AN EXPERIMENTAL INVESTIGATION OF HEAT TRANSFER FROM FINE WIRES TO STILL AIR AT LOW DENSITY

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SUMMARY

Heat transfer from electrically heated wires of 0.00025 cm nominal diameter to still air at room temperature has been investigated at pressures ranging from 1 to 0.0076 atmospheres. Three wires, having ratios of length to diameter in the approximate proportion 1:5:10, were tested in the vertical position. Due to time limitations, only the shortest of these was tested in a horizontal position. A check was made to determine whether the results were influenced by the geometry of the enclosing vessel.

For pressures at which the molecular mean free path is smaller than the wire diameter, the results appear to satisfy a relation of a form derived for free-molecule heat conduction. Small departures from this behavior at the higher pressures are attributed to the effects of natural convection. The Nusselt number was found not to be uniquely related to the product of the Grashof and Prandtl numbers, as has been proposed, for values of this product below 10^{-5}.

It was found that, at the lowest pressures reached, solid boundaries located at distances of the order of 3×10^4 diameters from the wire cannot be considered infinitely remote.

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I. INTRODUCTION

In hot-wire anemometry, use customarily is made of King's equation (Ref. 1) relating the Nusselt number and the Reynolds number,

$$Nu = A + B\sqrt{Re} \quad .$$

The constants A and B are determined by calibration, the value for A being obtained by extrapolating test results to zero velocity. The validity of this extrapolation comes into question at very low Reynolds numbers, such as are encountered in boundary layer investigations. For this reason, the limiting case of heat transfer from fine wires to still air is of interest.

Cole and Roshko (Ref. 2), in presenting test results for heat transfer from fine wires at very low Reynolds numbers, extrapolate to $\Delta T = 0$ results obtained at $50 < \Delta T < 200°C$, ΔT being the temperature difference between the wire and the air remote from the wire. The actual behavior of the Nusselt number as the temperature loading tends to zero was made a part of the investigation in order to check the validity of this extrapolation.

Collis and Williams (Ref. 3) have investigated free convection of heat from fine wires to air at atmospheric pressure. On the basis of their experimental results, they propose a correlation between the Nusselt number and the product of the Grashof and Prandtl numbers, extending a similar correlation recommended by McAdams (Ref. 4). Their proposed correlation covers the range $10^{-10} \leq (Gr.Pr) \leq 10^{-2}$, which includes the range covered in this investigation. It is of interest, therefore, to compare the variable-density results with those obtained at atmospheric pressure.

Kennard (Ref. 5) derives an expression for heat transfer between concentric cylinders under "free-molecule" conditions, utilizing the concept of a temperature "slip" or discontinuity at the interface between solid and gas. The slip is described by a characteristic length proportional to the mean free path, and hence inversely porportional to the density. It is assumed that the mean free path is small compared with the radius of the inner cylinder, that the slip is negligible at the outer radius, and that the thermal conductivity is independent of the pressure. Kennard's expression states that

$$\frac{1}{h} = \frac{r_1}{K} \log \frac{r_2}{r_1} + \frac{g}{pK}$$

where h = heat flux per unit area per degree temperature difference

r_1, r_2 = inner and outer radii

g/p = slip distance

K = thermal conductivity

In terms of the Nusselt number Nu = hD/K, this becomes

$$\frac{1}{Nu} = \frac{1}{2} \log \frac{r_2}{r_1} + \frac{g}{pD}$$

The quantity g is stated to be "at most a function of temperature". From its derivation, g should be proportional to the square root of the temperature of the gas in the immediate vicinity of the inner wall, but it is also a function of an accommodation coefficient whose behavior with temperature is not known.

In terms of the dimensionless temperature loading $\theta = (T_W - T_a)/T_a$, the square root of the temperature near the inner wall is approximately $T_W^{\frac{1}{2}} = T_a^{\frac{1}{2}}(1 + \theta)^{\frac{1}{2}}$, which for small values of θ becomes

$T_w^{1/2} = T_w^{1/2}(1 + 1/2\ \theta)$. Since the air temperature T_a was essentially constant during this investigation, $g(T)$ can be replaced by, say, $\phi(\theta)$.

The test configuration selected for this investigation differed from the coaxial cylinders for which Kennard's expression was derived. It was intended to represent cylinders of finite length-to-diameter ratio in an infinite conducting medium. For this case, the term $1/2 \log(r_2/r_1)$ in Kennard's expression required modification. This term represents the zero-slip condition - i.e., the solution for heat conduction in a homogeneous continuum, for which $\nabla^2\theta = 0$. For the two-dimensional case of a cylinder of infinite length in an infinite conducting medium, no solution to the Laplace equation is obtained for non-zero temperature difference, because no steady state is ever reached. Cole and Roshko (Ref. 2) point out, however, that a solution is obtained in the three-dimensional case of a body of finite aspect ratio. For a prolate ellipsoid of large but finite aspect ratio L/D, it is found that the Nusselt number approaches

$$Nu = \frac{2}{\log 2(L/D)} \quad .$$

The solution for a cylinder of finite aspect ratio should differ only by a constant factor from this.

With this modification, Kennard's expression may be rewritten as

$$\frac{1}{Nu} = A + \frac{P_o}{P}\,\phi(\theta)$$

where P_o/P = (dimensionless pressure)$^{-1}$, (atmospheres)$^{-1}$

$A = A_o \log 2(L/D)$

Here, A_o is a function of the geometry of the system comprising the heated wire and the vessel in which it is contained, and $\phi(\theta)$ is an unspecified function whose nature is sought in this experiment.

The above expression takes no account of buoyant effects due to thermal expansion, and hence of natural convection. The effect of natural convection is to increase the Nusselt number above the value predicted on the basis of conduction alone.

From the foregoing considerations, it is to be expected that within the range of validity of Kennard's assumptions the test results should satisfy a linear relation between (1/Nu) and (P_o/P) if conductive heat transfer is the dominant process involved.

In the interest of consistency with the results of others, particularly Collis and Williams (Ref. 3), Nusselt numbers have been evaluated at the arithmetic mean of the temperatures of the wire and the air remote from the wire. The Grashof number and Prandtl number have been evaluated at air conditions remote from the wire. Results were not corrected for end effects due to the metallic supports, nor for the effects of radiation. The end-effect correction is discussed under "Effect of Aspect Ratio" in Part III. A shortage of time forced considerable curtailment of the scope of the experimental work.

II. EQUIPMENT AND METHODS

A. Basic Arrangement

The test configuration is illustrated in Figure 1. The hot-wire probes were mounted above a disc of .051" aluminum alloy sheet which was supported near the bottom of a Pyrex desiccator, a thick-walled glass vessel of approximately 24 cm inside diameter fitted with a tight-sealing cover. A rubber stopper in the center of the cover provided access to the interior for electrical leads, a vacuum pump connection, and a laboratory thermometer. Sealing was accomplished with a Dow-Corning silicone grease intended for hig-vacuum use. The desiccator assembly was placed inside a perforated metal protective enclosure as a safety precaution.

Resistance of the wire was measured with a Wheatstone bridge. The current was determined from the voltage drop across a 0.500-ohm standard resistor, measured with a potentiometer. The pressure difference from the desiccator interior to atmosphere was read from a simple mercury manometer; this reading was subtracted from the local barometric pressure, read from a mercury barometer, to obtain the absolute pressure in the desiccator. Air temperature within the desiccator was measured with a high-precision mercury thermometer inserted through the stopper in the cover. The probes were mounted off the centerline of the desiccator so that the thermometer bulb would not be directly in the path of the column of heated air rising from the wires. A map of the temperature field in the vicinity of a hot wire, presented by Collis and Williams in Reference 3, was employed in this connection.

B. Probe Construction and Calibration

Three hot-wire probes were used in this investigation. Their basic characteristics are as follows:

Probe	Nominal Wire Diam. cm.	Length cm.	(Length)/(diam.)	Material	Resistance ohms (20°C)
A	0.00025	0.366	1440	Pt-10%Rh	132.2
B	0.00025	3.70	14600	Pt	622.7
C	0.00025	1.93	7600	Pt-10%Rh	751.4

All three were made of 0.0001" nominal diameter Wollaston wire of the sort commonly used in hot-wire anemometry. The wires were stretched between the tips of No. 8 sewing needles, to which they were attached by soldering.

In calibration of the probes, the resistance was assumed to be a linear function of the temperature:

$$R = R_o \left[1 + \alpha(T - T_o)\right]$$

Time did not permit determinations of the thermal coefficient of resistivity α. The following values, currently accepted and used in the laboratory, were accepted on faith:

Material	α, $(^{o}C)^{-1}$
Pt	0.0034
Pt-10%Rh	0.00166

The resistance R_o was measured with the probe mounted in a low-speed wind tunnel to assure adequate cooling of the wire. Resistance of the wire was first measured at a current of approximately 5 milliamperes. The current was then decreased, causing a decrease in the resistance. The process was continued until further decrease in current caused no detectable change in the observed resistance, as measured with the Wheatstone bridge. This limiting value was recorded as R_o (total), the total including the leads and the standard resistor in series with the hot wire. The air temperature in the tunnel was read from the same thermometer as used in the desiccator. The combined resistance of the leads and the standard resistor was measured separately by soldering the leads together at the ends connecting to the probe. The same leads were used in calibration and in subsequent testing.

From the information thus obtained, straight-line plots of total resistance versus temperature were prepared for the three probes. Wire temperatures based on resistance were read from these curves.

C. Probe Installation

The thermometer employed in the desiccator required a 75mm immersion length and had necessarily to be situated virtually on the centerline of the desiccator. In order to keep the hot wire reasonably remote from the thermometer bulb and also from the desiccator wall and the aluminum mounting disc, a compromise geometry was selected. The wires were mounted approximately 7.9 cm from the vertical wall, or 4.0 cm from the centerline. The vertical distance from the bulb tip to the mounting disc was 12.7 cm; the probes were installed with their midpoints just half this distance above the disc. The configuration is shown in Figure 1.

It was thought that under this arrangement all solid surfaces were sufficiently remote from the wire to be considered infinitely distant, so that the shape of the bounding surface would have no effect on the heat transfer. As a check on the validity of this assumption, Probe B was tested in the vertical position with and without a surrounding coaxial glass cylinder of 4.75 cm inside radius, 12.7 cm in length, reaching just to the tip of the thermometer bulb. The bulb was just outside the upper edge of the cylinder.

D. Method

The investigation was intended to determine the effects on the Nusselt number of the following parameters:

1. Temperature loading
2. Air density
3. Length-to-diameter ratio of the wire
4. Orientation of the wire with respect to gravity
5. Geometry of the bounding surface

Since the wire diameter cancels out of the expression for the Nusselt number, the diameter was not varied. For the same reason, no effort was made to check the actual wire diameter against the nominal size given by the manufacturer. A 10% error in the assumed size of the wire would result in a 14% error in the logarithm of the Grashof number, but this error would be constant for each probe.

As originally contemplated, the program of investigation called for testing three or more probes, each with a different ratio of length to diameter, under the following conditions:

1. Temperature loadings up to 100°C (temperature difference between wire and air), with particular emphasis on the behavior of the Nusselt number as the temperature difference tends to zero.
2. Pressure variation, by factors of 2, from one atmosphere down to the lowest attainable with the available pump.
3. Horizontal and vertical orientations.
4. Some systematic variation in geometry of the bounding surfaces.

Shortage of time forced some curtailment of these aims. The lower limit on pressure (about 5.8 mm Hg abs.) was imposed by limitations in measurement with the mercury manometer; time did not permit the refinement of the apparatus necessary for high vacua. The fine wire of Probe B was inadvertently parted during installation in the horizontal position, at a time when the supply of that particular wire size was temporarily exhausted. Here again, shortage of time prevented the testing of Probe B in the horizontal position. Probe C was tested to check an apparent discontinuity in the data from Probe B. Data were taken only to the extent necessary for this purpose.

In performing the testing at a given pressure, readings commenced with the measurement of resistance at the minimum available current (all variable resistance fully in). This resistance usually was found to correspond to a wire temperature only a few tenths of a degree above the observed air temperature. The corresponding current was then determined by balancing the potentiometer, which was connected to read the voltage drop across the standard resistor. The resistance setting of the Wheatstone bridge was then increased by a convenient increment, and the current adjusted to bring the bridge into balance. The new value of the current was determined after balancing the potentiometer as before. The process was repeated until the resistance corresponding to the maximum desired temperature loading was obtained. It usually proved necessary to re-check the balance of the bridge circuit after reading the potentiometer, and vice versa. This often led to a very tedious process of checking back and forth in an effort to balance both circuits simultaneously.

The sequence of test configurations was as follows:

1. Probe A, horizontal
2. Probe B, vertical
3. Probe B, vertical, with surrounding cylinder
4. Probe A, vertical
5. Probe C, vertical

Starting with Probe B, vertical, the practice was adopted of increasing the temperature loading as described above, up to the maximum, and then decreasing it stepwise back to the minimum-current limit as a check on repeatability of results.

E. <u>Accuracy</u>

1. Pressure.

The errors in pressure measurement were independent of the magnitude of the pressure measured, and so the percent error is inversely proportional to the pressure. The principal source of error lay in the reading of the U-tube manometer, in which the two levels were read against a meter stick. No mirror, hairline, or other reading aid was fitted to the manometer. Under the circumstances, the estimation of tenths of a millimeter in the height of the mercury meniscus is surely subject to an error of at least $\pm$0.1 mm, and quite probably more. For the difference in level between the two columns (requiring two independent readings), an error of $\pm$0.3 mm might reasonably be expected.

In reading the mercury barometer, some difficulty was experienced in judging the exact point at which contact was made between the pointer in the well and the surface of the mercury. The values of the ambient barometric pressure recorded were averages of repeated readings, including in every case at least three settings of the pointer in the reservoir and three or four readings of the column height for each such setting. Even so, it is considered likely that the barometric pressure might be in error by $\pm$0.1 mm Hg.

The total error in the absolute pressure is thus estimated to be $\pm$0.4 mm Hg. At a pressure of one atmosphere, this is only $\pm$0.05%, which is negligible for the purposes of this investigation. However, at 5.8 mm Hg abs. (the lowest pressure for which results are reported) the error amounts to about ±7%.

2. Temperature.

The thermometer, with scale graduations of 0.2°C, could be read with ease to the nearest tenth of a degree, and it always read very nearly the ambient temperature in the room. Of its total scale range from -10 to +110°C, only the portion in the immediate neighborhood of 20°C was utilized. The thermometer readings, then, are considered to be in error by no more than ±0.05°C, neglecting instrument error (which should be constant over the small range involved).

The possible error in the wire temperature as determined from the calibration curve includes the effects of errors in R_o, T_o, and the coefficient α, as well as errors in plotting and reading the calibration curves. G. T. Skinner, in Reference 6, reports variations of the order of 1% in the observed coefficient α for platinum wire of the type used in Probe B. Thus a systematic error of 1% is to be expected in the slope of the calibration curve for each probe, resulting in a 1% error in the temperature loading, consistent for each probe.

Random errors in the readings for any given run are estimated to be equivalent to an error of ±0.2° in the temperature loading, at least for Probes B and C and for the tests of Probe A in the vertical position. The earlier runs (Probe A, horizontal) show considerably more scatter than this. This error would result in a scatter in the Nusselt number inversely proportional to the temperature loading (±2% at 10°), in addition to that due to the use of a slide rule in data reduction.

3. Resistance.

The effect of random error in the resistance measurement is included in the estimate above, since an error in resistance is equivalent to an error in temperature loading. At low values of current the sensitivity of the bridge circuit decreased, so that errors of -0.1 ohm could have occurred in R_o. At higher currents, corresponding to higher temperature loadings, erratic fluctuations in the galvanometer reading made it difficult to balance the bridge and the potentiometer simultaneously. The effect of this unsteady behavior on the resistance and ultimately on the Nusselt number is best judged by the degree of scatter in the data at the higher temperature loadings (say, above 20^o).

Gross error in the bridge resistance is suspected in the case of Probe B. The Nusselt numbers showed a sharp discontinuity in a range of bridge settings between 638 and 650 ohms at all pressures. There is no consistency to the corresponding readings of the potentiometer. The bridge was checked against a decade resistance box in 2-ohm increments from 600 to 680 ohms; its readings were perfectly consistent over the range. Probe C, half the length of Probe B and of a different resistance, was constructed and tested with great care over a range of temperature loadings bracketing the apparent discontinuity. The Nusselt number was found to be smooth and continuous over the range. It was concluded that the discontinuities were the result of temporary or intermittent malfunctioning of the bridge circuit, and the corresponding data points were rejected at being grossly in error. They have been omitted from Figure 6.

The effect of random errors in resistance measurement (i.e., in balancing of the bridge) on the input I^2R can be neglected, since errors of the order of 0.1 ohm are beyond slide-rule accuracy for the resistances employed.

4. Potentiometer.

The accuracy of the potentiometer exceeded the requirements of this investigation. Its contribution to scatter in the test results can be neglected.

III. RESULTS AND DISCUSSION

A. Effect of Pressure

Data were taken at the discrete values of pressure listed in the following table. The air temperature remote from the wire was essentially room temperature, 294 ± 4 °K. The table also lists the corresponding values of the molecular mean free path and its ratio to the wire diameter. The mean free path for air at 15°C and 760 mm Hg abs was obtained from p. 149 of Reference 5.

Nominal Pressure (atm.)	Actual Pressure mm Hg abs	$\frac{P_{mm}}{760}$	Mean Free Path, cm.	$\frac{\text{(m.f.p.)}}{\text{wire diam.}}$
1	743 ± 2	.980	6.67×10^{-6}	0.027
1/2	372	.490	1.33×10^{-5}	0.053
1/4	186	.245	2.67×10^{-5}	0.11
1/8	93	.123	5.31×10^{-5}	0.21
1/16	46.5	.0612	1.07×10^{-4}	0.43
1/32	23.3	.0306	2.14×10^{-4}	0.86
1/64	11.6	.0153	4.27×10^{-4}	1.7
1/128	5.8	.0076	8.6×10^{-4}	3.4

Plots of (1/Nu) vs. (P_o/P) for constant temperature loadings are presented in Figures 2 and 3. The relation $(1/Nu) = A + (P_o/P)\,\phi(\theta)$ indicates that straight-line plots should be obtained. This is seen to be reasonably true of Probe A (L/D = 1440) in both horizontal and vertical positions for pressures above about 0.02 atmospheres ($P_o/P = 50$). At lower pressures, the curves are concave upward, indicating that the heat transfer is less than that predicted by Kennard's expression.

The slopes of the curves, $\partial(1/Nu)/\partial(P_o/P)$, give the values of the function $\phi(\theta)$, while their intercepts on the ordinate axis give the magnitude of the quantity A.

The curves for Probe B (L/D = 14,600) exhibit unusual behavior. In the absence of the vertical glass cylinder which was later placed around the wire, the curve of (1/Nu) vs. (P_o/P) breaks sharply upward in the neighborhood of 1/64 atmosphere, indicating an abrupt decrease in heat transfer. With the cylinder in place, the resulting curve is more nearly linear than that for Probe A, which was not tested with the cylinder. It is plain that the nature of the relation between Nusselt number and density is profoundly influenced by the boundary geometry when the density is low.

The curves for Probe B with and without the surrounding cylinder are complicated by inflection points in the neighborhood of 0.05 atm. (P_o/P = 20). The approximation of these curves by straight lines of (1/Nu) vs. (P_o/P) in the pressure range above 0.05 atm. is thus likely to yield misleading values of the slopes, although the intercepts of the curves on the ordinate axis can be obtained with reasonable accuracy.

Probe C (L/D = 7600) was tested only at pressures of 1 and 1/4 atmospheres, and only in the vertical position. All that can be said of it from the standpoint of pressure effect is that the slope $\partial(1/Nu)/\partial(P_o/P)$ in the neighborhood of one atmosphere is intermediate between those of Probes A and B (as might be expected from its intermediate aspect ratio). The ordinate intercept for this probe at $\Delta T = 10^\circ C$ occurs at 1/Nu = 3.55, approximately (3.40 at $\Delta T = 80^\circ C$). In the absence of further information, it cannot be said whether Probe C exhibits the inflection and the sharp upward break shown by Probe B.

B. Effect of Temperature Loading

Nusselt numbers are presented plotted against the temperature loading (temperature difference between wire and air) for constant values of pressure in Figures 4 through 7. Data are missing for Probe B (without cylinder) at temperature loadings above about 16°C, for pressures of 1 atm. and 1/4 atm.; those curves have been extrapolated on the basis of the corresponding curves for Probes A and C, proportionality being maintained in the rate of change of Nusselt number with temperature. The curves have been faired across the gaps apparently caused by a defect in the bridge circuit.

The Nusselt number is observed to break sharply downward as the temperature loading tends to zero, the break occurring in the vicinity of $\Delta T = 10°C$. Thus extrapolation of the curves from higher temperature loadings down to $\Delta T = 0$ does not appear warranted.

For temperature loadings in excess of 10°C, the slopes $\partial Nu/\partial \theta$ are seen to increase with pressure. Zero slope occurs at around 1/8 atmosphere; at higher pressures, positive slopes are observed, and at lower pressures, negative slopes. The slopes appear to tend to infinity as the temperature loading approaches zero.

From the curves of $(1/Nu)$ vs. (P_o/P) and from the information above, some statements can be made regarding the function $\phi(\theta)$. For Probe A, within the range of linearity of $(1/Nu)$ vs. (P_o/P) and for temperature loadings above 10°C, the measured slopes $\partial(1/Nu)/\partial(P_o/P)$ exhibit a linear variation with the temperature loading. Since these slopes are equal to the function $\phi(\theta)$, this implies that $\phi(\theta) = a_o + a_1 \theta$ within this range. In the case of Probe A, the following numerical values are obtained:

Horizontal: $\phi(\theta) = 0.060 + 0.059\ \theta$

Vertical: $\phi(\theta) = 0.062 + 0.076\ \theta$

By differentiation of the expression

$$\frac{1}{Nu} = A + (P_o/P)\ \ \phi(\theta)$$

one obtains

$$\frac{\partial Nu}{\partial \theta} = - \frac{(P/P_o) \cdot b}{(A \frac{P}{P_o} + \phi)^2}$$

which is inherently negative for positive values of b. Positive values of b are observed for all three probes. However, it has been remarked that at pressures well within the range of validity of the above expression, the slopes are observed to change from negative to positive as the pressure increases. This discrepancy is attributed to the effect of natural convection, which increases with increasing density and temperature loading.

In order to fit the observed behavior of Nusselt number with temperature loading below 10°C, the function $\phi(\theta)$ must be modified by the addition of a term in θ^{-1}. This causes the Nusselt number to vanish and its slope to become infinite as $\theta \rightarrow 0$. For Probe A, the data indicate a coefficient of approximately 0.002 for this term, which results in a negligible correction to the Nusselt number at $T > 10°C$.

It can be seen from Figures 1 and 2 that the ordinate intercepts of $(1/Nu)$ vs. (P_o/P), and hence the values of the quantity A in the conduction equation, are not entirely independent of the temperature loading, although the variation is slight. The tendency for the Nusselt number to increase with temperature loading is considered a natural convection effect.

An effect of boundary geometry on the slope $\partial Nu/\partial \theta$ is apparent in Figure 5, where Nusselt numbers for Probe B with and without its surrounding glass cylinder are compared. The effect disappears as the temperature loading is increased; at the higher loadings, the curves with and without the cylinder tend to become parallel.

Summarizing the effects of temperature and of pressure, it may be said that the data appear consistent (at pressures above about 0.02 atm.) with the relation

$$\frac{1}{Nu} = A + \frac{P_0}{P} \phi(\theta)$$

in which $\phi(\theta) = a_{-1}\theta^{-1} + a_0 + a_1\theta$ $(a_1 > 0)$. The coefficients a_i appear relatively insensitive to aspect ratio and are not greatly affected by orientation. The term A is affected only slightly by θ.

C. Effect of Aspect Ratio

The data as presented have not been corrected for end effects. The needle tips to which the hot wires are soldered are massive compared with the wires themselves. They thus act as heat sinks at the ends of the wires. The wire temperatures measured are mean temperatures averaged over the length of the wire; the actual temperature distribution along the wire was shown by King (Ref. 1) to be approximately a hyperbolic cosine. This causes the measured Nusselt numbers to exceed the true values. Kovásznay and Törmarck, in Reference 7, outline a linearized method for computing the required correction. This must be applied before the effects of aspect ratio can be discussed.

The end-effect correction clearly is greatest for the shortest wire. It is found also to increase as the density of the surrounding air decreases, this variation being implicit in an inverse dependence on the square root of the measured Nusselt number. For the shortest wire (Probe A, L/D = 1440) at the lowest pressure (1/128 atm.), the correction is approximately 10%; it changes only slightly with the temperature loading over the range involved. For the same wire at atmospheric pressure, the correction is approximately 5%. For Probe C (L/D = 7600) at atmospheric pressure the correction is 2%. For Probe B (L/D = 14,600) at one atmosphere, the correction is negligible; at 1/128 atmosphere, the measured Nusselt number for the probe without the surrounding cylinder is high by 4%. These corrections have the effect of increasing the nonlinearity of the plots of (1/Nu) vs. (P_o/P).

Data are available for three values of the length-to-diameter ratio of the wire, but only for wires oriented vertically.

The solution for the prolate ellipsoid indicates that the zero-slip term A in the conduction equation might be expected to vary as log 2(L/D). This term is given by the intercepts of the curves (1/Nu) vs. (P_o/P) on the ordinate axis, $P_o/P = 0$. Agreement of the test data with predictions based on proportionality between (1/Nu) and log 2(L/D) is shown in the following table, in which Probe A has been chosen arbitrarily as the basis for comparison. The curves for 10°C temperature loading (vertical position) were selected in order to minimize natural-convection effects. The measured intercepts are corrected for end effects before the comparison is made.

Probe	L/D	log 2(L/D)	$(1/Nu)_o$ Measured	$(1/Nu)_o$ Corrected	$(1/Nu)_o$ Predicted
A	1,440	7.96	2.85	3.00	--
C	7,600	9.64	3.55	3.62	3.62
B	14,600	10.27	3.80	3.80	3.86

Although the intercepts, being obtained by extrapolation, are subject to some uncertainty, these results certainly are not inconsistent with a relation of the form

$$A = A_o \log 2(L/D)$$

in which A_o may be a function of orientation and of boundary geometry.

D. Effect of Orientation

Data are available only for Probe A in both horizontal and vertical positions. For this wire the Nusselt numbers are about 6% lower in the vertical position than in the horizontal position. No consistent variation in this ratio with either pressure or temperature loading is apparent from the plot of Figure 5. The curves for horizontal and vertical orientations at each value of the pressure tested appear sensibly parallel.

The coefficients a_o and a_1 in the function $\phi(\theta) = a_o + a_1\theta$ show some change with orientation of the wire:

Position	a_o	a_1
Horizontal	0.060	0.059
Vertical	0.062	0.076

E. Effect of Boundary Geometry

Boundary geometry was varied only for Probe B in the vertical position. The glass cylinder placed around the probe in this test formed a "chimney", resting flush against the mounting plate at the bottom and open at the top. It was intended to yield only a qualitative answer to the question of whether boundaries situated at distances of the order of 3×10^4 wire diameters from the probe could be considered infinitely remote. They can not.

At pressures of the order of an atmosphere (say, down to 0.05 atm.) the presence of the surrounding cylinder was found to decrease the Nusselt number, and hence the heat transfer rate. The maximum decrease, 11%, was noted at one atmosphere and a temperature loading of 10°C. At pressures below 0.05 atmosphere, the presence of the cylinder increases the heat transfer rate by a factor that increases rapidly with decreasing pressure.

At the lowest pressure plotted (roughly 1/128 atm.) the Nusselt numbers with and without the surrounding cylinder bear a ratio of over 7:1.

Plotted as (1/Nu) vs. (P_o/P) the curve for Probe B with surrounding cylinder is more nearly linear than any of the other data. The remarkable behavior of the curve for the same probe without the cylinder has been mentioned previously. This emphasizes the profound effect of boundary geometry on the heat transfer from cylinders of high aspect ratio to air at low density. It suggests further the desirability of systematic variation of a simplified boundary geometry, such as coaxial cylinders, in any subsequent investigation of the problem.

F. <u>Correlation with the Grashof Number</u>

McAdams, (Ref. 4), presents a compilation of data from many sources on natural convection from horizontal cylinders. In presenting the data, the Nusselt number is correlated with the product of the Grashof free-convection parameter and the Prandtl number. The former is defined as

$$Gr = \frac{D^3 \rho^2 g \beta \Delta T}{\mu^2}$$

where β = coefficient of thermal expansion

= 1/T for ideal gas

The Prandtl number is defined as $Pr = \frac{c_p \mu}{k}$

The information is plotted in logarithmic coordinates and covers a range of $\log_{10}(Gr \cdot Pr)$ from -5 to $+9$. The correlation is quite good. Coordinates are given for a "recommended" curve extending down as far as $\log_{10}(Gr \cdot Pr) = -4$.

Collis and Williams (Ref. 3) present a similar plot for infinitely long wires, extending down to $\log_{10}(Gr \cdot Pr) = -10$. On the basis of their test results, they propose a single-valued correlation between the Nusselt number and the product of the Grashof and Prandtl numbers, for use at low values of the latter. They have faired their curve at its upper end into that recommended by McAdams, in the neighborhood of log(Gr · Pr) = 0. Their tests were performed at atmospheric pressure. Nusselt numbers were based on the arithmetic mean of the temperatures of the wire and of the air remote from the wire. The Grashof and Prandtl numbers were evaluated at the conditions remote from the wire.

Probe A has been selected for purposes of comparison with the dimensionless correlations proposed by Collis and Williams (Ref. 3). Although the wire was comparatively short (L/D = 1440), it was the only one tested in the horizontal position. The resulting plot is presented in Figure 7. The values of $\log_{10}(Gr \cdot Pr)$ extend from around -5 to somewhat below -10. The Nusselt number has been evaluated at the arithmetic mean temperature and the Grashof and Prandtl numbers at the temperature remote from the wire. Thus the only differences between these results and those of Collis and Williams (Ref. 3) should be those due to finite aspect ratio and to variable density.

Below $\log_{10}(Gr \cdot Pr) = -5$, the data for Probe A fan out into a family of curves, a separate curve being obtained for each value of the temperature loading. The shapes of these curves bear no apparent relation to the correlation proposed by Collis and Williams (Ref. 3). It appears, then, that the value of the Nusselt number corresponding to a given Grashof number (the Prandtl number may be considered constant) depends on the manner in which the Grashof number is achieved, and hence that the two parameters are not uniquely related in this range. On the other hand, the fact that the separate curves tend to merge into a single curve near $\log_{10}(Gr \cdot Pr) =$

-5 may indicate that a unique relationship exists above this value. This cannot be concluded definitely from these data because $\log_{10}(Gr \cdot Pr) = -5$ occurred only at a pressure of one atmosphere.

REFERENCES

1. King, L.V.: "On the Convection of Heat from Small Cylinders in a Stream of Fluid", Phil. Trans. Roy. Soc., A, Vol. 214 (1914), pp. 373-432.

2. Cole, J., and Roshko, A.: "Heat Transfer from Wires at Reynolds Numbers in the Oseen Range", Heat Transfer and Fluid Mechanics Inst., 13-23, University of California at Berkeley (1954).

3. Collis, D.C., and Williams, M.J.: "Free Convection of Heat from Fine Wires", Aero. Res. Lab. (Australia) Aerodynamics Note 140 (Sept. 1954).

4. McAdams, W.H.: "Heat Transmission", 2nd Edition, McGraw-Hill, N.Y. (1942), p. 243.

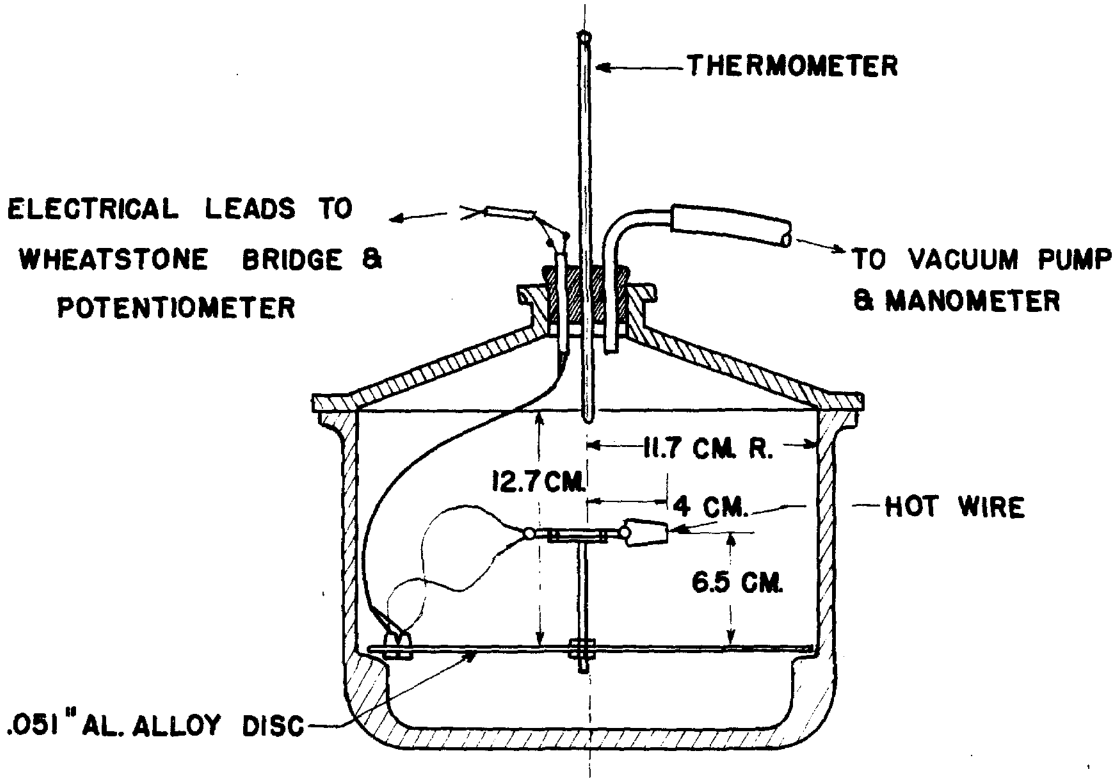
5. Kennard, E.H.: "Kinetic Theory of Gases", McGraw-Hill, N.Y. (1938), p. 323.

6. Skinner, G.T.: "Mean-Speed Measurements in Two-Dimensional, Incompressible, Fully-Developed Turbulent Channel Flow", Ae.E. Thesis, Calif. Inst. of Tech. (1951).

7. Kovásznay, L., and Törmarck, S.: "Heat Loss of Hot-Wires in Supersonic Flow", Johns Hopkins University, Bumblebee Series Report No. 127 (April 1950), pp. 12-13.

EQUIPMENT LIST

Wheatstone Bridge:	Shallcross Kelvin-Wheatstone Bridge, Model 638-2, Serial No. 8725
Potentiometer:	Leeds & Northrup No. 7552, Serial No. 1055661
Galvanometer:	Leeds & Northrup Catalogue No. 2420-B, Serial No. 430415
Standard Cell:	Eppley Laboratory, Inc., Catalogue No. 100, Serial No. 378365 (1.01894 v.)
St'd. Resistor:	Shallcross Mfg. Co., 1 Int. ohm - 0.02% (two in parallel)
Thermometer:	Cenco 19245B Extreme Precision, -10 to 110°C, 1/5° graduations
Hg Manometer:	Of local fabrication
Resistance Box:	Gray Instrument Co. No. 8320
Vacuum Pump:	Cenco Megavac (1/2 HP Craftsman motor)
Wollaston Wire:	Sigmund Cohn Mfg. Co.



INSTALLATION OF PROBE
IN DESICCATOR ASSEMBLY

FIGURE 1

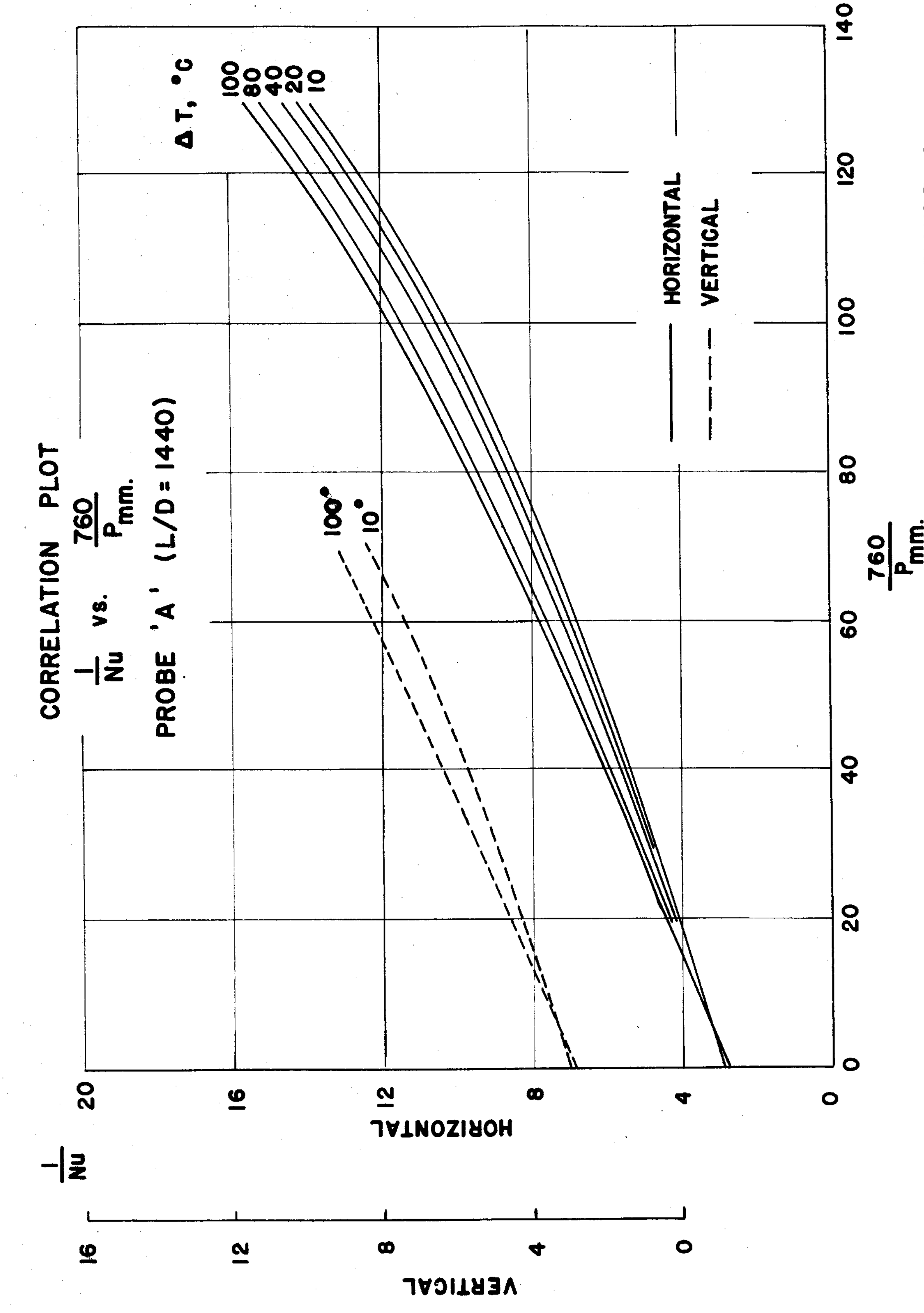

CORRELATION PLOT
1/Nu vs. 760/P mm.
PROBE 'A' (L/D = 1440)
ΔT, °C
100
80
40
20
10
100°
10°
HORIZONTAL
VERTICAL
1/Nu
HORIZONTAL
VERTICAL
0
4
8
12
16
20
0
20
40
60
80
100
120
140
760/P mm.

FIGURE 2

CORRELATION PLOT FOR PROBE 'B' (VERTICAL)
WITH AND WITHOUT SURROUNDING CYLINDER
(L/D = 14,600)

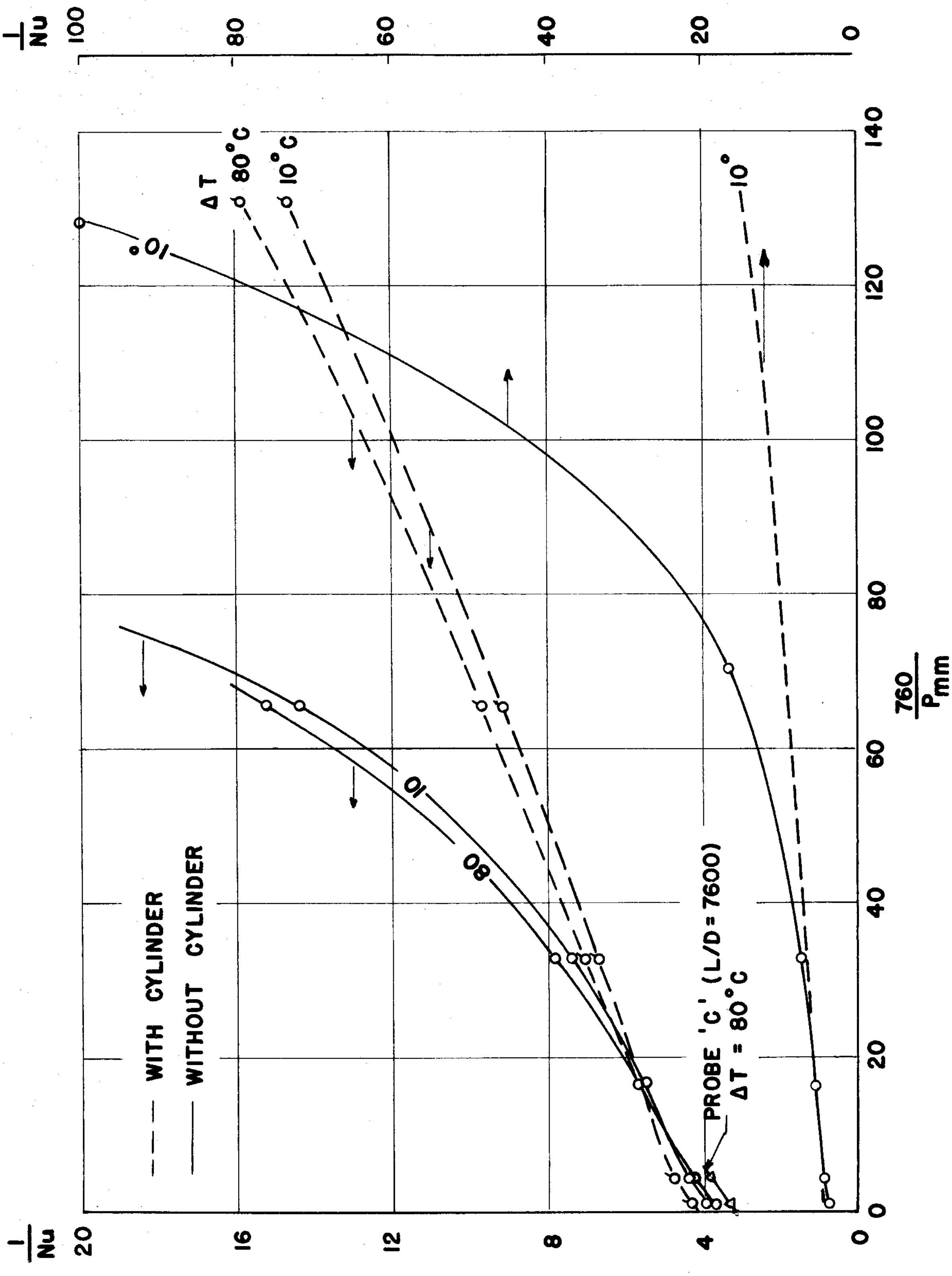


FIGURE 3

VARIATION IN NUSSELT NUMBER WITH TEMPERATURE LOADING
AT VARIOUS PRESSURES
PROBE 'A' (HORIZONTAL). L/D=1440

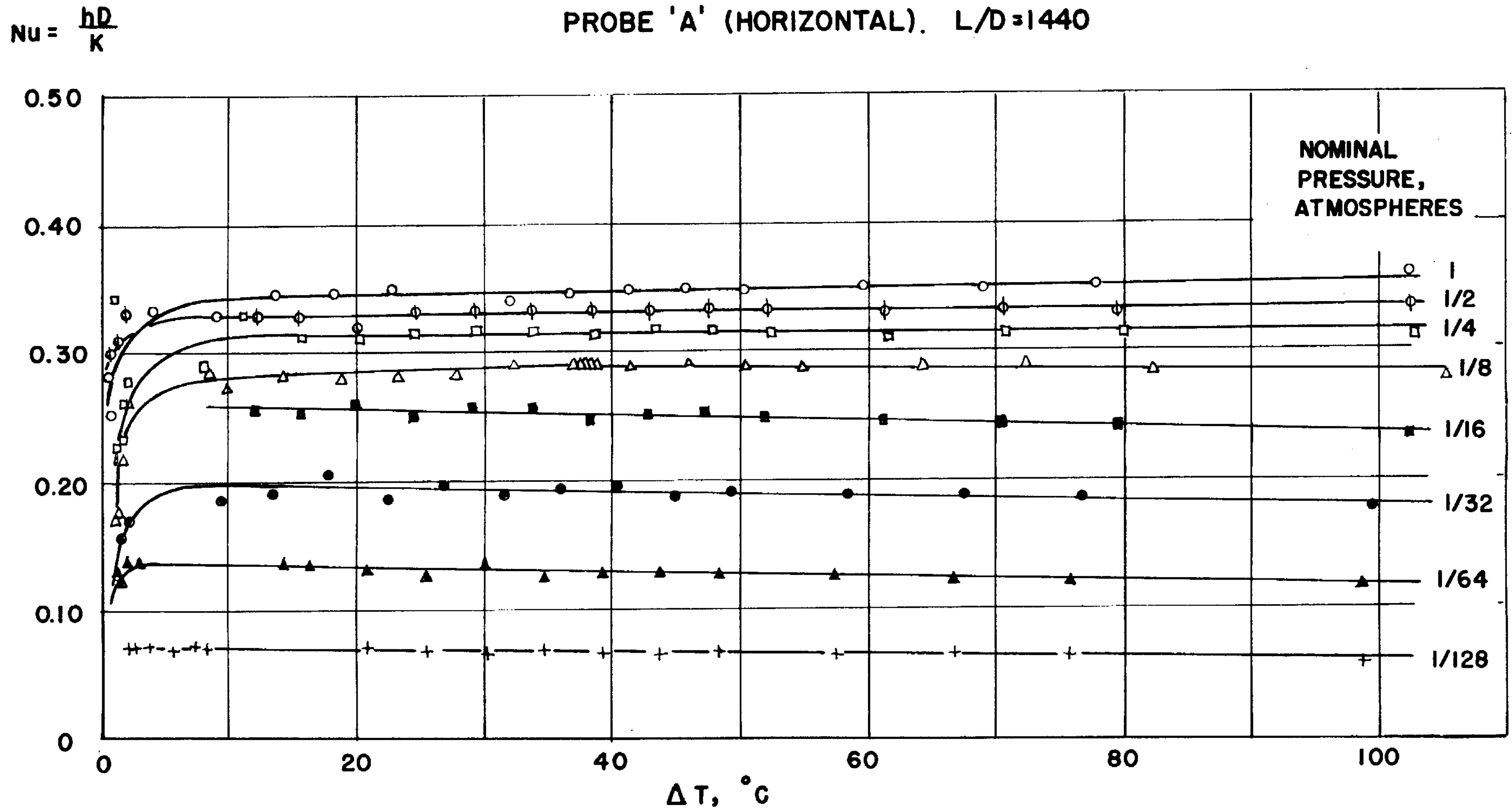


FIGURE 4

NUSSELT NUMBER VARIATION WITH TEMPERATURE LOADING
AT VARIOUS PRESSURES
SHOWING EFFECT OF ORIENTATION
PROBE 'A' (L/D = 1440)

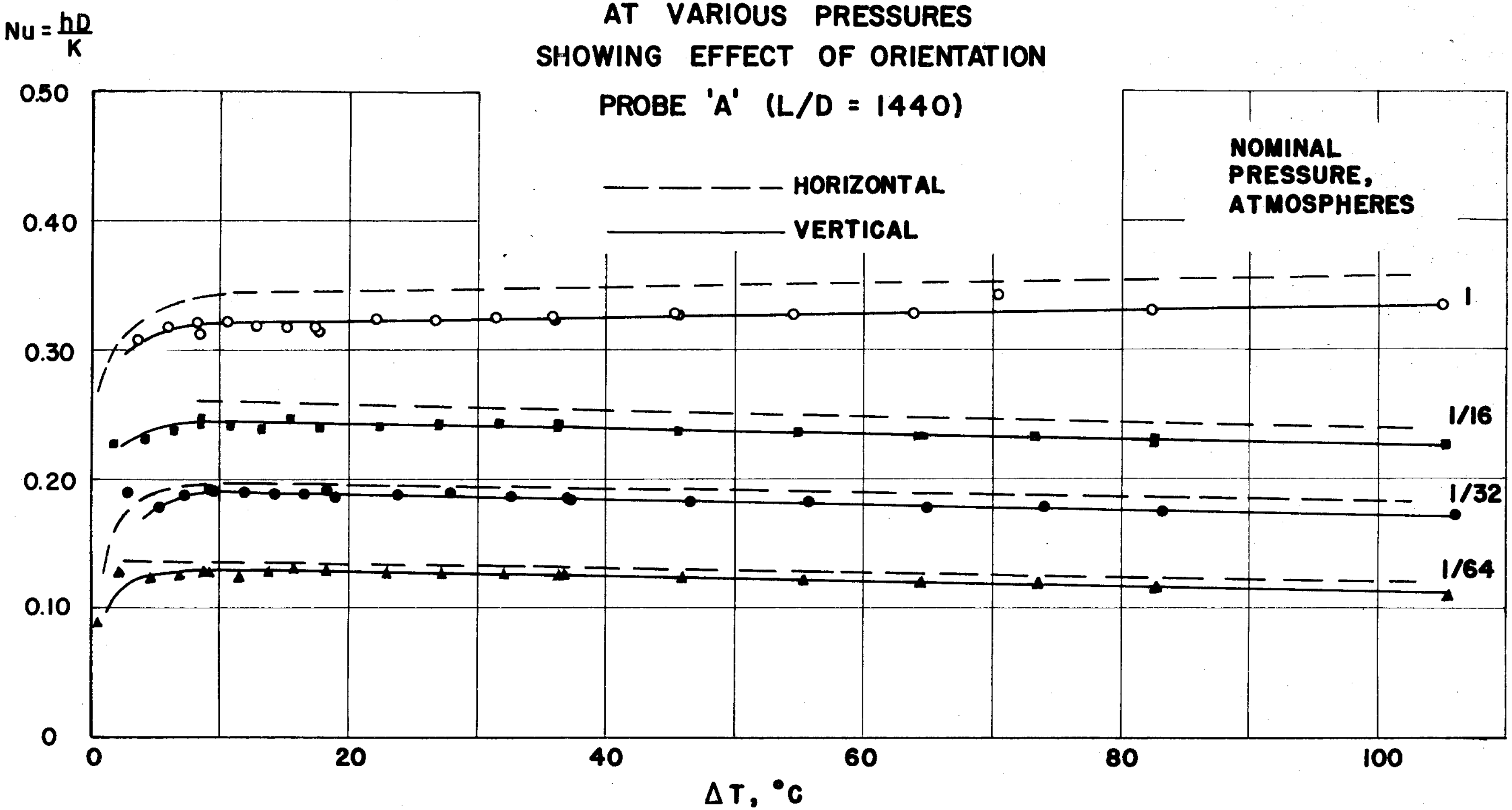
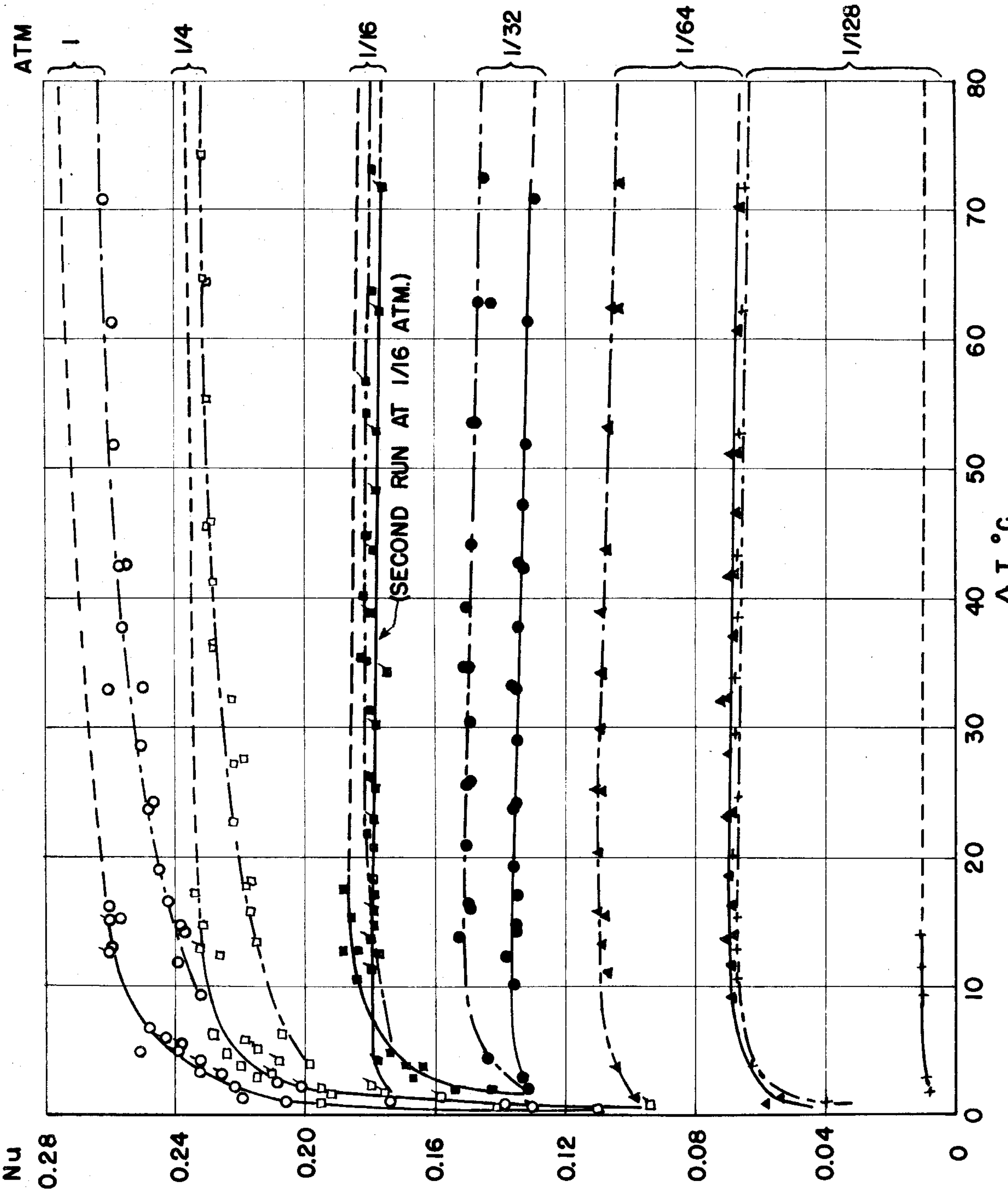


FIGURE 5

PROBE 'B' VERTICAL WITH AND WITHOUT SURROUNDING CYLINDER
L/D = 14,600



LEGEND:
——— PROBE 'B' WITHOUT CYLINDER
– – – EXTRAPOLATION BASED ON PROBES 'A' AND 'C'
—·— PROBE 'B' WITH CYLINDER

FIGURE 6

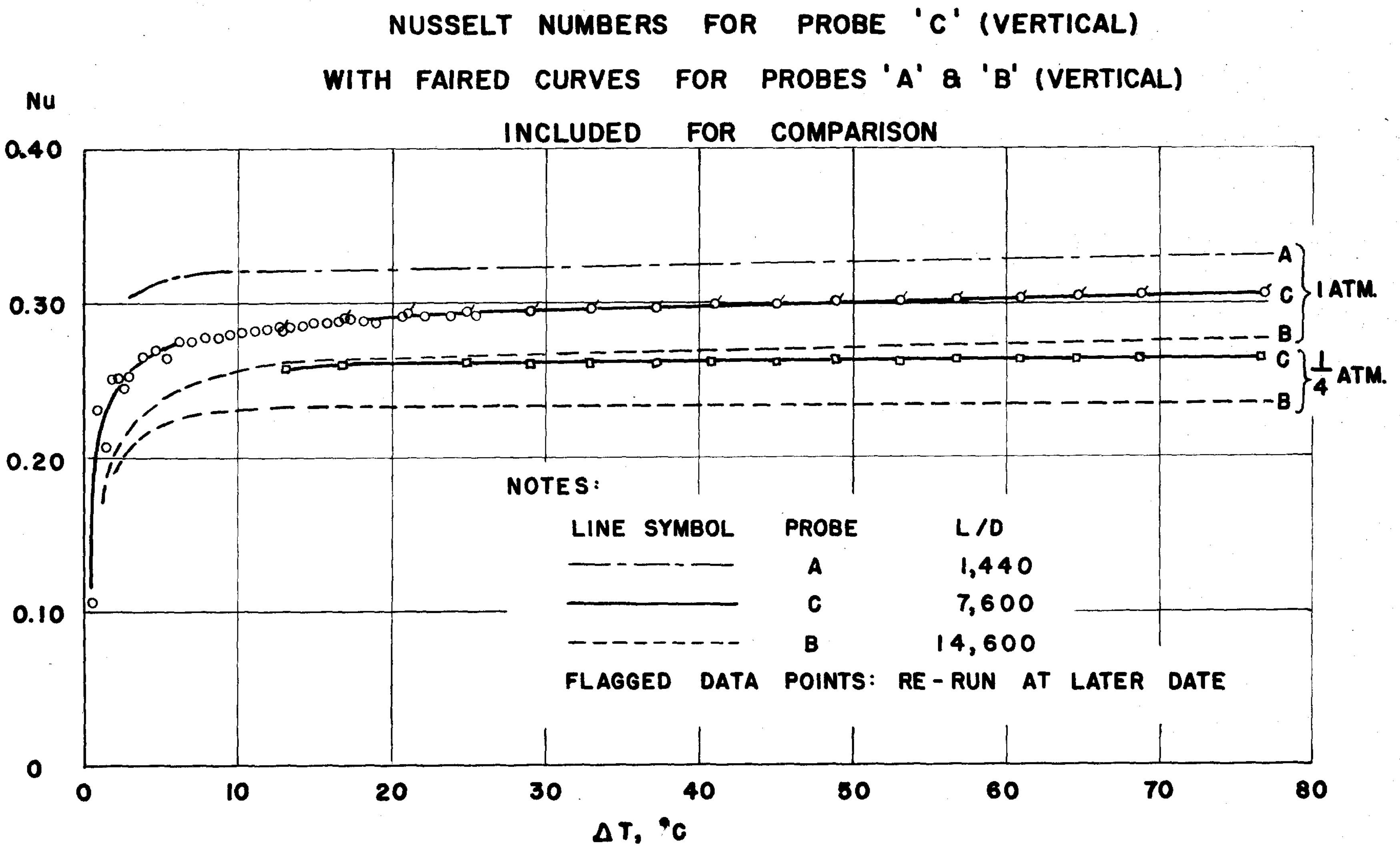

NUSSELT NUMBERS FOR PROBE 'C' (VERTICAL)
WITH FAIRED CURVES FOR PROBES 'A' & 'B' (VERTICAL)
INCLUDED FOR COMPARISON
Nu
0.40
0.30
0.20
0.10
0
0
10
20
30
40
50
60
70
80
ΔT, °C
A
C
B
1 ATM.
C
B
1/4 ATM.
NOTES:
LINE SYMBOL
PROBE
L/D
A
1,440
C
7,600
B
14,600
FLAGGED DATA POINTS: RE-RUN AT LATER DATE

FIGURE 7

CORRELATION WITH FREE - CONVECTION PARAMETER
DATA FOR PROBE 'A' (HORIZONTAL)