PROPAGATION OF COSMIC RAYS THROUGH INTERSTELLAR SPACE

Thesis by

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SUMMARY

The propagation of cosmic rays through interstellar space has been investigated with the view of determining what particles can traverse astronomical distances without serious loss of energy. The principal method of loss of energy of high energy particles is by interaction with radiation. It is found that high energy (10^{13} - 10^{18} ev) electrons drop to one-tenth their energy in 10^8 light years in the radiation density in the galaxy and that protons are not significantly affected in this distance. The origin of the cosmic rays is not known so that various hypotheses as to their origin are examined. If the source is near a star it is found that the interaction of electrons and photons with the stellar radiation field and the interaction of electrons with the stellar magnetic field limit the amount of energy which these particles can carry away from the star. However, the interaction is not strong enough to affect the energy of protons or light nuclei appreciably. The chief uncertainty in the results is due to the possible existence of a general galactic magnetic field. The main conclusion reached is that if there is a general galactic magnetic field, then the primary spectrum has very few photons, only low energy ($< 10^{13}$ ev) electrons and the higher energy particles are primarily protons regardless of the source mechanism, and if there is no general galactic magnetic field, then the source of cosmic rays accelerates mainly protons and the present rate of production is much less than that in the past.

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I. INTRODUCTION

The experimental determination of the nature of the cosmic ray primaries has been carried out by two main lines of attack. The first method is the investigation of the various geomagnetic effects with a view to obtaining their energy spectrum and charge. The results indicate that most of the primaries are charged, that the secondaries observed at sea level come chiefly from positive primaries and that the energy spectrum is approximately an inverse power law, but no direct evidence of the nature of the particles themselves is obtainable. The second approach is to make experiments on the incoming particles at the "top" of the atmosphere and, hence, to determine their character by their properties. Such experiments are difficult to carry out and equally difficult to interpret because of the many complicated interactions which occur with the air. There may be several types of particles present but there is no conclusive evidence pointing to the presence or absence of any type of particle although there is some evidence that protons generate the hard component. Auger showers are adduced to indicate the presence of high energy electrons at the top of the atmosphere but it is not clear whether they are primaries or secondaries. Several theoretical investigations of specific mechanisms for the generation of high energy particles have been made but only with the point of determining whether sufficient energy could be obtained from the mechanism. Consideration of the interaction of the primaries with interstellar gas have likewise led to no conclusion because of the rarity of the gas. In this paper the interaction of the particles with radiation is shown to lead to a restriction on the mean free path the various particles could have at high energies, and hence, to a restriction on the primaries which could reach the earth. The various hypotheses concerning the source of the cosmic rays are also investigated and it is shown that interaction with radiation and with magnetic fields give significant results here also. The body of the paper is divided into two parts. The first describes the contents of space and the diffusion of high energy particles through space, and the second investigates the loss of energy by interaction with radiation in transit and in the various proposed sources.

II. PROPERTIES OF SPACE AND DIFFUSION OF COSMIC RAYS

The diffusion of cosmic rays and their interaction with interstellar matter has been treated by several investigators so that it is only necessary to mention the salient facts here. The least well understood phenomena involve the magnetic fields in the galaxy, especially the magnitude of the general galactic field. The situation is complicated by the fact that the relaxation time for decay of a magnetic field of extension only one light year is many times the age of the galaxy and hence equilibrium with sources cannot be assumed.

2.1. Matter and Radiation Density

The amount of matter and energy in the galaxy is not known accurately but it may be estimated from the absorption of the light from stars. An upper limit to the total matter may be obtained from dynamical considerations involving its gravitational effect. Oort[1] gives the upper limit as 6×10^{-24} gm/cm^3, of which half is in the form of stars. Greenstein[2] gives a value of 2×10^{-25} gm/cm^3 for dust particles with mean diameter 10^{-5} cm. Struve and Elvey[3] measured the emission spectrum of hydrogen near hot stars, where the hydrogen is ionized, and obtained a density of three atoms/cm^3 or 5×10^{-24} gm/cm^3. Most of the hydrogen in space is unionized and unobservable since no absorption lines lie in the visible. Dunham[4] has determined the densities of calcium and sodium from measurements of the interstellar absorption lines and finds that the mass density is about 10^{-25} gm/cm^3. Spitzer[5] reviews the various determinations and concludes that reasonable values to use are one hydrogen atom per cm^3 or 1.7×10^{-24} gm/cm^3, and between 10^{-25} and 10^{-24} gm/cm^3 of dust.

The radiation density in interstellar space has been obtained by Dunham[4] from a weighted sum of the various stellar spectral types. His results for the radiation density per Angstrom is given in the first table.

TABLE 1

Mean Radiation Density Due to Stellar Radiation

λ in Angstrom units	$\rho_\lambda \times 10^{20}$ ergs/cm^3/A°
333	65.5
400	142.0
500	265
667	464
833	728
1000	1080
1250	1550
1667	2250
2000	2620
2500	3020
3333	3680
4000	4480
4250	4860
5000	6220
6667	9780
10000	10200

At longer wavelengths the radiation comes primarily from the interstellar gas. The contribution of the free-free and free-bound collisions of protons and electrons has been calculated by Henyey and Keenan(6). They show that these collisions are responsible for the observed "cosmic static" in the plane of the galaxy and that it just fails to give observable diffuse background at visible wave-lengths. Their results are given in Table 2, where the intensity is given in ergs/sec/cm^2/kilocycle band/square degree.

TABLE 2

Radiant Energy from Interstellar Gases

λ	$\log_{10} (I_\nu \times 10^{19})$
1000 cm.	1.0
100	1.75
10	1.7
1	1.6
10^{-1}	1.5
10^{-2}	1.3
10^{-3}	1.05
10^{-4}	.85
10^{-5}	.7

I_ν in ergs/sec/kc/cm^2/degree^2

These results have been checked in the radio range by direct measurement of "cosmic static" both from this galaxy and from Andromeda by Reber[7], Hey, Phillips and Parsons[8] and many others.

Hey, Phillips, and Parsons find that the mean intensity, averaged over all directions, is one-third that given in Table 2 and we shall use this factor to correct the curve throughout the spectrum. As pointed out by Henyey and Keenan[6] the energy in the visible must also be corrected for absorption by interstellar dust. Applying these corrections and changing units we obtain $\sigma(\lambda)$, the number of photons/cm^2/sec/A^o/steradian as given in Table 3. Also tabulated are several integrals related to $\sigma(\lambda)$ which we shall find useful later. These integrals were obtained by numerical integration using the trapezoidal rule and were smoothed where necessary.

Korff[9] has shown that the radiation density in intergalactic space is approximately .01 that in the galaxy by using Hubble's galactic counts and luminosities. For lack of better information we may assume the spectrum is the same. The amount of matter in intergalactic space is probably small since Hubble[10] finds that the absorption of light from distant galaxies is less than .1 magnitude. Assuming dust particles of 10^{-5} cm, we can have only a mass of one-tenth of a single layer of 10^{-5} cm thickness in a volume one cm^2 by 3×10^{26} cm. This corresponds to a density of less than 3×10^{-32} gm/cm^3. The density of ionized gas and atoms which give strong absorption lines in the visible is even less. There is no observational limit to the hydrogen density although it is probably small. The density of free electrons can be limited by the lack of observable Thompson scattering or by the lack of radio waves due to collisions, but neither limit is very stringent.

The expansion of the universe does not affect the conditions inside the galaxy since its diameter remains unaltered. The small change in density of radiation due to the crowding of the galaxies in the past may be neglected.

On the other hand the radiation density outside the galaxy is inversely proportional to the square of the radius of space. Assuming a linear expansion the age of the universe is roughly two billion years, so that the properties of intergalactic space have not changed significantly for our purposes in the last billion years.

TABLE 3

Photon Intensity in Interstellar Space and Related Functions

λ	$\sigma(\lambda)$	$\tau(\lambda)$	$\omega(\lambda)$	$L(\lambda)$	$M(\lambda)$
A°	photons/cm^2 /sec/A°/rad	photons/cm^2 /sec/A°	photons/cm^2 /sec/ A°	photon - A° /cm^2 / sec	photon - A° /cm^2 / sec
1×10^2	1.44	5.8×10	4.9×10	$.012 \times 10^7$	$.006 \times 10^7$
2	11.5	46	39	.37	.17
3	39	160	130	2.8	1.34
5	180	720	600	36	17.2
7	490	1700	1300	180	104
1×10^3	2.4×10^3	5.8×10^4	3.5×10^4	$.86 \times 10^{10}$	$.46 \times 10^{10}$
2	7.3	16.6	12.8	15	8.5
3	15	35	27.5	83	43
5	38	93	74	600	310
7	73	102	72	1850	800
1×10^4	140	70	42	4.0×10^{13}	2.5×10^{13}
2	20	25	10.3	9.9	6.6
3	6.6	12.6	4.5	14	10.6
5	2.2	5.7	1.9	20	14.5
7	1.3	3.6	1.3	24	17
1×10^5	.95	2.3	.95	32	20
2	.70	1.2	.72	55	29
5	.50	.65	.35	135	69
1×10^6	.33	.35	.20	$.30 \times 10^{16}$	$.15 \times 10^{16}$
2	.19	.22	.12	.71	.38
5	.09	.11	.06	2.2	1.27
1×10^7	.05	.06	.035	4.9	3.2
2	.027	.033	.020	11.2	7.8
5	.012	.015	.009	32	23
1×10^8	66×10^{-1}	77×10^0	43×10^0	70	48
2	36	43	23	150	110
5	15	19	10	420	260

Table 3 (Continued)

λ	σ(λ)	τ(λ)	ω(λ)	L(λ)	M(λ)
1 x 10^9	8 x 10^{-1}	10 x 10^0	5.5 x 10^0	.88 x 10^{19}	.58 x 10^{19}
2	4.3	5.2	2.8	1.9	1.3
5	1.8	2.2	1.1	5.1	3.2
1 x 10^{10}	.9	1.0	.5	10.5	7.0
2	.33	.44	.20	20.1	13
5	.07	.11	.04	42	29
1 x 10^{11}	170 x 10^{-5}	330 x 10^{-4}	85 x 10^{-4}	64 x 10^{19}	42 x 10^{19}
2	20	88	15	99	54
5	1.4	15	2.5	136	62
1 x 10^{12}	.2	3.8	.25	170	67
2	.02	1.0	.025	200	71
5	.002	.15	.0025	240	74
1 x 10^{13}	.0002	.04	.0004	280	75

$$\tau(\lambda) = \frac{2\pi}{\lambda^2} \int_0^{2\lambda} x\,\sigma(x)\,dx$$

$$\omega(\lambda) = \frac{\pi}{\lambda^3} \int_0^{2\lambda} x^2\,\sigma(x)\,dx$$

$$L(\lambda) = \int_0^{\lambda} x\,\tau(x)\,dx$$

$$M(\lambda) = \frac{4}{\lambda} \int_0^{\lambda} x^2\,\tau(x)\,dx - \frac{4}{\lambda^2} \int_0^{\lambda} x^3\,\tau(x)\,dx$$

2.2. Magnetic Fields

The terrestrial and solar magnetic fields have no effect on the general distribution of cosmic rays but merely keep low energy particles from reaching the earth. These fields also act as a spectrograph and allow an analysis of the charge and momentum distribution of the primary particles. The general field of the sun is of the order of 25 gauss at the pole and is primarily of dipole character. Recently Babcock(11) has measured the Zeeman effect in several early A-type stars with the results that the field strength at the pole is of the order of 1000 gauss and that the dipole moments are randomly oriented.

The most important unanswered question is whether there exists a uniform magnetic field throughout the galaxy due to its rotation. Extremely small fields can produce a large effect over astronomical distances; a field of 10^{-12} gauss would prevent charged particles of energy less than 10^{13} electron volts from reaching the earth from outside the galaxy, or vice versa, particles of less than this energy could not escape from the galaxy. Alfven(12) and Spitzer(13) have proposed fields of this magnitude but a careful analysis of the various possible sources of such a field and of the diamagnetic effect of the interstellar electrons is lacking. The diamagnetic effect of the electrons, in contrast to the degenerate electron gas in metals, is strongly field dependent so that there may be several equilibrium configurations. Furthermore the relaxation time of a large scale magnetic field is very low as may be seen from the formula given by Smythe(14)

$$\text{Relaxation time} = \frac{4\pi\mu}{\tau}\left(\frac{R}{3.2}\right)^2 \qquad (2.1)$$

for the lowest mode, where τ is the resistivity, μ is the permeability, R is the radius of the sphere, and 3.2 is the root of an equation involving

Bessel functions. In a sphere with radius one light-year and with a resistivity of 0.2 ohm-cm(15)* the relaxation time comes out to be 2×10^{20} years. For the whole galaxy the time would be about 10^{28} years, but the dynamical relaxation time of the galaxy is of the order of 10^{10} years and this time dominates the formation of the galaxy and the generation of a general magnetic field. However, the field is probably not in equilibrium at present so that we cannot compute its value without making assumptions concerning the initial conditions. Since many rotating astronomical bodies have a magnetic field, it is reasonable to assume that the galaxy has one, but the magnitude of the field cannot be estimated.

At the critical energy at which particles could escape from or enter the galaxy it is to be expected that there would be a discontinuity in the observed spectrum. No such discontinuity is observed and, although the experimental error is large, we may conclude that the field is less than 10^{-16} gauss (limiting energy 10^9 ev) or even greater than 10^{-10} gauss (10^{15} ev). Since we do not know which alternative is correct, we shall treat both cases separately for completeness.

2.3. Diffusion of Cosmic Rays

Because of the large distances in the galaxy, small charge or current unbalances can set up tremendous electric and magnetic fields, fields which can modify considerably the motion of charged particles. Swann(16) has shown that if the cosmic rays incident on the top of the atmosphere were all positively charged, and if the charge were not neutralized, there would exist interstellar electric fields of stupendous size. He showed that the difference of potential between the earth and a point on a sphere of radius $\underline{R}$ light years is greater

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*It should be noted that this value of the resistivity applies to the relaxation time of a magnetic field although it was derived assuming no magnetic field. It is probable that the conductivity of the interstellar medium was considerably smaller when the galaxy was formed than it is now, but it is unlikely that it was so small that the relaxation time of the magnetic field was less than the dynamical relaxation time.

than $7R^2 \times 10^{17}$ volts. Thus, even in traveling one light year, cosmic ray particles would lose or acquire much more than their mean energy. As pointed out by Alfven[17], however, this result does not rule out the possibility of only positively charged primaries since a very small increase in electron density would balance the field (the number of primaries/cm^3 is about 10^{-8}, while there is about one electron/cm^3). Evans[18] has computed the potential which a star would acquire if it intercepts only positively charged cosmic rays. He shows that the conductivity of interstellar space, which is due to the free electrons, reduces the potential to 10^{-9} volts. If there is a galactic magnetic field the conductivity perpendicular to the field is much reduced since charge can diffuse only by collisions which change the orbit in the field. However, it is still reasonable to assume that the potential is much less than one volt so that we may neglect all electrical fields in the galaxy.

The effects of magnetic fields are somewhat more complicated than the effects of electrical fields but they are not unwieldy. The stellar fields scatter the particles, but do not disturb the isotropy of the primary distribution[19]; in fact, they tend to make non-uniform distributions more nearly isotropic.

The isotropy of the distribution of cosmic rays allows us to draw certain conclusions concerning their source. The secular variations in intensity do not show a significant variation with sidereal time. Periods of a solar day and of 27 days (solar rotation period) are associated with the solar magnetic field[20,21]. The variation with sidereal day is below the experimental error of 0.1 percent[22]. Compton and Getting[23] suggested that the motion of the earth (280 km/sec) due to the rotation of the galaxy would increase the intensity on the front side of the earth and give rise to a variation with sidereal time. Vallarta, Graef and Kusaka[24] calculated the effect taking into account the earth's magnetic field and predicted an effect of 0.17 percent for all positive particles. The effect is probably reduced by scattering in stellar magnetic fields and may be masked by a small galactic magnetic field. Whether the small predicted variation exists or not, we may draw certain conclusions from the isotropy of the cosmic rays.

It is reasonable to assume that the rate of generation of cosmic rays is proportional to the density of matter on the average so that we should expect the greatest generation in the plane of the galaxy. Thus, if there is no galactic magnetic field, the intensity of cosmic rays should be large in the galactic plane just as visible light is distributed. Of course, the scattering by the stellar magnetic fields would blur this distribution but the anistropy would still be observable. Hence, we must assume that the cosmic rays were generated when the universe was young or that their generation is independent of the location of matter, a physically unattractive assumption. In either case, we may assume that their mean age is nearly that of the universe or conservatively 10^9 years.

If we assume the presence of a galactic magnetic field, the distribution should be isotropic since the particles would have completed many circles and would have been scattered several times by stellar fields. Hence, isotropy does not indicate any particular source. There are two attractive features of this assumption. First, the small fraction of neutral particles in the primaries (as seen from the geomagnetic effects) would be explained since they would leave the galaxies and be diluted by the large volume of intergalactic space. Second, the retention of the particles by the field would explain the large intensity of cosmic rays compared to light[25,28]. It is true that the mean life of the primaries is less in the galaxy due to the interaction with dust but there is still a net increase in intensity. It should be noted that the galactic field would not retain the particles indefinitely since scattering by the stellar fields would cause a slow diffusion outward.

Alfven[12,17] has calculated the effect of self produced magnetic fields upon a beam of cosmic ray particles and has obtained a maximum current which would not be destroyed by its own field. Actually, the large relaxation time of even a local field would prevent its formation and so his restrictions do not apply and we may assume that the distribution of the primaries is limited only by the general galactic field if it exists.

In passing through interstellar space the cosmic rays will interact with the dust and gas present. Assuming the density of 2×10^{-24} gm/cm^3, a particle would penetrate only 0.1 gm/cm^2 of matter in traversing the galaxy so that the matter would cause no effect (experiment indicates that about 100 gm/cm^2 are required to absorb one-half the primaries). The matter is probably denser toward the center of the galaxy and Zwicky[26] has suggested that the cosmic radiation may be less in this direction, but the effect is probably still less than experimental error. If the primaries are confined to the galaxy by a magnetic field, the effect of the matter is larger. A particle retained in the galactic plane would traverse 20 gm/cm^2 in 10^7 years. Actually, the particle would spend only a small part of its life in the plane of the galaxy if the field is roughly similar to that of a current loop. Hence, we may assume the mean life of the particle is of the order of 10^8 - 10^9 years. If the particles come from extra-galactic space, the interaction with matter is negligible.

Pomeranchuk[27] has pointed out that high energy charged particles will radiate due to their acceleration in a magnetic field and he gives the expression

$$\frac{dE}{dt} = -\frac{2}{3} c \left(\frac{e^2}{mc^2}\right)^2 \left|\frac{\overline{v}}{c} \times \overline{B}\right|^2 \left(\frac{E}{mc^2}\right)^2 . \tag{2.2}$$

For an electron with energy 10^{20} ev in a field of 10^{-10} gauss the fractional loss of energy in 10^7 years is 10^{-13} and for protons it is 10^{-26}. Hence, we may neglect the effect of a general galactic magnetic field on the energy of the particles.

In approaching the earth, the field varies and the expression must be integrated. Pomeranchuk[27] has done this and gives the approximate expression

(2.3) $$E = \frac{E_0 E_c}{E_0 + E_c} ,$$

where $\underline{E}$ is the energy of the particle at the surface of the earth, $\underline{E_0}$ is its initial energy and $\underline{E_c}$ is a constant involving the radius and the magnetic moment of the earth and the type of particle and its orbit. It is given by the expression

(2.4) $$E_c = \frac{\alpha\ R^5}{M^2} ,$$

where R is the radius of the earth, M the earth's dipole moment, and α is a constant, proportional to the fourth power of the particle's mass, which, for normal incidence at the equator, is

(2.5) $$\alpha = 4.0 \times 10^{25}\ \text{ev/gauss}^2\text{-cm.}$$

Using this value of α, the value of E_c is 7×10^{17} ev for electrons and 10^{30} ev for protons. As a particle is scattered by a stellar magnetic field it loses energy but the loss is insignificant unless it approaches within a few stellar radii. This means that the cross section for capture is increased by a small factor but stellar radii are so small compared to interstellar distances that capture may be neglected.

One other source of energy loss may be mentioned. Epstein[28] has shown that the energy of a high energy particle varies inversely as the radius of the universe. Hence, Lemaitre's assumption that the primaries were generated along with the universe in an explosion of a giant atom is untenable since their initial energies would have to have been tremendous. However, it is possible that they were generated in the early stages of the universe when the density and radiation pressure were high. Even in this case the initial energies must have been many times their present values and other forms of loss more probable. We shall come back to this point after studying the interaction with radiation. If the particles are confined to the galaxy by a magnetic field, their age is so short that expansion plays no role in their history.

III. INTERACTION OF PARTICLES WITH RADIATION

We are interested in determining the maximum energy that the primary cosmic rays can have. That there exists a maximum is not obvious since the interaction of high energy particles with matter and radiation might have a maximum and decrease at very high energies. Actually, this is not the case, since, even though the interactions we shall consider do have a maximum, there are other reactions which only become important at these high energies, which will keep the total interaction from decreasing.

We shall be primarily interested in the slowing down of the high energy particles by interaction with the radiation in interstellar space. It is to be noted that slowing down the high energy particles does not violate the principle of relativity since the stars from which the radiation is emitted define a unique inertial system. The interaction may also be looked upon as an approach to equi-partition of energy by degradation of the primary energy.

There are two different phases of the interaction of high energy particles with radiation; interaction with interstellar or intergalactic radiation while transversing space, and interaction with radiation while being generated, if the source is near a star. We must discuss each case for each possible type of primary particle, viz., photons, electrons, photons and light nuclei (neutrons and mesotrons are unstable and would decay; neutrinos are unobservable). Before investigating these cases, we shall discuss the theory of the various interactions. All these particles can interact with radiation in several ways but we shall discuss only the most important; photon-photon pair production, Compton scattering of electrons, pair production in the fields of the proton and the nuclei, and photo-disintegration of nuclei.

3.1. Theory of the Interaction of Protons and Nuclei

In considering the interaction of a high energy particle with radiation, it is convenient to transform to a coordinate system moving with the particle and to make all the computations in this system. The cross-sections are usually given with the particle at rest and the momentum transfer is usually simplest in such a system. Let us consider a particle of mass m moving with velocity v along the z axis of the coordinate system. Using hyperbolic functions we have

$$v = c \tanh \chi ,$$
$$p = mc \sinh \chi , \tag{3.1}$$
$$E = mc^2 \cosh \chi ,$$

where χ may be defined by any of the three equations and the other two are then consequences. A photon of frequency ν, whose direction of motion makes an angle α with the negative z axis, will, in the moving coordinate system, have the frequency

$$\nu' = \nu(\cosh \chi + \cos \alpha \sinh \chi), \tag{3.2}$$

and the direction

$$\sin \alpha' = \frac{\sin \alpha}{\cosh \chi + \cos \alpha \sinh \chi} . \tag{3.3}$$

The converse equations are

$$\nu = \nu' (\cosh \chi - \cos \alpha' \sinh \chi),$$
$$\sin \alpha = \frac{\sin \alpha'}{\cosh \chi - \cos \alpha \sinh \chi} . \tag{3.4}$$

It is to be noted that if $\cosh \chi >> 1$, then $\nu' >> \nu$ and $\alpha' << 1$ for almost all α. Hence, we may say that the particle sees a beam of high energy photons incident along the z axis. For this reason we can define the function $\tau(\lambda)$ which is defined so that $\tau(\lambda)d\lambda$ gives the number of photons/cm^2/sec* with wave length in the range $\frac{\lambda}{\cosh \chi}$ to $\frac{\lambda + d\lambda}{\cosh \chi}$, where λ is given in Angstroms. This function is convenient since it is nearly independent of the velocity of the particle at large velocities. In interstellar space where the radiation is isotropic $\tau(\lambda)$ may be determined from the integral:

$$(3.5) \qquad \tau(\lambda)\, d\lambda = 2\pi \int_0^\pi \sin\alpha\, d\alpha\, \sigma(\lambda') \frac{d\lambda'}{d\lambda}\, d\lambda,$$

where $\lambda' = \lambda(1 - \cos\alpha \tanh\chi)$. Setting $\tanh\chi = 1$ and changing to λ' as variable of integration, we have:

$$(3.6) \qquad \tau(\lambda) = \frac{2\pi}{\lambda^2} \int_0^{2\lambda} \lambda'\ \sigma(\lambda')\, d\lambda'.$$

This function is tabulated in Table 3.

In a coordinate system moving with the photon it is easy to see that the dominant interaction with light is pair production. If we assume that the electrons are ejected with equal velocities in the direction of the photon beam conservation of momentum and energy give us:

$$(3.7) \qquad \begin{aligned} h\nu' + Mc^2 &= Mc^2 \cosh\psi + 2mc^2 \cosh\phi \\ \frac{h\nu'}{c} &= Mc \sinh\psi + 2mc \sinh\phi, \end{aligned}$$

where $h\nu'$ is the photon energy, M the mass of the proton, $c \tanh\psi$ its velocity m the mass of the electrons and $c \tanh\phi$ the electron velocity.

*We measure time in the stationary coordinate system since the Lorentz factors will cancel in the final answer.

Writing

$$(3.8)\qquad \gamma = \frac{h\nu'}{mc^2}, \quad \mu = \frac{M}{m},$$

we have

$$(3.9)\qquad \begin{aligned} 2\cosh\phi &= \gamma - \mu(\cosh\psi - 1), \\ 2\sinh\phi &= \gamma - \mu\sinh\psi. \end{aligned}$$

Squaring and subtracting

$$(3.10)\qquad 4 = 2\gamma\mu(\sinh\psi - \cosh\psi + 1) + \mu^2(2 - 2\cosh\psi).$$

Since most of the momentum is absorbed by the electrons, ψ is small. Expanding the hyperbolic functions in (3.10) and solving the resultant quadratic equation, we have

$$(3.11)\qquad \psi = \frac{4}{\gamma\mu + \sqrt{\mu^2\gamma^2 - 4\mu^2 - 4\gamma\mu}}; \qquad \gamma > \frac{2}{\mu} + 2.$$

Transforming back to the stationary coordinate system the proton energy becomes

$$(3.12)\qquad E = Mc^2\cosh(\chi - \psi) \doteq Mc^2\cosh\, e^{-\psi} \doteq E_0\,(1 - \psi),$$

so that (3.11) gives the fraction of energy lost by the proton per pair produced. While (3.11) was derived under special assumptions, it is nearly correct for all probable directions of emission and division of the energy between the electrons so that we may use it with little error.

The cross section for pair production(29) is a complicated expression which, in the high energy range, simplifies to*

$$(3.13)\qquad \phi(\gamma) = \frac{1}{137}\left(\frac{e^2}{mc}\right)^2\left[\frac{28}{9}\ln 2\gamma - \frac{218}{27}\right].$$

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*We use the cross sections without screening by the atomic electrons because the atoms would be ionized very quickly by the interaction of the electrons with radiation (Paragraph 3.2) even if they were generated in an unionized state. It is to be noted that the cross sections are used in an experimentally verified region since, even though the particle energies are very large, the photon energy in the moving coordinate system is only a few Mev.

Since the fractional transfer of energy is small, we may neglect statistical fluctuations and write

$$(3.14) \qquad \frac{1}{E}\frac{dE}{dt} = \int_0^{\frac{h}{2mc}\cosh} \varphi(\gamma)\,\frac{4}{\gamma\mu+\sqrt{\gamma^2\mu^2-4\mu^2-4\gamma\mu}}\,\tau(\lambda)d\lambda,$$

where

$$(3.15) \qquad \gamma = \frac{h}{mc}\,\frac{\cosh\chi}{\lambda}.$$

In the case of atomic nuclei (3.13) must be multiplied by the square of the atomic number, Z, and (3.11) divided by the atomic number A, so that (3.14) is multiplied by (Z^2/A). The factor multiplying $\tau(\lambda)$ in (3.14) is very small so that the loss of energy is very slow. This factor is tabulated in Table 4 for later use.

TABLE 4

Cross Section for Energy Loss by Pair Production

γ	$\frac{4\,\phi(\gamma)}{\gamma\mu+\sqrt{\mu^2\gamma^2-4\mu^2-4\gamma\mu}}$	γ	$\frac{4\,\phi(\gamma)}{\gamma\mu+\sqrt{\mu^2\gamma^2-4\mu^2-4\gamma\mu}}$
2	0×10^{-32} cm^2	200	3.34×10^{-32} cm^2
3	2.0	500	1.69
4	5.33	1×10^3	.988
5	7.92	2	.555
6	9.48	5	.258
10	12.20	1×10^4	.142
20	11.21	2	.078
50	7.97	5	.035
100	5.33	1×10^5	.019

Photo-disintegration of the nuclei is also possible and may occur with high velocity nuclei. It requires 8 Mev (= 16 mc^2) to eject a neutron or proton from the nucleus so that, if the cross section is denoted by γ, the probability of disintegration per second is

$$(3.16) \qquad \int_0^{\frac{h}{16\ mc}\cosh} \sigma_\gamma \, T(\lambda)\, d\lambda.$$

The cross section σ_γ is proportional to γ^3 according to Weisskopf[30] and, according to Bothe and Gentner[31], the coefficient of proportionality is $.65 \times 10^{-29}$ $cm^2/(Mev)^3$. This result holds at energies such that the wave length is less than the nuclear circumference. At higher energies (approx. 100 Mev) the cross section again diminishes. In the case of deuterium we may use the theoretical expression for the cross section given by Bethe and Bacher[32],

$$(3.17) \qquad \sigma_\gamma = \frac{4\ e^2 h}{3Mc}\, \frac{I^{\frac{1}{2}}\ (h\nu - I)^{\frac{3}{2}}}{h\nu^3}$$

where I is the binding energy of deuterium and M is the mass of the proton.

3.2. Theory of the Interaction of Electrons and Photons

The interaction of electrons with radiation is just Compton scattering in the coordinate system moving with the electrons but the treatment is more complicated since large energy transfers can take place so that fluctuations are important. Hence, we must compute the probability for various fractional energy losses for all values of the energy. Similar considerations hold for the interactions of photons. The situation is further complicated by the fact that high energy electrons generate photons and high energy photons generate electrons so that a sort of "cosmic cascade" ensues.

If we now consider a photon γ in the moving coordinate system incident along the negative z - axis and assume a Compton scattering, then the energy of the scattered photon is given by

$$(3.18) \qquad \gamma' = \frac{\gamma}{1 + \gamma(1 - \cos\theta)},$$

where $\underline{\theta}$ is the angle of scattering. The cross section is[33]

$$(3.19)\quad d\phi = \frac{1}{2}\left(\frac{e^2}{mc^2}\right)^2 \frac{\sin\theta\, d\theta}{\left[1+\gamma(1-\cos\theta)\right]^2}\left\{(1-\cos\theta)+\cos^2\theta+\frac{1}{1+\gamma(1-\cos\theta)}\right\}$$

and the mean scattering angle is

$$(3.20)\quad \theta_{mean} = \sqrt{\frac{2}{\gamma}}\,.$$

Now in order that we may neglect the beam width (3.3) we must have $\alpha' << \theta_{mean}$. This is true since $\alpha' \approx \frac{1}{\cosh\chi}$, $\gamma = \cosh\chi \frac{h\nu}{mc^2}$, so that

$$(3.21)\quad \frac{\alpha'}{\theta_{mean}} \approx \sqrt{\frac{h\nu}{2mc^2\cosh\chi}} << 1.$$

Conversely (3.21) and (3.4) insure that the scattered photon will be along the $\underline{z}$ - axis in the stationary coordinate system. The energy of these photons is

$$(3.22)\quad \begin{aligned}\frac{h\nu'}{mc^2} &= \gamma'(\cosh\chi - \cos\theta\sinh\chi)\\ &\doteq \gamma'(1-\cos\theta)\sinh\chi\\ &= \frac{\gamma(1-\cos\theta)}{1+\gamma(1-\cos\theta)}\sinh\chi.\end{aligned}$$

Consequently, setting $z = \cos\theta$, the fractional energy loss, $\underline{\delta}$, of the electron is

$$(3.23)\quad \begin{aligned}\delta &= \frac{h\nu' - h\nu}{mc^2\cosh}\\ &= \frac{\gamma(1-z)}{1+\gamma(1-z)} = 1 - \frac{1}{1+\gamma(1-z)}.\end{aligned}$$

Solving for z we have the relations

$$z = 1 - \frac{1}{\gamma(1-\delta)},$$

(3.24) $$0 \leq \delta \leq \frac{2\gamma}{1+2\gamma}.$$

$$\frac{dz}{d\delta} = \frac{1}{\gamma^2}\,\frac{1}{(1-\delta)^2},$$

Substituting into (3.19) the cross section for a fractional loss between δ and $\delta + d\delta$ is

$$d\phi = \frac{1}{2}\left(\frac{e^2}{mc^2}\right)^2 (1-\delta)^2\left\{\frac{1}{1-\delta} + \left[1 - \frac{\delta}{\gamma(1-\delta)}\right]^2 + 1 - \delta\right\}\frac{dz}{d\delta}\,d\delta$$

(3.25)

$$= \frac{1}{2}\left(\frac{e^2}{mc^2}\right)^2\left\{\frac{1}{1-\delta} + 1 - \delta - \frac{2\delta}{\gamma(1-\delta)} + \frac{\delta^2}{\gamma^2(1-\delta)^2}\right\}d\delta,\quad 0 \leq \delta \leq \frac{2\gamma}{1+2\gamma}.$$

It may be seen that there is a large probability of large fractional energy loss since $(1-\delta)$ occurs in the denominator. An idea of the order of magnitude of the cross section may be obtained from the following table.

TABLE 5

The Values of δ and $d\phi$ from (3.23) and (3.25)

	γ \ θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
δ	0.1	0	.028	.091	.146	.167
	1.0	0	.227	.50	.63	.80
	10.	0	.745	.91	.945	.952
$d\phi \times 10^{26}\,cm^2$	0.1	80.0	60.4	40.4	61.2	84.8
	1.0	8.00	6.28	6.00	10.32	13.32
	10.	0.80	1.28	4.04	7.04	8.44

It may be noted from the table that for large γ the mean loss is nearly the maximum loss (for $\gamma = 10$, $d\phi$ drops to one-half when δ decreases from .95 to .90).

The effect of the large fractional loss leads to an interesting conclusion. We may say roughly that the mean fraction of energy retained is about equal to twice the minimum, i.e. $(1 - \delta)_{mean} \; \frac{2}{1 + 2\gamma}$. If we assume that the electron interacts with a 3 volt (4000 A°) photon then we have

$$\gamma = \frac{h\nu \cosh \chi}{mc^2} ,$$

(3.26)
$$\frac{2}{1 + 2\gamma} \doteq \frac{mc^2}{\cosh \chi} \frac{1}{h\nu} ,$$

$$E' = \frac{2E}{1 + 2\gamma} = \frac{(mc^2)^2}{h\nu} = 8.3 \times 10^{10} \text{ ev}$$

Thus a very high energy electron will drop to about 10^{11} ev in a single interaction; it does not share its energy equally with the photon as is usual in most high energy interactions.

To obtain the probable loss for a cosmic ray particle we must multiply (3.25) by the photon spectrum and integrate. If $P(\delta)\, d\delta$ is the probability of losing a fraction of energy δ per second, we have

(3.27)
$$P(\delta)\, d\delta = \int_0^\lambda \mathcal{T}(\lambda)\, d\phi\, d\lambda, \quad y = 2 \cosh \chi \frac{h}{mc} \frac{1 - \delta}{\delta}$$
$$= \frac{1}{2} \left(\frac{e^2}{mc^2}\right)^2 \frac{mc}{h \cosh \chi} \left\{ \left[\frac{1}{1 - \delta} + 1 - \delta \right] L(y) - M(y) \right\} d\delta/\text{second},$$

where the functions $L(y)$ and $M(y)$ only involve the photon spectrum and are defined by

(3.28)
$$L(y) = \int_0^y \lambda \mathcal{T}(\lambda)\, d\lambda,$$
$$M(y) = \frac{4}{y} \int_0^y \lambda^2 \mathcal{T}(\lambda)\, d\lambda - \frac{4}{y^2} \int_0^y \lambda^3 \mathcal{T}(\lambda)\, d\lambda.$$

These functions are tabulated in Table 3.

The cross section for photon-photon pair production has been worked out by Breit and Wheeler[34], who give the expression

$$(3.29)\quad \Phi(C) = 2\pi \left(\frac{e^2}{mc^2}\right)^2 \left[-SC^{-3} - SC^{-5} - \theta C^{-6} + 2\theta C^{-4} + 2\theta C^{-2}\right].$$

where

$$(3.30)\quad C = \cosh\theta = \frac{h(\nu_1\nu_2)^{\frac{1}{2}}\sin\phi/2}{mc^2}, \qquad S = \sinh\theta,$$

and ϕ is the angle between the directions of the photons. Φ must be multiplied by $\sin^2\phi/2$ when the photons do not collide head on. It is apparent from Table 6 that the magnitude of the cross section is comparable with that for Compton scattering so that the order of magnitude of the maximum energy should be about the same.

TABLE 6

Cross Section of Photon-Photon Pair Production from (3.29)

θ	.10	.5	1	2	3	4.605	6.908
S	.1002	.5211	1.1752	3.627	10.02	100	1000
C	1.050	1.1276	1.5431	3.762	10.07	100	1000
$\Phi \times 10^{26}$ cm^2	5.12	25.38	32.80	11.27	2.46	.0473	.0007

If C from (3.30) is less than one, the energy of the photons is insufficient to make a pair, but scattering of light by light, with the formation of a virtual pair as an intermediate state is possible, although small. Euler and Kockel[35] have estimated that for long wave lengths the cross section is approximately

$$(3.31)\quad Q \approx \left(\frac{e^2}{mc^2}\right)^4 \left(\frac{h}{mc}\right)^4 \frac{1}{\lambda_1^8}\,\frac{1}{\lambda_2^8}\,\sin^6\phi/2.$$

We may expect that Q is less than but comparable to the pair production cross section at energies at which pairs may be produced and that it drops off according to (3.31) at low energies. Hence, the scattering is only appreciable for a small range of energy at the limiting energy and has little effect on the maximum energy; we shall neglect it altogether.

The formula for (λ) was derived on the basis of a coordinate system moving with the high energy particle but (3.29) was derived in a center of momentum coordinate system, which varies with the energy of the light quantum. We must, therefore, compute the interaction directly from the photon density function (λ). Keeping γ constant, the probability of interaction per second is

$$(3.32) \quad 2\pi \int_0^\pi \int_0 \sin\phi\, d\phi\, d\lambda\, (\lambda) \left(\frac{h\gamma}{mc\lambda} \sin^2\phi/2 \right)^{\frac{1}{2}} \sin^2 \frac{\phi}{2} .$$

Transforming to the new variables of integration ϕ and x

$$(3.33) \quad \begin{aligned} \phi &= \phi, \\ x &= \frac{\lambda}{1 - \cos\phi}, \\ d\lambda &= (1 - \cos\phi)\, dx, \end{aligned}$$

(3.31) becomes

$$2\pi \int_0^\pi \int_0 \sin\phi\, d\phi (1 - \cos\phi)\, dx\, (x - x\cos\phi) \left(\frac{1 - \cos\phi}{2} \; \frac{h}{2mcx} \right)^{\frac{1}{2}}$$

$$(3.34) \quad = \int_0 \omega(x) \left(\frac{h}{2mcx} \right)^{\frac{1}{2}} dx,$$

$$= \int_0^{\frac{h\gamma}{2mc}} \omega(\lambda) \left(\frac{h}{2mc\lambda} \right)^{\frac{1}{2}} d\lambda,$$

where the upper limit is replaced by the largest value of λ for which does not vanish and

$$(3.35) \qquad \omega(\lambda) = \frac{\pi}{\lambda^2} \int_0^{2\lambda} \sigma(x)\, x^2\, dx.$$

This function is similar to $\tau(\lambda)$ as can be seen from Table 3 where it is tabulated.

The energy of the electrons is restricted between $\frac{\gamma mc^2}{2}(1 - \tanh\theta)$ and $\frac{\gamma mc^2}{2}(1 + \tanh\theta)$ but is strongly skewed and one particle takes most of the energy. In the center of mass system the distribution is isotropic but even a small forward component of velocity gives a large energy ratio; the ratio of the energies is $e^{2\theta\cos\beta}$ where $\underline{\theta}$ is defined in (3.30) and $\underline{\beta}$ is the angle the pair makes with the direction of motion of the coordinate system. Since both here and in the Compton scattering most of the energy is carried by one secondary the energy of a high energy particle is only slightly dissipated even though several secondaries are produced and there is no upper limit to the energy unless a large number of collisions are possible. However, if the light quantum has just sufficient energy the asymmetry is not marked; the integration over the spectrum introduces many such photons so that the distribution in energy is reasonably smooth.

3.3. Transmission through Space

It might be expected that the large radiation density in the vicinity of the earth would cause a significant interaction with high energy particles. This is not so since the time the particles spend in the radiation is quite small. The radiation field of the sun at one light year is nearly equal to the mean interstellar density due to stellar radiation. The mean radiation density over this light year as the particles approach the earth is given by

$$\overline{\tau(\lambda)} = \frac{1}{1 - R_o} \int_{R_o}^{1} \frac{\tau(\lambda)}{R^2} dR \doteq \frac{\tau(\lambda)}{R_o} , \tag{3.36}$$

where R_o , the radius of the earth's orbit, is 10^{-5} light years. Hence, we see that approaching the earth is equivalent to a path of 10^5 light years in interstellar space, and a trajectory reaching the sun would have an equivalent path of 10^7 years. Since we are interested in path lengths of the order of 10^8 light years, the solar effect is small.

The interaction of heavy particles with radiation is small because of the small cross section and because of the large value of γ required for pair production and for photo-disintegration. The fractional loss of energy of photons over a path of 10^9 light years has been obtained by numerical integration of (3.14). The results are given in Table 7 for various proton energies. It is apparent that the loss is not important. The photo-disintegration probabilities of deuterons and alpha-particles have been obtained by numerical integration of (3.16) and they are also given in the table. For deuterons the rate of disintegration is small and it is only important for particles with energy greater than 10^{14} ev, which were generated 10^9 years ago. For alpha-particles the disintegration probability is the same for an energy of 10^{16} ev but for an energy greater than 10^{17} ev the mean range drops to 10^7 light years.

TABLE 7

Heavy Particle Interaction with Radiation in Interstellar Space

Energy	Fractional loss of energy by protons/10^9 light years	Probability of photo-disintegration/10^9 light years	
		Deuterons	α-particles
10^{14} ev	$.05 \times 10^{-5}$	.004	.004
10^{16}	4.5	.4	.4
10^{18}	6	.5	150
10^{20}	8	.7	200

The interaction of electrons with radiation is much stronger. Let us assume that there is a galactic magnetic field such that the electrons are confined to the galaxy. Then the photons generated in the Compton scattering process leave the galaxy and are diluted by the large volume of extragalactic space to a negligible concentration and we may compute the final electron spectrum from the number of collisions that an electron has with a light quantum. Using the integrals listed in Table 3, the value of the function $P(\delta)$ from (3.27) has been computed for several values of δ and the electron energy.

Since the energy loss may occur through a small number of large transfers the actual loss of energy by a high energy electron may only be given on the average. The final energy is given by the initial energy times the product of the fraction retained in the various collisions undergone. Hence the probable fractional energy retained is given approximately by the expression

$$\exp\left\{\int_0^1 P(\delta) \ln(1-\delta)\, d\delta\right\} \tag{3.37}$$

The values of $P(\delta)$ and of the integral in (3.37) are given in Table 8.

TABLE 8

Interaction of Electrons with Radiation in Interstellar Space
Values of $P(\ )/10^8$ years from (3.27)

$E \backslash \delta$	.001	.01	.1	.5	.9	.99	.999	$\int_0^1 P(\delta)\ln(1-\delta)d\delta$
10^{10} ev	1160	114	14.2	.004	----	----	----	- .2
10^{11}	1700	116	11.4	1.9	.002	----	----	- .85
10^{12}	2380	170	11.6	1.5	1.0	.002	----	-1.5
10^{13}	3040	238	17.0	1.55	.78	1.04	.002	-1.75
10^{14}	3600	304	23.8	2.34	.75	.83	1.04	-2.15
10^{15}	2200	360	30.4	3.28	1.19	.78	.83	-2.75
10^{16}	700	220	36.0	4.2	1.6	1.27	.78	-3.5
10^{17}	125	70	22.0	4.9	2.1	1.8	1.27	-3.5
10^{18}	18	12.5	7.0	3.1	2.5	2.3	1.8	-3.0

It is apparent from the table that electrons of energy greater than 10^{13} ev will drop to about one-tenth of their energy in 10^8 years and that in 10^9 years there would be no electrons of energy greater than 10^{10} ev. It is possible that the electrons of energy much greater than 10^{18} ev could drop down into the vacant energy range but their number should be small since the energy spectrum at the source would be expected to decrease rapidly with energy in this range. From the values of $P(\delta)$ it is apparent that most of the photons generated have roughly the same energy as the electron but that there are some with much less energy. We may conclude from this result that no electrons of energy greater than 10^{10} ev can have been generated more than about 3×10^8 years ago and that few have been generated more than 10^8 years ago.

In case there is no galactic magnetic field so that the primaries come from outside the galaxy the high energy photons produced by the Compton scattering cannot be neglected. It is also necessary to apply a correction to the radiation density and hence the above calculation applies to an electron traveling 10^{10} years in intergalactic space at the present radiation density. Actually, however, the expansion of the universe and the corresponding change in radiation density reduces the actual time to 1.6×10^9 years. Likewise, 10^9 years in the galaxy is equivalent to 1.96×10^9 years in intergalactic space.

Now let us consider the interaction of photons with radiation. Table 9 gives the mean number of pairs/10^8 years due to photon-photon pair production as given by (3.34).

TABLE 9

Photon-photon Pair Production in Interstellar Space

E, ev	10^{12}	10^{13}	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}
No. pairs/10^8 years	1.25	2.05	3.70	6.65	9.8	11.7	12.7

It is apparent that over a period of 10^8 years the photons will have generated several pairs which will in turn generate more photons so that a regular cascade develops. Actually, over the course of 10^8 years the mean number of secondaries per primary particle will not be large and many will have a large energy. However, over the course of 10^9 years the cascade will develop sufficiently so that the mean particle energy will fall to about 10^{12} ev, and over 5×10^8 years it will drop to 10^{13} ev. An exact computation of the development of the cascade would be very difficult and is not justified since the fundamental data are so uncertain.

3.4. Proposed Sources of Cosmic Rays

We are now in a position to investigate the composition of cosmic rays with various assumptions as to their origin. Lemaitre has suggested that the universe started as a gigantic atom which exploded giving rise to cosmic rays among the other products of the explosion. As pointed out at the end of section two,the expansion of the universe decreases the energy of fast particles inversely as the radius of the universe. Hence, the rays cannot have been present at time zero but they may have been generated by the interaction of the matter with itself at a slightly later epoch. The latest time at which this could happen by a mechanism not possible today was when the galaxies started to separate since the intergalactic forces could have been large. Furthermore, probably only existing particles, i.e., protons and electrons, would be accelerated under these circumstances.

Since the mean spacing of galaxies is about twenty times their mean diameter today, these particles must have been generated at one-twentieth of the present age or 1.9×10^9 years ago. Such particles must have traveled a distance equivalent to 4×10^8 years in the galactic radiation field and hence, any electrons and photons would be slowed down by cascade formation to about

10^{13} ev. This slowing down would take place largely in the first 10^8 years. The expansion of the universe would then reduce the energy to about 10^{12} ev. Any heavy particles would lose only the factor of twenty in energy due to the expansion so that with this source we would expect chiefly protons as cosmic ray primaries at high energies. Furthermore, the geomagnetic effects show that the number of primary photons is small so that the number of electrons initially more energetic than 10^{13} ev must be small since otherwise a large number of Compton recoil photons would be present. Therefore the accelerating mechanism must be most efficient with protons, a condition difficult to satisfy by electromagnetic fields.

If the red shift of the galactic spectral lines is not a velocity shift, so that the universe is stationary, then the particles could come from much greater distances. Epstein[28] has pointed out that they must come from 10^{11} light years distance to explain their intensity relative to light. The interaction with radiation would slow the electrons down to 10^{12} ev in this case also.

If the galaxy has a general magnetic field, then it is possible that its formation, when the galaxy was formed, accelerated charged particles by the betatron principle. At that time the ionization of the interstellar gas may not have been so large as it is now but there should have been sufficient free electrons and protons to give the present intensity of cosmic rays. As shown in the last section, the high energy electrons would be slowed down to an energy of 10^{10} ev in the 1.9×10^9 years since the formation of the galaxy but the protons would have lost little of their energy.

Zwicky has proposed that cosmic rays are generated in novae or supernovae, but this is not possible in the case of electrons due to the interaction with the radiation. As pointed out, the solar radiation field is sufficient to equal

10^7 light years for a particle approaching it. For a particle leaving it, the effect is reduced by a factor of about two since the particle must overtake the photons most of the way. Thus a star 100 times as luminous or 5 magnitudes brighter would not be able to emit any high energy electrons since the equivalent path would be 10^9 light years. The stellar spectra do not contain as much infra red as used in the computations, but this radiation is not vital and we may conclude that bright stars such as the O-type and early B-type stars, and, a fortiori, novae and supernovae, could not emit electrons with energies greater than 10^{10} ev. In the case of supernovae the radiation is 10^7 - 10^8 times as intense as that of the sun so that the interaction with protons is not negligible. From Table 7 we see that

$$\frac{dE}{E} = 4.5\ dt \tag{3.38}$$

for a proton energy of 10^{16} ev and for a supernova 10^7 times as bright as the sun. The time in (3.38) is a fictitious time which measures the fraction of radiation passed. Integrating (3.38) we have

$$E = E_o\ e^{-4.5} . \tag{3.39}$$

Hence the energy drops to a value such that the exponent is small, that is, to about 10^{15} ev (it drops to about 10^{14} ev for a supernova 10^8 times as bright as the sun).

In the case of cool stars, the interaction with radiation does not limit the possible energies of emitted particles. Swann(36) has proposed that high energy electrons could be formed in sun spots by the associated varying magnetic field by the betatron principle. Actually, this process need not be restricted to electrons since protons could also be accelerated, nor need it be limited to the sun since other stars probably have similar features.* However, the

*Recent data(37) on a large increase in cosmic ray intensity in conjunction with a solar flare suggests that such a mechanism is possible.

energy of the emitted electrons is limited by the radiation of energy to the transverse acceleration in the solar magnetic field. Using Pomeranchuk's(2.4) calculations, the maximum energy comes out to be about 5×10^{12} ev, whereas the energy needed to penetrate the solar magnetic field is 10^{14} ev. If the magnetic field due to the sunspot is properly oriented, it can cancel the general solar field sufficiently so that lower energy particles can escape from the sun by a tortuous path, but the maximum energy is probably not altered appreciably. Since there are no measurements of stellar magnetic fields, for the class of stars similar to the sun, it is not possible to say whether they can emit high energy electrons but it seems reasonable to say that their limiting energy is nearly the same as that of the sun.

Alfven[38] has suggested that cosmic rays could be generated in the joint magnetic field of a double star. A charged particle follows a trochoidal orbit in the field of one star and as the other rotates the flux through the orbit changes and accelerates the particle. The particle must be in an orbit around the first star which requires a large energy to escape to infinity since otherwise it would escape with little energy. Since the periodic orbits in a dipole field lie near the plane of the equator, we shall approximate the orbit by the circular periodic orbit which lies farthest from the dipole in order to obtain an upper limit to the energy. If the dipole moment of the star is $\underline{M}$ and the radius of the oribt is $\underline{R}$ then the particle energy, $\underline{E}$, in the orbit is

$$E = \frac{M e}{R^2} . \qquad (3.40)$$

Now the radiation emitted due to the acceleration in the magnetic field is at least that emitted assuming the particle leaves in a straight line. Hence, we shall use the maximum energy appropriate to this case although it is to be expected that the maximum energy is reduced significantly by spiral path since it materially increases the time of travel in the field. From (2.4) the maximum energy is

$$E_o = \frac{aR^5}{M^2} . \qquad (3.41)$$

Equating E and E_o we have

$$R = \left(\frac{e}{a}\right)^{1/7} M^{3/7} , \qquad (3.42)$$

$$E = E_o = a^{2/7} e^{5/7} M^{1/7} . \qquad (3.43)$$

From (3.43) the maximum energy which could be generated in a dipole field equal to that of the sun ($M = 10^{34}$ gauss - cm^3) is $.9 \times 10^{14}$ ev. For protons the maximum energy would be 5×10^{17} ev but here the orbit would lie inside the sun. It must be remembered that these limits are the results of a very crude approximation and that the maximum energy that could be obtained is probably more nearly a tenth of these values. In the case of the early type stars the dipole moment may be as large as 10^{38} emu so that these energies may be multiplied by four, but here, and to a certain extent in cooler stars, the particles remain in orbits near the star for a sufficient time to lose energy by Compton scattering.

Millikan(39) has suggested the cosmic ray primaries are generated by the spontaneous transformation of the mass of atoms in interstellar space into energy which is given to a pair of electrons. There is no theoretical method of treating this mechanism so that the observed energy spectrum can be compared quantitatively with the density of the various elements present but it seems difficult to explain the high energies required to generate Auger showers. However, this mechanism generates the particles far from intense radiation and strong magnetic fields to that there is no theoretical limitation to the amount of energy they could bring to the earth.

IV. CONCLUSION

The general conclusion that may be drawn from these calculations is that the high energy cosmic ray primaries are not electrons or photons because their interaction with interstellar radiation and magnetic fields is sufficient to slow them down, but that they are probably protons, while the lower energy primaries may be any type of particle. This result is in agreement with the experimental results of Schein and coworkers[40] although the data are insufficient to prove the energetic primaries are protons.

It is difficult to draw quantitative conclusions about the maximum energy electrons or photons can have because of the uncertainty concerning their place of origin. It is not even clear whether their place of origin is inside the galaxy or not because the existence of a general galactic magnetic field is uncertain. There is no astrophysical evidence either for or against such a field except that all rotating astronomical bodies seem to have a permanent magnetic field. If there is a general galactic magnetic field the primary cosmic rays must come from the galaxy but the distribution in angle should be uniform because of the character of orbits in a magnetic field. The intensity relative to light would be higher since the particles are spread over a smaller volume. Furthermore the absence of primary photons (the geomagnetic effect shows that nearly all the primaries are charged) is explicable by the dilution of the photons by the large volume of intergalactic space.

The calculations of the interaction with radiation show that the mean free path for high energy (10^{13} - 10^{18} ev) electrons to lose nine-tenths of their energy is 10^8 light years in the galaxy, so that the number of such energetic electrons should be much smaller than the number of low energy electrons. If the primaries were all generated when the galaxy was formed there should be no electrons of energy greater than about 10^{10} ev, but if they are being generated steadily the number of high energy electrons will be finite but small. Furthermore the maximum energy of electron primaries may be limited in the source. Thus if they are formed in B or O-type stars or in novae then the interaction with radiation would limit the energy carried away to 10^{13} ev, while if they are generated in supernovae the interaction with radiation limits the electron energy to 10^{10} ev and the proton energy to 10^{14} ev. If they are generated by the magnetic fields associated with "sun-spots" in main sequence stars by induced electric fields the loss of energy by radiation due to the acceleration in the stellar magnetic fields would limit the electron energy to 5×10^{12} ev. If they are formed in the joint magnetic field of double stars the electron energy is limited to 10^{13} ev and the proton energy is limited to 10^{17} ev. Only in the case of particles generated in interstellar space far from intense radiation and magnetic fields will the energy not be limited in the source. However, even here, a large fraction of the high energy electrons would have been generated a sufficient time ago to be slowed down and the number of high energy electrons would be small. Only in the few cases mentioned would the heavy particle energy be limited so that the final spectrum would have mainly protons at high energy if the source is inherently symmetrical between protons and electrons.

If there is no galactic magnetic field the isotropy of the distribution of primaries can be explained only by the assumption that they were generated in the past, that their source bears no relation to the distribution of matter or that they come from a distance of 10^{11} light years. The last assumption is violated by the known age of the universe (about 2×10^{9} years) and the former two are contradicted by the recent evidence(37) that cosmic rays can be emitted by the sun in conjunction with a solar flare. It is possible, however, that the present rate of production is only a fraction of the total intensity. Furthermore, in order to explain the absence of photons, it is necessary to assume that no high energy electrons were present initially since they would generate high energy photons by interaction with radiation. If there were any appreciable number of energetic electrons present when the galaxy was formed they and their associated photons would still be present with an energy of about 10^{12} ev.

Thus the main conclusions of this work are that if there exists a general galactic magnetic field, then the primary spectrum has very few photons, only low energy ($< 10^{13}$ ev) electrons and the higher energy particles are primarily protons regardless of the source mechanism, and if there is no general galactic magnetic field, then the source of cosmic rays accelerates mainly protons and the present rate of production is much less than that in the past.

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