THE PERFORMANCE OF A SUPERSONIC AIRPLANE

Thesis by

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PART I

SUMMARY OF THESIS

This thesis is primarily a survey of the methods of calculating drag and performance of an airplane capable of supersonic flight. As a basis of presentation of the findings of this survey the drag and performance calculations are worked out for a single place airplane which has a rocket power plant, sweptback wings and other low drag features. These calculations reveal that this airplane should be able to attain a Mach number of 2 at an altitude of 42,000 feet.

PART II

DESCRIPTION OF THE AIRPLANE

The only claim made for the airplane about to be described is that it is feasable. It was designed on the basis of minimum drag with due consideration of stability and control and overall practicability. It is beyond the purpose of this thesis to delve deeply into the design details. Rather it is assumed that such an airplane is to be built and an attempt is made to compute its probable performance.

The fuselage is just large enough to contain a pilot and a reasonable quantity of fuel. It has a slenderness ratio of 11.5 and is conical at both ends. The optimum nose cone of semi-vertex angle of about $4^{\circ}00'$ was not used as the saving in drag was slight over the more pratical one of $7^{\circ}30'$. The optimum boattail cone of $7^{\circ}15'$ was, however, installed.

The wings are of NACA 0012 profile perpendicular to the leading edge with 60° sweepback at the leading edge and 45° sweepback at the trailing edge. With this amount of sweep the silerons can easly be employed as elevons and thus eliminate the necessity of a horizontal stabalizer.

The vertical fin has a NACA 0006 profile and a 60° sweepback at the leading edge.

The landing gear consists of a main double wheel located between the two fuselage fuel tanks, a nose wheel forward of the pilot and two small "outrigger" wheels, all retractable.

The power plant is a 7,000 pound regeneratively cooled acid-aniline rocket motor.

A brief weight statement is as follows:

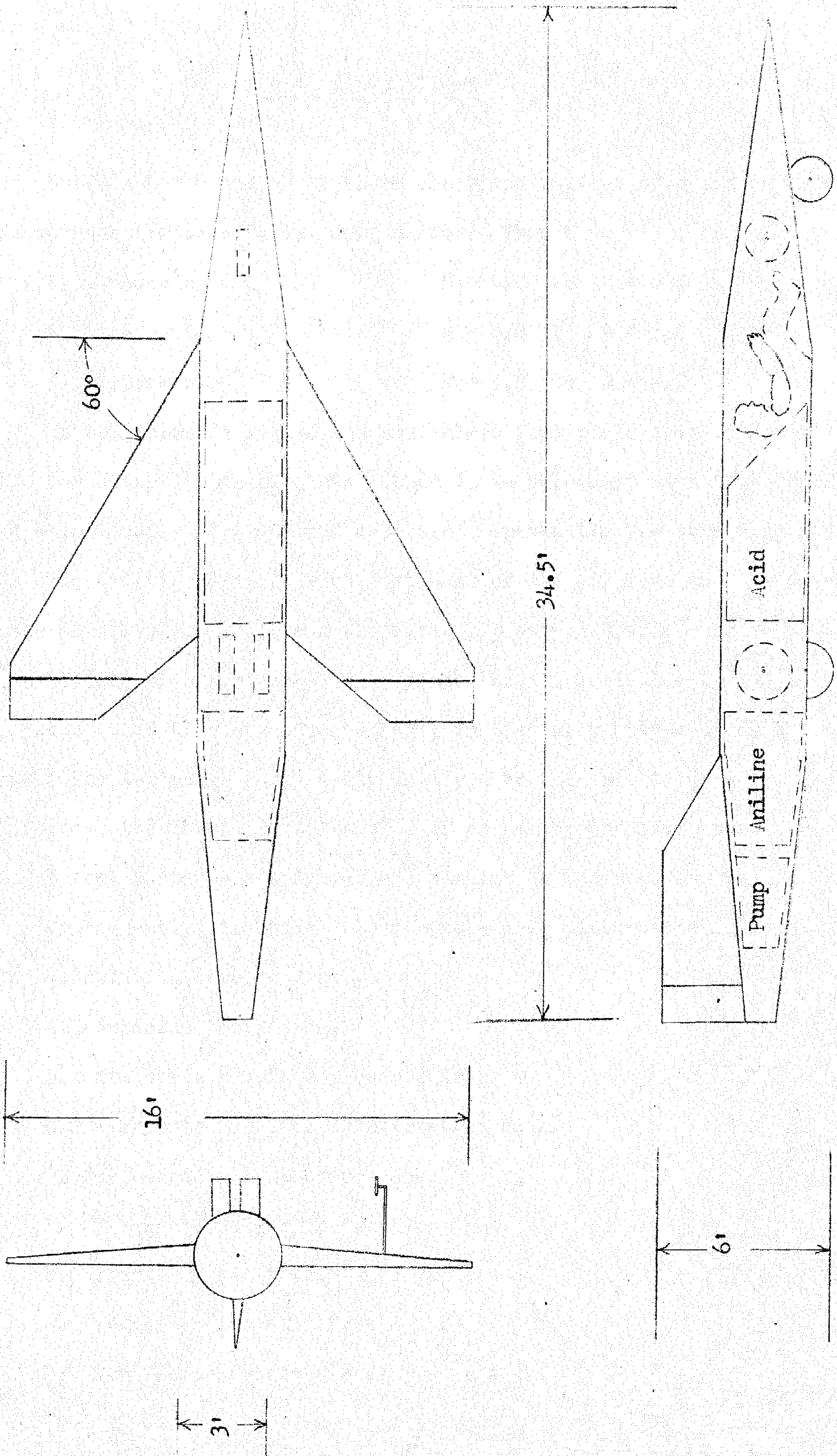
Fuselage hull	900	pounds
Wings	500	
Fin	50	
Landing gears	200	
Fuel tanks	150	
Rocket motor	200	
Pumping unit complete	200	
Plumbing	20	
Cockpit accessories and instrumentation	50	
Breathing oxygen equipment	30	
Pilot	200	
No fuel total weight	2500	pounds
Fuel weight	2900	
Starting weight	5400	pounds

Reference (1) was used as a guide in the above weight estimation.

Some essential data required for drag calculations is as follows:

Wind span	16	feet
Wing plan area external of fuselage	86	square feet
Average chord of wind parallel to line of flight	7	feet
Maximum wing loading	65.1	pounds per square foot
Minimum wing loading	29.1	pounds per square foot
Fin plan area	20	
Average chord of fin parallel to line of flight	7	feet
Fuselage frontal area	7.07	square feet
Rocket nozzle exit diameter	1	foot
Fuselage wetted area	243	square feet
Total wetted area	460	square feet

Figure 1. The Airplane



PART III

DRAG CALCULATIONS

Between M = 0 and M = 2 there are three regimes of speed in which the methods for calculating drag differ. They are:

1. Subsonic M = 0.0 to M = 0.8
2. Transonic M = 0.8 to M = 1.2
3. Supersonic M = 1.2 to M = 2.0

In the subsonic regime the equivalent flat plate area method is used for computing drag. This method is in agreement with both theory and experiment. At trans and supersonic speeds the pressure drag and the skin friction drag of each component of the airplane must be computed seperately. To obtain the various pressure drag forces we must lean heavily on experimental data since the theoretical methods of computing this kind of drag are still in the early stages of development. The technique necessarily employed to obtain this drag is, therefore, the "brute force" method of searching the literature for wind tunnel tests on configurations similar to those on the airplane.

Consequently, to obtain the complete drag picture the following drag calculations are necessary:

A. Subsonic total drag.

B. Transonic fuselage pressure drag.

C. Transonic fuselage skin friction drag.

D. Transonic wing pressure drag.

E. Transonic wing skin friction drag.

F. Transonic fin pressure drag.

G. Transonic fin skin friction drag.

H. Supersonic fuselage nose pressure drag.

I. Supersonic fuselage boattail pressure drag.

J. Supersonic fuselage skin friction drag.

K. Supersonic wing pressure drag.

L. Supersonic wing skin friction drag.

M. Supersonic fin pressure drag.

N. Supersonic fin skin friction drag.

The drag calculations will be confined to the speed range M = 0.2 to M = 2.0 and to the altitude range sea level to 60,000 feet.

The calculations are first made assuming an "ambient jet", i.e., it is assumed that at all times the pressure at the blunt part of the boattail is the same as the local atmospheric pressure. The jet-on and the jet-off drag are then computed from the ambient jet drag. There are two reasons for doing this: 1. The airplane is then completely isolated from its power plant during most of the calculations and alot of possible confusion is thereby avoided. 2. The nature of the drag data available is such that the jet-on drag is more easily obtained by this method of approach.

The drag due to lift is computed for the no fuel condition as this drag component has more significance during the glide and landing portion of the flight. The drag due to lift is so small at trans and supersonic speeds that it is disregarded.

Throughout the calculations certain working data are continually required. They are:

1. The equivalent flat plate area of the airplane. (Subsonic)
2. A table of feet persecond vs Mach number and altitude.
3. A graph of unit length Reynolds number vs Mach number at various altitudes.
4. A graph of skin friction coefficient vs Reynolds number.

A discussion of these items follows immediately in numerical order.

The equivalent flat plate area of the airplane is estimated from an "Aerodynamic Cleanliness" graph as shown in Figure 2.

A comparison is made with a hypothetical P-51 having the same wetted area as the XXX. Such a P-51 would have an equivalent flat plate area of 1.7 square feet. Since the XXX is even more "streamlined" than the P-51 its equivalent flat plate area will probably be a little less than this. However, in the interests of conservativism we will use this figure.

The speed of sound varies with altitude. Consequently the speed corresponding to a certain Mach number varies with altitude. From the table in Figure 3, one can obtain the speed for a given Mach number and altitude. These speeds were computed f rom the NACA standard atmosphere

The Reynolds number of the various parts of the airplane is frequently considered when calculating skin friction. A plot of unit length Reynolds number vs Mach number and altitude is found in Figure 4. To find the Reynolds number of a given body at a certain Mach number and altitude just multiply the length of the body in feet by the unit length Reynolds number fround on this graph.

There are a number of different empirical formulae and graphs in the literature expressing the relation between the smooth flat plate skin friction coefficient and the free stream Reynolds number in the subsonic regime. They are all in fairly close agreement. According to Charters, (3), there is also experimental evidence to show that these same relations hold true in the trans and supersonic regims. He recommends the graph which is reproduced here in Figure 5 and can also be found in reference (4).

Figure 2.

Aerodynamic cleanliness is defined by the quotient: $\frac{f}{w}$

where f = equivalent flat plate area and w = total wetted area.

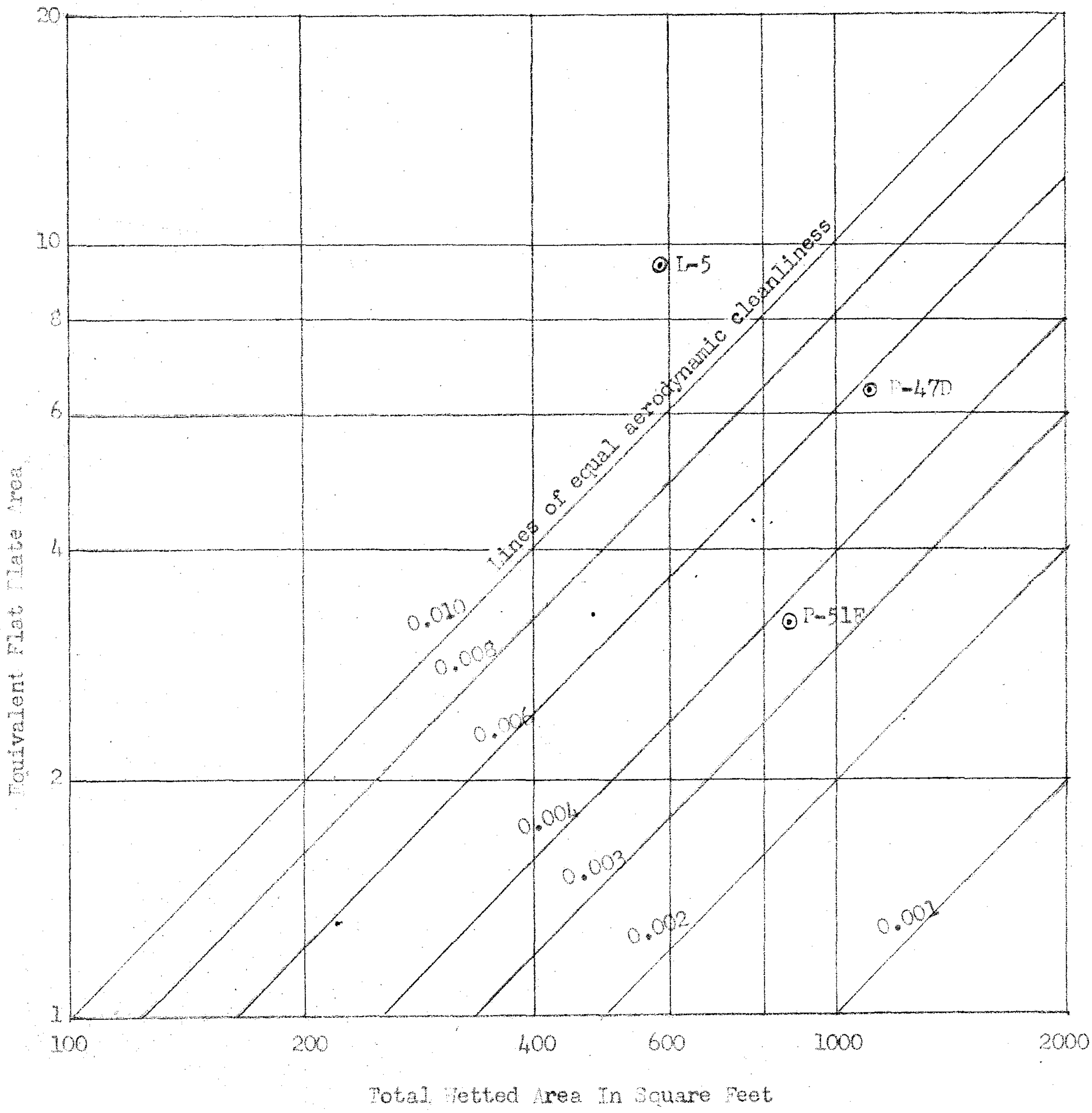


Figure 3.

Altitude	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
Mach No.							
0.2	223 Ft/Sec	215	207	199	194	194	194
0.3	335	323	311	298	291	291	291
0.4	446	431	414	398	388	388	388
0.5	558	538	518	497	485	485	485
0.6	669	646	620	596	582	582	582
0.7	781	754	725	696	679	679	679
0.8	892	861	827	795	777	777	777
0.9	1004	969	933	895	874	874	874
1.0	1116	1077	1037	994	971	971	971
1.1	1227	1184	1140	1083	1068	1068	1068
1.2	1339	1292	1254	1193	1165	1165	1165
1.4	1562	1508	1451	1392	1359	1359	1359
1.6	1785	1723	1657	1590	1553	1553	1553
1.8	2008	1938	1864	1789	1748	1748	1748
2.0	2231	2153	2072	1998	1941	1941	1941

Figure 4.

Unit length Reynolds number vs Mach number and altitude.

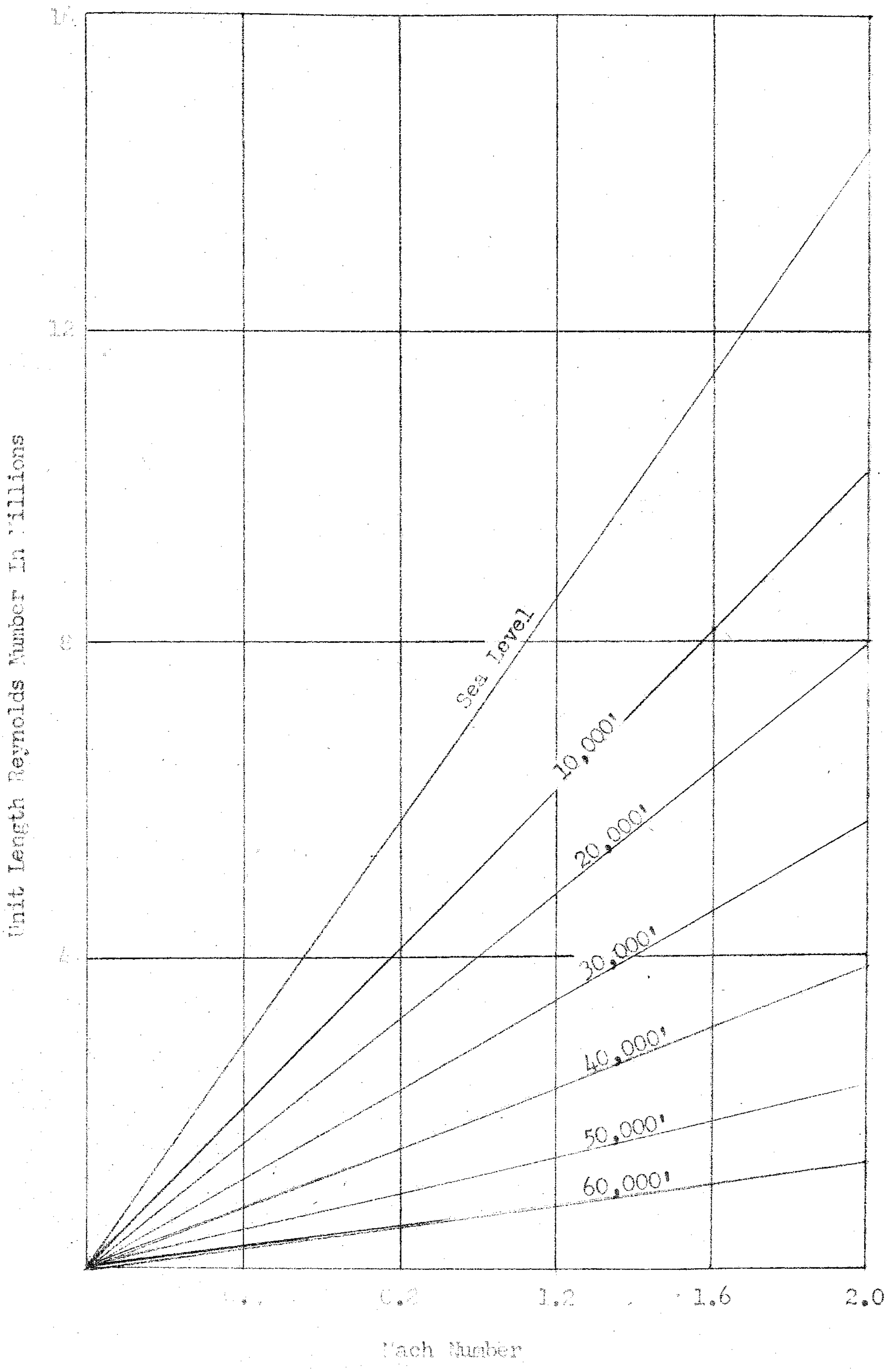
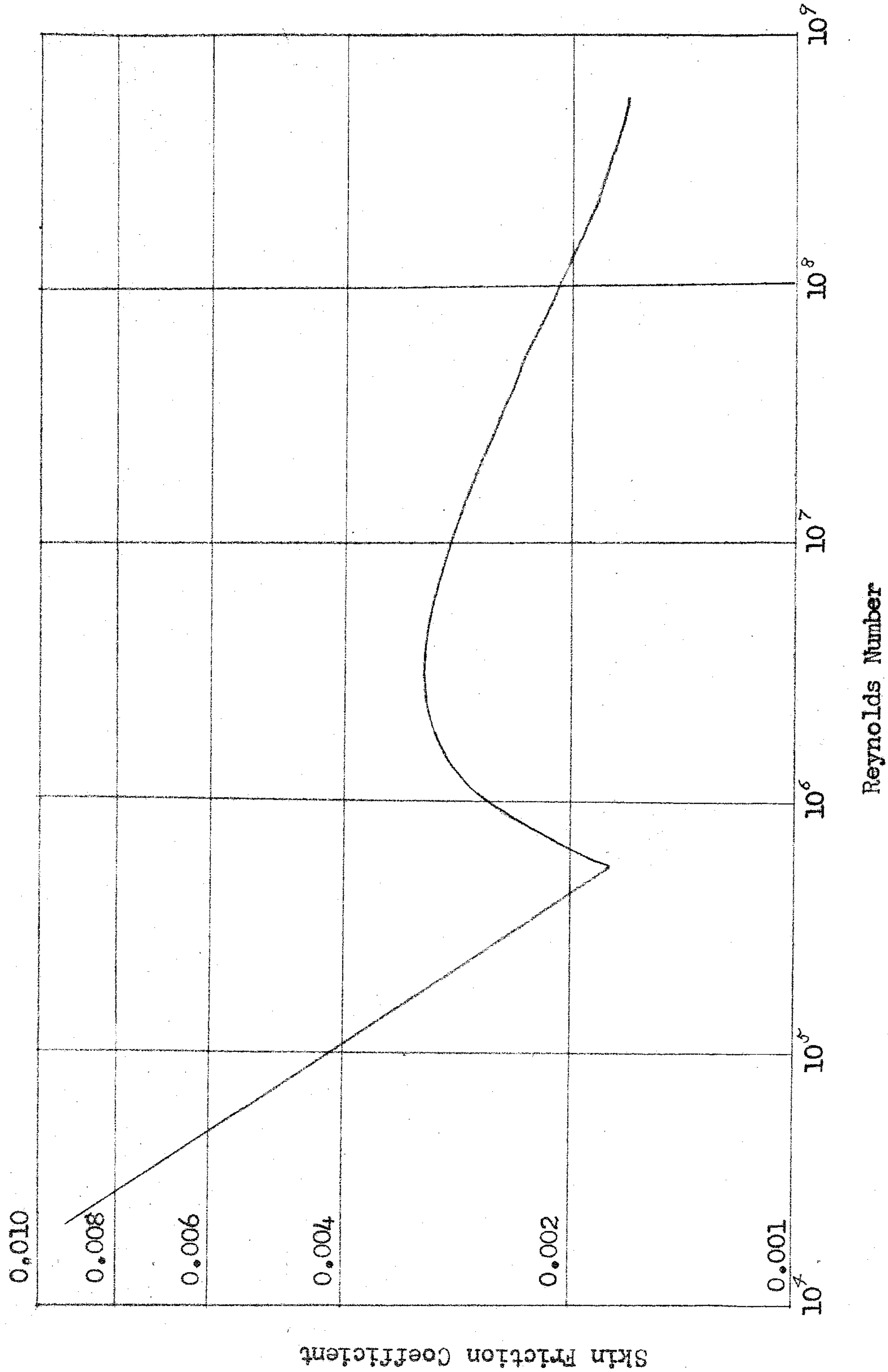


Figure 5.


0.010
0.008
0.006
0.004
0.002
0.001
10^4
10^5
10^6
10^7
10^8
10^9
Reynolds Number
Skin Friction Coefficient

DRAG CALCULATIONS A. SUBSONIC TOTAL DRAG

Total drag = D = parasite drag + drag due to lift. (5)

$$D = \frac{\rho f V^2}{2} + \frac{C_L^2 S \rho V^2}{2 \pi AR e}$$

$$D = \frac{\rho f V^2}{2} + \frac{2 W^2}{\rho V^2 \pi e b^2}$$

$$D = 2.01 \times 10^{-3} \sigma V^2 + \frac{9.35 \times 10^6}{\sigma V^2}$$

where f = 1.7 square feet

W = 2500 pounds

e = 0.7 (a conservative estimate)

b = 16 feet

ρ = 0.002378 slugs per cubic feet x σ

Using the formula just derived the following table of parasite drag forces is made:*

Altitude / Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
0.2	100	69	46	29			
0.3	226	150	106	63	42		
0.4	401	282	184	114	74	46	
0.5	625	436	306	178	115	73	45
0.6	904	620	412	256	168	107	65
0.7	1225	845	565	364	226	142	87
0.8	1600	1095	730	475	296	184	114

* All tabulated drag froces are in pounds.

Using the formula derived on the previous page the following table of induced drag forces is made:

Altitude	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
Mach No.							
0.2	184	277	443	632			
0.3	84	120	216	281	450		
0.4	47	68	121	158	253	408	
0.5	30	43	77	101	162	261	420
0.6	21	30	54	74	112	180	292
0.7	15	22	33	52	84	134	216
0.8	9	14	20	31	50	81	130

DRAG CALCULATIONS B. TRANSONIC FUSELAGE PRESSURE DRAG

The data used here is taken from reference (6) which concerns a body of revolution pointed at both ends and with a slenderness ratio of 6. A transcript of this data is as follows:

Mach No.	0.9	1.0	1.1
C_{D_π}	0.050	0.250	0.285

To obtain the pressure drag coefficients we must subtract from the above coefficients the respective skin friction coefficients accord-ind to the equation:

$$C_{D_{\pi p}} = C_{D_\pi} - C_F \times \frac{\text{wetted area}}{\text{frontal area}}$$

A typical calculation proceeds in the following manner. Let us find the drag at sea level at M = 1. The unit length Reynolds number at sea level at M = 1 is obtained from Figure 4. It is 7.0×10^6. The length of the model was five feet. Therefore the Reynolds number of the model is 3.5×10^7. Entering the graph of Reynolds number vs C_F we find C_F of model = 0.0025. Now $\frac{\text{Model wetted area}}{\text{Model frontal area}} = 16$.

Therefore the $C_{D_{\pi p}}$ of model = 0.250 - 0.0025 x 16 = 0.210. We use this same $C_{D_{\pi p}}$ for the fuselage of the XXX.

Therefore $D = 0.210 \times \frac{1}{2} \times 0.002378 \times (1116)^2 \times 7.07 = 2190$ pounds.

By similar computation we obtain the following table of fuselage pressure drag forces:

Altitude	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
Mach No.							
0.9	84	58	33	17	9	5	3
1.0	2190	1510	1005	642	391	243	149
1.1	3110	2130	1415	891	548	345	213

DRAG CALCULATION C. TRANSONIC FUSELAGE SKIN FRICTION DRAG

This drag component is obtained directly by use of the information contained in Figures 3, 4, and 5. As an example let us find the drag at M = 1.1 at 20,000 feet. The unit length Reynolds number under these circumstances is 4.5×10^6. The length of the fuselage is 34.5 feet. Consequently the fuselage Reynolds number is 1.5×10^8 and the corresponding C_D is 0.0020.

Therefore $D = 0.0020 \times \frac{1}{2} \times 0.001267 \times (1140)^2 \times 243 = 401$ pounds.

In this way we obtain the following table of fuselage skin friction drag forces:

Altitude / Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
0.9	577	419	288	198	124	83	56
1.0	692	503	343	239	151	100	67
1.1	827	601	401	280	179	117	79

DRAG CALCULATIONS D. TRANSONIC WING PRESSURE DRAG

The data used here is gathered from references (7), and (8). From the former are transcribed the following values of C_D for a wing of 52° sweepback at the leading edge and a similar plan form to the wing of the XXX.

Mach No.	0.9	1.2
C_D	0.02	0.05

To get the C_{D_p} of the model we use the formula : $C_{D_p} = C_D - 2C_F$.

The applicable Reynolds number of the model wing were:

500,000 at $M = 0.9$

850,000 at $M = 1.2$.

This results in the following skin friction coefficients:

At $M = 0.9$ $C_F = 0.0018$

At $M = 1.2$ $C_F = 0.0022$

Whence we arrive at the following model pressure drag coefficients:

At $M = 0.9$

At $M = 1.2$

No correction factor is made for the difference in sweepback between the model and the XXX, (only 8°), but a correction factor is introduced to account for the difference in thickness ratio. The airfoil described in reference (7) has a 12% thickness perpendicular to its maximum thickness line which has a sweep of 45°. The airfoil on the XXX has a 12% thickness perpendicular to the leading edge which has a sweep of 60°. Therefore the ratio of the thickness is $\frac{\text{cosine } 60^\circ}{\text{cosine } 45^\circ} = \frac{\sqrt{2}}{2}$.

The pressure drag coefficient at these velocities varies as the square of the thickness ratio. Consequently we have as the applicable pressure drag coefficients of the wing of the XXX:

At $M = 0.9$ $\quad C_{D_p} = 0.0082$

At $M = 1.2$ $\quad C_{D_p} = 0.0228$

These two figures are incomplete for our purposes, however, in that they fail to show in what manner C_{D_p} goes from 0.0082 at $M = 0.9$ to 0.0228 at $M = 1.2$. From reference (8) we find that for a wing of 45° sweepback, and a fortiori for a wing of larger sweep, the C_D curve is almost linear in the transonic region. Thus by linearly filling in the missing points we have:

Mach No.	0.9	1.0	1.1	1.2
C_{D_p}	0.0082	0.0131	0.0180	0.0228

Using these drag coefficients we construct the following table of wing pressure drag forces:

Altitude	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
Mach No.							
0.9	907	581	388	252	157	92	60
1.0	1800	1143	765	440	310	192	119
1.1	2775	1910	1274	807	512	318	197

DRAG CALCULATIONS E. TRANSONIC WING SKIN FRICTION DRAG

This drag component is derived in the same manner as in Drag Calculations C. We use as the characteristic length the average chord of the wing. The tabulated results are as follows:

Altitude / Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
0.9	477	354	242	159	116	68	35
1.0	590	437	298	196	125	82	44
1.1	707	507	352	224	148	99	53

DRAG CALCULATIONS F. TRANSONIC FIN PRESSURE DRAG

The same data is used here as was used to obtain the transonic wing pressure drag. Since the thickness ratio of the fin is one half that of the wing the correction factor 0.25 will have to be introduced. This results in the following:

Mach No.	0.9	1.0	1.1	1.2
C_{D_p}	0.00205	0.00327	0.00450	0.00570

Using these drag coefficients we construct the following table of fin pressure drag forces:

Altitude / Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
0.9	49	34	22	14	9	5	3
1.0	96	65	44	26	17	10	6
1.1	161	111	73	46	30	19	11

DRAG CALCULATIONS G. TRANSONIC FIN SKIN FRICTION DRAG

This drag component is computed in exactly the same way as in Drag Calculations E. The tabulated results are:

Altitude	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
Mach No.							
0.9	110	82	56	37	27	15	8
1.0	137	102	70	46	30	19	10
1.1	165	118	82	52	35	23	12

DRAG CALCULATIONS H. SUPERSONIC NOSE PRESSURE DRAG

The problem of pressure drag due to supersonic flow past a cone at zero angle of attack is the subject of references (9), (10), (11), and (12). Reference (13) discusses the merits of each of the above.

The method that results in the highest drag for the conditions of our problem is the simple application of Karman's approximate formula,

$$C_{D_{\pi np}} = 2\,\theta^2 \ln\left(\frac{2}{\theta\alpha}\right)$$

where θ = semi-vertex angle, and $\alpha = \sqrt{M^2 - 1}$

For a sample calculation let us find the nose pressure drag at $M = 1.8$ at an altitude of 30,000'.

$$C_{D_{\pi np}} = 2(0.1309)^2 \ln\left(\frac{2}{0.1309 \times \sqrt{(1.8)^2 - 1}}\right) = 0.0806$$

where 0.1309 radians = 7°30'.

$$D = 0.0806 \times \frac{1}{2} \times 0.000889 \times (1789)^2 \times 7.07 = 850 \text{ pounds}$$

By similar computation the following table is obtained:

Altitude	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
Mach No.							
1.2	1624	1135	768	487	299	188	117
1.4	1955	1345	900	580	356	225	140
1.6	2320	1595	1065	684	432	268	166
1.8	2730	1880	1355	800	497	314	195
2.0	3150	2170	1450	936	584	362	225

DRAG CALCULATIONS I. SUPERSONIC BOATTAIL PRESSURE DRAG

Reference (3) is used as a guide for finding this drag component. The recommended technique is to first compute the "square base" drag and then subtract the reduction, is drag due to boattailing. The square base drag is obtained from a graph of the ratio of square base pressure to atmospheric pressure vs Mach number. The reduction in drag due to boattailing is also obtained from a graph, namely ΔK_D vs Mach number. Both these graphs were constructed from empirical data.

A transcript of the pertinant information is as follows:

Mach No.	1.2	1.4	1.6	1.8	2.0
Sq. Base Press.	77%	72%	67%	62%	57%
ΔK_D	0.0570	0.0490	0.0425	0.0375	0.0340

The aquare base area of the fuselage is 7.07 square feet, but with the ambient jet assumption the effective square base area is 6.285 square feet.

For a sample calculation we will find the boattail pressure drag at $M = 1.2$ at sea level. The square base drag under these conditions is

$D = (1 - 0.77) \times 6.285 \times 2116 = 3050$ pounds.

The reduction in drag by use of boattailing under these conditions is $D = 0.057 \times \frac{8}{\pi} \times 6.285 \times \frac{1}{2} \times 0.002378 \times (1339)^2 = 1940$ pounds.

Therefore, the boattail drag = 3050 - 1940 = 1110 pounds.

By similar computation the following table is obtained:

Altitude / Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
1.2	1110	785	539	330	204	129	80
1.4	1490	1006	710	424	267	166	104
1.6	1890	1270	892	531	333	208	133
1.8	2220	1530	1070	644	399	250	155
2.0	2550	1760	1290	756	258	284	176

DRAG CALCULATION J. SUPERSONIC FUSELAGE SKIN FRICTION DRAG

The same technique is used here as in the previous skin friction computations.

Altitude / Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
1.2	938	658	460	307	209	137	93
1.4	1337	897	615	418	287	182	125
1.6	1700	1145	780	530	359	229	158
1.8	2150	1440	990	673	445	294	199
2.0	2580	1775	1215	839	533	363	246

DRAG CALCULATION K. SUPERSONIC WING PRESSURE DRAG

Due to the lack of applicable data in the literature, this drag component must be estimated by an extrapolation from the transonic pressure drag. The basis of this extrapolation is the fact that the pressure drag coefficient of a highly swept wing increases moderately in the region M = 0.9 to M = 1.2 and then remains fairly constant until the Mach wave crosses the maximum thickness line or the leading edge, whichever occurs first. At this point the drag coefficient increases about two and a half times in value. (Reference 14). Consequently the pressure drag coefficient will remain at about 0.0228 from M = 1.2 to M = 1.8, and then at M = 2.0 it will be about 0.0540. With this information we can construct the table of supersonic wind pressure drag forces.

Altitude Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
1.2	4160	2955	2018	1251	762	464	280
1.4	5625	3870	2580	1613	1020	645	400
1.6	7110	4882	3240	2100	1265	815	506
1.8	9260	6300	4231	2709	1705	1060	655
2.0	27400	18850	12650	8220	5090	3160	1960

DRAG CALCULATIONS L. SUPERSONIC WING SKIN FRICTION DRAG

Altitude Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
1.2	804	605	426	262	178	118	63
1.4	1115	788	526	355	240	151	93
1.6	1435	1003	670	462	314	202	128
1.8	1815	1245	867	584	381	239	157
2.0	2195	1540	1070	730	472	304	196

DRAG CALCULATIONS M. SUPERSONIC FIN PRESSURE DRAG

The same data is used here as was used to obtain the supersonic wing pressure drag. To account for the difference in thickness ratio the applicable pressure drag coefficients are multiplied by 0.25.

Altitude Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
1.2	242	172	107	73	44	27	16
1.4	327	225	150	93	59	38	23
1.6	414	285	189	122	74	47	29
1.8	535	367	246	158	99	62	38
2.0	1590	1095	735	276	296	183	114

DRAG CALCULATIONS N. SUPERSONIC FIN SKIN FRICTION DRAG

Altitude Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
1.2	183	161	98	61	41	28	15
1.4	258	182	122	82	55	35	22
1.6	334	233	156	107	73	47	30
1.8	423	290	202	136	89	56	37
2.0	510	358	249	170	110	71	46

DRAG SUMMATION

The following table gives the total drag on the airplane for straight and level flight at the various altitudes and Mach numbers. The drag due to lift is neglected after M = 0.8, and the "ambient jet" assumption is maintained throughout.

Altitude Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
0.2	284	346	489				
0.3	310	270	322	344			
0.4	448	350	305	272	327		
0.5	655	479	383	279	277	334	
0.6	925	650	466	330	280	287	357
0.7	1240	867	598	416	310	276	307
0.8	1612	1112	756	514	359	286	279
0.9	2204	1528	1029	777	442	269	165
1.0	5505	3760	2525	1589	924	646	395
1.1	7745	5292	3597	2300	1452	921	567
1.2	9061	6451	4416	2771	1737	1091	664
1.4	12107	8207	5603	3565	2284	1440	907
1.6	15203	10413	6982	4536	2840	1816	1150
1.8	19133	13152	8961	5704	3715	2275	1436
2.0	39975	27548	18661	12127	7540	4727	2963

Figure 6.

This figure presents in graphical form a plot of the total ambient jet fusalage drag coefficient and the total wing drag coefficient against Mach number at sea level. The characteristic area in both cases in the wing plan area.

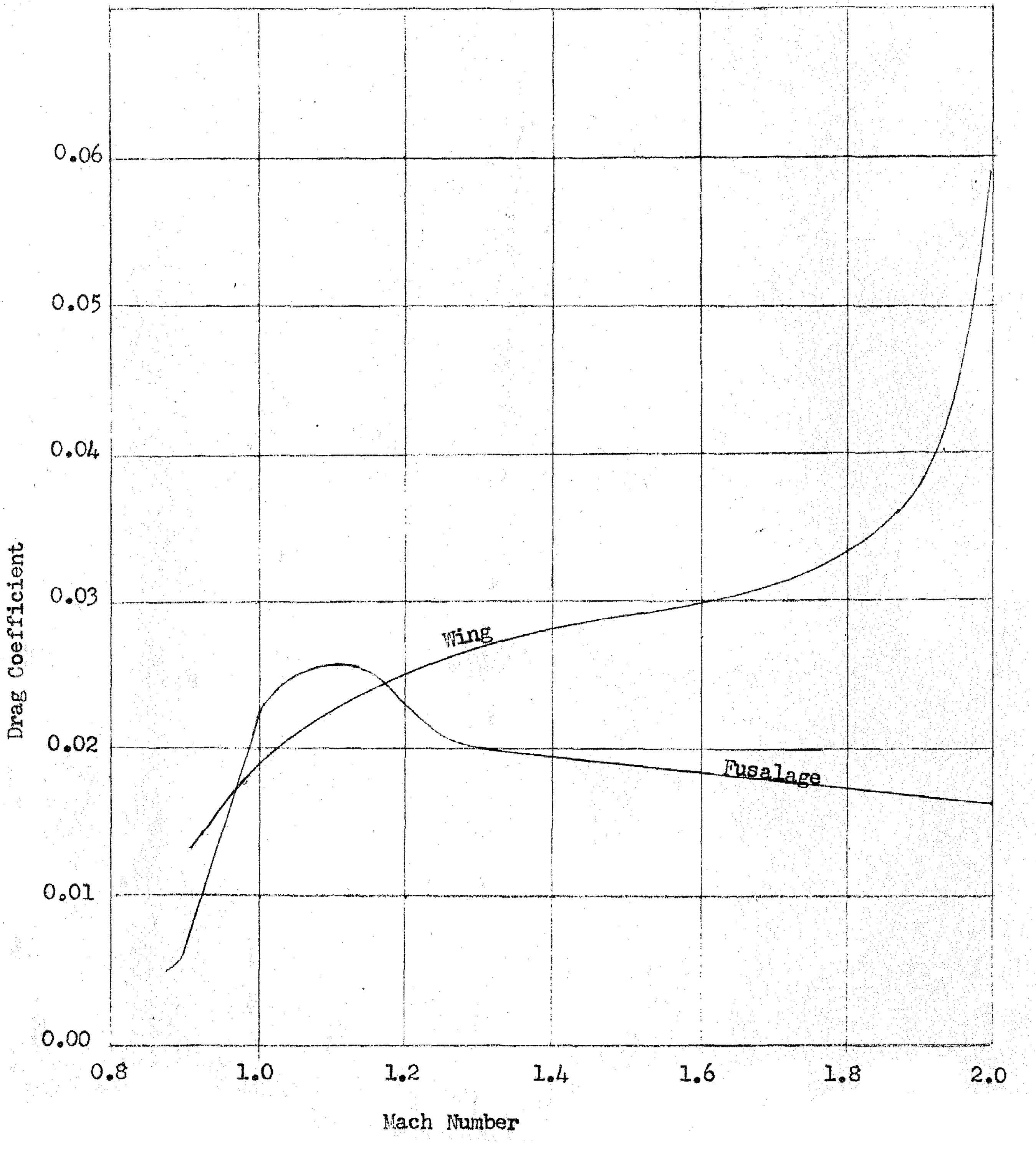
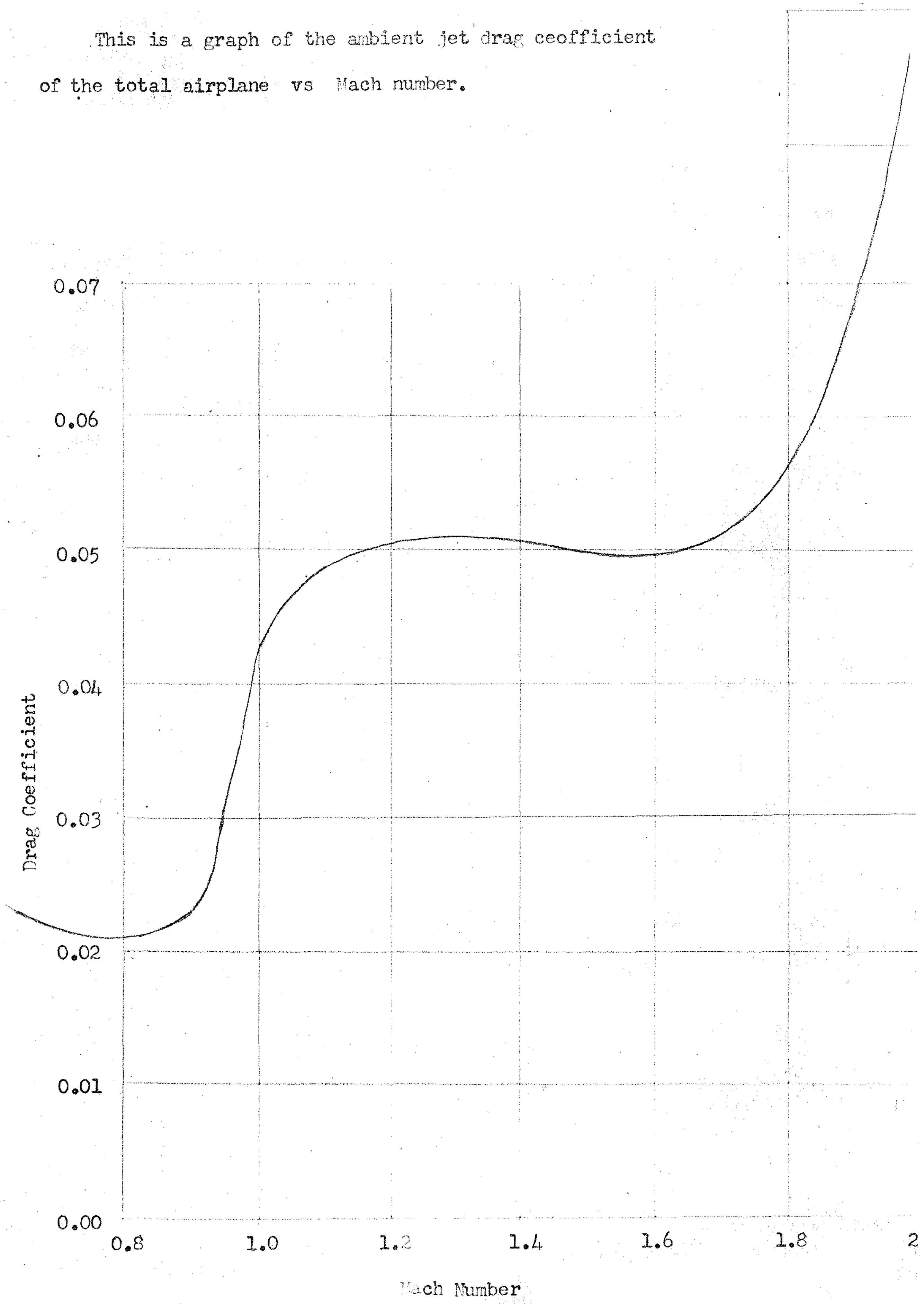


Figure 7.

This is a graph of the ambient jet drag ceofficient of the total airplane vs Mach number.



JET - OFF DRAG

Jet-off drag is equal to the "ambient jet" drag plus a certain amount of suction drag at the square base part of the boattail. To find this suction drag we have merely to know the base pressure as a percentage of ambient pressure for various speeds. This information is partly given in Drag Calculations I. It is found that in the region between M = 1.2 and M = 2.0 the base pressure varies linearly, and that the point M = 0, p/p_0 = 1 is almost on an extention of this line. By estimating the curve between M = 0 and M = 1.2 we arrive at the complete required data:

Mach No.		Mach No.	
0.0	1.000	0.90	0.845
0.2	0.980	1.0	0.820
0.3	0.970	1.1	0.795
0.4	0.950	1.2	0.770
0.5	0.930	1.4	0.720
0.6	0.910	1.6	0.670
0.7	0.890	1.8	0.620
0.8	0.870	2.0	0.570

using this information we calculate the following table of differences between ambient jet drag and jet-off drag.

THE DIFFERENCE BETWEEN AMBIENT JET DRAG AND JET OFF DRAG

Altitude Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
0.2	42	29	19	12	8	5	3
0.3	63	44	29	19	12	7	4
0.4	106	73	49	31	20	12	7
0.5	148	102	68	44	27	17	11
0.6	190	131	87	56	35	22	14
0.7	232	159	106	69	43	27	17
0.8	275	189	126	82	51	32	20
0.9	327	225	155	97	61	38	23
1.0	380	261	174	113	70	44	27
1.1	433	298	199	128	80	50	31
1.2	486	334	223	144	90	56	35
1.4	591	400	271	175	109	68	42
1.6	696	478	319	206	129	80	50
1.8	802	551	368	238	148	92	57
2.0	908	624	416	269	168	104	65

JET - OFF DRAG FOR STRAIGHT AND LEVEL FLIGHT

Altitude Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
0.2	326	375	508	673			
0.3	373	314	351	363	504		
0.4	554	423	354	303	347	466	
0.5	803	581	451	323	304	351	276
0.6	1115	781	553	386	315	309	371
0.7	1472	1026	704	485	353	303	324
0.8	1887	1301	882	596	410	318	299
0.9	2531	1753	1184	874	503	307	188
1.0	5885	4021	2699	1702	994	690	422
1.1	8178	5590	3796	2428	1532	971	598
1.2	9547	6985	4639	2915	1827	1147	699
1.4	12698	8607	5874	3740	2393	1508	949
1.6	15899	10891	7001	4742	2969	1896	1200
1.8	19935	13703	9329	5942	3863	2367	1493
2.0	40883	28172	17077	12396	7728	4831	3028

JET - ON DRAG

The difference between ambient jet drag and jet-on drag is caused by an added velocity at the rear and induced by the very rapid velocity of the exhaust. The only available data on this type of drag is the result of tests on a model V-2. (Reference 15). At M = 0.2 the jet-on parasite drag was twice the ambient jet parasite drag. This ratio decreased linearly to one at M = 0.9. In other words, at M = 0.9 the jet-on drag is the same as the ambient jet drag. For Mach numbers greater than 0.9 the ambient jet drag is greater than the jet-on drag and the difference increases linearly until at M = 2.0 the jet-on drag is 88% of the ambient jet drag. In tabular form we have:

At M = 0.2 the jet-on parasite drag	= 14/7	the ambient jet parasite drag.
At M = 0.3 " " " " "	= 13/7	" " " " "
At M = 0.4 " " " " "	= 12/7	" " " " "
At M = 0.5 " " " " "	= 11/7	" " " " "
At M = 0.6 " " " " "	= 10/7	" " " " "
At M = 0.7 " " " " "	= 9/7	" " " " "
At M = 0.8 " " " " "	= 8/7	" " " " "
At M = 0.9 " " " " "	= 7/7	" " " " "
At M = 1.0 " " " " "	= 98.9%	the ambient jet parasite drag.
At M = 1.1 " " " " "	= 97.8%	" " " " "
At M = 1.2 " " " " "	= 96.7%	" " " " "
At M = 1.4 " " " " "	= 94.5%	" " " " "
At M = 1.6 " " " " "	= 92.3%	" " " " "
At M = 1.8 " " " " "	= 91.1%	" " " " "
At M = 2.0 " " " " "	= 88.0%	" " " " "

Using this information we construct the following table of jet-on drag forces for straight and level flight.

JET - ON DRAG FOR STRAIGHT AND LEVEL FLIGHT

Altitude Mach No.	Sea Level	10,000'	20,000'	30,000'	40,000'	50,000'	60,000'
0.2	384	415	535				
0.3	504	398	413	398			
0.4	782	540	436	382	383		
0.5	1000	738	557	381	342	381	
0.6	1321	905	634	386	340	330	384
0.7	1585	1122	758	519	386	317	328
0.8	1834	1264	855	561	388	291	260
0.9	2204	1528	1029	777	442	269	165
1.0	5400	3680	2470	1565	905	633	387
1.1	7500	5130	3482	2230	1408	893	550
1.2	8700	6200	4220	2660	1665	1050	637
1.4	11390	7700	5270	3350	2150	1353	852
1.6	13990	9570	6420	4165	2280	1670	1060
1.8	17240	11830	8060	5140	2280	1670	1060
2.0	35080	24230	16440	10680	6640	4170	2640

PART IV

PERFORMANCE CALCULATIONS

Before actually investigating the performance of the XXX a brief discussion of its longitudinal stability and control seems appropriate.

Since there is no horizontal stabalizer it is important that the center of lift and center of gravity remain fairly close together for all operating speeds and weights. It is reasonable to assume that the subsonic lift is at the quarter chord and that the centroid of this lift is at one third the semi-span. This results in a center of lift at station 18, where station here means distance from the nose in feet. The airplane is designed so that its C.G. is fixed at station 17.9. Generally speaking, the airplane is then stable in the subsonic regime.

In the supersonic regime for the range of Mach numbers considered here, the flow over the wing is almost conical and the center of pressure may be readily computed since the pressure is constant along any radial line through the root of the leading edge. Thus for the wing of the XXX the center of pressure is at station 19.2, indicating even more stability than for subsonic flight. However, in the supersonic case the lift of the fuselage can play an important part since its lift usually may be considered as being concentrated at its nose, and therefore the complete stability is given by the equation

$$\frac{dC_M}{d\alpha} = l_f \frac{dC_{Lf}}{d\alpha} - l_w \frac{dC_{Lw}}{d\alpha}$$

where l_f is the distance from the nose of the C.G. and l_w is the distance from the C.G. to the center of lift. $\frac{dC_{Lf}}{d\alpha}$ will be approximately 2 per radian based on fuselage frontal area or 0.164 based on wing area. $\frac{dC_{Lw}}{d\alpha}$ varies with Mach number and for an example at M = 1.5

it is 3.1 per radian.

Thus at M = 1.5:

$$\frac{d\ C_m}{d\alpha} = 17.9 \times 0.164 - (19.2 - 17.9) \times 3.1 = -1.1 \text{ per radian.}$$

For straight and level flight the XXX will have to maintain an angle of attach given by:

$$L = \alpha \frac{dC_L}{d\alpha} \times \frac{\gamma}{2} M^2 p S$$

Assume for an example that M = 1.5, L = 3,000 pounds, and that p + 628 lb./sq. ft. corresponding to 30,000 feet altitude. Then α = 0.62 degrees.

To determine the necessary elevon diflection we use the expression:

$$C_M = 0 = \alpha \frac{dC_M}{d\alpha} - l_t C_{Le} \frac{Ae}{Aw}$$

where l_t is the distance from the C.G. to the center of the elevon and is here equal to 5.5 feet. Then

$$0 = 0.62 \times 0.01745 \times (-1.1) - 5.5\ C_{L_e} \times \frac{9}{86}$$

whence $C_{L_e} = -0.021$

Using the two dimentional $\frac{dC_{Le}}{d\alpha_e} = \frac{4}{\sqrt{M^2-1}}$ = 3.6, we have:

$$\alpha_e \frac{dC_{Le}}{d\alpha_e} = C_{L_e}$$

$$\alpha_e \times 3.6 = -0.021$$

$$\alpha_e = -0.0058 \text{ radians} = 0.33 \text{ degrees.}$$

We have thus verified the feasability of the XXX actually flying. We will now take up some aspects of its performance, which, in general, is centered around the differential equation:

$$W \times a + D + W \times \sin\psi = F$$

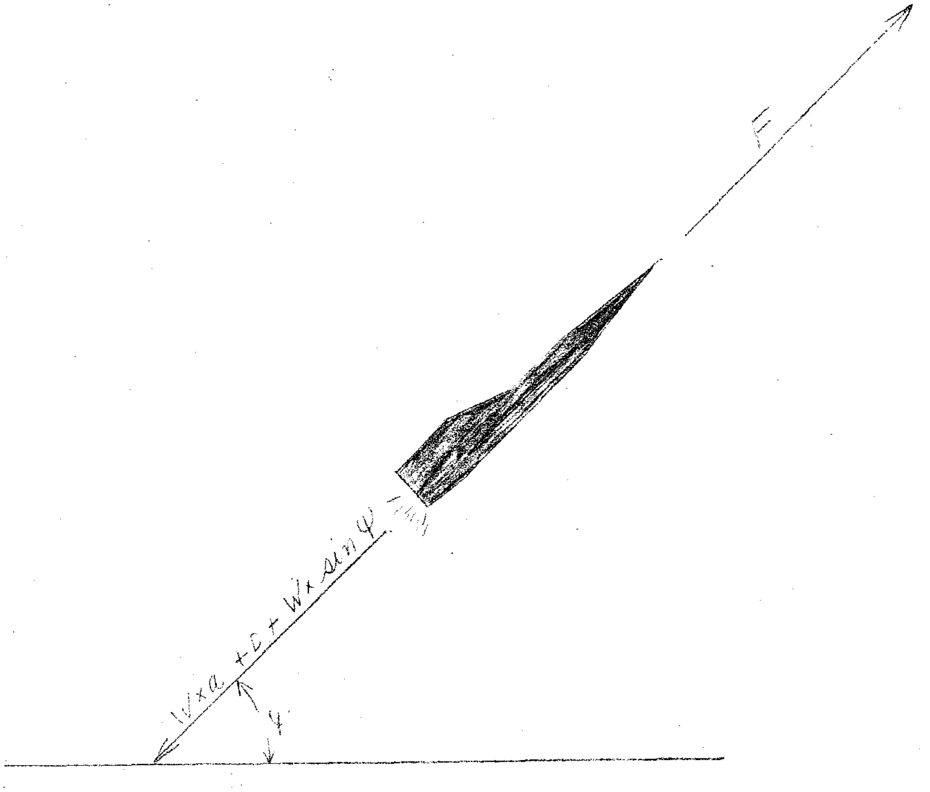
where W = the instantaneous weight

a = the acceleration

D = the instantaneous drag

ψ = angle from the horizontal

F = the instantaneous thrust.



In Part III we found D for the various conditions of flight. Before continuing our discussion of performance we will investigate F and W.

THRUST AVAILABLE

We use the simplified expression for thrust:

$$F = C_{fa} \times p_c \times f_t$$

where $C_{fa} = C_{ft} \times \quad \times \eta$. Both and η are usually equal to about 0.95.

The basic constants of the nitric acid and aniline rocket motor are:

p_c = 300 pounds per square inch and = 1.22.

With this information and Figure 9, which is copied from Reference (16) we can perform all the rocket computations necessary for our purposes here.

Arbitrarily selecting 25,000' as the altitude for optimum nozzle expansion we then have:

$$\frac{p_c}{p_o} = \frac{300 \times 144}{2116 \times 0.3709} = 55.$$

Entering the graph in Figure 9 with this number we arrive at an $\frac{f_e}{f_t}$ of 7.4 and a C_{ft} of 1.565 at 25,000'. Consequently the thrust at 25,000' is $F = 1.565 \times 0.95 \times 0.95 \times 300 \times 15.3 = 6,470$ pounds.

For other altitudes we have a different $\frac{p_c}{p_o}$ and consequently a different C_{ft} and a different thrust.

Altitude	$\frac{p_c}{p_o}$	C_{ft}	F
S.L.	20.45	1.34	5,540 pounds
10,000'	29.75	1.45	6,000 pounds

(continued)

Altitude	$\frac{p_c}{p_o}$	C_{ft}	F
20,000'	44.60	1.53	6,320 pounds
30,000'	68.90	1.59	6,570 pounds
40,000'	110.50	1.63	6,740 pounds
50,000'	178.00	1.66	6,860 pounds
60,000'	287.50	1.68	6,950 pounds

THE INSTANTANEOUS WEIGHT

The actual specific impulse of the acid-aniline combination is 200 lb. sec./lb., this gives a duration of 89.1 seconds and a mass fuel flow of 32.6 pounds per second. Therefore the instantaneous weight of the XXX is:

$$5400 - 36.6 \times T \qquad 0 \qquad T \qquad 89.1 \text{ seconds}$$

SOME SPECIFIC PERFORMANCE PROBLEMS

To find the maximum Mach number curve of the XXX we use only two terms of the performance equation: $D = F$.

These two terms are plotted independently against altitude and Mach number in Figure 10, and a cross plot yields the maximum Mach number curve in Figure 11.

To find the gliding characteristics of the XXX we again use only two terms of the performance equation: $D + w \times \sin \psi = 0$.

The gliding performance is presented graphically in Figures 12 and 13.

We will now find the maximum altitude attainable by taking off, making a 20 second turn to the vertical direction and continuing the climb straight up.

The sea level thrust is 5540 pounds and the starting weight is 5400 pounds, a thrust loading of a little over one. We estimate 200 mph or 293 ft./sec. as a safe take-off speed and also estimate that the drag forces during the take-off will be such that the actual acceleration will be one "g". Take-off speed will then be attained in 9.1 seconds after a run of 1335 feet at the expense of 297 pounds of fuel.

The upward turn after take-off will be considered in five segments of four seconds each. Each segment or leg has an average climb angle, average drag, average thrust, and consequently an average acceleration. The technique is to estimate a velocity at the end of a segment and from this, calculate the altitude at the end of the segment, the average drag, the average thrust and the average acceleration. Knowing the average acceleration one can then verify the estimated end velocity. Thus each step is essentially a cut and dry process. What we are acctually doing is a kind of a numerical integration of the original differential equation.

Presented in tabular form we have:

Leg	End T.	Av.	Av. W	Av. ($W \sin\psi$)	Av. D	Av. F	Av. a	End V	End Alt.
1	13.1 s	9°	5038 lbs.	787 lbs.	750 lbs.	5545 lbs.	25.6 f/s^2	401 f/s	220 ft.
2	17.1 s	27°	4907 lbs.	2235 lbs.	850 lbs.	5585 lbs.	16.4 f/s^2	466 f/s	1,070 ft.
3	21.1 s	45°	4777 lbs.	3360 lbs.	1025 lbs.	5625 lbs.	8.4 f/s^2	500 f/s	2,450 ft.
4	25.1 s	63°	4647 lbs.	4130 lbs.	1100 lbs.	5690 lbs.	3.2 f/s^2	513 f/s	4,265 ft.
5	29.1 s	81°	4516 lbs.	4440 lbs.	1000 lbs.	5780 lbs.	2.4 f/s^2	523 f/s	6,315 ft.

Thus we arrive at the vertical direction 29.1 seconds after starting, at 6,315 feet, at a speed of 523 ft./sec. at the expense of 950 pounds of fuel.

We shall consider the remaining 60 seconds of burning time in the same manner, only in intervals of six seconds each instead of four seconds.

Leg	End T	Av. W	Av. D	Av. F	Av. a	End V	End Alt.
6	35.1 s	4352 lbs.	1000 lbs.	5900 lbs.	4.0 f/s^2	547 f/s	9,500 ft.
7	41.1 s	4157 lbs.	1000 lbs.	6050 lbs.	6.9 f/s^2	586 f/s	12,850 ft.
8	47.1 s	3961 lbs.	1050 lbs.	6100 lbs.	8.8 f/s^2	639 f/s	16,400 ft.
9	53.1 s	3765 lbs.	1050 lbs.	6250 lbs.	12.1 f/s^2	712 f/s	20,000 ft.
10	59.1 s	3570 lbs.	1000 lbs.	6400 lbs.	16.5 f/s^2	811 f/s	24,600 ft.
11	65.1 s	3376 lbs.	1000 lbs.	6480 lbs.	19.5 f/s^2	928 f/s	29,850 ft.
12	71.1 s	3181 lbs.	1400 lbs.	6600 lbs.	20.4 f/s^2	1050 f/s	35,800 ft.
13	77.1 s	2985 lbs.	1800 lbs.	6650 lbs.	20.8 f/s^2	1174 f/s	42,470 ft.
14	83.1 s	2793 lbs.	1800 lbs.	6800 lbs.	26.5 f/s^2	1330 f/s	50,000 ft.
15	89.1 s	2598 lbs.	1700 lbs.	6900 lbs.	31.0 f/s^2	1516 f/s	58,500 ft.

Figure 9.

<u>Ideal Thrust Coefficient for $\gamma = 1.2$ and $\lambda = 1$.</u>

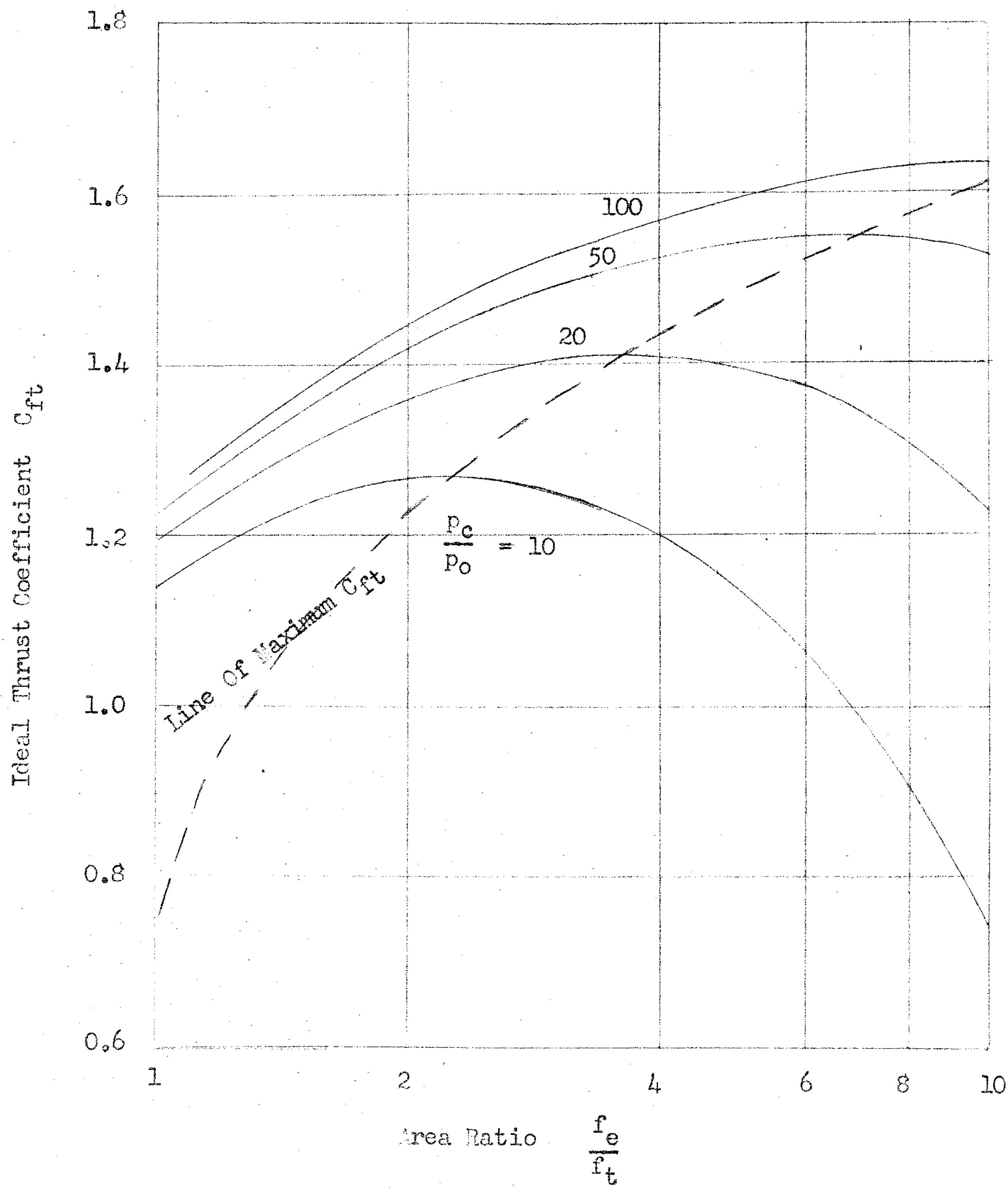


Figure 10

Thrust Available and Drag vs Mach Number

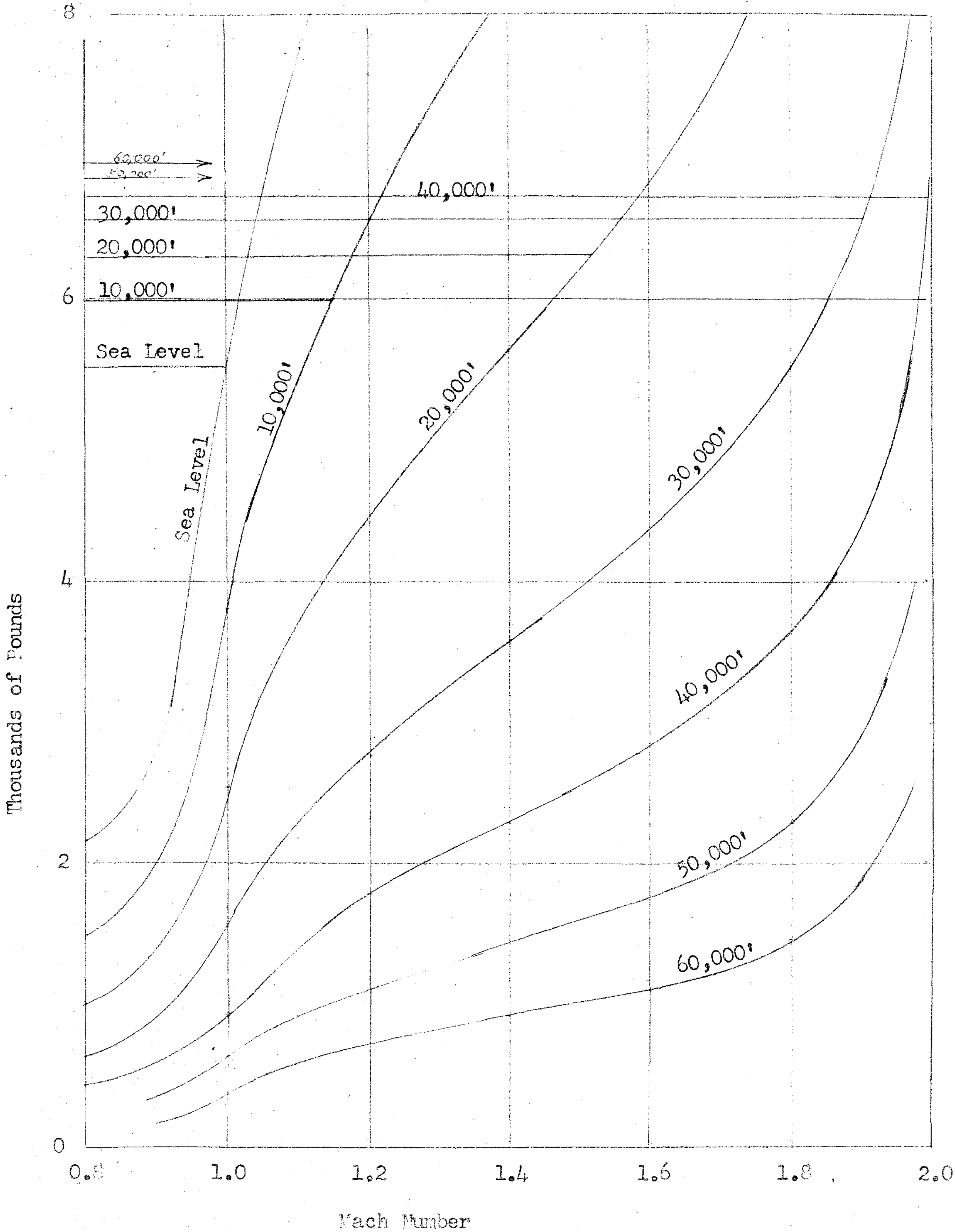


Figure 11

Maximum Mach Number For Unaccelerated Straight And Level Flight

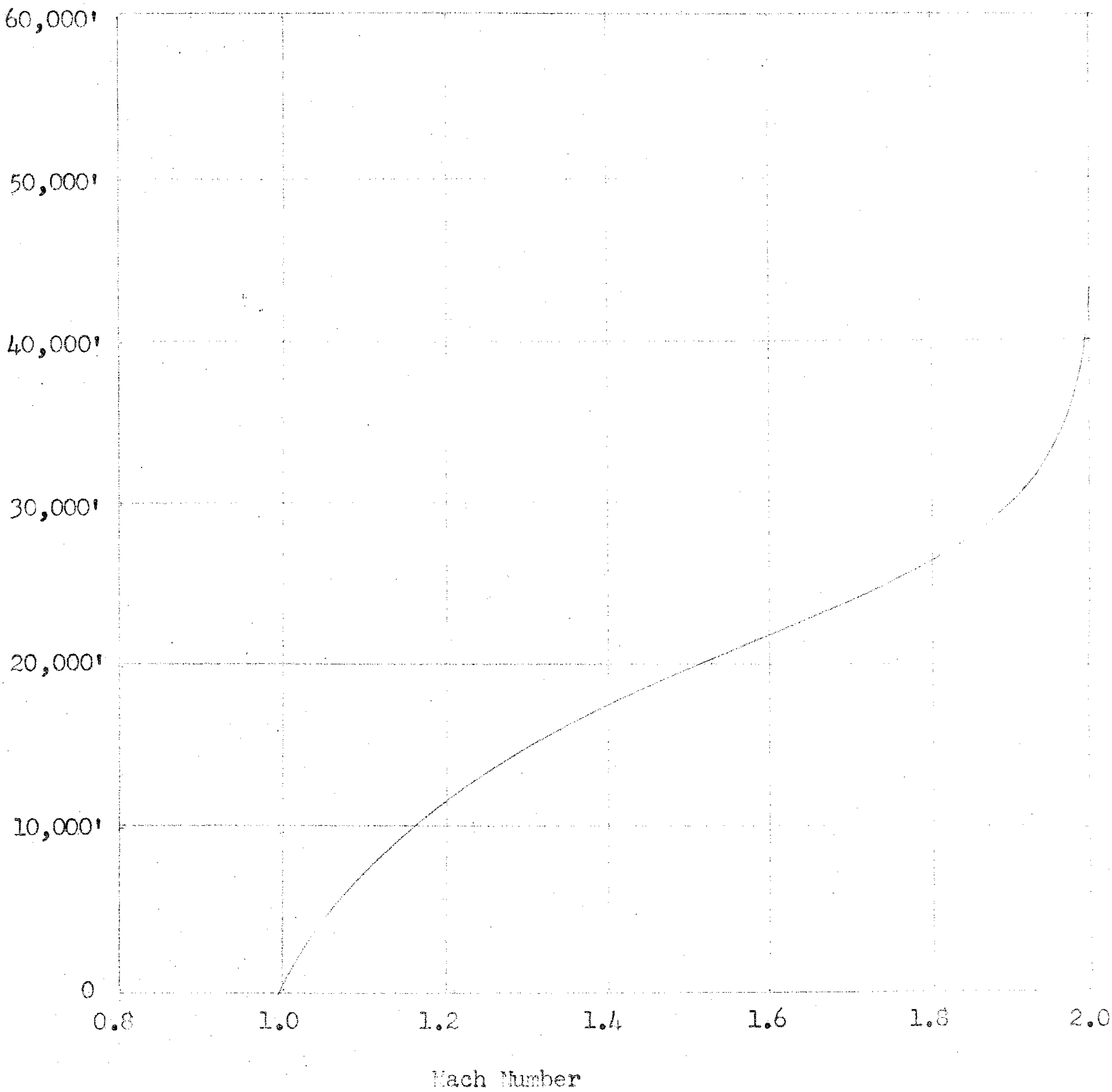


Figure 12.

Glide Angle Thrust vs Jet-off Drag

Glide angle thrust = 2500# x sin ψ

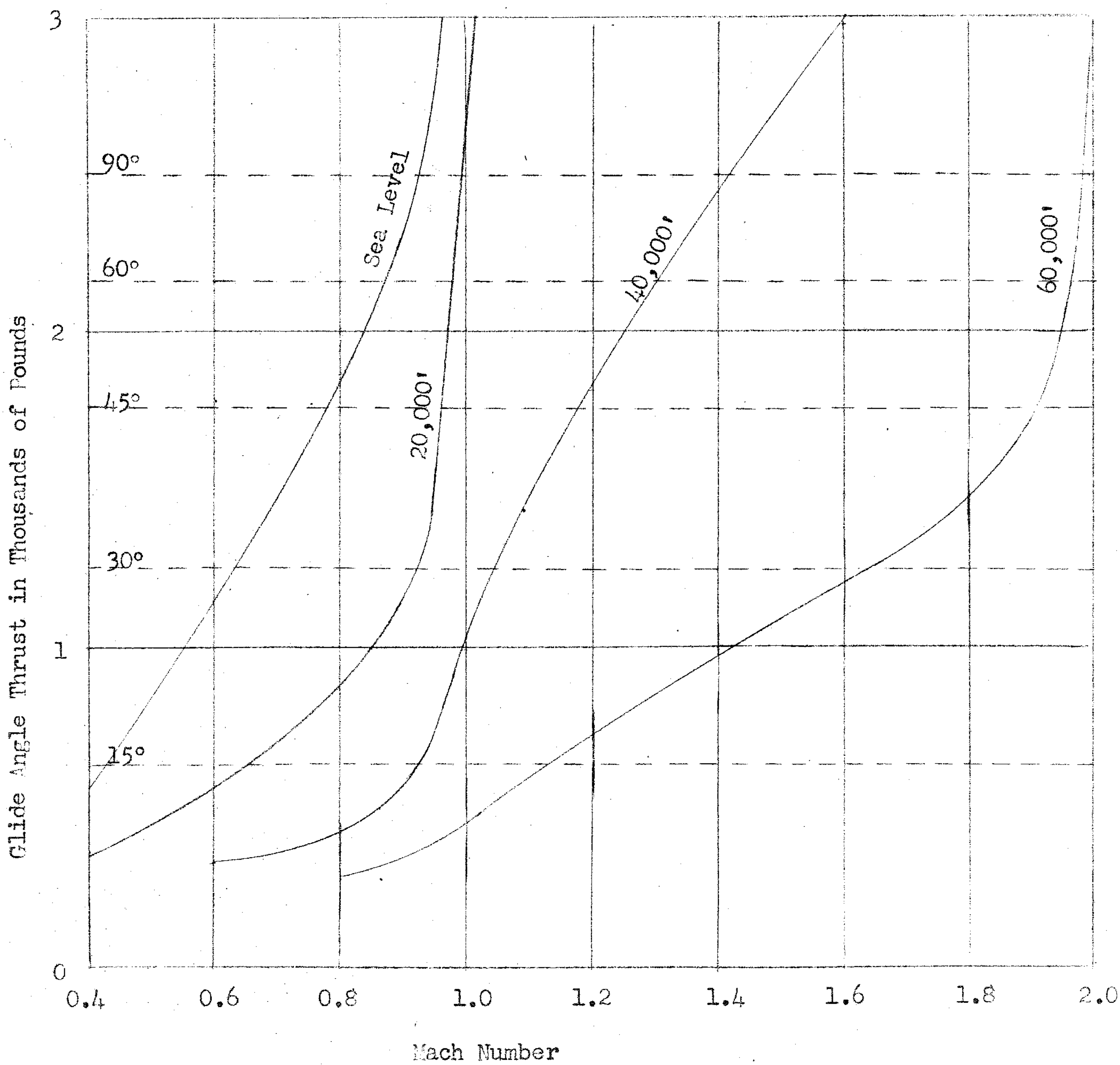
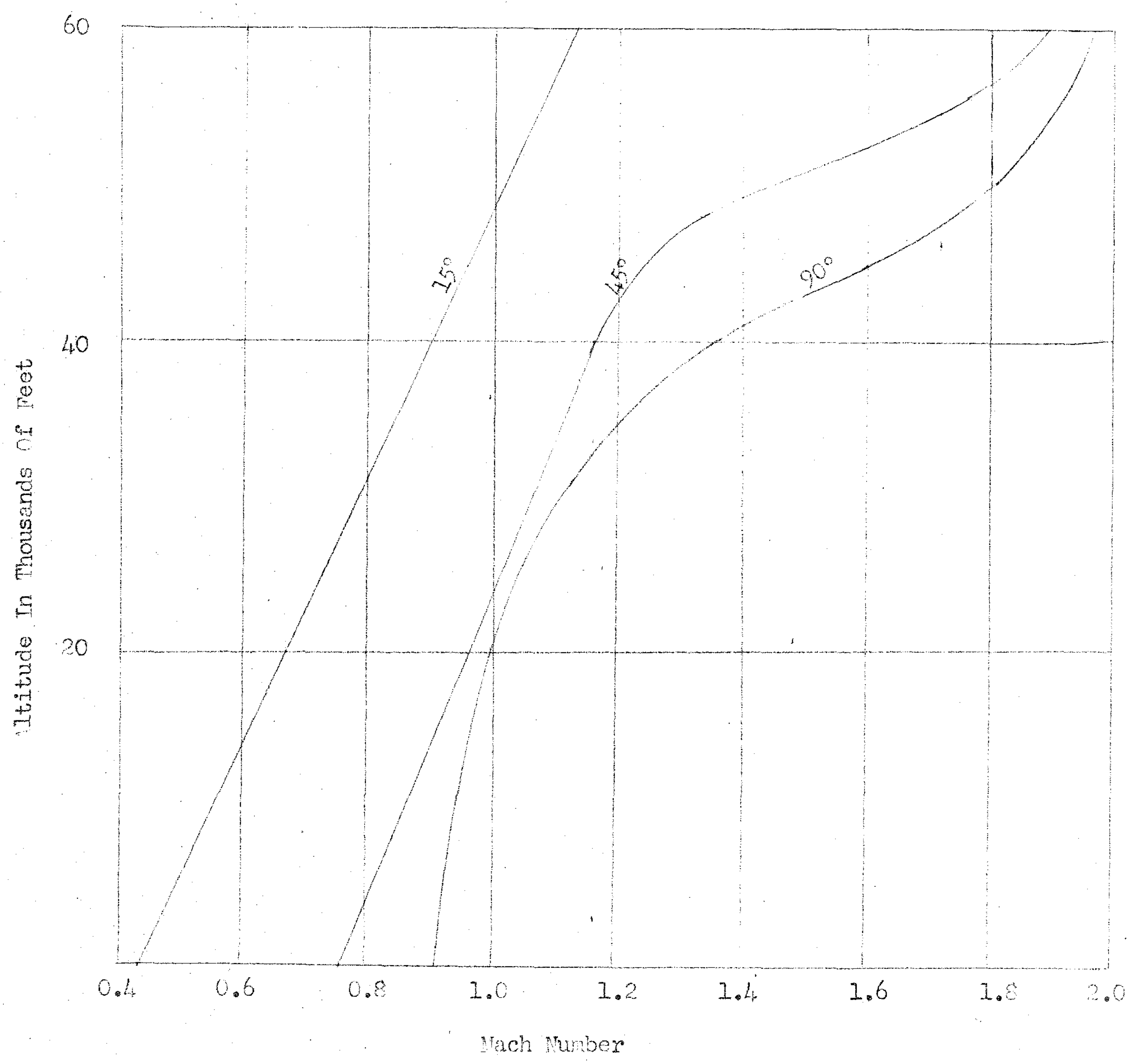


Figure 13

Maximum Mach Number For Unaccelerated Glide



Thus at end of burning we arrive at the height of 58,500 feet at a speed of 1516 ft./sec. Further altitude can be reached by allowing the airplane to coast to a stop in the vertical direction. We will assume that the effect of drag is to increase the gravity effect by 50%. An additional height of 23,800 feet or a total height of 82,300 feet can then be attained.

PART V

TABLE OF SYMBOLS

a	=	acceleration in feet per second per second
α	=	$\sqrt{M^2 - 1}$
α	=	angle of attack
α_e	=	angle of deflection of elevon
AR	=	aspect ratio
b	=	wing span
C.G.	=	center of gravity
C_D	=	drag coefficient
$C_{D_{\pi p}}$	=	drag coefficient based on frontal area
$C_{D_{\pi p}}$	=	pressure drag coefficient based on frontal area
$C_{D_{\pi np}}$	=	nose pressure drag coefficient based on frontal area
C_{D_p}	=	pressure drag coefficient
C_f	=	skin friction coefficient
C_{fa}	=	actual thrust coefficient
C_{ft}	=	theoretical thrust coefficient
C_M	=	moment coefficient
C_{lw}	=	lift coefficient of wing
C_{le}	=	lift coefficient of eleven
C_{lf}	=	lift coefficient of fuselage
e	=	airplane efficiency factor
η	=	a nozzle efficiency factor
F	=	thrust force
f	=	equivalent parasite area

f_e = nozzle exit area

f_t = nozzle throat area

γ = ratio of specific heats

K_d = $\frac{\pi}{8} \times C_d$, a drag coefficient found in ballistic literature

λ = a nozzle efficiency factor

L = left

ℓ_f = distance from C.G. to nose

ℓ_t = distance from C.G. to elevon

ℓ_w = distance from C.G. to center of lift

M = Mach number

P-51 = a fighter type military aircraft

p_c = rocket chamber pressure

p_o = ambient pressure

ρ = density of air in slugs per cubic foot

θ = semi-vertex angle of nose cone

σ = density ratio

S = wing plan area

V = velocity in feet per second

w = wetted area

W = weight of airplane

XXX = the airplane described in Part II

ψ = climb angle

PART VI

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