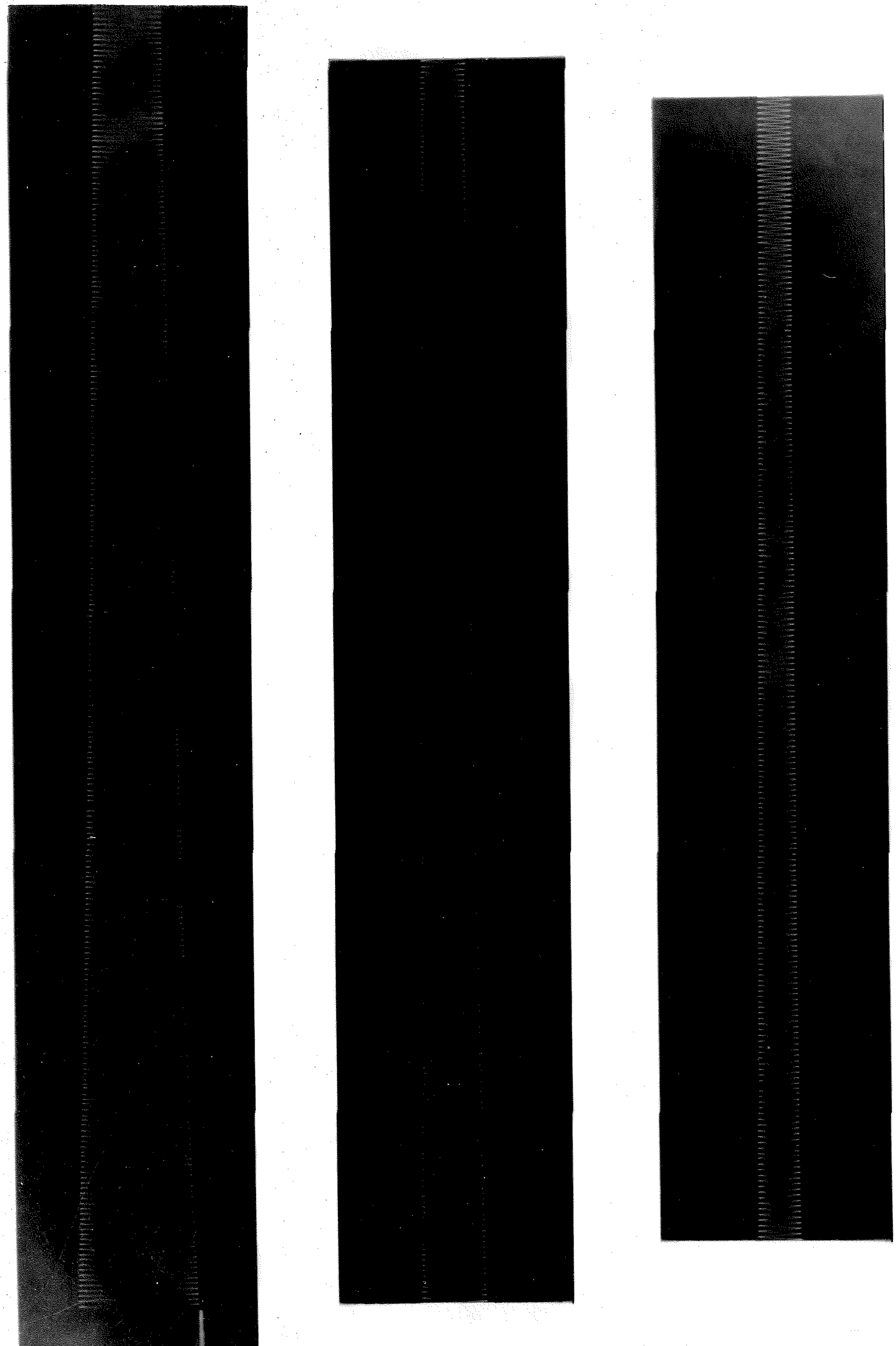
The axial loads were applied as gradually and as carefully as possible, in order to eliminate impact and bending in the specimen.

The type of record obtained from the instrument may be seen on pages 28 to 32. These samples are full-size prints made directly from the negative as it came from the instrument. The damping capacity can thus be accurately checked from the samples themselves.

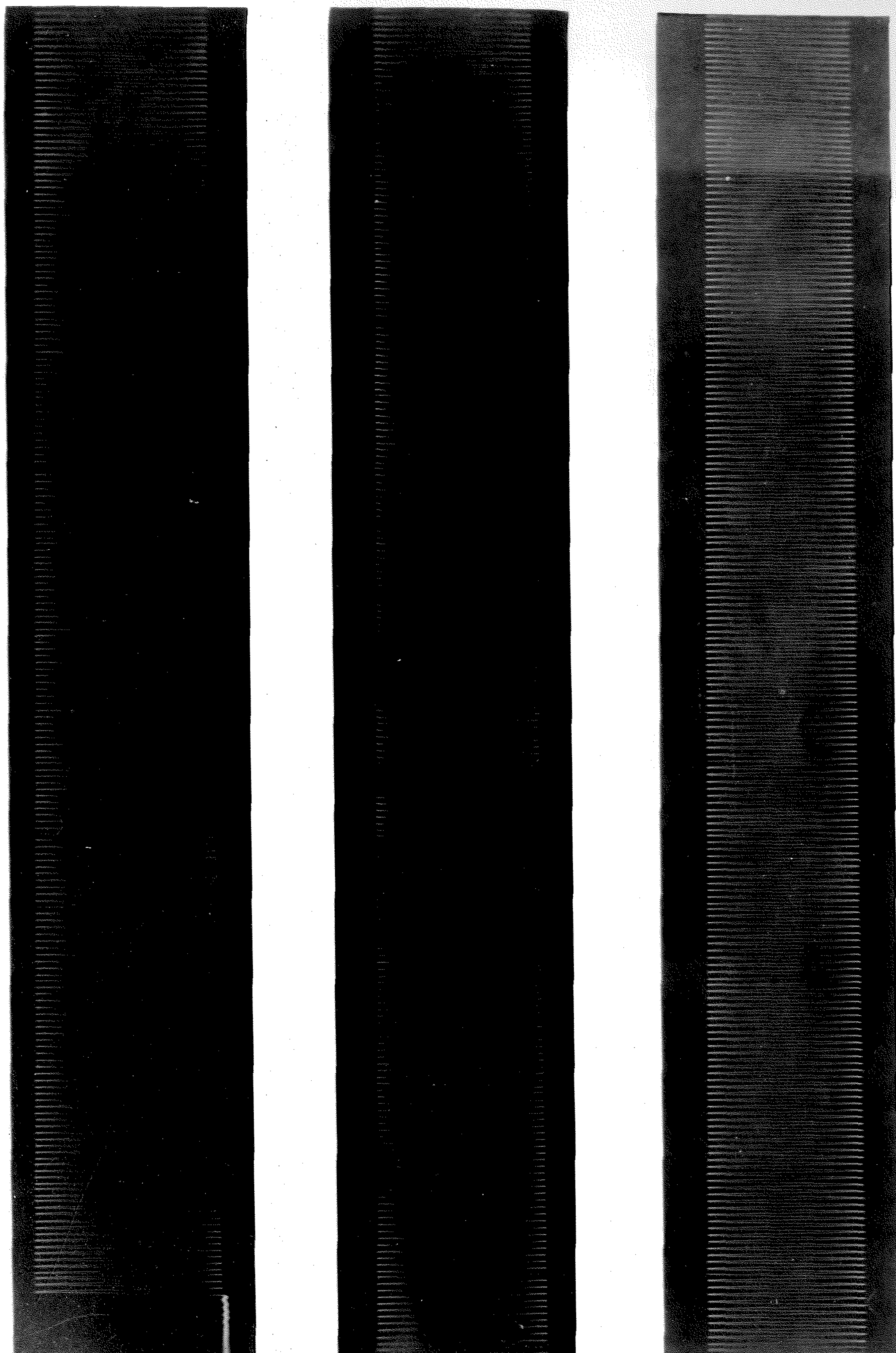


1.27" = 1.0 sec.

SOLID STEEL SPECIMEN

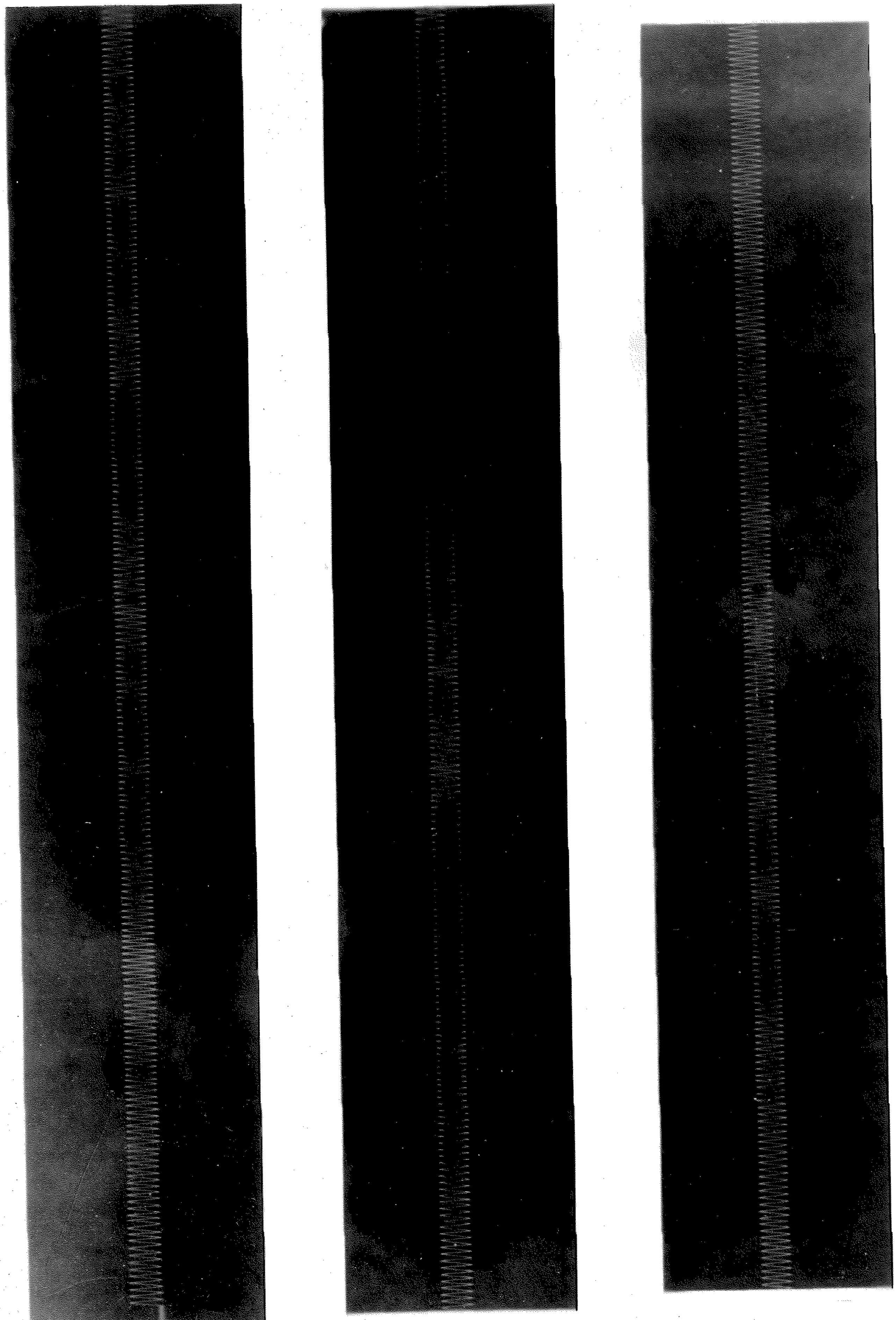


HOLLOW STEEL SPECIMEN



BRASS SPECIMEN

BRASS (CONT.)



BRASS (CONT.)

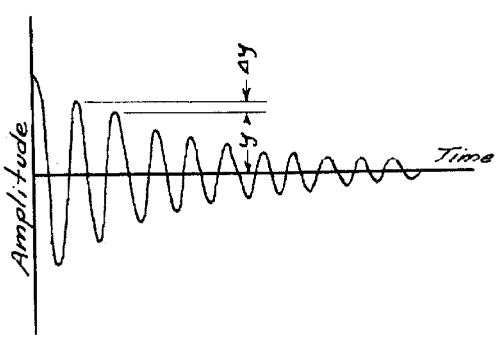
The three strips on any one page represent a continuous record, which has been cut up for convenience.

The records of pages 28 and 29 show, respectively, the damping in a solid and a hollow specimen of the same material. The increased apparent damping of the hollow specimen may be seen clearly. The records on pages 30 to 32 show the damping of brass in three stress regions.

E. The Method of Computation

As is shown in Appendix I, the specific damping capacity ψ may be very easily found from the vibration decay curve, since

$$\psi = 2\frac{\Delta y}{y}$$



For the materials tested, it was observed that the damping capacity was low enough so that for a small number of vibrations the amplitude of the vibration could be assumed to decrease linearly. The decrease in amplitude in any one cycle was found accordingly by dividing the total decrease in fifty cycles by fifty.

The record was first counted off into groups of fifty cycles each, and the amplitude of every fiftieth vibration was measured by means of dividers and a scale. When measuring the amplitude, the fiftieth cycle was always compared with neighboring cycles to insure against the measurement of an abnormal cycle.

The decrease in amplitude per cycle was then computed and the logarithmic decrement was found by dividing this decrease per cycle by the mean amplitude of the group. In no cases did the amplitudes change enough in any group to necessitate a distinction between the mean amplitude and the amplitude at any point within the fifty cycle group.

The above operations were combined into a computation sheet, by means of which the specific damping capacity as a function of stress may be quickly found. A computation sheet for one of the sample curves is shown as page 35.

SAMPLE COMPUTATION SHEET

SPECIMEN: Hollow Steel LOADING: No axial wt

INTERVAL: 50 Cycles DATE: 2-17-42

No.	2×Amp (cm)	Diff.	Sum	Mean	Diff./Mean	ψ D/M/25	$\tau_{max.}$ 633×M.
1.	2.22						
		0.26	4.18	2.090	0.1243	0.0050	1322
2.	1.96						
		0.25	6.37	1.835	0.1362	0.0055	1162
3	1.71						
		0.19	3.23	1.615	0.1177	0.0047	1022
4	1.52						
		0.14	2.90	1.450	0.0965	0.0039	919
5.	1.38						
		0.15	2.61	1.305	0.1150	0.0046	826
6	1.23						
		0.12	2.34	1.170	0.1025	0.0041	740
7	1.11						
		0.10	2.12	1.060	0.0944	0.0038	670
8	1.01						
		0.09	1.93	0.965	0.0933	0.0037	611
9	0.92						
		0.07	1.77	0.885	0.0791	0.0032	560
10.	0.85						
		0.07	1.63	0.815	0.0860	0.0034	516
11.	0.78						
		0.06	1.50	0.750	0.0800	0.0032	475
12.	0.72						
		0.04	1.40	0.700	0.0571	0.0023	444
13.	0.68						
		0.05	1.31	0.655	0.0764	0.0031	415
14.	0.63						
		0.03	1.23	0.615	0.0488	0.0020	390
15.	0.60						

THE RESULTS OF THE INVESTIGATION

A. Accuracy of the Method and Instrumental Losses

The first questions studied involved the errors inherent in the instrument and in the methods of measurement and computation.

(1) The Clamping Method

As mentioned above, it was found that the force required to slip the specimen in the chuck was many times greater than the maximum force which could be applied to the specimen by the apparatus. That the slipping, if any, was actually negligible, was indicated, first, by the fact that the results for the same specimen could be checked closely, and, second, by the fact that the damping measured for some of the specimens was extremely low, of a magnitude which left no possibility of any considerable additional loss in the supports.

(2) Foundation Losses

The energy lost in the instrument foundation is a function of the amplitude of motion of the foundation. Since the instrument was securely bolted to a heavy angle iron which in turn was fastened rigidly to a heavy concrete wall, such foundation losses cannot possibly be appreciable. Here, again, the extremely low values of damping which were measured for some of the specimens, indicates that the energy dissipation in the instrument itself must be very low, (see page 42).

The work of Flinn & Norton (Ref. 11) has demonstrated experimentally that foundation losses can be made entirely negligible.

(3) Air Damping

The work of several of the previous investigators (Ref. 5) has shown that for the type of materials under test the damping effect of the air is negligible. In the present case the air damping is known to be low, since it must be less than the smallest damping capacity measured by the instrument. Since the smallest damping capacity measured falls just on the limit of the computation errors, no corrections for losses of any kind need to be made, (page 42).

(4) Measurement and Computation Errors

An overall check on the accuracy of the amplitude measurements from the record and on the computation method in general was obtained in the following way:

The amplitudes of the fifty cycle groups were picked from the record and recorded on the data sheet in such a way that the record itself was not marked. The computations were then carried completely through and the damping capacities and stresses were recorded. On the following day the procedure was repeated, a completely independent set of readings being obtained, and a new set of damping capacities and stresses being computed from the new data. The two sets of damping capacities were then compared, and the differences between the two determinations at the same stress were computed for a dozen or so stresses. The mean difference in the damping capacity was then found.

This procedure was repeated for four different records, chosen to cover various amplitudes, frequencies, and general appearance of records. The mean differences in the damping capacity were found to be:

Record Number	Mean Difference
1	0.0004
2	0.0003
3	0.0002
4	0.0003

Hence the error in the computed damping capacity due to measurement from the record and to computation errors would seem to be about 0.0003. Since the damping capacities of most of the materials tested is many times this value, it is believed that this accuracy is sufficient for the present investigation.

Since the damping capacity of one of the materials tested was found to be about 0.0003, it is probable that the method of measurement of the amplitudes of vibration from the film is the limiting factor in the accuracy of the present equipment.

As was shown before, the losses in the instrument are less than 0.0003 Therefore, the overall accuracy of the determination of the damping capacity in the present study can be said to be of the order of 0.0003.

The small circles which mark the plotted points on the curves of the succeeding pages have diameters of approximately 0.0003, so that the degree of accuracy of the data is indicated on the curves.

B. Discussion of Results and General Conclusions

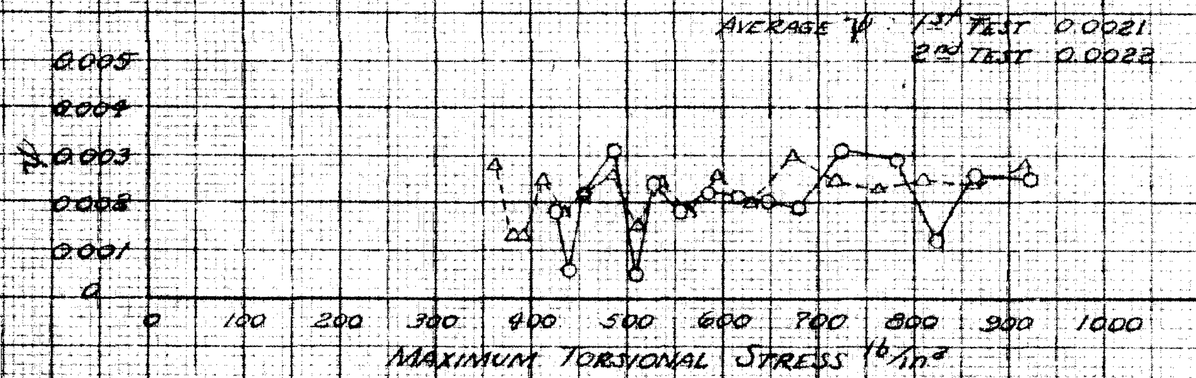
The data supporting the conclusions summarized at the beginning of this paper will now be discussed, taking up the points in the order in which they appear there.

(1) The curves and data of pages 40, 41, and 42 show the results obtained by making identical tests on the same specimen at different times. In all cases the specimen was completely removed from the instrument between tests, and in most cases the specimen was turned up-side down before re-insertion. With the present arrangement of the optical system the mirror must be removed when the specimen is taken out, hence any undesirable mirror movement or misadjustment would also be indicated by these tests.

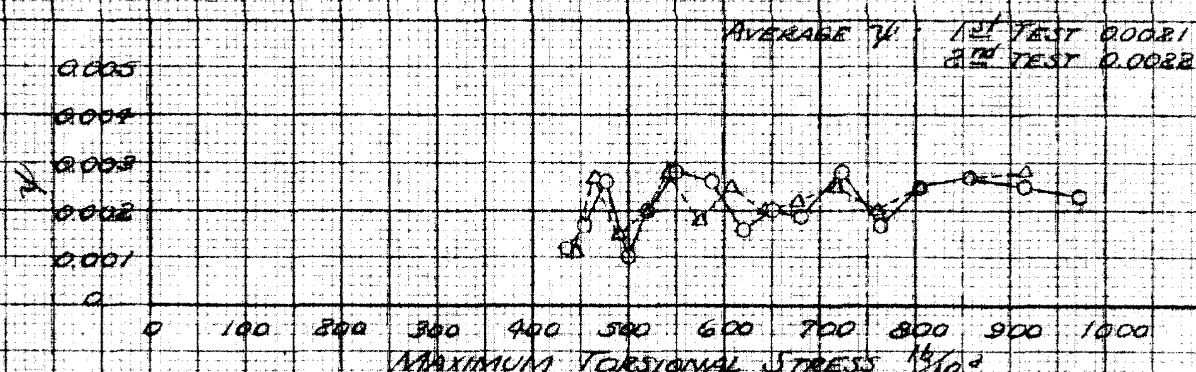
It will be noted from the curves that the different runs check closely as far as their general features are concerned. Although several fairly large discrepancies occur in the case of steel No. 1 for the individual points, it is clear that the average values of damping capacity over a stress-range of several hundred pounds per square inch agree very closely. In all cases the check on mean damping capacity is well within the limits of accuracy of the measurements.

We may thus conclude that the mean damping capacity of the steel over a stress-range of several hundred pounds per square inch is a physical property of the material whose value can be checked accurately by successive tests on the same specimen.

(2) The two steel specimens marked steel No. 1 and steel No. 2 were made from the same bar of steel. (Photomicrographs showing



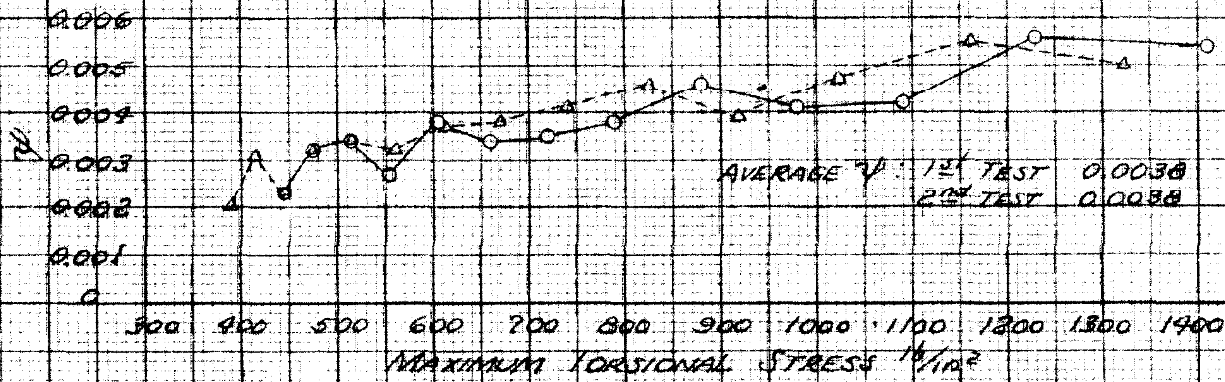
STEEL SPECIMEN No.1



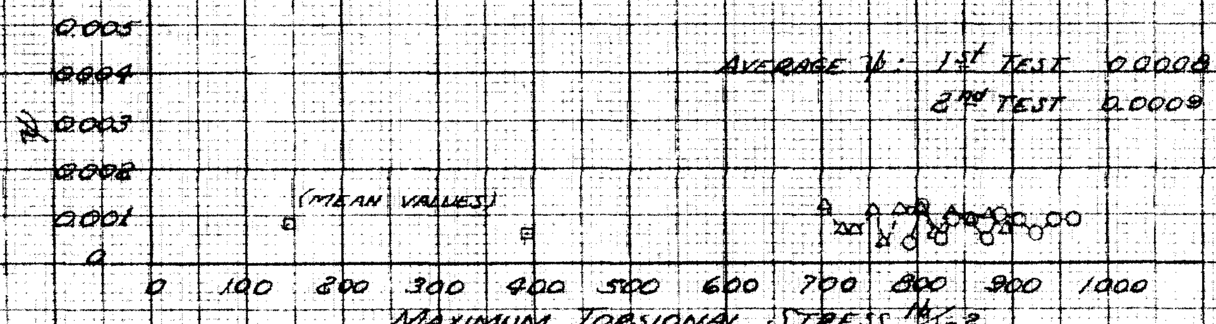
STEEL SPECIMEN No.2

REPRODUCIBILITY OF RESULTS

SPECIMEN COMPLETELY REMOVED FROM INSTRUMENT BETWEEN TESTS



HOLLOW STEEL SPECIMEN



BRASS SPECIMEN

REPRODUCIBILITY OF RESULTS

SPECIMEN COMPLETELY REMOVED FROM INSTRUMENT BETWEEN TESTS

RESULTS OF TESTS ON X4130 STEEL

Repeated damping tests were also made on a specimen of X4130 steel, furnished by the Kobe Company.

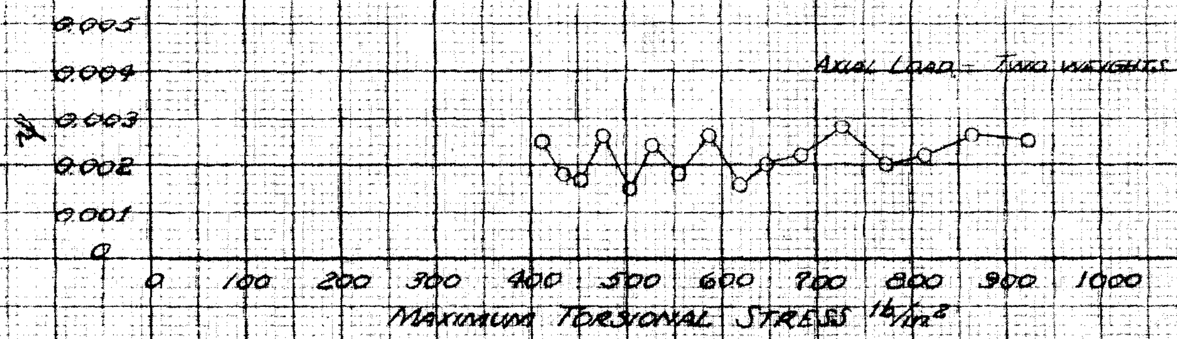
The damping capacity of this metal was so low that the amplitude did not decrease enough during the time covered by the test to make it possible to plot a very extensive stress-damping capacity curve.

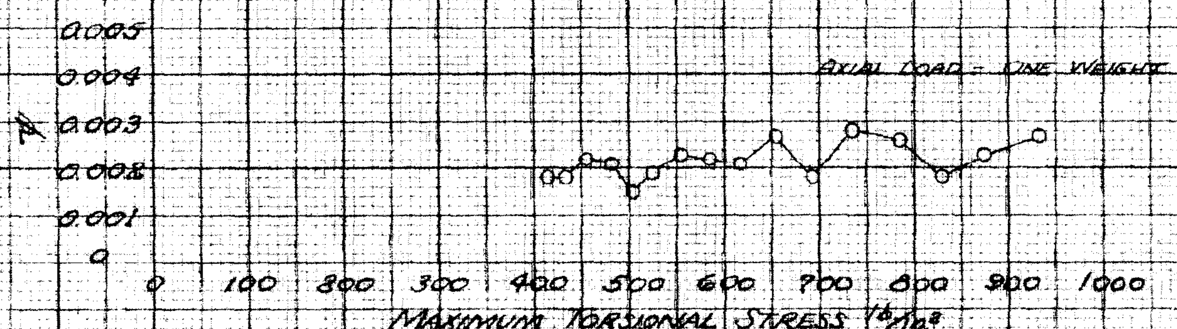
Two tests were made, the specimen being removed from the instrument between tests. The mean damping capacity over a stress range of from 700 to 900 pounds per square inch was found to be:

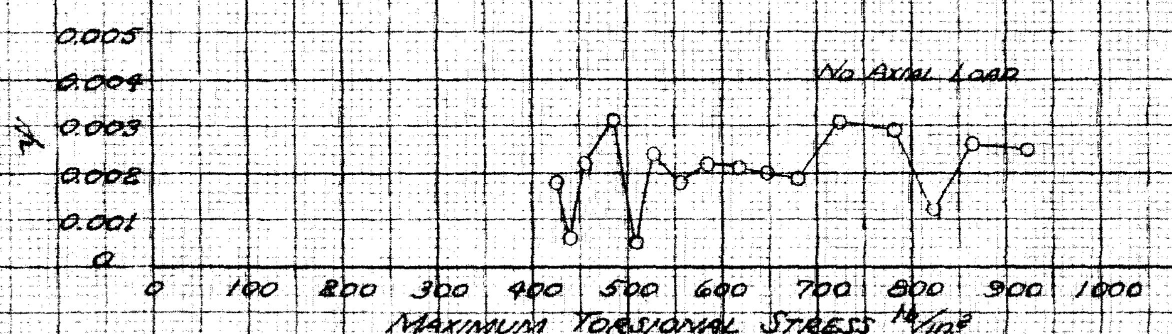
Test No. 1 - = 0.0003.

Test No. 2 - = 0.0003.

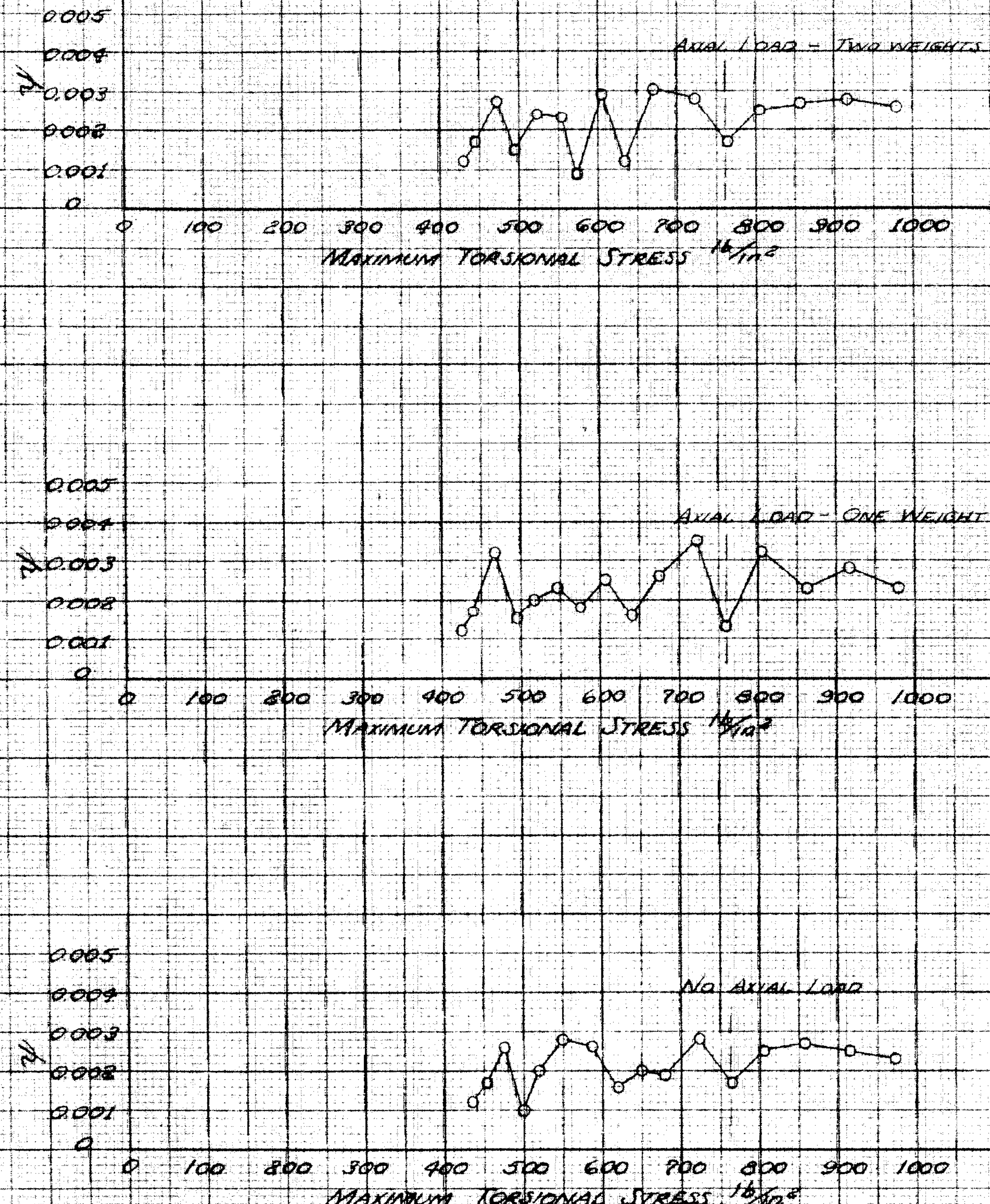
These values are just on the limit of experimental errors, and are interesting as an indication of the maximum possible loss which occurs in the instrument.



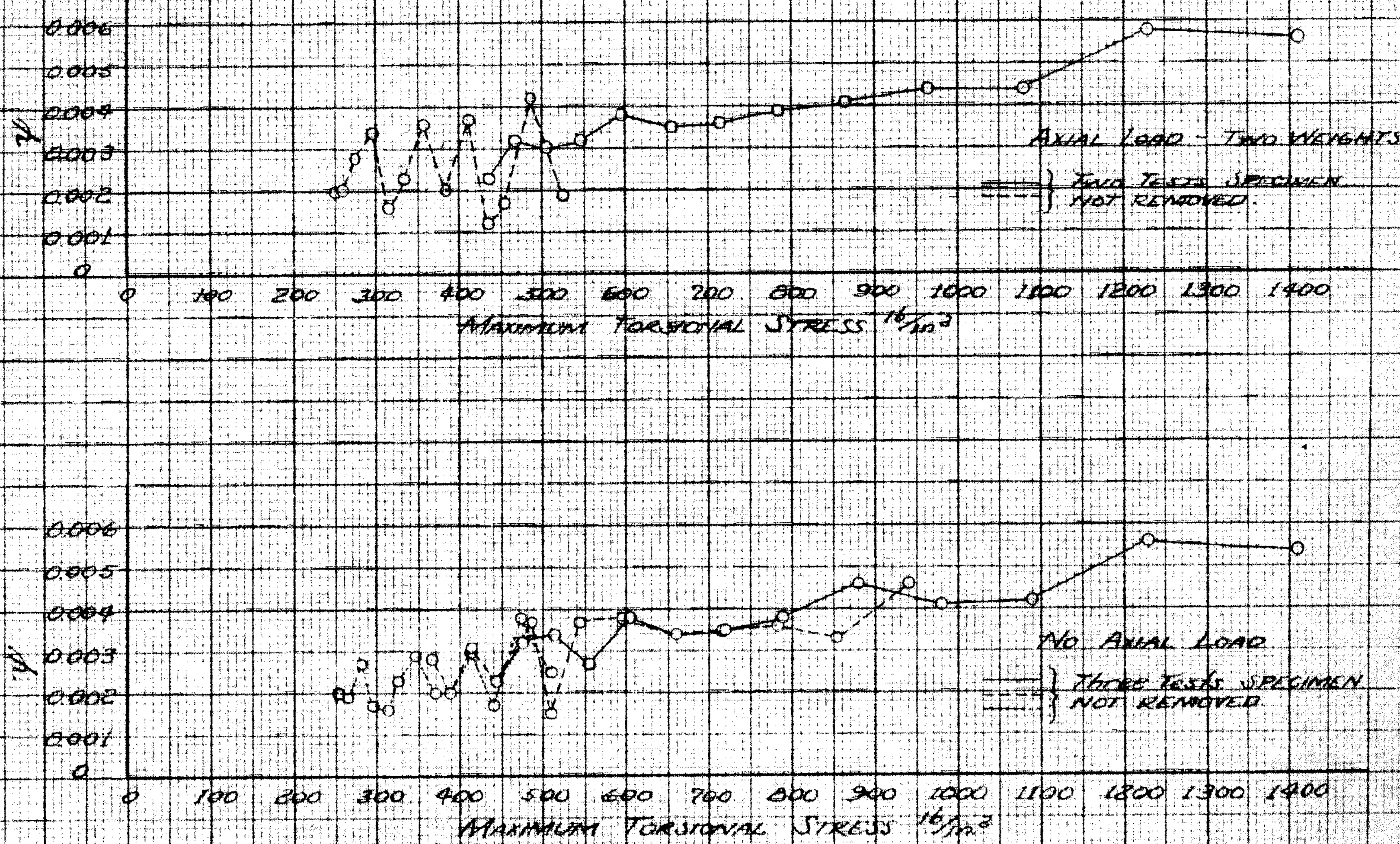




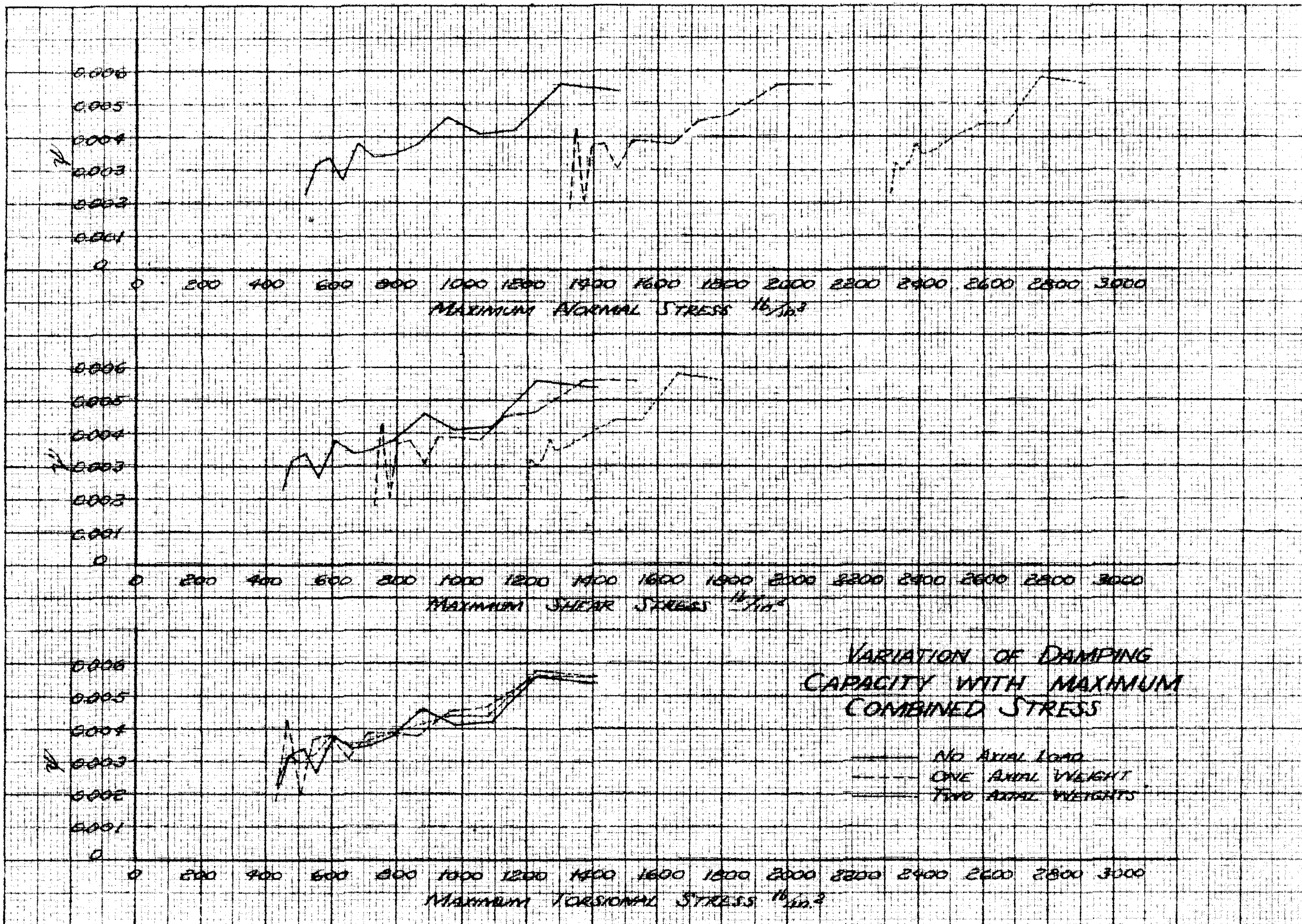
STEEL SPECIMEN No. 1



STEEL SPECIMEN No. 2



HOLLOW STEEL SPECIMEN


0.006
0.005
0.004
0.003
0.002
0.001
0
0 200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 2800 3000
MAXIMUM NORMAL STRESS lb/in.²
MAXIMUM SHEAR STRESS lb/in.²
MAXIMUM TORSIONAL STRESS lb/in.²
VARIATION OF DAMPING
CAPACITY WITH MAXIMUM
COMBINED STRESS
NO AXIAL LOAD
ONE AXIAL WEIGHT
TWO AXIAL WEIGHTS

the structure of the steel and brass specimens are shown on page 65.) From the data of page 40, it will be seen that the mean damping capacities over the range of stress covered by the tests check exactly for the two different specimens. Hence we may conclude that the mean damping capacity of the material over a stress range of several hundred pounds per square inch is a physical property of the material which can be checked for successive specimens with good accuracy.

(3) It will be seen from all of the plotted curves that the stress-damping capacity relationship is not a simple one, but that it apparently varies in a most erratic way. The departures of the individual points from a smooth curve following the general trend of the data are in a number of cases many times greater than the experimentally determined mean error. Furthermore, the departures have proved to be reproducible. On page 44, for example, it will be noted that the same low point in the damping capacity is found at a stress of 760 pounds per square inch for several different axial loadings of the specimen, and the same low spot is picked up again in the repeated test of page 40. Examination of the curves showing reproducibility of results, pages 40 and 41, will reveal a number of instances when large departures from the mean damping capacity have been exactly duplicated.

From these observations it would appear that there is a strong possibility that the true relationship between stress and damping capacity, for a material of a structure similar to that of steel, is of an extremely complex nature.

A further study of this particular point, using more accurate measuring methods, and trying the effects of annealing, cold-working, etc., on the relationships, would be most interesting.

(4) For the steel specimens the general trend of the stress-damping capacity relationship is towards lower damping capacities at lower stresses. This is seen most clearly for the hollow specimen, page 41 or 45, which, as will be shown later, gives a better picture of the damping capacity of the metal than the solid specimen.

The damping capacity of the brass specimen, page 41, was essentially constant, no marked decrease at the lower stress being evident within the accuracy of the present investigation.

(5) The results of the combined stress study may be seen on pages 43, 44, 45, and 46.

Pages 43, 44, and 45 show that the axial load has no measurable effect on the stress-damping capacity relationship.

On page 46 the damping capacity is plotted versus the maximum torsional stress set up in the specimen, the maximum normal stress, and the maximum shear stress, for three different combined stress conditions. It is immediately evident that the damping capacity for this type of loading is a function of the torsional stress and not of the shear or normal stress.

The nature of the combined stress calculations is shown in Appendix III.

(6) A comparison of the results obtained on the solid and on the hollow specimens may be made by referring to pages 40 and 41. The mean damping capacity of the hollow specimen below 1000 pounds per

square inch is 0.0034, while for the same stress range the solid specimen shows a mean damping capacity of 0.0022. The solid specimen, therefore, has an apparent damping capacity of approximately 65% that of the hollow specimen. Part of this difference can be accounted for by the following considerations:

We establish the following notation:

R = Outside radius of the specimen.

R_i = Inside radius of the specimen.

r = Radius of an elementary cylindrical shell.

τ = Shearing stress at the radius r.

S = The maximum shearing stress in the specimen at $r = R$.

l = Length of the deformed portion of the specimen.

ΔW = Loss of energy in the specimen in one cycle due to internal friction.

W = Strain energy of the specimen.

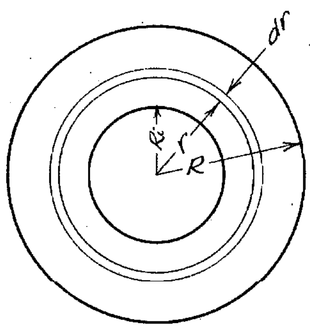
ψ_τ = True specific damping capacity of the material at the stress τ.

ψ_s = Specific damping capacity of the solid specimen as computed from $\psi_s = 2\frac{\Delta y}{y}$.

ψ_h = Specific damping capacity of the hollow specimen as computed from $\psi_h = 2\frac{\Delta y}{y}$.

$\psi = \frac{\Delta W}{W}$ by definition (see Appendix I.).

G = Modulus of elasticity in shear.



Consider a circular cylindrical element of the specimen of radius r, thickness dr, and length ℓ.
The strain energy per unit volume at a stress τ is:

$$dW' = \frac{\tau^2}{2G}$$

also $$\tau = \frac{r}{R} S$$

thus $$dW' = \frac{S^2}{2GR^2} r^2$$

The total strain energy for the shell is:

$$dW = \left(\frac{S^2}{2GR^2} r^2\right)(2\pi r dr \cdot \ell) = \frac{\pi \ell S^2}{GR^2} r^3 dr$$

Since $\psi_\tau = \frac{\Delta W}{W}$ the loss in energy during one cycle is:

$$d(\Delta W) = \psi_\tau \cdot \frac{\pi \ell S^2}{GR^2} r^3 dr = \psi_{(r)} \cdot \frac{\pi \ell S^2}{GR^2} r^3 dr$$

Thus the total loss in energy per cycle for the hollow specimen will be:

$$\Delta W = \frac{\pi \ell S^2}{GR^2} \int_{R_i}^{R} \psi_{(r)} \cdot r^3 dr$$

In order to evaluate this integral, the function $\psi_{(r)}$ must be known. The nature of this function may be seen by referring to the stress-damping capacity relationship for the hollow specimen where it will be noticed that a straight line will represent the facts closely.

Thus, we may assume:

$$\psi_\tau = A + B\tau$$

$$\therefore \quad \psi_{(r)} = A + \frac{BS}{R} r$$

Then
$$\Delta W = \frac{\pi l S^2}{G R^2}\int_{R_i}^{R}\left(A r^3 + \frac{BS}{R} r^4\right)dr$$

$$\Delta W = \frac{\pi l S^2}{G R^2}\left(\frac{AR^4}{4} + \frac{BSR^5}{5R} - \frac{AR_i^4}{4} - \frac{BSR_i^5}{5R}\right)$$

also
$$W = \frac{\pi l S^2}{G R^2}\int_{R_i}^{R} r^3 dr = \frac{\pi l S^2}{G R^2}\,\frac{(R^4 - R_i^4)}{4}$$

thus
$$\psi_h = \left(\frac{\Delta W}{W}\right)_h = A + \frac{4B}{5R}\frac{(R^5 - R_i^5)}{(R^4 - R_i^4)} S$$

For the solid specimen set $R_i = 0$.

Thus:
$$\psi_s = A + \frac{4B}{5} S$$

For the present specimen:

R = 0.1250 in.

R_i = 0.09375 in.

Substituting these values in the above expressions, we have:

ψ_h = A + 0.895 B S

ψ_s = A + 0.800 B S

Comparing the two expressions, it will be seen that both represent straight lines having the same ψ intercept, but that the slope of the hollow specimen is about 10% greater than the solid specimen slope. Hence the values of damping capacity at any point for the solid specimen would be expected to be about 90% of the ~~true~~ damping capacity of the hollow specimen.

The damping capacity as measured by a solid specimen would be expected, therefore, to be about 80% of the true damping capacity of the metal.

This theory thus indicates that the apparent damping capacity measured for a solid specimen should be lower than that measured for a hollow specimen. The discrepancy between the amount of lowering actually observed and the calculated amount may be due in part to: (1) the additional machine work done on the hollow specimen may have had an effect on the damping capacity; (2) the relationship between stress and damping capacity below about 400 pounds per square inch was extrapolated on the curve. If the function were to behave differently at low stresses, the above analysis would be in error.

Further experimental work comparing solid and hollow specimens would be desirable in order to clear up these questions.

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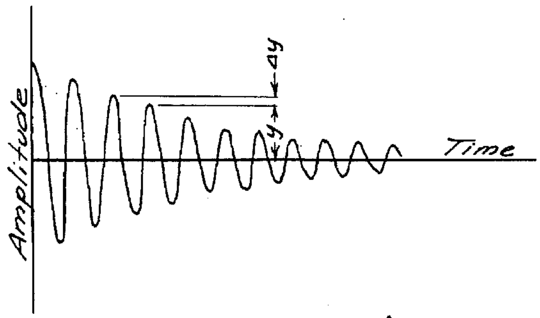
APPENDIX I.

Derivation of the Relationship between Specific Damping Capacity and the Rate of Amplitude Decay. (Ref. 31)



The strain energy stored up in a vibrating elastic system can be expressed as:

$$W = Ky^2$$

where: y = the amplitude of the vibration.

K = a constant depending on the mass, elastic properties, and dimensions of the vibrating body.

thus:

$$W_1 = K(y+\Delta y)^2$$

$$W_2 = Ky^2$$

where $y+\Delta y$, y are the amplitudes of successive cycles.

$$\Delta W = W_1 - W_2 = K[(y^2 + 2y\Delta y + \Delta y^2) - y^2]$$

for small damping, the Δy^2 term may be neglected.

$$\Delta W = (K)(2y\Delta y)$$

$$\psi = \frac{\Delta W}{W} = \frac{(K)(2y\Delta y)}{Ky^2}$$

$$\psi = 2\frac{\Delta y}{y}$$

The logarithmic decrement δ is defined as:

$$\delta = \log_e \frac{y+\Delta y}{y}$$

Expanding the logarithm in a series, we have:

$$\log_e\left(1+\frac{\Delta y}{y}\right) = \frac{\Delta y}{y} - \frac{1}{2}\left(\frac{\Delta y}{y}\right)^2 + \frac{1}{3}\left(\frac{\Delta y}{y}\right)^3 + \cdots$$

for small damping, the higher powers may be neglected.

thus: $$\delta = \frac{\Delta y}{y}$$

and $$\psi = 2\delta$$

APPENDIX II.

Relationships between Various Quantities Used in Characterizing Damping in Elastic Systems

Various authors have used a number of different expressions to measure the damping in mechanical systems. A comparison of some of the formulas frequently met with in the literature follows:

NOTATION:

- β = viscous damping constant, damping force = (β) (velocity).
- h = damping index used by seismologists in their instruments.
- k = spring constant.
- m = mass.
- x = displacement or amplitude of vibration.
- Δx = decrease in amplitude during one cycle.
- δ = logarithmic decrement.
- ΔW = energy dissipated by the damping forces per cycle.
- W = total energy of the system at the maximum amplitude of any particular cycle.
- ψ = "specific damping capacity".
- ξ = "solid friction coefficient".
- E = modulus of elasticity.
- f = resonant frequency of the system.
- Δf_x = width of the resonance curve at the amplitude x.
- Q = "sharpness of tuning" of an electric circuit.
- t_h = time required for a vibration to decay to $\frac{1}{2}$ of its original amplitude.

The following relationships may be shown to exist between the above quantities (relationships marked with a star* depend on a viscous friction relationship.)

(1)* $$h = \frac{\beta}{2\sqrt{km}}$$

If h = 1, the damping is critical, hence values of h will indicate the % of critical damping.

(2)* $$\delta = \frac{2\beta\pi}{\sqrt{4mk-\beta^2}}$$

(3) $$\delta = \log_e \frac{x+\Delta x}{x}$$

(4)* $$\delta = \frac{\beta}{2mf}$$

(5)* $$\delta = \frac{2\pi^2\beta f}{E}$$

(6) $$\delta = \frac{\Delta x}{x} \quad \text{(Small damping)}$$

(7)* $$\Delta W = 4\pi^2 x_{max.}^2 \beta f$$

(8) $$W = \frac{E x_{max.}^2}{2}$$

(9) $$\psi = \frac{\Delta W}{W}$$

(10) $$\delta = \frac{\psi}{2}$$

(11) $$\Delta W = \xi x_{max.}^2$$

(12) $$\delta = \xi E$$

(13)* $$\delta = \frac{\pi}{\sqrt{3}} \frac{\left(\Delta f_{x=\frac{x_{max.}}{2}}\right)}{f}$$

(14)* $$\delta = \frac{\pi\left(\Delta f_{x=\frac{X_{max}}{\sqrt{2}}}\right)}{f}$$

(15)* $$Q = \frac{f}{\Delta f_{x=\frac{X_{max}}{\sqrt{2}}}}$$

(16)* $$\delta = \frac{\pi}{Q}$$

(17)* $$\delta = \frac{0.693}{t_h f} \qquad (\log_e 2 = 0.693)$$

Summarizing the various ways in which the logarithmic decrement can be written, we have:

$$\delta = \log_e \frac{X+\Delta X}{X} \cong \frac{\Delta X}{X} = \frac{\beta}{2mf} = \frac{2\pi^2\beta f}{E} = \frac{2\beta\pi}{\sqrt{4mk-\beta^2}}$$

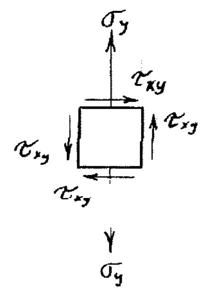
$$= \frac{\psi}{2} = \xi E = \frac{\pi}{\sqrt{3}}\frac{\left(\Delta f_{x=\frac{X_{max}}{2}}\right)}{f} = \frac{\pi\left(\Delta f_{x=\frac{X_{max}}{\sqrt{2}}}\right)}{f}$$

$$= \frac{\pi}{Q} = \frac{0.693}{t_h f}$$

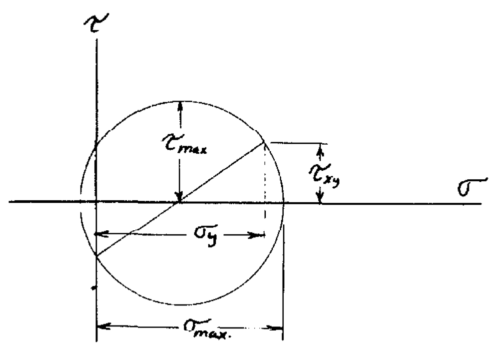
APPENDIX III.

Combined Stress Computations

The stress situation at the surface of the specimen as it is subjected to both axial load and torsion may be shown as:



The Mohr's circle representation of the stress on any inclined plane will then be:



From this circle the formulas for the maximum shear and the maximum normal stress are seen to be:

$$\sigma_{max.} = \frac{\sigma_y}{2} + \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

In the present case:

Axial load #1 = 22.6 lb.

Axial load #2 = 45.2 lb.

Total weight of disc, etc. = 3.0 lb.

Area of the specimen = 0.0216 sq. in. (hollow specimen).

and the formulas become:

<u>Case 1.</u> No axial load:

$$\sigma_{max.} = \tau_{xy}$$

$$\tau_{max.} = \tau_{xy}$$

(wt. of disc, etc. actually included)

<u>Case 2.</u> One axial load:

$$\sigma_{max} = 593 + \sqrt{(593)^2 + \tau_{xy}^2} \quad lb/in.^2$$

$$\tau_{max.} = \sqrt{(593)^2 + \tau_{xy}^2} \quad lb/in.^2$$

<u>Case 3.</u> Two axial loads:

$$\sigma_{max.} = 1118 + \sqrt{(1118)^2 + \tau_{xy}^2} \quad lb/in.^2$$

$$\tau_{max.} = \sqrt{(1118)^2 + \tau_{xy}^2} \quad lb/in.^2$$

In all cases:

$$\tau_{xy} = \frac{1}{2} G d \frac{\theta}{l}$$

where d = diameter of the specimen.

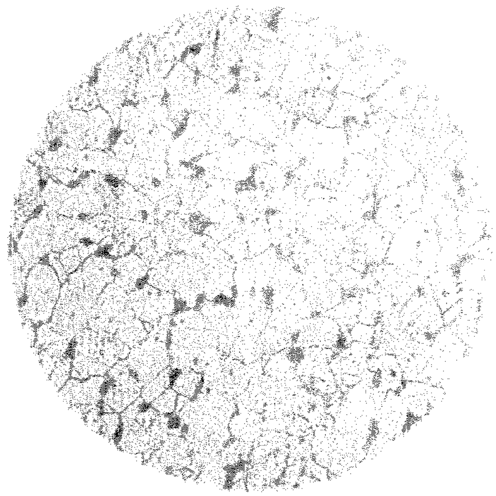
θ = total angle of twist in radians of a specimen of length l.

θ may be written in terms of the deflection of the light beam and the dimensions of the optical system.

For example, in the case of the steel specimens:

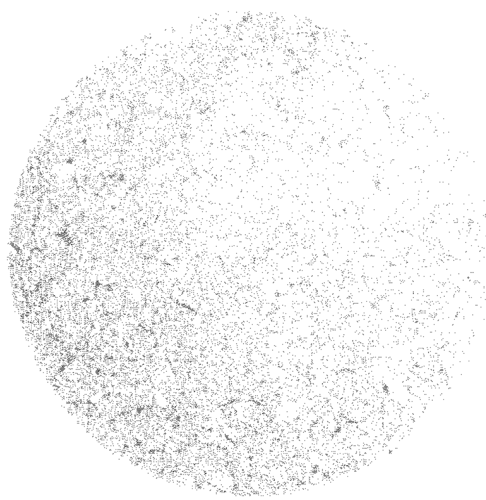
$$\tau_{xy} = 633\,x \quad lb/in.^2$$

where x is the double amplitude of the vibration as measured on the film.



STEEL 250X

ETCH: NITAL ROCK. HARD. 84B



BRASS 250X

ETCH: $NH_4OH + H_2O_2$ ROCK. HARD. 42B