THE DETERMINATION OF SOUND VELOCITY IN CORE SAMPLES

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INTRODUCTION

In an area that is being explored for the first time with the reflection seismograph, the manner in which the longitudinal wave velocity varies with depth---the so-called "velocity distribution"---is unknown. Structural features can be located without a knowledge of this function, but they cannot be measured. In the present advanced state of reflection seismology more than mere detection of structural detail is necessary. And in order to compute dips and depths from the reflections and thereby obtain a quantitative structural picture, the velocity distribution has to be known with a tolerably decent degree of accuracy.

There are in common use at present two methods of velocity distribution determination. The more direct of these consists simply of lowering a seismometer to successively different depths down a well already drilled, shooting at the surface near the casing head, and observing the travel times of the first impeti from the records. This method is simple, direct, and without doubt gives the best possible determination of velocity distribution. It necessitates, however, a well that is available for use in this way, as well as the expense of running a seismograph crew for a day or two. The second method of velocity measurement involves only surface shooting and recording. Travel time differences for different shooting distances are carefully measured from surface records of several hundred reflections, plotted, and an average velocity computed. This method involves a great amount of computing and inconveniences resulting from the fact that considerable shooting has to be done in an area before the velocity distribution becomes to be known.

Some sort of inexpensive laboratory measurement of velocity would apparently, therefore, be of value. Important wildcat wells of major companies are now being more or less continuously cored. If the sound velocity in representative samples of the corings at different depths could be measured some idea of the velocity-depth function might be obtained. It is the purpose of this thesis to examine the correlation, if any, between the velocity distribution as obtained in the customary manner as described above, and that obtained from laboratory measurement of velocity in core samples from a well in the same area.

METHODS

There are numerous ways of obtaining sound velocity in solids. These are sometimes divided into two classes, the static and dynamic methods. The former involve measurement of the strain of the solid under an applied stress, and from the elastic constants so determined, the wave velocity can be found from a simple formula. This principle was successfully applied as long ago as 1906 by Adams and Coker[1], who with rather simple apparatus determined the elastic constants of a number of different rocks. More recently static determinations on rock samples have been made by Zisman[2], who took advantage of Bridgman's lever piezometer and high pressure apparatus.

The dynamic methods have to do with setting the solid into vibration and measuring the frequency at which the specimen resonates with the applied oscillation. A classical example of this scheme is Kundt's tube[3]. It should be pointed out now that the two methods may not necessarily give the same results. This has been the subject of a recent paper by Ide[4] who measured the elastic constants for the same samples of rock with both methods. He found that the dynamically determined elastic moduli are systematically from 4 to 20 percent higher than those statically determined, due presumably to the presence of minute cracks and cavities in the specimens. The seismologically determined velocities, however, were found to agree closely with thos dynamically determined. In view of this disclosure, and because of the greater simplicity of the laboratory apparatus and procedure, some sort of dynamic methos seemed indicated for the present experiment.

Ide has done considerable work in this line at Harvard during the last 3 years[5]. His scheme was to stand the sample on a pair of condenser plates separated by a thin sheet of mica, connect the plates to a variable frequency oscillator, and thereby set them into vibration under electrostatic traction. The resulting vibration of the rock specimen was detected by ear, or by a piezoelectric pickup. At resonance the rock vibrates with increased amplitude, and from the frequency at which resonance occurs the velocity is calculable from a simple formula. His determinations of elastic constants and velocity were made on igneous, metamorphic , and well-indurated sedimentary rocks. The other extreme of hardness has been investigated by the Japanese, Ishimoto and Iida[6], who measured the sound velocity in soils and loose sands. To excite the specimen they used a mechanism similiar in principle to a telephone receiver, and for detecting the vibrations of the sample (in the form of long hollow cellophane cylinders filled with the loose material), an optical lever resting on the end of the cylinder. Most recently there has been published an article by Birch and Bancroft[7]. They set a long column of granite into resonance for its longitudinal, transverse, and torsional modes of vibration by an electromagnetic driving scheme. This paper contains the first published results on the magnitude of the internal damping in rocks.

No results appear to have been published as yet of work done with the poorly indurated sediments of some oil producing regions.

PRESENT APPARATUS

It was at first thought that the resonance frequency of the sample might be detected by using a driving device simiiliar to Ide's and measuring the impedance reflected back into the electrical system by the sample. A small amount of consideration showed that a condenser or piezoelectric crystal having a high enough capacitance for the reflected imperdance of the sample to make any appreciable difference was either not practical or unobtainable.

The system used here is shown diagramatically in Fig. 1.

The input voltage to the driving device was supplied by a General Electric 8-A variable frequency oscillator, capable of a maximum output of 80 volts in a frequency range of 100-50000 cycles. This variable voltage was fed into a crystal of Rochelle salt through shielded wire. A piezoelectric crystal instead of Ide's electrotractive condenser device was used because of its greater output and also because no high polarizing voltage such as Ide found necessary was needed. The crystal was firmly cemented on its lower side to an iron mass of 22 pounds; its upper surface carried the specimen. The core sample had to be very firmly cemented to the crystal, especially when long cores were used, in order to prevent any rocking of the sample on the crystal. At the upper end of the sample, and in light contact with it, was a piezoelectric pickup. A most convenient and sensitive detector for this purpose is a phonograph pickup unit from an RCA Record Player. This unit had to be shielded with tin foil to prevent capacitative coupling with the oscillator. The output of the pickup was led through shielded wire into a 3-stage pentode resistance-condenser coupled amplifier which had an overall voltage amplification of 16000. This amplification was great enough to cause overloading on some of the resonance peaks, so that a potentiometer between the pickup and the amplifier was found necessary. The voltage produced by the amplifier was read on an a.c. copper-oxide-rectifier type meter.

MATHEMATICAL THEORY

The following mathematical section considers the equations of motion of an ideal rod, very thin, perfectly elastic, and entirely free at one end. In a subsequent section will be mentioned the corredtions necessary to adapt this theory to actual rock samples.

Consider a long bar undergoing longitudinal vibrations. Every point in it is alternately displaced on either side of its rest position. Take the axis of x along the bar, and suppose u to be the displacement at time t of a plane whose equilibrium position is x. Then the displacement of a near-by plane at x dx is u du. The elongation of the section of the bar between the two planes is therefore $\frac{\partial u}{\partial x} dx$, and the proportional elongation is $\frac{\partial u}{\partial x}$. The force producing this strain is by Hooke's Law $f = E\frac{\partial u}{\partial x}$, where E is Young 's Modulus. The net force acting on the slice is $\frac{\partial f}{\partial x} dx$ and it must equal the mass of the slice times its acceleration. If ρ is the mass of the bar per unit area,

$$E\frac{\partial^2 u}{\partial x^2} dx = (\rho\, dx)\frac{\partial^2 u}{\partial t^2}$$

Placing $v^2 = E/\rho$, we obtain the familiar one-dimensional wave equation,

$$\frac{\partial^2 u}{\partial t^2} = v^2\frac{\partial^2 u}{\partial x^2}$$

It is well known that this equation has the general solution,

$$u = f_1(x - vt) + f_2(x + vt)$$

For sinusoidal waves, f_1 and f_2 must be trigonometric in form, for example,

$$u = A \sin k(x - vt) + A \sin k(x + vt)$$

This equation represents a pair of waves travelling in opposite directions. The term $A \sin k(x-vt)$ represents a wave travelling in the positive x direction with velocity v and wavelength $\lambda = 2\pi/k$ Similiarly $A \sin k(x+vt)$ denotes a wave advancing in the negative direction of x. Of course the foregoing equation can be written

$$u = A(\sin kx \cos kvt - \cos kx \sin kvt) + A(\sin kx \cos kvt + \cos kx \sin kvt)$$

$$= 2A \sin kx \cos kvt$$

In the present case of a rod standing on a piezoelectric crystal, one end of the rod is fixed, the other free. The last equation guarantees a node at $x = 0$ inasmuch as it contains the term sin kx. If the length of the bar is l, the displacement at the free end as a function of time is

$$u = 2A \sin kl \cos kvt$$

The displacement will be a maximum for a given frequency, and the rod will resonate, at a length l such that

$$\sin kl = 1$$

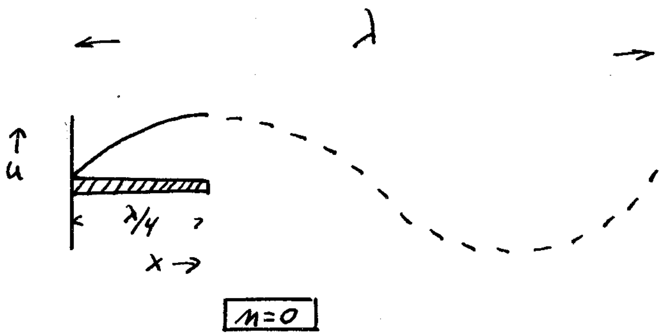
Or,

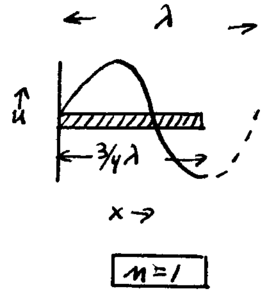
$$(2n+1)\pi/2 = kl = \frac{2\pi}{\lambda} l$$

So that,

$$l = (2n+1)\,\lambda/4$$

and at resonance, then, the length of the bar will be some odd multiple of a quarter wave length, thus:





If resonance occurs at a frequency f_r, we will have

$$v = f\lambda = \frac{4lf_r}{2n+1}$$

In this thesis only the lowermost resonant frequency was used to find v; in this case $n = 0$, and

$$\underline{v = 4lf_r}$$

This is the formula on which most dynamic methods of measuring sound velocity are based. The velocity is the product of the resonant frequency and 4 times the length of the rod.

The acceleration, which is the quantity measured by the piezoelectric pickup, is a maximum, for a fixed length of rod, at the same frequency the displacement is. For, on differentiating twice,

$$\frac{\partial^2 u}{\partial \tau^2} = -2Ak^2v^2 \sin kl \cos kv\tau$$

which becomes a maximum, as before, at a length such that $\sin kl$ 1.

The foregoing analysis neglects damping. In the physical case there will always be some damping, which, however, for rock turns out to be small. If R is the damping coefficient per unit area, the equation becomes

$$\rho\frac{\partial^2 u}{\partial \tau^2} + R\frac{\partial u}{\partial \tau} = E\frac{\partial^2 u}{\partial x^2}$$

Here a damping term proportionate to the velocity has been assumed.

$$\cancel{\rho \frac{d^2u}{dt^2} + R\frac{du}{dt} + Eu = p_0 e^{i\omega t}}$$

The solution of this equation is found in the usual way by assuming $u = u_0 e^{ik(vt-mx)}$, substituting in the equation and thereby determining m. The result is

$$u = u_0 e^{\frac{-R}{2\rho v}x} e^{ik(vt-x)}$$

At intervals of x one wavelength apart the ratio of the displacements is

$$\frac{u_1}{u_2} = e^{\frac{-R}{2\rho}\frac{\lambda}{v}} = e^{\frac{-R}{2\rho}\frac{1}{f}} = e^{-\frac{R}{\rho}\frac{\pi}{\omega}}$$

The quantity $\frac{\omega\rho}{R}$ is customarily denoted by the symbol Q, so that

$$\frac{u_1}{u_2} = e^{-\pi/Q}$$

and the logarithmic decrement δ, defined as $-\log u_1/u_2$, is therefore

$$\delta = \pi/Q$$

Q is a measure of the rate of decay of oscillations. A high Q for a given frequency means a slow decay of the vibrations; a low Q, a rapid decay.

So far we have found the displacement u as a function of both x and t. Let us now fix our ideas on a small section of the rod at some particular x and observe how the displacement varies with time. There will no longer be any partial derivatives in the equations. Suppose this section to be acted upon by a net force p which is a sinusoidal function of the time,

$$p = p_0 e^{i\omega t}$$

Opposing this force are 1) the inertial force of the section, 2) the damping force, assumed proportional to the velocity for mathematical tractability, $R\frac{du}{dt}$, and 3) the elastic force, Eu, where E is Young's Modulus. That is to say,

$$\rho \frac{d^2u}{d\tau^2} + k\frac{du}{d\tau} + Eu = \rho_0 e^{i\omega\tau}$$

For a steady state, this has the first integral

$$\frac{du}{d\tau} = \frac{\rho_0 e^{i\omega\tau}}{k + i(\omega\rho - E/\omega)}$$

of which the real part is

$$\left(\frac{du}{d\tau}\right)_{real} = \frac{\rho_0 \cos(\omega\tau - \alpha)}{\left[k^2 + \left(\omega\rho - \frac{E}{\omega}\right)^2\right]^{1/2}}$$

The graph of du/dt as a function of frequency has a maximum at the frequency of resonance. This frequency is that one which makes the denominator of the last equation a minimum, namely $\omega_{res} = \sqrt{E/\rho}$, which is just the undamped resonant period.

The analogies here with electrical circuit theory are considerable. The differential equation which ocurrs therein is similiar to the one at the top of this page, with replaced by inductance, k by resistance, and E by the reciprocal of capacitance. The displacement u is precisely analogous to charge. Most good texts on a.c. theory show how the Q of a circuit may be obtained from the current-frequency curve. If $f_2 - f_1$ is the width of the resonance curve at an ordinate equal to $1/\sqrt{2}$ times the maximum ordinate, then

$$Q = \frac{f_r}{f_2 - f_1}$$

where f_r is the resonant frequency. This method has been used in this thesis to find the Q of a number of samples. It should be pointed out that a cylinder of rock consists of distributed mass, damping, and elasticity, so that the electrical analogue to a fixed-free rod is a transmission line short-circuited at one end.

It is customary to imagine that internal friction in similiar to fluid friction, or viscosity, and that the damping force is proportional to velocity. This assumption yields a differential equation which, we saw, was easy to solve. It seems certain now that a number of different mechanisms are responsible for internal friction in solids, and that it is impossible to represent their action by a single general formula. An example of a modern theory is that of Zener[8], who calculated that part of the damping in solids due to stress inhomogeneities such as thermal effects and cavities, and checked it by experiments on thin reeds and wires. But some such simple assumption as the traditional one must be made in order to obtain a not-too-complex method for estimating the damping.

CORRECTIONS TO THE THEORY

Actually a number of small corrections should be made to measurements on real specimens.

The foregoing theory neglects the inertia of the parts of the rod not situated on the axis. If the radius of the rod is r and the length l, it is shown by Lord Rayleigh[9] that

$$f_c - f_o = f_o \left[\frac{i^2 \sigma^2 \pi^2}{4} \left(\frac{r}{l}\right)^2 \right]$$

where f_o is the observed resonant frequency of the ith harmonic and f_c the resonant frequency of a rod of the same length but infinitely thin. σ is as usual Poisson's ratio; if it is taken as $\frac{1}{4}$, the bracketed factor becomes $.15(r/l)^2$. For the test specimens used in this experiment, r/l never exceeded .2, so that the width correction was at most .6%; for the core samples, which were relatively thicker than these, the width correction amounted to 2% at a maximum.

A correction should also be make for the weight of the pickup as it rests on the rod. Lord Rayleigh[10] shows that this correction is simply the ratio of the load w of the pickup to the weight W of the rod itself, thus

$$f_c - f_o = f_o \frac{w}{W}$$

The pickup was always adjusted in position so that it just barely touched the rod. It is estimated that the weight of the pickup was never more than 1 gram, and that for the core samples w/W was never more than .5%.

The length l that is substituted in the formula v 4lf is the distance between the node and the free end of the bar. It accordingly includes the thickness (.70cm.) of the Rochelle salt crystal which drives the rod. This crystal had a resonant frequency sufficiently high so as not to affect the resonances of the samples themselves. An error is made by including this thickness of .70cm. in with the rod-length, but since the velocity in the crystal and in the sample is of the same order of magnitude, the appropriate correction may be safely neglected.

The velocity which remains after the corrections have been made is the velocity of longitudinal waves in a long thin rod of the material. Its magnitude is governed by Youngs Modulus and the density according to the relationship $v = \sqrt{E/\rho}$, and is the one-dimensional wave velocity of that material. This, of course , is something different from the seismic velocity which obtains in a three-dimensional medium. These seismic velocities are dependent on any <u>two</u> of the four commonly used elastic constants, namely Young's Modulus, bulk modulus, rigidity modulus, and Poisson's ratio. Only two of these constants are independent; the equations for velocity of body waves in terms of them are familiar and numerous, for they depend on which two are considered independent. In order to convert the one-dimensional velocity into bulk velocity, therefore, one elastic constant has to be arbitrarily assumed. This constant may conveniently be Poisson's ratio σ, which probably is more independent of the nature of the material than any of the others. In terms of σ and E, the equation for bulk velocity is

$$v_{bulk} = \sqrt{\frac{E}{\rho}\,\frac{1-\sigma}{(1+\sigma)(1-2\sigma)}} = v_l\sqrt{\frac{1-\sigma}{(1+\sigma)(1-2\sigma)}}$$

where v_1 is the one-dimensional velocity measured in this thesis. The magnitude of the correcting factor for typical values of σ is

σ	$\sqrt{\frac{1-\sigma}{(1+\sigma)(1-2\sigma)}}$
0	1.00
1/8	1.02
1/4	1.10

In view of the observed decrease of σ with pressure and of measurements of σ by Zisman[2], it is reasonably certain that σ is less than 1/8. The correction to convert the velocity measured by the present method into bulk velocity is accordingly less than 2%. Since it is small and nearly constant for all the core samples measured, it was not felt worth while to make this correction.

RESULTS ON TEST SPECIMENS

Before proceeding to the measurement of the core samples it was of course necessary to ascertain if the apparatus by itself was free from resonances, and if so, to make check determinations with a few materials of previously known velocity. An obvious way to check the crystal-pickup-amplifier system for uniformity of response is to place the pickup on the crystal itself, without a rock sample, and record the amplifier output. The results of this procedure are shown in Fig. 2. The response of the system is seen to be nearly constant between the frequencies 2- to 13000. We may therefore continue, with the assurance that there are no overall resonances in the working frequency range.

A cylindrical cobalt-steel rod, 17.95x1.80 cm., was carefully cemented to the crystal, and readings of the output meter taken for different applied frequencies. This frequency-response curve constitutes Fig 3, and shows a sharp resonance peak at a frequency of 6720. This corresponds to a velocity of 16450 ft/sec, which agrees with the one-dimensional velocity of sound in steel of 16360 as reported in the Smithsonian Physical Tables. The Q of the sample turns out to be in the neighborhood of 70, which is probably too low. However, with resonances as sharp as this the determination of Q is a delicate operation and is greatly influenced by irregularities in frequency calibration of the oscillator, plotting, and drafting. The values of Q as here determined should be taken only as indicators of the order of magnitude of the damping, and not as quantitatively reliable values.

The next sample tested was a thin cylindrical copper rod initially of length 17.95 cm and of diameter 1.15 cm. The velocity at this length was measured. Then a short length was cut off and the velocity of the remaining piece measured again.

The process was repeated once more, so that three frequency response curves for the same rod at different lengths were obtained. These constitute Figs 4,5,6, and the results are summarized in the following table:

length, cm.	f_r	v, ft/sec	Q
17.50	5010	12,000	50
11.65	7140	11,550	120
5.92	14,340	12,100	140

The longitudinal wave velocity as given in the Tables is 11670. The values of Q are not concordant, as is to be expected from the method of measurement; they ought, however, to show the magnitude of the internal damping in copper.

The same procedure was employed with a short core, 3 cm in diameter, of a coarse diorite from the Lake Bonneville dam site. The resonance curves are given in Figs 7,8 and the data in the following table:

length, cm	f_r	v, ft/sec	Q
10.95	9060	13,890	90
9.80	9840	13,450	100

Finally, a long core of a quartz-diorite from Sudbury, Ont., was obtained and the resonant frequencies for a number of different lengths of the core were measured. Since from the fundamental formula, $v = f_r\lambda$, $\frac{1}{\lambda} = \frac{1}{v} f_r$, the quantity can be plotted against f_r and a straight line whose slope is the reciprocal of the velocity should be obtained. This has been done in Fig 9 for eight different lengths of this particular core. The resulting straight line has a reciprocal slope of 14900 ft/sec. For this long sample a number of the third harmonics were obtained at measureable frequencies. In these instances the rod resonated at $\frac{3}{4}$ wave length. The computed velocity for these instances is 15600, somewhat higher than the other. The fact that the frequency-reciprocal wavelength curve is a straight line and shows a velocity of an expectable magnitude is a check on both the theory and the apparatus.

RESULTS ON ACTUAL CORE SAMPLES

It was realized before the work was begun that serious theoretical difficulties stood in the way of measuring in the laboratory by any simple experimental procedure a velocity that is comparable with the velocity actually observed seismically. For example it is well known that the elastic constants and density vary with pressure and temperature, and that so therefore does the velocity. Although this dependence is known for homogeneous isotropic solids, and could be used to correct velocities measured at one temperature and pressure to any other temperature and pressure, no work along this line seems to have as yet been done for sedimentary rocks. In order to get anything comparable with the velocity in the ground, the temperature-pressure conditions existing in the ground at the depth of the core sample would have to be duplicated in the laboratory. Another difficulty lies in the fact that only one elastic constant is being measured whereas two are needed to specify the bulk velocity. Fortuneately, it turns out, as we saw above on page 13, that the necessary correction is quite small. A more serious source of trouble lies in the possible alteration of the sample during and after being taken out of the hole. Certainly the rock is disturbed to a certain extent by the action of the core barrel during coring; moreover, on exposure to the atmosphere the sample will lose some of its contained water. The magnitude of these changes and their effect on the velocity seems impossible to estimate. A final difficulty arises from the dependence of velocity on the lithology of the rock sample. The velocity in shales, sands, and conglomerates at the same depth is not the same; even when the velocity is averaged over 500 foot intervals, as happens when a well is "shot" for velocity distribution, does the velocity show great variations. In order to get anything like a representative velocity distribution, therefore, a very large number of core samples would have to be used.

In spite of these troubles it was hoped that they would be small enough so that some sort of correllation of laboratory velocity with seismic velocity would be obtainable.

A total of 17 core samples were obtained from the recently completed Shell ~~K.C.L.~~ S. 87-4 well of the Shell Oil Co., located northwest of the Canal Oil Field, California, in Sec 4-30-25. These cores extended in depth between 7200 and 13800 feet, with all but three of them lying in the range 7200-10600 feet. They ranged in lithology from very coarse, almost conglomeritic, sands to some fine clays; some of them were ~~w~~soft enough to come off

CORE DATA

depth	diam. cm.	l cm.	f_r		V ft/sec	V_c	Description
7300	4.9	11.35	6500	1.56	10080	10150	fine soft massive shale
781[illegible]	4.9	8.85	6750	1.25	8480	8580	fine rather soft shale
9003	3.9	11.6	6030	1.61	16050	10090	soft shale
9090	3.9	8.8	7500	1.25	9350	9410	soft coarse ss.
10131	3.5	6.0	8520	.892	7600	7700	finely banded coarse ss.
10302	3.3	6.25	7860	.912	7170	7240	banded coarse ss.
10414	3.3	7.90	7245	1.125	8150	8200	coarse ss.
10500	3.6	9.50	7500	1.34	10000	10050	hard coarse ss.
10610	3.6	7.1	7880	1.02	8040	8120	hard shale
13497	3.6	9.10	7850	1.28	10010	10070	hard ss.
13659	3.6	9.40	7475	1.44	10080	10130	hard ss.
13735	3.3	8.37	8520	1.19	10020	10080	hard ss.
8909	4.9	6.7	8580	.972	8280	8440	soft coarse ss.
8700	4.9	9.9	7100	1.39	9880	9970	soft massive shale
8600	4.9	7.55	9700	1.08	10440	10610	rather hard shale
9202	4.9	11.2	7020	1.47	8610	8660	coarse ss.
8817	4.2	9.2	7820	1.20	9440	9500	hard sandy shale

on the fingers on being touched, while others were fairly well consolidated. In size they varied from 6.0 to 11.6 cm in length and from 3.3 to 4.9 cm in diameter.

One end of each core was ground down and smoothed on a lap, in order to form a plane surface to place in contact with the crystal. The dimensions and resonant frequencies of the cores were measured. The data are incorporated in the accompanying table, which also contains a rough description of each sample. Each calculated velocity was corrected for the finite diameter of the sample, and the final velocities tabulated in column v_c. These velocities are plotted against depth in Fig 10.

For two of the samples of a different kind the resonance curves were plotted. The core from depth 9003 is a soft shale; the other specimen, from depth 10414, is a coarse sandstone. A striking difference between these curves, shown in Figs 11,12, is their comparative flatness. This means, of course, a lower Q and greater damping; the values of Q are 30 for the shale and 20 for the sandstone. This greater damping is to be expected from the poorer compaction and greater inhomogeneity of these sediments as compared with the previously used materials. For some of the cores the response to the motion of the crystal was so meager that the output of the amplifier was only a few volts above the noise level resulting from building vibration, etc. In all cases, however, the resonance frequency could be measured with an accuracy of ½% or better.

The velocity-depth points in Fig 10 are divided into three groups on the basis of a rough lithologic classification. This figure also contains the seismic velocity-depth curve obtained from shooting the hole for velocity distribution. This is the curve with which the observed points are to be compared. The feature that is at once apparent and significant about the measured velocities is their scattering and apparently random distribution with depth. When some sort of subdivision is made on the basis of grain size, however, as is done in Fig 10, a slight degree of regularity in the data becomes indicated. An increase of velocity with depth is now gently suggested. The rate of this increase, however, would be impossible to estimate because of the fewness of the number of points lying on each curve.

The measured velocities are of the same order of magnitude as those ocurring in the ground. Those for the shales are nearly equal to the seismically determined velocities, while those for the sandstones are consistently lower. This difference may possibly be due to a greater decrease in velocity with decreasing pressure of the sandstones as compared with the shales.

It is seen that the measured data are not sufficient to establish a velocity distribution; unfortunately a larger number of core samples were not available. Yet it seems likely that if a considerable number of cores of the same kind of rock were chosen at closely spaced intervals and measured, the nature of the velocity-depth function could be ascertained by the present method.

SUMMARY

The results obtained from this work with cores may be summarized as follows:

1) The velocities scatter widely in cores of nearly the same depth.

2) The velocity on the average as measured in the laboratory is not much smaller than that in the ground.

3) The damping in sediments is high.

4) An increase of velocity with depth is indicated when rocks of the same general kind are grouped together.

5) If a large number of core samples of the same sort of rock are available, a rough velocity distribution may possibly be obtained.

ACKNOWLEDGEMENTS

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REFERENCES

1. Adams,F.D. & Coker, E.G.: "An Investigation into the Elastic Constants of Rocks", Carn.Inst.of Wash.Publ. 46, 1906.

2. Zisman, W.A.: "Young's Modulus and Poisson's ratio with Reference to Geophysical Applications", Proc.Nat.Acad.Sci., 19, p653, 1933.

-------------"Compressibility and Anisotropy of Rocks at and near the Earth's Surface", ibid, 19, p666, 1933.

-------------"Comparison of the Statically and Seismologically determined Elastic Constants of Rocks", ibid, p680, 1933

3. Barton, Textbook on Sound, Art.504.

4. Ide, J.M.: "An Experimental Study of the Elastic Properties of Rocks", Geophysics, I, p347, 1936.

--------- "The Elastic Properties of Rocks: A Correllation of Theory and Experiment", Proc.Nat.Acad.Sci., 22, 1936/

5. Ide, J.M.: "Comparison of Statically and Bynamically Determined Young's Modulus of Rocks", Proc.Nat.Acad.Sci., 22, 1936.

6. Ishimoto,M. & Iida,K.: "Determination of Elastic Constants of Soils by Means of Vibration Methods, Pt.I", Bull.Eq. Res.Inst., XIV, 1936, pt. 4.

------------ do., Pt II, ibid, XV, 1947, pt. 1

------------" Velocity of Elastic Waves in Sand", ibid, XVI, 1938, pt. 1.

7. Birch, F.& Bancroft, D: "Elasticity and Internal Friction in a long Column of Granite", Bull.Seis.Soc.Am., 28, p243, 1938

8. Zener, F. Physical Rev., vol 53, p90 & p582, 1938

9. Lord Rayleigh: Theory of Sound, 2nd Ed., p252

10. " " " " " p250

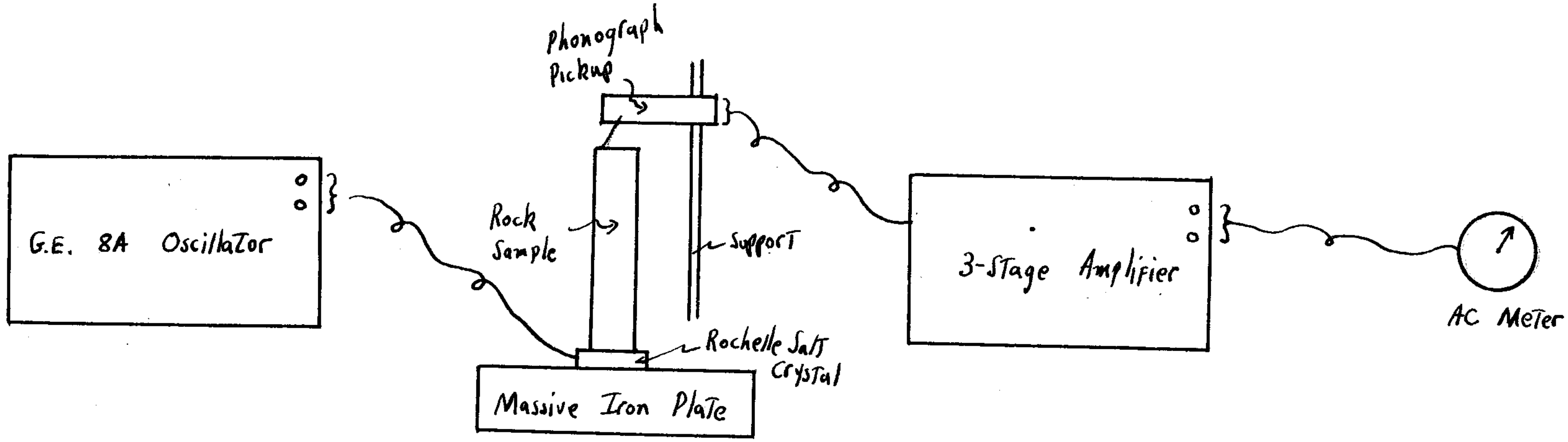
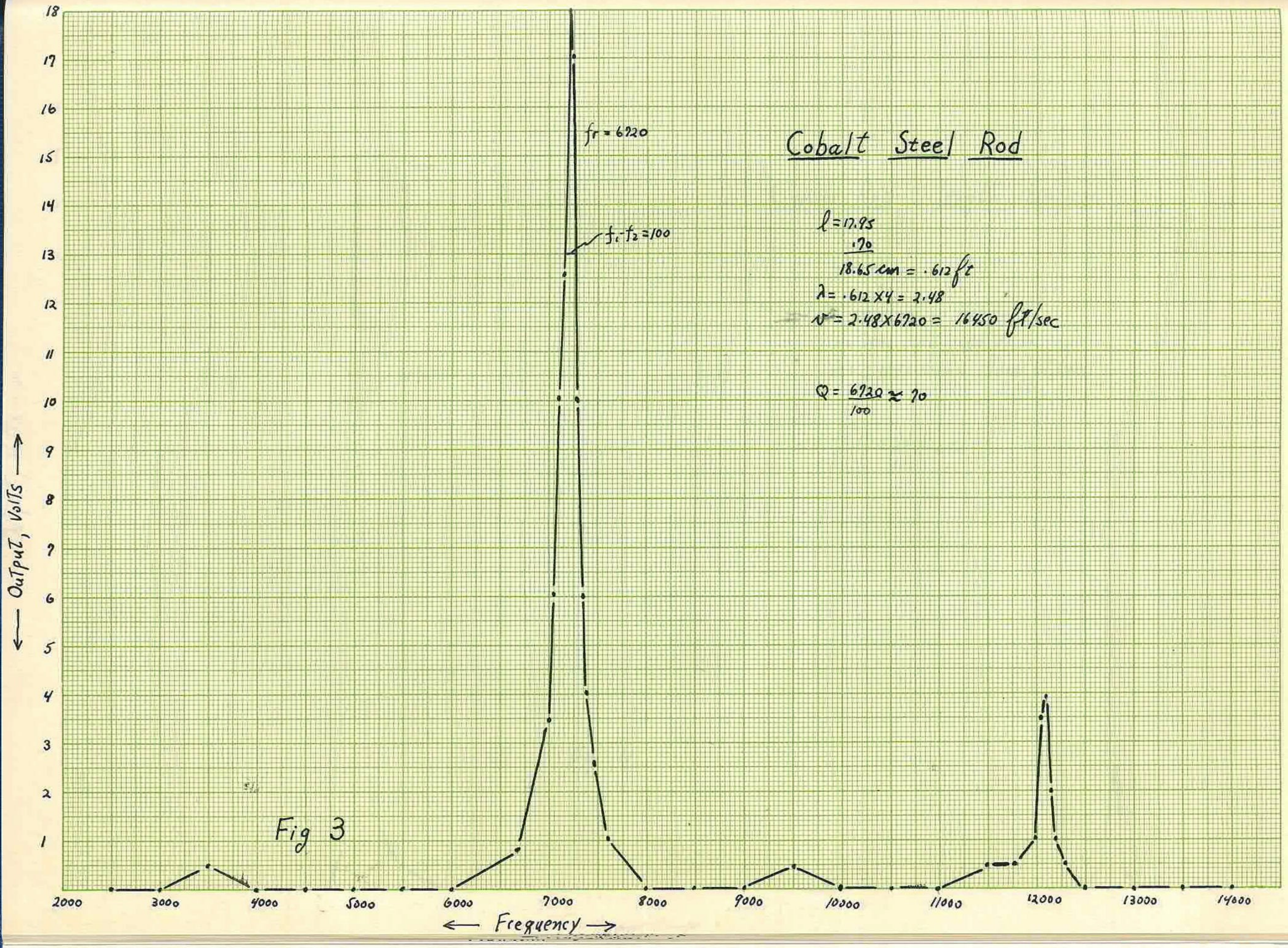

Phonograph
Pickup
G.E. 8A Oscillator
Rock
Sample
Support
3-Stage Amplifier
AC Meter
Rochelle Salt
Crystal
Massive Iron Plate

Fig 1


Output, Volts
16
15
14
13
Crystal Alone
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
14000
Frequency

Fig. 2



Cobalt Steel Rod
fr = 6720
f1-f2 = 100
l = 17.95
.70
18.65 cm = .612 ft
λ = .612 X 4 = 2.48
v = 2.48 X 6720 = 16450 ft/sec
Q = 6720/100 ≈ 70
Output, Volts
Frequency
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
14000

Fig 3

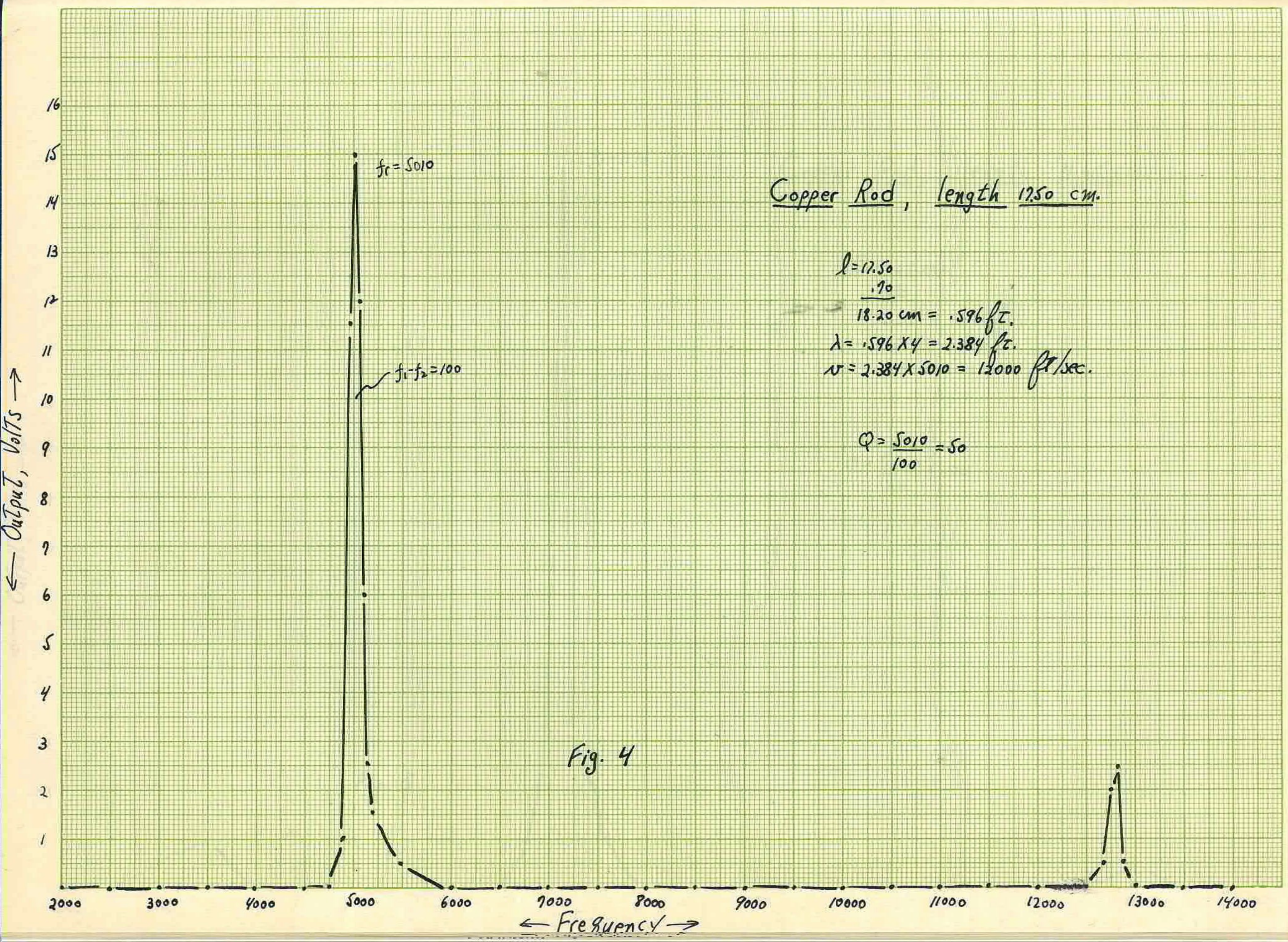
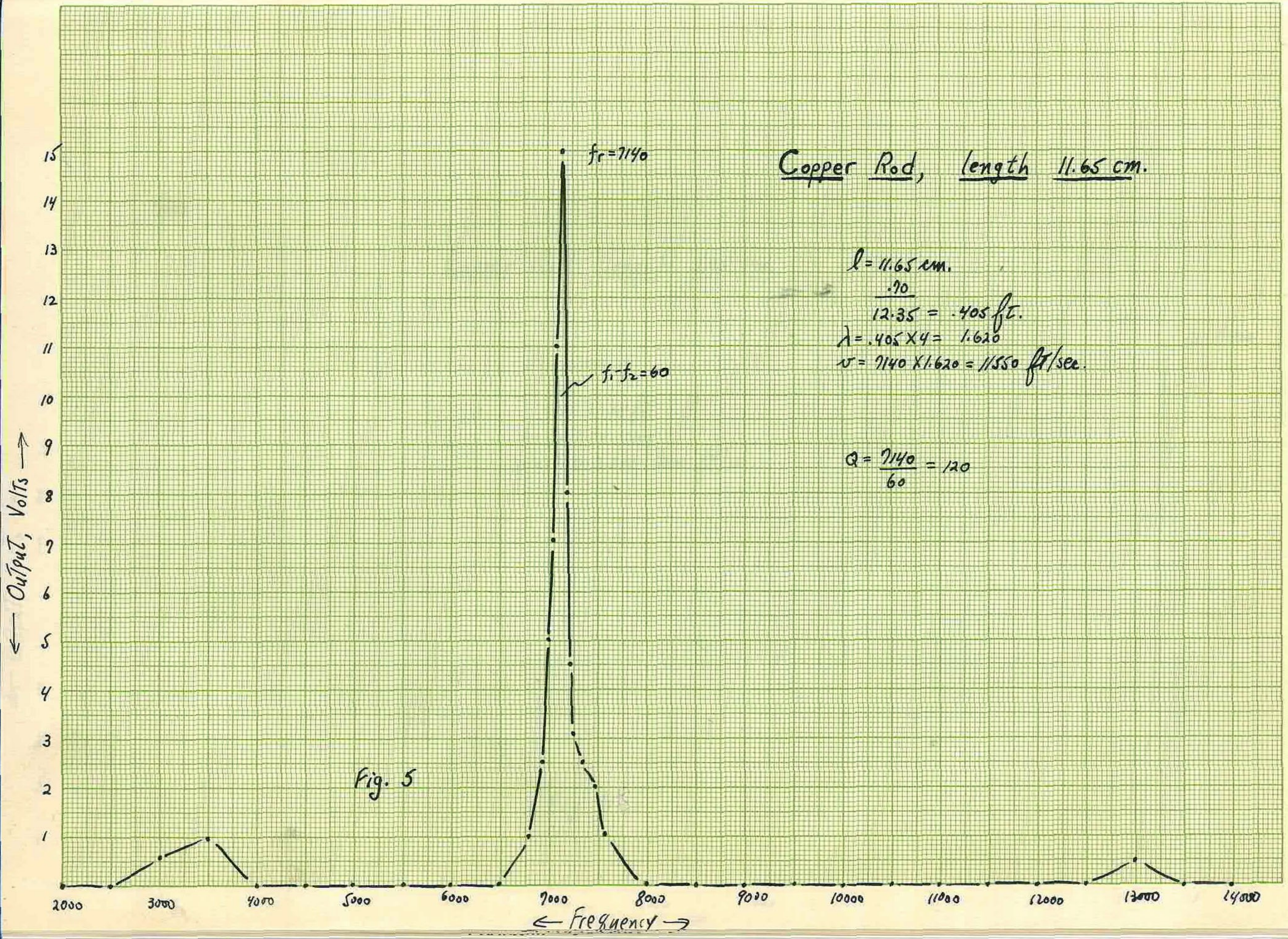

Copper Rod, length 17.50 cm.
l = 17.50
.70
18.20 cm = .596 ft.
λ = .596 X 4 = 2.384 ft.
v = 2.384 X 5010 = 12000 ft/sec.
Q = 5010/100 = 50
fr = 5010
f1-f2 = 100
← Output, Volts →
← Frequency →
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
14000

Fig. 4



Copper Rod, length 11.65 cm.
$l = 11.65$ cm.
$.70$
$12.35 = .405$ ft.
$\lambda = .405 \times 4 = 1.620$
$v = 7140 \times 1.620 = 11550$ ft/sec.
$Q = \frac{7140}{60} = 120$
$f_r = 7140$
$f_1 - f_2 = 60$
← Output, Volts →
← Frequency →
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
14000
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Fig. 5

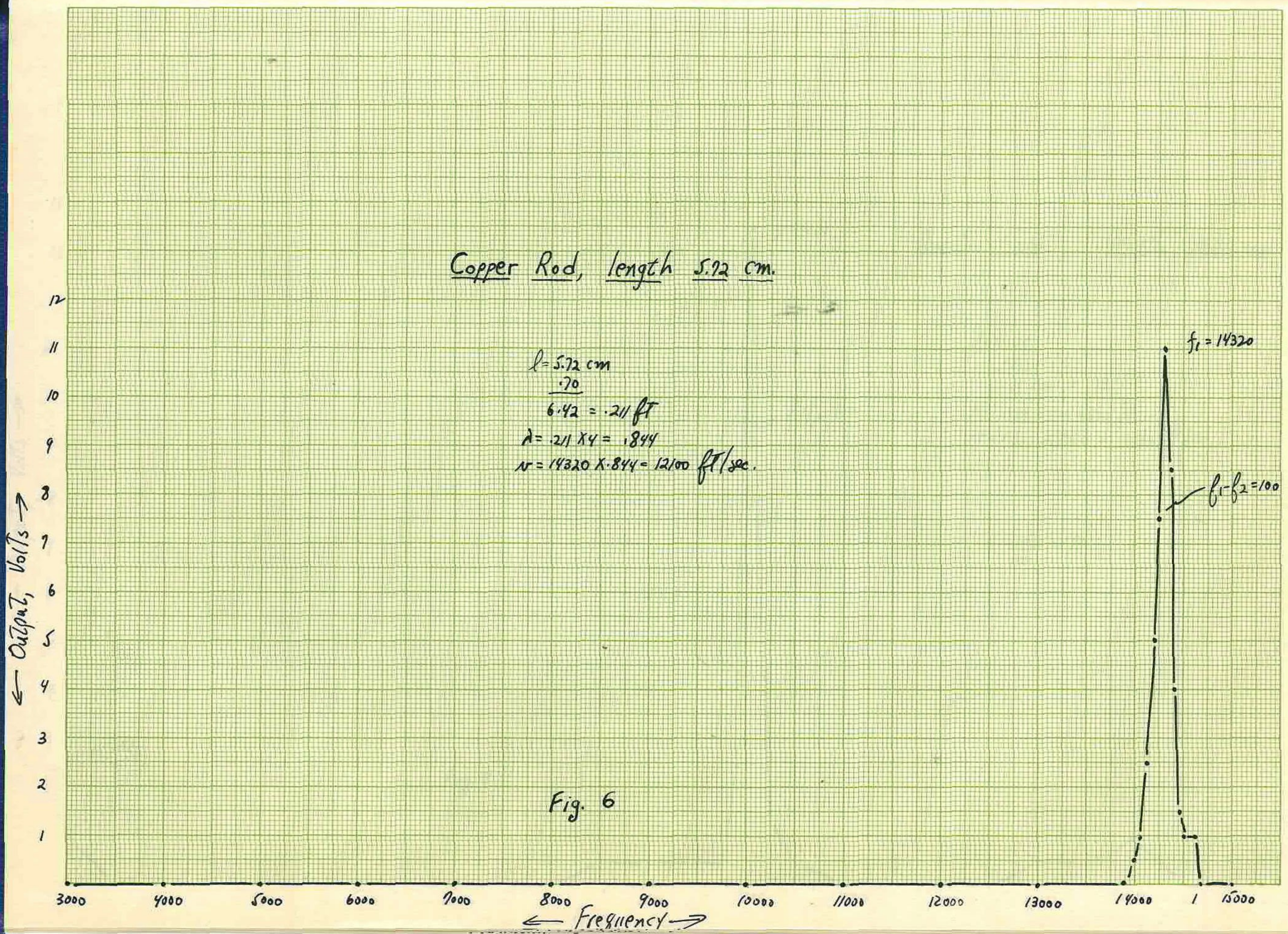
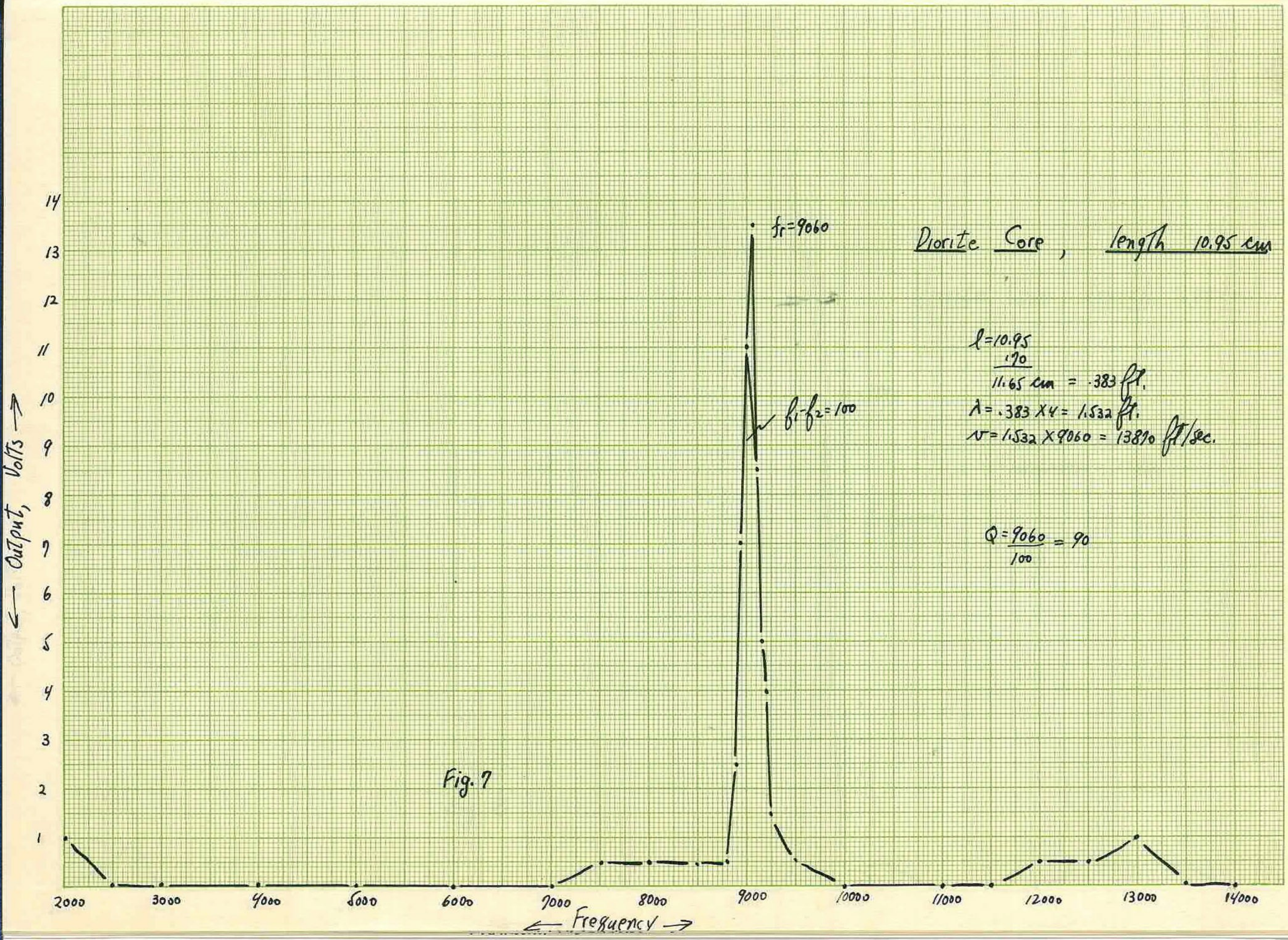
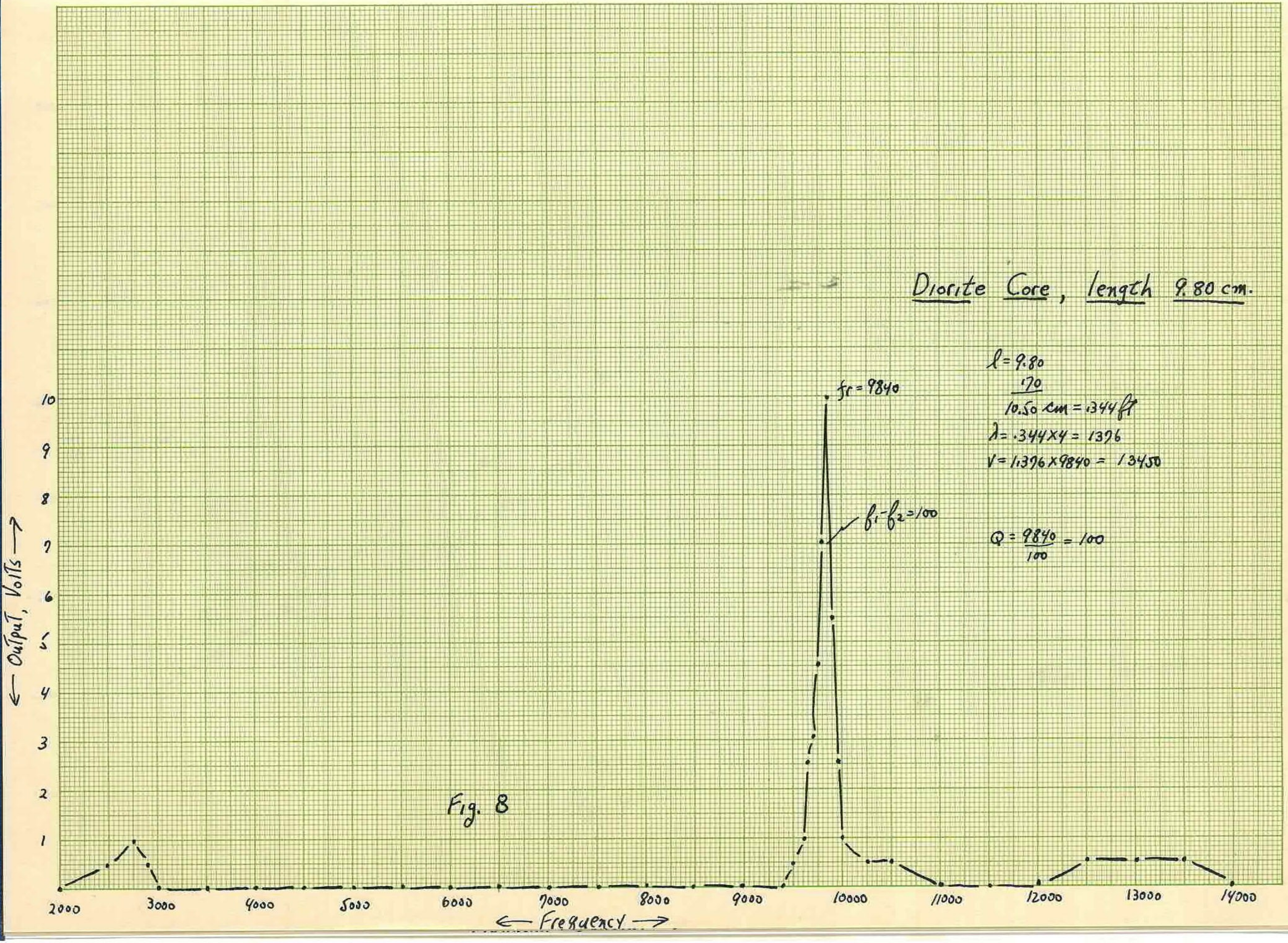

Copper Rod, length 5.72 cm.
l = 5.72 cm
.70
6.42 = .211 ft
λ = .211 X 4 = .844
v = 14320 X .844 = 12100 ft/sec.
f1 = 14320
f1 - f2 = 100
← Output, Volts →
← Frequency →
12
11
10
9
8
7
6
5
4
3
2
1
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
14000
1
15000

Fig. 6


Diorite Core, length 10.95 cm
$f_r = 9060$
$f_1 - f_2 = 100$
$l = 10.95$
.70
11.65 cm = .383 ft.
$\lambda = .383 \times 4 = 1.532$ ft.
$v = 1.532 \times 9060 = 13870$ ft/sec.
$Q = \frac{9060}{100} = 90$
← Output, Volts →
← Frequency →
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
14000
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Fig. 7



Diorite Core, length 9.80 cm.
l = 9.80
.70
10.50 cm = .344 ft
λ = .344 x 4 = 1.376
V = 1.376 x 9840 = 13450
Q = 9840/100 = 100
fr = 9840
f1-f2 = 100
10
9
8
7
6
5
4
3
2
1
← Output, Volts →
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
14000
← Frequency →

Fig. 8

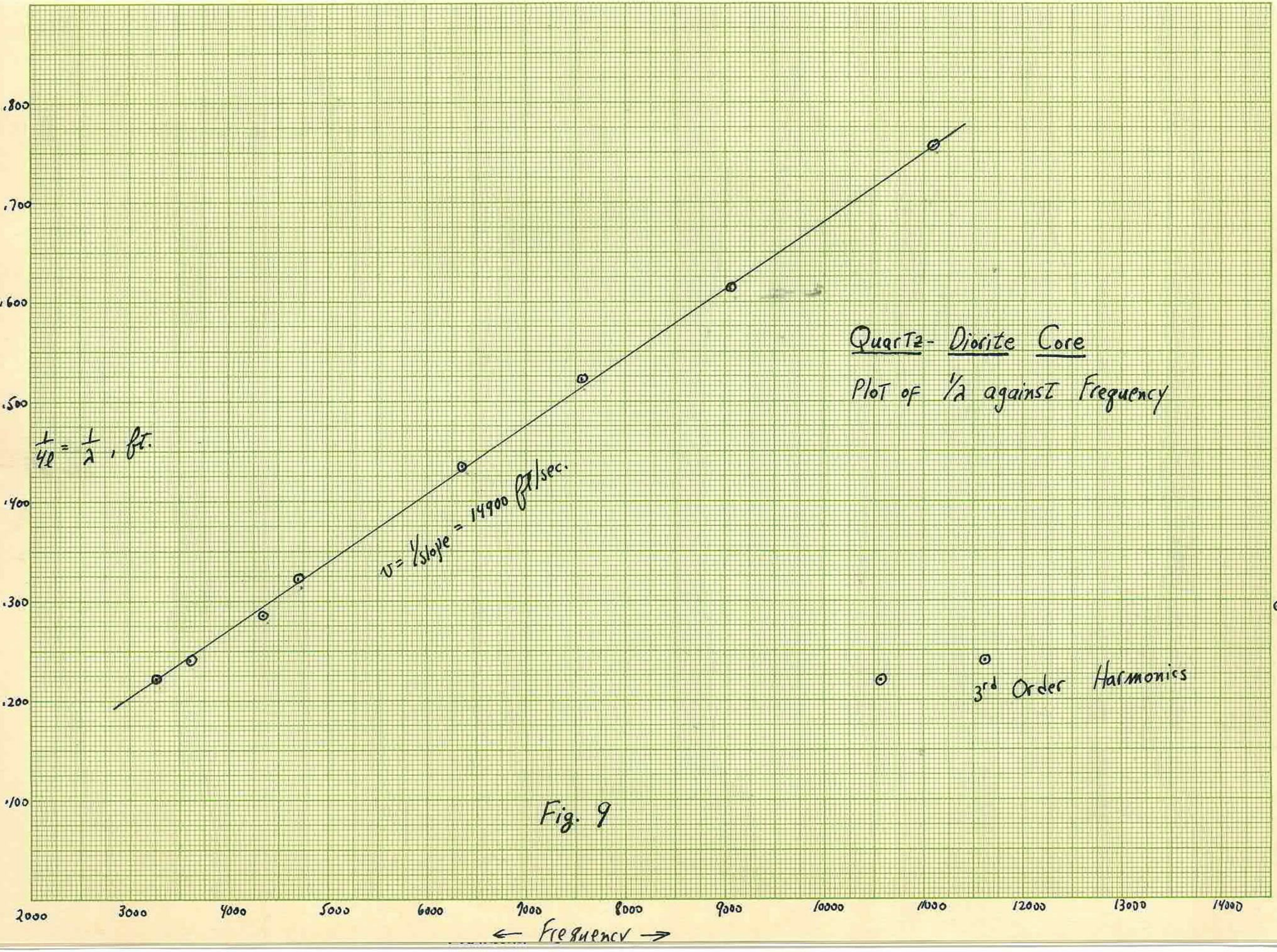
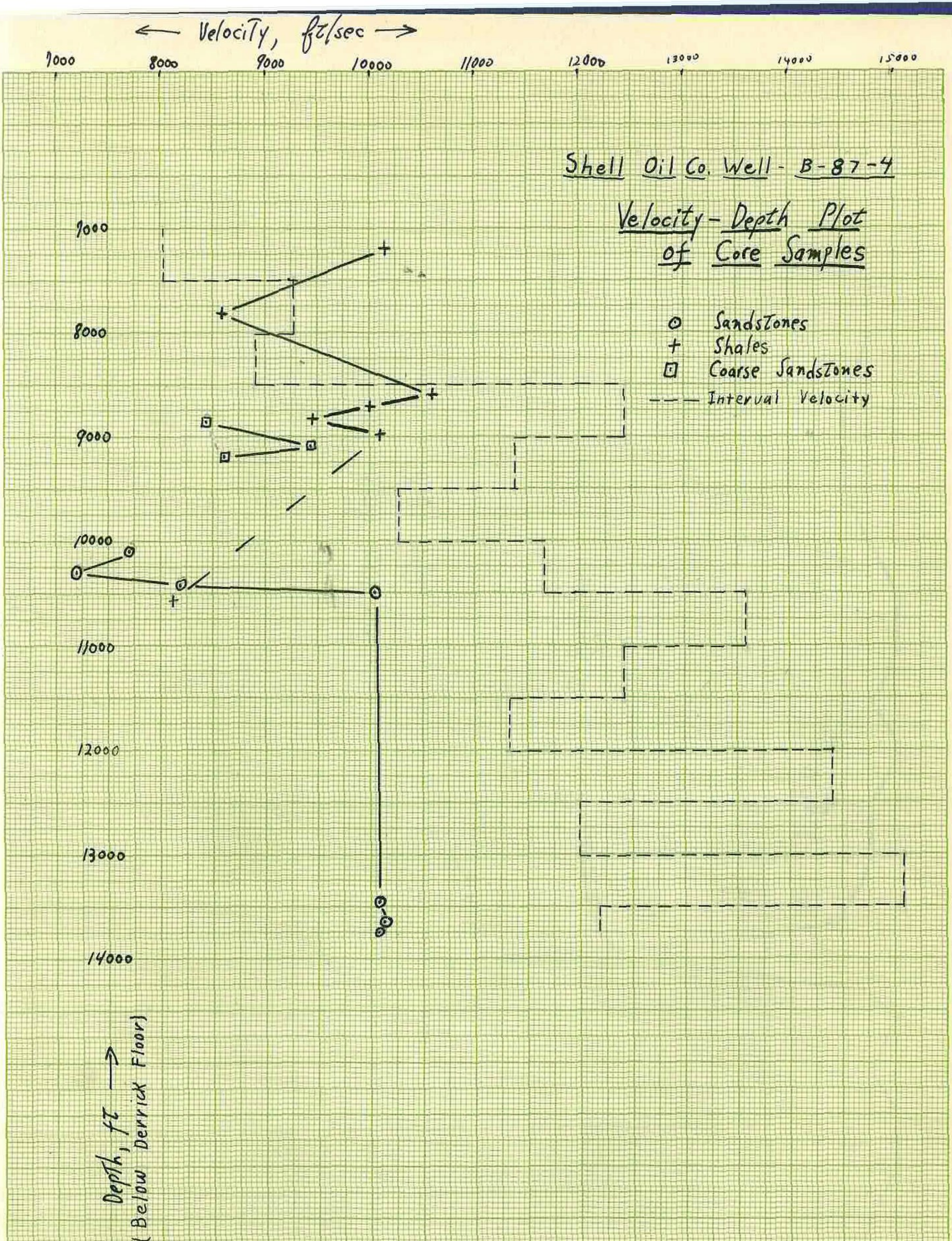

Quartz- Diorite Core
Plot of 1/λ against Frequency
$\frac{1}{4l} = \frac{1}{\lambda}$, ft.
v = 1/slope = 14900 ft/sec.
3rd Order Harmonics
.800
.700
.600
.500
.400
.300
.200
.100
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
14000
← Frequency →

Fig. 9



← Velocity, ft/sec →
7000
8000
9000
10000
11000
12000
13000
14000
15000
Shell Oil Co. Well - B-87-4
Velocity - Depth Plot of Core Samples
Sandstones
Shales
Coarse Sandstones
Interval Velocity
7000
8000
9000
10000
11000
12000
13000
14000
Depth, ft →
(Below Derrick Floor)

Fig 10

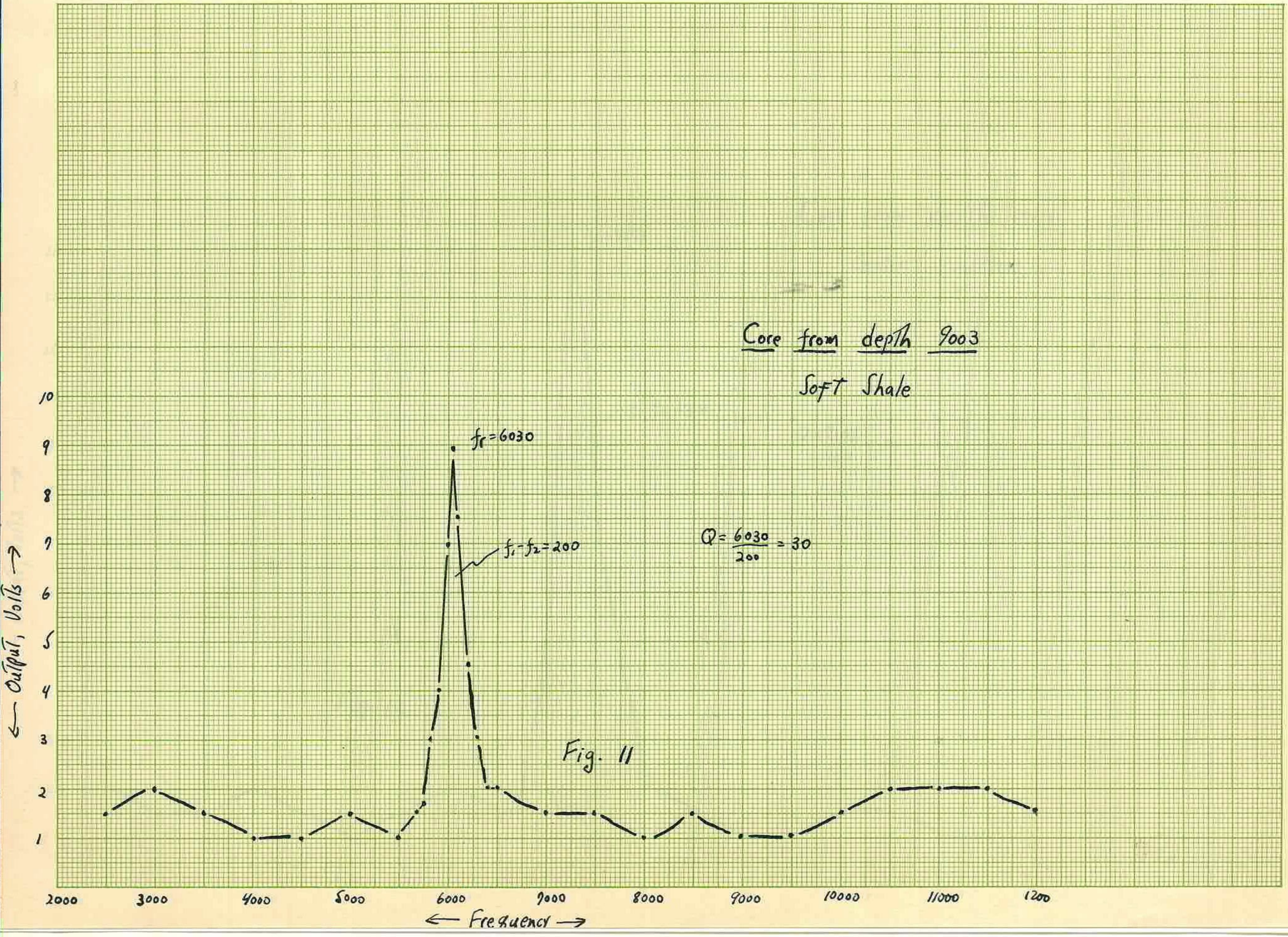
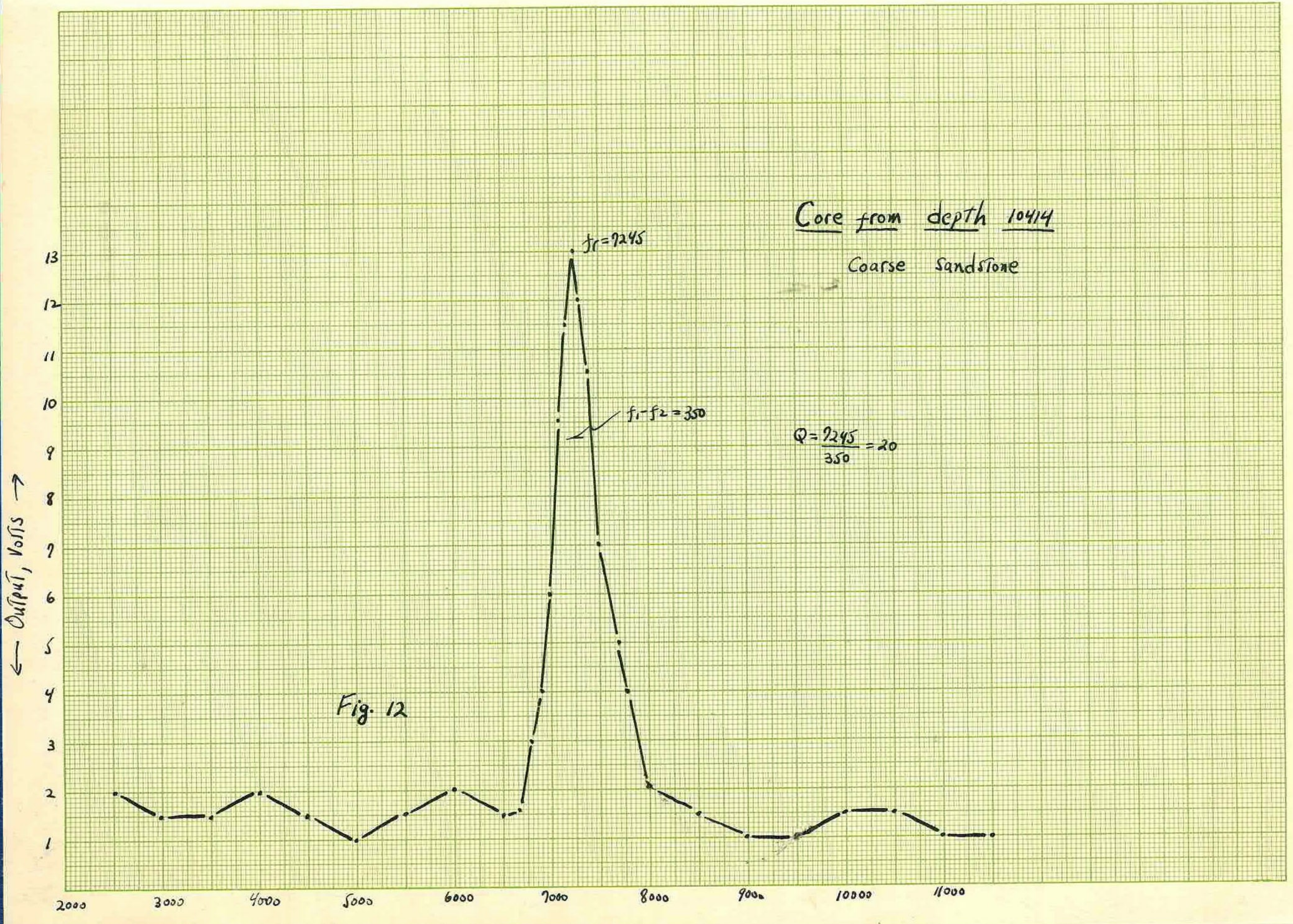

Core from depth 9003
Soft Shale
$f_r = 6030$
$f_1 - f_2 = 200$
$Q = \frac{6030}{200} = 30$
10
9
8
7
6
5
4
3
2
1
← Output, Volts →
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
1200
← Frequency →

Fig. 11



Core from depth 10414
Coarse sandstone
fr = 7245
f1-f2 = 350
Q = 7245/350 = 20
13
12
11
10
9
8
7
6
5
4
3
2
1
← Output, Volts →
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000

Fig. 12