Appendix A

Time-Domain Phase Analysis Using I/Q Demodulation

In this appendix we describe the in-phase and quadrature (I/Q) demodulation technique which is used for time-domain analysis of the locked-state OPLL phase error in chapter 6.

The goal of the technique is to separate the amplitude modulation $A(t)$ from the phase modulation $\theta(t)$ of a sinusoidal signal $y(t)$ with a known frequency ω_0,

$$y(t) = A(t)\sin\left[\omega_0 t + \theta(t)\right]. \tag{A.1}$$

We form the in-phase signal $y_i(t)$ and the quadrature signal $y_q(t)$ by multiplying $y(t)$ with sine and cosine waveforms at a frequency of ω_0, and low-pass filtering the results.

$$\begin{aligned} y_i(t) &= h(t) \star \left[y(t)\sin\omega_0 t\right] \\ &= h(t) \star \left\{\frac{A(t)}{2}\cos\theta(t) - \frac{A(t)}{2}\cos\left[2\omega_0 t + \theta(t)\right]\right\}, \text{ and} \end{aligned} \tag{A.2}$$

$$\begin{aligned} y_q(t) &= h(t) \star \left[y(t)\cos\omega_0 t\right] \\ &= h(t) \star \left\{\frac{A(t)}{2}\sin\theta(t) + \frac{A(t)}{2}\sin\left[2\omega_0 t + \theta(t)\right]\right\}, \end{aligned} \tag{A.3}$$

where $h(t)$ is the impulse response of the low-pass filter, and '$\star$' denotes the convolution operation. The filter is designed to average out the sum frequency terms at frequency $2\omega_0$, while retaining the difference frequency terms at DC, yielding

$$y_i(t) = \frac{A(t)}{2}\cos\theta(t), \text{ and} \tag{A.4}$$

$$y_q(t) = \frac{A(t)}{2}\sin\theta(t). \tag{A.5}$$

The amplitude and phase modulations are recovered using

$$A(t) = 2\sqrt{y_i^2(t) + y_q^2(t)}, \text{ and} \tag{A.6}$$

$$\theta(t) = \text{atan2}\left[y_q(t), y_i(t)\right], \tag{A.7}$$

where $\text{atan2}(y_q, y_i)$ is the four-quadrant inverse tangent function defined below.

$$\text{atan2}(y_q, y_i) \equiv \begin{cases} \tan^{-1}\left(\frac{y_q}{y_i}\right) & y_i > 0 \\ \tan^{-1}\left(\frac{y_q}{y_i}\right) + \pi & y_q \geq 0,\ y_i < 0 \\ \tan^{-1}\left(\frac{y_q}{y_i}\right) - \pi & y_q < 0,\ y_i < 0 \\ +\frac{pi}{2} & y_q > 0,\ y_i = 0 \\ -\frac{pi}{2} & y_q < 0,\ y_i = 0 \\ \text{undefined} & y_q = 0,\ y_i = 0 \end{cases} \tag{A.8}$$