AN EXPERIMENTAL INVESTIGATION OF PRESSURE GRADIENTS DUE TO TEMPERATURE GRADIENTS IN SMALL DIAMETER TUBES

Thesis by

Weston Morgan Howard

In Partial Fulfillment of the Requirements

For the Degree of

Aeronautical Engineer

California Institute of Technology

Pasadena, California

1955

ACKNOWLEDGMENTS

The author wishes to express his appreciation to Dr. H. T. Nagamatsu for his continued guidance throughout this investigation.

Sincere thanks are extended to the staff of the GALCIT Hypersonic Wind Tunnel for their cooperation and many helpful suggestions in fabricating the test equipment, and finally to Mrs. H. Van Gieson and Miss N. Waldron for their aid in the preparation of the final manuscript.

ABSTRACT

Results of an experimental investigation of pressure gradients due to axial temperature gradients in small diameter tubes are presented. The tests, which covered the region of Knudsen numbers (based on tube inside radius) of 0.01 to 6, indicate good correlation with theory.

It is of value to note that this correlation was obtained by using ΔT equal to the temperature difference between the hot and cold ends of the tubes and T_{ave} equal to the average of these two temperatures. In contrast, theory would dictate obtaining the temperature variation along the length of the tube and applying the formulas to small incremental ΔT's, then summing to get the total effect. Therefore, for normal laboratory conditions where pressure gradient corrections are to be computed, it is sufficient to record only the temperatures at the hot and cold ends rather than having to obtain a number of temperature readings along the tube.

In order to apply pressure corrections easily and rapidly, a system of correction curves is given. To simplify the procedure, the tube cold end temperature was assumed to be 80°F, and the correction curves drawn accordingly. However, for different laboratory conditions a similar system of curves could be drawn and used.

TABLE OF CONTENTS

LIST OF FIGURES

NOMENCLATURE

a	tube inside radius
Kn_a	Knudsen number = λ/a
L	tube length
p	pressure
p'	local pressure
p_{ave}	average pressure, $\frac{1}{2}(p_H + p_C)$
p_C	pressure at cold end of tube
p_H	pressure at hot end of tube
Q	mass flow rate
r	radius
R	gas constant
T	temperature
T'	local temperature
T_{ave}	average temperature, $\frac{1}{2}(T_H + T_C)$
T_C	temperature at cold end of tube
T_H	temperature at hot end of tube
u	creep velocity
x	axial distance
γ	ratio of specific heats (1.40 for air)
Δp	increment of pressure, $p_H - p_C$
ΔT	increment of temperature, $T_H - T_C$
θ	coefficient of slip
λ	molecular mean free path
μ	absolute viscosity

I. INTRODUCTION

Thermal creep, a gaseous flow phenomenon which occurs at the boundary of a surface and a gas at low pressure, is gaining considerable interest in the field of hypersonics. An important effect of this thermal creep is to cause a pressure gradient along the length of a tube. Thus, appreciable error may result at high temperatures and low pressures when attempting to measure pressures on the surface of a hypersonic wind tunnel model, since the pressure lead tubes are generally small and have an axial temperature gradient due to high model surface temperatures.

Theory for regions of relatively large and small Knudsen numbers has been derived (Refs. 1 and 2) and is well established. However, some doubt exists concerning the transition region where the Knudsen number range is approximately 0.1 to 10. For this transition region a semi-empirical equation, based on experiments with hydrogen, was derived by Knudsen.

With these facts in mind, an experimental program was planned to check the theory for the range of Knudsen numbers which might be encountered in hypersonic wind tunnel testing. The range chosen was from 0.01 to 10.

II. BASIC THEORY

Works on kinetic theory (Ref. 1) show that thermal creep along a surface is due to the diffusely-reflected molecules at different temperatures impinging upon the wall and imparting to it a tangential force in the opposite direction to the temperature gradient. Therefore, the gas near the wall receives an equal force in the direction of the thermal gradient. The resulting creep velocity is:

$$u = \frac{3}{4} \frac{\mu R}{p'} \cdot \frac{\partial T'}{\partial x} \tag{1}$$

where

u = creep velocity

μ = coefficient of viscosity

R = gas constant

p' = local pressure

$\partial T'/\partial x$ = local temperature gradient of the wall

If the mean free path, λ, of the molecules is small (λ/a approximately between 0.01 and 0.1) compared to the inside radius, a, of the tube, and if the tube length, L, is very large compared to a, then a combination of Hagen-Poiseuille type flow and slip boundary conditions will occur. The creep velocity can be combined with the slip condition to obtain the boundary condition, to the first order:

$$u = -\theta \frac{\partial u}{\partial r} + \frac{3}{4} \frac{\mu R}{p'} \cdot \frac{\partial T'}{\partial x} \; ; \text{ at } r = a \tag{2}$$

where θ = coefficient of slip, approximately equal to λ.

Using this boundary condition the mass flow rate, Q, is found to be

$$Q = -\frac{\pi a^4 p'}{8\mu RT'} \left(1 + 4\frac{\lambda}{a}\right) \frac{dp'}{dx} + \frac{3}{4}\pi \frac{\mu a^2}{T'} \frac{dT'}{dx} \tag{3}$$

For the case of a measuring tube, the mass flow is zero for equilibrium conditions, thus giving

$$\frac{dp'}{dx} = \frac{6\mu^2 R}{a^2 p'(1 + 4\frac{\lambda}{a})} \cdot \frac{dT'}{dx} \tag{4}$$

but from kinetic theory

$$p' = 1.48 \sqrt{\frac{R}{\gamma}} \cdot \frac{\mu\sqrt{T'}}{\lambda} \tag{5}$$

where γ = ratio of specific heat (1.40 for air).

Substituting Eq. (5) in Eq. (4),

$$\frac{dp'}{p'} = \frac{3.8\ (\frac{\lambda}{a})^2}{1 + 4\frac{\lambda}{a}} \cdot \frac{dT'}{T'} \tag{6}$$

The Knudsen number, Kn_a, can be defined as λ/a, in which case Eq. (6) reduces to

$$\frac{dp'}{p'} = \frac{3.8\ Kn_a^2}{1 + 4\ Kn_a} \frac{dT'}{T'} \tag{7}$$

Eq. (7) gives the effect of a temperature gradient along the length of a tube for small values of Kn_a.

For the case of very large values of Kn_a (Kn_a approximately greater than 10), the phenomenon of transpiration occurs (Refs. 1 and 2) where the gas molecules are affected primarily by collisions with the tube walls rather than by collisions among the molecules. The result is that $p'/\sqrt{T'}$ is made constant along the tube for equilibrium conditions.

Therefore,

$$\frac{dp'}{p'} = \frac{1}{2} \frac{dT'}{T'} \tag{8}$$

Eq. (8) describes the effect of a temperature gradient for large values of Kn_a, or for the free molecule flow regime.

M. Knudsen (Refs. 2, 3, and 4) derived a semi-empirical equation for the transition region between slip flow and free molecule flow by matching experiments with hydrogen to theory. The derivation is too long and involved to be reproduced here, but the final result is described by the following equation:

$$\frac{dp'}{p'} = \frac{1}{2} \left[1 + \frac{2.46\ (Kn_a + 3.15)}{Kn_a\ (Kn_a + 24.6)}\right]^{-2} \cdot \frac{dT'}{T'} \tag{9}$$

Eqs. (7), (8), and (9), therefore, describe the effect of a temperature gradient for a complete range of Knudsen numbers. In order to see more clearly the implications of the equations it is useful to plot $(dp'/p')/(dT'/T')$ versus Kn_a. This is shown in Fig. 4.

III. EQUIPMENT AND PROCEDURE

A. Test Apparatus

The test apparatus consisted of stainless steel tubes of inside radius 3.33 cm, 0.0241 cm, and 0.0784 cm. The tube lengths were about 11 inches. Provisions were made for evacuating the tubes and for heating one end and cooling the other. The large tube served as a reference, since no appreciable pressure gradient would be built up for such a large diameter tube. Therefore, the pressure drop for the small diameter tubes could be found by comparing the small tube cold end pressure with the large tube reference pressure. A schematic diagram of the apparatus and over-all system is shown in Fig. 3. Photographs of the system are shown in Figs. 1 and 2.

B. Instrumentation

Instruments used in connection with the test apparatus were as follows: An alphatron and a Wallace and Tieman vacuum gage were used to measure the absolute reference pressures. A precision manometer (Ref. 5) was used to measure the pressure difference between the small tube cold end and the reference pressure. This device is accurate to within about 0.5 microns of mercury. To measure temperatures, thermocouples were placed near the hot and cold ends of the tubes, and readings were taken using a Brown potentiometer. A thermometer was also placed in the cooling water at the cold end of the tubes. Figs. 1, 2, and 3 show relative placement of the instruments.

C. Test Procedure

The most important operation in the test was to control effectively leaks and outgassing. To do this, the tubing was cleaned carefully, and the number of joints was held to a minimum. Wherever possible, joints were soldered, but it was necessary in several cases to use a plastic type of tubing to join the lines. These joints were sealed with glyptal and, in addition, encased in a mercury bath for absolute leak control.

The initial outgassing was accelerated by evacuating the system to a very low pressure and applying heat to the test apparatus. Approximately a week was taken for this initial outgassing operation. Even after a long period at low pressure, some outgassing was still present in the system. The effect of this slight outgassing was eliminated by trial-and-error movement of the system cutoff valve until the outgassing in the reference pressure line exactly balanced the outgassing in the small tube pressure line. This was possible since Δp should be zero (1) for a ΔT of zero and any pressure and (2) for the test value of ΔT and a pressure high enough so that Kn_a is less than 0.01. It was found that a change in pressure and temperature (within the test range) did not affect the valve "zero" position. The balancing operation was performed prior to each series of tests so that any slight change in the outgassing characteristics of the tubes was corrected.

The general procedure for obtaining experimental data consisted of setting a desired temperature difference and observing the pressure differential for a number of reference pressures. The temperature was controlled by adjusting the flow of cooling water for the cold end and setting the oven thermostat for the hot end. T_C was taken to be the cooling water temperature at water jacket exit. T_H was taken to be the temperature indicated by the thermocouple near the tube hot end. The desired pressures were obtained by closing the vacuum pump valve and bleeding air into the system until the desired pressure was reached.

The test covered temperatures from 80°F to 1080°F and pressures ranging approximately from 0.03 to 50 mm of Hg.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

The results are most clearly described by plotting the data in the form $(\Delta p/p_{ave})/(\Delta T/T_{ave})$ versus Kn_a, as shown in Fig. 5. As can be seen from the plot, the data fall closely on the theoretical curve except for point scatter at large values of Kn_a. Due to the method of plotting and the characteristic limitations in the accuracy of obtaining pressures, the percentage error increases rapidly with Kn_a. This is due, of course, to the fact that the pressure differential is divided by the average pressure, which is very small for large Knudsen numbers. Also, drift and outgassing are more detrimental at the low pressures. Therefore, positive correlation with theory is questionable for Kn_a greater than 2. However, it appears that the data would follow Knudsen's transition curve up to at least a Knudsen number of 10 if more precise measurements could be made.

It is interesting to note that the experimental data describe the theory for a range of temperature differences up to at least 1000°F, even with the approximation that ΔT equals the temperature difference between the hot and cold ends of the tube. However, for cases with unusual temperature distributions this simplification may not be valid. For the test cases (which would approximate typical laboratory conditions) the temperature distribution along the tube did not vary in an unusual way but was roughly linear. The fact that several different ΔT's and two different tube diameters were used indicates that the approximations involved are not serious for these conditions. Thus, for the usual cases encountered the approximation will give good results. If a problem is encountered with wide or unusual temperature distributions, a more satisfactory approach would be to measure the actual temperature distribution along the tube, apply the previous experimental results to incremental lengths, and then sum to obtain the over-all result.

In reducing and plotting the data it was necessary to know the mean free path of air at the various test conditions. These were calculated, using an average of the values given by Boltzmann and Meyer for nitrogen and oxygen (Ref. 6) and applying the relationship

$$\lambda = (\text{const.}) \frac{T}{p}$$

Fig. 6 is a laboratory correction chart designed for the typical case where the tube cold end temperature is 80°F. Therefore, for the typical case, pressure corrections can be found by following through the curves for the given hot end temperature, pressure, and tube inside diameter. The manometer fluid on which the chart is based is Dow Corning 200 silicone fluid, which has a specific gravity of 0.934 at 75°F. This, of course, must be taken into account if the pressures are measured in units other than cm of Si.

V. CONCLUSIONS

1. In the region of Knudsen numbers from 0.01 to 0.1, the experimental results accurately support the theory. The relationship can be described for this region by the following equation:

$$\frac{\Delta p}{p_{ave}} = \frac{3.8\ Kn_a^2}{1 + 4\ Kn_a} \cdot \frac{\Delta T}{T_{ave}}$$

2. In the transition region, for Knudsen numbers of 0.1 to 10, the relationship is described by the following equation:

$$\frac{\Delta p}{p_{ave}} = \frac{1}{2} \left[1 + \frac{2.46\ (Kn_a + 3.15)}{Kn_a\ (Kn_a + 24.6)} \right]^{-2} \cdot \frac{\Delta T}{T_{ave}}$$

The experimental results are conclusive only for the region of Knudsen numbers from 0.1 to about 2, but it is clear that points falling between Knudsen numbers of 2 and 10 will not deviate appreciably, if at all, from theory.

3. Corrections can usually be applied by taking ΔT equal to the temperature difference between the hot and cold ends, rather than obtaining the actual temperature distribution along the tube.

4. For typical laboratory conditions a simplified chart can be designed and used to find the pressure correction for a given hot end temperature, pressure and tube diameter.

REFERENCES

1. Kennard, E. H., "Kinetic Theory of Gases", McGraw-Hill Book Co., Inc., New York, 1938.

2. Loeb, L. B., "The Kinetic Theory of Gases", McGraw-Hill Book Co., Inc., New York, 1934.

3. Knudsen, M., "Eine Revision der Gleich gewichtshedingung der Gase. Thermische Molekulorstromung", Annalen der Physik, Vol. 31, 1910, p. 205.

4. Knudsen, M., "Thermischer Molekulardruck in Rohren", Annalen der Physik, Vol. 83, 1927, p. 797.

5. Baloga, P. and Nagamatsu, H. T., "Instrumentation of GALCIT Hypersonic Wind Tunnels", California Institute of Technology, Guggenheim Aeronautical Laboratory, Hypersonic Wind Tunnel Memorandum to be published, Contract No. DA-04-495-Ord-19.

6. Handbook of Chemistry and Physics, 34th Edition, p. 2876, Chemical Rubber Publishing Co., Cleveland, Ohio.

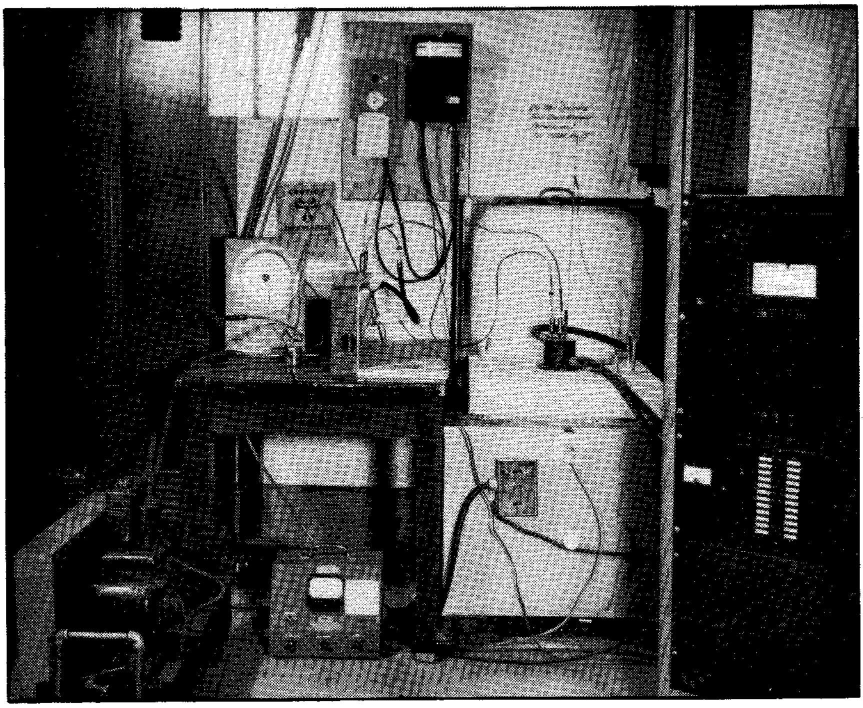

FIGURE 1

TEST APPARATUS AND INSTRUMENTATION

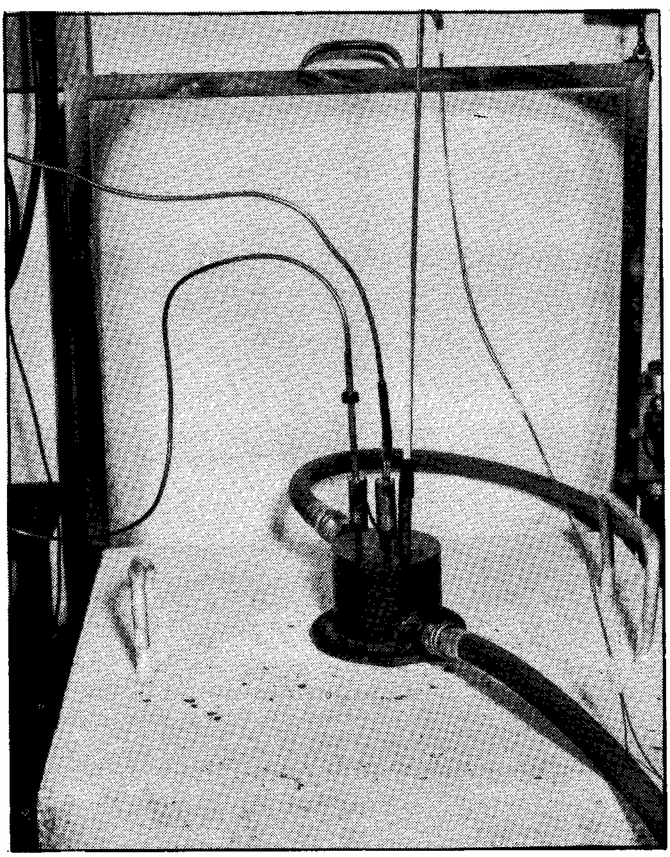

FIGURE 2

CLOSE-UP OF PRESSURE GRADIENT DEVICE INSERTED IN OVEN AND READY FOR TESTS

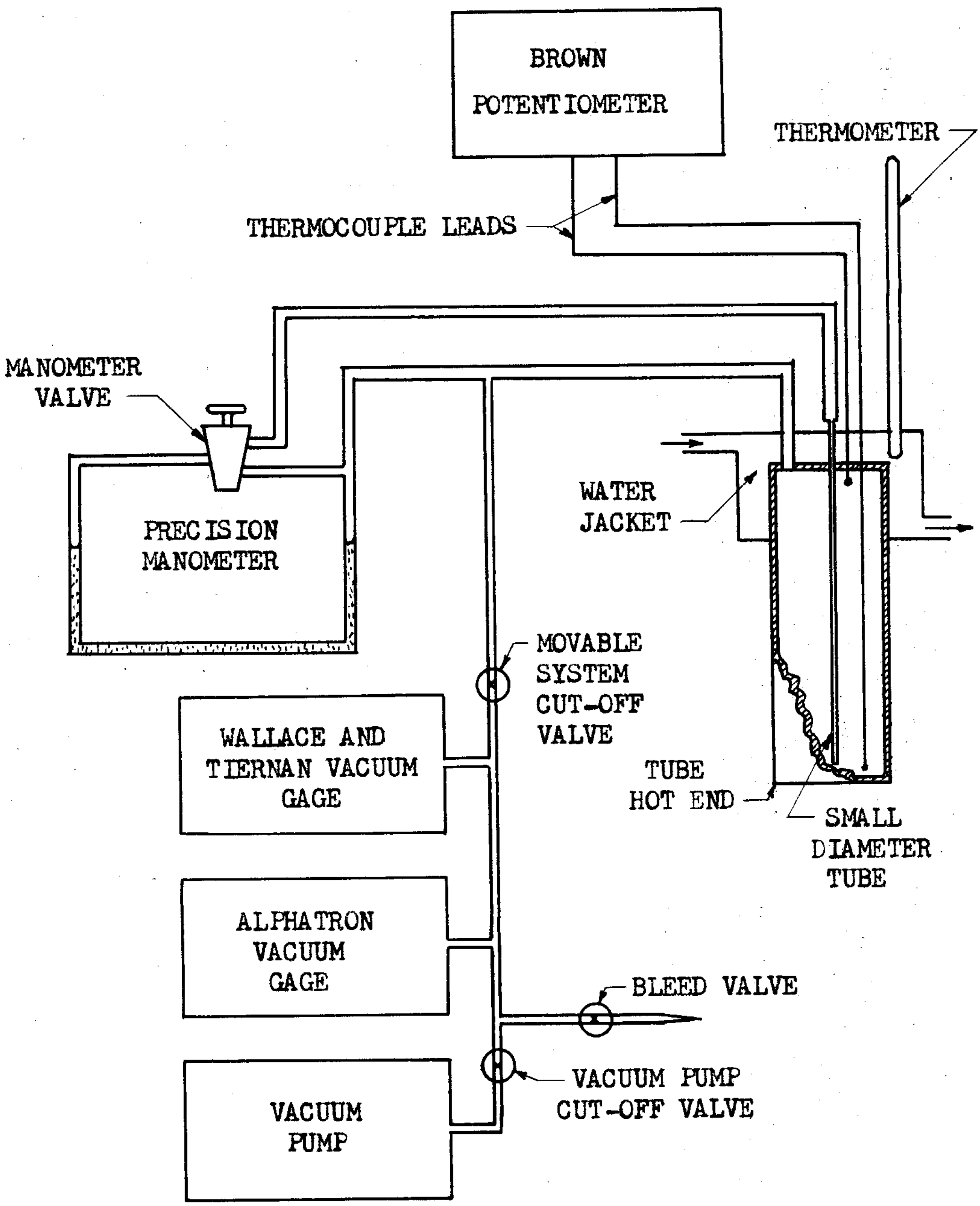

FIGURE 3

SCHEMATIC DIAGRAM OF PRESSURE GRADIENT
TEST APPARATUS AND INSTRUMENTATION

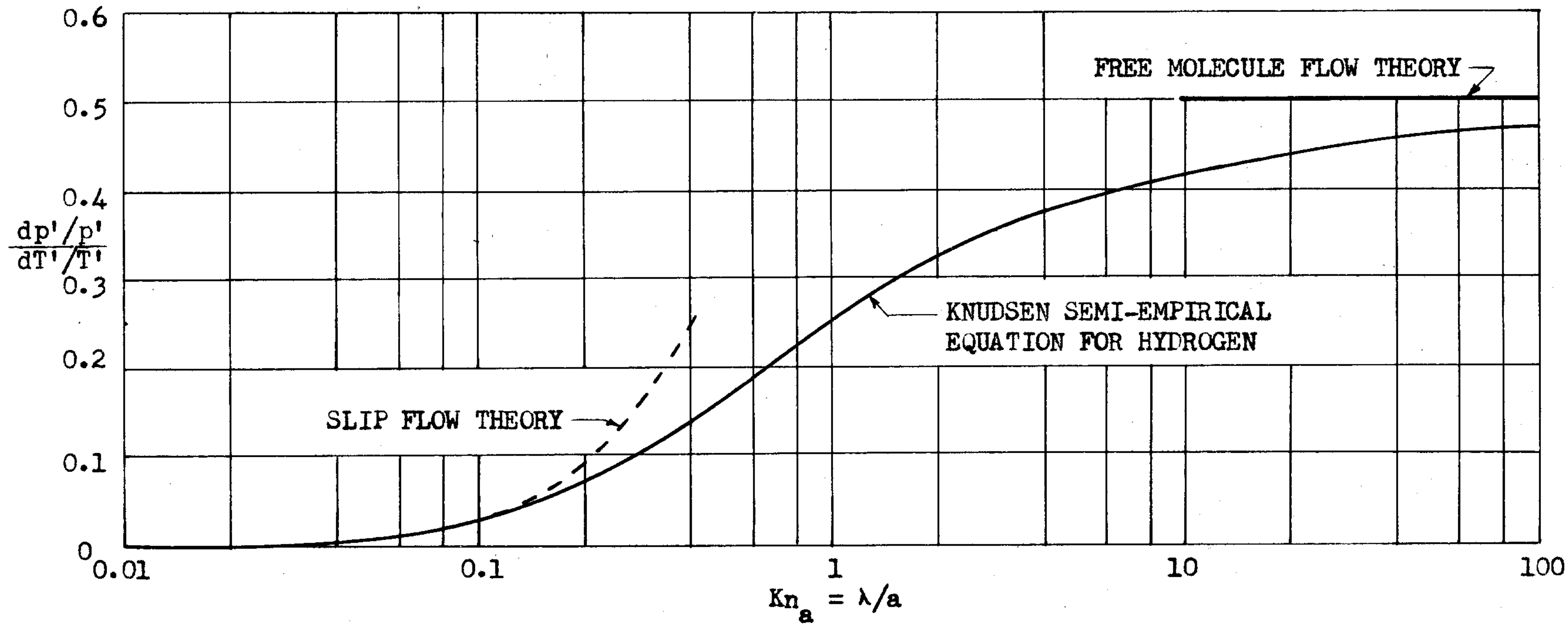

FIGURE 4

THEORETICAL EFFECT OF THERMAL GRADIENTS ON THE PRESSURE MEASURED THROUGH CIRCULAR TUBES

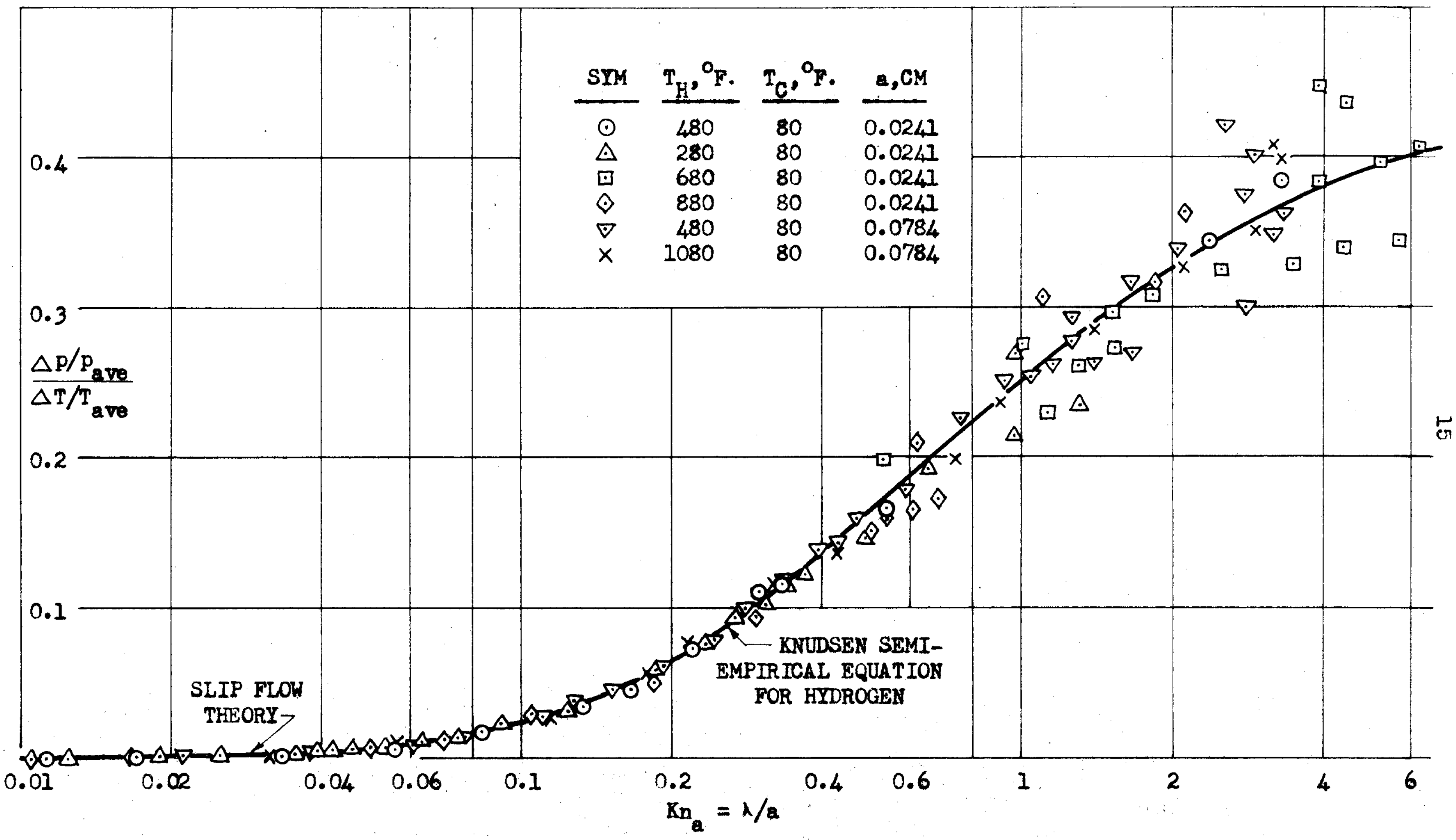

FIGURE 5

EXPERIMENTALLY DETERMINED EFFECT OF THERMAL GRADIENTS ON THE PRESSURE MEASURED THROUGH CIRCULAR TUBES

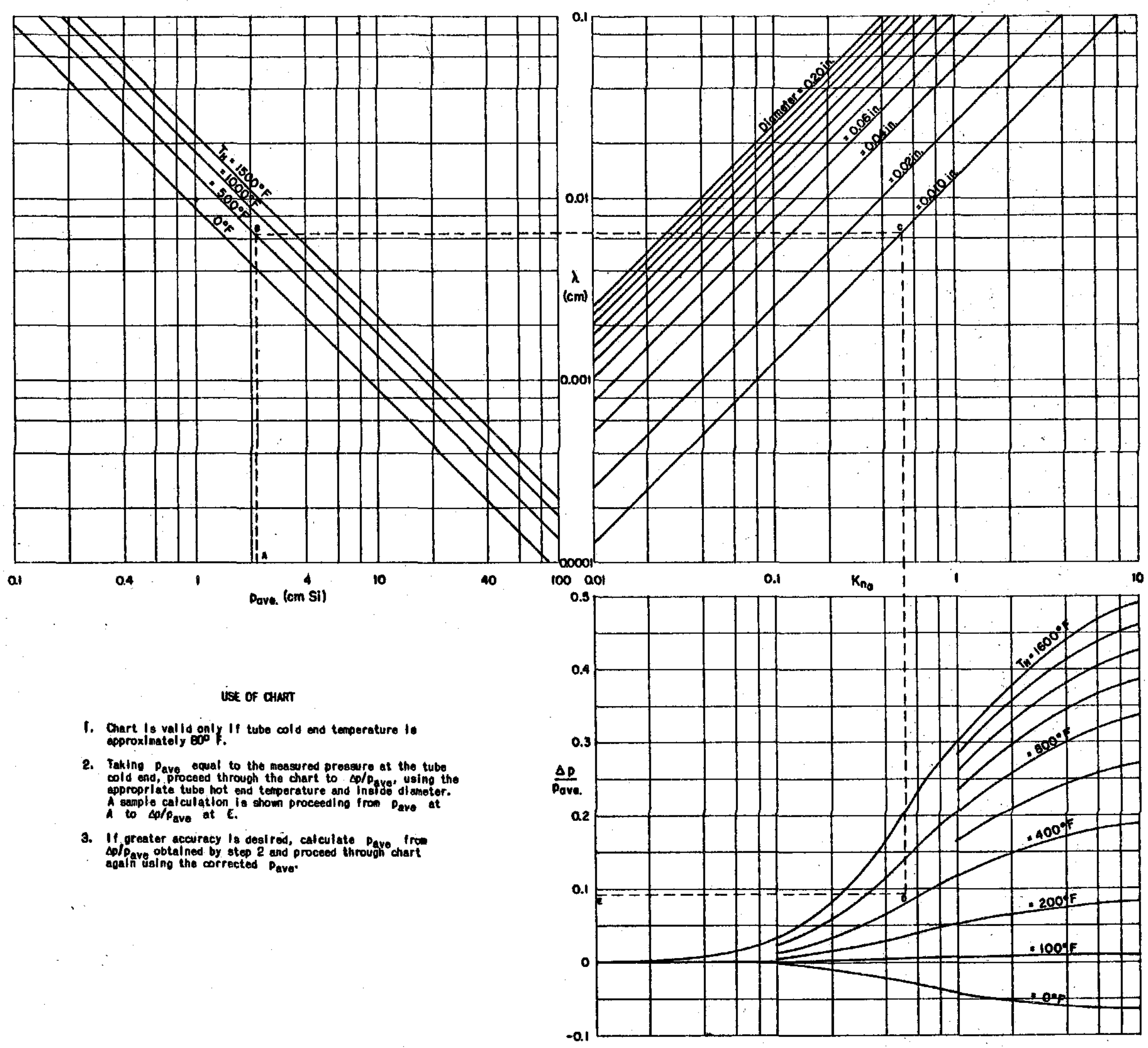

FIGURE 6

LABORATORY CHART FOR COMPUTING THE EFFECT OF THERMAL GRADIENTS ON PRESSURE MEASURED THROUGH CIRCULAR TUBES